Why is it so Difficult to Find An Effect of Exchange Rate Risk on Trade?
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Why is it so Difficult to Find
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Abstract
The question whether exchange rate risk affects trade has received considerable attention in the literature. However, the conclusions are still mixed. This paper analyzes why it is so difficult to obtain a clear answer from time series analyses. We use data on bilateral aggregate US exports to the other G7 countries. The results show that export decisions are mostly affected by the exchange rate about one year later. The riskiness of the exchange rate at such a long horizon appears fairly constant over time with only short-term fluctuations. This makes it difficult to discover the true effect of exchange risk on trade from the limited time series data that are typically available.

Key words: Exports, risk measurement, imperfect substitutes, distributed lags
JEL classification: C22, C51, F14, F31.

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1 Introduction

The effect of exchange rate risk on international trade has attracted much research in international economics. This is not surprising, because the issue has important implications for the choice of an international monetary system. For instance, it is one of the main economic arguments for monetary unification in Europe, as it is generally believed that exchange rate risk hampers international trade (EU Commission (1990)). The standard argument for such a negative relation is that greater exchange risk increases the riskiness of trade profits, leading risk averse traders to reduce trade.

The voluminous theoretical and empirical literature on this topic, however, has resulted in ambiguous findings concerning the effect of exchange risk on trade.¹ We first empirically re-examine the effect of risk on trade for our data set, which concerns monthly bilateral aggregate US exports to the other G7 countries from 1978 to 1996. The paper pays special attention to several methodological issues. For instance, compared to existing studies, we reduce measurement error in the crucial exchange risk measure by using daily exchange rates to quantify multi-months-ahead real exchange risk. Moreover, to enhance the dynamic structure of our distributed lag model and to determine which exchange risk horizon is relevant for goods traders, we introduce a new parsimonious lag structure using the Poisson probability (mass) function to distribute the total effect of a regressor over time. Both methodological issues will be discussed in more detail later on in this introduction. Our results on the effect of real exchange risk on exports confirm the ambiguity found in the literature.

Next, we address the main focus of the paper, that is, we analyze why it is so difficult to find a clear effect. We concentrate on time series analyses, as they are used in the vast majority of existing empirical studies. The estimates show that export decisions are mostly affected by the exchange rate distribution about one year later. The riskiness of the exchange rate at such a long horizon appears fairly constant over time with only short-term fluctuations. This makes it so difficult to discover the true effect of risk on trade from the limited time series data that are typically available.

In general, there can be several reasons for the ambiguity found in the empirical literature on the effect of exchange rate risk on trade. Here, we discuss three of them (see Côté (1994) for additional reasons). First, the effect may indeed be absent, for instance, because firms can avoid all exchange risk by hedging. However, Wei (1999)

finds no support for the hedging argument. The absence of any effect would also be in contrast with the widespread view of a negative effect.

A second reason, stressed by Bini-Smaghi (1991), may be that the empirical tests are subject to methodological problems. One issue concerns the measurement of exchange rate risk or, in other words, exchange rate volatility or variability. Quite surprisingly, this question has received only moderate attention in the trade literature, despite the central role of this variable. Many authors use the moving standard deviation of the past, say, 24 monthly exchange rate changes for simplicity. Others use a generalized autoregressive conditional heteroskedasticity (GARCH) model, given the popularity of this model to capture the strong volatility clustering in high-frequency time series. We demonstrate that both measures have conflicting implications for the evolvement of risk over time. The moving standard deviation measure implies that exchange rate shocks persist in risk for a considerable period of time (24 months in our example), suggesting high serial correlation in risk. The GARCH measure, on the other hand, yields a low or even zero persistence of shocks in monthly risk, suggesting low or no serial correlation in risk. To solve this contradiction we use an alternative risk measure based on Merton (1980) and Andersen and Bollerslev (1998). Instead of taking monthly squared changes, we compute monthly exchange rate volatilities by cumulating squared daily changes in the month. Then we estimate an autoregressive model of order two on the monthly (estimated) volatilities, and we use the AR(2) forecasts to define multi-months-ahead exchange rate risk. We show that this measure describes the serial correlation in risk better than the two measures commonly used in the trade literature. Hence, our measure yields a reduction in measurement error for the crucial exchange risk variable, making the estimated effect of risk on exports more accurate.

Another methodological issue we address concerns the dynamic specification of the trade model. We employ a distributed lag model and introduce a new way to impose structure on the lag coefficients. Our method separates the total effect of a regressor from the distribution of the effect over time. The latter part appears to be a probability function, which can be freely chosen. For convenience, we use the Poisson probability function. This lag structure turns out to be more appropriate than the commonly used geometric and polynomial lags, because the Poisson lag structure can capture hump shaped lag patterns and it avoids sign-switching of the estimated lag coefficients. The

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3 See Bollerslev, Chou and Kroner (1992) for an overview of GARCH. GARCH risk measures are used in Pozo (1992), Kroner and Lastrapes (1993), Caporale and Doroodian (1994) and Qian and Varangis (1994), among others.
estimates for the Poisson parameter show that foreign income has the largest effect on domestic exports after about one quarter, while for the exchange rate this occurs only after about one year. Such time lags underscore the importance of allowing for dynamics in trade equations.

In summary, we take account of some important methodological issues that may explain the ambiguous results in existing trade studies. Nevertheless, we still find no clear effect of real exchange rate risk on trade. Hence, methodological problems are no sufficient explanation.

A third reason for the empirical ambiguity may come from the characteristics of exchange risk. Gagnon (1993) shows in a simulation experiment that the exchange risk level currently observed among industrial countries is too low to yield statistically detectable effects on trade. Our paper is complementary to Gagnon (1993) in the sense that we study the time-variation instead of the level of risk. We empirically demonstrate that the time-variation in risk at the long horizon relevant for goods traders is rather low and that deviations from average risk do not persist long. Therefore, even if risk affects exports, the effect captures only a minor part of the time-variation and the long-term swings in exports; other shocks to exports are likely to overshadow any risk effect. We conclude that the two characteristics of long-term real exchange rate risk just mentioned make it difficult to discover the true effect of risk on exports from the limited time series data that are typically available.

The paper is organized as follows. In section 2 we use an economic model to introduce the variables we think are important for the empirical work. Section 3 describes how we measure these variables. Given the importance of the exchange risk variable, we explain its measurement in detail in subsection 3.2. Section 4 presents the empirical model with special attention to the Poisson lag structure in subsection 4.2. In section 5 we report the empirical results and explain why we think it is so difficult to find the true effect of risk on trade from time series analyses. Section 6 concludes.

2 Economic Model

In this section we develop an economic model for the determination of exports. It provides a motivation for the choice of explanatory variables in the econometric model for US exports that will be used in this paper.

The economic model is based on the popular two-country imperfect substitutes model (see Goldstein and Khan (1985)), which considers domestic exports and goods produced abroad as imperfect substitutes. The extension we make to the standard imperfect substitutes model is that we explicitly account for the lag between the time
of the trade decision and the time of the actual trade flow and payment. This time lag is an important characteristic of international trade, as Goldstein and Khan (1985) and Sawyer and Sprinkle (1997) argue. Its existence implies that exchange rate risk may affect trade, as the exchange rate needed to convert foreign currency payments is unknown at the time of decision making.

Let $t$ denote the time (month) of observing a nominal export flow $X_t$, expressed in domestic currency, from the home to the foreign country. Exports are, supposedly, the result of a contract signed $l$ months earlier, stating both the export quantity $Q_{xt}$ and price $P_{xt}$. For simplicity, we assume that the price is specified in the home currency, so that $X_t = Q_{xt}P_{xt}$.\(^4\)

Our focus variable is (the logarithm of) the real value of exports, using the price $P_t$ of domestically produced goods as deflator:

$$x_t = q_{xt} + p_{xt},$$

where $x_t = \log(X_t/P_t)$, $q_{xt} = \log(Q_{xt})$ and $p_{xt} = \log(P_{xt}/P_t)$. We concentrate on the value $x_t$ rather than the quantity $q_{xt}$, as is often done in the literature, because we study bilateral exports for which $x_t$ is directly observable, while there are no observations on the bilateral prices needed to derive $q_{xt}$ from $x_t$.

The determinants of $x_t$ follow from the assumptions regarding export supply and demand. Supply is an unknown function $q^s_{xt}$ of only the price of exports relative to the price of domestic output in month $t$:\(^5\)

$$q^s_{xt} = q^s_x(P_{xt}).$$

To solve for the unobservable price $p_{xt}$, we have to specify foreign demand for domestic exports. Demand depends on two components. First, we suppose that it depends on real foreign income. Since the trade decision is made $l$ months before the actual trade flow in month $t$, we use (the logarithm of) lagged real foreign income $y^f_{t-l}$.

The second determinant of foreign demand is the price of traded goods relative to the price $P^f_t$ of foreign produced goods, both in foreign currency. Since the traded goods

\(^4\)The model can be extended to allow for invoicing in foreign currency as well. In that case, $X_t$ also depends on the contemporaneous nominal exchange rate, which converts the foreign currency invoiced part of exports into domestic currency. It can be shown that the collection of export determinants in the final model equation (5) should then be extended by the contemporaneous real exchange rate. We can avoid this extra complexity, because in the empirical part of the paper we study US exports and these are almost completely invoiced in US dollars (see Page (1981) for empirical evidence).

\(^5\)We take the price level $P_t$ of the month of the export flow, month $t$, to deflate the export price, because we assume that the exporter receives payment in the same month as the delivery of the goods. This assumption is quite reasonable, as Stokman (1995) reports that payments peak in the month of delivery.
are invoiced in domestic currency, this relative price equals $P_{xt}/S_{t}\cdot 1/P^*_t$, where $S_t$ is the nominal (spot) exchange rate, that is, the domestic currency price of one unit of foreign currency. In logarithms, the relative price equals $p_{xt} - s_t$, where $s_t = \log(S_tP^*_t/P_t)$ is the real exchange rate.

Although it is implicitly assumed that $P_t$ and hence $p_{xt}$ are perfectly forecastable at time $t-l$, such an assumption is not realistic for $s_t$, at least not for floating exchange rates. Hence, we account for the randomness of $s_t$ at the time $t-l$ the trade decision is made. As usual in the trade literature, we assume that the mean and standard deviation of $s_t$, conditional on information $I_{t-l}$ available at time $t-l$, are sufficient to capture the effects of exchange rates on export demand.\(^6\)

Combining the income and price components just discussed, we specify the demand for domestic exports as

$$q^d_{xt} = q^d_x \left( y^*_{t-l}, E_{t-l}\{p_{xt} - s_t\}, V^{1/2}_{t-l}\{p_{xt} - s_t\} \right),$$

where $E_{t-l}$ and $V^{1/2}_{t-l}$ denote the mean and standard deviation conditional on $I_{t-l}$.

The market for domestic exports is in equilibrium if

$$q_{xt} = q^d_{xt} = q^d_{xt}. \quad (4)$$

Solving (2)-(4) for $p_{xt}$ and $q_{xt}$ and substitution in (1) then yields

$$x_t = x \left( y^*_{t-l}, E_{t-l}\{s_t\}, V^{1/2}_{t-l}\{s_t\} \right). \quad (5)$$

Hence, the determinants of real (domestic output) exports are real foreign income (with an expected positive effect), the expected real exchange rate level (positive effect) and real exchange rate risk (unknown effect). The inclusion of income and the real exchange rate level is standard in trade models, in particular models that are also based on the imperfect substitutes model (see Goldstein and Khan (1985)). The extra real exchange risk term in (5) originates from the lag between the contract time $t-l$ and the time $t$ of delivery and payment and from the fact that foreign demand depends on the exchange rate, which is unknown at time $t-l$.

\section{Data Characteristics}

In this section we first describe the data we use to measure the variables in (5), as these are the variables that will appear in the econometric model later on. We then pay

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\(^6\)For simplicity, we abstract from the existence of a forward market to hedge exchange rate risk. Because the forward exchange rate is highly dependent on the mean and standard deviation of the future spot rate (see Viaene and De Vries (1992)), which we both take account of, the benefits from including the forward rate are likely small.
specific attention to the measurement of the conditional standard deviation $V_{t-l}^{1/2}\{s_t\}$. Finally, we study the stationarity of the variables.

### 3.1 Data

The data is monthly bilateral aggregate US exports to the six other G7 countries, namely Canada, France, Germany, Italy, Japan and the UK. We use bilateral instead of the often used multilateral data to avoid the difficult construction of multi-country explanatory variables. Moreover, by considering several export flows that are selected in a rather natural manner we hope to get robust results. The fact that we use aggregate instead of product-specific trade data is not important for the focus of the paper, as shown in subsection 5.2.

The export time series span January 1978 through November 1996, leading to 227 monthly observations. For the other variables we require longer series because of the lags in (5); they are available from April 1974 to November 1996.

The source for the data on the dollar value of exports is the US Bureau of the Census. To convert nominal exports into real (domestic output) exports $x_t$ we use the US wholesale price index from the OECD Main Economic Indicators. This is also the source for foreign industrial production, which is commonly used to proxy $y_t^*$, because real national income is only available at the quarterly frequency. The monthly nominal exchange rate is taken from IMF’s International Financial Statistics and the OECD wholesale price indices are used to convert it to the real exchange rate $s_t$ (except for the French real exchange rate, which is based on French and US consumer price indices, because French WPI is not available).

To obtain a measure for $E_{t-l}\{s_t\}$ we simply take the lagged rate $s_{t-l}$. Although there appears to be some predictability in real exchange rate changes in the long-run (see Mark and Choi (1997), among others), a random walk based forecasting rule is a good approximation, as Diebold and Nason (1990) show in a nonparametric analysis that it is difficult to improve on the random walk in point prediction.

Measuring exchange rate risk, $V_{t-l}^{1/2}\{s_t\}$, is less obvious. Given the importance of this variable, we discuss it extensively in the next subsection. Our preferred measure will appear an AR(2) based forecast using past monthly real exchange rate volatilities, where monthly volatility is defined as the square root of the sum of squared daily percentage changes in that month.\footnote{Although daily nominal exchange rates are observable (from Datastream), daily real exchange rates are not perfectly observable, because price ratios $P_t^*/P_t$ are only observed once a month. However, given the stability of the price ratios, we use good proxies of daily price ratios by linear interpolation of the monthly ratios.}
3.2 Real Exchange Rate Risk Measure

In this subsection we first discuss and compare two risk measures that are commonly used in the trade literature. Then we introduce an alternative measure, based on daily exchange rates, which provides a more appropriate description of risk. Two characteristics of this risk measure will play a crucial role in the derivation of the conclusion of the paper.

The measures used in the trade literature so far are typically one-period-ahead volatility measures, that is, \( V_{t-1}^{1/2} \{ s_t \} \) instead of \( V_{t-l}^{1/2} \{ s_t \} \) for some positive \( l \). Hence, in case of monthly data it is one-month-ahead risk and for quarterly data it is one-quarter-ahead risk that is allowed to affect trade flows. Although one should not a priori impose such a time lag, for ease of exposition we first discuss the various risk terms for one-period-ahead risk.

The first commonly used risk measure is the moving sample standard deviation of past percentage real exchange rate changes. The window width is prespecified and is usually about two years (for instance, Chowdury (1993) uses eight quarters). For illustrative purposes, let us therefore assume that the window width is 24 months, so that the moving standard deviation measure becomes

\[
V_{t-1}^{1/2} \{ s_t \} = \sqrt{\frac{1}{24} \sum_{m=1}^{24} [100(s_{t-m} - s_{t-m-1})]^2}, \tag{6}
\]

where it is implicitly assumed that the average real exchange rate change is zero. One can interpret measure (6) as first approximating volatility in month \( t \) by \([100(s_t - s_{t-1})]^2\) and then smoothing by taking the average over 24 months. Of course, taking a 24-months equally-weighted average is rather ad hoc, but usually the authors report that the results are not very sensitive to other weighting schemes (see Chowdury (1993), among others).

The main characteristic of the moving standard deviation measure (6) is that it implies a high (24 months) persistence of real exchange rate shocks and, therefore, considerable serial correlation in risk. This is illustrated by figures 1A and 2A, in which the thick lines plot measure (6) for the two most important trading partners of the US, namely Canada and Japan, respectively. Apart from the monthly shocks, there are some long swings in the risk measure, particularly for Japan. Later on in this subsection we will check whether the high autocorrelation is real or spuriously induced by definition (6).

The second measure of exchange rate risk that is commonly used in the trade literature is based on a GARCH model to smooth monthly volatilities \([100(s_t - s_{t-1})]^2\).
For instance, if one uses a GARCH(1,1) model, the risk measure is

\[ V_{t-1}^{1/2} \{ s_t \} = \sqrt{\omega_0 + \omega_1 [100(s_{t-1} - s_{t-2})]^2 + \omega_2 V_{t-2} \{ s_{t-1} \}}, \]

where we assume for the surprise term \([100(s_{t-1} - s_{t-2})]^2\) that the mean real exchange rate change is zero.

The main characteristic of measure (7) regarding our purpose of measuring volatility at the monthly frequency is illustrated by the thin lines in figures 1A and 2A. They show that there is low persistence of shocks in risk; for Canada the GARCH approach even results in constant risk. The reason for this becomes clear from table 1. The top half of that table presents the first-order autocorrelation, \(\rho_1\), and the Box-Pierce combination \(Q_{10}\) of the first ten autocorrelations of monthly volatility \(\sqrt{[100(s_t - s_{t-1})]^2}\).

It demonstrates that squared real exchange rate changes exhibit zero or small autocorrelation at the monthly frequency (we always use a significance level of 5%). This result is well-known from the GARCH literature (see Bollerslev, Chou and Kroner (1992)) and causes the low or zero autocorrelation in the monthly GARCH risk measures.

The low serial correlation in risk found by measure (7) is not consistent with the high correlation suggested above by the moving standard deviation measure (6). Hence, what is the true degree of serial correlation?

To analyze this question we start from an idea presented by Merton (1980) and formalized by Andersen and Bollerslev (1998). The latter authors argue that the ex-post squared change in a period is a very noisy indicator for the latent variance in that period. They propose to measure volatility by cumulating squared high-frequency changes in the period, so as to decrease measurement error. Under the reasonable assumption of no autocorrelation in the high-frequency changes, they argue that, as the observation frequency tends to infinity, the cumulative measure converges to the true volatility.

We use this idea to reduce the noise in the monthly volatilities \([100(s_t - s_{t-1})]^2\) used in (6) and (7). More specifically, we take the sum of squared daily percentage real exchange rate changes over all \(D_t\) days in month \(t\), \(\sqrt{\sum_{d=1}^{D_t} [100(s_d - s_{d-1})]^2}\), to measure monthly volatility (see also Merton (1980) on stock returns). As each monthly volatility is now based on about 21 daily volatilities, it is not surprising that this measure is more accurate than the monthly volatility measure based on a single monthly change.

We can now re-examine the serial correlation in monthly volatility with the new measure. The second half of table 1 shows that there is clear evidence of serial correlation. This indicates that our GARCH based claim of no or low autocorrelation is wrong, a result previously documented by Andersen, Bollerslev, Diebold and Lahys (1999).
To analyze whether the serial correlation in volatility is high, as the moving standard deviation measure (6) suggests, we estimate an autoregressive model for the monthly volatilities (based on daily data). As table 2 demonstrates, AR(2) models with moderate AR coefficients suffice to capture all serial correlation. Hence, the suggestion of high persistence of shocks from the moving standard deviation measure is not correct either. We conclude that there is significant autocorrelation in monthly volatilities, but that it dies out rather quickly.

Given the drawbacks of the moving standard deviation and GARCH measure for our purpose of studying the effect of exchange rate risk on trade, we propose an alternative risk measure. It is based on the AR(2) estimates just presented. More specifically, our measure is the AR(2) forecast based on past monthly volatilities obtained from daily data, that is,

$$V_{t-1}^{1/2} = \mu_v + \sum_{p=1}^{2} \alpha_p \left( \sum_{d=1}^{D_{t-p}} \frac{[100(s_d - s_{d-1})]^2}{2} - \mu_v \right),$$  \hspace{1cm} (8)

where $\mu_v$, $\alpha_1$ and $\alpha_2$ are substituted by the estimates presented in table 2. Because this measure takes account of the serial correlation in monthly volatilities in a better way than the two commonly used risk measures described above, thereby reducing measurement error for the important exchange risk variable, we use it in the remaining part of the paper.

An additional advantage of our measure is that multi-months-ahead risk, $V_{t-l}^{1/2}$, for some positive $l$, which is the measure we actually need in (5), is easy to compute. Assuming that monthly real exchange rate changes are uncorrelated, $V_{t-l}^{1/2}$ is the square root of $V_{t-l}^{1/2}$, $V_{t-l}^{1/2} = \mu_v + \sum_{p=1}^{2} \alpha_p \left( \sum_{d=1}^{D_{t-p}} \frac{[100(s_d - s_{d-1})]^2}{2} - \mu_v \right)$, where each term is a standard multi-period-ahead AR(2) forecast, which can be obtained in a recursive manner.

Two characteristics of (the multi-months-ahead version of) risk measure (8) will play a crucial role in subsection 5.2, where we derive the final conclusion of the paper. These characteristics concern the variation in risk over time and the duration of deviations from average risk. Figures 1B and 2B illustrate the risk measure for Canada and Japan, respectively, for both $l = 1$ and $l = 12$. They show that real exchange rate risk is time-varying, but that shocks do not persist very long in risk. Moreover, particularly for $V_{t-12}$, the time-variation in risk is small relative to the risk level. This conclusion is supported by table 3, as the standard deviation of risk is on average only 5% of the mean.
3.3 Non-Stationarity and Cointegration

To specify a time series model for exports in section 4, we first have to investigate the stationarity of the four variables in economic model (5). It is common to assume that two of these, real exports \( x_t \) and foreign industrial production \( y_t^* \), have a unit root. In contrast, measure (8) for exchange rate risk \( V_{t-i}^{1/2} \{ s_t \} \) is stationary, as the AR(2) estimates in table 2 are positive and their sum is well below unity (see Hamilton (1994, p. 57)). Stationarity is confirmed by plots of the risk measure; see figures 1B and 2B for Canada and Japan, respectively. Finally, we assume that the expected real exchange rate, \( E_{t-i} \{ s_t \} = s_{t-i} \), is stationary. This is based on the recent literature on purchasing power parity (PPP), which provides more and more evidence of long-run relative PPP, in other words, of stationarity of the real exchange rate (for instance, see Abuaf and Jorion (1990), Koedijk, Schotman and Van Dijk (1998) and Klaassen (1999)).

Next, we check for cointegration between the two unit root variables \( x_t \) (exports) and \( y_t^* \) (foreign industrial production). From an economic point of view one expects that they are cointegrated. This is confirmed by the empirical results in Sawyer and Sprinkle (1997), among others. But obtaining statistical evidence for our data is not so obvious, as augmented Dickey-Fuller unit root tests (not reported) on the residuals from a regression of \( x_t \) on \( y_t^* \) do not show evidence of cointegration.

The insignificant Dickey-Fuller test results, however, do not imply the absence of cointegration, as it is well-known that standard unit root tests may have problems with power. To examine this, we inspect the residual plots concerning the regression of \( x_t \) on \( y_t^* \). They demonstrate that there is no trend in the residuals and that the residuals exhibit long swings. For instance, for all six flows the residuals swing downwards for some years before 1986 and follow an upward swing in the years after that. These long swings, taking several years, in combination with the moderate length of our export series (19 years) may well be the reason for the insignificant Dickey-Fuller tests. After all, the swings in the residual series have a similar shape as those in the real exchange rates (which are likely to be the cause of the residual swings), and from the PPP literature we know that standard unit root tests have great difficulties in finding stationarity from short stationary series exhibiting long swings (slow mean reversion).

Although economic intuition argues for cointegration, we still have no conclusive statistical evidence. Obtaining such evidence requires a much more detailed cointegration analysis, which goes beyond the scope of this paper. Instead, we follow an indirect approach. First, we simply assume cointegration and specify the econometric model.

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8 If one is not willing to assume stationarity of the real exchange rate, the main conclusion of the paper, which concerns the stationary risk measure, is still valid; this follows from subsection 5.1.
Afterwards, having estimated the model, we examine the residuals of that model. We will show in subsection 5.1 that they are stationary, so that, given the stationarity of $E_{t-l}\{s_t\}$ and $V_{t-l}^{1/2}\{s_t\}$, it is very likely that $x_t$ and $y_t^*$ are cointegrated, as economic intuition suggests.

4 Econometric Model

In this section we develop the econometric model to be estimated later on. Its main elements are the export equation, described in subsection 4.1, and the restrictions placed on its dynamic structure, discussed in 4.2.

4.1 Export Equation

To specify an econometric equation for real US exports we use the variables that appear in economic model (5). That model takes explicit account of the important dynamic nature of international trade by specifying the determinants of exports in month $t$ when the export contract was signed $l$ months before. However, the data on US exports are aggregated across all products and it is likely that for different products the lags are different. To account for this, we use a distributed lag model, where the effect of a change in a regressor is allowed to be distributed over time.

Given the assumed cointegration between real exports $x_t$ and foreign industrial production $y_t^*$, the stationarity of $E_{t-l}\{s_t\}$ and $V_{t-l}^{1/2}\{s_t\}$, and assuming linearity, we specify real exports as

$$x_t = \beta_0 + \sum_{l=1}^{\infty} \left( \beta_{yl}y_{t-l} + \beta_{El}E_{t-l}\{s_t\} + \beta_{Vl}V_{t-l}^{1/2}\{s_t\} \right) + \varepsilon_t,$$

(9)

where the disturbance term $\varepsilon_t$ is allowed to follow an AR(2) process with autoregressive coefficients $\theta_1$ and $\theta_2$ and with conditionally normal innovations having variance $\sigma^2$. Although $x_t$ concerns bilateral exports, we suppress the index indicating the partner country for notational simplicity. We also do not explicitly write down the eleven monthly dummies that we include to correct for seasonal effects.

Of course, unrestricted estimation of (9) is not feasible because of the infinite number of parameters. In the next subsection we introduce the restrictions on $\beta_{yl}$, $\beta_{El}$ and $\beta_{Vl}$ that complete the econometric model.

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9 For Canada we allow for a break in $\beta_y$ from 1991 onwards to account for the increase in trade openness due to the Free Trade Agreement between the US and Canada. Moreover, we use an AR(5) instead of AR(2) process to capture all autocorrelation in the disturbance term.
4.2 Poisson Lag Structure

Careful investigation of the lag structure is important for dynamic trade equations such as (9). This subsection pays special attention to the lags. We first discuss two popular lag structures. After that, we introduce an alternative structure based on the Poisson probability (mass) function, which we argue is more appropriate. Moreover, the Poisson lag structure allows us to let the data reveal the exchange risk horizon that is relevant for goods traders, which is an important element in the derivation of the main conclusion of the paper in subsection 5.2.

In the literature there exist several ways of restricting the infinite number of coefficients \( \beta_1, \beta_2, \ldots \) in (9) to obtain a parsimonious model. For instance, one can use a geometric lag specification, that is, \( \beta_l = \beta \cdot w_l \), where \( w_l = \gamma \cdot (1 - \gamma)^{l-1} \) is the geometric probability function translated one unit to the right (0 < \( \gamma < 1 \)). It implies that the \( \beta_l \) are decreasing over \( l \). This may be appropriate for the income effects \( \beta_{yl} \), as there appears to be some agreement in the literature that income effects are large for small lags and decline rapidly thereafter (see Goldstein and Khan (1985)). However, according to Goldstein and Khan there is much less of a consensus on the lag pattern for the expected exchange rate effects \( \beta_{El} \); that may well be hump shaped, as Sawyer and Sprinkle (1997) claim. Hence, it is not appropriate to impose a geometric lag specification a priori.

A second example of a popular lag structure is the polynomial or Almon lag specification. It assumes that the \( \beta_l \) fall on a polynomial of a prespecified order. Such a specification is more flexible with respect to the dynamics of \( \beta_l \) than the geometric model, as it allows for both a declining and a hump shaped lag pattern. However, it may well occur that the polynomial structure forces some \( \beta_l \) to be positive and others to be negative. This is difficult to justify theoretically (see Goldstein and Khan (1985)).

Given the importance of a satisfactory lag structure, we introduce an alternative approach to avoid the problems just described. Let us suppose that all \( \beta_l \) have the same sign. Then, one can write \( \beta_l = \beta \cdot w_l \), where \( w_l \geq 0 \) and \( \sum_{l=1}^{\infty} w_l = 1 \). Hence, \( \beta \) gives the total, long run effect of the regressor. The \( w_l \), on the other hand, describe how the total effect is distributed over time; by definition, they form a probability function with support \( \{1, 2, \ldots \} \).

Besides the convenient interpretation of \( \beta \) and the \( w_l \), the main attractive feature of our class of probability function based lag specifications is its flexibility. One can choose any probability function for the \( w_l \), depending on the specific needs. For instance, the approach encompasses the geometric lag specification as the special case where the \( w_l \) are defined by a translated geometric probability function (see above). It can also
capture, for instance, hump shaped or bimodal lag patterns.

Within the class of lag specifications just described, we take “Poisson lags” for our export model (9). That is,

$$\beta_{il} = \beta_i \cdot \frac{(\lambda_i - 1)^{l-1}}{(l-1)!} \exp[-(\lambda_i - 1)], \quad \text{for } \lambda_i \geq 1 \text{ and } i = y, E, V. \quad (10)$$

Note that we have to translate the Poisson probability function one unit to the right, because $l$ starts at one instead of zero. The parameter $\lambda$ is close to the mode of the translated Poisson distribution.\(^{10}\) Hence, we give $\lambda$ the convenient interpretation of the lag at which the maximal effect occurs, that is, the lag with the largest coefficient $\beta_i$. Because $\lambda_E$ and $\lambda_V$ both concern the exchange rate distribution (mean and variance) and to avoid identification problems if $\beta_E$ or $\beta_V$ is zero, we impose that $\lambda_E$ and $\lambda_V$ are equal to, say, $\lambda_{EV}$ (this restriction will be tested in subsection 5.1). We allow $\lambda_y$ and $\lambda_{EV}$ to be different.

The Poisson lags (10) can capture a declining lag structure as well as a hump shaped one, while nevertheless being very parsimonious. Moreover, by definition all $\beta_i$ have the same sign. Hence, Poisson lags avoid the disadvantages concerning geometric lags and polynomial lags discussed above. We can let the data decide whether a declining or hump shaped lag structure is more appropriate and how long it takes for industrial production and exchange rates to have the strongest effect on exports, an issue that is also unresolved in the literature (see Sawyer and Sprinkle (1997)). Figure 3 illustrates the Poisson lags for $\lambda = 3.38$ and $\lambda = 12.85$ (with $\beta = 2.23$ and $\beta = 0.62$, respectively; the numbers are based on the estimation results to be discussed below).

This completes the description of the econometric model for the determination of exports. It is given by (9) and (10).

5 Empirical Results

In this section we first report the estimates of the parameters in the model just developed. As in the existing literature, we find an ambiguous effect of exchange rate risk on exports. In subsection 5.2 we provide an explanation for that.

\(^{10}\)The exact mode of the translated Poisson distribution with parameter $\lambda$ is the largest integer $l$ less than $\lambda$; if $\lambda$ itself is an integer, then $l = \lambda - 1$ and $l = \lambda$ are tie modes.
5.1 Estimation Results

We estimate the econometric model of section 4 with maximum likelihood (ML) on each of the six US export flows separately.\textsuperscript{11} Table 4 present the results.

The focus parameter of this paper is $\beta_V$, the total impact of real exchange rate risk on exports. We find that the estimate of $\beta_V$ is significantly positive for Canada, significantly negative for Italy and insignificant for the other four countries. Hence, as in the existing literature, we find no clear effect of risk on exports.

Table 4 also demonstrates that foreign industrial production has the expected positive effect on the real (domestic output) value of US exports. This holds for all six series. The average estimate of $\bar{\beta}_y$ is 2.23.\textsuperscript{12}

An attractive implication of the Poisson lag specification (10) is that we can directly estimate the time lag $\lambda_y$ between a change in industrial production and the maximal change in exports. Table 4 shows that the maximal effect occurs after about one quarter (the average estimate of $\lambda_y$ is 3.38, ignoring the outlying estimate for the UK). This conclusion is robust to the use of another lag specification, as a preliminary analysis with polynomial lag structures of various degrees points in the same direction. Hence, the effect of foreign income on US exports goes quite rapidly; this corroborates Goldstein and Khan (1985) and Sawyer and Sprinkle (1997). The dots in figure 3 illustrate the implication of the average $\lambda_y$ for the distribution of the average $\bar{\beta}_y$ over the lags.

The remaining regressor is the expected real exchange rate. As table 4 demonstrates, all six estimates for $\beta_E$ are significantly positive. This is not surprising, as a US dollar depreciation generally lowers the foreign currency price of (dollar denominated) US exports, thereby increasing the quantity and dollar value of exports. The average estimate of $\bar{\beta}_E$ is 0.62. It is remarkable that the values of our estimates are so consistent across countries given the wide range of estimated export price elasticities in the literature, as analyzed by Marquez (1999). This consistency is a sign of robustness of our model.

From the Poisson lag structure we find that the maximal effect of the exchange rate distribution occurs after about one year (the average $\lambda_{EV}$ is 12.85).\textsuperscript{13} This conclusion

\textsuperscript{11} Multivariate ML is theoretically possible. However, the cross-sectional correlation in the univariate residuals turns out to be low (the average absolute correlation between the residuals of two equations is only 0.12, and the maximum is 0.25), so that the efficiency gains from multivariate estimation are likely to be small. Moreover, multivariate estimation involves more than one hundred parameters, so that there is a serious danger of ending up in a local maximum of the likelihood function.

\textsuperscript{12} The estimates for $\bar{\beta}_y$ are not directly comparable with the income elasticities of US exports that are typically reported in the literature, since the endogenous variable in (4.1) is the value of exports, not the quantity, and because the explanatory variable is industrial production, not real national income.

\textsuperscript{13} Recall that $\lambda_{EV}$ determines the lag distribution of both $\beta_E$ and the risk coefficient $\beta_V$ (see as-
is again supported by a preliminary analysis with polynomial lags of various orders. Therefore, the short-run effect of changes in the exchange rate distribution on exports is small, while in the longer run there is a clear effect. This supports the view of a hump shaped instead of a declining lag pattern and hence helps solve the question on the true lag pattern for exchange rates (Goldstein and Khan (1985)). The stars in figure 3 illustrate the distribution of the average $\beta_E$ over the lags as implied by the average $\lambda_{EV}$.

The final estimation results presented in table 4 concern the autoregressive parameters of the AR process for the error term $\varepsilon_t$ in (9). The moderate values for the estimates of $\theta_1$ and $\theta_2$ show that the systematic part of export equation (9) describes the dynamics of exports quite well. Moreover, the fact that the estimates of $\theta_1$ and $\theta_2$ are positive and that their sum is well below unity ensures that the estimated AR process is stationary (see Hamilton (1994, p. 57)). Stationarity is also confirmed by the residual plots (not in the paper). This supports our assumption of cointegration between the trending variables exports and industrial production, as made in subsection 3.3.

Table 4 also reports some diagnostic statistics. There is no sign of remaining autocorrelation or conditional heteroskedasticity in the residuals, so that we have no reason to extend the model.

### 5.2 Why is the Effect of Exchange Risk on Exports Ambiguous?

As just discussed, we find no clear evidence of an effect of real exchange rate risk on the real (domestic output) value of exports. In this subsection we try to explain this.

We distinguish two points of view. First, it may be that there is no effect of risk on trade; this would imply that the common idea of a negative effect is wrong. Second, there is an effect, but it is overshadowed by the variation in the unsystematic part of the model in such a way that one cannot discover the true effect of risk on trade from the limited time series that are typically available.

In the literature there is a tendency towards the first point of view, because the many studies on this issue have not yet come to a conclusive answer. We, however, argue that the second point may be more relevant.

This claim is based on the estimated Poisson parameter $\lambda_{EV}$ (the lag with the maximal exchange rate effect on exports) and on the two main characteristics of real

---

sumption $\lambda_E = \lambda_V = \lambda_{EV}$ below (10)). To test the restrictiveness of that assumption we perform a likelihood ratio test. The likelihood ratios [p-value] are 0.91 [0.34] for Canada, 3.80 [0.05] for France, 0.36 [0.55] for Germany, 0.01 [0.92] for Italy, 3.62 [0.06] for Japan, and 0.55 [0.46] for the UK. Hence, we do not reject the restriction.
exchange rate risk as discussed at the end of subsection 3.2. From the estimated $\lambda_{EV}$ we concluded that the maximal effect of exchange rates on trade occurs after about one year. We have seen that, at such a long horizon, the variation of exchange risk over time is rather small (see table 3 and figures 1B and 2B, particularly the one year ahead risk measure). Moreover, the second characteristic of risk discussed in subsection 3.2 shows that deviations from average risk are short-lived, since AR(2) processes with moderate autoregressive parameters are already sufficient to capture the autocorrelation in risk (see table 2 and figures 1B and 2B).

These three properties imply that, even if risk affects exports, the effect explains only little of the variation and the long-term movements in exports over time; other shocks to exports are likely to dominate and overshadow such an effect. Loosely speaking, risk is too constant to identify its effect on exports from time series analysis. We conclude it is unlikely that one will discover the true effect of risk on exports from the limited time series data that are typically available, no matter whether the true effect is zero or not.

6 Conclusion

This paper presents an empirical study on monthly bilateral aggregate US exports to the other G7 countries from 1978 to 1996. To motivate the choice of variables in the econometric model we develop an economic model, where we explicitly account for the time lag between the export decision and the actual trade flow and payment. The model implies that not only foreign income and the expected future real exchange rate are important, but also that real exchange rate risk may be relevant for exports. This latter effect is the main focus of the paper.

From a methodological point of view, the paper yields two contributions to the trade literature. First, we improve on currently used risk measures by using daily exchange rates to construct multi-months-ahead risk. This reduces measurement error and makes the estimated effect of risk on exports more accurate. In addition, we pay special attention to the dynamic structure of the model by introducing a convenient Poisson lag structure for the distributed lag model.

The empirical results demonstrate that, as expected, foreign income affects US exports positively and rather quickly, since the maximal effect in the Poisson lag structure occurs after about one quarter. Exports react much slower to changes in the real exchange rate distribution, as the maximal effect happens only after about one year. The expected real exchange rate level has the normal positive effect, but real exchange rate risk has no clear effect.
To explain this latter, commonly reported finding, we examine the long-term (about one year) risk that is relevant for goods traders in more detail. Such long-term risk appears rather constant over time with only short-term deviations from average risk. In our opinion, this is the reason why it is so difficult to find an effect of exchange rate risk on trade from time series data.

It is important to realize that our conclusion concerns countries with low time-variation in long-term real exchange rate risk, such as most developed countries over the post Bretton Woods period. It would be interesting to study the effect of risk on trade flows between countries with more time-variation in risk, for instance, developing countries. In addition, employing cross-sectional variation in exchange risk may be fruitful. Such panel or pure cross-sectional studies may benefit from the few cross-sectional papers that already exist and that tend to be more supportive for a negative effect of exchange risk on trade (see Côté (1994)). This is left for future research.
References


Table 1: Autocorrelation in monthly real exchange rate volatility

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<tr>
<th></th>
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<th>Ger</th>
<th>Ita</th>
<th>Jap</th>
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<td>Using monthly data i.e.</td>
<td>$\rho_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\sqrt{\frac{100 \cdot (s_t - s_{t-1})^2}{100}}$</td>
<td>0.06</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.21*</td>
<td>0.12*</td>
<td>0.15*</td>
</tr>
<tr>
<td>$Q_{10}$</td>
<td>15.07</td>
<td>10.68</td>
<td>14.73</td>
<td>52.82*</td>
<td>17.11</td>
<td>21.42*</td>
</tr>
<tr>
<td>Using daily data i.e.</td>
<td>$\rho_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{\frac{\sum_{d=1}^{T}100(s_d - s_{d-1})^2}{100}}$</td>
<td>0.27*</td>
<td>0.48*</td>
<td>0.48*</td>
<td>0.50*</td>
<td>0.43*</td>
<td>0.55*</td>
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<tr>
<td>$Q_{10}$</td>
<td>48.14*</td>
<td>255.19*</td>
<td>215.76*</td>
<td>243.81*</td>
<td>253.43*</td>
<td>320.22*</td>
</tr>
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</table>

Standard errors in parentheses and p-values in square brackets; * is significant at 5% level. The symbol $\rho_1$ denotes the first-order autocorrelation and $Q_{10}$ is the Box-Pierce statistic of order 10.

Table 2: AR(2) estimation results for monthly real exchange rate volatility

<table>
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<td>2.71</td>
<td>2.85</td>
<td>2.68</td>
<td>2.72</td>
<td>2.68</td>
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<tr>
<td></td>
<td>0.23*</td>
<td>0.41*</td>
<td>0.44*</td>
<td>0.41*</td>
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<td>0.43*</td>
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<tr>
<td></td>
<td>(0.06)</td>
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<td>(0.06)</td>
<td>(0.06)</td>
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<tr>
<td></td>
<td>$\alpha_2$</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>0.14*</td>
<td>0.16*</td>
<td>0.10</td>
<td>0.18*</td>
<td>0.18*</td>
<td>0.23*</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
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<td>Autocorrelation</td>
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<tr>
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<tr>
<td>$Q_{10}$</td>
<td>9.34</td>
<td>15.89</td>
<td>10.71</td>
<td>14.22</td>
<td>15.34</td>
<td>3.44</td>
</tr>
<tr>
<td></td>
<td>[0.50]</td>
<td>[0.10]</td>
<td>[0.38]</td>
<td>[0.16]</td>
<td>[0.12]</td>
<td>[0.97]</td>
</tr>
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Standard errors in parentheses and p-values in square brackets; * is significant at 5% level. Definitions of $\rho_1$ and $Q_{10}$: see notes of table 1.
Table 3: Descriptive statistics of real exchange rate risk measure

<table>
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<th>US dollar real exchange rate versus currency of</th>
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<tr>
<td>$V_{t-1}^{1/2}{s_t}$ mean</td>
<td>1.35</td>
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<tr>
<td></td>
<td>coeff. of variation</td>
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<tr>
<td>$V_{t-12}^{1/2}{s_t}$ mean</td>
<td>4.61</td>
</tr>
<tr>
<td></td>
<td>coeff. of variation</td>
</tr>
</tbody>
</table>

The risk measure $V_{t-l}^{1/2}\{s_t\}$ ($l = 1, 12$) is the $l$-months-ahead forecast based on the AR(2) process that has been estimated for the monthly real exchange rate volatilities from daily data. See the discussion below (8) for an exact description.
Table 4: Estimation results for export equations

<table>
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<th></th>
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<th>US exports to</th>
<th>US exports to</th>
<th>US exports to</th>
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<tr>
<td>Constant</td>
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<td>6.54*</td>
<td>5.43*</td>
<td>1.71*</td>
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<td></td>
<td>(1.55)</td>
<td>(0.82)</td>
<td>(0.42)</td>
<td>(0.67)</td>
<td>(0.41)</td>
<td>(0.49)</td>
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<td>Foreign industr. prod.</td>
<td>1.80*</td>
<td>4.18*</td>
<td>2.25*</td>
<td>1.38*</td>
<td>1.28*</td>
<td>2.49*</td>
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<tr>
<td></td>
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<td>(0.04)</td>
<td>(0.17)</td>
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<td>Lag of max. effect</td>
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<td>5.63</td>
<td>2.87</td>
<td>2.51</td>
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<td>(1.54)</td>
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<td>Expected exch. rate</td>
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<td>0.50*</td>
<td>0.65*</td>
<td>0.52*</td>
<td>0.95*</td>
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<td></td>
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<td>Lag of max. effect</td>
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<td>10.34</td>
<td>9.91</td>
<td>8.50</td>
<td>12.97</td>
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<td>(1.19)</td>
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<td>(0.07)</td>
<td>(0.07)</td>
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<td>0.30*</td>
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<td>Error variance</td>
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Residual diagnostics

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<td>$Q_{10}$</td>
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<td>$Q_{10}^*$</td>
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<tr>
<td></td>
<td>[0.12]</td>
<td>[0.82]</td>
<td>[0.88]</td>
<td>[0.09]</td>
<td>[0.06]</td>
<td>[0.67]</td>
</tr>
</tbody>
</table>

Standard errors in parentheses and p-values in square brackets; * is significantly different from zero at 5% level.

The estimated equation is (9) with the Poisson lag restriction (10); we do not report the estimates for the monthly seasonality dummies.

The significance of the estimates for $\beta_y$ is based on the cointegration between $x_t$ and $y_t^c$. The significance of the estimates for $\beta_E$ is based on t-ratios. Because of the slow mean reversion in real exchange rates, the asymptotic 5% critical value of about two is possibly different from the critical value relevant for our finite sample. Nevertheless, we consider the t-ratios to be sufficiently large to conclude that the estimates are significant.

For exports to Canada we have allowed for a break in $\beta_y$ from 1991 onwards to account for the increase in trade openness due to the Free Trade Agreement between the US and Canada; the estimated increase in $\beta_y$ is 0.04* (0.005). Moreover, we have estimated an AR(5) instead of AR(2) process to capture all autocorrelation in the error term; the three extra AR parameter estimates are 0.26* (0.08), -0.14* (0.07) and 0.17* (0.07).

Definitions of $\rho_1$ and $Q_{10}$: see notes of table 1; $\rho_1^*$ and $Q_{10}^*$ are similarly defined, except that they concern the squared residuals.
A: Common risk measures: using monthly data up to one month ago

- Moving standard deviation of monthly changes over 24 months
- GARCH on monthly changes

B: Alternative risk measure: AR(2) based on sum of squared daily changes in a month

- Using daily data up to one month ago
- Using daily data up to one year ago

Figure 1: Risk measures for Canadian dollar real exchange rate
A: Common risk measures: using monthly data up to one month ago

- Moving standard deviation of monthly changes over 24 months
- GARCH on monthly changes

B: Alternative risk measure: AR(2) based on sum of squared daily changes in a month

- Using daily data up to one month ago
- Using daily data up to one year ago

Figure 2: Risk measures for Japanese yen real exchange rate
Figure 3: Distribution of total effect $\beta$ of regressors on exports over time according to a Poisson($\lambda$) lag structure.