Customized Pull Systems for Single-Product Flow Lines

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Traditionally pull production systems are managed through classic control systems such as Kanban, Conwip, or Base stock, but this paper proposes ‘customized’ pull control. Customization means that a given production line is managed through a pull control system that in principle connects each stage of that line with each preceding stage; optimization of the corresponding simulation model, however, shows which of these potential control loops are actually implemented. This novel approach may result in one of the classic systems, but it may also be another type: (1) the total line may be decomposed into several segments, each with its own classic control system (e.g., segment 1 with Kanban, segment 2 with Conwip); (2) the total line or segments may combine different classic systems; (3) the line may be controlled through a new type of system. These different pull systems are found when applying the new approach to a set of twelve production lines. These lines are configured through the application of a statistical (Plackett-Burman) design with ten factors that characterize production lines (such as line length, demand variability, and machine breakdowns).

(Pull Production / Inventory; Sampling; Optimization; Evolutionary Algorithm)
JEL classification: C0

1. Introduction

Much research has already been performed in the field of production control and inventory management. Production control has a major influence on the performance of a manufacturing company, notably in terms of inventory, production delays, and makespan. Thus, the competitiveness
of a company relies partly on its production control strategy. There are two general types of strategy: push control and pull control. The former has a production plan made according to estimates for demand and lead-time. The latter uses demand occurrences to control production. This paper focuses on pull control applied to production lines processing a single part type. In such a system, a workstation can produce if and only if the workstation is idle and an authorization (e.g., a card or Kanban) is available. An authorization originates from a workstation downstream in the production line, when a part is removed from its input inventory; see Figure 1 for a simple illustration. The number of authorizations circulating between machines $i$ and $i+k$ determines the maximum Work In Process (WIP) between these two stages.

Kimura and Terada (1981) claim that the multiple objectives of a pull system are to minimize WIP's average and fluctuations, shorten production lead-time, prevent amplified transmission of demand fluctuations from stage to stage, enhance control through decentralization, react faster to demand changes, and reduce defects. These objectives explain why pull production has received such a growing interest from both companies and researchers, after its formalization at the end of the 1950s.

Pull systems can be specified by the way demand information flows through the production system. Gstettner and Kuhn (1996) identified three basic pull systems, namely Kanban, Conwip, and Base stock; they also considered the serial combination of these pull systems within a production line. In the next section, we describe these three basic systems and their possible serial combinations; we call these combinations, segmented systems; we also consider joint systems, that is, combinations of control systems on the same segment of a production system. In section 3 we propose 'customized' pull control. Customization means that a given production line is managed through a pull control system that in principle connects each stage of that particular line with each preceding stage; optimization, however, shows which of these potential control loops are actually implemented. In section 4 we define a set of twelve production lines, which are compatible with models found in the literature on pull systems. Section 5 describes the solutions obtained when applying our customization to the set of lines. In section 6 we analyze these solutions, and propose new structures of pull control that generalize our results.

2. Pull production control systems

2.1. Three basic pull systems

(i) Kanban

Dr. Taichi Ohno, manager at the Toyota Motors company, has developed the Kanban strategy. The principle is to limit the inventory level at each stage of a process, by defining control loops between each pair of consecutive stages (Monden, 1993); see figure 2. Cards materialize authorizations, but
there are many implementation forms of Kanban. Indeed, several papers propose adaptations of the original strategy. Berkley (1992) proposes a classification of Kanban system models; he uses operational design criteria such as the blocking mechanism, the withdrawal strategy, and the type of Kanban cards. Huang and Kusiak (1996) survey various Kanban systems and alternatives, and classify the previous studies. Chu and Shih (1992) compare numerous simulation studies on Just-In-Time (JIT) production systems. Price, Gravel, and Nsakanda (1994) review optimization models of Kanban systems. Sing and Brar (1992) classify various models. Gaury, Pierreval, and Kleijnen (1997) take stock of the way modeling techniques and simulation are used to study Kanban systems.

The benefits that Japanese companies obtained through the Kanban method were reported to be important, so researchers tried to compare the Kanban method with classical methods such as Material Requirement Planning (MRP), order point systems, and other push-type systems. These studies agree on one fact: a Kanban system is very efficient in an ideal environment, that is, an environment with low process and demand variability, few breakdowns, etc. In a typical Western environment, however, the Kanban method is much less efficient. Some studies even conclude that in such an environment push systems perform better than pull systems. Huang, Rees, and Taylor (1983) emphasize that high production rates can be realized only when the number of Kanban is chosen optimally. Thus, even in unfavorable environments, optimized Kanban systems may perform almost as well as push systems, in terms of output, but with a WIP level always lower in terms of average and variability.

(ii) Conwip
Conwip stands for Constant Work In Progress. Spearman, Woodruff, and Hopp (1990) proposed the name Conwip, but Bertrand (1983) proposed a similar approach under the name of workload control. Both approaches can be considered as capacity-based order review/release (ORR) strategies; see Philipoom and Fry (1992) for a short review of ORR strategies.

The objective of Conwip is to combine the low inventory levels of Kanban with the high throughput of Push. One way to achieve this objective is to consider a Push system that has only a limited number of parts allowed into it: raw materials can be released into the system only when the last stage asks for it (Pull principle). This limitation is implemented through a single control loop that
links the last stage to the first one. Within the system, each stage can produce as fast as it can (Push principle). Similarly, workload control is an order release strategy that aims at maintaining a specific workload level of the production system. Comparing figures 2 and 3 shows that Conwip's implementation is much simpler than Kanban's: there are fewer control loops. Thus, modeling and optimization are easier. Actually, a Conwip system can be viewed as a Kanban system with a single loop that controls the whole production line.

Roderick, Phillips, and Hogg (1992) and Roderick, Toland, and Rodriguez (1994) compare Conwip with MRP and three order release strategies. They conclude that Conwip gives the best performance measured in mean WIP, mean throughput, and proportion of tardy jobs. Therefore, Roderick et al. (1992) recommend Conwip as a 'strategy that should be seriously considered by practitioners for implementation in actual shop environments'. De Koster and Wijngaard (1989) compare the performance of local control (Kanban) and integral control (workload control and base stock) for production lines. They did not find any significant difference in performance between these types of control.

(iii) Base stock
There are several definitions of base stock. This policy was developed in the 1950s, and has been extensively used by practitioners ever since. Bonvik, Couch and Gershwin (1998) and Lee and Zipkin (1992) consider the base stock policy described in Kimball (1988); see figure 4. Each stage of the production line holds an initial amount of inventory $S_i$, called the base stock level. Any demand for finished products immediately triggers demands at each preceding stage: demand information is broadcasted from the last stage to each stage in the production line. Demands that cannot be filled from stock, are backordered. There is no upper bound for the inventory level at each stage. The base stock levels $S_i$ are the only parameters of the base stock policy.

An advantage of base stock is its responsiveness to demand: as soon as a demand occurs, all the stages can start working simultaneously. A drawback, however, is that consecutive stages are not
coordinated: if one stage fails, preceding stages do not stop working. A solution is to limit the amount of inventory, using authorizations as in Kanban. When a finished good is delivered, one card is sent to each stage of the production line, thereby allowing stages to produce. Such a system is called an \textit{Integral Control System}; see Buzacott and Shanthikumar (1993).

2.2. Segment systems

The three basic pull systems (described in section 2.1) can be combined along a production line. The idea is to segment the production system into cells, and use a specific control strategy per cell. This is a recent approach, and much research remains to be done. The main issues concern the definition of cells and the configuration of the control strategy per cell.

Figure 5 shows a variety of segmented control systems, introduced in the literature. Ettl and Schwehm (1994) and Di Mascolo, Frein, and Dallery (1996) consider Kanban systems with each loop controlling a set of machines. Such systems can be viewed as segmented Conwip systems. Gstettner and Kuhn (1996) propose segmented Kanban/Conwip and Base stock systems, but they do not study such systems nor do they propose a design procedure. Segmented push/pull systems have also been
investigated. For instance, Cochran and Kim (1998) consider a production line that is controlled partly through MRP and partly through a pull strategy; they use simulated annealing to optimize inventory levels and the junction point, which separates the production line into two subsystems. Olhager and Ostlund (1990) emphasize that the junction point can be the customer order point, a bottleneck resource, or a point derived from the product structure.

2.3. Joint systems

Control mechanisms can also be combined to control the same part of a system: superimpose several control systems, in order to obtain their respective benefits. The main issue is to evaluate the performance of the new systems, relatively to other control strategies. Such comparisons should not be limited to pure performance: it should also include issues such as implementation complexity (obviously, Conwip is simplest). One aspect of complexity is the control strategy's number of parameters (such as number of cards, base stock levels) compared with the number of stages of the production system (say) $N$. Kanban and Base stock both have $N$ parameters, whereas Conwip has only one parameter (so, Conwip's complexity is independent of the number of stages).

Bonvik et al. (1998) propose a control system called (two-boundary) Hybrid that combines local control using Kanban, and integral control using Conwip; see figure 6. Hybrid performs better than Base Stock, Kanban, Conwip, and Minimal Blocking (variation of Kanban), not only in terms of average overall inventory and service level but also in robustness to demand rate changes. As for complexity, Hybrid is easy to implement as a modification of Kanban; its number of parameters is $N$.

Buzacott (1989) and Zipkin (1989) propose Generalized Kanban, which combines Base stock and Kanban. The objective is to associate Base stock's rapid reaction to demand and the initial amounts of inventory with Kanban's coordination between consecutive stages and local control of inventory; see figure 7. A general approach to Base stock/Kanban joint systems is proposed in Dallery and
Liberopoulos (1995). The resulting system is called *Extended Kanban control system*. It has the same objective as Generalized Kanban, but is claimed to be conceptually clearer and potentially easier to implement. Both systems, however, are rather complex since they both have $2N$ parameters. Liberopoulos and Dallery (1997) propose variations of Extended Kanban for various production environments.

This short review of the literature on pull control shows that researchers have improved pull production control systems over the years. By combining existing control systems, they have created more efficient systems. However, there is a limit to this approach since it is based on three basic systems and almost all possible combinations have already been investigated. Yet, it is possible to imagine information flows that cannot be obtained with simple combinations of the three basic systems. Therefore, we propose a new approach that investigates such control systems.

### 3. Customized pull production control system

Gaury, Kleijnen, and Pierreval (1997) propose a methodology for choosing among Kanban, Conwip, and Hybrid. The idea is to optimize a *generic* system that aggregates the information flows of the three systems; see figure 8. Thus, the generic pull system can model a Kanban, a Conwip, or a Hybrid system, depending on the choice of the various parameters; see table 1. A control loop with an infinite number of authorizations does not impose any constraint on the flow of parts. Such a loop has no effect on the performance of the production line, so it can be removed from the generic system; it does not need to be implemented. For a given production system, the optimal configuration of the generic system not only shows which type of pull strategy is preferred, but also which values should be selected for the various numbers of cards. Suppose that for the example of a four-stage production line, optimization of the generic model gives $k_1 = 2$, $k_2 = 3$, $k_3 = 5$, $k_4 = 4$, and $c = \infty$. According to table 1, this configuration of the generic model is equivalent to a Kanban system. Thus, Kanban provides the best performance; its optimal configuration is $k_1 = 2$, $k_2 = 3$, $k_3 = 5$, and $k_4 = 4$.

#### Table 1  Generic system for three pull production control systems

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>...</th>
<th>$k_{n-1}$</th>
<th>$k_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kanban</td>
<td>$\infty$</td>
<td>$k_1$</td>
<td>$k_2$</td>
<td>...</td>
<td>$k_{n-1}$</td>
<td>$k_n$</td>
</tr>
<tr>
<td>Conwip</td>
<td>$C$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>...</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Hybrid</td>
<td>$C$</td>
<td>$k_1$</td>
<td>$k_2$</td>
<td>...</td>
<td>$k_{n-1}$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

**Figure 8**  The generic model for Kanban, Conwip and Hybrid in Gaury *et al.* (1997)
Gaury, Pierreval, and Kleijnen (1998) use an evolutionary algorithm to optimize the generic model for a production line with four stages inspired by a Toyota factory. They conclude that the resulting system cannot be classified as Kanban, Conwip, or Hybrid; it may be considered to be a simplified Hybrid system. In general, the best solution found through the optimization of the generic model does not necessarily correspond to any predefined system.

Huang and Kusiak (1998) criticize predefined strategies such as Kanban, for not considering the specific characteristics of manufacturing systems. They propose an algorithm based on decision rules for choosing which strategy (push or pull) should be adopted at each stage of a manufacturing system. Even though they consider only local control (no combination), they are the first to investigate what we call *customized* control systems: each production system has its own specificity and requires a special control system; predefined systems (such as Kanban and Conwip) might not be good enough.

Our objective is to design a pull production control system for a given production system, without using any *predefined* type of pull system a priori. Instead, we search for a control system in the whole set of possible pull systems, including local and integral control, segmented and joint systems. For this purpose we design a new generic system that accounts for all possible types of pull production control systems: *through control loops we link each stage to all its preceding stages*. Figure 9 shows this new generic system for a production line with four stages, where $k_{i,j}$ denotes the number of authorizations circulating in the loop linking stage $i$ to stage $j$. $M_i$ and $I_i$ denote respectively the machine and the inventory at stage $i$. There are $4(4 + 1)/2 = 10$ loops. As in the former generic model (Gaury et al., 1997), we do not implement a control loop with an infinite number of authorizations. A major difference, however, is that the former generic model was limited to Kanban, Conwip, and Hybrid.

Let us consider a general production line with $N$ stages. If we want the generic model in figure 9 ($N = 4$) to be a Kanban system, then we select the numbers of authorizations as follows: $k_{i,j} = \infty$, $\forall (i, j) \in \{1...N\}^2 / i \neq j$ (meaning: $k_{i,j}$ has an infinite value for all $i$ and $j$ such that $i \neq j$), and $k_{i,i} << \infty$, $\forall i \in \{1...N\}$ (meaning: $k_{i,i}$ has a finite value for all $i$). Similarly, Conwip is obtained by choosing $k_{N,1} << \infty$, and $k_{i,j} = \infty$, $\forall (i, j) \in \{1...N\}^2 / (i, j) \neq (N, 1)$. The generic model cannot represent the Base stock strategy of section 2.1 because that strategy uses information about demand occurrences, whereas the generic model focuses on information about actual deliveries. The Integral Control (variant of Base stock; see section 2.1.iii), however, is a possible instantiation of the generic model. Also, the generic model can represent control systems that have not been investigated in the literature. For instance, one can imagine a system for which control loops link each machine to the first one; this system requires authorizations from all machines to release raw materials.

Hence, we can use the generic system of figure 9 to find the best pull production control system for a given production system. Indeed, optimizing the various authorization numbers $k_{i,i}$ in this generic system should give a solution with both finite and infinite numbers of authorizations, showing which
loops must be implemented. The advantage of our approach is that we do not try to apply any predefined type of systems. This advantage, however, seems costly in terms of complexity. For a production line with \( N \) machines, the optimization model contains \( N(N + 1)/2 \) control loops. Thus, the optimization procedure has to deal with a rather large search space. For instance, for ten machines, the optimization model has 55 control loops. If the various authorization numbers are chosen from \( \{1...20\} \cup \{\infty\} \), then the search space includes \( 21^{55} = 5.27 \times 10^{72} \) configurations of the generic system. Obviously, it does not seem reasonable to attempt solving the exact optimization problem for large systems. Instead, our objective is to derive rules that can be generalized to larger systems. In this paper we choose to select a sample of simple line configurations for which the generic model will be configured and optimized. In the next section we define a sample of production lines compatible with models studied in the literature.

4. A sample of twelve production lines

In order to build a sample of production lines, we analyze the characteristics of pull production systems (mainly Kanban) studied in the literature. This analysis yields a list of ten factors, with typical values. We define three factor classes: process, demand, and performance. Below, we discuss each class individual factors, and their values used in the literature; for each factor we choose two values or levels.

4.1. Process factors

Line length or number of stages

Chu and Shih (1992) point out that most of the Kanban models in the literature are relatively small. Actually, Krajewski, King, Ritzman, and Wong (1987) study a 50-stage manufacturing system, but the usual length of the modeled lines is four or five stages, and the largest is nine. Because Conwip models are simpler, they can easily be studied for large production systems. Table 2 gives some references for the line length factor. For our sample of production lines we choose two levels: a low level of four stages, and a high level of eight stages.
Table 2  Line length in the pull literature

<table>
<thead>
<tr>
<th>Reference</th>
<th>Control system</th>
<th>Line length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Krajewski et al. (1987)</td>
<td>Kanban</td>
<td>50</td>
</tr>
<tr>
<td>Sarker and Fitzsimmons (1989)</td>
<td>Kanban</td>
<td>9</td>
</tr>
<tr>
<td>Spearman et al. (1990)</td>
<td>Conwip</td>
<td>10</td>
</tr>
<tr>
<td>Meral and Erkip (1991)</td>
<td>Kanban</td>
<td>3, 4, 5, and 6</td>
</tr>
<tr>
<td>Bonvik et al. (1997)</td>
<td>Kanban, Conwip, Hybrid, Base stock</td>
<td>4</td>
</tr>
</tbody>
</table>

Line imbalance

A production line is said to be non-balanced (bottlenecked) when resources do not have the same production rate. Sarker and Harris (1988) and Gupta and Gupta (1989) claim that balanced pull systems outperform non-balanced ones. The literature, however, does not agree on this point: for instance, Villeda, Dudek, and Smith (1988) show that some imbalance patterns can improve output rates.

A production line can be non-balanced in several ways. (i) The mean processing times in different stages can differ. This form of imbalance is studied most often. Various patterns are defined and analyzed; for example, bowl (the machines at the two ends of the line have the highest mean processing times), funnel (mean processing times are getting shorter from stage to stage), and reversed funnel (mean processing times are getting longer from stage to stage); see Hillier and Boling (1966). (ii) A line can also be non-balanced in its processing time variances, and its machine breakdown rates. Thus, the imbalance factor is very complex. We focus on the mean processing times: because process variability and machine reliability will be considered as factors, the other forms of imbalance will also be studied indirectly. We refer to Opwell and Pyke (1998) for an analysis of imbalance in both processing time means and variances in assembly systems.

Meral and Erkip (1991) define an interesting measure of imbalance in mean processing times: the Degree of Imbalance \( DI \) of a line is \( DI = \max \{ \frac{TWC}{N} - \min(PT_i); \ \max(PT_i) - \frac{TWC}{N} \} \cdot N/TWC \), where \( PT_i \) is the mean Processing Time at workstation \( i \) in an \( N \)-station line, and \( TWC/N \) is the mean processing time at a workstation on the balanced \( N \)-station line. Table 3 reviews some of the values for imbalance in the Kanban literature.

Table 3  Degree of imbalance in the Kanban literature

<table>
<thead>
<tr>
<th>Reference</th>
<th>( DI )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Villeda et al. (1988)</td>
<td>0 to 1.4 (step 0.2)</td>
</tr>
<tr>
<td></td>
<td>0 to 0.7 (step 0.1)</td>
</tr>
<tr>
<td>Meral and Erkip (1991)</td>
<td>0, 0.1, 0.2, 0.45</td>
</tr>
<tr>
<td>Yavuz and Satir (1995)</td>
<td>0, 0.1, 0.3, 0.5</td>
</tr>
</tbody>
</table>

\( DI \), however, does not account for the imbalance pattern (bowl, funnel, reversed funnel). Meral and Erkip (1991) and Yavuz and Satir (1995) study the bowl phenomenon. For our sample of production lines, we define two factors, one for \( DI \), and one for the imbalance pattern. The levels of the \( DI \) factor are 0 (a perfectly balanced line) and 0.5. For the latter \( DI \) level, the two imbalance
patterns are funnel and reversed funnel; we see the bowl pattern as a combination of funnel and reversed funnel, so we do not study bowl. The bottleneck will be the first or last resource in four stage lines, and the third or sixth resource in eight stage lines; see section 5.

**Processing time variability**

It is well known that performance is sensitive to processing time variability: inventory has to be raised in order to maintain acceptable service. The variance by itself does not fully characterize the variability of processing times: it has to be considered together with the mean. Thus, as a measure of variability we use the Coefficient of Variation (CV) defined as the standard deviation divided by the mean. Table 4 gives a review of CVs used in the study of Kanban systems. For our sample, we use a low level of 0.1 and a high level of 0.5.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Coefficients of variation, CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarker and Fitzsimmons (1989)</td>
<td>0, 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 1.0</td>
</tr>
<tr>
<td>Meral and Erkip (1991)</td>
<td>1, 1.5, 2</td>
</tr>
<tr>
<td>Swinehart and Blackstone (1991)</td>
<td>0.2, 0.5, 1.0</td>
</tr>
<tr>
<td>Savsar and Al-Jawini (1995)</td>
<td>0.2, 0.6, 1.2, 1.8</td>
</tr>
<tr>
<td>Yavuz and Satir (1995)</td>
<td>0, 0.1, 0.5, 0.9</td>
</tr>
</tbody>
</table>

**Machine reliability**

Two random variables are needed to model machine breakdowns: Time Between Failures (TBF) and Time To Repair (TTR). Many different distributions have been used in the literature, often without justification. Table 5 reviews distributions in the Kanban literature. Yavuz and Satir (1995) use a maintenance model instead of a reliability model: they model a policy of preventive maintenance. Preventive maintenance should be widely used in companies. Practice, however, is often different. Therefore, we consider a reliability model rather than a maintenance model. For the time between failures we use a popular distribution, namely the exponential, in order to make comparisons with previous literature. Time to repair is not really predictable: it can be very short, but also extremely long. An exponential distribution is then appropriate. We define two levels for the machine reliability factor: first we assume that the machines are perfectly reliable; next we generate TBF and TTR through exponential distributions.

<table>
<thead>
<tr>
<th>Reference</th>
<th>TBF Distribution</th>
<th>TTR Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Krajewski <em>et al.</em> (1987)</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>So and Pinault (1988)</td>
<td>Exponential</td>
<td>Exponential</td>
</tr>
<tr>
<td>Sarker and Fitzsimmons (1989)</td>
<td>Normal</td>
<td>Exponential</td>
</tr>
<tr>
<td>Wang and Wang (1990)</td>
<td>Exponential</td>
<td>Exponential</td>
</tr>
<tr>
<td>Yavuz and Satir (1995)</td>
<td>Uniform</td>
<td>Uniform</td>
</tr>
<tr>
<td>Bonvik <em>et al.</em> (1997)</td>
<td>Exponential</td>
<td>Exponential</td>
</tr>
</tbody>
</table>
4.2. Demand factors

**Demand rate**

In a production system the demand rate may change frequently (this happens more often, since the design of new products requires less time). So it is a key issue to know whether the choice of a control system depends on the demand rate: once a control system is implemented, it might not be changed easily. In order to have real meaning, the demand rate has to be defined relatively to line capacity: define the ratio of demand rate and long-term capacity of the line. We select two levels: a low level of 0.8, and a high level of 0.9.

**Demand variability**

For demand variability we use the same principle as for processing time variability: we take CV. A sample of values in studies on pull control systems is given in table 6.

<table>
<thead>
<tr>
<th>Table 6 Demand variability in the Kanban literature</th>
<th>Demand CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berkley (1996)</td>
<td>0.1, 0.045</td>
</tr>
<tr>
<td>Yavuz and Satir (1995)</td>
<td>0.052, 0.075, 0.115, 0.133</td>
</tr>
<tr>
<td>Savsar and Al-Jawini (1995)</td>
<td>0, 0.1, 0.316, 0.447, 0.707, 1</td>
</tr>
</tbody>
</table>

Next, we choose two levels: CV = 0, and CV = 0.5. The case CV = 0 may correspond to dependent demand (internal customers); in Bonvik et al. (1997), for instance, the production line feeds an assembly system, which processes parts at a constant rate.

**Customers' attitude**

We define customers' attitude as willingness to wait for finished products. We consider two extreme cases. In the first one customers do not accept to wait at all, and do not order if they cannot be satisfied from stock: lost sales scenario. In the second case, customers are always willing to wait, and orders that cannot be filled from stock are backlogged: backorders scenario. Most publications on pull production consider the backorders scenario. Bonvik et al. (1997), however, use the lost sales scenario.

4.3. Performance factors

Chu and Shih (1992) identify three performance measures frequently used in the Just-In-Time literature: facility utilization, output rate, and WIP. Goldrat and Fox (1986), however, argue that facility utilization should not be considered as a performance measure, since the goal of a production system is not to keep resources (workers and machines) busy. Output rate should be measured relatively to demand rate: a production system should not overproduce; yet it should meet demand requirements on time. Thus, we use the proportion of demand actually met from stock as one performance criterion. We also use a weighted sum of the average WIP levels along the line.
**Targeted service level**

The service level (also called fill rate) is the proportion of demand supplied from stock. Intuition tells that the higher the level of finished good products, the higher the service level. Thus, the choice of a target value for the service level affects the overall WIP amount. The service level should be set by managers. It may vary from one production system to another, so it is necessary to consider the service level as a factor (not only as a performance measure). Service level targets close to 100% may be used for systems with lost sales, whereas lower targets correspond to systems with backorders. Setting a target for the service level has not often been done in the literature; yet, Bonvik *et al.* (1997) do use a target, namely 99.9%. We will also use service levels close to 100%; a low level of 95% and a high level of 99%.

**Inventory value (added value)**

An objective of pull production is to minimize inventory levels. Thus, the inventory level is a major performance indicator. In some cases, however, managers may need to account for the value of inventory. Indeed, whereas keeping a high finished good inventory may be good for the service level, it may be prohibitive in terms of cost. Goldrat and Fox (1986) emphasize that inventory is invested money; improved competitiveness can be achieved by minimizing this investment.

We use the total value of inventory as a performance measure. After each operation, the value of a part increases. We assume that this added value is the same at each stage, and we define the total value factor as the ratio of the finished good value and the raw material value. We choose two levels: 1.0 (value not of interest to managers; total inventory value is equal to total amount of inventory) and 2.0. We expect these two cases to yield different inventory allocations along the production line, and also different information flows.

**4.4. Summary of factors and levels**

Table 7 gives an overview of our ten factors, now labeled from A through J, and their levels. A sample of line configurations can now be built through Design Of Experiment (DOE), which combines the various factor levels (see Kleijnen 1998 for an introduction to DOE in simulation). In order to minimize the number of combinations (configurations), we use the Plackett-Burman design (see Kleijnen, 1987 p. 329-336) displayed in table 8.
Table 7 Design factors and levels

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>Letter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line length</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Line imbalance</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Imbalance pattern</td>
<td>Funnel</td>
<td>Reversed funnel</td>
</tr>
<tr>
<td>Processing time CV</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Machine reliability</td>
<td>Perfect</td>
<td>Breakdowns</td>
</tr>
<tr>
<td>Demand CV</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Demand rate/capacity</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Service level target (%)</td>
<td>99</td>
<td>95</td>
</tr>
<tr>
<td>Inventory value ratio</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Customers' attitude</td>
<td>Lost sales</td>
<td>backorders</td>
</tr>
</tbody>
</table>

Table 8 Sample of production lines configurations, using Plackett-Burman design

<table>
<thead>
<tr>
<th>Line #</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
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<td>+</td>
<td>+</td>
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<td>-</td>
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<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
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<td>+</td>
</tr>
<tr>
<td>6</td>
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<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
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<tr>
<td>10</td>
<td>+</td>
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<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Many of the factors selected to define a line configuration, are variability sources with specific probability distributions. Thus, it may be difficult to perform analytic studies on most of the twelve configurations specified in table 8. Therefore we choose simulation for the performance evaluation of these production lines. The next step is to determine the optimal configuration of the generic pull control system for each production line; we use a heuristic optimization procedure based on an evolutionary algorithm (EA). Other optimization techniques have been used to configure pull systems: simulated annealing (Chang and Yih, 1994), RSM (Davis and Stubitz, 1987), neural networks (Hurion, 1997), and exhaustive search in a restricted search space (Duri, 1997; Bonvik et al., 1997). An advantage of EAs, however, is that infinity can be used as an explicit value for the card numbers. We refer to appendix A for details about this optimization procedure.

5. Simulation experiments

To evaluate the performance of the generic control system, we built twelve models corresponding to the twelve production lines in table 8. We use the SIMAN simulation language; see Pegden, Shannon, and Sadowski (1991). We estimate performance measures through single-run simulations, each with 240,000 time units, after eliminating a transient period of 10,000 time units.
The search space of the optimization procedures has $21^{10}$ and $21^{36}$ (21 is the number of integer values including infinity for $k_{i,j}$, 10 and 36 are the number of loops) possible solutions for four- and eight-stage production lines respectively, except for line #11, for which the search space has to be widened (the optimal number of Conwip cards is 27). Solutions may be simplified using a simple procedure: we measure the WIP level within each portion of the line controlled through a loop; if the maximal WIP level is below the corresponding number of cards, then the control loop does not have to be implemented. Conwip will be the reference for discussing our results because Conwip is the simplest strategy and it has proven to be very efficient. We find the optimal number of Conwip cards by running the simulation models for values chosen by dichotomy. Next we discuss the results for the first four line configurations. Other results are detailed in appendix B.

**Line configuration #1**

Surprisingly, the best control system found by the EA is a Kanban system; see figure 11. An interesting characteristic of our solution is that each control loop has only one card. Thus, manufacturing is highly synchronized: as soon as a machine stops, the machines upstream are not authorized to produce anymore and the machines downstream starve. The advantage is that inventory is extremely low. Such a system, however, is highly sensitive to variations (demand, process variability, breakdowns, etc). In fact, it is well known that there can be a single Kanban card per control loop if the production environment is ideal. Indeed, Monden (1983) and Sugimori, Kusunoki, Cho, and Uchikawa (1977) reported that Japanese managers use the following formula to determine the number of Kanban: $y_i \geq D_i L_i (1 + \alpha_i)/a$, where $y_i$ is the number of Kanban at stage i, $D_i$ is the average demand per time unit at stage i, $L_i$ is the average production lead-time at i, $\alpha_i$ is a variable for safety stock at i, and $a$ is the container capacity (a single Kanban is attached to each container). If demand is met with probability one, then $D_i L_i = 1$. If there is no variation, then safety stocks are not needed, so $\alpha_i = 0$. Moreover, if transport and switchover costs are unimportant, then there is no need for containers so $a = 1$. Thus, in an ideal production environment, the minimum number of Kanban at each stage is one: $y_i \geq 1$. In line configuration #1, demand is indeed constant, processing time

![Figure 11 Line configuration #1 and best solution](image-url)
variability is low, and customer pressure is low (service level of 95% and no backlog); see table 7 and line 1 of table 8. Thus, it seems reasonable to have a single Kanban per loop.

For this line configuration, the best Conwip system has four cards, which is as many as the total in the best generic system. However, the performance of the Conwip system is not as good in terms of average inventory value: see table 9. The WIP allocation shows that Conwip pushes inventory to the end of the line, whereas Kanban guarantees a uniform allocation along the line. Since the value of inventory is increasing along the line, Conwip is relatively expensive. However, we note that Conwip yields the same average overall inventory level, but with a higher service level and simpler implementation. Thus, if the inventory value is not an important issue, then Conwip is preferred.

<table>
<thead>
<tr>
<th>Service</th>
<th>WIP allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target is 95%</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 9 Performance of the best Generic and Conwip systems for line configuration #1

<table>
<thead>
<tr>
<th>Value</th>
<th>Service Target is 95%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kanban 1,1,1,1</td>
<td>5.997</td>
<td>98.91%</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Conwip 4</td>
<td>6.38</td>
<td>100.00%</td>
<td>0.80</td>
<td>0.81</td>
<td>0.81</td>
</tr>
</tbody>
</table>

**Line configuration #2**

Line configuration #2 and the best control system found by the EA are shown in figure 12. This is a simple control system: only two control loops. It is also a new type of pull system, in the sense that it is neither a segmented nor a joint system. The tradeoff between service and inventory is as follows: the first machine is the bottleneck, but it is quick enough to satisfy more than 95% of demands; see table 10. Thus, it authorizes more products into the line than necessary: see Conwip 5, for which service is closer to 100% but at the cost of higher total inventory value. The average WIP allocation along the line shows clearly that the best Generic system maintains a higher WIP level than Conwip at the beginning of the line, and a lower WIP level at the end. The best Generic system would be better if inventory value increases along the line. In fact, this example illustrates one of the flaws of Conwip: its lack of control. Indeed, a Conwip system with four cards yields a total inventory value of 3.742 units and a service level of 94.73%, which is below target. Thus, a change in the number of Conwip cards can have dramatic effects on the performance.

**Figure 12  Line configuration #2 and best solution**
Table 10  Performance of the best Generic and Conwip systems for line configuration #2

<table>
<thead>
<tr>
<th></th>
<th>Service Value</th>
<th>WIP allocation</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Service Target is 95%</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Best Generic</td>
<td>4.10</td>
<td>96.31%</td>
<td>1.28</td>
<td>0.62</td>
<td>0.44</td>
<td>1.75</td>
</tr>
<tr>
<td>Conwip 5</td>
<td>4.73</td>
<td>99.21%</td>
<td>0.80</td>
<td>0.62</td>
<td>0.44</td>
<td>2.86</td>
</tr>
</tbody>
</table>

Line configuration #3

Figure 13 shows that the result for configuration #3 is rather complex: it has 11 authorization cards. The first stage requires authorizations from five other stages! The last stage authorizes production at three stages upstream. The total inventory value is only 3% lower than for Conwip: see table 11. Thus, Conwip would be implemented for this production line. The average WIP levels at the various stages in Conwip are almost equal, except for the last stage. The main difference introduced by the generic system is that part of the finished good inventory is shifted to the middle of the line, namely stage 4.

Despite the complexity of the control system, implementation is possible for certain types of production. In automated lines, for instance, authorizations may be transmitted through a computer network and counters may replace cards.

Table 11  Performance of the best Generic and Conwip systems for line configuration #3

<table>
<thead>
<tr>
<th></th>
<th>Service Value</th>
<th>WIP allocation</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Service Target is 99%</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Best Generic</td>
<td>17.42</td>
<td>99.01%</td>
<td>0.85</td>
<td>0.85</td>
<td>0.83</td>
<td>0.97</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>Conwip 11</td>
<td>17.96</td>
<td>99.17%</td>
<td>0.89</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.86</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Figure 13  Line configuration #3 and best solution

STAGE 1  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.0
STAGE 3  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.286
STAGE 5  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.571
STAGE 7  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.857
DEMANDS  Time between: Logn(1.25, 0.625)  Workload: 80%  Service target: 99%
Lost sales  
STAGE 2  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.143
STAGE 4  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.428
STAGE 6  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 1.714
STAGE 8  Processing times: Logn(1.0, 0.1)  Breakdowns: TBF exp(1000)  TTR exp(3)  Value of inventory per unit: 2.0
Line configuration #4

Figure 14 shows the ‘best’ control system found by the EA. One would expect the total WIP level in the Conwip system to be closer to 11 units, and local WIP levels to be almost equal at the first stages (as for lines #1 and #2); see table 12. An explanation might be that the demand/capacity ratio is high (0.9) and processing time variability is also high, so demands are frequent and the resource at stage 1 may not be able to keep up the pace at some times because of variability. However, the inventory accumulated at the first stage seems to be redundant because WIP levels at the next two stages are much lower. The generic system solves this problem by adding local control to Conwip: two control loops at the beginning of the line further reduce the release of raw materials. Thus, this is a joint system as defined in section 2.3. The result is a 6% decrease of the total inventory value, at the expense of an increase in complexity (four control loops instead of one).

Table 12 Performance of the best Generic and Conwip systems for line configuration #4

<table>
<thead>
<tr>
<th></th>
<th>Service WIP allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
</tr>
<tr>
<td>Best Generic</td>
<td>9.36</td>
</tr>
<tr>
<td>Conwip</td>
<td>9.95</td>
</tr>
</tbody>
</table>

6. Discussion

The overall WIP level can change drastically from one production line to another. Indeed, the number of authorization cards in the best Conwip system vary between 4 and 12 for four-stage lines, and between 6 and 27 for eight-stage production lines. Analysis of variance (ANOVA), using table 8 as input (normalized values) and the optimized number of Conwip cards as output, shows that five of the ten factors have an important effect; see table 13:

- Line length (factor A). The longer the line, the more WIP is needed.
- Line imbalance (B). More WIP is necessary when the line is balanced; in fact, non-balanced lines have overcapacity, except at the bottleneck, and quick resources require lower inventories.
- Processing time CV (D). The higher the CV, the more WIP is needed.
- Ratio demand rate/capacity (G). The closer the ratio to one, the higher the WIP level.
- Customers' attitude (J). Production lines allowing backorders require a higher WIP level. An explanation is that backordered demands must be fulfilled, so more throughput is required than in the lost sales case.

Efforts to improve the performance of a line should focus on these five important factors, if possible. Surprisingly, factors such as the demand CV (F) and the service level target (H) do not have any effect. The other factors (imbalance pattern, machine reliability, and inventory value ratio) are more important.

Table 13 Regression analysis of the optimal number of Conwip cards as a function of ten factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression coefficient</td>
<td>-2.58</td>
<td>3.08</td>
<td>0.92</td>
<td>-2.42</td>
<td>1.08</td>
<td>0.08</td>
<td>2.08</td>
<td>-0.08</td>
<td>-0.75</td>
<td>-2.25</td>
</tr>
</tbody>
</table>

Optimization of the generic model leads to much simpler control systems than we expected. Indeed, whereas the generic model has \(N(N+1)/2\) loops, the ‘optimized’ solutions have rarely more than \(N+1\) loops. We also found systems that are neither segmented nor joint, such as line configuration #2 (figure 12). In many cases, however, Conwip is preferred over the ‘optimized’ generic system because the gains in terms of total inventory value are small. Most of these cases have eight stages. More generally, we observe that the benefit of the generic system depends partly on how close to target is the service level in the best Conwip system. Indeed, the largest gains are obtained in configurations for which Conwip has a service level well above target; see lines #2 and #7 in table 14. However, this is not always so: for line #6, Generic reduces inventory by 7.16%, while losing only 0.17% in service (table 14). Thus, significant improvement can be obtained through a more efficient control of WIP along the line, at little expense in terms of service.

There seems to be two main ways of achieving savings in inventory value. First, the release of raw materials can be limited so that the overall inventory level is reduced: the first stage requires authorizations from several other stages, in most cases. Second, when the value of inventory increases along the line, the total inventory value can be reduced by allocating more WIP to early stages (instead of pushing parts to the last stage, as Conwip does): several control loops link the last stage to upstream stages. Obviously these two ways can be combined. In fact, many of the solutions described in the previous section are such combinations; a typical example is line configuration #4 (see figure 14).
Table 14  Performance comparison of Generic versus Conwip for each line configuration

<table>
<thead>
<tr>
<th>Line #</th>
<th>Generic</th>
<th>Conwip</th>
<th>Gain (%)</th>
<th>Service level</th>
<th>Conwip (above target)</th>
<th>Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Generic</td>
<td>Conwip</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5.99</td>
<td>6.38</td>
<td>6.00</td>
<td>98.91</td>
<td>100.00 (5.00)</td>
<td>1.09</td>
</tr>
<tr>
<td>2</td>
<td>4.10</td>
<td>4.73</td>
<td>13.32</td>
<td>96.31</td>
<td>99.21 (4.12)</td>
<td>2.92</td>
</tr>
<tr>
<td>3</td>
<td>17.42</td>
<td>17.96</td>
<td>3.00</td>
<td>99.01</td>
<td>99.17 (0.17)</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>9.36</td>
<td>9.95</td>
<td>5.93</td>
<td>95.13</td>
<td>95.49 (0.49)</td>
<td>0.38</td>
</tr>
<tr>
<td>5</td>
<td>11.34</td>
<td>11.70</td>
<td>3.07</td>
<td>99.05</td>
<td>99.31 (0.31)</td>
<td>0.26</td>
</tr>
<tr>
<td>6</td>
<td>15.18</td>
<td>16.35</td>
<td>7.16</td>
<td>99.00</td>
<td>99.17 (0.17)</td>
<td>0.17</td>
</tr>
<tr>
<td>7</td>
<td>5.30</td>
<td>6.00</td>
<td>11.67</td>
<td>97.48</td>
<td>99.07 (4.07)</td>
<td>1.60</td>
</tr>
<tr>
<td>8</td>
<td>7.91</td>
<td>7.99</td>
<td>1.00</td>
<td>99.03</td>
<td>99.12 (0.12)</td>
<td>0.09</td>
</tr>
<tr>
<td>9</td>
<td>14.90</td>
<td>15.14</td>
<td>1.58</td>
<td>99.04</td>
<td>99.12 (0.12)</td>
<td>0.08</td>
</tr>
<tr>
<td>10</td>
<td>8.52</td>
<td>10.33</td>
<td>9.51</td>
<td>99.00</td>
<td>99.36 (0.36)</td>
<td>0.36</td>
</tr>
<tr>
<td>11</td>
<td>41.21</td>
<td>42.36</td>
<td>2.71</td>
<td>95.11</td>
<td>96.00 (1.00)</td>
<td>0.93</td>
</tr>
<tr>
<td>12</td>
<td>16.68</td>
<td>17.28</td>
<td>3.47</td>
<td>95.06</td>
<td>96.01 (1.01)</td>
<td>0.99</td>
</tr>
</tbody>
</table>

7. Conclusion

In this paper we proposed a novel approach to the design of pull production control systems for single-product flow lines. Instead of using predefined systems such as Kanban, Conwip, or Base stock, we design customized systems using a generic model that in principle connects each stage of a line with each preceding stage. This generic model integrates not only the three predefined systems, but also their combinations. Moreover, it extends the concept of pull control to systems that have never been investigated in the literature. Optimization of the model shows which potential control loops are actually implemented. To derive general results, we designed twelve production lines, using an experimental design with ten factors (such as line length, demand variability, and machine breakdowns). For each production line we optimized the control system. Despite the complexity of the generic system, results are quite simple most of the time. Simplicity, however, is the main advantage of Conwip, which is preferred for most eight-stage production lines. Nevertheless, customization may yield an inventory value 13% lower than Conwip, while maintaining service on target. Moreover, analysis of our results reveals two important patterns of information flows: one linking each stage to the first stage, and another one linking the last stage to each preceding stage (Integral Control; see section 2.1.iii); these two patterns (possibly simplified: \( k_{i,j} = \infty \)) fully characterize many solutions. The first pattern and the combination of the two patterns have not been mentioned in the literature. Further research is required to investigate the mechanisms involved in these types of control systems.

We see many other extensions of our research. First, it should be possible to extend the concept of customization to other types of production systems, such as assembly/disassembly systems and multi-product flow lines. Second, robustness issues could be considered during the design of customized pull systems. Indeed, a major concern is to design patterns of information flows that are insensitive to variations in the production environment. In Gaury and Kleijnen (1998) we used risk analysis to compare the robustness of four pull systems for a four-stage production line inspired by Toyota. Measuring robustness/risk, however, has a high computational cost, so it is not yet possible to design
customized pull systems using a robustness/risk criterion. Nevertheless, it is possible to compare the robustness of several good solutions.

References


Appendix A. Evolutionary algorithm

Evolution is a method of searching for solutions among a large set of possibilities (search space). Random variations obtained through such operators as mutation and recombination, make species evolve. Natural selection tends to let the fittest individuals survive and reproduce, thus propagating their good genes to the next generations. Evolutionary Algorithms (EAs) reproduce this stochastic process. They begin their search from a set of potential solutions, called a population. Each solution is coded as a chromosome, which has components (also called genes) that are the parameters of the problem to be solved. Generally, the initial population is chosen randomly. EAs make the initial population evolve toward a population that is expected to contain the best solution. For this purpose, EAs use the following reproduction-evaluation cycle: for each iteration – called a generation – solutions from the current population are selected with a given probability, and copies of these solutions are created. This selection is based on the fitness of solutions, relatively to the current population, in the sense that the strongest solutions will have a higher probability of being copied (“survival of the fittest”). In a simulation-optimization approach, a solution is the input value vector of a simulation model; its fitness is a function of the output variables (Pierreval and Tautou, 1997). Part of the copied solutions is submitted to the mutation and recombination operators (which we shall describe later); this process is called reproduction. The recombination mechanism mixes parental information, while passing information on to the descendants or offspring. Mutation introduces innovation into the population. From one generation to another, solutions have higher fitness, globally speaking.

Our goal is to achieve a predetermined service level, while minimizing the total value of inventory (see section 4.3). There are techniques that use EAs for such optimization problems with a constraint (like the service level). The most widely used technique penalizes chromosomes that do not respect the constraint, artificially either decreasing or increasing the fitness of these chromosomes, according to the objective (Michalewicz, 1992). If the objective is to minimize the fitness, then penalizing a solution that does not respect the constraint implies increasing its fitness. Michalewicz, Dasgupta, Le Riche, and Schoenauer (1996) mention the main difficulty is to choose the right level of penalty: if the penalty is too low, the final solution might not respect the constraint; if the penalty is too high, the search might be confined to a small part of the search space and converge to a local optimum.

We evaluate the fitness of individuals through simulation: the EA sends chromosomes as input to the simulation model, which returns a fitness estimation of the corresponding individuals. To estimate the fitness in case of stochastic simulation, we can use either several replications or a single long simulation run.
The main steps of an EA are as follows:

Step 0. Start with the generation counter equal to zero.
Step 1. Initialize a population of individuals.
Step 2. Evaluate the fitness of all initial individuals in population.
Step 3. Increase the generation counter.
Step 4. Rank the individuals according to their fitness.
Step 5. Copy the individuals in the best half of the population and keep them for the next generation.
Step 6. Select individuals among the cloned ones for children reproduction (selection).
Step 7. Recombine selected parents (recombination).
Step 8. Perturb the mated population stochastically (mutation). The new individuals added to the copied ones (step 5) are the new generation.
Step 9. Evaluate the fitness of the new individuals (evaluation).
Step 10. Test the termination criterion (number of generations, fitness, etc.), and stop or return to step 3.

When implementing an EA, five main choices must be made: encoding of solutions, fitness, selection mechanism, evolutionary operators, and parameters of the algorithm. Let us discuss these choices in the case of pull systems. Other studies of pull systems based on EAs and simulation can be found in the literature. Paris and Pierreval (1997) optimize number of Kanbans, container sizes, sequencing rules, and sizes of safety storage for a four-stage production line making three product types. Bowden, Hall, and Usher (1996) determine the number of Kanbans and the order sizes (30 parameters) for an assembly line inspired by a factory of the Whirlpool Corporation.

A.1. Encoding of solutions

In order to identify a pull system, we need to know which stations are linked by control loops, and how many cards each control loop has. In fact, these two types of information may be expressed as follows. When a resource does not need authorizations to produce, it is allowed to produce as long as it has parts in its input inventory. For instance, in a Conwip system all resources except the first one, work according to this principle: they always have the authorization to produce. This can be modeled through a control loop with an infinite number of cards. Thus, a Conwip system is equivalent to a Hybrid system with all Kanban control loops having an infinite number of cards. In this way we build a common representation for all the pull systems represented by the generic model: a chromosome is the list of genes \((k_{1;1}, k_{2;2}, k_{2;1}, k_{3;3}, k_{3;2}, \ldots, k_{N;1})\) where \(k_{1;1}, \ldots, k_{N;1}\) are the numbers of cards shown in figure 9 (section 3). We are interested in genes with a domain of the type \(D \cup \{\infty\}\), where \(D\) is a finite set of integer values.
A.2. Fitness
We minimize the objective function $f$ inspired by Michalewicz et al. (1996), which we define as follows:
if service $\geq$ target, $f =$ inventory value,
if service $<$ target, $f =$ inventory value/(service/$k$)$^p$
where $p$ is the penalty parameter and $k$ is a scaling factor chosen such that service/$k$ <1. Thus, when a solution does not respect the service constraint, the objective function is increased by a penalty factor equal to (service/$k$)$^p$. When the service constraint is respected, the optimum is the solution with the lowest total inventory value.

A.3. Selection
One of the most popular selection systems is the roulette wheel (Goldberg, 1989). In that system the decision whether to select a chromosome is made according to a probability assigned to each chromosome. That probability is based on the fitness of the chromosome, such that the one with the best fitness has a higher chance of surviving. The literature provides many alternatives to the roulette-wheel principle:
- sigma-scaling also accounts for the standard deviation of the individual fitness (Forrest, 1985);
- elitism preserves a number of the best individuals, from one generation to another (De Jong, 1975);
- Boltzmann selection uses the principle of crystallization that is also used in simulated annealing (Goldberg, 1990, De la Maza and Tidor, 1993);
- rank selection maintains the pressure of selection, even when the fitness of individuals gets very close to each other (Baker, 1985);
- tournament selection makes individuals compete against each other (Goldberg and Deb, 1991).

We choose to implement the elitism and the roulette wheel principles. A part of the new population is an exact copy of the best solutions in the previous generation (elitism), whereas another part is selected randomly from the previous population (roulette wheel) and is changed by evolutionary operators.

A.4. Evolutionary operators
The combination of EA and simulation is rather time-consuming. In order to save computer time, uninteresting solutions should be avoided. The understanding of card-based control systems can help identifying such solutions. Local control affects the performance of the production system only if there is no stronger constraint at a higher level (it is a necessary not a sufficient condition). In other words, a control loop should have a smaller number of cards than any control loop above it. In a Hybrid system, for instance, each Kanban number should be smaller than the number of Conwip cards. Otherwise, the corresponding control loop does not have any effect on the performance of the
production system, and the control system can be simplified by removing the control loop (infinite number of cards). Thus, whenever we need to choose a number of cards, we search for the strongest constraint \( c_{\text{max}} \) above the corresponding card loop. Then, the number of cards should be selected from \( \{1 \ldots c_{\text{max}}, \infty\} \). We use this property for the creation of the initial population, the mutation operator, and the recombination operator (partial repair of the offspring).

**Recombination**

As a recombination operator we use single-point crossover (Goldberg, 1989), which replaces with probability \( p_{\text{cross}} \) two parents \( X^i \) and \( X^j \) by their offspring \( X^i' \) and \( X^j' \), as follows. An integer \( \text{pos} \), representing the position at which the solutions \( X^i \) and \( X^j \) are cut, is selected randomly between 1 and \( q - 1 \) where \( q \) is the number of genes in the chromosome. The inversion of the two parts of each chromosome leads to the offspring shown in figure A-1. To improve computing efficiency, the offspring may be partially ‘repaired’, avoiding uninteresting solutions.

![Figure A-1: Recombination operator](image)

**Mutation**

In the natural world, each gene in the chromosome of an offspring has a small chance \( p_{\text{mut}} \) of mutating. We use such a “small” mutation operator. The value of the new gene is chosen randomly. The following selection strategies can be found in the literature (Michalewicz, 1992, and Pierreval and Tautou, 1997): selection of the bounds of the domain, use of a uniform probability distribution, use of a triangular or Gaussian probability distribution, etc.

The value of the new gene must be chosen within the domain \( \{1 \ldots c_{\text{max}}, \infty\} \). The probability distribution for the selection of a gene’s value within its domain is chosen as follows: \( \infty \) with probability \( p_{\infty} \), and any integer value of \( \{1 \ldots c_{\text{max}}\} \) with constant probability \( (1 - p_{\infty})/\text{Card}(\{1 \ldots c_{\text{max}}\}) \) where \( p_{\infty} \) denotes the probability of selecting \( \infty \) as the value of a gene, and \( \text{Card}(\{1 \ldots c_{\text{max}}\}) \) is the number of integer values contained in the set \( \{1 \ldots c_{\text{max}}\} \). The same probability distribution is used to define the initial population (step 1 in the algorithm).

This mutation operator can randomly generate any solution (or simplified equivalent) of the search space in the initial population. Furthermore, any solution of the search space can be reached from any other solution, using a finite sequence of mutations.
A.5. Parameters

The last decision concerns the choice of values for the various parameters in the EA. De Jong (1975) gives many experiments that quantify how much the parameter values influence the EA performance. He concludes that the best population size is 50 to 100 individuals, the best single-point crossover rate is approximately 0.6 per pair of parents, and the best mutation rate is 0.001 per bit. Obviously these values depend on his experimental conditions; for instance, a population of 50 to 100 individuals does not seem reasonable when fitness is estimated through stochastic simulation. More generally, Mitchell (1996, pp. 175-177) suggests that crossover, mutation, and selection should be balanced, depending on both the fitness function, and the encoding. Therefore, she recommends to choose the parameter values according to a trial and error strategy. Using this approach, we choose the following parameters:

- penalty parameter $p$ is 2 and scaling factor $k$ is 200;
- $p_\infty$ is 0.25;
- domains for all card numbers are $D = \{1,..,20\}$;
- population size, mutation and recombination probabilities are selected appropriately for each of the 12 line configurations.

Appendix B. More optimization results

Line configuration #5

Figure B-1 shows that the ‘optimal’ generic system for configuration #5 is very similar to the one obtained for configuration #4. The main difference between the two lines is that the demand/capacity ratio is now lower (0.8). Thus, demands are less frequent, and the resource at stage 1 seems to be able to follow the pace. The first effect is that the overall WIP level is close to the number of Conwip cards. The second effect is that the generic system cannot improve performance much. Table B-1 shows that the WIP allocations along the line are similar for Conwip and generic, and the gain in terms of total inventory value is only 3%. Thus, Conwip may be preferred.

Figure B-1 Line configuration #5 and best solution

![Diagram of line configuration #5 and best solution]

STAGE 1
Processing times: Logn(1.0, 0.5)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

STAGE 2
Processing times: Logn(1.0, 0.5)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

STAGE 3
Processing times: Logn(1.0, 0.5)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

STAGE 4
Processing times: Logn(1.0, 0.5)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

DEMANDS
Time between: 1.25 (constant)
Workload: 80%
Service target: 99%
Backorders

STAGE 1
STAGE 2
STAGE 3
STAGE 4
12
### Table B-1 Performance of the best Generic and Conwip systems for line configuration #5

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Service Target is 99%</th>
<th>WIP allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Generic</td>
<td>11.34</td>
<td>99.05%</td>
<td>1.26 1.32 1.45 7.32</td>
</tr>
<tr>
<td>Conwip</td>
<td>11.70</td>
<td>99.31%</td>
<td>1.31 1.42 1.46 7.50</td>
</tr>
</tbody>
</table>

### Line configuration #6

The best control system found by the EA for configuration #6 shows interesting characteristics; see figure B-2.

- The second machine of the production line does not need authorizations to produce: it can produce as long as parts are available in its input inventory. Thus, the two first machines can be grouped into a production cell.
- The control system is based mainly on an integral control policy (variant of base stock; see section 2.1): the last machine, which is the bottleneck, controls production at the previous stages through three control loops. Since the service target is close to 100% and demand is highly fluctuating (CV = 0.5), inventory has to be kept at a high level, especially at the last stage; see table B-2. Thus, in the ‘optimal’ generic system, finished good inventory has an upper bound of seven parts, whereas the upper bound for the whole line is nine.
- The fastest machines are at the beginning of the line, and the inventory values increase along the production line. Hence, it is advantageous to delay the release of raw materials into the line as much as possible. The ‘optimal’ generic system achieves this advantage through two control loops at the beginning of the line.
- Inventory allocations in table B-2 show that the generic system has a slightly higher WIP level at the second stage, and a much lower WIP at the end of the line. This yields a total inventory value saving of about 7%.

![Figure B-2 Line configuration #6 and best solution](image-url)
Table B-2  Performance of the best Generic and Conwip systems for line configuration #6

<table>
<thead>
<tr>
<th></th>
<th>Service Value</th>
<th>Target is 99%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Generic</td>
<td>15.18</td>
<td>99.00%</td>
<td>0.35</td>
<td>0.90</td>
<td>1.87</td>
<td>5.26</td>
</tr>
<tr>
<td>Conwip 9</td>
<td>16.35</td>
<td>99.17%</td>
<td>0.37</td>
<td>0.82</td>
<td>2.18</td>
<td>5.63</td>
</tr>
</tbody>
</table>

**Line configuration #7**

Figure B-3 shows that the inventory level at the first four machines is limited to two parts only. This means that at all times at least two of the first four machines are idle. The inventory at the last five stages is controlled by a loop with four cards. Two other control loops are added. They guarantee good coordination between the two halves of the production line: one loop informs the beginning of the line about the bottleneck consummation, while the other loop transmits information concerning finished good deliveries. The slowest machine and the consecutive ones do not need any authorizations to produce. Thus, they can be grouped within a production cell. The same grouping can be implemented for stages 2 and 3.

As table B-3 shows, the behavior of Generic and Conwip are quite similar. The main difference is that the generic system operates with a much lower finished good inventory. If inventory value is considered, then the gains obtained by switching from Conwip to Generic (11.6%) are more important. The generic system has low complexity (only four control loops for eight stages), but is still more complex than Conwip.

Table B-3  Performance of the best Generic and Conwip systems for line configuration #7

<table>
<thead>
<tr>
<th></th>
<th>Service Value</th>
<th>Target is 95%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Generic</td>
<td>5.30</td>
<td>97.48%</td>
<td>0.35</td>
<td>0.41</td>
<td>0.59</td>
<td>0.65</td>
<td>0.81</td>
<td>0.89</td>
<td>0.74</td>
<td>0.87</td>
</tr>
<tr>
<td>Conwip 6</td>
<td>6.00</td>
<td>99.07%</td>
<td>0.30</td>
<td>0.42</td>
<td>0.54</td>
<td>0.67</td>
<td>0.84</td>
<td>0.90</td>
<td>0.75</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Figure B-3 Line configuration #7 and best solution

```
<table>
<thead>
<tr>
<th>STAGE 1</th>
<th>STAGE 3</th>
<th>STAGE 5</th>
<th>STAGE 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing</td>
<td>Processing</td>
<td>Processing</td>
<td>Processing</td>
</tr>
<tr>
<td>times:</td>
<td>times:</td>
<td>times:</td>
<td>times:</td>
</tr>
<tr>
<td>Logn(0.5, 0.05)</td>
<td>Logn(0.9, 0.09)</td>
<td>Logn(1.3, 0.13)</td>
<td>Logn(1.25, 0.125)</td>
</tr>
<tr>
<td>Breakdowns:</td>
<td>Breakdowns:</td>
<td>Breakdowns:</td>
<td>Breakdowns:</td>
</tr>
<tr>
<td>TBF exp(1000)</td>
<td>TBF exp(1000)</td>
<td>TBF exp(1000)</td>
<td>TBF exp(1000)</td>
</tr>
<tr>
<td>TTR exp(3)</td>
<td>TTR exp(3)</td>
<td>TTR exp(3)</td>
<td>TTR exp(3)</td>
</tr>
<tr>
<td>Value of</td>
<td>Value of</td>
<td>Value of</td>
<td>Value of</td>
</tr>
<tr>
<td>inventory</td>
<td>inventory</td>
<td>inventory</td>
<td>inventory</td>
</tr>
<tr>
<td>per unit: 1.0</td>
<td>per unit: 1.0</td>
<td>per unit: 1.0</td>
<td>per unit: 1.0</td>
</tr>
</tbody>
</table>

DEMANDS

Time between: 1.666 (constant)
Workload: 90%
Service target: 95%

Lost sales

STAGE 2
Processing times: Logn(0.7, 0.07)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

STAGE 4
Processing times: Logn(1.1, 0.011)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

STAGE 6
Processing times: Logn(1.5, 0.15)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0

STAGE 8
Processing times: Logn(0.75, 0.75)
Breakdowns: TBF exp(1000)
TTR exp(3)
Value of inventory per unit: 1.0
```
Figure B-4 shows that the solution is rather complex: it has nine control loops (as much as a Hybrid system would require), so the information flows would not be easy to implement. The first stage requires authorizations from four other stages to produce, and the last stage releases authorizations to two stages. Other loops guarantee control at intermediary stages. Table B-4 shows that the performance of Generic and Conwip are equivalent: Generic yields only a 1% gain in terms of inventory value, and the WIP allocations of Generic and Conwip along the line are almost identical. Thus, Conwip is preferred for this production line.

Table B-4  Performance of the best Generic and Conwip systems for line configuration #8

<table>
<thead>
<tr>
<th>Service Value</th>
<th>WIP allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8</td>
<td>0.49 1.06 1.05 0.82 0.65 0.51 0.38 2.94</td>
</tr>
</tbody>
</table>

Line configuration #9

Figure B-5 shows that the solution is a complex control system. The release of raw materials requires authorizations from four downstream stages; the last stage authorizes production at four stages; two other loops control production at intermediary stages. This system does not yield many benefits. Indeed, the total value of inventory is reduced by only 1.5% and the average WIP allocation along the line is almost unchanged: see table B-5. Thus, Conwip should be implemented.
Table B-5 Performance of the best Generic and Conwip systems for line configuration #9

<table>
<thead>
<tr>
<th>Value</th>
<th>Service WIP allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Target is 99%</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Best Generic</td>
<td>14.90</td>
</tr>
<tr>
<td>Conwip</td>
<td>16</td>
</tr>
</tbody>
</table>

Line configuration #10

Figure B-6 shows that the control system is very close to the one obtained for line configuration #2. Control loops have been added at the beginning of the line, to limit the release of raw materials and at the end of the line (because finished good inventories have high value). Another difference with the
control system for line configuration #2 is that the card numbers are slightly higher. Possible factors are that the service target and the workload are both higher (99% and 90% respectively, instead of 95% and 80%). The benefits of Generic compared with Conwip are clearly shown by table B-6: the average overall inventory is lower in the Conwip system, but the WIP allocation along the line is such that the inventory value in Conwip is much higher. Total inventory value is reduced by 17.5%. Thus, Generic might replace Conwip, even though its implementation is rather complex (five loops, which is as complex as for Hybrid).

| Table B-6 Performance of the best Generic and Conwip systems for line configuration #10 |
|----------------------------------|----------------|----------------|----------------|----------------|
| Value   | Service Target is 99% | WIP allocation |
|         | 1   | 2   | 3   | 4   |
| Best Generic | 8.52 | 99.00% | 1.94 | 0.99 | 1.96 | 0.99 |
| Conwip   | 10.33 | 99.36% | 0.91 | 0.71 | 0.50 | 3.82 |

**Line configuration #11**

Figure B-7 shows that the result of the optimization procedure is a system with lower complexity (in terms of number of loops) than some of the systems obtained for eight-stage production lines: see configuration #3, #8, and #9. Control seems to be guaranteed mainly by the last stage, which releases authorizations to three other stages upstream. Three Kanban loops control production in the first half of the line. A surprising result is the high level of inventory needed by the line to operate properly (see section 6).

Even though the inventory value increases along the line, Generic yields only a 2.7% improvement. Generic shifts a large amount of inventory from the last stage to stages 4 and 6: see table B-7. However, since the difference in inventory values at stages 4 and 8 is not important enough,
the gain is small. It might be much more important for other inventory values. Conwip should be implemented.

**Table B-7 Performance of the best Generic and Conwip systems for line configuration #11**

<table>
<thead>
<tr>
<th>Value</th>
<th>Service Target is 95%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Generic</td>
<td>41.21 95.11%</td>
<td>2.22</td>
<td>2.76</td>
<td>2.36</td>
<td>3.64</td>
<td>2.38</td>
<td>4.18</td>
<td>2.37</td>
<td>6.13</td>
</tr>
<tr>
<td>Conwip</td>
<td>42.36 96.00%</td>
<td>2.28</td>
<td>2.44</td>
<td>2.49</td>
<td>2.59</td>
<td>2.62</td>
<td>2.65</td>
<td>2.61</td>
<td>8.43</td>
</tr>
</tbody>
</table>

**Line configuration #12**

Figure B-8 shows that the result is the simplest system obtained so far for an eight-stage production line: it has five control loops only, three controlling the release of raw materials. As table 20 shows, average WIP allocation for Generic is very close to Conwip. The main difference is that part of the inventory is kept at stage 6, to avoid storing more valuable finished goods. However, the gain in terms of total inventory value is 3.5% only, which is not large enough to be an alternative to Conwip for implementation.

**Table B-8 Performance of the best Generic and Conwip systems for line configuration #12**

<table>
<thead>
<tr>
<th>Value</th>
<th>Service Target is 95%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Generic</td>
<td>16.68 95.06%</td>
<td>0.29</td>
<td>0.42</td>
<td>0.61</td>
<td>0.91</td>
<td>1.41</td>
<td>1.89</td>
<td>0.70</td>
<td>3.54</td>
</tr>
<tr>
<td>Conwip</td>
<td>17.28 96.01%</td>
<td>0.29</td>
<td>0.45</td>
<td>0.65</td>
<td>0.95</td>
<td>1.45</td>
<td>1.08</td>
<td>0.70</td>
<td>4.42</td>
</tr>
</tbody>
</table>

**Figure B-8 Line configuration #12 and best solution**

- **STAGE 1**
  - Processing times: Logn(0.5, 0.25)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.0

- **STAGE 2**
  - Processing times: Logn(0.7, 0.35)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.143

- **STAGE 3**
  - Processing times: Logn(0.9, 0.45)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.286

- **STAGE 4**
  - Processing times: Logn(1.1, 0.55)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.429

- **STAGE 5**
  - Processing times: Logn(1.3, 0.65)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.571

- **STAGE 6**
  - Processing times: Logn(1.5, 0.75)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.714

- **STAGE 7**
  - Processing times: Logn(1.25, 0.625)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 1.857

- **STAGE 8**
  - Processing times: Logn(0.75, 0.375)
  - Breakdowns: TBF exp(1000), TTR exp(3)
  - Value of inventory per unit: 2.0

- **DEMANDS**
  - Time between: Logn(1.87, 0.937)
  - Workload: 80%
  - Service target: 95%
  - Backorders