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Klaassen, F.J.G.M.

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Long Swings in Exchange Rates: Are They Really in the Data?

by Franc Klaassen *

CentER and Department of Economics

Tilburg University

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Abstract

The random walk is often used to model exchange rates. According to the Lucas critique, however, policy shifts may lead to breaks in the trend of exchange rates and hence to long swings. We use a Markov regime-switching model to allow for such swings and we reject the random walk in favor of the regime-switching model. Earlier papers report this result too, but the authors are concerned about the reliability of their Wald based tests in the strongly nonlinear regime-switching model. We show that these tests are indeed not very robust. Hence, we use a likelihood ratio test for which the (non-standard) critical values have been computed recently.

Key words: Markov regime-switching, testing, forecasting, exchange rates.

JEL classification: F31, C52, C53.

*Correspondence to: Franc Klaassen, Department of Economics, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, the Netherlands; tel: +31-13-4668229; fax: +31-13-4663280; E-mail: F.J.G.M.Klaassen@kub.nl.

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1 Introduction

Modeling exchange rates has been a main endeavor for economists. Since the work of Meese and Rogoff (1983), many researchers have used the random walk model. The empirical quality of this model has also been stressed by Diebold and Nason (1990), who find in a nonparametric analysis that it is difficult to improve on the random walk in point prediction.

The random walk, however, is unsatisfactory from an economic point of view. It ignores any effect of observed changes in economic policy, and, according to the Lucas (1976) critique, such policy shifts may well affect the exchange rate generating process. For instance, regarding monetary policy, Kaminsky (1993) shows theoretically that a change from a contractionary to an expansionary monetary policy increases the exchange rate depreciation. Moreover, the relevance of international policy coordination appears from the 1985 Plaza agreement, in which the G-5 countries announced to bring about a U.S. dollar depreciation after the sharp dollar appreciation during the five years before; the dollar indeed depreciated strongly from 1985 to 1987. Both examples show that policy shifts can lead to changes in the trend of exchange rates and thus to long swings.

The idea of long swings is further supported by time plots of exchange rates. Figures 1A, 2A and 3A plot the dollar price of one German mark, Japanese yen and U.K. pound, respectively, from April 1974 to July 1997. Exchange rates indeed seem to be characterized by long swings.

In this paper we formally examine whether long swings exist. We test the random walk against the long swings model and find that long swings are indeed a systematic part of the exchange rate generating process. Engel and Hamilton (1990), among others, report this conclusion too, but the authors are concerned about the reliability of their Wald tests in the strongly nonlinear regime-switching model. We show that the Wald test is indeed not very robust in regime-switching models. This problem does not apply to our test approach, as we use a likelihood ratio test for which the (non-standard) critical values have been computed recently by Garcia (1995). Hence, we can conclude that there is really evidence of long swings in exchange rates.

To formalize the concept of long swings, we use the Markov regime-switching model introduced in the seminal paper of Hamilton (1989). According to the basic regime-switching model, the expected exchange rate change is one of two constants depending on the regime the process is in. Persistence of such "mean regimes" then leads to the long swings.

In the literature so far, regime-switching models have been used in various ways.

Hamilton (1989), Goodwin (1993), Durland and McCurdy (1994), Filardo (1994) and Ghysels (1994) successfully use regimes to capture recessions and expansions in the U.S. business cycle. In contrast to these papers, which concentrate on the mean of a series, regime-switching models can also be useful to describe the variance. Persistence of regimes with different unconditional variances can explain part of the conditional heteroskedasticity which we often find in high-frequency data. Cai (1994), Hamilton and Susmel (1994), Gray (1996a) and Klaassen (1998) use such "variance regimes" to model the variance of changes in interest rates, stock indices, interest rates and exchange rates, respectively.

Most related to our paper are Engel and Hamilton (1990), Kaminsky (1993), Engel (1994) and Dewachter (1997), since they also use regimes to capture long swings in exchange rates. Engel and Hamilton (1990) and Engel (1994) have quarterly data of several major exchange rates, Kaminsky (1993) uses monthly data of the dollar-pound rate, while Dewachter (1997) has weekly data of the dollar versus three European currencies. In all four papers, the authors argue that long swings exist. However, Kaminsky (1993) does not formally test the null hypothesis of a random walk against the regime-switching alternative, while Engel and Hamilton (1990) admit that there is some concern with their Wald test statistics, which are also used in Engel (1994) and Dewachter (1997).

The test problems originate from identification problems under the null of interest, the random walk. Under this null, only one regime governs the exchange rate, so that the parameters for the second regime are not identified. This makes the asymptotic distribution of the usual tests (likelihood ratio, Wald and Lagrange multiplier) no longer χ^2, as Hansen (1992) shows.

Engel and Hamilton (1990), Engel (1994) and Dewachter (1997) circumvent this problem by taking the slightly more general null that the current regime is independent of the previous one. This hypothesis implies that there are no long swings, as in the random walk. Under the more general null, however, all parameters are identified, and the authors use Wald statistics to test it. Gallant (1987), however, argues that Wald statistics are less robust than, for instance, likelihood ratios in nonlinear models such as regime-switching models. This is clearly illustrated by our computations for the weekly dollar-mark exchange rates: the likelihood ratio for the general null is 9, while the Wald test is extremely high, namely 3,866. Hence, it is useful to test for long swings with another statistic than the Wald test.

Recently, Garcia (1995) has solved the identification problem mentioned above by deriving the correct asymptotic distribution of the likelihood ratio statistic. Hence, we are able to test the exact null of interest without much difficulty. Remarkably, we find no significant evidence of long swings using similar quarterly and monthly data as in Engel and Hamilton (1990), Kaminsky (1993) and Engel (1994). This may be caused by the data frequency: even if swings exist and last for some quarters, sampling at the quarterly or monthly frequency may result in too few observations per swing to distinguish the swings from a random walk. Therefore, we use weekly data to enhance the power of the test. Our empirical results now reject the random walk. Hence, the data suggest that long swings are really a systematic part of the exchange rate generating process.

In the next section, we formally describe the regime-switching model. In section 3 we describe the data and the empirical results. There, we actually test for the existence of long swings. Section 4 concludes.

2 Regime-Switching Model

To be able to test for the existence of long swings, we need a model that allows for such swings. In this section we develop that model. It is an extended version of the Engel and Hamilton (1990) regime-switching model, as we explicitly take account of the conditional heteroskedasticity that is present in our weekly data.

We need the following notation. Let S_t denote the logarithm of the spot exchange rate at time t, that is, the domestic currency price of one unit of foreign currency. We concentrate on the exchange rate change $s_t = 100(S_t - S_{t-1})$, so that s_t is the percentage depreciation of the domestic currency from time $t-1$ to t.

The regime-switching model consists of four elements, namely the regime process, mean, variance and distribution. We now discuss these elements subsequently, and we relate our specification to the one used by Engel and Hamilton (1990).

The regime process we use is the same as in Engel and Hamilton (1990). It is based on two (unobservable) regimes. Let $r_t \in \{1, 2\}$ denote the regime at time t. Within this regime, the mean exchange rate change is μ_{r_t}, which we assume to be constant over time. Across regimes, however, the means are allowed to differ, and we identify the first regime as the low mean regime: $\mu_1 \leq \mu_2$. This provides the basis for the swings. After all, being in the first and then in the second regime for a while leads to a period of appreciation followed by depreciation, that is, to swings in the exchange rate. Note, however, that we do not impose this kind of exchange rate behavior; we do allow for $\mu_1 = \mu_2$, so that exchange rates can have a constant mean.

Whether swings are long or not depends on the regime staying probabilities. Let $p_{t-1}(r_t|\widetilde{r}_{t-1}) = p(r_t|I_{t-1}, \widetilde{r}_{t-1})$ denote the probability of going to regime r_t at time t

conditional on the information set of the data generating process, which consists of two parts. The first part, $I_{t-1} = (s_{t-1}, s_{t-2}, \dots)$, denotes the information that is observed by the econometrician; the second part, $\widetilde{r}_{t-1}$, is the regime path $(r_{t-1}, r_{t-2}, \dots)$, which is not observed by the econometrician. Note that we use the subscript $t-1$ below an operator (probability, expectation or variance) as short-hand notation for conditioning on I_{t-1}.

As in Engel and Hamilton (1990), we assume that r_t follows a first-order Markov process with constant staying probabilities, so that

$$p_{t-1}(r_t \,|\, \widetilde{r}_{t-1}) = p(r_t \,|\, r_{t-1}) = \begin{cases} p_{11} & \text{if } r_t = r_{t-1} = 1 \\ p_{22} & \text{if } r_t = r_{t-1} = 2. \end{cases} \tag{1}$$

Hence, if p_{11} and p_{22} are high, regimes are persistent and exchange rate swings are long.

Whereas persistence in mean regimes is supposed to take account of the long swings, or "long-run autocorrelation", there may still be short-run dynamics within a mean regime. In the conditional mean specification we take account of this "short-run autocorrelation" by an autoregressive part, as has been done by Hamilton (1989). We use only one autoregressive term, as it is generally believed that the short-run autocorrelation in exchange rates is small (see West and Cho (1995)):

$$s_t = \mu_{r_t} + \theta(s_{t-1} - \mu_{r_{t-1}}) + \varepsilon_t, \tag{2}$$

where the conditional expectation of the innovation is $E_{t-1}\{\varepsilon_t | \widetilde{r}_t\} = 0$.

Equations (1) and (2) are fundamental, as they relate to the long swings directly. For a complete model specification, however, we also have to define the two other elements, namely the conditional variance of ε_t and its distribution. This is the subject of the remaining part of this section.

To specify the conditional variance of ε_t, $V_{t-1}\{\varepsilon_t | \widetilde{r}_t\}$, Engel and Hamilton (1990) assume that it is constant within a mean regime, but different across the two regimes. This allows for some time-variation in volatility. However, as the authors admit, the perfect dependence between mean and variance can be problematic. For instance, if the appreciation regime is associated with high volatility, a period of unusual volatility can force the process into this appreciation regime, even when the currency is actually depreciating. Moreover, economists are not convinced that there is any relation between the mean and the variance of exchange rates (for instance, see Engle, Ito and Lin (1990)).

A second restriction of the Engel and Hamilton (1990) variance specification is that the variance is constant during mean regimes. As mean regimes are very persistent (a few years according to Engel and Hamilton), the variance is also constant for a long time. In particular for high-frequency data, such as the weekly data that we will use, this is problematic, as it is well-known that there is conditional heteroskedasticity.

To solve both problems, we disconnect the mean and the variance, and let the popular generalized autoregressive conditional heteroskedasticity (GARCH) model govern the conditional variance (see Bollerslev, Chou and Kroner (1992) for an overview of GARCH). A direct application of the standard GARCH(1,1) formula in our regime-switching model would define the conditional error variance as

$$V_{t-1}\{\varepsilon_t|\widetilde{r}_t\} = \omega + \alpha\varepsilon_{t-1}^2 + \beta V_{t-2}\{\varepsilon_{t-1}|\widetilde{r}_{t-1}\}. \tag{3}$$

This specification, however, appears practically infeasible when estimating the model. In building the sample log-likelihood, the econometrician first expresses the unobserved previous surprise term ε_{t-1}^2 in terms of the conditioning variables by using $\varepsilon_{t-1}^2 = \{s_{t-1} - [\mu_{r_{t-1}} + \theta(s_{t-2} - \mu_{r_{t-2}})]\}^2$. Hence, $V_{t-1}\{\varepsilon_t|\widetilde{r}_t\}$ depends on the unobserved regimes r_{t-1} and r_{t-2}. However, it also depends on the lagged variance $V_{t-2}\{\varepsilon_{t-1}|\widetilde{r}_{t-1}\}$, which depends on r_{t-2}, r_{t-3} and $V_{t-3}\{\varepsilon_{t-2}|\widetilde{r}_{t-2}\}$, where the latter depends on r_{t-3}, r_{t-4} and $V_{t-4}\{\varepsilon_{t-3}|\widetilde{r}_{t-3}\}$, and so on. Consequently, the conditional variance in (3) depends on the entire sequence of regimes up to time $t-1$. Since the number of possible combinations grows exponentially with $t-1$, this leads to an enormous number of regime paths to $t-1$. The econometrician, who does not observe regimes, has to integrate out all possible regime paths. This renders estimation intractable.

To avoid the path-dependency problem, it is interesting to realize that the same problem also hampered the application of regime-switching GARCH models, where the conditional variance depends on the volatility regime the process is in and where the conditional variance within each regime is governed by a GARCH process (see Cai (1994) and Hamilton and Susmel (1994)). For such models, Gray (1996a) and Klaassen (1998), who adjusts Gray's model, have introduced a way to remove the path-dependence from the likelihood. We apply the basic idea behind their techniques to solve the problem also in our regime-switching mean model. That is, we directly average out the regimes r_{t-1} and r_{t-2} in the source of the path-dependence, $\varepsilon_{t-1}^2 = \{s_{t-1} - [\mu_{r_{t-1}} + \theta(s_{t-2} - \mu_{r_{t-2}})]\}^2$, instead of only in the likelihood. This removes the regime-dependence of $V_{t-1}\{\varepsilon_t|\widetilde{r}_t\}$. Because we use the observed information I_{t-1} when averaging out the regimes, $V_{t-1}\{\varepsilon_t|\widetilde{r}_t\}$ becomes equal to $V_{t-1}\{\varepsilon_t\}$:

$$V_{t-1}\{\varepsilon_t|\widetilde{r}_t\} = V_{t-1}\{\varepsilon_t\} = \omega + \alpha E_{t-1}\{\varepsilon_{t-1}^2\} + \beta V_{t-2}\{\varepsilon_{t-1}\}. \tag{4}$$

This specification is, of course, more restrictive than (3). However, the only purpose of the variance specification is to make the long swing results robust to conditional heteroskedasticity. Subsection 3.4 shows that (4) is sufficient for that.

We complete the conditional variance specification by imposing the usual GARCH restrictions $\omega > 0$ and $\alpha,\ \beta \geq 0$ to ensure $V_{t-1}\{\varepsilon_t\} > 0$ for all t. We also assume that $\alpha+\beta<1$, so that the unconditional variance is $\sigma^2 = \frac{\omega}{1-\alpha-\beta}$.

The fourth and final element of the regime-switching model concerns the conditional distribution of exchange rate changes. Engel and Hamilton (1990) choose a normal distribution. However, to allow for extra leptokurtosis in our weekly data, we follow other papers by taking a t-distribution (see Bollerslev, Chou and Kroner (1992)). It has ν degrees of freedom, zero mean, and variance $V_{t-1}\{\varepsilon_t\}$:

$$\varepsilon_t \,|\, I_{t-1}, \widetilde{r}_t \quad \sim \quad t\,(\nu\ ,\ 0\ ,\ V_{t-1}\{\varepsilon_t\})\,. \tag{5}$$

Equations (1), (2), (4) and (5) describe the complete regime-switching model. As in Engel and Hamilton (1990), we estimate it by maximum likelihood. The likelihood function, which has a convenient recursive structure, is derived in appendix A.

3 Empirical Results

In this section we use the regime-switching model developed above to address the central question of this paper, namely whether long swings really exist. First, we describe the data. In subsection 3.2 we test for the swings. After that, we analyze the estimates of the regime-switching model and in subsection 3.4 we present some checks on its specification. In the last subsection, we examine whether taking account of the long swings leads to better exchange rate forecasts than the simple random walk model.

3.1 Data

We use three U.S. dollar exchange rates, namely, the dollar vis-à-vis the German mark, the Japanese yen and the U.K. pound. These exchange rates have been chosen because of their important role on foreign exchange markets and because they behave relatively independently, for instance, compared to several dollar-EMS exchange rates. We have 1,216 weekly observations for the percentage dollar depreciations s_t over the post-Bretton-Woods period from April 2, 1974 to July 22, 1997. They have been obtained from Datastream. In this subsection we provide some characteristics of the data and use them to motivate our model specification empirically.

In panel A of figures 1, 2 and 3 we show the behavior of the three exchange rates over the sample period. The figures contain the exchange rate levels in U.S. dollars, not in logarithms. At first sight, exchange rates indeed seem to be characterized by long swings.

In table 1 we report some descriptive statistics of the three exchange rate changes. There is significant first-order autocorrelation in the weekly German mark changes (we always use a significance level of 5%). For this reason, we have extended the Engel and Hamilton (1990) model by a first-order autoregressive term in the mean equation (2). Estimates for higher-order autocorrelations are not reported separately, but are combined in Box-Pierce type statistics $\widetilde{Q}_{10}$. They show that higher-order autoregressive terms are unnecessary.

Table 1 also presents two autocorrelation tests for the squared exchange rate changes. Both tests point at conditional heteroskedasticity for all three series. This is why we have extended the Engel and Hamilton (1990) model with GARCH specification (4) for the conditional error variance.

3.2 Long Swings in Exchange Rates: Are They Really in the Data?

As we have just seen from figures 1A, 2A and 3A, exchange rates seem to exhibit long swings. In this section we analyze the main point of the paper, namely whether long swings are a systematic part of the exchange rate generating process, as Engel and Hamilton (1990), Kaminsky (1993), Engel (1994) and Dewachter (1997) claim. After all, the long swings may be only a pattern imposed by the eye on the realization of a random walk. More formally, we test the null hypothesis that exchange rates follow a random walk (with drift) against the regime-switching alternative.

The null hypothesis of interest is nested in the regime-switching model, as the null restriction $\mu_1 = \mu_2$ implies that exchange rates follow a random walk.[1] However, the asymptotic distribution of the usual tests (likelihood ratio, Wald and Lagrange multiplier) is no longer χ^2, since the regime-switching probabilities are not identified under the null. Garcia (1995) solves this problem by deriving the correct asymptotic distribution of the likelihood ratio (LR) statistic. Moreover, he shows that this asymptotic distribution is very close to the small-sample distribution. We first use his results to test the random walk against the regime-switching model for similar quarterly data as in Engel and Hamilton (1990) and Engel (1994). After that, we enlarge the quarterly series to 1997, and check whether the results change. Finally, we increase the data frequency from quarterly to monthly and then to weekly; this leads to our final answer to the question whether long swings really exist.

To start our series of tests of the random walk, we take quarterly data from 1974:I to 1987:I, similar to the data in Engel and Hamilton (1990) and Engel (1994). The top row of table 2 contains the LR tests of the random walk for the three currencies

[1] For the moment, we neglect the autoregressive term in the mean equation (2).

and the 5% critical value from Garcia (1995). We find significant evidence against the random walk for the U.K. However, the random walk is not rejected for the other two currencies. This latter conclusion is opposite to the one of Engel and Hamilton (1990) and Engel (1994), who claim to have found evidence of long swings for the U.K. as well as Germany and Japan.[2] However, Engel and Hamilton admit that there is some concern with their test approach. As discussed in the introduction, they use a Wald test for a slightly more general null than the random walk, so as to circumvent the identification problems associated with the null of a random walk. We have shown in the introduction that the Wald statistic is indeed not the most appropriate statistic to use in a regime-switching model.

So far, we have not found conclusive evidence of the existence of long swings, at least not for Germany and Japan. Of course, this may be due to the absence of swings, but it may also be that a sample period of thirteen years is too short to detect long swings. To analyze this, we extend our sample period by including data from 1987:II to 1997:III. As table 2 shows, all LR tests become insignificant now. Hence, our finding based on the quarterly data is that we have no evidence of long swings.

Our inability to reject the random walk can be due to a lack of power of the tests. One reason for this may be that the data frequency is too low. After all, even if swings are part of the exchange rate generating process and last for a number of quarters, quarterly data may result in too few observations per swing to distinguish swings from a random walk. To examine this, we first increase the data frequency from quarterly to monthly. As table 2 shows, all LR tests are still insignificant, although they are generally higher than for the quarterly data.

As a final attempt, we use the weekly series described in subsection 3.1. The results change completely: all LR statistics are significant now. Hence, the previous inability to reject the random walk using quarterly or monthly data has pure statistical reasons: the low data frequency leads to too few observations to significantly distinguish long swings from a random walk. Weekly data give the LR test enough power. Our final conclusion is thus that the data really suggest that long swings exist. Note that this conclusion is entirely based on the results of Garcia (1995), not on the problematic Wald tests that have been used by others.

[2] Although Engel and Hamilton (1990) and Engel (1994) allow for different variances across mean regimes (see section 2), we assume a constant variance over the complete sample period, since we find no conditional heteroskedasticity at the quarterly frequency. Our homoskedasticity assumption is not the reason behind the opposite conclusions; even if we allow for different variances across mean regimes, the likelihood ratio is insignificant.

3.3 Estimation Results

We now present the estimation results for our regime-switching model and, for comparison, the results for the random walk. We first consider the mean equation (2). Then we extensively discuss the regime process in (1). Finally, we briefly address the error distribution (5) with variance (4).

As table 3 shows, all three exchange rates are characterized by an appreciation and a depreciation regime. Moreover, there is significant first-order autocorrelation for the German mark only.

Despite the minor importance of short-run autocorrelation, all three exchange rates exhibit long-run autocorrelation caused by the long swings, that is, by the persistence of regimes with different means. The high persistence of regimes is represented by the large regime-staying probabilities p_{11} and p_{22}, which all exceed 0.975.

To get a better idea about the degree of persistence that the staying probabilities imply, we first compute the expected duration of a regime r, which is $(1 - p_{rr})^{-1}$ (see Hamilton (1989)). The average estimates of p_{11} and p_{22} imply an expected duration of somewhat more than one year for the low mean regime and about two years for the high mean regime.[3]

A second way to examine the persistence of regimes is by inspecting estimated regime probabilities. Following Gray (1996a), we use two types of regime probabilities, namely ex ante and smoothed probabilities. The ex ante probability of a particular regime at time t is the conditional probability that the process was in that regime at time t using only information available to the econometrician at time $t-1$, that is, I_{t-1}. The smoothed regime probability, on the other hand, uses the complete data set I_T, thereby smoothing the ex ante probabilities.[4] Hence, it gives the most informative answer to the question which regime the process was likely in at time t.

To illustrate the effect of smoothing the ex ante probabilities, figures 1B, 2B and 3B show the ex ante probabilities of being in the high mean regime for the German

[3] These durations, which are comparable to the ones in Engel and Hamilton (1990), are remarkably different from the Dewachter (1997) results. For instance, his estimates for the German mark give an expected duration of two and three months instead of years for the low and high mean regime, respectively. Such short durations are difficult to reconcile with his (Wald-based) conclusion that there are long swings. The reason for this result is that he does not take account of short-run autocorrelation. The regimes, which are supposed to model long-run autocorrelation, are then exploited to capture the short-run autocorrelation as well. This leads to unstable regimes and thus to short instead of long swings.

[4] In appendix B we show how to compute the smoothed probabilities in a recursive manner. The algorithm is based on Gray (1996b). It links the ex ante probabilities, which are used during estimation (see appendix A), directly to the smoothed probabilities by iterating forward from the ex ante to the smoothed probabilities.

mark, Japanese yen and U.K. pound, respectively, while figures 1C, 2C and 3C give the corresponding smoothed probabilities. The ex ante probabilities are, of course, more volatile, in particular the ones for the two European currencies in the first half of the eighties. At that time there were several short periods of depreciation, which were viewed ex ante as indications of regime-switches. However, they appeared to be only temporary depreciations afterwards, as the dollar continued to strengthen until 1985. Using this information to update the ex ante probabilities smoothes away the temporary deviations and makes the smoothed probabilities much more stable.

The smoothed probabilities in figures 1C, 2C and 3C confirm that regimes are persistent. Moreover, they show that the regime-classification is in general as one would have expected. For instance, the well-known dollar appreciation against the European currencies in the first half of the eighties and the subsequent depreciation against all three currencies are well captured by the regime-switching model.

After this extensive discussion of the regime process, we now briefly address the error variance (4) and distribution (5). The lower part of table 3 contains the estimates. We find that conditional homoskedasticity and conditional normality are strongly rejected. Furthermore, for all three series the results are very robust across the two models, indicating that the variance is rather independent of the specification of the mean equation.

3.4 Diagnostics

To check whether our model sufficiently captures the autocorrelation and conditional heteroskedasticity in the data, we analyze the normalized residuals. Table 4 presents tests for autocorrelation and heteroskedasticity in them. From the first-order autocorrelations and the Box-Pierce statistics, we conclude that there is no remaining autocorrelation, at least for the regime-switching model. The random walk, which has no autoregressive term, misses some autocorrelation. Furthermore, the autocorrelation tests for the squared normalized residuals show no reason to extend the variance specifications of the two models.

3.5 Forecasting Performance

Knowing that long swings really exist, a natural question is whether this can be exploited to predict future exchange rates better than a random walk. In this subsection we focus on this issue.

We first compare the in-sample and then the out-of-sample forecasts generated by the random walk and the regime-switching model. We examine both point predictions and predictions of the direction of exchange rate changes by comparing the actual (log of the) exchange rate level at some future time τ, S_τ, with the predicted level based on information available at time $t-1$, $\widehat{E}_{t-1}\{S_\tau\}$. For the random walk, this forecast is the previous exchange rate S_{t-1} plus an estimated drift term. For the regime-switching model, $\widehat{E}_{t-1}\{S_\tau\}$ follows from (14) in appendix C, after substitution of the estimation results of subsection 3.3. The forecasts are computed for three horizons, namely the one-week, which corresponds to the data frequency, the one-quarter (13-week), and the one-year (52-week) horizon.

Starting with the in-sample forecasts, the first, often-used forecasting statistics we consider are the root mean squared error (RMSE), which is defined as the square root of $\frac{1}{T}\sum_{t=1}^{T}(S_\tau - \widehat{E}_{t-1}\{S_\tau\})^2$, and the mean absolute error (MAE) $\frac{1}{T}\sum_{t=1}^{T}\left|S_\tau - \widehat{E}_{t-1}\{S_\tau\}\right|$. Table 5 shows that the regime-switching model beats the random walk in 12 out of 18 cases, so that there is only a slight preference for our regime-switching model.

Our model, however, clearly outperforms the random walk at predicting the direction of change. In eight out of nine cases the estimated probability of a correct prediction is higher than for the random walk. In even seven out of nine cases our model predicts the direction of change correctly in significantly more than half of the observations, while for the random walk this happens only once.[5] Apparently, taking account of long swings improves the in-sample forecast quality, particularly regarding the direction of change.

We now turn to the out-of-sample forecasts. We reestimate the two models using only the first three quarters of the sample. Holding the parameters fixed, we then use the 304 observations in the final quarter (from November 1, 1991 to July 22, 1997) to generate the forecasts $\widehat{E}_{t-1}\{S_\tau\}$.

From table 6 we see that the marginal superiority of the regime-switching model in terms of RMSE and MAE has vanished. In only two out of eighteen cases the regime-switching model outperforms the random walk and in the other cases it does worse. This conclusion is also drawn by Engel (1994) and is in line with Diebold and Nason (1990), who find in a nonparametric analysis that it is difficult to beat the random walk in point prediction.

Nevertheless, we still see that our model outperforms the random walk at predicting the direction of change, as it does better in seven out of nine cases and does worse only

[5]This conclusion about significance is robust to the autocorrelation originating from the fact that for the one-quarter and one-year horizon the forecast horizon exceeds the one week period between observations. The standard errors of the percentages in table 5 are based on the Newey and West (1987) asymptotic covariance matrix. Following West and Cho (1995), we have taken Bartlett weights and have used the same data-dependent automatic lag selection rule. This rule, introduced by Newey and West (1994), has certain asymptotic optimality properties.

once. This is also concluded by Engel (1994) and is supported by our in-sample results.

4 Conclusion

The random walk is often used to model exchange rates. We test the validity of this model against the more general Markov regime-switching model. The latter model explicitly allows for long swings in exchange rates. The central question of the paper is whether such long swings actually exist.

In the literature so far, the conclusion is that long swings do exist. However, we demonstrate that the commonly used Wald tests are not very reliable in the highly nonlinear regime-switching model. Moreover, we find from a more robust likelihood ratio test that, based on similar quarterly data as in Engel and Hamilton (1990), the random walk cannot be rejected in favor of the long swings.

However, this is not our final conclusion. Sampling at the quarterly or monthly frequency appears to result in too few observations to significantly distinguish long swings from a random walk. After all, we find evidence of long swings when we use weekly data.

Given our evidence of long swings, we also analyze whether this feature can be exploited to forecast exchange rates better than a random walk. As already suggested by Diebold and Nason (1990), beating the random walk in point prediction is difficult. Nevertheless, we find evidence that the long swings model predicts the direction of change better than the random walk.

Our model can be extended in various respects. First, other variables, such as forward rates, can be included in the mean equation to improve exchange rate forecasts. Second, the assumption of time-constant regime-switching probabilities can be relaxed. Deviations of exchange rates from fundamental rates may be informative about the likelihood of regime-switches, so that it may prove useful to make the regime-switching probabilities depend on them. For instance, one can include deviations from purchasing power parity (PPP), as in Klaassen (1999), and deviations from trade balance equilibrium. This is left for future research.

Appendices

A Estimation

We estimate the regime-switching model introduced in section 2 by maximum likelihood. In this appendix we derive the likelihood function and show that it has a convenient recursive structure.

To obtain the likelihood function, we first need the density of the exchange rate change at time t conditional on only observable information. Let $p_{t-1}(s_t)$ denote this density evaluated at an exchange rate change equal to s_t.[6] It can be split up as

$$p_{t-1}(s_t) = \sum_{r_t, r_{t-1}=1,2} p_{t-1}(s_t \,|\, r_t, r_{t-1}) \cdot p_{t-1}(r_t, r_{t-1}). \tag{6}$$

We now discuss how to compute both terms on the right-hand-side.

The first term, $p_{t-1}(s_t \,|\, r_t, r_{t-1})$, denotes the density of the exchange rate change at time t evaluated at the value s_t conditional on I_{t-1} and on the current and previous regimes having values r_t and r_{t-1}. This t-density follows from formulas (2), (4) and (5). It is, however, not straightforward how to compute the conditional variance in (4), as this requires integrating out the regimes r_{t-1} and r_{t-2} in $\varepsilon_{t-1}^2 = \{s_{t-1} - [\mu_{r_{t-1}} + \theta(s_{t-2} - \mu_{r_{t-2}})]\}^2$. For that, we need $p_{t-1}(r_{t-1}, r_{t-2})$, the conditional probability that the two most recent regimes have values r_{t-1} and r_{t-2}. This probability is crucial, since all regime probabilities in the paper can be derived from it. Using similar techniques as in Gray (1996a), the following formula shows that this probability has a first-order recursive structure, which simplifies its computation a lot:

$$\begin{aligned} p_{t-1}(r_{t-1}, r_{t-2}) &= p_{t-2}(r_{t-1}, r_{t-2} \,|\, s_{t-1}). \\ &= \frac{p_{t-2}(s_{t-1} \,|\, r_{t-1}, r_{t-2}) \cdot p_{t-2}(r_{t-1}, r_{t-2})}{p_{t-2}(s_{t-1})} \\ &= \frac{p_{t-2}(s_{t-1} \,|\, r_{t-1}, r_{t-2}) \cdot p_{t-2}(r_{t-1} | r_{t-2}) \cdot \sum_{r_{t-3}=1,2} p_{t-2}(r_{t-2}, r_{t-3})}{p_{t-2}(s_{t-1})}. \end{aligned} \tag{7}$$

Hence, the variables to compute $p_{t-1}(r_{t-1}, r_{t-2})$ are its previous values $p_{t-2}(r_{t-2}, r_{t-3})$ for $r_{t-3} = 1, 2$, the constant $p_{t-2}(r_{t-1} \mid r_{t-2})$ and the previous densities $p_{t-2}(s_{t-1} \mid r_{t-1}, r_{t-2})$ and $p_{t-2}(s_{t-1})$. This makes the computation of $p_{t-1}(r_{t-1}, r_{t-2})$ a first-order recursive process.

[6] We use the same symbol p_{t-1} for several densities (see (1) and (6)). The specific meaning of p_{t-1} is uniquely determined by the symbols used in its argument. This results in a concise notation, which will prove useful in the remaining part of the paper.

The second term on the right-hand-side in (6), $p_{t-1}(r_t, r_{t-1})$, is the conditional probability that the current and previous regimes have values r_t and r_{t-1}, respectively. It can be calculated from

$$p_{t-1}(r_t, r_{t-1}) = p_{t-1}(r_t \,|\, r_{t-1}) \cdot \sum_{r_{t-2}=1,2} p_{t-1}(r_{t-1}, r_{t-2}), \tag{8}$$

where the switching probability $p_{t-1}(r_t \,|\, r_{t-1})$ follows directly from (1) and $p_{t-1}(r_{t-1}, r_{t-2})$ is given by (7).

Having discussed both terms on the right-hand-side of (6), we can now compute the density of interest, $p_{t-1}(s_t)$, being a mixture of four t-densities. This density can then be used to build the sample log-likelihood $\sum_{t=1}^{T} \log(p_{t-1}(s_t))$ with which all parameters in the regime-switching model can be estimated.

From a practical point of view, it is important to realize that the log-likelihood has a second-order recursive structure, similar to that of a standard one-regime AR(1)-GARCH(1,1) model. After all, for (8) one needs the constant $p_{t-1}(r_t \mid r_{t-1})$ and the first-order recursive probability $p_{t-1}(r_{t-1}, r_{t-2})$ in (7) for all eight combinations of (r_t, r_{t-1}, r_{t-2}); density (6) can then be computed from (8), the previous changes s_{t-1} and s_{t-2}, (7) and the previous variance $V_{t-2}\{\varepsilon_{t-1}\}$ in (4). This second-order recursiveness of $p_{t-1}(s_t)$ makes the calculation of the sample log-likelihood quite fast. To start up the recursive process, we set the required variables equal to their expectation without conditioning on the information set.

B Regime Inference

As stated in footnote 4, the smoothed probability that the regime was r_t at time t, $p_T(r_t)$, can be computed recursively. More generally, any ex post regime probability $p_\tau(r_t)$, for a given future time $\tau \in \{t, t+1, \ldots, T\}$, can be calculated in a recursive manner. This claim, which we prove in this appendix, is based on the following recursive process for the two-regime ex post probability $p_\tau(r_t, r_{t-1})$ starting from the ex ante probability $p_{t-1}(r_t, r_{t-1})$.

We can write $p_\tau(r_t, r_{t-1})$ for the four regime combinations as

$$\begin{aligned} p_\tau(r_t, r_{t-1}) &= p_{\tau-1}(r_t, r_{t-1} \,|\, s_\tau) \\ &= \frac{p_{\tau-1}(s_\tau | r_t, r_{t-1}) \cdot p_{\tau-1}(r_t, r_{t-1})}{\sum_{r_t, r_{t-1}=1,2} p_{\tau-1}(s_\tau | r_t, r_{t-1}) \cdot p_{\tau-1}(r_t, r_{t-1})}. \end{aligned} \tag{9}$$

Suppose first that $\tau = t$. Then $p_\tau(r_t, r_{t-1})$ follows directly from (9), as $p_{\tau-1}(r_t, r_{t-1})$ and $p_{\tau-1}(s_\tau | r_t, r_{t-1})$ are known from the estimation process (see appendix A).

Let us suppose from now on that $\tau > t$. The computation of (9) requires two inputs. The first one is the previous ex post probability $p_{\tau-1}(r_t, r_{t-1})$, which is known from the previous recursion for all combinations of r_t and r_{t-1}. The second ingredient of (9) is the density $p_{\tau-1}(s_\tau|r_t, r_{t-1})$ for all regime outcomes. Its computation requires a number of steps. We first write it as

$$p_{\tau-1}(s_\tau|r_t, r_{t-1}) = \sum_{r_\tau, r_{\tau-1}=1,2} p_{\tau-1}(s_\tau|r_\tau, r_{\tau-1}) \cdot p_{\tau-1}(r_\tau, r_{\tau-1}|r_t, r_{t-1}), \tag{10}$$

where we use that the conditional distribution of s_τ given $r_\tau, r_{\tau-1}$ does not depend on the earlier regimes r_t and r_{t-1}. This formula itself has two ingredients. The first one is the density $p_{\tau-1}(s_\tau|r_\tau, r_{\tau-1})$ for all regime combinations, which is known from the estimation process.

The second term needed in (10) is the $(\tau-t)$-period-ahead regime-switching probability $p_{\tau-1}(r_\tau, r_{\tau-1}|r_t, r_{t-1})$ for all regime combinations. Once it has been computed, it should be saved, since it will be needed in the next recursive step. Making use of the Markov structure of the regime process, it can be written in terms of $(\tau-1-t)$-period-ahead switching probabilities:

$$p_{\tau-1}(r_\tau, r_{\tau-1}|r_t, r_{t-1}) = \sum_{r_{\tau-1}, r_{\tau-2}=1,2} p_{\tau-1}(r_\tau, r_{\tau-1}|r_{\tau-1}, r_{\tau-2}) \cdot p_{\tau-1}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1}). \tag{11}$$

Again, we have two ingredients. First, we need $p_{\tau-1}(r_\tau, r_{\tau-1}|r_{\tau-1}, r_{\tau-2})$ for all regime combinations. Due to the Markov property of the regime process, this switching probability does not depend on $r_{\tau-2}$. It equals

$$p_{\tau-1}(r_\tau, r_{\tau-1}|r_{\tau-1}, r_{\tau-2}) = p_{\tau-1}(r_\tau|r_{\tau-1}), \tag{12}$$

which is constant and follows from (1).

The second ingredient of (11) is $p_{\tau-1}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1})$ for all regime combinations:

$$\begin{aligned} p_{\tau-1}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1}) &= p_{\tau-2}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1}, s_{\tau-1}) \\ &= \frac{p_{\tau-2}(s_{\tau-1}|r_{\tau-1}, r_{\tau-2}) \cdot p_{\tau-2}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1})}{\sum_{r_{\tau-1}, r_{\tau-2}=1,2} p_{\tau-2}(s_{\tau-1}|r_{\tau-1}, r_{\tau-2}) \cdot p_{\tau-2}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1})}, \end{aligned} \tag{13}$$

where we use that the conditional density of $s_{\tau-1}$ is independent of the regimes r_t and r_{t-1} once $r_{\tau-1}$ and $r_{\tau-2}$ are given. We have two ingredients. First, the conditional density $p_{\tau-2}(s_{\tau-1}|r_{\tau-1}, r_{\tau-2})$ for all regime combinations. It is known from the estimation process. Second, we need the $(\tau-1-t)$-period-ahead switching probability $p_{\tau-2}(r_{\tau-1}, r_{\tau-2}|r_t, r_{t-1})$ for all regime combinations. This one was saved during the previous recursion, if $\tau > t+1$. If $\tau = t+1$, it equals one.

This completes the algorithm to compute (10), which is the second ingredient of (9). For each recursion one has to compute (13), use it together with (12) to compute (11) and use this to compute (10). Using this in (9) yields the ex post probability $p_\tau(r_t, r_{t-1})$ from $p_{\tau-1}(r_t, r_{t-1})$. Therefore, starting from the ex ante probability $p_{t-1}(r_t, r_{t-1})$ one can recursively compute the ex post probability $p_\tau(r_t, r_{t-1})$ and eventually the probability of interest $p_\tau(r_t)$.

C Forecasting

Subsection 3.5 deals with forecasting exchange rate levels S_τ at time $t-1$, where $\tau \geq t$. This appendix explains how to compute these forecasts.

As usual, we first decompose the exchange rate forecast as

$$E_{t-1}\{S_\tau\} = S_{t-1} + \sum_{i=t}^{\tau} E_{t-1}\{s_i\}. \tag{14}$$

To calculate $E_{t-1}\{s_i\}$, we rewrite s_i by repeated substitution of lags of (2) for the lagged changes. Since the innovations have zero expectation, this yields

$$E_{t-1}\{s_i\} = \sum_{r_i, r_{t-1}=1,2} p_{t-1}(r_i, r_{t-1}) \cdot \left(\mu_{r_i} + \theta^{i-(t-1)}(s_{t-1} - \mu_{r_{t-1}})\right). \tag{15}$$

The only probability involved in (15) is $p_{t-1}(r_i, r_{t-1})$ for $i = t, \ldots, \tau$. We have

$$p_{t-1}(r_i, r_{t-1}) = p_{t-1}(r_{t-1}) \cdot p_{t-1}(r_i \,|\, r_{t-1}), \tag{16}$$

where the first term on the right-hand-side follows after summation of $p_{t-1}(r_{t-1}, r_{t-2})$ in (7) over r_{t-2}.

To compute the multi-period-ahead switching probability on the right-hand-side of (16), we first form the one-period-ahead Markov transition matrix M:

$$M = \begin{bmatrix} p(r_t{=}1 \,|\, r_{t-1}{=}1) & 1 - p(r_t{=}2 \,|\, r_{t-1}{=}2) \\ 1 - p(r_t{=}1 \,|\, r_{t-1}{=}1) & p(r_t{=}2 \,|\, r_{t-1}{=}2) \end{bmatrix}, \tag{17}$$

where its elements follow from (1). The theory of Markov processes for multi-period-ahead switching probabilities then implies that

$$p_{t-1}(r_i \,|\, r_{t-1}) = \left(M^{i-(t-1)}\right)_{r_i r_{t-1}}. \tag{18}$$

Having explained how to calculate (16), we can now compute (15). Computation of (15) for all i and substitution in (14) then gives the forecast of interest $E_{t-1}\{S_\tau\}$.

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Table 1: Moments of exchange rate returns and autocorrelation tests.

	GERMANY	JAPAN	U.K.
Mean	0.03	0.07	−0.03
Variance	2.14	2.11	2.13
Skewness	−0.14	0.53	−0.40
Excess Kurtosis	1.70	2.01	3.00
Autocorr. ρ_1	0.07∗	0.05	0.04
	(0.03)	(0.04)	(0.04)
Autocorr. $\widetilde{Q}_{10}$	14.07	22.57∗	6.05
	[0.17]	[0.01]	[0.81]
Autocorr. squares ρ_1^s	0.11∗	0.20∗	0.20 ∗
	(0.03)	(0.03)	(0.03)
Autocorr. squares Q_{10}^s	57.60∗	92.03∗	151.82 ∗
	[0.00]	[0.00]	[0.00]

Standard errors in parentheses and p-values in square brackets; * is significant at 5% level.
The first-order autocorrelation, ρ_1, is estimated as the slope coefficient in a regression of the change, s_t, on the first lagged change, s_{t-1}, and a constant. The standard errors are based on White's (1980) heteroskedasticity-consistent asymptotic covariance matrix.
$\widetilde{Q}_{10}$ denotes a modified Box-Pierce type statistic that combines the first ten autocorrelations. Following Pagan and Schwert (1990), it is defined as the sum of the first ten squared normalized autocorrelation estimates, where the normalizing factors are the heteroskedasticity-consistent standard errors of the autocorrelation estimates. $\widetilde{Q}_{10}$ is asymptotically χ^2_{10} distributed.
The first-order autocorrelation in the squared changes, ρ_1^s, and the Box-Pierce type statistic Q_{10}^s are similarly defined, although without the correction for heteroskedasticity.

Table 2: Likelihood ratio tests of long swings.

Data frequency and period		GERMANY	JAPAN	U.K.	Critical value
Quarterly data	1974-1987	3.17	6.43	8.95*	8.60
	1974-1997	2.42	3.39	5.37	8.60
Monthly data	1974-1997	5.31	4.12	5.37	8.68
Weekly data	1974-1997	9.00*	10.93*	12.16*	8.68

* is significant at 5% level.
The null hypothesis is the random walk model; the alternative is the regime-switching model.
For the quarterly data, the autoregressive term has been removed from the regime-switching model, because there is no autocorrelation at the quarterly frequency. Likewise, we assume conditional homoskedasticity. Finally, the error is normally distributed, as in Engel and Hamilton (1990) and Engel (1994).
For the monthly and weekly data we have extended both models (including the random walk) with a first-order autoregressive term and a t-distributed GARCH(1,1) error term, as in section 2, to correct for autocorrelation and conditional heteroskedasticity.
The 5% asymptotic critical values are from Garcia (1995).

Table 3: Estimation results.

		GERMANY		JAPAN		U.K.	
		RW	Regime	RW	Regime	RW	Regime
Mean of regime	μ_1	0.03 (0.04)	$-0.27*$ (0.09)	0.01 (0.03)	-0.30 (0.15)	0.01 (0.03)	-0.30 * (0.09)
	μ_2		0.15* (0.07)		0.13 (0.07)		0.14 * (0.06)
Autocorr.	θ		0.07* (0.03)		0.04 (0.03)		-0.01 (0.03)
Regime stay prob	p_{11}		0.992 (0.010)		0.976 (0.028)		0.981 (0.021)
	p_{22}		0.996 (0.007)		0.983 (0.019)		0.992 (0.013)
Uncond. variance	σ^2	2.89 (1.08)	3.11 (1.41)	1.82 (0.87)	1.62 (0.84)	2.86 (1.11)	2.81 (1.11)
ARCH	α	0.13* (0.03)	0.14* (0.03)	0.07* (0.02)	0.07* (0.02)	0.11* (0.02)	0.10 * (0.02)
GARCH	β	0.84* (0.04)	0.83* (0.04)	0.92* (0.02)	0.92* (0.02)	0.88* (0.02)	0.89 * (0.02)
T-dist.	ν^{-1}	0.12* (0.03)	0.14* (0.03)	0.20* (0.02)	0.21* (0.02)	0.20* (0.02)	0.22 * (0.03)
Log-likelihood minus RW		-2126 0	-2116 9.34	-2053 0	-2044 8.91	-2068 0	-2062 6.34

Standard errors in parentheses; * is significant at 5% level.
"RW" denotes the random walk, "Regime" the regime-switching model.
We report the inverse of the degrees of freedom of the t-distribution, because testing for conditional normality then boils down to simply testing whether ν^{-1} differs significantly from zero.

Table 4: Diagnostic statistics for normalized residuals and their squares.

	GERMANY		JAPAN		U.K.	
	RW	Regime	RW	Regime	RW	Regime
Autocorr. ρ_1	0.10*	0.01	0.08*	0.01	0.06*	0.04
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Autocorr. Q_{10}	24.40*	6.47	34.11*	17.37	16.32	6.37
	[0.01]	[0.78]	[0.00]	[0.07]	[0.09]	[0.78]
Autocorr. ρ_1^s	-0.05	-0.05	0.06*	0.06*	0.03	0.04
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Autocorr. Q_{10}^s	16.32	15.87	11.13	11.16	9.31	9.91
	[0.09]	[0.10]	[0.35]	[0.35]	[0.50]	[0.45]

Standard errors in parentheses and p-values in square brackets; * is significant at 5% level.
"RW" denotes the random walk, "Regime" the regime-switching model.
The residual is the exchange rate change minus the estimate of its conditional expectation $E_{t-1}\{s_t\}$. The regime probability to integrate out the unobserved regimes in this expectation can be found in appendix A. The residual is normalized using the estimate of its variance $V_{t-1}\{s_t\}$. Note that this variance is not equal to the error variance $V_{t-1}\{\varepsilon_t\}$, since the possibility of regime-switches is an additional source of variance of the residuals besides the error variance.
All autocorrelation statistics have been defined below table 1, although the standard error of ρ_1 and the value of Q_{10} are no longer corrected for heteroskedasticity.

Table 5: In-sample forecasting statistics.

	GERMANY		JAPAN		U.K.	
	RW	Regime	RW	Regime	RW	Regime
Panel A: One-week horizon						
RMSE	1.464	1.458	1.454	1.449	1.459	1.455
MAE	1.095	1.085	1.041	1.033	1.043	1.038
Correct direction	0.527*	0.562*	0.484	0.548*	0.507	0.560 *
	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)
Panel B: One-quarter horizon						
RMSE	5.941	5.959	6.305	6.368	5.974	5.944
MAE	4.814	4.757	4.956	4.916	4.585	4.485
Correct direction	0.530	0.576*	0.539	0.586*	0.492	0.579 *
	(0.045)	(0.041)	(0.047)	(0.038)	(0.046)	(0.039)
Panel C: One-year horizon						
RMSE	12.945	13.487	14.059	14.751	12.891	12.911
MAE	10.585	10.338	11.042	11.581	10.722	10.280
Correct direction	0.534	0.597*	0.609*	0.535	0.480	0.589
	(0.065)	(0.056)	(0.063)	(0.049)	(0.065)	(0.054)

Standard errors in parentheses; * is significantly greater than 0.5 at 5% level.
"RW" denotes the random walk, "Regime" the regime-switching model.
"Correct direction" denotes the fraction of forecasts that yield the correct direction of change of the exchange rate level. For the one-quarter and one-year horizon the standard errors have been corrected for autocorrelation as explained in footnote 5.

Table 6: Out-of-sample forecasting statistics.

	GERMANY		JAPAN		U.K.	
	RW	Regime	RW	Regime	RW	Regime
Panel A: One-week horizon						
RMSE	1.523	1.526	1.511	1.515	1.465	1.473
MAE	1.133	1.136	1.097	1.099	1.000	1.006
Correct direction	0.512	0.531	0.454	0.484	0.459	0.502
	(0.029)	(0.029)	(0.029)	(0.029)	(0.029)	(0.029)
Panel B: One-quarter horizon						
RMSE	5.612	5.680	6.490	6.562	5.638	5.759
MAE	4.589	4.663	5.106	5.026	3.671	3.784
Correct direction	0.438	0.486	0.503	0.545	0.490	0.483
	(0.075)	(0.076)	(0.081)	(0.071)	(0.074)	(0.048)
Panel C: One-year horizon						
RMSE	10.151	11.033	12.765	12.803	9.470	9.515
MAE	8.807	9.489	11.059	10.787	6.995	7.297
Correct direction	0.455	0.498	0.605	0.628	0.522	0.522
	(0.101)	(0.095)	(0.106)	(0.080)	(0.095)	(0.095)

Standard errors in parentheses; * is significantly greater than 0.5 at the 5% level.
"RW" denotes the random walk, "Regime" the regime-switching model.
The whole series except for the last quarter has been used for estimation, while the last quarter (304 weeks from November 1, 1991 to July 22, 1997) has been used for forecasting. This means that for the one-quarter (one-year) horizon there are 292 (253) comparisons between the actual and predicted values.
"Correct direction" denotes the fraction of forecasts that yield the correct direction of change of the exchange rate level. For the one-quarter and one-year horizon the standard errors have been corrected for autocorrelation as explained in footnote 5.

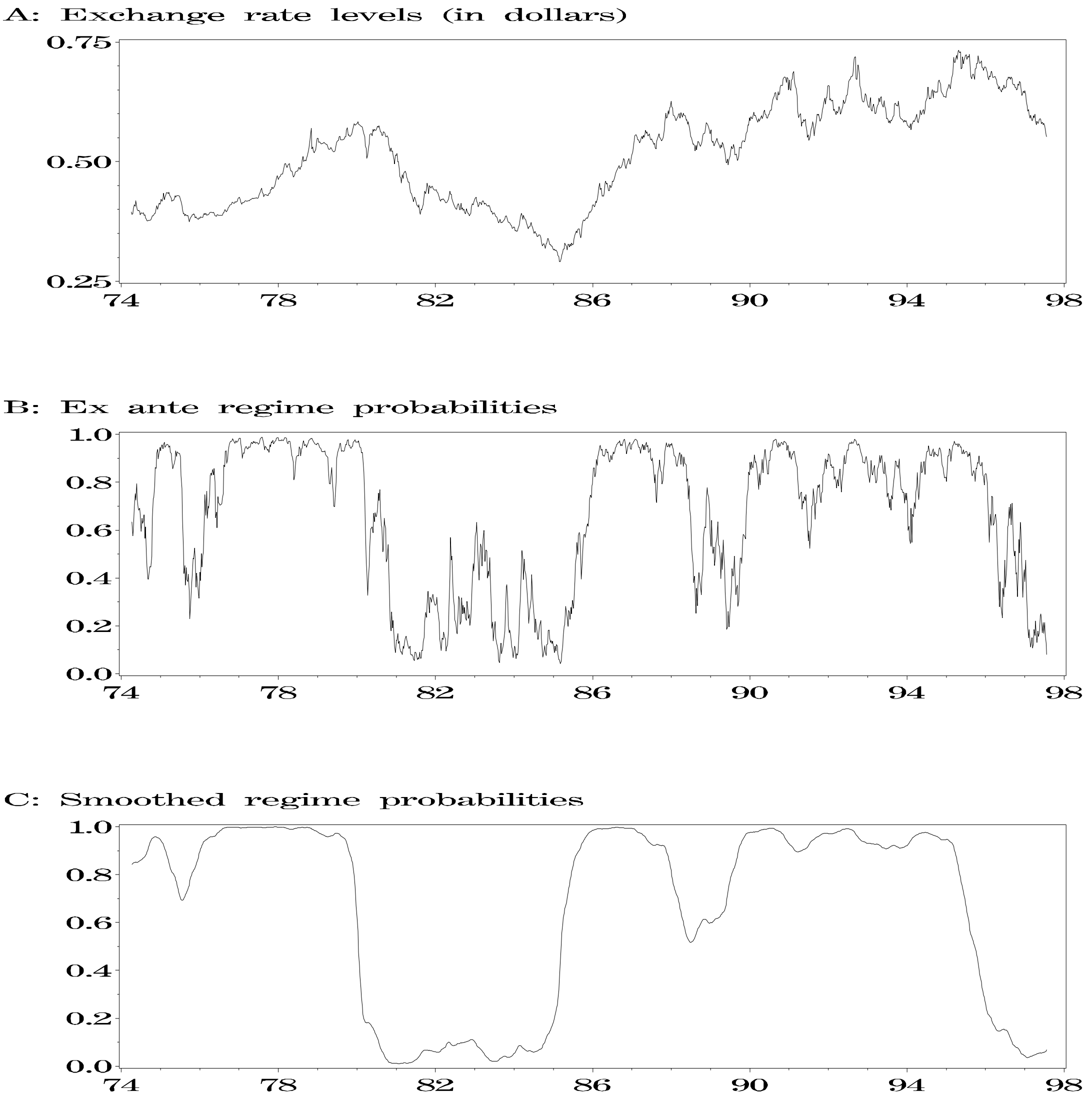

Figure 1: German mark over the sample period April 1974 to July 1997.

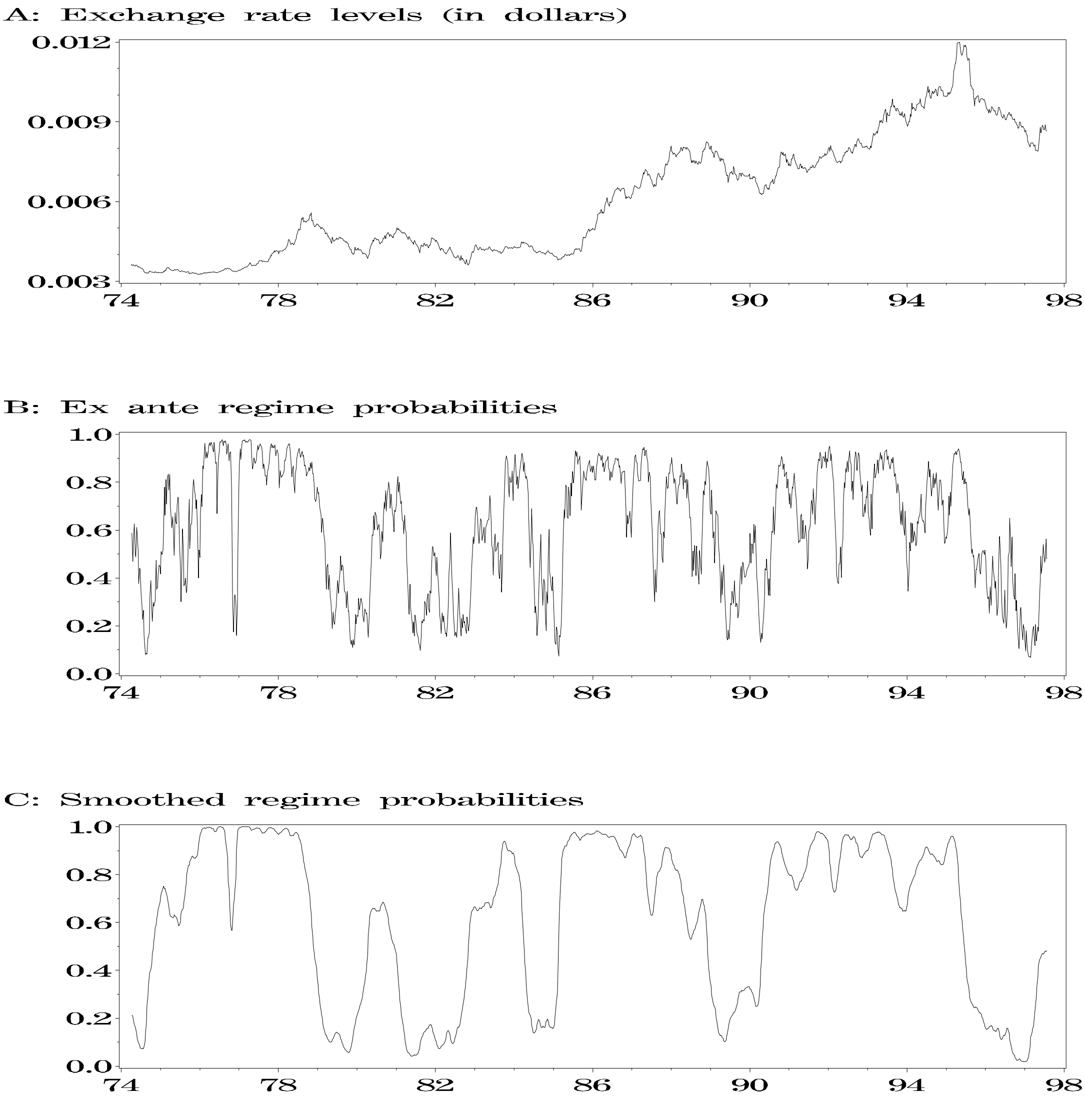

Figure 2: Japanese yen over the sample period April 1974 to July 1997.

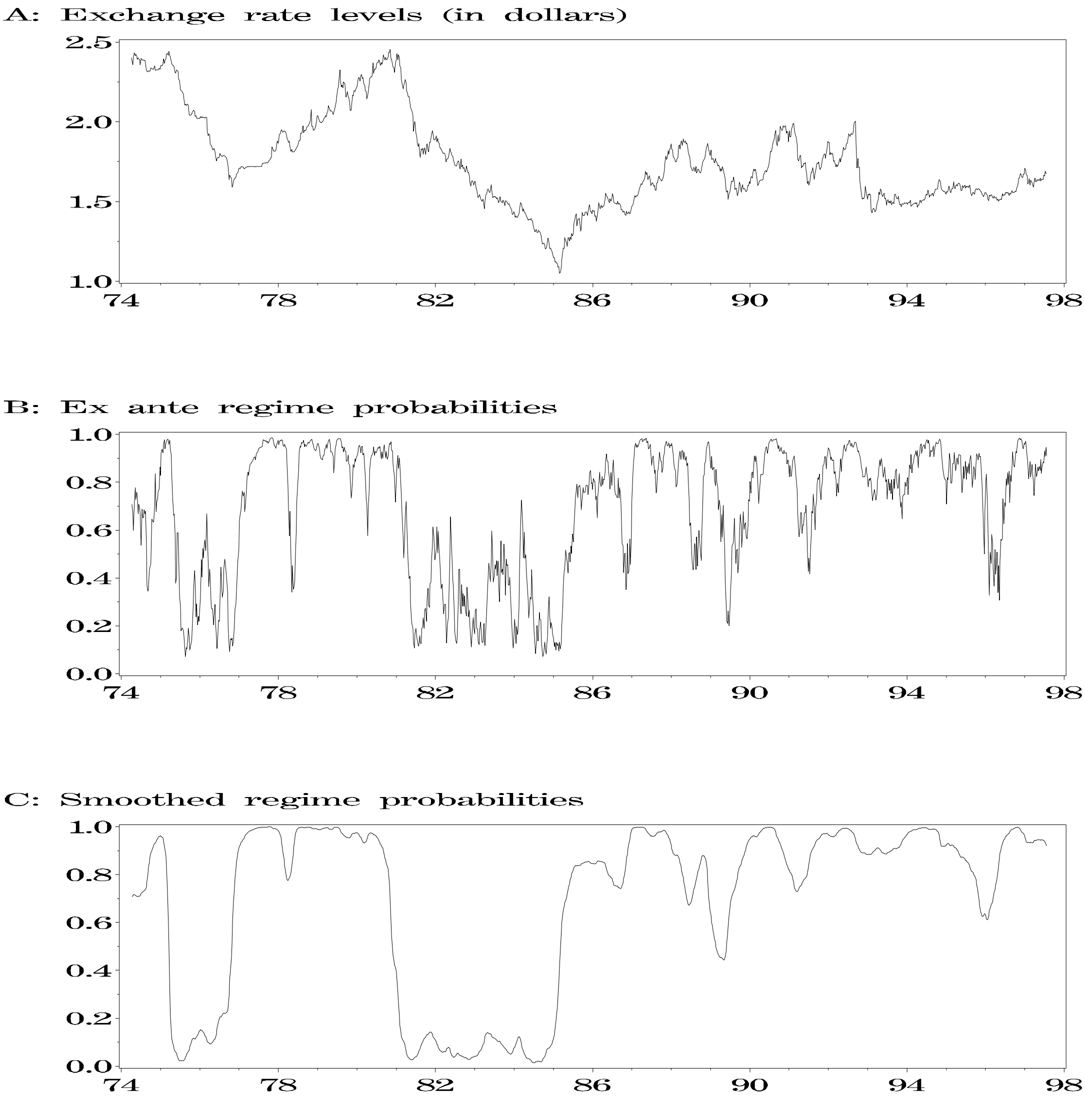

Figure 3: U.K. pound over the sample period April 1974 to July 1997.