Tilburg University

Tax Policy in a Model of Search with Training

Boone, J.; de Mooij, R.A.

Publication date:
2000

Citation for published version (APA):
Boone, J., & de Mooij, R. A. (2000). *Tax Policy in a Model of Search with Training*. (CentER Discussion Paper; Vol. 2000-07). Macroeconomics.

Center
for
Economic Research

No. 2000-07

TAX POLICY IN A MODEL OF SEARCH WITH TRAINING

By Jan Boone and Ruud A. de Mooij

February 2000

Tax policy in a model of search with training

Jan Boone[¤]and Ruud A. de Mooij[y]

24-1-2000

Abstract

This paper develops a model of search on the labour market with training. The model reveals how the tax system can restore the social optimum if the Hosios condition is not satis¯ed in the private equilibrium. Furthermore, the e®ects are explored of a second-best reform from average to marginal taxes when a given amount of public revenue has to be raised. We ¯nd that (i) a marginal wage tax is less distortionary to raise revenue than is an average tax per job, provided that training is not distorted initially; (ii) this conclusion may reverse in the presence of training distortions; (iii) marginal wage taxes are less distortionary in economies characterized by commitment in wage bargaining, such as the European labour market. Hence, tax reforms that reduce the average tax per job and raise the marginal wage tax, such as an EITC or a negative income tax, are more attractive in Europe than in the US.

1 Introduction

Many European countries su®er from structural labour-market problems, such as high unemployment and low participation. To tackle these problems, various tax proposals have been put forward, including cuts in payroll

---

taxes on employers, earned income tax credits and negative income taxes (see Snower and De la Dehesa (1996), Haveman (1996), Sørensen (1997), Bovenberg et al. (1999)). These proposals aim to reduce unemployment and stimulate participation, without seriously damaging the incomes of transfer recipients or cutting government spending. However, these measures typically make the tax system more progressive. This is the case if the lower average tax burden on work is financed by a higher marginal tax burden on higher incomes. Or, as in the case of an EITC, if a tax reduction at low incomes is phased out to reduce the budgetary costs of the tax reduction and to get back to the original tax schedule at high incomes. Accordingly, the government typically faces a trade-off between the positive effects of the lower average tax per job on participation and the potential adverse incentive effects of higher marginal taxes.

This paper illustrates this trade-off in a model of search on the labor market with training. On the one hand, our model is a special case of the framework developed by Pissarides (1990) and Mortensen and Pissarides (1999). In particular, we use a static (or one shot) version of their models, where agents incur search costs to find a vacancy and firms post vacancies at a fixed cost. After being matched, a worker and a firm Nash bargain about the wage. It is well-known that in this type of models taxes influence the division of the surplus between worker and firm (see for instance Mortensen and Pissarides (1998) and Pissarides (1999)). Hence, taxes influence search intensity, the number of vacancies and unemployment. On the other hand, we extend the Mortensen-Pissarides framework by introducing training decisions. As stressed by the training literature, high marginal tax rates typically reduce the inc1entive to acquire skills (see e.g. Trostel (1993), Dupor et al. (1996), Sørensen and Nielsen (1997), Bovenberg and van Ewijk (1997), Heckman et al. (1999)). With our model, we aim to illustrate the trade-off between the possible beneficial effect of an EITC on unemployment and the possible detrimental effect on training.

The advantage of focusing on a static version of a search and matching model is that we are able to derive analytical solutions for the optimal taxes. Pissarides (1999) and Mortensen and Pissarides (1998) evaluate the effects of tax reform in dynamic search models numerically using simulation techniques. In particular, Pissarides (1999) considers the effects of wage tax reform on unemployment and wages in four different models. In the case of constant real unemployment benefits, he finds that a higher marginal tax rate reduces unemployment. This is comparable to the results we find below by

2

assuming that training costs are completely tax deductible. If training costs are only partly tax deductible, however, increasing the marginal tax rate will reduce labour market tightness and employment. Mortensen and Pissarides (1998) consider a far broader array of taxes and subsidies, including hiring subsidies and ¯ring taxes. However, they do not derive analytical results on the optimal use of these instruments.

In our model, the tax system serves a threefold task. First, the tax system should correct for distortions in training. Subsidies on training are able to alleviate these distortions. Second, taxes need to restore ine±ciencies in labour-market tightness. The latter distortion arises from the mismatch between the marginal productivity of search (vacancies) in the matching process, and the corresponding bargaining power of the worker (¯rm) in wage negotiations, which determine the private marginal bene¯t from job matching.[1] We show that a combination of average taxes per job and marginal wage taxes can always be used to restore the Hosios condition by redistributing the surplus from a match between the worker and the ¯rm. Finally, the tax system aims to raise public revenue with least cost to the private sector. We ¯nd that, if labour-market tightness and training are not distorted initially, the marginal wage tax is always less distortionary than the average tax per job as an instrument to raise public revenue. The reason is that, although both taxes distort search through adversely a®ecting the expected surplus for the worker, the average tax distorts labour-market tightness as well. We relate this result to the Diamond and Mirrlees (1971) intuition of production e±ciency. The result has important implications for the welfare e®ects of an EITC. Indeed, if the government would adopt average taxes initially to ¯nance its spending (in addition to using them to correct for labour-market tightness), a small reduction in the average tax per job ¯nanced by a higher marginal wage tax { which can be interpreted as an EITC { is welfare improving.

This result may change if training is taxed on a net basis, e.g. because the government does not have access to training subsidies or because the costs of training are not fully tax deductible. One might guess that adding training distortions to the model would always call for a lower marginal tax rate. However, we ¯nd that the opposite may be true as well. In particular, initial training distortions introduce two additional e®ects of tax reforms on wel-

---

[1]Equality between the marginal productivity and marignal private bene¯ts is known as the Hosios condition.

fare. On the one hand, higher marginal taxes exacerbate the initial training distortion by further reducing training effort. On the other hand, a higher marginal tax reduces the surplus for the firm because the net tax burden on training reduces the value of a match. This distorts the ratio of vacancies to search if labour-market tightness is sub-optimal initially. Whereas the first effect raises the distortionary impact of marginal wage taxes, the second effect can work in both directions, depending on whether labor-market tightness is too high or too low. Accordingly, training distortions can make the introduction of an EITC either more or less attractive. This result originates in the second-best character of the model. In particular, marginal taxes not only exacerbate initial training distortions, but may also alleviate distortions associated with labour-market tightness.

The paper explores two alternative assumptions regarding training. First, bargaining parties may commit to the wage profile before training takes place. In that case, workers and firms share the costs and benefits of training. In a second model, wage bargaining occurs after the training. In this latter framework, workers bear the entire cost of training, while the benefits are shared across the worker and the firm.[2] It turns out that the wage profile is flatter in the no-commitment case than in the commitment case. This causes two additional distortions in the no-commitment case. First, training is too low since the social benefits of training exceed the private benefits for the worker. This distortion can be alleviated by setting training subsidies above the marginal wage tax. Second, since training costs do not reduce the surplus from a match to the firm, they distort labour-market tightness. This calls for a higher average tax and a lower marginal tax, compared to the commitment case.

Our comparison of the commitment and no commitment cases is related to the analysis of Acemoglu and Shimer (1999). They present a model in which firms can increase the value of a match by investing in capital before being matched with a worker. They show that with (Nash) bargaining over wages after the match, the private outcome is never efficient. In particular, if bargaining gives firms the right incentives to post vacancies, there is underinvestment in capital. Also, if firms' incentives lead to efficient capital stocks, there is overinvestment in vacancies. Acemoglu and Shimer show that

---

[2] The inefficiency associated with the gap between the marginal private cost and benefits of training would disappear if the cost of training are shared across the worker and the firm in the same way as the benefits are distributed.

if ¯rms can commit to a wage before being matched and post that wage to workers searching for a job, then the private outcome can be e±cient. In our model, it is the worker who can increase the value of the match by investing in training. Although the training decision is taken after the worker and ¯rm are matched, it is taken before wages are (re)negotiated. If the ¯rm and worker can commit to a (Nash bargained) wage schedule before the training decision is taken, the worker chooses the e±cient training e®ort. However, if the parties cannot commit and (re)negotiate after the training decision, it is not possible that both the training and ¯rms' vacancy decisions are e±cient. This is similar to the Acemoglu and Shimer result. Whereas they consider di®erent ways of market organization to solve the problem, we focus on the use of tax instruments to restore e±ciency.

The model with commitment may be more relevant for European labour markets while the labour market in the US may be characterized by less commitment. Indeed, a higher degree of corporatism in European labour markets precludes that individuals renege on ex-ante agreed wage outcomes. This commitment in EU labour markets encourages investment in ¯rm-speci¯c training (Teulings and Hartog, 1998). Our analysis reveals that tax reforms which reduce the average tax per job and raise the marginal wage tax { such as an EITC or a negative income tax { tend to be more attractive in Europe than in the US.

The rest of this paper is organized as follows. The next section elaborates on the search model with training. In that model, bargaining parties commit to the wage pro¯le before the training decision is made. Section 2 also illustrates how taxes in the private outcome may restore the social optimum and explores a reform from average to marginal taxes if the government has a positive revenue requirement. Section 3 presents how the model changes if there is no commitment with respect to the wage pro¯le and reveals how the tax system can restore the social optimum in this case. Section 4 concludes. The appendix contains the proofs of all results in the paper.

2   Model with commitment

This section develops a model of search on the labour market. The model describes the matching process between vacancies posted by ¯rms, and workers that search for a job. A job match yields a surplus that is divided across the worker and the ¯rm through a bargaining process. Workers can also engage

in training in order to raise their skill level and thus to receive a higher wage. Firms may in°uence the training decision by changing the wage pro¯le.

In contrast to most search models, our framework is of a one-shot nature. This is a considerable simpli¯cation of the dynamic models in Mortensen and Pissarides (1999). However, the static model captures a number of features of the dynamic models. Most notably, it captures the Hosios condition discussed below. The simpli¯cation allows us to introduce a training decision in the model and still derive analytical solutions for the optimal tax rates. This focuses the analysis on the three distortions in the model: the hold up problem in search and vacancy creation, the distortion in training and the positive revenue requirement of the government. Moreover, the results can be interpreted in the light of the Diamond and Mirrlees (1971) production e±ciency result, which appears to be new in the search and matching literature.

In the model, sequencing in the decision process is important. In particular, the commitment model has the following timing structure.

time t	agents	¯rms
0	search intensity s_i	vacancies v
1	matches $m(v; s)$	
2	bargaining V_e	
3	wage pro¯le $(w_0; w_1)$	
4	training e®ort ¾	
5	output $(y_0; y_1)$	

At $t = 0$, ¯rms decide whether or not to open up a vacancy at a ¯xed cost k: At the same time, workers choose their search intensity s_i at cost $°(s_i)$, where $°(0) = 0; °^0(0) = 0; °^{00}(s_i) > 0$ and $\lim_{s_i \to 1} °^0(s_i) = +1$. The set of workers is modelled as the unit interval, hence $i \in [0; 1]$.

At $t = 1$, ¯rms and workers are matched, where the total number of matches equals $m(v; s)$ with $s = \int_0^1 s_i di$. The matchingsfunction is homogenous of degree one in v and s and is increasing and concave in each (separate) argument. We denote the matching elasticity of search by ´ and the elasticity of vacancies by $1 ¡ ´$. As argued by, among others, Blanchard and Diamond (1989) and Broersma and Van Ours (1999), a Cobb-Douglas matchingsfunc-

tion $m(v;s) = m_0 v^{1-\gamma}s^{\gamma}$ is a reasonable approximation in reality. We assume that the matchingfunction is of this form and hence γ is a constant. Firms and workers that are not matched have value 0.

At $t = 2$, firms and workers bargain about the value of being employed for a worker V_e.

At $t = 3$, the firm determines the wage profile $(w_0; w_1)$ where w_0 is the net (of taxes) wage for an untrained worker and w_1 the net wage of a trained worker. The profile has to satisfy the property that the expected value of being employed equals (at least) the bargained value V_e.

At $t = 4$, the worker chooses his training effort σ at a cost $c\sigma$: A training effort σ brings the worker in the trained state with probability $p(\sigma)$ and leaves the worker in the untrained state with probability $1 - p(\sigma)$, where $p'(\sigma) > 0$ and $p''(\sigma) < 0$. This training technology assumption simplifies the analysis in two dimensions. First, it captures a dynamic decision in a static framework. The literature on training generally models the incentive to train as the gain in future wage income compared to the current disutility of training, often modelled as the income forgone due to the time spend on training (see e.g. Heckman et al. (1999) and the references therein). Our formalization captures these incentives, although in a somewhat different manner. In particular, our static framework does not include the time lag between the moment of training and the appearance of the benefits in the form of higher wages.[3] Furthermore, the disutility of training is captured by effort costs, rather than time. The crucial element in the training literature, however, is not whether training cost refer to time, but rather the degree of tax deductibility of training costs (Trostel, 1993). We capture tax deductibility in our analysis by means of a training subsidy.[4] Second, our training technology has only two states, trained and untrained, while the training decision σ is a continuous variable. Alternatively, one could have modelled training as a deterministic process where a worker's productivity is a continuous function of training effort σ. Since workers are ex ante identical and risk neutral this is equivalent to the model above.[5] However, in this case one would have had to model how the wage depends on productivity for an interval of productivities

---

[3]The model is thus a special case of the dynamic framework by assuming a zero discount rate.

[4]We turn to the possible limitations of our approach in the concluding remarks.

[5]This can be seen as follows. Let $y(\sigma)$ denote a worker's productivity as a function of his training effort σ. Further, let y_1 (y_0) denote a worker's productivity in trained (untrained) state. Then defining $y(\sigma) \equiv p(\sigma)y_1 + (1 - p(\sigma))y_0$ shows the equivalence.

instead of just two levels of productivity.

At $t = 5$, output is produced and wages are paid, where a trained worker produces y_1 and receives net wage w_1 and an untrained worker produces y_0 (with $y_1 > y_0$) and receives w_0.

This sequencing in the decision making process implies that bargaining parties commit to the wage profile before the training decision is made. Hence, the model does not give rise to the hold-up problem in training investments. Section 4 will elaborate on the alternative case in which bargaining parties can renegotiate the wage, after the training decision is made. However, the model in this section contains another hold-up problem, because firms and agents cannot commit to the return to search or vacancies. Hence each of the parties invests to create a joint surplus, while the division of the surplus is bargained over after these investments are sunk. Correcting this hold up problem turns out to be an important function of the tax system.

2.1   The private outcome

The process of job matching

The probability for a worker with search intensity s_i of being matched with a firm equals

$$\frac{s_i}{s}m(v;s) = s_i m(\frac{v}{s};1) = s_i m(\mu)$$

where the first equality follows from the assumption that $m(s;v)$ is homogenous of degree 1 in v and s, $\mu \equiv \frac{v}{s}$ denotes the labour market tightness and $m(\mu) \equiv m(\mu;1)$. A worker chooses s_i to maximise the expected surplus from search

$$\max_{s_i} \{s_i m(\mu)V_e - \gamma(s_i)\}$$

where the value of not being matched is assumed to be equal to 0. Looking at a symmetric equilibrium where each agent chooses the same $s_i = s$ we find that s solves

$$\gamma'(s) = m(\mu)V_e \tag{1}$$

Hence, marginal search costs equal the expected marginal gain from search. Since we assume a strictly convex cost function for search (i.e.

$\sigma''(s) > 0$), the net expected surplus for the worker, $sm(\mu)V_e \,_\text{¡}\, \sigma(s) = s\sigma'(s) \,_\text{¡}\, \sigma(s)$, is positive.

The probability that a ¯rm is matched with a worker equals $\frac{m(v;s)}{v} = \frac{m(\mu)}{\mu}$. Assuming free entry into the vacancy posting business, the value for a ¯rm of employing a worker J_e satis¯es

$$\frac{m(\mu)}{\mu}J_e \,_\text{¡}\, k = 0 \tag{2}$$

where we use the assumption that the value of not being matched equals 0.

Hence, ¯rms earn zero pro¯ts in equilibrium.[6]

The value of a job match

Given a certain training e®ort ¾ and wage pro¯le $(w_0; w_1)$, to be determined below, the value of being employed for a worker depends on the expected wage minus training costs

$$V_e = p(¾)w_1 + (1 \,_\text{¡}\, p(¾))w_0 \,_\text{¡}\, [1 \,_\text{¡}\, \frac{s¾}{1 + s¾}]c¾ \tag{3}$$

where $\frac{s¾}{1+s¾}$ stands for a subsidy on training. The value of employing a worker for a ¯rm equals

$$
\begin{aligned}
J_e &= p(¾)(y_1 \,_\text{¡}\, (1 + ¿)w_1) + (1 \,_\text{¡}\, p(¾))(y_0 \,_\text{¡}\, (1 + ¿)w_0) \,_\text{¡}\, ¿_a \\
&= p(¾)y_1 + (1 \,_\text{¡}\, p(¾))y_0 \,_\text{¡}\, ¿_a \,_\text{¡}\, (1 + ¿)V_e \,_\text{¡}\, (1 + Ã)c¾ \tag{4}
\end{aligned}
$$

where $(1 + ¿)w + ¿_a$ equals the total wage cost for the ¯rm of paying a net wage w to the worker Hence, $¿$ denotes the marginal tax rate on wages while $¿_a$ stands for the tax per job. We refer to this as the average tax.[7]

Furthermore, $1 + Ã \,´\, \frac{1+¿}{1+s¾}$ so that $Ã$ measures the net tax burden on training (see below).

The worker and the ¯rm bargain about V_e. This is modelled with a Nash bargaining function with the threat points for both worker and ¯rm equal to

---

[6] If the cost of opening a vacancy would be a function of v (i.e. $k(v)$, where $k(:)$ is a strictly convex function), there would be a positive surplus for the ¯rm. The e®ect on optimal wage taxes of such a generalization is analyzed by Boone and Bovenberg (1999).

[7] This is in line with the Mortenson and Pissarides (1999) and others. In public ¯nance, the average tax generally refers to the tax burden on wages. In our model, it might be viewed as a tax on value-added, which is equivalent to the total surplus from a match.

0.[8] Hence V_e solves

$$\max_{V_e} V_e^{\bar{\beta}}\,(p(\theta)y_1 + (1 - p(\theta))y_0 - \tau a - (1 + \tau)V_e - (1 + \tilde{A})c\theta)^{1-\bar{\beta}}$$

It is routine to verify that this yields the following solution

$$V_e = \frac{\bar{\beta}}{1 + \tau}\,(\bar{y} - (1 + \tilde{A})c\theta - \tau a) \qquad (5)$$

$$J_e = (1 - \bar{\beta})\,(\bar{y} - (1 + \tilde{A})c\theta - \tau a) \qquad (6)$$

where $\bar{y} \equiv p(\theta)y_1 + (1 - p(\theta))y_0$ denotes the expected output of a worker. The bargaining outcome reveals that the total surplus from a match, $\bar{y} - (1 + \tilde{A})c\theta - \tau a$, is divided between the employee and the employer on the basis of their respective bargaining powers, $\frac{\bar{\beta}}{1+\tau}$ and $1 - \bar{\beta}$, respectively. The marginal wage tax, τ, reduces the value of a job for the worker. This is a well known result, see for instance Pissarides (1999: 142, 143). The reason is that a high marginal tax rate makes workers less aggressive in bargaining since a rise in the gross wage translates in a small change in the net wage. Similarly, a high τ makes employers more aggressive in bargaining, since a rise in w translates into a big rise in wage costs. In the Nash bargaining solution this effect of τ on the wage exactly cancels the higher τ for employers.

The surplus from a match, $\bar{y} - (1 + \tilde{A})c\theta - \tau a$, is sum of three terms. First, there is expected output $\bar{y}$. Second, training costs are subtracted because bargaining parties commit to the bargaining solution before the training decision is made. Hence, the incidence of the training costs is divided between the worker and the firm. Apart from the gross cost of training, there is a net tax burden on training, denoted by $\tilde{A}$. On the one hand, training costs are subsidized through s_θ. On the other hand, the compensation to employees for the costs of training occurs through wage payments that are subject to the marginal wage tax. If $\tilde{A} > 0$, the marginal wage tax exceeds the training subsidy so that the compensation for training costs is taxed on a net basis.[9]

---

[8] This follows from the one shot nature of the game. If the firm and worker cannot agree on the value V_e, the match is dissolved. Then the worker and firm receive the same pay off as the firms and workers that were not matched at time 1. In equilibrium such disagreement never happens.

[9] An alternative interpretation of the training subsidy is tax deductibility. In particular, if $s_\theta = \tau$, training costs would be fully tax deductible. This would be the case if training costs consist of foregone working time (compare Boskin (1976)). If part of the training costs is not tax deductible, e.g. because tuition is not tax deductible or if leisure is included in the model, we find that $s_\theta < \tau$ or equivalently $\tilde{A} > 0$ (see e.g. Trostel (1993)).

This reduces the value of a match. Accordingly, the net tax burden on training is shared across the worker and the ¯rm. The ¯nal term is the average tax, τ_a. By reducing the value of a match, this tax also bears on both the employer and the employee.

Using the free entry condition (2) together with (5) and (6), we can rewrite V_e as

$$V_e = \frac{\bar{}}{1 \mathbin{i} \bar{}} \frac{1}{1+\tau} \frac{k\mu}{m(\mu)} \tag{7}$$

Substituting this into the ¯rst order condition for s, equation (1), yields

$$\circ^0(s) = \frac{\bar{}}{1 \mathbin{i} \bar{}} \frac{k\mu}{1+\tau} \tag{8}$$

Training

Given the (pre-commited) value V_e, the ¯rm will set the wage pro¯le $(w_0; w_1)$ in such a way that ¾ maximizes J_e in equation (4). Hence ¾ solves

$$p^0(\text{¾}) = \frac{c(1 + \tilde{A})}{y_1 \mathbin{i} y_0} \tag{9}$$

The way that the ¯rm implements this value for ¾ is to choose the di®erence $w_1 \mathbin{i} w_0$ such that the worker, choosing ¾ to maximize V_e, selects this value. Hence, equation (9) determines $w_1 \mathbin{i} w_0$ and (5) determines V_e. It is routine to verify that this implies the following wage pro¯le

$$w_0 = \frac{\bar{}}{1+\tau}(y_0 \mathbin{i} \tau_a) + \frac{1 \mathbin{i} \bar{}}{1+\tau}((1 + \tilde{A})c\text{¾} \mathbin{i} p(\text{¾})(y_1 \mathbin{i} y_0)) \tag{10}$$

$$w_1 = \frac{\bar{}}{1+\tau}(y_1 \mathbin{i} \tau_a) + \frac{1 \mathbin{i} \bar{}}{1+\tau}((1 + \tilde{A})c\text{¾} + (1 \mathbin{i} p(\text{¾}))(y_1 \mathbin{i} y_0)) \tag{11}$$

Using equation (9) to rewrite $(1 + \tilde{A})c\text{¾} = p^0(\text{¾})\text{¾}(y_1 \mathbin{i} y_0)$ in equations (10) and (11), one veri¯es that the concavity of $p(:)$ implies that w_0 is below the renegotiation wage $\frac{\bar{}}{1+\tau}(y_0 \mathbin{i} \tau_a)$ and w_1 above the renegotiation wage $\frac{\bar{}}{1+\tau}(y_1 \mathbin{i} \tau_a)$. This is discussed in section 4.

At the level of ¾, determined in (9), the free entry condition yields

$$\frac{k\mu}{m(\mu)} = (1 \mathbin{i} \bar{})\,(\bar{y} \mathbin{i} (1 + \tilde{A})c\text{¾} \mathbin{i} \tau_a) \tag{12}$$

11

The three equations (8), (9) and (12) determine the three endogenous variables s, ¾ and μ. From these, we can derive the total number of matches sm(μ) and total unemployment u = 1¡ sm(μ). Total output equals sm(μ)(p(¾)y$_1$+ (1 ¡ p(¾))y$_0$):

2.2   The social optimum

As a measure of welfare, we use total expected output minus the total costs of training, search and vacancies. Since workers and ¯rms are assumed to be risk neutral, this is not an unreasonable criterion. Also note that the risk neutrality assumption implies that there is no reason to redistribute income among trained, untrained and unemployed workers. When below we consider the case with positive government expenditure, g > 0, this welfare criterion implies that government expenditure has the same social value as private income and consumption.

$$ sm(\mu)\,[p(¾)y_1 + (1 ¡ p(¾))y_0 ¡ c¾] ¡ °(s) ¡ ks\mu \qquad (13) $$

Assume that the social planner chooses s; μ and ¾ to maximize welfare (13). Then the following result is a slight generalization of the Hosios (1990) condition to the case with training. For the case without training it is well known (see for instance Pissarides (1990)) that ¯ = ´ and ¿ = ¿$_a$ = s¾ = 0 causes the private outcome to coincide with the social optimum.

Lemma 1 If ¯ = ´ then
  (i) ¿ = ¿$_a$ = s¾ = 0 implies that the private outcome coincides with the social optimal outcome;
  (ii) ¿ > 0; ¿$_a$ ¸ 0 and Ã ¸ 0 implies that agents underinvest in search;
  (iii) Ã > 0 (Ã < 0) implies that workers underinvest (overinvest) in training and if further ¿$_a$ ¸ 0 then ¯rms underinvest (overinvest) in vacancies;
  (iv) ¿$_a$ > 0 (¿$_a$ < 0) and ¿ = s¾ = 0 implies that agents underinvest (overinvest) in search and ¯rms underinvest (overinvest) in vacancies.
  If ¿ = ¿$_a$ = s¾ = 0 then
  (v) ¯ < ´ (¯ > ´) implies that tightness μ is too high (low) in the private outcome;
  (vi) ¯ 6 ´ implies that agents underinvest in search.

The intuition for these results is as follows.

(i) If there are no distortionary taxes and the Hosios condition, $\bar{} = $ ´, is satis¯ed (see for instance Hosios (1990) or Mortensen and Pissarides (1999)) then the private returns to training, search and creating vacancies coincide with the social returns. The intuition for the Hosios condition is that an agent's (¯rm's) bargaining power should coincide with the marginal contribution of search (vacancies) in establishing a match. In particular, the bargaining solution determines the marginal private bene¯ts of a match for the ¯rm and the worker. If the marginal private bene¯ts equal the marginal social bene¯ts, the hold up problem is solved as each party gets the social return on its sunk investments in search, respectively, vacancies.

(ii) If there are positive wage tax distortions ($\iota > 0$), the private return of search falls short of the social return. This follows from (8) because the marginal tax ι does yield social surplus but is not captured by the worker.

(iii) If the training decision is not distorted ($\iota = s_{¾}$ so that $\bar{A} = 0$), the marginal tax rate has no e®ect on ¯rms' incentives to create vacancies. This follows immediately from (6). As mentioned above, the intuition is that a high marginal tax rate makes workers less aggressive in bargaining. If the wage tax exceeds the training subsidy (i.e. $\iota > s_{¾}$ so that $\bar{A} > 0$), the marginal tax rate reduces the ¯rm's incentive to create vacancies. This is because the net tax burden on training raises the ¯xed compensation that employers provide to workers to compensate them for the costs of training. In other words, ι creates an average tax burden comparable to ι_a. Accordingly, the total surplus available declines so that a rise in ι reduces the number of vacancies and hence labour-market tightness, µ. This is the main di®erence with a model without training where ι does not a®ect µ.

(iv) The tax ι_a a®ects the value of the match and hence the agents' search activity and ¯rms' incentives to create vacancies. According to (13) the tax ι_a does create social value, but it reduces the private return to ¯rms and workers. Hence it causes underinvestment in vacancies and search. Since the training decision is taken after the match, it is only a®ected by the marginal tax rate ι and the training subsidy, $s_{¾}$, not by ι_a:

(v) If the Hosios condition is not satis¯ed, labor market tightness is too high compared to the social optimum if the workers' bargaining power $\bar{}$ is too low compared to their productivity in establishing matches ´. Tightness is too low if $\bar{}$ exceeds ´, because ¯rms do not receive a big enough share of the surplus and hence create too few vacancies.

(vi) Finally, agents always underinvest in search irrespective of whether their bargaining power is too big or too small. The idea is that too much

13

bargaining power for the worker reduces the number of vacancies created and hence reduces the return to searching. Too little bargaining power reduces a worker's share of the surplus and hence reduces his search intensity.

The next lemma reveals that, if the Hosios condition is not satis¯ed, taxes ι; $s_{¾}$ and ι_a can be used to restore the social optimal outcome, in the case with zero government expenditure, $g = 0$.

Lemma 2 If $g = 0$ and $^{-} \neq ´$ then there are taxes ι; ι_a and $s_{¾}$ that restore the social optimum in the private outcome. Moreover, these taxes balance the government budget. If training subsidies are not available and $^{-} \neq ´$ then the tax instruments ι and ι_a are insu±cient to restore the social optimum. In particular,

$$1 + \iota \;=\; \frac{^{-}\,\frac{1 \,_\text{i}\, ´}{´}}{1 \,_\text{i}\, ^{-}} \tag{14}$$

$$\iota_a \;=\; \frac{´ \,_\text{i}\, ^{-}}{1 \,_\text{i}\, ^{-}}(y \,_\text{i}\, c_{¾}) \tag{15}$$

$$s_{¾} \;=\; \iota \tag{16}$$

make the private outcome in (8), (9) and (12) equivalent to the social outcome.

The intuition is as follows. The training subsidy $s_{¾} = \iota$ ensures that training decisions are not distorted. The tax instruments ι and ι_a can be used to redistribute the surplus of a match in such a way that ¯rms and workers get their correct share determined by $´$. To illustrate, if the workers' bargaining power $^{-}$ exceeds their productivity $´$, the government uses a positive marginal tax rate ι to reduce the workers' bargaining power and subsidises the ¯rm by setting $\iota_a < 0$. The reason that this can be done with a balanced budget is that there are no external e®ects in the model. The total private value created equals the total social value created. The value just needs to be (re)distributed in the right way.[10]

If the training subsidy would not be available { e.g. because the government cannot observe training e®orts and hence they are not tax deductible {

---

[10]Note that the balanced budget result depends on our assumption of constant returns to scale in the matchingsfunction. In particular, with decreasing returns to scale in the matchingsfunction the taxes needed to induce the social optimum in the private case yield a budget surplus for the government. With increasing returns to scale the taxes yield a de¯cit.

it would not be possible in general to arrive at the social optimum. Indeed, the government would have access to just two instruments, τ, τ_a to correct three variables, s; ¾ and μ. This is not possible in general.

In the next section, we explore the e®ects of a tax reform, starting from an equilibrium that is not necessarily optimal. Furthermore, we analyze the case of a positive revenue requirement for the government.

2.3 Tax reform

In this section, public expenditures, g, are positive in the initial equilibrium. Hence, we start from a sub-optimal equilibrium since the social optimum is always characterized by zero public revenues (see Lemma 2). We assume that the level of g remains ¯xed, e.g. because political barriers preclude budgetary cuts. The government does not have access to a non-distortionary lump-sum tax to ¯nance its spending. This brings us into a second-best world. Indeed, the government has to rely on three distortionary tax instruments, τ; τ_a and $s_{¾}$, to raise public revenue. This section analyzes the optimal mix between these taxes. Furthermore, we explore the welfare e®ects of a balanced budget tax reform from the average tax, τ_a, towards the marginal tax, τ, if we start from a sub-optimal initial equilibrium. This reform is interpreted as an earned income tax credit (EITC).

The government budget constraint is of the form

$$g = sm(\mu)[p(¾)(\tau w_1 + \tau_a) + (1 - p(¾))(\tau w_0 + \tau_a) - \frac{s_{¾}}{1 + s_{¾}}c_{¾}]$$

The government spends g and has to ¯nance this with a wage tax $\tau w_1 + \tau_a$ ($\tau w_0 + \tau_a$) per matched and trained (untrained) worker. Further, from equation (3), the government pays a training subsidy $\frac{s_{¾}}{1+s_{¾}}c_{¾}$ per matched worker. Using equation (3), the government budget constraint can be written as

$$g = sm(\mu)[\tau V_e + \tau_a + \tilde{A}c_{¾}] \qquad (17)$$

Now consider the folowing two optimization programs.

$$\max_{\tau;\tau_a;s_{¾}} sm(\mu)\left[p(¾)y_1 + (1 - p(¾))y_0 - c_{¾}\right] - °(s) - ks\mu \qquad (P_{C1})$$

subject to equations (8),(9),(12) and (17)

15

and

$$\max_{\substack{\tau;\iota_a;s_{\%}}} s \tag{P_{C2}}$$

subject to equations (8),(9),(12) and (17)

The following proposition substantially simpli¯es our welfare analysis.

Proposition 3 Optimization programs (P_{C1}) and (P_{C2}) are equivalent.

Hence, instead of maximizing the expression for welfare in (P_{C1}) we just need to maximize workers' search intensity s subject to the same equations determining $s;\%;\mu$ and government budget balance. The intuition why s is a su±cient statistic for welfare (13) is as follows. Total surplus is divided between workers, ¯rms and the government. Since ¯rms make zero expected pro¯ts by the free entry condition and because we maximize welfare for given government expenditure g, only workers' surplus creates welfare. As mentioned above, substituting (1) into workers' expected surplus yields

$$sm(\mu)V_e \; _i \; ^{\circ}(s) = s^{\circ0}(s) \; _i \; ^{\circ}(s)$$

which is positive and increasing in s due to the assumption that $^{\circ}(:)$ is convex. Hence s is a su±cient statistic for total welfare.

E®ects on search

To arrive at the reduced form for search, we log-linearize the model around an initial equilibrium and solve the linearized model analytically. In particular, we solve the model for two exogenous variables ι and $s_{\%}$. The government budget constraint is used to adjust ι_a endogenously. Intuitively, the revenues from a higher marginal tax (or a lower training subsidy) are used to reduce the average tax per job, such that the government budget remains balanced.

Lemma 4 The reduced form for search can be written as

$$"_s \Phi s = [(^- \; _i \; \acute{})(\acute{y} \; _i \; c_{\%}) + (1 \; _i \; ^-)(\iota_a + \bar{A}c_{\%})]\iota \; _i \; \frac{\bar{A}c_{\%}}{"_{\%}}(\iota \; _i \; s_{\%}) \tag{18}$$

where

$$\Phi \; \acute{} \; \acute{} (\acute{y} \; _i \; c_{\%}) \; _i \; \frac{g}{m(\mu)s}(1 + \frac{1}{"_s}) > 0$$

and $"_s \; \acute{} \; \frac{^{\circ00}(s)s}{^{\circ0}(s)} > 0$, $"_{\%} \; \acute{} \; _i \; \frac{p^{00}(\%)\%}{p^{0}(\%)} > 0; s = \frac{ds}{s}; \iota = \frac{d\iota}{1+\iota}$ and $s_{\%} = \frac{ds_{\%}}{1+s_{\%}}$:

16

The determinant, $\mathfrak{C}$, should be strictly positive for the model to be stable. This means that we should be on the left side of the La®er curve. It requires that the size of the public sector, g, is not too large. Since, at the optimum, the marginal change in search is zero, the reduced form shows some direct implications for the optimal second-best tax structure in case of a positive revenue requirement for the government, g > 0.

Proposition 5 Consider the case where g > 0:
(i) If $^{-}$ = $^{´}$ then it is optimal for the government to set $\tau_a = 0$ and $\tilde{A} = 0$, irrespective of the level of g;
(ii) if $^{-}$ $\neq$ $^{´}$ then it is optimal for the government to set $\tau_a = \frac{´ \mathbin{-} }{1 \mathbin{-}}(\mathring{y} \mathbin{-} c_{\frac{3}{4}})$ and $\tilde{A} = 0$, irrespective of the level of g;
(iii) at the optimal taxes it is the case that $V_e = ´(\mathring{y} \mathbin{-} c_{\frac{3}{4}}) \mathbin{-} \frac{g}{s m(\mu)}$.

Hence, in a second-best framework with a positive revenue requirement, the government should always set the average tax per job and the training subsidy at their ¯rst-best levels derived in equations (15) and (16). Public revenues should thus be raised by the marginal wage tax alone. The intuition for this result is the following. As ¯rms earn zero pro¯ts, they cannot bear the burden of taxation. Indeed, the incidence of all taxes is ultimately borne by workers in the form of a lower net surplus from search. The government can tax away this surplus through either τ or τ_a (we ignore the training subsidy for the moment). It turns out that taxing the surplus from search directly through the wage tax τ is more e±cient than taxing it indirectly through the average tax per job. Intuitively, whereas both taxes distort the search intensity of the worker, the average tax distorts also labour-market tightness by reducing vacancies. Accordingly, the average tax per job is a relatively ine±cient instrument to tax away the surplus from search of the worker.

This result resembles that of Diamond and Mirrlees (1971). Their model assumes perfect competition, which ensures zero pro¯ts for the ¯rm, and implies that there is no such distortion as a deviation from the Hosios condition, because each production factor receives its marginal product. If the government relies on distortionary taxes to raise public revenue, Diamond and Mirrlees show that it should always maintain production e±ciency, i.e. the government should not impose taxes that distort the input mix in production. In our framework, we ¯nd that the government should not impose taxes that distort the ratio between vacancies and search, i.e. labour-market tightness. Hence, only deviations from the Hosios condition call for an average tax rate, τ_a, not the need to ¯nance g > 0.

17

In terms of the optimal incidence, the expression for V_e under (iii) summarizes this discussion. The worker gets his e±cient (Hosios) share ´ of the surplus ($\dot{y}$ ¡ c¾) and bears the burden of g per match completely.

Armed with the optimal structure, the reduced form reveals under which conditions an EITC improves welfare if we start from a sub-optimal equilibrium. The following result follows immediately from equation (18).

Proposition 6 Introducing an EITC, here interpreted as a rise in ¿ where the government budget is balanced by a reduction in $¿_a$, has the following implications for welfare:

(i) in case Ã = 0, then it raises welfare if and only if $¿_a > \frac{´¡¯}{1¡¯}(\dot{y} ¡ c¾)$;

(ii) in case $¿_a = \frac{´¡¯}{1¡¯}(\dot{y} ¡ c¾)$ and Ã > 0, then it raises welfare if and only if $1 ¡ ¯ > \frac{1}{"_¾}$;

(iii) in case $¿_a = \frac{´¡¯}{1¡¯}(\dot{y} ¡ c¾)$ and Ã > 0, it always raises welfare if $¿ = s_¾$.

The intuition for the result is the following.

(i) If training subsidies are set at their ¯rst-best level (i.e. Ã = 0), the introduction of an EITC would raise welfare if and only if the initial $¿_a$ exceeds the optimal second-best level in proposition 5 (ii) (see also 15).

(ii) If $¿_a$ is set at its optimal level in (15), a marginal increase in ¿ yields an ambiguous e®ect on welfare if s¾ is ¯xed at too low a level, that is Ã > 0. This case with ¿ > s¾ seems relevant in practice. In particular, not all costs of training are tax deductible. Examples include books, materials, training hours during leisure time and pure e®ort costs.

With ¿ > s¾; training is taxed on a net basis. This net tax burden on training causes two distortions. First, it distorts the distribution of the surplus from a match across the worker and the ¯rm. In particular, the tax burden on training reduces the surplus for the ¯rm so that a lower level of $¿_a$ would su±ce to maintain the Hosios condition. Indeed, it is always optimal to avoid taxing the ¯rm to raise revenu. If one starts from the optimal second-best level of $¿_a$ (i.e. the level without a training distortion), a marginal reduction thus yields a ¯rst-order welfare improvement. Second, the net tax burden on training implies that training is too low. A marginal increase in ¿ exacerbates this distortion if the training subsidy is not adjusted accordingly. Hence, the overall welfare e®ect of an EITC is ambiguous and depends on the magnitude of a reform on the two distortions, i.e. the distortion in labour-market tightness and the distortion in training. The e®ect on the distortion in labour-market tightness is measured by $1 ¡ ¯$, which denotes the extend

to which the tax burden on training reduces the surplus for the ¯rm. If ¯rms have relatively much bargaining power (i.e. 1_i ¯ is large), they also bear a large part of the net tax burden on training since these reduce the surplus from a match. The e®ect on the training distortion depends on ^{2}¾, which measures the speed with which the rate of return to training declines with the level of training. In particular, if rate of return to training declines rapidly with the level of training (i.e. "¾ is large), the e®ect on training will be small. A net tax burden on training thus makes an EITC more attractive if the ¯rst e®ect dominates the second e®ect. Otherwise, an EITC becomes less attractive.

(iii) If the government would raise the training subsidy in line with ¿, or equivalently all training costs would be tax deductible, it would o®set the effect of ¿ on the training distortion. Hence, an EITC (i.e. a marginal increase in ¿ and s¾ and a corresponding reduction in ¿$_a$) would unambiguously raise welfare since it alleviates the distortion in labour-market tightness.

3   Model without commitment

In the previous model, agents choose training after bargaining parties committed to the wage pro¯le. In this section, we explore an alternative sequence of decision making. In particular, we consider the following timing structure.

time		
	agents	¯rms
0	search intensity s_i	posting vacancies v
1	matches $m(v; s)$	
2	training e®ort ¾	
3	bargaining $(w_0; w_1)$	
4	output $(y_0; y_1)$	

Hence, households decide about their training e®ort before negotiating about the wage pro¯le.

3.1   The private outcome

If ¯rms cannot commit to a wage pro¯le $(w_0; w_1)$, the worker and ¯rm (re)negotiate the wage in each state, trained and untrained. Again assuming

zero threat points, w_i is determined by the Nash bargaining function in state $i = 0, 1$: That is, w_i solves

$$\max_{w_i} w_i^{\bar{\ }} (y_i - (1 + \zeta)w_i - \zeta a)^{1-\bar{\ }}$$

Consequently

$$w_i = \frac{\bar{\ }}{1 + \zeta}(y_i - \zeta a) \tag{19}$$

$i = 0, 1$.

Hence, if bargaining parties negotiate the wage outcome after the training decision, the wage profile differs from the model with commitment above. Indeed, comparing the wages in (10) and (11) reveals that the wage profile is less steep in the model without commitment, because in the commitment case w_0 is below $\frac{\bar{\ }}{1+\zeta}(y_0 - \zeta a)$ and w_1 above $\frac{\bar{\ }}{1+\zeta}(y_1 - \zeta a)$. The intuition is the following. Although the division of the surplus may not be optimal in the commitment case above (if Hosios is not satisfied), the firm has still an incentive to set the wage difference $w_1 - w_0$ in such a way as to maximize the total surplus. In the no commitment case here, the training costs are already sunk when the firm and worker bargain about the surplus. Hence the relevant surplus for the firm is $y_i - \zeta a$, not the total surplus $\bar{y} - \zeta a - (1 + \bar{A})c_{\frac{3}{4}}$. In other words, the firm has neither the means nor the incentive to maximize the total surplus.

Note that the commitment case assumes commitment from both the firm and the worker not to renegotiate wages. In particular, in the untrained state the worker has an incentive to renegotiate because the wage w_0 in (10) is below the renegotiation wage $\frac{\bar{\ }}{1+\zeta}(y_0 - \zeta a)$. Similarly, in the trained state the firm has an incentive to renegotiate because w_1 in (11) is above the renegotiation wage $\frac{\bar{\ }}{1+\zeta}(y_1 - \zeta a)$. In other words, the commitment case requires an institutional setting where both parties have an incentive to stick to their agreement. Reasons may be that individual workers and firms try to build a reputation for being able to commit. This is most likely if relationships are expected to last long. Alternatively, unions and representatives of firms bargain at a centralized level and force their constituencies to stick to the agreement because the aggregate surplus is bigger with commitment. Either way, in our interpretation, the commitment case is more likely to come about on the European continent than in the Anglosaxon countries (Teulings and Hartog, 1998). We return to this below.

It follows that

$$V_e = p(\tau)w_1 + (1 - p(\tau))w_0 - \frac{1}{1 + s_\tau}c_\tau \qquad (20)$$

Hence workers choose τ to solve

$$\max_\tau \{ p(\tau)\frac{-}{1 + \rho}(y_i - y_0) - \frac{1}{1 + s_\tau}c_\tau \}$$

Thus

$$p'(\tau) = \frac{c(1 + \bar{A})}{\overline{(y_1 - y_0)}} \qquad (21)$$

Comparing this expression to (9), for given $\bar{A}$, workers invest less in training in the no commitment case than in the commitment case. The intuition is that in the no commitment case, workers pay all training costs while they share the returns of training with ¯rms. Further,

$$
\begin{aligned}
J_e &= p(\tau)(y_1 - (1 + \rho)w_1) + (1 - p(\tau))(y_0 - (1 + \rho)w_0) - \rho a \\
&= \bar{y} - c_\tau - V_e - \frac{g}{sm(\mu)} \qquad (22)
\end{aligned}
$$

where the last equality follows from the government budget constraint (17). The interpretation is that the total surplus after a match, $\bar{y} - c_\tau$, is distributed over employers, J_e, employees, V_e, and the government, $\frac{g}{sm(\mu)}$. Combining this equation with the free entry condition (2) yields

$$\frac{k\mu}{m(\mu)} = \bar{y} - c_\tau - V_e - \frac{g}{sm(\mu)} \qquad (23)$$

The ¯rst order condition (1) for search intensity s remains unchanged

$$\circ'(s) = m(\mu)V_e \qquad (24)$$

3.2 The social optimum

To ¯nd the solution for the optimal taxes, we again simplify the analysis by proving the equivalence of two optimization programs. Consider the following optimization programs

$$\max_{\rho,\rho a,s\tau} sm(\mu)[p(\tau)y_1 + (1 - p(\tau))y_0 - c_\tau] - \circ(s) - ks\mu \qquad (P_{NC1})$$

subject to (21), (23), (24), (19) and (20)

and

$$\max_{\iota,\iota_a,s_{\frac{3}{4}}} fsg \qquad\qquad (P_{NC2})$$

subject to (21), (23), (24), (19) and (20)

Proposition 7 The optimization programs (P_{NC1}) and (P_{NC2}) are equivalent.

The intuition for this equivalence is the same as in proposition 3. In fact, the proof of proposition 3 does not depend on the commitment assumption.

Proposition 8 With optimal taxes, the commitment and no commitment cases yield the same values for $s, \mu, \frac{3}{4}$ and V_e. Comparing the optimal taxes in each case yields

$$
\begin{aligned}
\iota^{NC} &= \iota^C - \frac{\bar{c}\frac{3}{4}}{V_e}\\
\iota_a^{NC} &= \iota_a^C + c\frac{3}{4}\\
\tilde{A}^{NC} &= -(1-\bar{\ })\\
\tilde{A}^C &= 0
\end{aligned}
$$

where the solution to (P_{C1}) is denoted by $(\iota^C, \iota_a^C, s_{\frac{3}{4}}^C)$ and $1+\tilde{A}^C = \frac{1+\iota^C}{1+s_{\frac{3}{4}}^C}$ and the solution to (P_{NC1}) is denoted by $(\iota^{NC}, \iota_a^{NC}, s_{\frac{3}{4}}^{NC})$ and $1+\tilde{A}^{NC} = \frac{1+\iota^{NC}}{1+s_{\frac{3}{4}}^{NC}}$.

The three tax instruments ι, ι_a and $s_{\frac{3}{4}}$ can solve the problem of lack of commitment, in the sense that the values for $s, \mu, \frac{3}{4}$ and V_e are the same with optimal taxes in the commitment and no commitment case. In both cases we find that training effort $\frac{3}{4}$ maximizes the total surplus of a match $p(\frac{3}{4})y_1 + (1-p(\frac{3}{4}))y_0 - c\frac{3}{4}$. In the commitment case we see this by substituting the optimal value $\tilde{A}^C = 0$ in proposition 5 into equation (9). In the no commitment case the result follows from substituting $\tilde{A}^{NC} = -(1-\bar{\ })$ into equation (21). The reason why the government revenue requirement does not affect the value of the surplus $\dot{y} - c\frac{3}{4}$ is again production efficiency. If financing g would distort $\frac{3}{4}$, it indirectly reduces tightness (and search s) by reducing the value of the match. As argued above, the demand side of the labour market cannot bear the burden of taxation because of the free entry condition. Since the burden of taxation falls on the labour supply side

22

anyway, it is more e±cient to tax s directly and leave ¾ and µ at the their e±cient level rather than distorting ¾ and thereby µ and s as well.

Given this optimal value of $\mathring{y}$ ¡ c¾, the question is how this surplus net of government expenditure, $\mathring{y}$ ¡ c¾ ¡ $\frac{g}{sm(\mu)}$, is distributed over workers, V_e, and ¯rms, J_e. This distribution then a®ects s and µ. The instruments ¿ and ¿$_a$ are used to achieve this distribution. (P_{C2}) and (P_{NC2}) make clear that V_e is chosen to maximize search s or equivalently welfare (13). The way to do this is to give workers their Hosios share ´ of surplus $\mathring{y}$ ¡ c¾ minus the government expenditure requirement per match $\frac{g}{sm(\mu)}$. Thus $V_e = $ ´ ($\mathring{y}$ ¡ c¾) ¡ $\frac{g}{sm(\mu)}$. The reason why workers bear the incidence of g completely is (again) that ¯rms earn zero (expected) pro¯ts and hence cannot bear any incidence at all. The value of V_e in the no commitment case corresponds to the value in proposition 5 (iii) for the commitment case. Hence V_e; s and µ are the same in the commitment and no commitment case. However, the values of the tax instruments di®er in the two cases.

The intuition for the tax results is the following. The model without commitment contains two additional distortions compared to the commitment case. First, training is too low in the no-commitment case due to the hold-up problem. Indeed, the social bene¯ts of training exceed the private bene¯ts for the worker so that the private outcome yields too little incentives for training. In the commitment case, training is distorted only if it is taxed on a net basis. Proposition 8 reveals that the hold-up problem can be alleviated in the no commitment case by setting the training subsidy above the marginal wage tax, that is training is subsidized on a net basis (Ã < 0). In particular, the net subsidy on training should raise the return to training for the worker so that the marginal costs and bene¯ts coincide. This calls for a net subsidy equal to 1 ¡ ¯, i.e. the share of the bene¯ts of training that °ow to the ¯rm instead of the worker. The second distortion due to the lack of commitment in wage bargaining appears because training costs do not reduce the surplus from a match to the ¯rm. Accordingly, it distorts labour-market tightness. Compared to the commitment case, this calls for a higher average tax and a lower marginal tax in order to redistribute surplus from the ¯rm to the worker. Intuitively, redistributing the surplus creates the right incentives for search and vacancy creation by equating the marginal private bene¯ts with the corresponding marginal productivities in the matching process.

If we start from an arbitrary tax system, the introduction of an EITC is thus more likely to improve welfare in an economy characterized by com-

mitment on the labour market than in an economy without commitment. Interpreting the US economy as being characterized by less commitment than the economies on the European continent, we conclude that an EITC in Europe would be more attractive than in the US. In the US, a number of economists already consider the EITC as a successful instrument to stimulate participation. Indeed, Eissa and Liebmann (1995) find that the EITC significantly raises the participation of women with children. Furthermore, the adverse incentive effects of high marginal taxes on hours worked are found to be negligible.

4 Concluding remarks

This paper formalizes the trade-off between labour-market participation and training in a model of search on the labour market. In particular, it illustrates the optimal tax structure in the presence of three distortions, namely a training distortion, a public revenue requirement that has to be financed by distortionary taxes, and a hold-up problem that arises because firms and workers cannot pre-commit to the return to search and vacancies. We are able to derive analytical results because we use a simplified, static representation of the search model in the tradition of Pissarides (1990) and Morensen and Pissarides (1999). Although our approach captures some important features of these dynamic models, it may also have some limitations. In particular, since we ignore dynamics, our results should be interpreted as steady-state solutions. Especially for human capital formation, this may be too simplistic, although we believe that our model captures the main incentive effects of training. An important assumption in our model is that tax policies have no implications for the fall back position of workers and firms, which always have a value equal to zero. If these values were positive, which is the case in a dynamic model of search, taxes may have a different impact on labour-market decisions, including the training decision. This type analysis may be left for future research, but it seems unlikely that one would be able to derive analytical solutions from such a model.

References
Acemoglu, D. and R. Shimer, (1999), Holdups and efficiency with search frictions, International Economic Review, Vol. 40 (no. 4), 827-850.
Blanchard, O.J. and P.A. Diamond, (1989), The Beveridge curve, Brook-

ings Papers on Economic Activity 1, 1-60.

Boone, J., and A.L. Bovenberg, (1999), Optimal taxation in search models, mimeo.

Boskin, M., (1975), Notes on the tax treatment of human capital, NBER working paper no. 116.

Bovenberg, A.L. en C. van Ewijk, (1997), Progressive taxes, equity, and human capital accumulation in an endogenous growth model with overlapping generations, Journal of Public Economics 64, 153-179.

Bovenberg, A.L., J.J. Graa°and and R.A. de Mooij, (2000), Tax reform and the Dutch labor market: an applied general equilibrium approach, Journal of Public Economics (forthcoming).

Broersma, L. and J.C. van Ours (1999), Job searchers, job matches and the elasticity of matching, Labour Economics, 6, 77-93.

Diamond, P.A. and J.A. Mirrlees, (1971), Optimal taxation and public production: I. Production e±ciency, American Economic Review 61, 8-27.

Dupor, B., L. Lochner, C. Taber and M.B. Wittekind, (1996), Human capital and taxes: some e®ects of taxes on schooling and training, American Economic Review 86 (Papers and Proceedings), 340-346.

Eissa, N. and J.B. Liebman, (1995), Labor supply response to the earned income tax credit, Quarterly Journal of Economics 111, pp. 605-637.

Haveman, R., (1996), Reducing poverty while increasing employment: a primer on alternative strategies and a blueprint. OECD Economic Studies No. 26, Paris.

Heckman, J.J., L. Lochner and C. Taber, (1999), Human capital formation and general equilibrium treatment e®ects: a study of tax and tuition policy, Fiscal Studies, pp 25-40.

Hosios, A.J., (1990), On the e±ciency of matching and related models of search and unemployment, Review of Economic Studies 57, 279-298.

Mortensen, D.T. and C.A. Pissarides, (1998), Taxes, Subsidies and Equilibrium Labor Market Outcomes, mimeo.

Mortensen, D.T. and C.A. Pissarides, (1999), New developments in models of search in the labor market, forthcoming in the Handbook of Labor Economics.

Pissarides, C.A., (1990), Equilibrium Unemployment, Blackwell.

Pissarides, C.A, (1999), The impact of employment tax cuts on unemployment and wages; The role of unemployment bene¯ts and tax structure, European Economic Review 42, No.1, 155-183.

Snower, D.J, and G. de la Dehesa, (1996), Unemployment policy: Government options for the Labour Market. CEPR and Cambridge University Press, Cambridge.

S¿rensen, P.B., 1997. Public ¯nance solutions to the European unemployment problem. Economic Policy, 221-251.

S¿rensen, P.B. en S.B. Nielsen, (1997), On the optimality of the Nordic system of dual income taxation, Journal of Public Economics 63, 311-329.

Teulings, C.N. and J. Hartog, (1998), Corporatism or competition: an international comparison of labour market structures, and their impact on wage formation, Cambridge University Press.

Trostel, P.A., (1993), The e®ect of taxation on human capital, Journal of Political Economy 2, 327-350.

Appendix

This appendix contains the proofs of the results in this paper.

Proof of Lemma 1

The ¯rst order conditions for s; μ and ¾ in maximizing welfare (13) are respectively

$$m(\mu)(\ring{y} \text{ ; } c¾) \text{ ; } °^0(s) \text{ ; } k\mu \; = \; 0 \tag{25}$$

$$m^0(\mu) \, [\ring{y} \text{ ; } c¾] \text{ ; } k \; = \; 0 \tag{26}$$

$$p^0(¾)(y_1 \text{ ; } y_0) \text{ ; } c \; = \; 0 \tag{27}$$

where, as above, $\ring{y} = p(¾)y_1 + (1 \text{ ; } p(¾))y_0$. Rewriting equation (26) as

$$(1 \text{ ; } ´)(\ring{y} \text{ ; } c¾) = \frac{k\mu}{m(\mu)} \tag{28}$$

Substituting this into (25) yields

$$°^0(s) = \frac{´}{1 \text{ ; } ´}k\mu \tag{29}$$

(i) Comparing (27), (28) and (29) to the private outcome in equations (8), (9) and (12) shows that the private and social outcomes coincide if ¿ = s¾ = ¿a = 0 and ¯ = ´.

(ii) Comparing (29) with (8) shows that for given μ, ¿ > 0 implies that the private s is lower than the socially optimal s. Further, (12) together with Ã ¸ 0; ¿a ¸ 0 implies that private μ does not exceed socially optimal μ. This implies that s in (8) does not exceed s in (29).

(iii) The e®ect on training follows immediately from (9) and (27). Since the value of ¾ in (27) maximizes ỹ ¡ c¾, it follows from (28) and (12) (with the assumption that $\iota_a \geq 0$) that private µ is below socially optimal µ.

(iv) ¿ = s¾ = 0 implies that Ã = 0 and hence both private and socially optimal ¾ maximize ỹ ¡ c¾. Now comparing equations (28) and (12) shows that $\iota_a < 0$ implies that µ is higher in private outcome than in social optimum. Comparing (29) and (8) shows that the higher private µ implies that private search s is higher than socially optimal search.

(v) The e®ects of ¯ ⋼ ´ are as follows. First, note that ¾ is una®ected by ¯ in the private outcome (9). Second, the e®ect of ¯ on µ can be derived from (12) as

$$\frac{d\mu}{d\bar{} } = \frac{¡}{´}\frac{1}{1 ¡}\frac{\mu}{\bar{}} < 0$$

(vi) Using this in equation (8) to ¯nd the e®ect of ¯ on s yields

$$\frac{ds}{d\bar{}} = \frac{k\mu}{\circ^{00}(s)}\frac{1}{(1 ¡ \bar{})^2}\left[1 ¡ \frac{\mu}{\bar{}}\right]^{\frac{1}{2}} \quad \begin{array}{l} > 0 \text{ if } \bar{} < ´ \\ < 0 \text{ if } \bar{} > ´ \end{array}$$

Hence s is always too low if ¯ ⋼ ´. QED

Proof of Lemma 2
It is routine to verify that ¿; s¾ and ι_a satisfying

$$1 + ¿ = \frac{\bar{}}{1 ¡ \bar{}}\frac{1 ¡ ´}{´}$$

$$\iota_a = \frac{´ ¡ \bar{}}{1 ¡ \bar{}}(ỹ ¡ c¾)$$

$$s¾ = ¿$$

make the private outcome in (8), (12) and (9) equivalent to the social outcome in (29), (28) and (27). To check budget balance we show that ¿ (p(¾)w₁ + (1 ¡ p(¾))w₀) + ι_a ¡ $\frac{s¾}{1+s¾}$c¾ = 0 as follows

$$¿(p(¾)w_1 + (1 ¡ p(¾))w_0) + \iota_a ¡ \frac{s¾c¾}{1 + s¾} = ¿V_e + \iota_a + Ãc¾$$

$$= \frac{¿}{1 + ¿}\bar{}(ỹ ¡ c¾ ¡ \iota_a) + \iota_a$$

$$= (ỹ ¡ c¾)\frac{(1 ¡ ´) ¡ (1 ¡ \bar{})´}{1 ¡ ´}\left[1 ¡ \frac{´ ¡ \bar{}}{1 ¡ \bar{}}\right]$$

$$= 0$$

27

where the ¯rst equality follows from (3), the second equality from $\tilde{A} = 0$ and (5) and the third equality follows from the expressions for $\dot{\iota}$ and $\dot{\iota}_a$ above. QED

Proof of proposition 3
Using the expressions for the value of a match for the worker and the ¯rm (5) and (6), we write

$$(1 + \dot{\iota})V_e + J_e = [\dot{y} \mathbin{\text{i}} \dot{\iota}_a \mathbin{\text{i}} (1 + \tilde{A})c\frac{3}{4}]$$

Using this expression to eliminate $\dot{y} \mathbin{\text{i}} c\frac{3}{4}$ from (13), we write welfare as

$$sm(\mu)[\dot{\iota}V_e + \dot{\iota}_a + \tilde{A}c\frac{3}{4}] + s[m(\mu)V_e \mathbin{\text{i}} \frac{^{\circ}(s)}{s}] + sm(\mu)[J_e \mathbin{\text{i}} \frac{k\mu}{m(\mu)}]$$

Using (1) and (2) and the government budget constraint (17), we can write welfare as follows

$$g + s[^{\circ\,0}(s) \mathbin{\text{i}} \frac{^{\circ}(s)}{s}]$$

Hence, welfare is determined by public expenditures, the net surplus from search and the net surplus from vacancies (which equals zero). Since we explore only balanced budget reforms (dg = 0), the change in welfare can be written as

$$s^{\circ\,00}(s)ds$$

Because $^{\circ\,00}(s) > 0$ for each s $\lrcorner$ 0, welfare maximization is equivalent to maximization of search. Note that this holds true for both the case with and without commitment (see below). QED

Proof of lemma 4
Log linearizing equations (8), (9) and (12) yields

$$
\begin{pmatrix} "_s & & \\ 0 & \acute{} [\dot{y} \mathbin{\text{i}} \dot{\iota}_a \mathbin{\text{i}} (1 + \tilde{A})c\frac{3}{4}] & 0 \\ 0 & 0 & "_{\frac{3}{4}} \end{pmatrix}
\begin{pmatrix} \hat{s} \\ \hat{\mu} \\ \hat{\frac{3}{4}} \end{pmatrix}
=
\begin{pmatrix} \mathbin{\text{i}} \hat{\dot{\iota}} \\ \mathbin{\text{i}} \dot{\iota}_a \hat{\dot{\iota}}_a \mathbin{\text{i}} (1 + \tilde{A})\frac{3}{4}c(\hat{\dot{\iota}} \mathbin{\text{i}} \hat{s}_{\frac{3}{4}}) \\ \mathbin{\text{i}} (\hat{\dot{\iota}} \mathbin{\text{i}} \hat{s}_{\frac{3}{4}}) \end{pmatrix}
$$

(30)

where $"_s \acute{} \frac{^{\circ\,00}(s)s}{^{\circ\,0}(s)}$; $"_{\frac{3}{4}} \acute{} \mathbin{\text{i}} \frac{p^{00}(\frac{3}{4})\frac{3}{4}}{p^0(\frac{3}{4})}$; $\hat{\dot{\iota}} \acute{} \frac{d\dot{\iota}}{1+\dot{\iota}}$ and $\hat{s}_{\frac{3}{4}} \acute{} \frac{ds_{\frac{3}{4}}}{1+s_{\frac{3}{4}}}$.

28

Using equation (7), government budget constraint (17) can be written as

$$g = sm(\mu)(\iota V_e + \tilde{A}c\tfrac{3}{4} + \iota a)$$
$$= s\frac{\iota}{1+\iota}\frac{-}{1\,\mathrm{i}}k\mu + sm(\mu)\tilde{A}c\tfrac{3}{4} + sm(\mu)\iota a$$

Hence

$$dg = m(\mu)s\left[\frac{\iota}{1+\iota}\frac{-}{1\,\mathrm{i}}\frac{k\mu}{m(\mu)} + (1\,\mathrm{i}\,\acute{})(\tilde{A}c\tfrac{3}{4} + \iota a)\right]\mu + gs + sm(\mu)\tilde{A}c\tfrac{3}{4}\tfrac{3}{4} +$$
$$sm(\mu)\left[\frac{1}{1+\iota}\frac{-}{1\,\mathrm{i}}\frac{k\mu}{m(\mu)} + \tfrac{3}{4}c(1+\tilde{A})\right]\iota + sm(\mu)\iota a\iota a\,\mathrm{i}\;sm(\mu)c\tfrac{3}{4}(1+\tilde{A})s\tfrac{3}{4}$$

Using dg = 0 we get

$$\iota a\iota a + \tfrac{3}{4}c(1+\tilde{A})(\iota\,\mathrm{i}\;s\tfrac{3}{4}) = \mathrm{i}\;\frac{1}{1+\iota}\frac{-}{1\,\mathrm{i}}\frac{k\mu}{m(\mu)}\iota +$$
$$+ \left[\frac{\iota}{1+\iota}\frac{-}{1\,\mathrm{i}}\frac{k\mu}{m(\mu)} + (1\,\mathrm{i}\,\acute{})(\tilde{A}c\tfrac{3}{4} + \iota a)\right](\mathrm{i}\;\mu) +$$
$$+ \frac{g}{m(\mu)s}\frac{1}{"s}(\mathrm{i}\;\mu + \iota) + \tilde{A}c\tfrac{3}{4}\frac{(\iota\,\mathrm{i}\;s\tfrac{3}{4})}{"\tfrac{3}{4}} \qquad (31)$$

or equivalently, using equations (7) and (17) to gather the term $\mathbb{C}$,

$$\mathbb{C}\mu = \left[\frac{1}{1+\iota}\frac{-}{1\,\mathrm{i}}\frac{k\mu}{m(\mu)}\,\mathrm{i}\;\frac{g}{m(\mu)s}\frac{1}{"s}\right]\iota\,\mathrm{i}\;\frac{\tilde{A}c\tfrac{3}{4}}{"\tfrac{3}{4}}(\iota\,\mathrm{i}\;s\tfrac{3}{4})$$

where

$$\mathbb{C}\;\acute{}\;\acute{}(\acute{y}\,\mathrm{i}\;c\tfrac{3}{4})\,\mathrm{i}\;\frac{g}{m(\mu)s}(1+\frac{1}{"s})$$

Using the first row of (30), we find the following reduced form for search

$$"s\mathbb{C}s = ([\,\bar{}\,\mathrm{i}\,\acute{}](\acute{y}\,\mathrm{i}\;c\tfrac{3}{4}) + (1\,\mathrm{i}\,\bar{})(\iota a + \tilde{A}c\tfrac{3}{4}))\iota\,\mathrm{i}\;\frac{\tilde{A}c\tfrac{3}{4}}{"\tfrac{3}{4}}(\iota\,\mathrm{i}\;s\tfrac{3}{4})$$

QED

Proof of proposition 5

29

Proposition 3 says that maximizing welfare subject to (8),(9),(12) and (17) is equivalent to maximizing search subject to these four equations. The reduced form in (18) takes these four equations already into account. To find the socially optimal taxes, we need (only) to solve $\frac{ds}{d\tau} = 0$ and $\frac{ds}{ds_\tau} = 0$. From (18), it follows immediately that $\frac{ds}{ds_\tau} = 0$ if and only if $\frac{\tilde{A}c_\tau}{\tau_\tau} = 0$, that is $\tilde{A} = 0$, or equivalently $\tau = s_\tau$. Using this in the first order condition $\frac{ds}{d\tau} = 0$ yields

$$(\bar{\ } - \acute{\ })(\bar{y} - c_\tau) + (1 - \bar{\ })\tau_a = 0$$

or equivalently

$$\tau_a = \frac{\acute{\ } - \bar{\ }}{1 - \bar{\ }}(\bar{y} - c_\tau)$$

Note that these expressions for $\tilde{A}$ and τ_a are independent of the level of g. In other words, only τ is affected by the level of g.

(i) and (ii) follow immediately from $\tilde{A} = 0$ and $\tau_a = \frac{\acute{\ }-\bar{\ }}{1-\bar{\ }}(\bar{y} - c_\tau)$.

(iii) Noting that the total surplus $\bar{y} - c_\tau$ is divided among employed workers, firms and the government, we can write

$$V_e + J_e + \frac{g}{sm(\mu)} = \bar{y} - c_\tau$$

or equivalently

$$
\begin{aligned}
V_e &= \bar{y} - c_\tau - J_e - \frac{g}{sm(\mu)} \\
&= \bar{y} - c_\tau - (1 - \bar{\ })\left[\bar{y} - c_\tau - \frac{\acute{\ }-\bar{\ }}{1-\bar{\ }}(\bar{y} - c_\tau)\right] - \frac{g}{sm(\mu)} \\
&= \acute{\ }(\bar{y} - c_\tau) - \frac{g}{sm(\mu)}
\end{aligned}
$$

where the second equality follows from equation (6) and the expressions for optimal taxes $\tilde{A} = 0$ and $\tau_a = \frac{\acute{\ }-\bar{\ }}{1-\bar{\ }}(\bar{y} - c_\tau)$. QED

Proof of proposition 6

The reduced form (18) is derived under the assumption that changes in τ (and s_τ) are offset by changes in τ_a to keep a balanced budget for the government. In particular, this follows from the $dg = 0$ assumption in equation (31) above. The results (i), (ii) and (iii) follow immediately from equation

(18). QED

Proof of proposition 7
It is straightforward to verify that the proof of proposition 3 also applies in the no commitment case. Hence (P_{NC1}) is equivalent to

$$\max_{\xi;\xi_a;s_{¾}} s$$

subject to (21),(23),(24),(19) and (20)

QED

Proof of proposition 8
Instead of maximizing with respect to $\xi;\xi_a$ and $s_{¾}$, we determine ¯rst the wage level w_0 and the wage di®erential $\Phi \equiv w_1 - w_0$ which maximize s. From these we derive the corresponding optimal values of the tax instruments $\xi;\xi_a$ and $s_{¾}$.

We write optimization program (P_{NC2}) as

$$\max_{w_0;\Phi} \{s\} \text{ subject to}$$

$$\phi'(s) = m(\mu)(w_0 + p(¾)\Phi - c¾)$$

$$\frac{k\mu}{m(\mu)} = y_0 - w_0 + p(¾)(y_1 - y_0 - \Phi) - \frac{g}{sm(\mu)}$$

$$p'(¾) = \frac{c}{\Phi}$$

where the three equations determining $s;\mu$ and $¾$ follow from (20), (21), (23) and (24). Linearizing these three equations in $s;\mu;¾;w_0$ and Φ yields

$$\begin{pmatrix} \phi''(s) & _3-(1-´)\phi'(s)\frac{1}{\mu} & 0 \\ -\frac{g}{s^2 m(\mu)} & \frac{1}{\mu}-\frac{k\mu}{m(\mu)}-(1-´)\tilde{g} & p'(¾)(y_1-y_0-\Phi) \\ 0 & 0 & p''(¾) \end{pmatrix} \begin{pmatrix} ds \\ d\mu \\ d¾ \end{pmatrix}$$

$$= \begin{pmatrix} m(\mu)[dw_0 + p(¾)d\Phi] \\ -dw_0 - p(¾)d\Phi \\ -\frac{c}{\Phi^2}d\Phi \end{pmatrix}$$

where $\tilde{g} \equiv \frac{g}{sm(\mu)}$. De¯ning det as the determinant of the matrix at the left hand side, we ¯nd

$$\det = \frac{p''(¾)\phi'(s)}{s\mu}\left[´''_s\frac{k\mu}{m(\mu)} - (1-´)\tilde{g}(1+''_s)\right]$$

31

Using Cramer's rule to determine ds yields

$$ds = \frac{1}{det}\left\{ m(\mu)\,[dw_0 + p(¾)d¢]\,p''(¾)\frac{1}{\mu}\left[´\frac{k\mu}{m(\mu)} ¡ (1 ¡ ´)ĝ\right] + \right.$$
$$\left. + (1 ¡ ´)\frac{°'(s)}{\mu}\,p''(¾)\,[¡\,dw_0 ¡ p(¾)\,d¢] + \frac{c}{¢^2}p'(¾)(y_1 ¡ y_0 ¡ ¢)\,d¢\right\}$$

Hence

$$\frac{ds}{dw_0} = \frac{p''(¾)}{\mu\,det}\left\{ m(\mu)\left[´\frac{k\mu}{m(\mu)} ¡ (1 ¡ ´)ĝ\right] ¡ (1 ¡ ´)°'(s)\right\}$$

Using $°'(s) = m(\mu)V_e$ in $\frac{ds}{dw_0} = 0$ yields

$$V_e = ´\frac{k\mu}{1 ¡ ´\,m(\mu)} ¡ ĝ$$

or equivalently

$$V_e = ´(ẏ ¡ c¾) ¡ ĝ$$

Further, $\frac{ds}{d¢} = 0$ yields, in a similar way,

$$m(\mu)\,p''(¾)\,p(¾)\frac{1}{\mu}\left[´\frac{k\mu}{m(\mu)} ¡ (1 ¡ ´)ĝ\right] ¡ (1 ¡ ´)°'(s)\frac{p''(¾)p(¾)}{\mu} + \frac{c}{¢^2}p'(¾)(y_1 ¡ y_0$$
$$= 0$$

or equivalently $¢ = y_1 ¡ y_0$. Hence in the no commitment case we ¯nd $p'(¾) = \frac{c}{¢} = \frac{c}{y_1 ¡ y_0}$: That is, ¾ is at its socially optimal level. Further, the expression for the incidence $V_e = ´(ẏ ¡ c¾) ¡ ĝ$ is the same as in the commitment case (see proposition 5 (iii)).

Now we turn to the problem of determining the values of the tax instruments in the commitment and no commitment case. Proposition 5 implies that $Ã^C = 0$. Hence it follows from (9) that the private $¾^C$ in the commitment case satis¯es $p'(¾) = \frac{c}{y_1 ¡ y_0}$. Thus we ¯nd $¾^{NC} = ¾^C$. Let ¾ denote this value $¾ ´ ¾^{NC} = ¾^C$. Whereas $Ã^C = 0$ achieves this value of ¾ in the commitment case, equation (21) implies that $Ã^{NC} = ¡(1 ¡ ¯)$.

In the commitment case, equations (1), (2) and proposition 5 (iii) imply

$$°'(s^C) = m(\mu^C)V_e^C$$
$$\frac{k\mu^C}{m(\mu^C)} = ẏ ¡ c¾ ¡ V_e^C ¡ \frac{g}{s^C m(\mu^C)}$$
$$V_e^C = ´(ẏ ¡ c¾) ¡ \frac{g}{s^C m(\mu^C)}$$

where $\tilde{y}^C = \tilde{y}^{NC} = \tilde{y} \equiv p(¾)y_1 + (1 - p(¾))y_0$.

For the no commitment case we find:

$$°^0(s^{NC}) = m(\mu^{NC})V_e^{NC}$$

$$\frac{k\mu^{NC}}{m(\mu^{NC})} = \tilde{y} - c¾ - V_e^{NC} - \frac{g}{s^{NC}m(\mu^{NC})}$$

$$V_e^{NC} = ´(\tilde{y} - c¾) - \frac{g}{s^{NC}m(\mu^{NC})}$$

Comparing the equations for s^C, μ^C and V_e^C with these for s^{NC}, μ^{NC} and V_e^{NC} shows immediately that $s^C = s^{NC} = s$, $\mu^C = \mu^{NC} = \mu$ and $V_e^C = V_e^{NC} = V_e$. However, the values of the tax instruments $\tau; \tau_a; \tilde{A}$ to achieve these values differ in the commitment and no commitment case.

From the government budget constraint (17) and the expression for $\tilde{A}^{NC} = - (1 - ¯)$ we find

$$\frac{g}{sm(\mu)} = \tau^{NC}V_e + \tau_a^{NC} - (1 - ¯)c¾$$

Using $V_e = ´(\tilde{y} - c¾) - \frac{g}{sm(\mu)}$ this can be written as

$$(1 + \tau^{NC})\frac{g}{sm(\mu)} = \tau^{NC}´(\tilde{y} - c¾) + \tau_a^{NC} - (1 - ¯)c¾$$

Combining this with the expression $(1 + \tau^{NC})V_e = ¯(\tilde{y} - c¾ - \tau_a^{NC})$ yields

$$\tau_a^{NC} = \frac{´ - ¯}{1 - ¯}(\tilde{y} - c¾) + c¾$$

$$= \tau_a^C + c¾$$

where the expression for τ_a^C follows from proposition 5 (ii).

Finally, the expression for τ^{NC} is derived from the government budget constraint (17) written as

$$\frac{g}{sm(\mu)} = \tau^{NC}V_e + \frac{´ - ¯}{1 - ¯}(\tilde{y} - c¾) + c¾ - (1 - ¯)c¾$$

$$\frac{g}{sm(\mu)} = \tau^C V_e + \frac{´ - ¯}{1 - ¯}(\tilde{y} - c¾)$$

Substracting these two equations yields

$$0 = (\tau^{NC} - \tau^C)V_e + ¯c¾$$

33

or equivalently

$$\dot{c}^C = \dot{c}^{NC} + \frac{\overline{c}_{3/4}}{V_e}$$

QED