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Network formation models with costs for establishing links

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Abstract

In this paper we study endogenous formation of communication networks in situations where the economic possibilities of groups of players can be described by a cooperative game. We concentrate on the influence that the existence of costs for establishing communication links has on the communication networks that are formed. The starting points in this paper are two game-theoretic models of the formation of communication links that were studied in the literature fairly recently, the extensive-form model by *Aumann* and *Myerson* (1988) and the strategic-form model that was studied by *Dutta*, *van den Nouweland* and *Tijs* (1998). We follow their analyses as closely as possible and use an extension of the Myerson value to determine the payoffs to the players in communication situations when forming links is not costless. We find that for both models, the pattern of structures formed as costs increase depends on whether the underlying coalitional game is superadditive and/or convex.

1 Introduction

In this paper we study endogenous formation of communication networks in situations where the economic possibilities of groups of players can be described by a cooperative game. We concentrate on the influence that the existence of costs for establishing communication links has on the communication networks that are formed. The starting

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points of this paper are two game-theoretic models of the formation of communication links that were studied in the literature fairly recently, the extensive-form model by *Aumann* and *Myerson* (1988) and the strategic-form model studied by *Dutta*, *van den Nouweland* and *Tijs* (1998).[1] In both of these papers forming communication links is costless and, once a communication network has been formed, an external allocation rule is used to determine the payoffs to the players in different communication networks. The external allocation rule used by *Aumann* and *Myerson* (1988) is the Myerson value (cf. *Myerson* (1977)) and *Dutta*, *van den Nouweland* and *Tijs* (1998) considered a class of external allocation rules that contains the Myerson value. We follow their analyses as closely as possible and use a natural extension of the Myerson value to determine the payoffs to the players in communication situations with costs for establishing links.

The goal of this paper is to study the influence that costs of forming communication links have on the structures that are formed. In order to be able to clearly isolate the influence of the costs, we assume that costs are equal for all possible communication links. Starting from costs equal to zero, we increase the costs and see how these increasing costs induce different equilibrium communication structures. Throughout the paper, we almost always restrict our analysis to situations in which the underlying cooperative games are 3-player symmetric games.

In the extensive-form game of link formation we consider communication structures that are formed in subgame perfect Nash equilibria. We find that for this game, the pattern of structures formed as costs increase depends on whether the underlying coalitional game is superadditive and/or convex. We find that in case the underlying game is not superadditive or in case it is convex, increasing costs for forming communication links result in the formation of fewer links in equilibrium. However, if the underlying game is superadditive but not convex, then increasing costs initially lead to the formation of fewer links, then to the formation of more links, and finally lead to the formation of fewer links again.

For the strategic-form game of link formation we briefly discuss the inappropriateness of Nash equilibria and strong Nash equilibria and then consider coalition-proof Nash equilibria. We find that for this game the pattern of structures formed as costs increase also depends on whether the underlying coalitional game is superadditive and/or convex. In contrast to the results for the extensive-from game of link formation, we find that in all cases increasing costs for forming communication links result in the formation of fewer links in equilibrium. We restrict our analysis of the formation of networks to symmetric

[1] The model studied by *Dutta*, *van den Nouweland* and *Tijs* (1998) was actually first mentioned in *Myerson* (1991).

3-player games for reasons of clarity of exposition, but we prove the existence of coalition-proof Nash equilibria for 3-players games in general to show that the analysis using the coalition-proof Nash equilibrium concept can be extended to such games.

The importance of network structures in the organization of many economic relationships has been extensively documented (see e.g. the references in *Goyal* (1993) and *Jackson* and *Wolinsky* (1996)). The game-theoretic literature on communication networks was initiated by *Myerson* (1977), who studied the influence of restrictions in communication on the allocation of the coalitional values in TU-games. The influence of the presence of communication restrictions on cooperative games has been studied by many authors since and an extensive survey on this subject can be found in *van den Nouweland* (1993).

In the current paper we study the formation of communication networks. Formally, a communication network (cf. *Myerson* (1977)) is a graph in which the players are the nodes and in which two players are connected by a communication link (an edge in the graph) if and only if they can communicate with each other in a direct and meaningful way. The game theoretic literature on the formation of communication networks includes a number of papers, including *Aumann* and *Myerson* (1988), *Goyal* (1993), *Dutta* and *Mutuswami* (1997), *Jackson* and *Wolinsky* (1996), *Watts* (1997), *Bala* and *Goyal* (1998), *Dutta*, *van den Nouweland* and *Tijs* (1998), and *Slikker* and *van den Nouweland* (1998).

The current paper is most closely related to *Aumann* and *Myerson* (1988) and *Dutta*, *van den Nouweland* and *Tijs* (1998). Both of these two papers study the formation of communication links in situations where the economic possibilities of the players can be described by a cooperative game. It is the models in these two papers that we use to study the formation of communication links in the current paper. However, in these two papers establishing communication links is costless, whereas we impose costs for forming communication links.

To our knowledge, the formation of communication networks when there are costs for forming communication links has only been studied in specific parametric models, as is the case in *Goyal* (1993), *Watts* (1997), *Bala* and *Goyal* (1998), and some examples in *Jackson* and *Wolinsky* (1996). The first three of these papers study the formation of networks within the framework of a simple parametric model of information transmission. These papers employ different processes of network formation and study the efficiency and stability of networks. *Jackson* and *Wolinsky* (1996) do not specify a specific model of network formation, but they study the stability and efficiency of networks in situations where self-interested agents can form and sever links. In their paper, a value function gives the value of each possible network and an exogenously given allocation rule is used to determine the payoffs to individual players for each possible network structure. They show that for anonymous and component balanced allocation rules efficient graphs need not be stable. The value function used by *Jackson* and *Wolinsky* (1996) allows for costs of communication links to be incorporated in the model in an indirect way.

Our paper differs from *Goyal* (1993), *Watts* (1997), and *Bala* and *Goyal* (1998) in the sense that we do not restrict ourselves to a specific parametric model. On the other hand, it is less general than *Jackson* and *Wolinsky* (1996) because, following *Aumann* and *Myerson* (1988) and *Dutta*, *van den Nouweland* and *Tijs* (1998), we restrict ourselves to situations in which the economic possibilities of the players can be described by a coalitional game. However, we explicitly model the costs of establishing communication links, rather than having those implicit in the value function. This allows us to study the influence of these costs.

The outline of the paper is as follows. In section 2 we provide general definitions concerning communication situations and allocation rules. In section 3 we compute the payoffs allocated to the players in different communication situations according to the extension of the Myerson value that we use as the external allocation rule in this paper. We describe and study the linking game in extensive form in section 4 and section 5 contains our study of the linking game in strategic form. We conclude in section 6.

2 Communication situations

In this section we will describe communication situations and an allocation rule for these situations, the Myerson value. Additionally, we will introduce communication costs and describe how these costs will be divided between the players.

A communication situation (N, v, L) consists of a cooperative game (N, v), describing the economic possibilities of all coalitions of players, and a communication graph (N, L), which describes the communication channels between the players. The characteristic function v assigns a value $v(S)$ to all coalitions $S \subseteq N$, with $v(\emptyset) = 0$. We will restrict ourselves to zero-normalized non-negative games, i.e. $v(\{i\}) = 0$ for all $i \in N$ and $v(S) \geq 0$ for all $S \subseteq N$. Communication is two-way and is represented by an undirected communication graph, i.e. the set of links L is a subset of $\bar{L} := \{\{i, j\} \mid \{i, j\} \subseteq N,\ i \neq j\}$. The communication graph (N, L) should be interpreted as a way to model restricted cooperation between the players. Players can only cooperate with each other if they are directly connected with each other, i.e. there is a link between them, or if they are indirectly connected, i.e. there is a path via other players that connects them. Note that indirect communication between two players requires the cooperation of the players on a connecting path between them as well. The communication structure (N, L) gives rise to a partition of the player set into groups of players who can communicate with each other. Two players belong to the same partition element if and only if they are connected with each other, directly or indirectly. A partition element is called a *communication component* and the set of communication components is denoted N/L. The communication graph (N, L) also induces a partition of each coalition $S \subseteq N$. [2] This partition is denoted by S/L and it consists of the communication components of the subgraph $(S, L(S))$, where $L(S)$ contains the communication links within S, i.e. $L(S) := \{\{i, j\} \in L \mid \{i, j\} \subseteq S\}$.

The restricted communication within a coalition $S \subseteq N$ influences the economic possibilities of the coalition. Cooperation between players in different communication components is not possible, so benefits of cooperation can only be realized within communication components. We define the value of coalition S in the communication situation (N, v, L) as the sum of the values of the communication components of S,

$$v^L(S) := \sum_{T \subseteq S/L} v(T).$$

The game (N, v^L) is usually called the *graph-restricted game*. The Myerson value of the communication situation (N, v, L) coincides with the Shapley value Φ (see *Shapley* (1953)) of the graph-restricted game,

$$\mu(N, v, L) = \Phi(N, v^L).$$

Myerson (1977) characterizes this rule using two properties, component balancedness and fairness.[3] Component balancedness states that the players in a communication component C divide the value of this communication component, $v(C)$, between them. Fairness states that the addition (deletion) of a link in a communication situation should have the same cardinal effect on the two players that form this link.

In the description of the model above, it is assumed that there are no costs for establishing communication links. In the following we will introduce such costs and integrate these in the analysis of the communication situations described above.

We will assume that the formation of a communication link between any two players results in a fixed cost $c \geq 0$. A straightforward way to distribute these costs among the two players is to divide them equally among the players who form the link. *Jackson* and *Wolinsky* (1996) study value functions, functions that assign a value to all subsets

[2] $S \subseteq N$ denotes that S is a subset of N, $S \subset N$ denotes that S is a strict subset of N.

[3] *Myerson* (1977) refers to component balancedness as component efficiency. We prefer to use component balancedness to avoid confusion with efficiency of graphs.

of links. They show that there is a unique fair and component balanced allocation rule for these situations. This allocation rule results in dividing the cost of each link equally among the players who form it if the value function that describes the costs is considered, i.e. the value function described by $v(A) = c|A|$ for all $A \subseteq L$. So, if the philosophy of the Myerson value is applied to divide the profits, then the payoff to player $i \in N$ in communication situation (N, v, L) will be the amount he would get according to the Myerson value minus his share of the costs of establishing communication links. Denoting the set of links player i is involved in by $L_i := \{\{i, j\} \mid \{i, j\} \in L\}$, his payoff is

$$\nu_i(N, v, L, c) := \mu_i(N, v, L) - \frac{1}{2}|L_i|c, \quad \text{for all } i \in N.$$

We will refer to allocation rule ν as the *cost-extended Myerson value*. In the following sections we will introduce costs of establishing communication links in two well-known models of link formation, the model of link formation in extensive form introduced by *Aumann* and *Myerson* (1988) and the model of link formation in strategic form introduced by *Myerson* (1991). Throughout this paper we mostly restrict ourselves to 3-person cooperative games where the worth of a coalition only depends on how many members it has and not on the identities of these members.

3 The cost-extended Myerson value for symmetric 3-player games

In this section we will compute the payoffs according to the cost-extended Myerson value for symmetric 3-player games and all possible communication structures between the three players of these games. Due to symmetry, we need to distinguish only 5 different positions a player can have in a communication graph. We will analyze the preferences of the players over these positions, depending on the underlying cooperative game and the costs of establishing communication links.

Let (N, v) be a symmetric 3-player game, i.e. there exist $w_1, w_2, w_3 \in \mathbb{R}$ such that $v(S) = w_{|S|}$ for all $S \subseteq N$ with $S \neq \emptyset$, and let c denote the non-negative communication costs.

In figure 1 on page 7 we distinguish 5 different positions in the set of graphs with 3 vertices. Position 1 is the isolated position. An isolated player receives zero payoff.[4] Note that in the graph with one link, one of the players is isolated:

$$\nu_i(N, v, \emptyset, c) = \nu_i(N, v, \{\{j, k\}\}, c) = 0 \quad \text{if } i \notin \{j, k\}. \tag{1}$$

[4] Recall that we restrict ourselves to zero-normalized games.

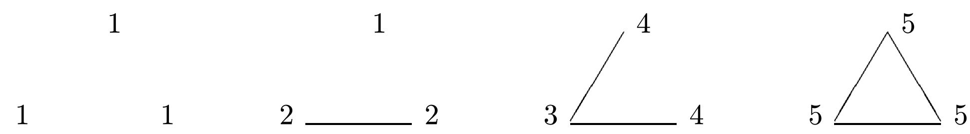

Figure 1: Different positions

Position 2 denotes the linked position in a graph with one link. The two linked players divide the value of a 2-person coalition and the costs,

$$\nu_j(N, v, \{\{j, k\}\}, c) = \frac{1}{2}w_2 - \frac{1}{2}c. \tag{2}$$

Position 3 is the central position in the graph with two links. A player in this position receives

$$\nu_i(N, v, \{\{i, j\}, \{i, k\}\}) = \frac{1}{3}w_3 + \frac{1}{3}w_2 - c. \tag{3}$$

Position 4 is the non-central position in the graph with two links. The payoff a player in this position receives equals

$$\nu_j(N, v, \{\{i, j\}, \{i, k\}\}, c) = \frac{1}{3}w_3 - \frac{1}{6}w_2 - \frac{1}{2}c. \tag{4}$$

Finally, position 5 represents a position in the graph with three links. In the graph with three links, every player receives

$$\nu_i(N, v, \bar{L}, c) = \frac{1}{3}w_3 - c. \tag{5}$$

Obviously, depending on (N, v) and c a player will have different preferences over the positions 1 through 5. The preferences of a player are described in table 1 on page 8.

If a condition in table 1 holds with equality then a player is indifferent between the positions while a reverse preference holds if the reverse inequality holds. In the next sections we will consider the influence of costs of establishing communication links in link formation games.

4 Linking game in extensive form

In this section we will introduce a slightly modified version of the linking game in extensive form that was introduced and studied by *Aumann* and *Myerson* (1988). The modification consists of the incorporation of costs of establishing communication links. Subsequently, following *Aumann* and *Myerson* (1988), we will study the subgame perfect Nash equilibria (SPNE) in this model. We provide an example that illustrates some curiosities that can arise and we also provide a systematic analysis of 3-player symmetric games.

Preference	Condition independent of c	Condition dependent on c
$1 \succ 2$		$c > w_2$
$1 \succ 3$		$c > \frac{1}{3}w_3 + \frac{1}{3}w_2$
$1 \succ 4$		$c > \frac{2}{3}w_3 - \frac{1}{3}w_2$
$1 \succ 5$		$c > \frac{1}{3}w_3$
$2 \succ 3$		$c > \frac{2}{3}w_3 - \frac{1}{3}w_2$
$2 \succ 4$	$2w_2 > w_3$	
$2 \succ 5$		$c > \frac{2}{3}w_3 - w_2$
$3 \succ 4$		$c < w_2$
$3 \succ 5$	$w_2 > 0$	
$4 \succ 5$		$c > \frac{1}{3}w_2$

Table 1: Preferences over different positions

4.1 The game

We will now describe a slightly modified version of the game in extensive form as it was introduced by *Aumann* and *Myerson* (1988). A TU-cooperative game (N, v) and a cost per link c are exogenously given and initially there are no communication links between the players. According to some exogenously given rule of order pairs of players get the opportunity to form a link. This link is actually formed if both players agree on forming this link. If a link is formed, it cannot be broken in a further stage of the game. After a pair of players have decided on whether or not to form a link, the next pair of players who have not yet formed a link get the opportunity to do so. After the last pair of players in the order have had the opportunity to form a link, the first pair of players in the order who have not yet formed a link get a new opportunity to form the link between them. The process stops when, after the last link that has formed, all pairs of players that have not yet formed a link have had a final opportunity to do so and declined this offer. This process results in a set of links. We will denote this set by L. The payoff to the players is then determined by the cost-extended Myerson value, i.e. if (N, L) is formed player i receives

$$\nu_i(N, v, L, c) = \mu_i(N, v, L) - \frac{1}{2}|L_i|c.$$

In the original model of *Aumann* and *Myerson* (1988) there are no costs for links and player i receives $\mu_i(N, v, L)$.

Aumann and *Myerson* (1988) already noted that the order in which two players in a pair decide whether or not to form a link has no influence. Furthermore, since the game is of perfect information it has subgame perfect Nash equilibria (see *Selten* (1965)).

4.2 An example

In this section we will consider the 3-player symmetric game (N, v) with

$$v(S) := \begin{cases} 0 & \text{if } |S| \leq 1 \\ 60 & \text{if } |S| = 2 \\ 72 & \text{if } S = N \end{cases} . \tag{6}$$

This game was analyzed by *Aumann* and *Myerson* (1988) who showed that in the absence of costs of establishing communication links every subgame perfect Nash equilibrium results in the formation of exactly one link. We will analyze the influence of link formation costs on the subgame perfect Nash equilibria of the model. The payoffs for the four classes of structures that can result follow directly from section 3. It holds that

$$\nu_i(N, v, \emptyset, c) = 0 \text{ for all } i \in N.$$

If one link results, the two players forming this link equally divide the value of a two-player coalition and the cost of the link:

$$\begin{aligned} \nu_i(N, v, \{\{j,k\}\}, c) &= 0; \\ \nu_j(N, v, \{\{j,k\}\}, c) &= 30 - \tfrac{1}{2}c. \end{aligned}$$

In case two links form, the payoffs are

$$\nu_j(N, v, \{\{i,j\}, \{j,k\}\}, c) = 44 - c;$$
$$\nu_i(N, v, \{\{i,j\}, \{j,k\}\}, c) = 14 - \tfrac{1}{2}c.$$

Finally, if all three links are formed all players receive the same payoff,

$$\nu_i(N, v, \bar{L}, c) = 24 - c \text{ for all } i \in N.$$

Aumann and *Myerson* (1988) study this example with $c = 0$. If two players, say i and j, form a link, they will each receive a payoff of 30. Certainly, both would prefer to form a link with the remaining player k, provided the other player does not form a link with player k, and receive 44. However, if player i forms a link with player k he can anticipate that subsequently players j and k will also form a link to get 24 rather than 14. So, both players i and j know that if one of them forms a link with player k they will end up with a payoff of 24, which is strictly less than 30, the payoff they receive if only the link between players i and j is formed. Hence, every subgame perfect Nash equilibrium results in the formation of exactly one link.

What will happen if establishing a communication link with another player is not free any more? One would expect that relatively small costs will not have very much influence and that larger costs will result in the formation of fewer links.

For small costs, say $c = 1$, we can repeat the discussion above and conclude that exactly one link will form. However, if the costs are larger the analysis changes. Assume for example that $c = 22$. Then, forming one link will result in a payoff of 19 for the two players forming the link, and the remaining player will receive 0. Forming two links will give the central player 22 and the other two players will receive 3 each. Finally, the full cooperation structure will give all players a payoff 2. We see that this changes the incentives of the players. Once two links are formed, the two players that are not linked with each other yet, prefer to stay in the current situation and receive 3 instead of forming a link and receive only 2. In case one link has been formed, a player who is already linked is now willing to form a link with the isolated player since this would increase his payoff (from 19 to 22) and the threat of ending up in the full cooperation structure has disappeared. Obviously, all players prefer forming some links to no link at all. Similar to the argument that in the absence of costs all three stuctures with one link are supported by a subgame perfect Nash equilibrium (see *Aumann* and *Myerson* (1988)), it follows that with communication costs equal to 22 all three structures with two links are supported by a subgame perfect Nash equilibrium.

The surprising result in this example is that an increase in the costs of establishing a communication link results in more communication between the players (2 links rather than 1). In the following subsection we will again see this result. We will also show that a further increase in the costs will result in a decrease in the number of links between the players.

4.3 Symmetric 3-player games

In this subsection we will describe the communication graphs that will result in 3-player symmetric games and with various levels of costs. The payoffs and preferences of the players can be found in section 3.

We will distinguish three classes of games that result in different communication structure patterns with changing costs of establishing communication links. First, assume that the game (N, v) satisfies $w_2 > w_3$. Then, combining the inequalities in table 1 we find the structures that are supported by subgame perfect Nash equilibria as a function of the costs of communication links. The results are summarized in figure 2 on page 11.

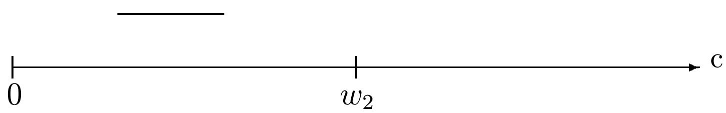

Figure 2: Communication structures according to SPNE in case $w_2 > w_3$

We note that on the boundary, i.e. $c = w_2$, both structures that appear for $c < w_2$ and for $c > w_2$ are supported by a subgame perfect Nash equilibrium. If $w_2 > w_3$ the full communication structure, in which all players are connected directly, will never form. Checking the preferences of the players, we see that the full communication structure would be formed only if $c < \frac{2}{3}w_3 - w_2$. Since $\frac{2}{3}w_3 - w_2 < 0$ and since costs of establishing a communication link are non-negative the full cooperation structure will not be formed.

Secondly, assume the game (N, v) satisfies $2w_2 > w_3 > w_2$. The structures resulting from subgame perfect Nash equilibria for this class of games are summarized in figure 3.

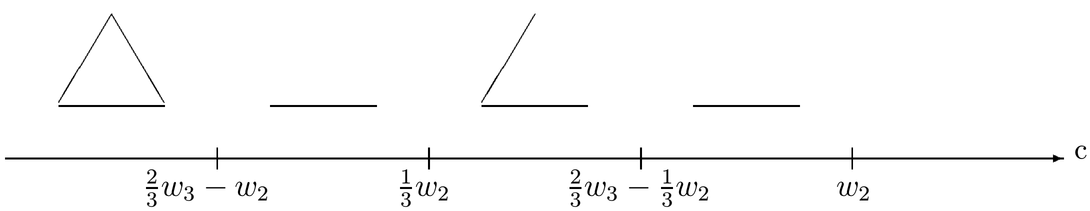

Figure 3: Communication structures according to SPNE in case $2w_2 > w_3 > w_2$

The example in subsection 4.2 belongs to this class of games. In that example $\frac{2}{3}w_3 - w_2 < 0$. Since the condition $2w_2 > w_3 > w_2$ can result in $\frac{2}{3}w_3 - w_2 < 0$ but also in $\frac{2}{3}w_3 - w_2 > 0$, we have not explicitly indicated $c = 0$ in figure 3.

Thirdly, consider the class of games satisfying $w_3 > 2w_2$. For these games the structures supported by subgame perfect Nash equilibria are summarized in figure 4 on page 12.

The discussion above makes a distinction between three classes of games. Note that if $w_2 = w_3$ then figures 2 and 3 lead to the same results since some of the boundaries coincide. If $w_3 = 2w_2$ then figures 3 and 4 lead to the same results.

The communication structure patterns above result in three classes of games. The first class, with games satisfying $w_2 > w_3$, contains only non-superadditive games. The

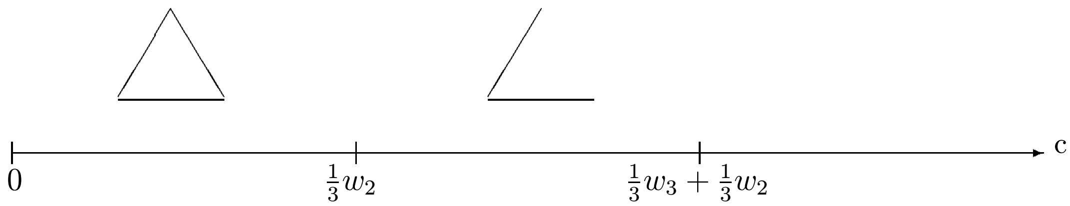

Figure 4: Communication structures according to SPNE in case $w_3 > 2w_2$

second class, defined by $2w_2 > w_3 > w_2$, contains only superadditive games that are not convex. The last class, with $w_3 > 2w_2$, contains only convex games.

We conclude that for non-superadditive games and for convex games increasing costs of establishing communication links results in a decreasing number of communication links, while for superadditive non-convex games increasing costs can result in more communication links.

5 Linking game in strategic form

In this section we will introduce costs of establishing communication links in the link formation game in strategic form that was introduced by *Myerson* (1991) and subsequently studied by *Qin* (1996), *Dutta*, *van den Nouweland*, and *Tijs* (1998), and *Slikker* (1998). We will analyze this model by means of the Nash equilibrium, strong Nash equilibrium, and coalition proof Nash equilibrium concepts.

5.1 The game

Let (N, v) be a cooperative game and c an exogenously given cost per link. The link formation game $\Gamma(N, v, c, \nu)$ is described by the tuple $(N; (S_i)_{i\in N}; (f_i^\nu)_{i\in N})$. For each player $i \in N$ the set $S_i = 2^{N\setminus\{i\}}$ is the strategy set of player i. A strategy of player i is an announcement of the set of players he wants to form communication links with. A communication link between two players will form if and only if both players want to form the link. The set of links that form according to strategy profile $s \in S = \prod_{i\in N} S_i$ will be denoted by

$$L(s) := \{\{i, j\} \subseteq N \mid i \in s_j,\ j \in s_i\}.$$

The payoff function $f^\nu = (f_i^\nu)_{i\in N}$ is defined as the allocation rule ν, the cost-extended Myerson value, applied to $(N, v, L(s), c)$,

$$f^\nu(s) = \nu(N, v, L(s), c).$$

In the original model of *Myerson* (1991) the players receive $\mu(N, v, L) = \nu(N, v, L, 0)$. *Dutta*, *van den Nouweland*, and *Tijs* (1998) study the undominated Nash and coalition proof Nash equilibria in this game. They show that in superadditive games the full communication structure will form or a structure that is payoff equivalent to it. *Slikker* (1998) shows a similar result for (strictly) perfect and (weakly/strictly) proper equilibria.

5.2 Nash equilibria and Strong Nash equilibria

In this section we study Nash equilibria and strong Nash equilibria. We present an example showing that many communication structures can result from Nash equilibria, while strong Nash equilibria do not always exist.

Recall that a strategy profile is a Nash equilibrium if there is no player who can increase his payoff by unilaterally deviating from it. A strategy profile is called a strong Nash equilibrium if there is no coalition of players that can strictly increase the payoffs of all its members by a joint deviation (*Aumann* (1959)).

Consider the following example. Let (N, v) be the symmetric 3-player game with

$$v(S) := \begin{cases} 0 & \text{if } |S| \leq 1 \\ 30 & \text{if } |S| = 2 \\ 48 & \text{if } S = N \end{cases} . \tag{7}$$

The payoffs to the players for the five positions we distinguished in figure 1 are summarized in table 2.

Position	Payoff
1	0
2	$15 - \frac{1}{2}c$
3	$26 - c$
4	$11 - \frac{1}{2}c$
5	$16 - c$

Table 2: Payoffs in different positions

If $c = 0$ every structure can be supported by a Nash equilibrium, since nobody wants to break a link and two players are needed to form a link.[5] If costs rise, fewer structures are supported by Nash equilibria. However, since a communication link can only be formed if two players want to do so, the communication structure with zero links is always supported by a Nash equilibrium.

[5] This was already proven for all superadditive games by *Dutta*, *van den Nouweland*, and *Tijs* (1998).

Since Nash equilibria can result in a fairly large set of structures, we consider the refinement to strong Nash equilibria for this game. Suppose that the costs per link are 10. We will show that no strong Nash equilibrium exists by considering all possible communication structures. First the communication structures with zero and three links cannot result from a strong Nash equilibrium since two players can deviate to a strategy profile resulting in only the link between them, improving their payoffs from 0 or 6 to 10. A structure with two links is not supported by a strong Nash equilibrium since the two players in the non-central positions can deviate to a strategy profile resulting in only the link between them and improve their payoffs from 6 to 10. Finally, a communication structure with one link is not supported by a strong Nash equilibrium since one player in a linked position and the player in the non-linked position can deviate to a strategy profile resulting in an additional link between them, increasing both their payoffs by 6. We conclude that strong Nash equilibria do not always exist.[6]

5.3 Coalition proof Nash equilibria

The multiplicity of structures resulting from Nash equilibria and the non-existence of strong Nash equilibria for many communication situations with costs for establishing communication links inspires us to consider an intermediate concept, coalition proof Nash equilibria. After giving the definition of coalition proof Nash equilibria, we first study an example and then continue with general 3-player symmetric games. We also show that coalition proof Nash equilibria always exist for 3-player games.

Before we can define the concept of *coalition proof Nash equilibrium* (CPNE) we will introduce some notation. Let $(N, (S_i)_{i \in N}, (f_i)_{i \in N})$ be a game in strategic form. For every $T \subset N$ and $\hat{s}^{N \setminus T} \in S_{N \setminus T}$, let $\Gamma(\hat{s}^{N \setminus T})$ be the game induced on the players of T by the strategies $\hat{s}^{N \setminus T}$, so

$$\Gamma(\hat{s}^{N \setminus T}) = (T, (S_i)_{i \in T}, (f_i^*)_{i \in T})$$

where for all $i \in T$, $f_i^* : S_T \to \mathbb{R}$ is given by $f_i^*(s^T) := f_i(s^T, \hat{s}^{N \setminus T})$ for all $s^T \in S_T$.

Now, coalition proof Nash equilibria are defined inductively. In a one-player game with player set $N = \{i\}$, $\hat{s}^i \in S = S_i$ is a CPNE of $\Gamma = (\{i\}, S_i, f_i)$ if $\hat{s}^i$ maximizes f_i over S_i. Let Γ be a game with $n > 1$ players. Assume that coalition proof Nash equilibria have been defined for games with less than n players. Then a strategy profile $\hat{s} \in S_N$ is called *self-enforcing* if for all $T \subset N$, $\hat{s}^T$ is a CPNE of $\Gamma(\hat{s}^{N \setminus T})$. Now, the strategy vector $\hat{s}$ is a CPNE of Γ if $\hat{s}$ is self-enforcing and there is no other self-enforcing strategy profile $s \in S_N$ with $f_i(s) > f_i(\hat{s})$ for all $i \in N$.

[6] *Dutta et al.* (1998) already note that in case $c = 0$ strong Nash equilibria might not exist.

The set of coalition proof Nash equilibria is a superset of the set of strong Nash equilibria. The strong Nash equilibrium concept demands that no coalition can deviate to a profile that strictly improves the payoffs of all players in the coalition. The coalition proof Nash equilibrium concept has similar requirements, but the set of allowed deviations is restricted. Every player in the deviating coalition should strictly improve his payoff and the strategy profile of the deviating players should be stable with respect to further deviations by subcoalitions.

5.3.1 CPNE in the linking game in strategic form

We start with an example to illustrate coalition proof Nash equilibria in the link formation game in strategic form. Consider the 3-player symmetric game (N, v) studied in subsection 4.2. Note that the payoffs to the players in the four classes of structures are also listed in that subsection.

If $c = 0$, it follows from *Dutta*, *van den Nouweland*, and *Tijs* (1998) that the full communication structure (with 3 links) is formed in the unique coalition-proof Nash equilibrium of the linking game in strategic form. To understand this, we consider the four classes of structures one-by-one. First note that the players would unilaterally like to form any additional links they can, which implies that in a Nash equilibrium s there can be no two players i and j such that $i \in s_j$ and $j \notin s_i$.

Hence, the structure with no links can only be formed in a Nash equilibrium if all 3 players state that they do not want to communicate with any of the other players, i.e. $s_i = s_j = s_k = \emptyset$. This strategy is not a CPNE, because two players i and j can deviate to $t_i = \{j\}$ and $t_j = \{i\}$ and form the link between them to get 30 rather than 0 and then neither one of these players wants to deviate further.

A structure with one link, say link $\{i, j\}$, can only be formed in a Nash equilibrium if $s_i = \{j\}$, $s_j = \{i\}$, and $s_k = \emptyset$. But players i and k have an incentive to deviate to the strategies $t_i = \{j, k\}$ and $t_k = \{i\}$ and form an additional link. This will give player i 44 rather than 30 and player k 14 rather than 0 and neither i nor k wants to deviate further because they do not want to break links and they cannot form new links. This shows that a structure with one link will not be formed in a CPNE.

In a Nash equilibrium, a structure with two links, say $\{i, j\}$ and $\{j, k\}$, can only be formed if $s_i = \{j\}$, $s_j = \{i, k\}$, and $s_k = \{j\}$. But players i and k have an incentive to deviate to the strategies $t_i = \{j, k\}$ and $t_k = \{i, j\}$ and form an additional link, so that they will each get 24 rather than 14. They will not want to deviate further, since this can only involve breaking links.

So, the only structure that can possibly be supported by a CPNE is the full communication structure. Suppose $s_i = \{j,k\}, s_j = \{i,k\}$, and $s_k = \{i,j\}$. The only deviations from these strategies that give all deviating players a higher payoff, are deviations by two players who break the links with the third player and induce the structure with only the link between themselves. Suppose players i and j deviate to the strategies $t_i = \{j\}$ and $t_j = \{i\}$ which will give both players 30 rather than 24. Then player i has an incentive to deviate further to $u_i = \{j,k\}$, in which case links $\{i,j\}$ and $\{i,k\}$ will be formed and player i will get 44 instead of 30. This shows that deviations from s by two players are not stable against further deviations by subcoalitions of the deviating coalition. Hence, s is a CPNE.

What will happen in this example if establishing communication links is not costless? Of course, for small costs, there will only be minor changes to the discussion above and the conclusion will be unchanged. But if the costs are larger, then some of the deviations that were previously taken into consideration will no longer be attractive. Suppose for instance that $c = 24$. Then *all* players will prefer a structure with two links above the structure with three links, in which they all get 0. In a structure with two links, no player wants to break any links, since this will reduce his or her payoff by 2. Hence, for these costs, exactly two links will be formed in a CPNE.

We now continue with the description of coalition proof Nash equilibria in symmetric 3-player games. Table 3 provides an overview of coalition proof Nash equilibria depending on the position a player prefers most.

Most preferred position	Additional condition	Structure resulting from CPNE
1		0 links
2		1 link
3	$5 \succ 4$	3 links
3	$4 \succ 5$	2 links
4	$\nu_3(N,v,L,c) > 0$	2 links
4	$\nu_3(N,v,L,c) < 0$	0 links
5		3 links

Table 3: Coalition proof Nash equilibria depending on preferences of the players

This table can be used to determine the coalition proof Nash equilibria for the three classes of games we distinguished in subsection 4.3. The following figures describe the communication structures resulting from coalition proof Nash equilibria. Figure 5 on page 17 describes the structures resulting from CPNE for the class of games containing only non-superadditive games, figure 6 on page 17 is for the class of games containing only superadditive but non-convex games, and figure 7 deals with the class of games containing only convex games.

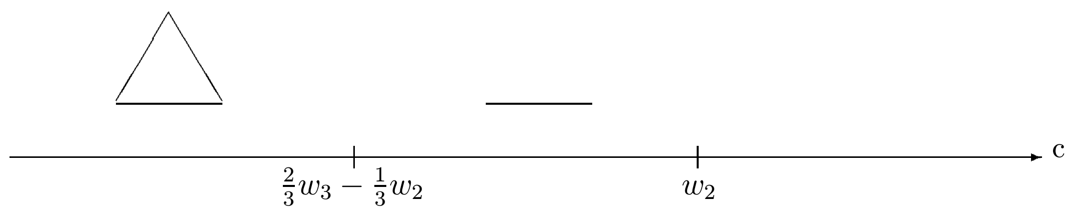

Figure 5: Communication structures according to CPNE in case $w_2 > w_3$

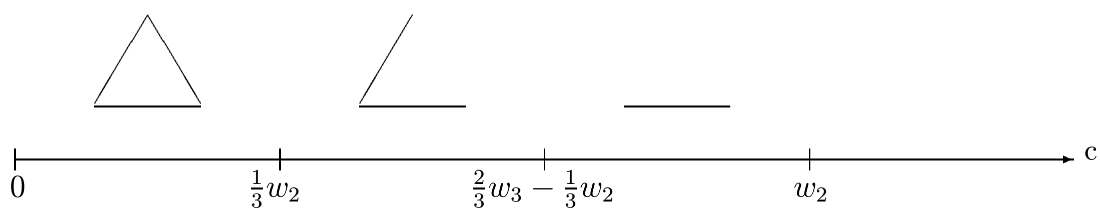

Figure 6: Communication structures according to CPNE in case $2w_2 > w_3 > w_2$

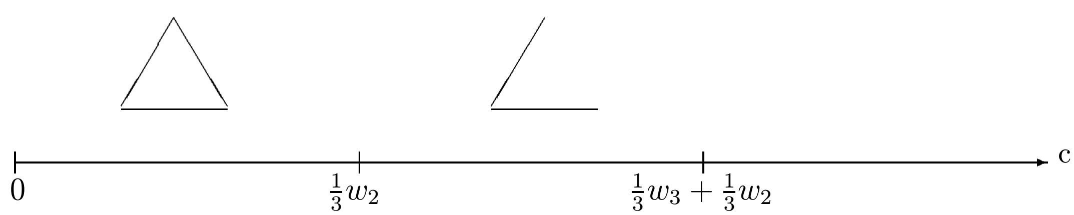

Figure 7: Communication structures according to CPNE in case $w_3 > 2w_2$

5.3.2 Comparison to the linking game in extensive form

Comparing figures 2, 3, and 4 to figures 5, 6, and 7, respectively, we find that the predictions according to SPNE in the extensive-form model and those according to CPNE in the strategic-form model are remarkably similar.

For a class containing only convex games ($w_3 > 2w_2$), both models generate exactly the same predictions (see figures 4 and 7).

For non-superadditive games, we get almost the same predictions. The only difference between figures 2 and 5 is that the level of costs that marks the transition form the full communication structure to a structure with one link is possibly positive ($\frac{2}{3}w_3 - \frac{1}{3}w_2$) in the extensive-form model, whereas it is negative ($\frac{2}{3}w_3 - w_2$) in the strategic-form model.[7]

The predictions of both models are most dissimilar for a class containing only superadditive non-convex games ($2w_2 > w_3 > w_2$). In the extensive-form model we get a structure with one link in case $\frac{2}{3}w_3 - w_2 < c < \frac{1}{3}w_2$ (see figure 3), whereas in the strategic-form model for these costs we get the full communication structure (see figure 6). For lower costs we find the full communication structure for both models.

5.3.3 Existence of CPNE in general 3-player games

This subsection is devoted to the existence of CPNE for general 3-player games. Hence, we extend the scope of our investigation beyond symmetric games. We do, however, still restrict ourselves to zero-normalized non-negative games. For convenience, we will assume (without loss of generality) that

$$v(\{1,2\}) \geq v(\{1,3\}) \geq v(\{2,3\}).$$

Throughout the rest of this section we call a deviation by a coalition profitable if it strictly improves the payoffs of all deviating players. A deviation is called stable if the deviation is a coalition proof Nash equilibrium in the subgame induced on the coalition of deviating players by the strategies of the other players, i.e. a deviation from strategy profile s by coalition T is stable if it is a CPNE in the game $\Gamma(s^{N\backslash T})$ as defined on page 14. A deviation is called self-enforcing if this deviation is self-enforcing in the subgame induced on the coalition of deviating players by the strategies of the other players, i.e. a deviation from strategy profile s by coalition T is self-enforcing if it is self-enforcing in the game $\Gamma(s^{N\backslash T})$

The following lemmas will be used in the proof of existence of coalition proof Nash equilibria in 3-player games.

Lemma 5.1 Let $\Gamma(N, v, c, \nu)$ be a 3-player link formation game with $c < \frac{2}{3}v(N) + \frac{1}{3}v(\{1,3\}) - \frac{2}{3}v(\{1,2\})$. Let $\bar{s}$ be the strategy profile with $\bar{s}_i = N\backslash\{i\}$ for all $i \in N$ which results in the full communication structure. Let $i, j \in N$. Then the deviation from $\bar{s}$ by $\{i,j\}$ given by $(s_i, s_j) = (\{j\}, \{i\})$ is not stable.

[7] See the discussion of figure 2 on page 11.

Proof: We will show that there exists a further deviation of (s_i, s_j) which is profitable and stable, implying that (s_i, s_j) is not stable. First, assume $\{i, j\} = \{1, 2\}$. Consider a further deviation $t_1 = \{2, 3\}$ by player 1. Then[8]

$$f_1^\nu(t_1, s_2, \bar{s}_3) = \nu_1(\{\{1,2\},\{1,3\}\}) = \frac{1}{3}v(N) + \frac{1}{6}v(\{1,3\}) + \frac{1}{6}v(\{1,2\}) - c$$
$$> \frac{1}{2}v(\{1,2\}) - \frac{1}{2}c = \nu_1(\{\{1,2\}\}) = f_1^\nu(s_1, s_2, \bar{s}_3),$$

where the inequality follows since $c < \frac{2}{3}v(N) + \frac{1}{3}v(\{1,3\}) - \frac{2}{3}v(\{1,2\})$. Since the strategy space of a player is finite there exists a strategy of player 1 that maximizes his payoff, given strategies $(s_2, \bar{s}_3)$ of players 2 and 3. This strategy is a profitable and stable deviation from (s_1, s_2). We conclude that (s_1, s_2) is not stable.

Similarly, by considering $t_1 = \{2, 3\}$ we find that there exists a profitable and stable further deviation if $\{i, j\} = \{1, 3\}$ and considering $t_2 = \{1, 3\}$ implies that there exists a profitable and stable further deviation if $\{i, j\} = \{2, 3\}$. In both cases we use that $v(\{1,2\}) \geq v(\{1,3\}) \geq v(\{2,3\})$. □

Lemma 5.2 Let $\Gamma(N, v, c, \nu)$ be a link formation game. Let s be a strategy profile. If there exists a profitable and self-enforcing deviation from s by N, then the game has a CPNE.

Proof: Suppose t^1 is a deviation from s by N that is profitable and self-enforcing. Since t^1 is a self-enforcing deviation by N, there exists no profitable and stable deviation from t^1 by any $S \subset N$. If there is no profitable and self-enforcing deviation from t^1 by N then t^1 is a CPNE. If t^2 is a profitable and self-enforcing deviation from t^1 by N, then

$$f^\nu(t^2) > f^\nu(t^1) > f^\nu(s).$$

Repeat the process above to find a sequence $(t^1, t^2, t^3, \ldots)$ such that t^k is a profitable and self-enforcing deviation from t^{k-1} for all $k \geq 2$. It holds that

$$f^\nu(t^1) < f^\nu(t^2) < f^\nu(t^3) < \ldots.$$

Since the strategy space of every player is finite this process has to end in finitely many steps. The last strategy profile in the sequence is a CPNE. □

We can now prove that coalition proof Nash equilibria exist in 3-player link formation games in strategic form.

[8]If there is no ambiguity about (N, v, c) we simply write $\nu(L)$ instead of $\nu(N, v, L, c)$.

Theorem 5.1 Let (N, v) be a 3-player cooperative game and let $c \geq 0$ be the costs of establishing a communication link. Then there exists a coalition proof Nash equilibrium in the link formation game $\Gamma(N, v, c, \nu)$.

Proof: If $(\emptyset, \emptyset, \emptyset)$ is a CPNE we are done. From now on assume $(\emptyset, \emptyset, \emptyset)$ is not a CPNE.

Hence, there exists a profitable and stable deviation from $(\emptyset, \emptyset, \emptyset)$ by some $T \subset N$ or a profitable and self-enforcing deviation by N. If there exists a profitable and self-enforcing deviation by N it follows by lemma 5.2 that we are done. So, from now on assume there exists no profitable and self-enforcing deviation from $(\emptyset, \emptyset, \emptyset)$ by N. Hence there exists a profitable and stable deviation from $(\emptyset, \emptyset, \emptyset)$ by some $T \subset N$. Since a player cannot unilaterally enforce the formation of a link, we conclude that there exists a profitable and stable deviation by a coalition with (exactly) two players.

So, there exists a profitable and stable deviation from $(\emptyset, \emptyset, \emptyset)$ by 2 players, say i and j. The structures players i and j can enforce are the structure with no links and the structure with link $\{i, j\}$. Since the structure with no links does not change their payoffs, it follows that this profitable and stable deviation results in link $\{i, j\}$. This deviation is profitable and stable iff $v(\{i, j\}) > c$.[9] Since, $v(\{1, 2\}) \geq v(\{i, j\}) > c$ it follows that $(s_1, s_2) = (\{2\}, \{1\})$ is a profitable and stable deviation from $(\emptyset, \emptyset, \emptyset)$ and that $s = (\{2\}, \{1\}, \emptyset)$ is a Nash equilibrium. If s is a CPNE in the game $\Gamma(N, v, c, \nu)$ we are done. So, from now on assume that s is not a CPNE.

Hence, there exists a profitable and stable deviation from s by some $T \subset N$ or a profitable and self-enforcing deviation by N. However, no profitable and self-enforcing deviation by 3 players exists, since this would be a profitable and self-enforcing deviation from $(\emptyset, \emptyset, \emptyset)$. Since s is a Nash equilibrium, we derive that there exists a profitable and stable deviation by a coalition with (exactly) two players. Since coalition $\{1, 2\}$ can only break link $\{1, 2\}$, it follows that there exists a profitable and stable deviation from s by coalition $\{1, 3\}$ or by coalition $\{2, 3\}$. We will distinguish between these two cases.

CASE A: There exists a profitable and stable deviation from s by coalition $\{1, 3\}$, say (t_1, t_3). Since $v(\{1, 2\}) \geq v(\{1, 3\})$ it follows that the deviation from s cannot result in link $\{1, 3\}$ alone, since this would not improve the payoff of player 1. Hence, the deviation results in links $\{1, 2\}$ and $\{1, 3\}$, the only two links that can be enforced by players 1 and 3, given the strategy of player 2. Note that such a deviation is profitable if and only if

$$c < \frac{2}{3}v(N) - \frac{2}{3}v(\{1, 2\}) + \frac{1}{3}v(\{1, 3\}). \tag{8}$$

[9] We remind the reader that we restrict ourselves to zero-normalized games.

So, inequality (8) must hold. Since a further deviation by player 1 or player 3 can only result in breaking links, it follows that $(t_1, t_3) = (\{2,3\}, \{1\})$ is a profitable and stable deviation from s. Also, $\nu_2(\{\{1,2\},\{1,3\}\}) \geq \nu_3(\{\{1,2\},\{1,3\}\}) > 0$, where the weak inequality follows since $v(\{1,2\}) \geq v(\{1,3\})$ and the strict inequality follows by inequality (8). It follows that (t_1, s_2, t_3) is a Nash equilibrium, since unilaterally player 2 can only break link $\{1,2\}$. If (t_1, s_2, t_3) is a CPNE in the game $\Gamma(N, v, c, \nu)$ we are done. From now on assume (t_1, s_2, t_3) is not a CPNE.

Since coalitions $\{1,2\}$ and $\{1,3\}$ cannot enforce an additional link, they cannot make a profitable and stable deviation from (t_1, s_2, t_3). There exists no profitable and self-enforcing deviation by N from (t_1, s_2, t_3) since this would be a profitable and self-enforcing deviation from $(\emptyset, \emptyset, \emptyset)$. So, there exists a profitable and stable deviation from (t_1, s_2, t_3) by coalition $\{2,3\}$, say (u_2, u_3). Since both players receive a positive payoff according to (t_1, s_2, t_3), any profitable deviation results in at least the formation of link $\{2,3\}$. Since player 3 receives at least as much in the structure with links $\{1,2\}$ and $\{1,3\}$ as in the structure with links $\{1,2\}$ and $\{2,3\}$ this last structure will not form after deviation (u_2, u_3). Similarly, since player 2 receives at least as much in the structure with links $\{1,2\}$ and $\{1,3\}$ as in the structure with links $\{1,3\}$ and $\{2,3\}$ this last structure will not form after deviation (u_2, u_3). Finally, player 2 prefers the communication structure with links $\{1,2\}$ and $\{2,3\}$ above the communication structure with link $\{2,3\}$ since

$$c < \frac{2}{3}v(N) - \frac{2}{3}v(\{2,3\}) + \frac{1}{3}v(\{1,2\}),$$

where the inequality follows from inequality (8) and $v(\{1,2\}) \geq v(\{1,3\}) \geq v(\{2,3\})$. So, the deviation by players 2 and 3 to the communication structure with link $\{2,3\}$ alone will not be stable. We conclude that t_1 and deviation (u_2, u_3) together result in the full communication structure. We will show that (t_1, u_2, u_3) is a CPNE in the game $\Gamma(N, v, c, \nu)$. The deviation (u_2, u_3) from (t_1, s_2, t_3) is profitable iff $v(\{2,3\}) > 3c$. But, if $v(\{2,3\}) > 3c$ there is no profitable deviation from (t_1, u_2, u_3) to a structure with two links since $v(\{1,2\}) \geq v(\{1,3\}) \geq v(\{2,3\}) > 3c$. By lemma 5.1 and inequality (8) it follows that there is no profitable and stable deviation from (t_1, u_2, u_3) to a structure with one link. Since $v(\{1,2\}) \geq v(\{1,3\}) \geq v(\{2,3\}) > 3c > 0$ it follows that a deviation to the communication structure with no links cannot be stable. We conclude that (t_1, u_2, u_3) is a CPNE, showing the existence of a CPNE in the game $\Gamma(N, v, c, \nu)$ in CASE A.

CASE B: There exists a profitable and stable deviation from s by coalition $\{2,3\}$, say (t_2, t_3). Since $v(\{1,2\}) \geq v(\{2,3\})$ it follows that the deviation from s cannot result in link $\{2,3\}$ alone. Hence, the deviation results in links $\{1,2\}$ and $\{2,3\}$, the only two links that can be enforced by players 2 and 3, given the strategy of player 1. Note that such a deviation is profitable if and only if

$$c < \frac{2}{3}v(N) - \frac{2}{3}v(\{1,2\}) + \frac{1}{3}v(\{2,3\}). \tag{9}$$

However, since $v(\{2,3\}) \leq v(\{1,3\})$ it follows that inequality (9) implies inequality (8). Hence, there exists a profitable and stable deviation from s by coalition $\{1,3\}$. Then CASE A applies and we conclude that a CPNE in the game $\Gamma(N, v, c, \nu)$ exists.

This completes the proof of the theorem. □

6 Conclusions

In this paper, we explicitly studied the influence of costs for establishing communication links on the communication structures that are formed in situations where the underlying economic possibilities of the players are given by a cooperative game. To do so, we considered two existing models of the formation of communication networks, the extensive-form model of *Aumann* and *Myerson* (1988) and the strategic-form model studied by *Dutta*, *van den Nouweland* and *Tijs* (1998). For these models, we studied how the communication networks that are formed change as the costs for establishing links increase. In order to be able to isolate the influence of the costs, we assumed that costs are equal for all possible communication links. We restricted our analysis to 3-player symmetric games because our proofs involve explicit computations and this of course puts severe restrictions on the type of situations that we can analyze while not loosing ourselves and the readers in complicated notation. The proof of the existence of coalition-proof Nash equilibria in the strategic-form game of link formation for general 3-players games provides a glimpse of the type difficulties that we would have to deal with if we extended our analysis beyond symmetric games.

In the extensive-form game of link formation of *Aumann* and *Myerson* (1988), we considered communication structures that are formed in subgame perfect Nash equilibria. We find that for this game, the pattern of structures formed as costs increase depends on whether the underlying coalitional game is superadditive and/or convex. In case the underlying game is not superadditive or in case it is convex, increasing costs for forming communication links result in the formation of fewer links in equilibrium. However, if the underlying game is superadditive but not convex, then increasing costs initially lead to the formation of fewer links, then to the formation of more links, and finally lead to the formation of fewer links again. This is, in our view, the most surprising result of the paper. It shows that subsidizing the formation of links does not necessarily lead to more links being formed. Hence, authorities wishing to promote more cooperation cannot always rely on subsidies to accomplish this goal. In fact, such subsidies might have an adverse effect.

For the strategic-form game of link formation studied by *Dutta*, *van den Nouweland* and *Tijs* (1998) we briefly discussed the inappropriateness of Nash equilibria and strong Nash equilibria and went on to consider coalition-proof Nash equilibria. We find that for this game the pattern of structures formed as costs increase also depends on whether the underlying coalitional game is superadditive and/or convex. In contrast to the results for the extensive-from game of link formation, we find that in the strategic-form model in all cases increasing costs for forming communication links result in the formation of fewer links in equilibrium. The discussion of subsection 5.3.2 shows that the results we obtain for the two models are otherwise remarkably similar. Even though we restrict our analysis of the formation of networks to symmetric 3-player games for reasons of clarity of exposition, we prove the existence of coalition-proof Nash equilibria for 3-players games in general to show that the analysis using the coalition-proof Nash equilibrium concept can be extended to such games.

In order to follow the analyses in *Aumann* and *Myerson* (1988) and *Dutta*, *van den Nouweland* and *Tijs* (1998) as closely as possible, we extended the Myerson value to situations in which the formation of links is not costless. We did so in a manner that is consistent with the philosophy of the Myerson value. The Myerson value was introduced by *Myerson* (1977) as the unique allocation rule satisfying component balancedness and fairness. Myerson's analysis was restricted to situations in which the formation of communication links is costless. *Jackson* and *Wolinsky* (1996) note that Myerson's result can be extended to situations in which a value function describes the economic possibilities of the players in different networks (see *Jackson* and *Wolinsky*, theorem 4 on page 65). It seems reasonable to view the unique allocation rule for such situations that is component balanced and fair as the natural extension of the Myerson value. Since costs for forming links can be implicitly taken into account using value functions, this extension of the Myerson value can be used to determine allocations when an underlying cooperative game describes the economic possibilities of the players and in which there are costs for forming links. It is this allocation rule that we use.

Jackson and *Wolinsky* (1996) establish that there is a conflict between efficiency and stability if the allocation rule used is component balanced. Indeed, we see many illustrations of this result in the current paper. For example, for the strategic-form model of link formation we find in section 5.3 that for small costs all links will be formed.[10]

[10]For costs equal to zero, this follows directly from the results obtained by *Dutta*, *van den Nouweland*, and *Tijs* (1998).

This is clearly not efficient, because the (costly) third link does not allow the players to obtain higher economic profits. Rather, building this costly link diminishes the profits of the group of players as a whole. It is formed only because it influences the allocation of payoffs among the players. The formation of two links in case the game is superadditive (see figures 6 and 7) is promising in this respect. However, from an efficiency point of view these should be formed if $w_3 - 2c > w_2 - c$, or $c < w_3 - w_2$, and the cutoffs in figures 6 and 7 appear at different values for c.

7 References

Aumann, R. (1959). *Acceptable points in general cooperative n-person games.* In: Contributions to the theory of games IV (Eds. A. Tucker and R. Luce), Princeton University Press, 287-324.

Aumann, R. and R. Myerson (1988). *Endogenous formation of links between players and coalitions: an application of the Shapley value.* In: The Shapley value (Ed. A. Roth), Cambridge University Press, Cambridge, United Kingdom, 175-191.

Bala, V. and S. Goyal (1998). *Self-Organization in Communication Networks.* Working paper.

Dutta, B., A. van den Nouweland, and S. Tijs (1998). *Link formation in cooperative situations.* International Journal of Game Theory 27, 245-256.

Dutta, B. and S. Mutuswami (1997). *Stable Networks.* Journal of Economic Theory 76, 322-344.

Goyal, S. (1993). *Sustainable Communication Networks.* Discussion Paper TI 93-250, Tinbergen Institute, Erasmus University, Rotterdam.

Jackson, M. and A. Wolinsky (1996). *A Strategic Model of Social and Economic Networks.* Journal of Economic Theory 71, 44-74.

Myerson, R. (1977). *Graphs and cooperation in games.* Mathematics of Operations Research 2, 225-229.

Myerson, R. (1991). *Game Theory: Analysis of Conflict.* Harvard University Press, Cambridge, Massachusetts.

Qin, C. (1996). *Endogenous formation of cooperation structures.* Journal of Economic Theory 69, 218-226.

Selten, R. (1965). *Spieltheoretische behandlung eines oligopolmodells mit nachfrage-traegheit.* Zeitschrift für die gesamte staatswissenschaft 121, 301-324, 667-689.

Shapley, L. (1953). *A value for n-person games.* Contributions to the theory of games II (Eds. A. Tucker and H. Kuhn), 307-317.

Slikker, M. (1998). *A note on link formation.* CentER Discussion Paper 9820, Tilburg University, Tilburg, The Netherlands.

Slikker, M. and A. van den Nouweland (1998). *A one-stage model of link formation and payoff division.* Working paper.

van den Nouweland, A. (1993). *Games and graphs in economic situations.* Ph. D. Dissertation, Tilburg University Press, Tilburg, The Netherlands.

Watts, A. (1997). *A Dynamic Model of Network Formation.* Working paper.