The Trade-Off between Sclerosis and Hold Up Problems

Boone, J.; Smulders, J.A.

The trade o® between sclerosis and hold up problems: Rhenish vs. Anglosaxon Economies

Jan Boone[¤] and Sjak Smulders[y]

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Abstract

This paper presents a simple model to explain the relative advantages of market economies with high turnover of ¯rms and those with low turnover rates but long-term relationships. The two types of economies, labeled Anglosaxon and Rhenish respectively, arise as two equilibria that can simultaneously exist in the model. We show that welfare is not necessarily higher in one of the two. A trade o® exists between sclerosis and a hold up problem. Our main result is that deregulation in a Rhenish economy yields smaller e®ects on output than in an Anglosaxon economy.

Keywords: Corporate Governance, Long Term Relationships, Deregulation, Organisation and Markets

1. Introduction

Economic institutions di®er signi¯cantly among market economies. Often the Anglosaxon organisation of economic acticity is contrasted to the European, Japanese, East Asian, or Rhenish institutional setting (see e.g. Albert 1991, Gerlach 1992,

¤ Corresponding author, Dept. of Economics, Tilburg University P.O.Box 90153, 5000 LE Tilburg, The Netherlands and CPB Netherlands Bureau of Economic Policy Analysis. Email: j.boone@kub.nl.

y Dept. of Economics, Tilburg University. Email J.A.Smulders@kub.nl. Smulders thanks the Dutch Science Foundation NWO for ¯nancial support. The views in this paper are the authors' personal views and not necessarily those of the organisations they work for. Comments by Lans Bovenberg and Theo van de Klundert are much appreciated.

Rajan and Zingales 1998, The Economist 1999).[1] In the Anglosaxon system, dynamism and turnover in markets for labour and corporate control are high. David (1995: 15) calls this impatient capitalism and observes that 'no other rich country gives companies quite such a free hand to ... shift resources from declining industries into growing ones'. In the Rhenish system, long-term relationships and dynamism within firms are relatively more important. The main advantage of the Anglosaxon system is fast redeployment of resources: if a firm becomes relatively unproductive the resources in the firm are released and can be used more productively elsewhere in the economy. The main advantage of Rhenish (and the disadvantage of Anglosaxon) economies is that agents are (not) willing to make long-term investments (cf. Jacobs 1993). In the words of Porter (1990: 528), in the US 'employees are often not committed ... to their company, partly because ... their company is not committed to them'.

Especially in Europe, there have been launched proposals to try to combine the best of the two worlds. It is argued that countries like Germany, France and the Netherlands should introduce more competition in their economies. One often points to the US and UK where privatisation and liberalisation of economic sectors has led to notable successes. The Dutch government has commited to a deregulation program for the Dutch economy, in which the results from a comparison between the Dutch and US economy play an influencial role (McKinsey 1997).

In this paper we present a simple model which focuses on two issues in comparing the two systems. First, there is the sclerosis problem in the Rhenish economy, because inefficient firms continue production while it would be socially desirable that their resources would be reallocated to more efficient firms. Second, there is a hold up problem. Agents can make relationship specific investments, but due to contractual incompleteness they can only reap part of the benefits. It turns out that the Rhenish economy can (partly) solve the hold up problem due to the fact that relationships last longer precisely because inefficient firms continue production. Hence, the two externalities interact: in the Rhenish system, the sclerosis problem helps to mitigate the hold-up problem.

In particular, in our model the rate of turnover of firms, or `speed' of the economy, is endogenous. Firms are interpreted as a match between agents which can be established only after a time-consuming search process (as in labour market search models, see for instance Pissarides 1990). In particular, we assume that

[1] Alternative labels that are used to characterize the two systems include: allience capitalism, crony capitalism, or relation-based system vs arm's length system.

the probability that an agent is matched with another agent increases with the number of searching agents. This captures the idea that it is socially desirable for inefficient firms to dissolve the match and release their resources. The assumption that the match probability of an agent increases in the number of searching agents also creates the two equilibria: the Renish equilibrium where inefficient firms stay in production and the expected search costs are high and the Anglosaxon equilibrium where inefficient firms stop production and expected search costs are low. Further, once matched agents can invest to raise the value of this match (only). But an agent only appropriates a fraction $\alpha \in \langle 0; 1 \rangle$ of the additional revenues due to this match specific investment. This captures the hold up problem.

We show that if firm specific investments are absent, welfare in the Anglosaxon equilibrium exceeds welfare in the Rhenish equilibrium. And if firm specific investments are important but hard to appropriate, Rhenish welfare exceeds Anglosaxon welfare because relationships last longer in the Rhenish economy which helps to solve the hold up problem.

Further, we examine the welfare effects of attempts to incorporate elements from the Anglosaxon model into the Rhenish model. In particular, we study the effects of reductions in the cost of starting a new firm. We interpret this as a liberalisation or deregulation policy that is aimed at increasing output in the economy. The main result is that in high speed economies liberalisation is much more effective than in slower economies. Intuitively, in the low speed economy, start-up costs are incurred less frequently and a reduction in set-up costs is accordingly less effective than in an economy with high turnover of firms. We also discuss how a large deregulation program could move a Rhenish system to the Anglosaxon mode. We argue that a switch from the Rhenish to the Anglosaxon mode might involve serious short-run costs despite possibly large long-run gains. Similarly, it may not be optimal for an Anglosaxon economy to switch to the Rhenish system even if the latter is better able to solve hold-up problems. Hence, we explain why the two systems tend to stably coexist, even if policy makers are aware of the welfare differences between the two.

The paper is organised as follows. In Section 2 we present the core of the model and show how the two equilibria arise because of increasing returns in aggregate matching. In Section 3, we compare welfare over the two equilibria and extend the model with firm-specific investment. Section 4 studies the effect of deregulation. Section 5 considers some dynamic aspects. Section 6 concludes. Proofs of all results are relegated to the Appendix.

2. The model

Consider an economy with a set of identical agents [0,1]. At each point in time, two agents can be matched to produce output. Once matched they produce an output level y each. With a poisson arrival rate λ they become inefficient and produce only εy (with $0 < \varepsilon < 1$). Once this happens the partners can decide whether to continue producing or split up.

If they continue producing, there is a probability δdt that the match is dissolved exogenously and the partners become unemployed. There is also a probability μdt that the productivity is restored to y instead of εy. If the match is dissolved (either endogenously or exogenously) the agents become unemployed.

Unemployed agents face a probability $m(u)dt$ of being matched with another unemployed agent, where u denotes the number of unemployed agents. The matching function satisfies $m(0) = 0; m'(u) > 0$ and $m''(u) < 0$. That is, the probability that an agent finds another unemployed agent is increasing in the number of unemployed agents. This matching function is in line with Diamond (1982). But it differs from the labour market search literature, where a distinction is made between vacancies and unemployed agents. Our focus is not so much on the labour market, but more generally on how resources flow through the economy. The assumption $m'(u) > 0$ implies that matched partners are more willing to break up if there are more unemployed resources available in the economy with which they can be matched. In other words, there is a positive search externality.

If agents are matched, they pay a set up cost ϕ each and, as mentioned above, they produce output level y each. Both partners equally share costs and benefits. This process is illustrated in Figure 1, where n_1 denotes the number of agents producing at full productivity and n_0 the number of agents producing εy.

figure 1 around here

If all agents decide to continue producing after the negative productivity shock, matches, or relationships, obviously last longer than when they decide to stop producing. We will therefore label the former situation as a Rhenish equilibrium and the latter as an Anglosaxon equilibrium to capture the stylized fact that long-term relationships prevail in Rhenish economies. One way to think of this is the predominance of the Japanese "life-time employment systems" (see for example Mincer and Higuchi 1988) or the close long-term relationships with suppliers in the Keiretsu. Another interpretation relates to the nature of job creation. Navarro

(1998) documents from the OECD (1995) jobs study that in Europe gross job flows stem relatively more from job creation within existing establishments, while in the US there is more job creation from new establishments. In our model, shifts between the high-productivity state and the ε-state (occuring in the Rhenish economy only) can be seen as job creation and destruction within firms, while new matches (occuring more frequently in the Anglosaxon equilibrium) represent job creation from new firms.

In order to determine whether agents will continue or dissolve the match we need to solve the following Bellman equations

$$
\begin{aligned}
\rho V_u &= m(u)(V_1 - V_u - \phi) && (2.1)\\
\rho V_1 &= y + \lambda(V_0 - V_1) && (2.2)\\
\rho V_0 &= \max\{\rho V_u; \varepsilon y + \mu(V_1 - V_0) + \delta(V_u - V_0)\} && (2.3)
\end{aligned}
$$

where ρ denotes the discount rate, V_u denotes the value of being unemployed, V_1 the value of being in the high productivity state and V_0 the value of the low productivity state. At that moment the partners choose whether to dissolve the match and receive V_u or whether to continue and receive $\varepsilon y + \mu(V_1 - V_0) + \delta(V_u - V_0)$:

Lemma 2.1. *If agents choose once and for all whether to stick to a low productivity match or whether to dissolve such a match, the condition for dissolving is $m(u) > \bar{m}$ and that for continuing is $m(u) < \bar{m}$ where $\bar{m}$ is defined as*

$$\bar{m} = \frac{(\rho + \lambda)\varepsilon + \mu}{1 - \varepsilon - (\rho + \lambda + \mu)\frac{\phi}{y}}$$

According to this lemma, a Rhenish equilibrium arises if agents face a small matching probability, while the Anglosaxon equilibrium is characterized by a high matching probability. In Rhenish economies low matching probabilities cause search frictions to be high, thus making it more attractive to continue production than to stop and search for partners to start a new firm. Few new firms are started and more resources are bound to low productivity activities in the Rhenish equilibrium relative to the Anglosaxon equilibrium. In contrast, in the Anglosaxon equilibrium, search costs are low, new matches are easily found, and existing firms find it attractive to give up low productivity activities in order to search for new high productivity activities. Accordingly, turnover in the Anglosaxon equilibrium is higher.[2] In short, the matching probability m represents the 'speed' at which resources are redeployed in the economy.

Note that $\bar{m}$ is increasing in $\varepsilon; \mu; \lambda; \psi/y$, and ρ. The Rhenish equilibrium is more likely to arise than the Anglosaxon equilibrium if negative productivity shocks are less severe (ε large) and less long-lasting (μ large) because this reduces the costs of continuing production in a low productivity state. The Anglosaxon equilibrium is more likely if average time in high productivity state ($1/\lambda$) is large, if the cost to set up a new firm (ψ/y) are low, and if the discount rate ρ is low because this reduces the costs of searching and establishing a new firm that reaps high productivity returns.

In the Anglosaxon case where the ε state is dissolved immediately, steady-state unemployment satisfies $um = \lambda(1 - u)$ or equivalently

$$m = \lambda \frac{1 - u}{u} \tag{2.4}$$

In the Rhenish case where firms continue in the ε state, steady-state unemployment satisfies $um = \delta(1 - n_1 - u)$ and $um + \mu(1 - n_1 - u) = \lambda n_1$. Solving for m yields

$$m = \frac{\lambda}{1 + \frac{\lambda + \mu}{\delta}} \frac{1 - u}{u} \tag{2.5}$$

Figure 2 depicts (2.4) and (2.5) in the $(u; m)$ plane as the downward sloping curves AE and RE respectively. From lemma 2.1 we have taken into account that the former applies for $m > \bar{m}$ and the latter for $m < \bar{m}$, which is why we show the curves partly broken. In equilibrium the relation between m and u is furthermore pinned down by the matching function $m(u)$ which is upward sloping. Hence, a point of intersection between the solid part of the AE curve (RE curve) and the matching curve represents an Anglosaxon (Rhenish) steady-state.

figure 2 around here

The long-run matching probability is endogenously determined by (2.4) and the matching function in the Anglosaxon equilibrium. Hence, λ and the parameters of the matching function determine the long-run speed of resource redeployment in the Anglosaxon economy. Similarly, λ, μ, δ and the parameters of the matching function determine the speed in the Rhenish economy.

[2] International comparisons of gross entry rates are rare. Some evidence for a lower rate of gross entry in Europe relative to Anglosaxon economies is provided in Cable and Schwalbach (1991).

Interestingly, both equilibria may arise in the steady-state as is stated in the following proposition.

Proposition 2.2. For each set of parameters, $\rho; \phi; \lambda; \mu; \delta; y$ and ε, there exists a matching function $m(u)$ such that both equilibria exist and that both are stable.

Graphically, proposition 2.2 says that is always possible to find an increasing and concave function $m(u)$ that intersects the RE curve below $\bar{m}$ and intersects the AE curve above $\bar{m}$ in figure 2.

The intuition behind the existence of multiple equilibria is as follows. If partners face a high matching probability, they are willing to dissolve when hit by a negative productivity shock (Anglosaxon equilibrium). This implies high inflows in the pool of unemployed (u). Since search becomes easier with many searching agents ($m'(u) > 0$), the high inflows imply high u and favourable matching probabilities, thus reinforcing the reasons behind the Anglosaxon equilibrium. In contrast, in the Rhenish equilibrium, matching probabilities are low, firms are reluctant to go bankrupt and the inflow in the pool of unemployed is small. Smaller numbers of searching agents reinforce the Rhenish equilibrium by lowering matching probabilities.[3] In sum, a thin market externality and the associated coordination failure similar to that in Diamond (1982) causes the multiplicity of equilibria.

3. Welfare effects

In this section we compare the welfare outcomes in the Anglosaxon and Rhenish economies. We show that in the model of the previous section, the Anglosaxon equilibrium yields higher welfare than the Rhenish outcome. However, introducing relationship specific investments with returns that cannot be completely appropriated can overturn this result.

[3] The fact that unemployment in the US is lower than in European Economies seems to be inconsistent with our model. However, we argue that the European employment problems should be interpreted as a participation problem, which is studied in detail below. Indeed, once a participation decision is included in the model, it predicts lower participation in the Rhenish economy. Of course, in Japan and other Asian Economies (to be classified as a Rhenish economy) average unemployment is low. Furthermore, in the model u not only represents unemployed labour but also other unemployed assets ("firms"). Finally, this paper does not attempt to explain differences in social security systems that might explain differences in employment and turnover rates.

3.1. Without relationship speci¯c investments

Proposition 3.1. Steady-state welfare in Anglosaxon equilibium is always higher than in Rhenish equilibrium.

To understand why the high speed economy outperforms the low speed economy, we have to consider the 'thin market externality'. Agents take conditions in the labour market as given, i.e. take m as given. This implies that they do not take into account that dissolving a ¯rm improves the matching probability for all unemployed resources in the economy. A higher matching probability obviously improves welfare: friction decreases and spells of idleness of resources are shorter. Hence, the social returns to voluntary dissolving of a match are higher than its private returns, or, in other words, if voluntary bankruptcy is privately optimal, it is socially so for sure. Assuming that both a Rhenish and Anglosaxon equilibrium exist, voluntary bankruptcy is apparently privately optimal (in Anglosaxon equilibrium) and hence socially. However, in the Rhenish equilibrium, there is no such bankruptcy, so that the Anglosaxon economy reaches the social optimum and the Rhenish economy is suboptimal.[4]

3.2. With relationship-speci¯c investments

The model so far ascribes one-sided advantages to Anglosaxon economies. However, as noted in the Introduction, Rhenish economies may provide better incentives for long-term investments.[5] To investigate this claim, we extend the model to allow for relationship-speci¯c investment.

At the moment that the two parties meet, each of them can invest © in addition to Á to raise ouput from y to $y + Y$ in the high productivity state and from "y to "$(y + Y)$ in the low productivity state for the current match. However, this investment is non-contractable and each agent can only expect to appropriate a share ® of the returns of his investment. When deciding upon whether to invest or

[4] Note that the Anglosaxon equilibrium coincides here with the social optimum because there is only one bad state. If we assume that ¯rms face shocks with poisson arrival rate ¸dt and then get a productivity " which is drawn from a nondegenerate distribution (as in Mortensen and Pissarides (1998)), there will be a reservation productivity below which the match is dissolved. In that case the Anglosaxon outcome does not necessarily coincide with the social optimum.

[5] Often ¯rm-speci¯c investments in workers' skills are said to be more important in Rhenish economies than in Anglosaxon economies. For example, formal on-the-job training is more important in Germany and Japan than in the US, see OECD 1995, Table 7.4 and 7.12.

not the agent takes the investment and match dissolving decisions of his partner as given.

Proposition 3.2. For given parameters $\rho; \phi; \Phi; \lambda; \mu; y; Y$ and ε there exist a matching function and values for α and δ satisfying

$$\frac{\rho\frac{\rho+\mu+\lambda}{\delta}+\rho+\lambda}{\frac{\rho+\mu+\lambda}{\delta}\varepsilon+1}\frac{\Phi}{Y}<\alpha<(\rho+\lambda)\frac{\Phi}{Y}<1 \tag{3.1}$$

such that (i) both equilibria exist and are stable and (ii) steady-state welfare in the Rhenish economy exceeds steady-state welfare in the Anglosaxon economy.

The intuition for this result is as follows. Since the returns to investment are only partly appropriable ($\alpha < 1$), the private incentive for investment is too low. This is the hold up problem, where the investor cannot appropriate the whole surplus created by his investment. In the Rhenish equilibrium the hold up problem is mitigated because matches last longer and the sunk cost of investment can be spread over a longer time horizon. In other words, it is precisely because the inefficient firms continue production (sclerosis) that the hold up problem is solved[6].

In particular, if investment is socially optimal (a sufficient condition for which is that the costs Φ are less than discounted returns $\frac{Y}{\rho+\lambda}$: the last inequality in (3.1)), and if appropriablity conditions are not too bad (α is not too small: the first inequality in (3.1)), the Rhenish economy invests, while the Anglosaxon economy does not. This solution to the hold up problem provides the advantage for the Rhenish economy. If solving the hold up problem offsets its sclerosis disadvantage, the Rhenish economy outperforms the Anglosaxon. In particular, we prove that this happens for a large enough probability of failure in the low productivity state (δ). If δ becomes larger, average time spent in the low productivity state in the Rhenish economy is short and the Rhenish economy becomes more like the Anglosaxon economy where no time is spent in the low productivity state. Hence, if δ is large[7], sclerosis is small and the Rhenish solution to the hold up problem dominates the welfare comparison.

[6] Note that because investment is a binary decision here, the Renish equilibrium with investment is socially optimal. If investment is a continuous variable (with Y increasing in the amount invested), then $\alpha < 1$ implies that there is underinvestment in the Rhenish equilibrium as well as in the Anglosaxon equilibrium.

[7] Of course, as δ increases the lower bound on α in proposition 3.2 increases as well.

From the perspective of Anglosaxon economies, high turnover now not only has the advantage of speeding up the e±cient redeployment of resources but also the disadvantage of reducing the incentives to invest in socially valuable ¯rm-speci¯c capital. To overcome this problem, Anglosaxon economies may try to mitigate appropriability problems. The model suggest that policy measures or institutional arrangements that increase ® improve welfare in the Anglosaxon economy. In this sense, the model sheds some light on the casual observations that in Anglosaxon economies contracts and bargaining are more formal, the role of law ¯rms is more important, and reliance to courts is more frequent than in Rhenish economies.

Caballero and Hammour (1998) also study sclerosis and hold-up problems. In their model, the hold-up problem allows one production factor to appropriate returns from the other factor, thus depressing investment and creation of new ¯rms and thus depressing factor rewards in new ¯rms. As a result, hold-up problems reduce the opportunity costs of factor use in new ¯rms and depress redeployment of resources (which amounts to sclerosis) compared to an e±cient economy with complete contracts.[8] However, since private agents decisions are based on too high opportunity costs (the appropriating factor gets too big a reward in newly created ¯rms), destruction of old ¯rms is excessive if contracts are incomplete. Also in our model, excessive destruction may occur. Private agents fail to internalize the negative e®ect of destruction on ¯rm-speci¯c investments. Hence, if condition (3.1) holds, the Anglosaxon economy destroys too many ¯rms.

4. Deregulation within systems

This section analyses the e®ects of deregulation on output and welfare. It shows that deregulation increases output and welfare in both systems, but the gain is bigger in the Anglosaxon equilibrium than in the Rhenish outcome. For this analysis (within systems), we ¯rst return to the simple model in section 2.

Deregulation is modelled here as a reduction in Á. Assume that Á is a pure e®ort cost of agents to start a match. One can think of the e®ort to register a company, to get permission to build a ¯rm on a plot of land and the e®ort to qualify for some license. Deregulation then means less government intervention

[8] In our model, in contrast, hold-up problems do not a®ect the rate of creation or destruction, since the hold-up problem a®ects all agents symmetrically and does not a®ect the relative incentives for agents in di®erent states. While in Caballero and Hammour's setting new ¯rms are costlessly created, we model set-up costs and search costs. Indeed, it is the search externality (thin market externality) that creates sclerosis in our model.

in the sense of a reduction in such e®ort costs.

There may well be good reasons why the government has introduced such registering procedures. For example the government may want to in°uence the quality of the products traded in the market. Our paper concentrates on the positive output e®ects of deregulation which have to be weighed against such costs. The point is that the positive output e®ects are smaller in the Rhenish equilibrium and in this sense there is less reason to deregulate in the Rhenish equilibrium as compared to the Anglosaxon equilibrium.

In order to analyse the e®ects of liberalisation, we introduce an equation determining participation. Suppose agents can decide whether or not to participate in the economy. If they decide to participate they start out as unemployed until they are matched with another agent. Assume agents have an outside value (of leisure say) equal to !, which is distributed with distribution function $F(!)$ and density function $f(!)$. Then the number of agents participating in the economy is given by $F(V_u)$, the number of agents with outside value smaller than V_u.

Proposition 4.1. Assume that the matching function has a constant elasticity ´ ´ $\frac{dm(u)}{du}\frac{u}{m(u)}$. Then, unless $\frac{f(V_u^R)}{F(V_u^R)} >> \frac{f(V_u^A)}{F(V_u^A)}$, a fall in Á yields a bigger proportional rise in steady-state output in the Anglosaxon equilibrium than in the Rhenish equilibrium.

In other words, the bene¯cial output e®ects of deregulation in the US (as documented by for instance Whinston(1993)) form only an upperbound on the bene¯ts that can be expected in continental Europe or Japan. This does not imply that deregulation should not be tried there, but only that the bene¯ts should not be exaggerated.

The intuition for this result is clear. As the speed in the Anglosaxon economy is higher than in the Rhenish economy, the cost Á of starting a new ¯rm is more often incurred. A reduction in Á thus yields a bigger rise in the welfare level of the unemployed, V_u. Hence, the e®ect of a fall in Á on participation and thus total output is higher in the Anglosaxon equilibrium (unless the number of agents that are just about to participate, $f(V_u)$, is far bigger in the Rhenish than in the Anglosaxon equilibrium).

Finally, the expression for $\hat{m}$ in lemma 2.1 is increasing in Á. Hence, for a given matching function reducing Á may imply eliminating the Rhenish equilibrium. Such a move between systems is the last topic of this paper.

5. Moving between systems: a discussion

Up to now we have assumed that a Rhenish economy remains stuck in a Rhenish equilibrium and similarly for an Anglosaxon economy. We have shown in the appendix that each equilibrium is locally stable. However, it may be possible and even desirable from a welfare perspective, to switch from one regime to another. The parameter we use here to move between systems is ϕ. Roughly speaking, for a given matching function by reducing ϕ far enough, the Rhenish equilibrium disappears, by increasing it far enough the Anglosaxon equilibrium disappears.

First consider the possibility of regime switches. A large decrease in set-up costs ϕ may move a Rhenish economy to the Anglosaxon system by lowering the critical level μ (see Figure 1). However, small shocks to set-up costs do not shake up a Rhenish steady state. Similarly, a large enough positive shock to ϕ may bring an Anglosaxon economy to the Rhenish system. In short, policy measures aimed at moving to another regime require major changes and half-hearted reforms are deemed to fail completely. This provides another reason why the European efforts to considerably raise efficiency may prove to be difficult, in addition to the reason in proposition 4.1.

A second issue is the desirability of a regime switch. If the value of solving the hold up problem for the Rhenish economy is relatively small, long-run welfare is higher by switching to the Anglosaxon system. However, there is the following short run transition cost. Moving to the Anglosaxon equilibrium implies forcing all firms in the low (ε) productivity state to dissolve. This leads to a big inflow in unemployment and it will take time before all these agents are matched again. That is, shifting from the Rhenish to the Anglosaxon equilibrium leads to an initial unemployment rate that exceeds the steady state unemployment rate[9]. Thus, if the discount rate is high and speed of the Rhenish economy differs considerably from the crital level μ, then the short-run costs of a regime switch can outweigh the long-run gains and there is no reason to pursue a switch. Although it may still be welfare enhancing to pursue smaller reductions in ϕ.

As firm specific investments become more important, Anglosaxon economies may perform worse than Rhenish economies and a big reduction in ϕ may reduce welfare in the Rhenish economy. In fact, Anglosaxon economies may consider to engineer a switch to the Rhenish system. This requires increases in search

[9] Note that the steady state unemployment rate in the new Anglosaxon equilibrium exceeds that in the previous Rhenish equilibrium. However this is not a problem since the shift is pursued because welfare is higher in the Anglosaxon equilibrium.

friction (e.g. by raising set-up costs Á) so as to stimulate long-term contracts to solve the hold up problem. Note that in this case there is not even a short-run cost, since relationships start to last longer so that unemployment gradually falls, and new ¯rms start to undertake socially desirable investment. However, raising search friction is a second-best policy, since it stimulates investment at the cost of sclerosis. The ¯rst-best policy would be to mitigate the appropriability problems associated with speci¯c investment and at the same time maintain the high speed of recource deployment. We already mentioned that tendencies in court reliance and the growth of law ¯rms can be seen as institutional changes to cope with appropriability. The debate on corporate governance in the US also focuses on arrangements to improve incentives to undertake investment without reducing the extent of competition in the market for corporate control and without relying on distortionary industry policies.

6. Conclusion

Our simple model points out the trade o® between sclerosis and hold up problems. The model generates two equilibria. The Rhenish equilibrium features sclerosis but can solve the hold up problem through long-term relationships. The Anglosaxon equilibrium features fast reallocation of resources from ine±cient ¯rms to e±cient ones but at the cost of smaller ¯rm speci¯c investments. Though one system may be superior over the other from a long-run perspective, a regime switch may be di±cult to engineer or may even be socially unattractive because of high short-run costs. Moreover, certain policies that have proven to be e®ective in one regime (like deregulation in Anglosaxon countries) may be less e®ective in the other. We have shown that the two externalities that have been separately studied before (e.g. Diamond 1982 and Caballero and Hammour 1998) crucially interact: since the sclerosis problem mitigates the hold-up problem, an economy su®ering from sclerosis is not necessarily worse o® than a high speed economy that nevertheless su®ers imperfect appropriability of investment.

The model may provide a framework to analyse some other important differences in economic structure and institutions between the Anglosaxon and the European/Japanese economies. First, the model may explain why the welfare state is in some respect more generous in Rhenish economies. In the Rhenish equilibrium, the unemployed are worse of (V_u is low) than in the Anglo-Saxon equilibrium. Hence, unemployment insurance is more attractive in the former than in the latter. Second, the model can be extended to allow for innovation and growth. Di®erences in innovation regimes can then be studied. Rhenish economies may have a comparative advantage in inhouse R&D aimed at improving and building on existing products and processes, while Anglosaxon economies may devote relatively more innovation e®orts to innovations that imply creative destruction of existing products and processes. Trade between two regions that are in di®erent regimes may thus yield substantial gains, because of the following specialization argument. Firms in the Anglosaxon economy specialize in products where creative desctruction is important, while ¯rms in the Rhenish economy specialize in products where inhouse R&D and long term relationships are productive. Another extension is to disaggregate the model and show that for some industries the Anglosaxon equilibrium is optimal while for other industries the Rhenish equilibrium dominates. For instance, Porter (1990: 108) points in this direction. Finally, the model can also be extended to comparison of business cycle patterns. We may investigate whether Anglosaxon economies are likely to experience faster recovery after recessions.

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Appendix

For notational convenience write m instead of $m(u)$. Then the value functions evaluated at a given m are written as $V_x^j(m)$ with $x = u, 1, 0$ and with $j = A$ ($j = R$) if the value function is evaluated conditional on matches being dissolved (continued) in the ε state. A value function evaluated at the equilibrium value of $m = m(u)$ is denoted by V_x^A if $m = m(u^A)$ and V_x^R if $m = m(u^R)$ where u^A (u^R) is the steady-state unemployment level in the Anglosaxon (Rhenish) equilibrium.

Proof of lemma 2.1

As shown by equation 2.3 there are two possible cases: $\rho V_0 = \rho V_u$ and $\rho V_0 > \rho V_u$. We start by considering the first case, that is the case that matches are dissolved in the ε-state. Then it follows that

$$\rho V_u^A(m) = y\left(\frac{m}{\rho+m}\right)\left(\frac{1-(\rho+\lambda)\frac{\phi}{y}}{1+\frac{\lambda}{\rho+m}}\right) \tag{.1}$$

$$\rho V_1^A(m) = y\left(\frac{1-\lambda\frac{m}{\rho+m}\frac{\phi}{y}}{1+\frac{\lambda}{\rho+m}}\right) \tag{.2}$$

Turning to the second case, $\rho V_0 > \rho V_u$ (matches are continued in the ε-state), and again writing m instead of $m(u)$, we find

$$\rho V_u^R(m) = y\left(\frac{m}{\rho+m}\right)\left(\frac{\rho+\lambda\varepsilon+\mu+\delta-[\rho(\rho+\lambda+\mu)+\delta(\rho+\lambda)]\frac{\phi}{y}}{\rho+\lambda+\mu+\delta(1+\frac{\lambda}{\rho+m})}\right) \tag{.3}$$

$$\rho V_1^R(m) = y\left(\frac{\rho + \lambda\varepsilon + \mu + \delta\left(1 - \lambda\frac{m}{\rho+m}\frac{\phi}{y}\right)}{\rho + \lambda + \mu + \delta\left(1 + \frac{\lambda}{\rho+m}\right)}\right) \tag{.4}$$

$$\rho V_0^R(m) = y\left(\frac{(\rho + \lambda)\varepsilon + \mu + \delta\frac{m}{\rho+m}\left(1 - (\rho + \lambda)\frac{\phi}{y}\right)}{\rho + \lambda + \mu + \delta\left(1 + \frac{\lambda}{\rho+m}\right)}\right) \tag{.5}$$

Now consider an agent who chooses once and for all whether he will continue a match in the ε state or whether he will dissolve it. Of course if all other agents decide to dissolve, he has no choice but to dissolve as well. But if all other agents decide to continue in the ε state, he can consider whether he wants to dissolve or not[10]. Taking m as given, he compares $\rho V_u^A(m)$ in equation (.1) with $\rho V_0^R(m)$ in equation (.5). He decides to always dissolve if and only if $V_u^A(m) > V_0^R(m)$, that is, if

$$\left(\frac{m}{\rho+m}\right)\left(\frac{1 - (\rho + \lambda)\frac{\phi}{y}}{1 + \frac{\lambda}{\rho+m}}\right) > \left(\frac{(\rho + \lambda)\varepsilon + \mu + \delta\frac{m}{\rho+m}\left(1 - (\rho + \lambda)\frac{\phi}{y}\right)}{\rho + \lambda + \mu + \delta\left(1 + \frac{\lambda}{\rho+m}\right)}\right)$$

which can be rewritten as

$$\frac{m}{\rho+m}\left(\frac{1}{\rho+\lambda} - \frac{\phi}{y}\right)(\rho + \lambda + \mu) > \left(1 + \frac{\lambda}{\rho+m}\right)\left(\varepsilon + \frac{\mu}{\rho+\lambda}\right) \tag{.6}$$

Letting $\bar{m}$ denote the value of m for which the left hand side is equal to the right hand side, this condition can be written as $m(u) > \bar{m}$ as in lemma 2.1, because the left hand side of inequality (.6) is increasing in m while the right hand side is decreasing in m. Matches are dissolved if $V_u^A(m) < V_0^R(m)$ which implies $m(u) > \bar{m}$ ■

Lemma .1. The following inequalities hold
(1) $m(u^A) \equiv m^A > m^R \equiv m(u^R)$
(2) $n_1^A > n_1^R$
(3) $V_1^A > V_u^A$
(4) $V_1^R > V_0^R > V_u^R$
(5) $V_u^A > V_0^R$
(6) $V_1^A > V_1^R$.

Proof

[10] In the equilibria we consider all agents play the same (pure) strategy.

(1) Follows immediately from figure 2.

(2) In the Anglosaxon equilibrium it is the case that

$$\dot{n}_1 = (1 - n_1)m^A - \lambda n_1 \tag{.7}$$

Hence in steady state

$$n_1^A = \frac{m^A}{\lambda + m^A} \tag{.8}$$

Similarly, in the Rhenish equilibrium we have

$$\begin{aligned} \dot{n}_1 &= um^R + \mu(1 - u - n_1) - \lambda n_1 && (.9) \\ \dot{u} &= \delta(1 - u - n_1) - um^R && (.10) \end{aligned}$$

Hence in steady state

$$n_1^R = \frac{m^R(\delta + \mu)}{m^R(\delta + \mu + \lambda) + \lambda\delta} \tag{.11}$$

$$n_0^R = \frac{m^R \lambda}{m^R(\delta + \mu + \lambda) + \lambda\delta} \tag{.12}$$

It is routine to verify that n_1^R is increasing in m^R: Hence we find the following inequalities

$$n_1^R < \frac{m^A(\delta + \mu)}{m^A(\delta + \mu + \lambda) + \lambda\delta} < \frac{m^A}{\lambda + m^A} = n_1^A$$

where the first inequality follows from $m^A > m^R$ in (1) and the second inequality follows from $m^A > \bar{m} > \mu$, where $\bar{m}$ is defined in lemma 2.1.

(3) Follows immediately from equations (2.1) and (2.2) with $V_0 = V_u$.

(4) $V_1^R > V_0^R$ follows from $\varepsilon < 1$. We prove the second inequality, $V_0^R > V_u^R$; by contradiction. We know from lemma 2.1 that at $m < \bar{m}$ it is the case that $V_0^R(m) > V_u^A(m)$. Now suppose by contradiction that $V_0^R < V_u^R$; but then a worker is better off to dissolve a match immediately at $m = m^R$; this contradicts $V_0^R(m) > V_u^A(m)$ at $m^R < \bar{m}$.

(5) $V_u^A > V_0^R$ follows from on the one hand that $V_u^A(m) > V_0^R(m)$ at $m > \bar{m}$ (lemma 2.1) and on the other that $V_0^R(m)$ is increasing in m. Hence (1) implies that

$$V_u^A = V_u^A(m^A) > V_0^R(m^A) > V_0^R$$

(6) $V_1^A > V_1^R$ follows from (5) $V_u^A > V_0^R$ and equation (2.2):

$$\begin{aligned} \rho V_1^A &= y + \lambda(V_u^A - V_1^A) \\ \rho V_1^R &= y + \lambda(V_0^R - V_1^R) \end{aligned}$$

■

Proof of proposition 2.2

From figure 2 it is immediately clear that there always exists an upward sloping concave matching function (like curve OM) that cuts RE for $m < \bar{m}$ and AE for $m > \bar{m}$. This proves existence of both equilibria. To proof stability, we need to consider the equations of motion for the two regimes. For $m > \bar{m}$ (Anglosaxon regime), the dynamics are governed by (.7), hence u increases (falls) for values of u to the left (right) of the AE curve and the Anglosaxon equilibrium is (locally) stable. For $m < \bar{m}$ (Rhenish regime), the dynamics are governed by (.9) and (.10). The Jacobian of this system of differential equations has a positive determinant $\delta\lambda + (m + um'(u))(\lambda + 2\delta)$ and a negative trace $-(\lambda + 2\delta + m + um'(u))$ so that the Rhenish equilibrium is (locally) stable. ■

Proof of proposition 3.1

Welfare is defined as the weighted average of value functions where the weights equal the number of agents in the corresponding state. Denoting welfare in the Anglosaxon (Rhenish) equilibrium by W^A (W^R), we can write

$$
\begin{aligned}
W^A &= n_1^A V_1^A + (1 - n_1^A) V_u^A \\
W^R &= n_1^R V_1^R + (1 - n_1^R) V_0^R + u^R (V_u^R - V_0^R)
\end{aligned}
$$

Using lemma .1 above we find that (3) $V_1^A > V_u^A$, (4) $V_1^R > V_0^R$, (5) $V_u^A > V_0^R$ and (6) $V_1^A > V_1^R$. Further the weight on the highest value $V_1^{\cdot}$ is higher in the Anglosaxon case since (2) $n_1^A > n_1^R$, therefore $n_1^A V_1^A + (1 - n_1^A) V_u^A > n_1^R V_1^R + (1 - n_1^R) V_0^R$. Finally, by (4) $V_u^R - V_0^R < 0$. ■

Proof of proposition 3.2

Specific investment, if any, will be made immediately after the firm is set up, since the cost is sunk and the returns can be reaped as long as the match lasts. We find firm value that arises from a partner's own investment by replacing y in (.4) and (.2) by $y + \alpha Y + (1 - \alpha) Y_p$, where Y_p equals 0 if the other partner does not invest and Y if she invests. By replacing y by $y + (1 - \alpha) Y_p$ we find firm value that a partner gets without own investment. The difference between the two is the marginal value of investment given the other partner's investment decision. Investment is privately attractive if this difference exceeds the sunk cost of investment γ. Using (.2), we can write this condition in the Anglosaxon equilibrium as:

$$
\frac{\alpha Y - \lambda \frac{m}{\rho + m} \gamma}{\rho + \lambda \frac{\rho}{\rho + m}} > \gamma
$$

or:

$$\frac{\alpha Y}{\rho+\lambda} > \Phi \qquad (.13)$$

Using (.4), we can write the condition for investment in the Rhenish equilibrium as:

$$\left(\frac{\rho+\mu+\varepsilon\lambda+\delta}{\rho(\rho+\mu+\lambda+\delta)+\delta\lambda}\right)\alpha Y > \Phi \qquad (.14)$$

The socially optimal investment decision is found by setting $\alpha = 1$. Hence, there is underinvestment in the Anglosaxon economy if (.13) is violated for the specified value of α but at the same time holds for $\alpha = 1$. This boils down to:

$$\alpha < (\rho+\lambda)\frac{\Phi}{Y} < 1 \qquad (.15)$$

The right investment decision is made in the Rhenish economy if (.13) holds both for the specified value of α and for $\alpha = 1$. This boils down to:

$$\left(\frac{\rho\frac{\rho+\mu+\lambda}{\delta}+\rho+\lambda}{\frac{\rho+\mu+\lambda\varepsilon}{\delta}+1}\right)\frac{\Phi}{Y} < \alpha < 1 \qquad (.16)$$

Combining the last two equations, we find the first set of inequalities in the proposition.

Now we calculate welfare in the situation that there is no investment in A, but there is in R. As shown[11] in Boone and Smulders (1999) the weighted average of the value functions, where the weights correspond to the steady-state proportions of agents in the state, equals the weighted average of per period discounted pay offs. That is, $n_1^A V_1^A + u_1^A V_u^A = n_1^A y/\rho - u^A m^A \phi/\rho$ and $n_1^R V_1^R + n_0^R V_0^R + u^R V_u^R = n_1^R(y+Y)/\rho + n_0^R \varepsilon(y+Y)/\rho - u^R m^R(\phi+\Phi)/\rho$. Using the steady-state relationships $n_1^A = \frac{m^A u^A}{\lambda}$, $n_1^R = \frac{m^R u^R}{\lambda}\frac{\delta+\mu}{\delta}$, and $n_0^R = \frac{m^R u^R}{\lambda}\frac{\lambda}{\delta}$ (see (.7), (.9) and (.10)), we find:

$$\begin{aligned}\rho\lambda(W^R - W^A) &= (u^R m^R - u^A m^A)(y - \lambda\phi) + u^R m^R\left[(Y - \lambda\Phi) + \frac{\mu+\varepsilon\lambda}{\delta}(y+Y)\right] \\ &\geq (u^R m^R - u^A m^A)(y - \lambda\phi) + u^R m^R (Y - \lambda\Phi) \\ &> (u^R m^R - u^A m^A)(y - \lambda\phi) + u^R m^R \rho\Phi\end{aligned}$$

[11] See also note for the referee.

where the last (strict) inequality follows from equation (.15). We will show by construction that there exists a matching function such that for δ big enough both equilibria still exist and $(u^R m^R - u^A m^A)(y - \lambda\phi) + u^R m^R \rho\Omega$ becomes positive.

First, let $\hat{u}_\delta$ denote the value of u in between the intersection of equation (2.4) and $\hat{m}$ (defined in lemma 2.1) and equation (2.5) and $\hat{m}$ in figure 2. That is, $\hat{u}_\delta \equiv \frac{1}{2}\frac{\lambda}{\lambda+\hat{m}} + \frac{1}{2}\frac{\lambda}{\lambda+\hat{m}(1+\frac{\lambda+\mu}{\delta})}$. Now define the matching function[12] as

$$m(u) = \begin{cases} \hat{m} + \frac{1}{\delta} & \text{if } \; u > \hat{u}_\delta \\ \hat{m} - \frac{1}{\delta} & \text{if } \; u < \hat{u}_\delta \end{cases}$$

By construction this matching function generates both equilibria for each $\delta > 0$: We can write $u^A = \frac{\lambda}{\lambda+\hat{m}+\frac{1}{\delta}}$ and $u^R = \frac{\lambda}{\lambda+(\hat{m}-\frac{1}{\delta})(1+\frac{\mu+\lambda}{\delta})}$ and a sufficient condition for $W^R - W^A > 0$ is

$$\left(\frac{\lambda(\hat{m}-\frac{1}{\delta})}{\lambda+(\hat{m}-\frac{1}{\delta})(1+\frac{\mu+\lambda}{\delta})} - \frac{\lambda(\hat{m}+\frac{1}{\delta})}{\lambda+\hat{m}+\frac{1}{\delta}}\right)(y-\lambda\phi) + \frac{\lambda(\hat{m}-\frac{1}{\delta})\rho\Omega}{\lambda+(\hat{m}-\frac{1}{\delta})(1+\frac{\mu+\lambda}{\delta})} > 0$$

Now it is clear that there is a value of δ big enough such that this inequality holds since $\lim_{\delta\to+\infty}\left[\frac{\lambda(\hat{m}-\frac{1}{\delta})}{\lambda+(\hat{m}-\frac{1}{\delta})(1+\frac{\mu+\lambda}{\delta})} - \frac{\lambda(\hat{m}+\frac{1}{\delta})}{\lambda+\hat{m}+\frac{1}{\delta}}\right] = 0$. Further, for this value of δ we can choose a value of α in the interval defined by (3.1). ■

Proof of proposition 4.1

For the Anglosaxon case we differentiate equation (.1) with respect to ϕ which yields

$$\left|\frac{dV_u^A(m)}{d\phi}\right| = \frac{1}{\rho}\left(\frac{m}{\rho+m}\right)\frac{\rho+\lambda}{1+\frac{\lambda}{\rho+m}} \tag{.17}$$

which is clearly increasing in m. For the Rhenish case we differentiate (.3) with respect to ϕ and solve for $\left|\frac{dV_u}{d\phi}\right|$, which yields

$$\left|\frac{dV_u^R(m)}{d\phi}\right| = \frac{1}{\rho}\left(\frac{m}{\rho+m}\right)\frac{\rho+\lambda+\left(\frac{\rho+\lambda+\mu}{\delta}\rho\right)}{1+\frac{\lambda}{\rho+m}+\frac{\rho+\lambda+\mu}{\delta}} \tag{.18}$$

This expression is also increasing in m:

[12]Strictly speaking we required in section 2 that the matching function is differentiable. The proof can be extended, at the expense of heavy notation, by defining a differentiable concave matchingsfunction which intersects the RE curve in figure 2 (slightly) below $\hat{m}$ and the AE curve (slightly) above $\hat{m}$.

The expression for $\left|\frac{dV_u(m)}{d\phi}\right|$ in equation (.17) is bigger than in (.18) even if we use the same value for m. But in fact m is higher in the Anglosaxon equilibrium than in the Rhenish equilibrium. So $\left|\frac{dV_u}{d\phi}\right|$ is bigger in the Anglosaxon equilibrium.

Total (per period) steady-state output in Anglosaxon equilibrium equals $Q^A = (n_1^A y)F(V_u^A)$ and in Rhenish equilibrium $Q^R = (n_1^R y + n_0^R \varepsilon y)F(V_u^R)$, where $n_1^A; n_1^R$ and n_0^R denote the proportion of agents in the respective states.

Because of the thin/thick market externality in the matching function, there is an effect of participation $F(V_u)$ on $n_1^A; n_1^R$ and n_0^R as well. The reason is that these proportions depend on matchings probabilities which is in turn a function of the total number of unemployed $m = m(Fu)$.

First, write the fraction of unemployed among the participating agents as

$$u = 1 - n_1\left(1 + \frac{n_0}{n_1}\right)$$

and note that $\frac{n_0}{n_1} = 0$ in an Anglosaxon equilibrium and, from (.12) and (.11), that $\frac{n_0}{n_1} = \frac{\lambda}{\delta+1}$, so that n_0/n_1 does not depend on m, ϕ or F. Now we can calculate the effect of F on n_1:

$$\frac{dn_1}{dF} = \frac{dn_1}{dm}\frac{dm}{d(Fu)}\frac{d\left[\left(1 - n_1\left(1 + \frac{n_0}{n_1}\right)\right)F\right]}{dF}$$

Note from (.8), (.12) and (.11) that $\frac{dn_1}{dm} = \frac{un_1}{m}$ in both equilibria. Furthermore denote the elasticity of the matching function by $\eta = \frac{dm}{d(Fu)}\frac{Fu}{m}$ which is assumed to be constant. Substituting these results we find:

$$\frac{dn_1}{dF} = \frac{un_1}{m}\frac{\eta m}{Fu}\left[(1 - n_0 - n_1) - (n_0 + n_1)\frac{F}{n_1}\frac{dn_1}{dF}\right]$$

or

$$\frac{F}{n_1}\frac{dn_1}{dF} = \frac{\eta u}{1 + \eta(1 - u)}$$

Output is given by

$$Q = F(n_1 y + n_0 \varepsilon y) = \left[\left(1 + \frac{n_0}{n_1}\varepsilon\right)y\right]n_1 F$$

where the term in brackets at the RHS of the second equality is independent of m, ϕ or F. Now we easily find the effect of ϕ on output:

$$\frac{dQ}{d\phi}\frac{1}{Q} = \left[\frac{\eta u}{1 + \eta(1 - u)} + 1\right]\frac{f(V_u)}{F(V_u)}\frac{dV_u}{d\phi}$$

Since this expression applies to both the Rhenish and the Anglosaxon equilibrium, we can now compare the output effect in the two regimes. First, we know from figure 2 that $u^R < u^A$. Hence the expression in square brackets is higher for the Anglosaxon than for the Rhenish equilibrium. Second, we have shown above that $\left|\frac{dV_u^A}{dÁ}\right| > \left|\frac{dV_u^R}{dÁ}\right|$. Therefore the only way that the proportional rise in output (due to a fall in Á) in the Rhenish equilibrium can exceed that in the Anglosaxon equilibrium is when $\frac{f(V_u^R)}{F(V_u^R)} >> \frac{f(V_u^A)}{F(V_u^A)}$ which is excluded in the proposition. ■

Note for the referee

In this note we provide a brief and simplified summary of our 1999 paper to explain the equivalence between welfare and income.

Consider a representative agent who can at each moment in time be in one of S states $s \in \{1, 2, \ldots, S\}$. In state s at time t he can choose an action $x_{s,t} \in X_s$. This action affects his pay off in state s, $p_s(x_s) \geq 0$, and his transition rate (Poisson arrival rate) from state s to state s', $m_{s,s'}(x_{s,t})$.

The value function for an agent in state s at time t can now be written as

$$\rho V_{s,t} = \max_{x_a \in X_s} \{p_s(x_a) + \sum_{s' \neq s} m_{s,s'}(x_a)(V_{s',t} - V_{s,t}) + \dot{V}_{s,t}\} \tag{.19}$$

To simplify notation, let $p_{s,t} \equiv p_s(x_{a,t})$ denote the pay offs and $m_{s,s',t} \equiv m_{s,s'}(x_{a,t})$ the transition rates in state s at time t that follow from the Bellman equation .19.

Definition .2. Define intertemporal welfare at time t as the weighted average of the value functions $V_{s,t}$ in the states at time t with the weights equal to the proportion of agents in each state s at time t, $n_{s,t}$

$$W_t \equiv \sum_{s=1}^{S} n_{s,t} V_{s,t}.$$

In the following proposition we state how welfare and pay-offs are related:

Proposition .3. $\rho W_t = \sum_{s=1}^{S} n_{s,t} p_{s,t} + \frac{dW_t}{dt}.$

Proof. It turns out that the proposition is most easily proved using matrix notation. Define a matrix A_t as $a_{ii,t} = -\sum_{j \neq i} m_{i,j,t}$ and $a_{ij,t} = m_{i,j,t}$ $i \neq j$. Then one can see that $\sum_{j=1}^{S} a_{ij,t} = 0$. It follows that over time the number of agents in the states evolves as

$$\dot{n}_t = A_t^T n_t \tag{.20}$$

where $n_t = (n_{1,t}, n_{2,t}, \ldots n_{S,t})^T$. Let V_t denote the vector of values $V_t = (V_{1,t}, V_{2,t}, \ldots V_{S,t})^T$ and P_t the vector of per period pay offs $P_t = (p_{1,t}, p_{2,t}, \ldots p_{S,t})^T$ and I the identity matrix. Then equation .19 can be written as

$$(\rho I - A_t)V_t = P_t + \dot{V}_t \tag{.21}$$

With this notation we want to prove that

$$n_t^T \rho V_t = n_t^T P_t + \frac{d\left(n_t^T V_t\right)}{dt}$$

or equivalently

$$n_t^T \rho V_t = n_t^T P_t + n_t^T \dot{V}_t + \dot{n}_t^T V_t$$

Using equation (.21) and (.20), this can be written as

$$n_t^T \left(\rho V_t - (\rho I - A_t) V_t\right) - \left(A_t^T n_t\right)^T V_t = 0$$

which indeed holds. ■

Clearly, in model in the main text, $\frac{d\left(n_t^T V_t\right)}{dt} = 0$ in the steady-state. Hence we have $\rho W_t = \sum_{s=1}^{S} n_{s;t} p_{s;t}$ in the steady state.

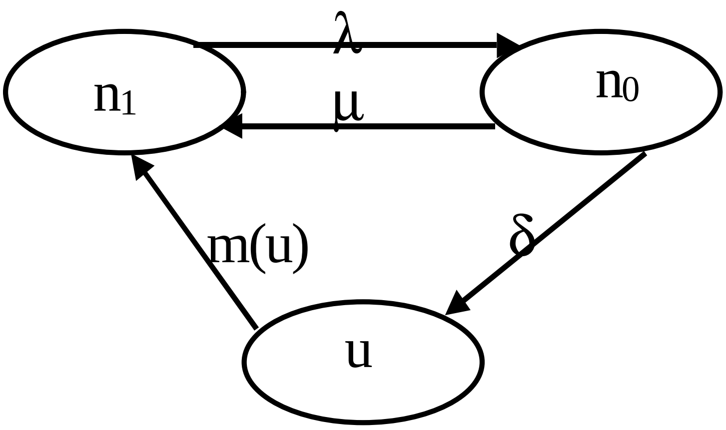

Figure 1

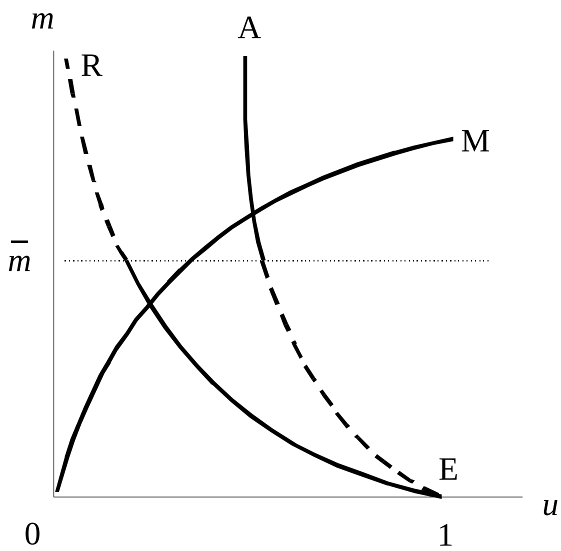

Figure 2