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The Term Structure of Interest Rates and Inflation Forecast Targeting

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Abstract

This paper examines the implications of the expectations theory of the term structure for the implementation of inflation targeting. We show that the term structure weakens the transmission of short term interest rates to ultimate policy objectives. Therefore, short term interest rates in the central bank's forward looking monetary policy rule need to respond more strongly to the output gap and deviations of inflation from its target. Thus, in general the term structure implies a higher degree of policy activism. Next, we show that both the sensitivity of the term spread to economic fundamentals, and the extent to which the spread predicts future output, are increasing in the duration of the long bond and the degree of structural output persistence. If the central bank becomes relatively less concerned about inflation stabilisation the term spread becomes less sensitive to fundamentals, and the spread will be less successful in predicting real economic activity.

Keywords: term structure of interest rates, inflation targets

JEL Codes: E43, E58

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1. Introduction

Since the early 1990’s the conduct of monetary policy in many countries has switched to a regime of direct inflation targeting. This change was triggered either as a result of the breakdown of the relationship between growth rates in monetary aggregates and inflation (New Zealand and Canada) or because of the disappointment following the use of exchange rates as an intermediate target for monetary policy (United Kingdom, Sweden and Finland). The use of inflation targets derives its theoretical rationale from the fact that they can be used to overcome credibility problems since they can mimick the results of optimal performance incentive contracts (see Walsh (1995) and Svensson (1997a)). However, these theories assume that central banks can instantaneously choose the rate of inflation. Contrary to this assumption, in practice central banks rarely directly set some desired level for the money supply but rather conduct monetary policy by deciding upon the price at which the banking system’s systematic shortage of central bank balances on the interbank money market will be relieved. This gives the central bank perfect control over the day-to-day interbank interest rate. Moreover, this interest rate will affect the rate of inflation with a considerable time lag. From a theoretical perspective this raises the issue as to how explicit inflation targets should be translated into monetary policy instruments. A first contribution to this question was made by Svensson (1997b) who has shown that, because of lags in the transmission process, inflation targeting implies inflation forecast targeting. In this analysis the inflation forecast produced by the central bank’s structural model of the economy becomes an ideal intermediate target since it is by definition closely related to the ultimate policy goal and since it can be perfectly controlled by the central bank. Furthermore, the inflation forecast will lead to an endogenous optimal interest rate reaction function which has the same form as the Taylor rule (Taylor (1993)).

Also, the past few years have seen a revival of interest in the importance of the term-structure of interest rates for the transmission of monetary policy (e.g. Turnovsky (1989), Goodfriend (1997)). Changes in the central bank’s key interest rate are transmitted to longer term interest rates through the term structure. In particular, the expectations hypothesis of the term structure assumes that these longer term interest rates will be equal to a weighted average of expected future one-period interest rates (i.e. the expected future values of the central bank’s instrument). These longer term interest rates will, in turn, affect the determinants of aggregate demand. Recently, research in this respect has focussed on explanations for the failure of predictive content of

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3 Bernanke and Woodford (1997) have argued that inflation forecast targeting can only work if the inflation forecast is based on the central bank’s own structural model of the economy. They show that responding to private sector forecasts may lead to indeterminacy or non-existence of a rational expectations equilibrium.
long-short spread for future movements in interest rates (McCallum (1994) and Rudebusch (1995)), and on the interaction between the term structure and shifts in the conduct of monetary policy in VAR-models (Fuhrer and Moore (1995), Fuhrer (1996)).

The purpose of this paper is to incorporate the term structure of interest rates into the Svensson (1997b) inflation forecast targeting framework. To this end, Section 2 will present a model in which monetary policy affects the real economy via the term structure. Section 3 will derive the endogenous optimal interest rate reaction function and will discuss the determinants of policy activism. Next, in Section 4 we will discuss the determinants of the sensitivity of the real term spread with respect to the underlying state of the economy and the factors which affect the covariance between the term spread and output in the subsequent period. Finally, Section 5 will analyse the implication of the interaction between the optimal monetary policy rule and the term structure of interest rates for the predictive value of the term spread as far as future movements in long-term interest rates are concerned.

2. Monetary Policy and the Term Structure

The purpose of this section is to incorporate the term-structure of interest rates in the Svensson (1997b) inflation forecast targeting framework. To this end we assume that the short real rate \( r_t \) and the long real rate \( R_t \) are related by the following version of the Pure Expectations Hypothesis (PEH)

\[
r_t = R_t - D(E_t R_{t+1} - R_t)
\]

(2.1)

Here \( r_t \) represents the real yield to maturity on a one-period bond which is traded on the interbank money market. The LHS denotes the (one-period) real holding period return on a long-term bond. The latter’s real yield to maturity \( R_t \) is the long-term real interest rate. The parameter \( D \) is defined such that \( D + 1 \) is equal to Maccaulay’s duration. 4

\[1 + r_t = E_t \left( \frac{P_{N-1,t+1}}{P_{N,t}} \right) = E_t \left( \frac{1 + R_{N,t}}{(1 + R_{N-1,t+1})^{N-1}} \right)\]

Since \( R_{t+1} \) is a stochastic variable, Jensen’s Inequality will affect this equation because \( \ln E_t(X) \neq E_t(\ln X) \) and because \( E_t(1/X) \neq 1/E_t(X) \). However, since equation (2.1) yields a very convenient linear approximation, it is quite standard in the term structure literature to ignore this effect (see e.g. Fuhrer and Moore (1995)) Ignoring the effect of Jensen’s Inequality, omitting \( N \) and \( N-1 \) from the yield to maturity subscripts and taking logs on both sides of the equation then yields equation (2.1).
For our purposes it turns out to be convenient to rewrite (2.1) to express the current long real rate as a convex combination of the current short real rate and the expected long real rate in the next period:

\[ R_t = (1-k) r_t + k E_t R_{t+1} \quad \text{with} \quad k \equiv \frac{D}{1+D} \]  

(2.2)

Note that the long and short real interest rates will be equal if the parameter \( k \) is equal to zero. In that case the duration of the long-term bond will be equal to one and the model will collapse into the original Svensson (1997b) model in which there is no distinction between short and long term interest rates.

The current short-term real interest rate will be equal to:

\[ r_t = i_t - E_t \pi_{t+1} \]  

(2.3)

Here \( i_t \) is the instrument of the central bank (i.e. the nominal interest rate on the interbank money market) and \( E_t \pi_{t+1} \) represents the expected rate of inflation in period \( t+1 \) conditional on the information set in period \( t \).

Following Svensson (1997b) we assume that inflation and output are linked by the following short-term accelerationist Phillips-curve relationship:

\[ \pi_{t+1} = \pi_t + \alpha_1 y_t \]  

(2.4)

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5 This equation implies: \( R_t = (1-k) \sum_{\tau=1}^{\infty} k^{\tau-1} E_t r_{t+\tau} \), i.e. the current long-term real rate is a weighted average of expected future short-term real rates (see Schiller et al (1983)).

6 If the long term bond pays coupon interest, then Macaulay’s duration will be strictly smaller than maturity and will actually depend on \( R_t \). However, for reasons of tractability we will treat \( D \) as a constant. In this case equation (2.1) can be shown to be a log-linear approximation of the condition for equalisation of expected holding period returns (see Shiller, Campbell and Schoeholtz (1983)).

7 As noted amongst others by Ball (1997), this equation ignores the Lucas critique and therefore violates the natural rate hypothesis since an ever increasing rate of inflation can coexist with a systematic positive value of the output gap.
where \( \pi_t \equiv p_t - p_{t-1} \), the inflation rate in period \( t \) (\( p_t \) is the (log of the) price level). The variable \( y_t \) represents the (log of the) output gap in period \( t \) where potential output has been normalised to zero. Finally, the parameter \( \alpha_1 \) measures the slope of the Phillips-curve. The output gap is determined by the following dynamic relationship:

\[
y_{t+1} = \beta_1 y_t - R_t + x_{t+1}
\]

(2.5)

Following Svensson (1997b) we assume that output is serially correlated and stationary \((0 < \beta_1 < 1)\). However, whereas in the Svensson model output is decreasing in the \textit{short-term} real interest rate with a lag of one period, here we assume that next period’s output gap is decreasing in the \textit{long-term} real interest rate \( R_t \). This assumption can be justified on the grounds that the interest rate sensitive components of aggregate demand generally do not depend directly on the day-to-day interbank money market interest rate but rather on the yield on \textit{some} financial asset with a longer maturity\(^8\). For simplicity we assume that there is only one long-term interest rate in the output equation which pertains to the real yield to maturity on a long-term indexed bond the duration of which is equal to \( D+1 \). Finally, output is increasing in an exogenous demand shock \((x_{t+1})\) which itself is also serially correlated and stationary \((0 < \beta_2 < 1)\):

\[
x_{t+1} = \beta_2 x_t + \varepsilon_{t+1} ; \quad \varepsilon_{t+1} \sim N\left(0, \sigma^2\right)
\]

(2.6)

Having described the structure of the economy it remains to specify the preferences of the central bank. Monetary policy is conducted by a central bank with an explicit inflation target \( \pi^* \) (say 2.5% per year) that aims to minimise deviations of inflation from this assigned target, on the one hand, and fluctuations of output around the natural rate (which is normalised to zero), on the other.\(^9\)

Consequently, the central bank will choose a sequence of current and future short-term nominal rates to minimise the following loss function:

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However, given the preferences of the central bank such policies will never be optimal.

\(^8\) Of course, there will be many interest rates which pertain to debt instruments of both short and long maturities which affect aggregate demand. In this respect, the parameter \( D \) can be seen as an indicator of the relative share of long-term private debt in the economy.

\(^9\) As noted by Svensson (1997b) this means that the central bank does have a long run inflation target \((\pi^*)\) but no long run output target (other than the natural rate of output). In other words, even though the central bank wishes to limit short-term output variability, in the long run its only objective is price stability.
\[ L^{CB} = E_T \sum_{t=0}^{\infty} \delta^{-t} \left[ \frac{1}{2} (\pi_t - \pi^*)^2 + \frac{\lambda}{2} y_t^2 \right] \]  

(2.7)

Here \( \lambda \) represents the central bank’s relative weight on output stabilisation while the parameter \( \delta \) (which fulfils \( 0 < \delta < 1 \)) denotes the discount factor (i.e. a measure of the policy horizon). The expectation is conditional on the central bank’s information set in period \( t \) which contains current output \( (y_t) \) the current inflation rate \( (\pi_t) \) and the structure of the economy as described by equations (2.2) - (2.6).

3. Derivation of the Optimal Instrument Rule

From the structure of the model it can be seen that next period’s conditional expectation for output \( (E_t y_{t+1}) \) can be regarded as an indirect control variable for the central bank. Equation (2.3) shows that for a given\(^{11} \) value of \( E_t R_{t+1} \), the central bank can attain any desired value for the current long rate by manipulating its instrument. This, in turn, allows it to target \( E_t y_{t+1} \) perfectly. Hence, as in Svensson (1997b) the central bank’s problem can be reformulated as follows:

\[
V(E_t \pi_{t+1}) = \text{Min}_{E_t \pi_{t+1}} \left\{ \left[ \frac{1}{2} (E_t \pi_{t+1} - \pi^*)^2 + \frac{\lambda}{2} (E_t y_{t+1})^2 \right] + \delta E_t V(E_t \pi_{t+2}) \right\}
\]

subject to

\[ E_{t+1} \pi_{t+2} = \pi_{t+1} + \alpha_t \ y_{t+1} \]

As shown in Appendix A, the first-order condition for the minimisation problem in equation (3.1) will yield a rule for the central bank’s conditional one-to-two year inflation forecast \( (E_t \pi_{t+2}) \). This conditional inflation forecast thus becomes the central bank’s intermediate target for monetary policy

\( ^{10} \) Note that here the central bank is conducting monetary policy from a clear forward looking perspective. This means that - as elegantly stated by Greenspan in his Congressional testimony on 22 February 1995 - “…monetary policy will have a better chance of contributing to meeting the nation’s macroeconomic objectives if we look forward as we act, however indistinct our view of the road ahead. Thus over the past year (1994) we have firmed policy to head off inflation pressures not yet evident in the data…”

\( ^{11} \) As we will show later on this given value of \( E_t R_{t+1} \) is unambiguously pinned down by the expectation that in every future period the central bank will implement monetary policy so as to minimise its loss function.
and can be expressed as follows:

\[
E_t,\pi_{t+2} = \pi^* + n[E_t,\pi_{t+1} - \pi^*] \quad ; \quad n \equiv \frac{\lambda}{\delta \alpha^*_t \mu + \lambda}
\]  \hspace{1cm} (3.2)

The reduced-form parameter \( \mu \) is a function of the parameters \( \alpha_1, \delta \) and \( \lambda \) (see Appendix A). If the central bank engages in strict inflation targeting (i.e. if the relative weight on output stabilisation (\( \lambda \)) is equal to zero), it will set its intermediate target equal to the inflation target. However in the more realistic case of flexible inflation targeting, it will allow \( E_t,\pi_{t+2} \) to adjust gradually towards the assigned inflation target \( \pi^* \) (note that \( n \in [0,1] \)). The speed of adjustment will then depend negatively on the central bank’s relative weight on output stabilisation (i.e. \( \partial n/\partial \lambda > 0 \), see Appendix A).

From equations (2.4) and (2.5) it can be seen that the actual one-to-two year inflation forecast dictated by the structure of the economy will be equal to:

\[
E_t,\pi_{t+2} = \pi_t + \alpha_1 (1 + \beta_1) y_t - \alpha_1 R_t + \alpha_1 \beta_2 x_t
\]  \hspace{1cm} (3.3)

In Appendix B it is shown that, on the assumption that the central bank seeks to attain the optimal intermediate target in each and every period (i.e. (3.2) holds for all \( \tau \geq t \)), the long real interest rate can be expressed as follows:

\[
R_t = \frac{1 - k}{1 + k \beta_1} r_t + \frac{kn(1 - n)}{\alpha_1 (1 + k \beta_1)} (\pi_t - \pi^*) + \frac{k(n(1 - n) + \beta_2^2)}{1 + k \beta_1} y_t + \frac{k \beta_2 (\beta_1 + \beta_2)}{1 + k \beta_1} x_t
\]  \hspace{1cm} (3.4)

Substituting this equation in equation (3.3) we can express the actual one-to-two year inflation forecast in terms of period \( t \) state variables and the central bank’s instrument:
\[ E_{t+2} = \left[ 1 + \frac{\alpha_1(1-k)}{(1+k\beta_1)} \right] \pi_t - \frac{kn(1-n)}{(1+k\beta_1)}(\pi_t - \pi^*) + \frac{\alpha_1\beta_2(1-k\beta_2)}{(1+k\beta_1)} x_t \]

\[ + \frac{\alpha_1}{(1+k\beta_1)} \left[ (1+\beta_1)(1+k\beta_1) + \alpha_1(1-k) - k(n(1-n) + \beta_1^2) \right] y_t - \frac{\alpha_1(1-k)}{(1+k\beta_1)} i_t \]

(3.5)

Obviously, the central bank will choose \( i_t \) such that the one-to-two year conditional inflation forecast in equation (3.5) will be equal to the optimal intermediate target specified in equation (3.2). Hence, by combining these two equations we find the following endogenous optimal instrument rule:

\[ i_t = \pi_t + \frac{(1-n)}{\alpha_1(1-k)}(1+k\beta_1 - kn)(\pi_t - \pi^*) + \]

\[ \frac{\{(1+\beta_1-n)(1+k\beta_1) + \alpha_1(1-k) - k(n(1-n) + \beta_1^2)\}}{(1-k)} y_t + \frac{\beta_2(1-k\beta_2)}{(1-k)} x_t \]

(3.6)

This equation has the same form as the Taylor rule and explicitly allows for an effect of the term structure of interest rates on the optimal instrument rule (see Proposition 1 below). Note that this endogenous interest rate rule will collapse into the Svensson rule if the output gap is directly determined by the central bank’s instrument (i.e. if \( k=0 \)).

The effect of several parameters on the extent to which the central bank’s instrument will respond to economic fundamentals can be summarised by the following proposition:

**PROPOSITION 1:**

The optimal response of the short-term interest rate to its determinants will become stronger if:

1. the duration of the long bond (D) increases
2. the relative weight on output stabilisation (\( \lambda \)) decreases
3. the degree of output persistence (\( \beta_1 \)) increases

12 In that case we obtain: \( i_t = \pi_t + [(1-n)/\alpha_1](\pi_t-\pi^*) + (1+\beta_1+\alpha_1-n)y_t + \beta_2 x_t \) which can easily be shown to be the solution to a particular variant of this model where it holds that \( R_t=r_t \).
The proof of this proposition is given in Appendix D. The first part of Proposition 1 summarises the effect of the term structure on the central bank’s optimal reaction function. The response of monetary policy with respect to the output gap and the deviation of inflation from its target (in short the degree of policy activism) will increase as the duration of the long term bond increases. This is caused by a decrease in policy leverage over the long real rate since the latter will now to a greater extent be determined by expected future short real rates at the expense of the present short real rate (see Goodfriend (1997)). However, provided central bank preferences are constant over time, a change in duration will not alter the central bank’s optimal intermediate target as expressed in equation (3.2). Therefore, the central bank will have to manipulate its instrument more actively in order to attain the same desired effect on the long-term real interest rate.

The second part of this Proposition is equivalent to Svensson’s finding that the extent to which the central bank’s instrument will react to the output gap and the deviation of inflation from its target will decrease as the central bank cares more about short-term output stabilisation. The intuition is that this will reduce the speed with which the central bank wants inflation to return to its target after the economy has been hit by a shock. This will induce a less activist response of interest rates. Obviously, this result is insensitive to the question whether or not the term structure of interest rates constitutes an important part of the monetary transmission mechanism. Finally, an increase in output persistence ($\beta_1$) will increase the effect of current exogenous shocks ($x_t$) on next period’s output gap ($y_{t+1}$). To offset this effect, interest rates will have to be manipulated more actively if the central bank is to attain its objectives for output and inflation stabilisation.

### 4. The Behaviour of the Term Spread under the Optimal Rule

This Section will study the implications of inflation forecast targeting for the spread between the real short and the real long rate. First of all, from equations (2.3) and (3.6) the equilibrium real short -term interest rate will be equal to:

$$r_t = \frac{(1-n)}{\alpha_i(1-k)}(1+k\beta_1 - kn)(\pi_t - \pi^*) +$$

$$+ \frac{\{(1+\beta_1 - n)(1+k\beta_1) - k(n(1-n) + \beta_1^2)\}}{(1-k)}y_t + \frac{\beta_2(1-k\beta_2)}{(1-k)}x_t$$

Furthermore, using this equation in the expression for the equilibrium long-term real interest interest rate obtained in equation (3.4) we get:
\[ R_\tau = \frac{(1-n)}{\alpha_1}(\pi_\tau - \pi^*) + (1 + \beta_1 - n)y_\tau + \beta_2 x_\tau \quad (4.2) \]

In our model the long term real interest rate is exactly the same as the ex ante real short term interest rate \( (i_t - \alpha_1 y_t - \pi_t) \) obtained in the model where the term structure is absent (see footnote 12). This result should not be surprising since in both models the central bank seeks to attain the same value for \( \text{E}_t \pi_{t+2} \) which implies that in both models \( \text{E}_t y_{t+1} \) will be the same. The only difference is that \( \text{E}_t y_{t+1} \) will be directly influenced by \( i_t \) in the absence of the term structure while in our model the central bank will set \( i_t \) such as to attain that specific value of \( R_t \) consistent with its intermediate target.

Next, we can examine the economic determinants of the real term spread by subtracting equation (3.1) from (3.2) which yields:

\[
R_t - r_t = \frac{k(1-n)(1 + \beta_1 - n)}{\alpha_1(1-k)}(\pi_t - \pi^*) - \\
\frac{k(1 + 2\beta_1 + n(n - (2 + \beta_1)))}{(1-k)}y_t - \frac{k\beta_2(1 - \beta_2)}{(1-k)}x_t 
\quad (4.3)
\]

Since optimal monetary policy entails an endogenous Taylor-rule-like response to inflation, the output gap, and an exogenous demand shock, the real term spread will also be driven by these factors. The effect of these variables can be summarised by the following proposition (see Appendix D for a proof):

**PROPOSITION 2:**

The real short term interest rate will rise relative to the real long rate (i.e. \( R_t - r_t \) will decrease) if:

1. the current inflation rate is higher than its target \( (\pi_t > \pi^*) \)
2. the current output gap is positive \( (y_t > 0) \)
3. the current aggregate demand shock is positive \( (x_t > 0) \)

If one or more of the determinants of the term spread take on a positive value this will mean increase in future inflation. We have assumed that the central bank’s exclusive long run objective is to maintain a level of inflation equal to its assigned target (i.e. the model does not deal with such issues
as a persistent temptation to increase output above its natural rate and the concomitant credibility problems which follow). The implication of such a fully credible inflation targeting regime is that the central bank will not allow inflation to deviate \textit{systematically} from its target and that, therefore, output will not \textit{systematically} differ from potential. Since, in addition, the unconditional expectation of the output shock \((x_t)\) is assumed to be equal to zero, this means that the real short rate will not \textit{systematically} differ from zero either (see equation (3.1))\(^{13}\). Because the long real rate is essentially a weighted average of current and future expected real short rates, the increase in the current real short rate necessary to reduce inflationary pressures will be larger than the induced increase in the long real rate.

Furthermore, as a result of the optimal reaction of monetary policy to the state of the economy, the central bank will induce a positive relationship between the real term spread in period \(t\) and the output gap in period \(t+1\). This can be inferred from the following expression for the covariance between these two variables (see Appendix C)\(^{14}\):

\[
\text{Cov}[(R_t - r_t) y_{t+1}] = \frac{k(1 + 2\beta_1 - n)}{(1 + n)(1 - k)} \sigma_e^2
\]  \hspace{1cm} (4.4)

This result lines up with the literature on the effect of monetary policy on future output (e.g. Bernanke and Blinder (1992), Fuhrer and Moore (1995)) and is essentially the consequence of a systematic ‘leaning against the wind’ policy.

Even though these studies indicate that the term spread contains information about future output growth, they also show that are substantial differences between countries. For instance, Smets and Tsatsaronis (1997, p 4) present evidence that ‘(...) the correlation between annual output growth and the lagged term spread is higher in Germany than in the United States(…)’. They attribute part of this difference to the fact that the influence of inflation scares on the US \textit{nominal} term spread in the is much more significant than in Germany as a result of the fact that the Bundesbank enjoys a higher degree of credibility. Furthermore, they present evidence that the Bundesbank reacts more vigorously in real terms to various shocks than the US. In this respect it is interesting to investigate what the practice of inflation forecast targeting implies for the \textit{sensitivity} of the real term spread to

\(^{13}\) In other words, both the steady state real short rate and the steady state real long rate are normalized to zero in this model (in the steady state it holds that \(y_t = x_t = 0\) and \(\pi_t = \pi^*\)), i.e. real interest rates will not be \textit{systematically} affected by monetary policy.

\(^{14}\) For reasons of tractability, in computing this expression we have assumed that the stochastic output shock \((x_t)\) is purely white noise (i.e. it holds that \(\beta_2 = 0\)).
its determinants. Evidently, a more vigorous response of the real term spread to these determinants will lead to a stronger relationship between movements in the term spread and future output. The results are summarised in the following proposition (see Appendix D for a proof):

**PROPOSITION 3:**

The real term spread will react more strongly to the current output gap ($y_t$) and the current deviation of inflation from its target ($\pi_t - \pi^*$) and as a result the covariance between this spread and future output ($\text{Cov}[(R_t-r_t)y_{t+1}]$) will increase if:

1. the duration of the long bond (D) increases
2. the relative weight on output stabilisation ($\lambda$) decrease
3. the degree of output persistence ($\beta_1$) increases

First of all, an increase in the duration of the long-term bond does not affect the real long rate itself but it does require a more active manipulation of the short rate to attain that specific value of the long real rate consistent with the central bank’s objectives. For given realisations of the underlying determinants, the induced increase in policy activism will widen the gap between the real short and the real long rate. The practical implication of this result is that the duration of the debt instrument which affects aggregate demand will be one of the determinants of the response of the real term spread to economic developments. Because of this, the financial structure of the economy (i.e. the extent to which spending depends on long term interest rates) will influence the observed correlation between the term spread and future output growth. Hence, the fact that many VAR-studies (e.g. Estrella and Mishkin (1997), Smets and Tsatsaronis (1997)) indicate that the real term spread seems to be more strongly related to future output in Germany than in the US can also be partly explained by the fact that the financial structure of the German economy incorporates a larger relative share of long-term debt than the US.

Next, as the central bank attaches a larger weight to short-term output stabilisation relative to inflation stabilisation (i.e. if $\lambda$ increases), it will exert less influence on the output gap in any given period. As a consequence, the term spread will be less sensitive to changes in monetary policy and its predictive value for future output will diminish because of the weaker link between the central bank’s instrument in period t and output in the subsequent period.

Finally, an increase in the degree of output persistence ($\beta_1$) will increase the effect of current stochastic shocks to the output gap on output and inflation in subsequent periods. To offset this effect the central bank will display a more activist response to current economic variables which will induce a closer relationship between monetary policy and future output.
5. Implications for the Predictive Ability of the Term Structure

One of the implications of the expectations hypothesis of the term structure of interest rates is that the spread between long and short rates should have predictive power for future movements in short as well as long rates. After all, rational expectations implies:

\[ E_t R_{t+1} = R_{t+1} + \nu_{t+1} \]  \hspace{1cm} (5.1)

Here, the error term \( \nu_{t+1} \) has an unconditional expectation which is equal to zero and its distribution is orthogonal to both \( R_t \) and \( r_t \). Using this in equation (2.1) we can write:

\[ D(R_{t+1} - R_t) = (R_t - r_t) - D\nu_{t+1} \]  \hspace{1cm} (5.2)

Hence, it seems that in a regression of \( D(R_{t+1} - R_t) \) on the real term spread \( (R_t - r_t) \), the slope coefficient should have a probability limit of one. Nevertheless, such regressions systematically yield and estimate of this slope coefficient which is significantly lower than one (for a survey see Rudebusch (1995)). This result seems to either reject the assumption of rational expectations or the expectations hypothesis of the term structure. However, McCallum (1994) has shown that the expectations theory of the term structure can be reconciled with a lack of predictive content in the term spread by explicitly taking the conduct of monetary policy into account. In particular, he postulates an exogenous interest rate reaction function which features interest rate smoothing combined with a response to the term spread. Assuming that the term spread is also influenced by an exogenous serially correlated term premium and solving for this equation and the postulated central bank reaction function simultaneously, he finds a slope coefficient with a probability limit which is significantly smaller than one. This reflects the general idea that future short rates (and therefore also future long rates) will be determined by future monetary policy and that, consequently, expectations of future policy will be reflected in the term spread (see also Fuhrer and Moore (1995)).

In this respect it is interesting to see what our endogenously derived interest rate reaction function implies for the probability limit of the slope coefficient in equation (5.2). Leading equation (4.2) by

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15 McCallum justifies this by invoking the predictive content of the term spread for future output. This may, however, be a monetary policy rule with undesirable consequences since this predictive content observed in practice is at least to some extent induced by monetary policy. Bernanke and Woodford (1997) show that adherence to such a rule may lead to sunspot equilibria.
one period and using equations (2.4)-(2.6) the long real rate in period $t+1$ can be shown to be equal to:

$$R_{t+1} = \frac{(1-n)(n-\beta_i)}{\alpha_i} (\pi_t - \pi^*) + (1-n)(n-\beta_1)y_t$$

$$+ \beta_2^2 x_t + (1 + \beta_1 - n + \beta_2) \epsilon_{t+1}$$

(5.3)

Subtracting equation (3.2) from this equation and pre-multiplying the result by $D = k/(1-k)$ yields:

$$D(R_{t+1} - R_t) = -\frac{k(1-n)(1+n-\beta_1-n)}{\alpha_i(1-k)}(\pi_t - \pi^*) - \frac{k(1+2\beta_1+n(n-(2+\beta_1))}{(1-k)}y_t$$

$$-\frac{k\beta_2(1-\beta_2)}{(1-k)} x_t + \frac{k(1+\beta_1+\beta_2-n)}{(1-k)} \epsilon_{t+1}$$

(5.4)

$$= R_t - r_t + \frac{k(1+\beta_1+\beta_2-n)}{(1-k)} \epsilon_{t+1}$$

Hence, in this model the slope coefficient will have a probability limit which is equal to one. In other words, within the inflation forecast targeting framework the conduct of monetary policy will not affect the predictive content of the real term spread for future movements in long term real interest rates. The reason is that the central bank always sets short-term interest rates according to the optimal rule. Since the response coefficients in this rule do not change over time this means that next period’s long term interest rate will be predicable (up to a random error term) as well as can be seen from equation (5.3).

Consequently, the empirical failure of the predictive content of the term structure cannot be explained in the context of inflation forecast targeting framework. In order to explain this failure either some degree of interest rate smoothing (McCallum (1994)) or changes in the conduct of monetary policy over time (Fuhrer (1996) must be relied upon. These two features probably do play a significant role in practice. However, the problem is how to account for them theoretically since neither is optimal given our standard description of central bank preferences involving a short-term (time-invariant) trade-off between inflation and output variability.
6. Summary and Concluding Remarks

This paper incorporates the term structure of interest rates into the Svensson (1997b) inflation forecast targeting framework. In this respect it is found that if the term structure is relevant for the transmission process, the reaction coefficients in the optimal instrument rule will have to be higher than the ones implied by a model in which there is no distinction between long-term and short-term interest rates.

We identified three parameters which affect the degree of policy activism. First of all, the duration of the long-term interest rate which affects aggregate demand has a positive effect on activism since this parameter is inversely related to the central bank’s leverage over long term interest rates. The same holds for the degree of structural output persistence. On the other hand, the relative weight on output stabilisation will exert a moderating influence on the degree of activism since it decreases the speed with which the central bank brings actual inflation back in line with the inflation target. Since in this model both the real term spread and future output are to a large extent driven by monetary policy we also looked at the implication of changes in the afore-mentioned parameters for the sensitivity of the term spread to the underlying economic situation and the covariance between the term spread and output in the subsequent period. In general, a higher degree of policy activism will also serve to increase both the afore-mentioned sensitivity and the correspondence between movements in the term spread and future output.

Finally, we have looked at the question whether or not the conduct of monetary policy implied by inflation forecast targeting can be used to explain the empirical failure of the predictive content of the term spread for future interest rates. This question is interesting since it is generally agreed that such an explanation requires that the monetary policy reaction function be taken into account explicitly. However, we found that the optimal interest rate policy in this model implies that the term spread should yield estimates of the slope coefficient in the term structure which are not significantly different from one. The reason for this result is that theoretical explanations for the afore mentioned empirical failure all depend on either some degree of interest rate smoothing and/or shifts in the conduct of monetary policy over time. Neither of these is optimal within the present framework. We leave the question whether or not the model can be altered in such a way as to yield these phenomena as part of the central bank’s optimal policy rule for future research.
Appendix A: Derivation of the Optimal Intermediate Target

This appendix provides a brief description of the derivation of the optimal intermediate target. For a more elaborate treatment see Svensson (1997b). First of all, from equation (2.7) we realise that the indirect loss function will be of the general form:

\[ V(E, \pi_{t+1}) = \mu_0 + \frac{1}{2} \mu (E, \pi_{t+1} - \pi^*)^2 \]  

(A.1)

Here, \( \mu_0 \) and \( \mu \) are coefficients which remain to be determined. Next, using equation (2.7) in the main text, the first-order condition for the central bank's optimisation problem will read as follows:

\[ \lambda y_{t+1} \alpha \delta \partial \pi \left( E, \pi_{t+1} \right) = 0 \]  

\[ \left( E, \pi_{t+2} - \pi^* \right) = -\frac{\lambda}{\delta \alpha \mu} E y_{t+1} \]  

(A.2)

Using equation (A.1) to find an expression for the partial derivative between brackets, we can write:

\[ E_{t+1} \partial \pi = -\frac{\lambda}{\delta \alpha \mu} E y_{t+1} \]  

(A.3)

From equation (2.4) the conditional forecast of next period’s output will be equal to:

\[ E y_{t+1} = \frac{1}{\alpha_1} (E_{t+1} \pi_{t+2} - E_{t+1} \pi_{t+1}) \]  

(A.4)

Plugging this equation into equation (A.3) and rearranging will yield equation (3.2) in the main text.

Next, in order to identify the coefficients \( \mu_0 \) and \( \mu \) in equation (A.1) we realise that using equation (3.1) we can compute:

\[ \frac{\partial V(E, \pi_{t+1})}{\partial E_{t+1} \pi_{t+1}} = E_{t+1} \pi_{t+1} - \pi^* \]  

\[ \frac{\partial V(E_{t+1} \pi_{t+2})}{\partial E_{t+1} \pi_{t+1}} \left( \frac{\partial E_{t+1} \pi_{t+2}}{\partial E_{t+1} \pi_{t+1}} \right) \]  

(A.5)

As far as this equation is concerned we note that from equations (A.1) and (2.4) respectively, we can derive:

\[ \frac{\partial V(E_{t+1} \pi_{t+2})}{\partial E_{t+1} \pi_{t+2}} = \mu (E_{t+1} \pi_{t+2} - \pi^*) \]  

(A.6)

\[ \frac{\partial E_{t+1} \pi_{t+2}}{\partial E_{t+1} \pi_{t+1}} = 1 \]  

Plugging these equations into (A.5) and using the expression obtained for \((E_{t+1} \pi_{t+2} - \pi^*)\) in equation (3.2) we can rewrite (A.5) as follows:
\[
\frac{\partial V(E, \pi_{t+1})}{\partial E, \pi_{t+1}} = \left[ 1 + \frac{\lambda \delta \mu}{\delta \alpha_i^2 \mu + \lambda} \right] (E, \pi_{t+1} - \pi^*)
\]  
\hspace{1cm} (A.7)

Identification for the coefficient for \((E, \pi_{t+1} - \pi^*)\) yields:
\[
\mu = \left[ 1 + \frac{\lambda \delta \mu}{\delta \alpha_i^2 \mu + \lambda} \right] \equiv F(\mu)
\]  
\hspace{1cm} (A.8)

From this equation it can be seen that for \(k \in [0, \infty)\) it will hold that \(F(\mu) \in [1, \lambda/\alpha^2]\). Using this it can be shown that the unique positive solution for \(\mu\) will be:
\[
\mu = \frac{1}{2} \left( 1 - \frac{\lambda (1 - \delta)}{\delta \alpha_i^2} + \sqrt{\left( 1 + \frac{\lambda (1 - \delta)}{\delta \alpha_i^2} \right)^2 + 4 \frac{\lambda}{\alpha_i^2}} \right) \geq 1
\]  
\hspace{1cm} (A.9)

To prove that \(\partial n/\partial \lambda > 0\) we first realize that we can write:
\[
n = \frac{1}{\delta \alpha_1^2 \mu/\lambda + 1} ; \quad \frac{\partial n}{\partial \left( \frac{\mu}{\lambda} \right)} = -\frac{\delta \alpha_i^2}{(\delta \alpha_i^2 \mu/\lambda + 1)^2} < 0
\]  
\hspace{1cm} (A.10)

Furthermore, using equation (A.9) we can compute:
\[
\frac{\partial \left( \frac{\mu}{\lambda} \right)}{\partial \lambda} = \frac{1}{2} \left\{ \begin{array}{l}
\left( \frac{-4}{\alpha_i^2 \lambda^2} - 2 \left( \frac{(1 - \delta)}{\alpha_i^2 \delta} + \frac{1}{\lambda} \right) \right) - \frac{1}{\lambda^2} \\
2 \sqrt{\left( \frac{(1 - \delta)}{\alpha_i^2 \delta} + \frac{1}{\lambda} \right)^2 + \frac{4}{\alpha_i^2 \lambda}}
\end{array} \right\} < 0
\]  
\hspace{1cm} (A.11)

Consequently, it will hold that: \(\partial n/\partial \lambda = \partial n/\partial (\mu/\lambda) \ast \partial (\mu/\lambda)/\partial \lambda > 0\).

**Appendix B: Derivation of the Long Real Rate under the Optimal Monetary Policy Rule**

Leading equation (2.5) by one period, using equation (2.5) in the resulting expression, taking expectations conditional on the information in period \(t\) and rearranging we obtain:
\[
E_i R_{t+1} = \beta_1 y_i - E_i y_{t+2} - \beta_1 R_t + \beta_2 (\beta_1 + \beta_2) x_i
\]  
\hspace{1cm} (B.1)
Furthermore, by leading equation (2.4) one period and taking expectations conditional on the information in period $t$, $E_t y_{t+2}$ can be expressed as follows:

$$E_t y_{t+2} = \frac{1}{\alpha_1} [ E_t \pi_{t+3} - E_t \pi_{t+2} ]$$  \hspace{1cm} (B.2)

Since the central bank will follow its optimal target rule in every period we can find an expression for the term between brackets by leading equation (3.2) one period and subtracting equation (3.2) from the result:

$$E_t \pi_{t+3} - E_t \pi_{t+2} = -n(1-n) [ E_t \pi_{t+1} - \pi^* ]$$  \hspace{1cm} (B.3)

Note that the RHS of this equation will be equal to zero if it holds that $\lambda = 0$. Using equations (B.2) and (B.3) we can rewrite (B.1) as follows:

$$E_t R_{t+1} = \frac{n(1-n)}{\alpha_1} (\pi_t - \pi^*) + (n(1-n) + \beta_1^2) y_t + \beta_2 (\beta_1 + \beta_2) x_t - \beta_1 R_t$$  \hspace{1cm} (B.4)

Plugging this equation into equation (2.3) and rearranging will yield equation (3.4).

**Appendix C: Derivation of the Covariance between the Real Term Spread and Future Output**

Plugging the equilibrium solution for $R_t$ obtained in equation (4.2) into the dynamic equation for output (2.5) and subtracting $\pi^*$ on both sides of the Phillips-curve relationship (2.4) we have the following two-dimensional VAR(1) system:

$$Z_{t+1} = A Z_t + \phi_{t+1}$$

where:

$$Z_t = \begin{bmatrix} y_t \\ \pi_t - \pi^* \end{bmatrix}; \quad A = \begin{bmatrix} -(1-n) & -(1-n) \\ \alpha_1 & 1 \end{bmatrix}; \quad \phi_{t+1} = \begin{bmatrix} \varepsilon_{t+1} \end{bmatrix}$$

The system has two distinct and real Eigenvalues: $e_1 = 0$ and $e_2 = n$. These indicate that the two column vectors in the matrix $A$ are linearly dependent and that the system is stationary provided $n$ is strictly smaller than one (which is the case as long as the relative weight on output stabilisation $\lambda$ is finite). Let $\text{vec}(\Phi) = (\sigma_e^2 \ 0 \ 0 \ 0)^\top$ be the vector form of the variance-covariance matrix of $\phi$ and let $\text{vec}(V) = (\sigma_y^2 \sigma_{ny} \sigma_{ny} \sigma_{\pi}^2)^\top$ be the vector form of the variance-covariance matrix of $Z$. Assuming $0 \leq n < 1$ we can compute:
\[
vec(V) = (I - A \otimes A)^{-1} \vec(\Phi) \quad \Rightarrow \quad \vec(V) = \begin{bmatrix}
\frac{2\sigma_{\epsilon}^2}{(1+n)} \\
-\alpha_t \sigma_{\epsilon}^2 \\
\frac{-\alpha_t \sigma_{\epsilon}^2}{(1+n)} \\
\frac{\alpha_t^2 \sigma_{\epsilon}^2}{(1-n^2)}
\end{bmatrix}
\]

(C.2)

From this equation it can be seen that: \( \partial \sigma_y^2 / \partial n < 0 \), \( \partial \sigma_{\pi y} / \partial n < 0 \) and \( \partial \sigma_{\pi}^2 / \partial n > 0 \), i.e. \( \lambda \) (and therefore \( n \)) affects the trade-off between inflation variability and output variability (see Ball (1997))

The expression for \( \text{Cov}[ (R_t-r_t) y_{t+1}] \) can be obtained as follows: First of all, we compute the product of the expression for \( y_{t+1} \) found in equation (C.1) and the real term spread in equation (4.3) (setting \( \beta_2 \) equal to zero in the latter equation). Subsequently, we take the unconditional expectation of the resulting expression where we use the fact that: \( E(\pi_t-\pi^*) = \sigma_{\pi}^2 \), \( E(y_t^2) = \sigma_y^2 \) and \( E((\pi_t-\pi^*) y_t) = \sigma_{\pi y} \) and the fact that \( \epsilon_{t+1} \) is exogenous with respect to \( (\pi_t-\pi^*) \) and \( y_t \).

Appendix D: Proofs of Propositions

1. Proposition 1

Define

\[
b_0 = \frac{(1-n)(1+k\beta_1 - kn)}{\alpha_t(1-k)} \\
b_1 = \frac{(1+\beta_1-n)(1+k\beta_1)+\alpha_t(1-k)-k(n(1-n)+\beta_1^2)}{(1-k)} \\
b_3 = \frac{\beta_2(1-k\beta_2)}{(1-k)}
\]

Then it can be shown that

\[
\frac{\partial b_0}{\partial k} = \frac{(1-n)(1+\beta_1-n)}{\alpha_t(k-1)^2} > 0 \\
\frac{\partial b_1}{\partial k} = \frac{(n-1)^2 + \beta_1(2-n)}{(k-1)^2} > 0 \\
\frac{\partial b_3}{\partial k} = \frac{\beta_2(1-k\beta_2)}{(k-1)^2} > 0
\]

Since it holds that \( \partial k/\partial D = 1/(1+D)^2 > 0 \) it follows that \( \partial b_0/\partial D > 0 \) for \( i = 0,1,2 \)
Next, as far as the parameter n is concerned, we can compute:\(^{16}\)

\[
\frac{\partial b_0}{\partial n} = -\frac{1 + k(1 + \beta_1 - 2n)}{\alpha_1(1 - k)} < 0 \quad ; \quad \frac{\partial b_1}{\partial n} = -\frac{1 + k(1 + \beta_1 - 2n)}{(1 - k)} < 0
\]

In Appendix A it is shown that \(\partial n/\partial \lambda > 0\). Therefore, we can conclude: \(\partial b_0/\partial \lambda < 0\) and \(\partial b_1/\partial \lambda < 0\). Finally, we can show:\(^{17}\)

\[
\frac{\partial b_0}{\partial \beta_1} = \frac{k(1-n)}{\alpha_1(1-k)} > 0 \quad ; \quad \frac{\partial b_1}{\partial \beta_1} = \frac{1+k(1-n)}{(1-k)} > 0
\]

2. Proposition 2

The coefficient for \((\pi_t - \pi^*)\) will be smaller than or equal to zero since \(0 \leq n \leq 1\). The same holds for the coefficient for \(x_t\) since \(0 \leq \beta_2 < 1\). Finally, for the coefficient for \(y_t\) to be negative, the denominator of this coefficient needs to be positive, which will be the case since this condition can be rewritten as follows:

\[
\beta_1(2-n) > -1 - n^2 + 2n \iff \beta_1 > \frac{(n-1)^2}{(2-n)}
\]

3. Proposition 3

Setting \(\beta_2\) equal to zero in equation (3.3), define the absolute value of the reaction coefficients for inflation and output respectively as follows:

\[
a_0 = \frac{k(1-n)(1+\beta_1-n)}{\alpha_1(1-k)} ; \quad a_1 = \frac{k[1+2\beta_1 + n(n-(2+\beta_1))]}{(1-k)}
\]

Then it can be shown that:\(^{18}\)

\(^{16}\) Note that the inequality \(1+k(1+\beta_1-2n) > 0\) can be rewritten as \(\beta_1 > 2n - (1/k) - 1\). The RHS of this expression is strictly increasing in both \(k\) and \(n\). Since in addition it holds that \(0 \leq k \leq 1\) and \(0 \leq n \leq 1\), we know the inequality will always be satisfied if it holds for the special case in which \(k=n=1\). Substituting this into the inequality yields: \(\beta_1 > 0\).

\(^{17}\) Note that it holds that \(\partial b_2/\partial n = \partial b_2/\partial \beta_1 = 0\) because of which it can be concluded that a decrease in \(n\) and/or an increase in \(\beta_1\) will unambiguously increase the degree of activism.

\(^{18}\) Note that it holds that: \(0 \leq k \leq 1\) ; \(0 \leq n \leq 1\) ; \(0 < \beta_2 < 1\) and that the proof that the denominator in the expression for \(\partial a_1/\partial k\) is greater than zero is given in the proof of Proposition 2.
\[
\frac{\partial a_0}{\partial k} = \frac{(1-n)(1+\beta_1-n)}{\alpha_1(k-1)^2} > 0 \quad ; \quad \frac{\partial a_0}{\partial n} = -\frac{k(2(1-n)+\beta_1)}{\alpha_1(1-k)} < 0 \\
\frac{\partial a_i}{\partial k} = \frac{1+2\beta_1+n(n-(2+\beta_1))}{(k-1)^2} > 0 \quad ; \quad \frac{\partial a_i}{\partial n} = -\frac{k(2(1-n)+\beta_1)}{(1-k)} < 0 \\
\frac{\partial a_0}{\partial \beta_1} = \frac{k(1-n)}{\alpha_1(1-k)} > 0 \quad ; \quad \frac{\partial a_i}{\partial \beta_1} = \frac{k(2-n)}{(1-k)} > 0
\]

Since it is shown in Appendix A that \( \frac{\partial n}{\partial \lambda} > 0 \), we conclude that \( \frac{\partial a_i}{\partial \lambda} < 0 \) for \( i = 0,1 \). Similarly, since \( \frac{\partial D}{\partial k} > 0 \) it will hold that \( \frac{\partial a_i}{\partial D} > 0 \) for \( i = 0,1 \). Finally, from equation (3.4) we can compute:

\[
\frac{\partial \text{Cov}[(R_t-r_t)y_{t+1}]}{\partial k} = \frac{1+2\beta_1-n}{(k-1)^2(1+n)} > 0 \quad ; \quad \frac{\partial \text{Cov}[(R_t-r_t)y_{t+1}]}{\partial \beta_1} = \frac{2k}{(1-k)(1+n)} > 0 \\
\frac{\partial \text{Cov}[(R_t-r_t)y_{t+1}]}{\partial n} = -\frac{2k(1+\beta_1)}{(1-k)(1+n^2)} < 0
\]
References:


