Capital Mobility and Catching Up in a Two-Country, Two-Sector Model of Endogenous Growth

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Capital Mobility and Catching Up in a Two-Country, Two-Sector Model of Endogenous Growth

by

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Abstract

Global knowledge spillovers allow backward countries to catch up by accumulating knowledge faster than leading countries. International growth differentials will fall as diminishing returns with respect to the national knowledge stock apply. The domestic growth rate will finally equal the world-wide growth rate, which is endogenous if constant returns apply with respect to all world knowledge stocks taken together.

In our two-country two-sector model with firm-specific knowledge, we show that there is convergence to a symmetric steady state under balanced trade. Under perfect mobility of financial capital, however, there is leapfrogging in the sense that the initially backward country reaches a higher productivity level in the steady state than the initially leading country. This leapfrogging result is found for countries that are perfectly symmetric in all respects except for initial knowledge stocks. The result is therefore path-dependent, implying that overtaking is higher the larger the initial productivity gap. The existence of a nontradables sector is driving the result. During the catch-up process, the backward economy accumulates a foreign debt to smooth consumption. To service this debt, labor is shifted from nontradables to tradables production and investment in firm-specific knowledge.

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1. Introduction

Low per capita income countries may catch up under suitable conditions. In the neoclassical theory of economic growth low income levels are associated with relative capital scarcity. By investing their own savings, backward countries may realize a growth rate that is temporarily higher than in the steady state. However, convergence in terms of GDP per capita is conditioned by the structural characteristics of each country. Things are quite different under perfect international capital mobility. Then backward countries adjust without delay and attain the common level of GDP per capita in the world economy.

These results are clearly at odds with the facts, which can be explained in two ways. Either international capital mobility does not matter in reality or the neoclassical theory in its strictest form must be rejected. There are good reasons to take international capital mobility in the past as well as in the present seriously. As observed by Taylor (1996), international capital mobility played an important role in the nineteenth century processes of convergence and divergence. In modern times we witness a substantial integration of international capital markets with funds flowing to rapidly growing economies. As a consequence, the challenge lies on the theoretical level.

There are two ways to proceed. One can adjust the neoclassical theory or look for an alternative. The first route is followed in Barro, Romer and Sala-i-Martin (1992), who introduce a form of imperfect international capital mobility by assuming that foreign borrowing for investment in human capital is not feasible. Human capital cannot be used as collateral for international borrowing. Therefore, the amount of foreign debt cannot exceed the value of physical capital. Countries which are credit-constrained in this sense convergence to their steady state gradually, but at a higher speed than in the case of a closed economy.

Buiter and Kletzer (1991) follow the second route by providing a theoretical alternative for the neoclassical model. In their view there are important local or homegrown inputs (like different modes of infrastructure), which are essential for the production process. To capture this idea the authors introduce a non-traded capital good ("human capital") produced with a non-traded input (labour) that has an alternative use in consumption. In the Buiter-Kletzer model countries can realize different long-run growth rates of output per worker because growth rates of human capital per worker may be different. However, Buiter and Kletzer (1991) acknowledge that their assumptions capture but very partially the notion of a homegrown infrastructure.

In Turnovsky and Sen (1995), instantaneous adjustment under capital mobility is prevented by the introduction of a non-tradables sector that provides for capital goods in the domestic economy. The rate of investment remains finite because production is subject to increasing marginal costs.

In this paper, we show that nontradables have a fundamental effect on international

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productivity convergence under capital mobity. In contrast with Turnovsky and Sen (1995), we focus on the role of nontraded consumption goods (which can be interpreted as leisure). Production of tradables requires, apart from labour, knowledge inputs which are strictly firm-specific and therefore nontradable as well. Each firm has to accumulate its own knowledge stock by employing labour in R&D activities. The assumption that the stock of knowledge capital is immobile (or homegrown) while financial capital is perfectly mobile is sufficient to avoid the implication of the neoclassical model that adjustment of income levels is instantaneous.

The existence of nontradable consumption goods crucially affects long-run relative productivity levels. Capital mobility allows poor countries to smooth consumption and attract foreign savings to finance their investment. Long-run consumption, both of tradables and nontradables, as a fraction of national product is lower because of the burden of interest payments on the foreign debt that is accumulated. Other things equal, demand for nontradable consumption goods is lower in a debtor country than in a country holding net foreign assets so that the former has more domestic resources (labour) available for accumulation. In our two-country model, the initially poor country grows faster than the rich country, accumulates foreign debt, and continues to grow faster when it has reached the rich country productivity level. Leapfrogging in terms of productivity levels is the result.

International knowledge spillovers in the R&D process explain why the formerly backward economy does not grow faster than the rich one indefinitely. A steady state with equal growth rates in both countries arises since a faster growing country will benefit proportionally less and less from foreign spillovers while the opposite holds for the slower growing country. From the point of view of a single country, there are diminishing returns with respect to the accumulation of knowledge, and in the long run the domestic rate of growth equals the world growth rate. However, we also assume that there are constant returns to scale in the R&D process with respect to domestic and foreign knowledge taken together. Hence, the rate of growth of the world economy is endogenous.

We find that the long-run world growth rate as well as the world distribution of productivity depends on the initial distribution of productivity and financial wealth. To sharpen the analysis we abstract from all other sources of long-run productivity differences. The more unequal the initial distribution of productivity is, and the more skewed the initial distribution of financial wealth is towards high productivity countries, the more leapfrogging arises and the more world growth is affected. (With small international spillovers and high substitution between tradables from different countries, world growth is adversely affected). Initial positions are therefore more important than is assumed in traditional convergence models.

The paper is organized as follows. In Section 2 we present separately the structural and behavioural relationships of the model. Section 3 is devoted to a discussion of the steady-state properties of the system. Dynamic aspects for different balance of payments regimes are

considered in Section 4 by using a linearized version of the model. In Section 5 the implications of the model are illustrated by presenting and discussing numerical examples. The final section summarizes the main conclusions of the analysis. Algebraic derivations are relegated to appendices.

2. A two-country endogenous growth model

2.1. Structure of the model

There are two countries that are characterized by identical preferences, technological opportunities, and primary factor endowments. However, one country, indexed by superscript 1, starts at a lower productivity level than country 2. Each country has one primary factor of production in fixed supply (labour), which is allocated over two sectors, tradables and non-tradables. Non-tradables are homogenous. Tradables are differentiated and each variety is produced by a single monopolistic firm. These firms control and accumulate firm-specific knowledge (as in Smulders and Van de Klundert, 1995). Within each country, there is a continuum of symmetric firms on the unit interval. This allows us to save on notation by formulating the model for a single representative firm.

The structural relationship are given in Table A. Countries are denoted by superscript i = 1, 2 (and if necessary also by superscript j for the other country). Each line in the table represents two equations, one for each country. Eqs. (A.1) tells that one unit of labour L_Y produces one unit of tradables Y. Labour productivity in the tradables sector is denoted by h as appears from eqs. (A.2), relating output X to input L_X.

Firms have an opportunity to increase labour productivity h by performing R&D according to eqs. (A.3). Knowledge can be increased by allocating labour (L_R) to R&D. Productivity in the R&D division depends on a fixed coefficient ξ and a knowledge component consisting of three elements. First, firms build upon specific knowledge accumulated in the past. Second, all firms benefit from knowledge spillovers emanating from domestic firms. Third, there are knowledge spillovers from abroad. Knowledge spillovers relate to the average level of knowledge in the different economies ($\hbar$). Productivity levels may differ across countries, but are identical across firms within a country. For this reason average knowledge levels are equal to the knowledge levels of firms in each country ($\hbar^i = h^i$).

Table A Structural relationships

Technology

  Non-tradables sector

$$Y^i = L_Y^i \tag{A.1}$$

  Tradables sector

$$X^i = h^i L_x^i \tag{A.2}$$

$$\dot{h}^i = \xi (h^i)^{1-\alpha_h-\alpha_f} (\bar{h}^i)^{\alpha_h} (\bar{h}^j)^{\alpha_f} L_R^i \tag{A.3}$$

Preferences

$$U^i(0) = \int_0^\infty \ln C^i(t) \, e^{-\vartheta t} \, dt \tag{A.4}$$

$$C^i = (C_X^i)^\sigma \, (C_Y^i)^{1-\sigma} \tag{A.5}$$

$$C_X^i = \left[(D^i)^{(\varepsilon-1)/\varepsilon} + (M^i)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)} \tag{A.6}$$

Market clearing

$$Y^i = C_Y^i \tag{A.7}$$

$$X^i = D^i + M^j \tag{A.8}$$

$$L_Y^i + L_x^i + L_R^i = L \tag{A.9}$$

Symbols

C	aggregate consumption index	h	labour productivity tradables
C_X	consumption index tradables	L	labour supply
C_Y	consumption non-tradables	L_k	labour allocated in sector/division k
			($k = Y, X, R$)
D	consumption domestically produced tradables	M	imported tradables

All equations apply to $i=1,2$ and $j \neq i$.

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Intertemporal preferences in the consumption index C are given in eqs. (A.4). Infinitely-lived households apply a constant utility discount rate $\hat{\upsilon}$. The relative rate of risk aversion is unity as appears from the logarithmic instantaneous utility function. The choice of tradables versus non-tradables is governed by a Cobb Douglas preference function as shown in eqs. (A.5). Consumption of non-tradables (C_Y) relates to homogenous products.[1] The consumption index of tradables (C_x) combines consumption of domestically produced varieties (D) and imported varieties (M) by way of a CES sub-utility function, with an elasticity of substitution denoted by ε, eqs. (A.6).

Markets for non-tradables clear, eqs. (A.7). So do all markets for tradables, eqs. (A.8). The supply of labour (L) equals total demand for labour, as written down in eqs. (A.9).

2.2. Consumer and firm behaviour

The behavioural equations of our model are summarized in Table B. Consumers maximize intertemporal utility over an infinite horizon. The decision problem consists of three stages subject to the usual budget constraints. In the first stage, each consumer decides on the path of aggregate consumption over time. This gives rise to the familiar Ramsey rule, shown in eqs. (B.1). The growth rate of consumption equals the difference between the real consumption rate of interest and the pure rate of time preference. In the second stage total expenditure on consumption is divided over non-tradables, eqs. (B.2), and tradables, eqs. (B.3). In the third stage consumers decide about spending the amount of money reserved for tradables on domestically produced varieties, eqs. (B.4) and foreign varieties, eqs. (B.5). The price elasticity of demand is equal to ε in all cases considered. Eqs. (B.6) and (B.7) define respectively the price index of tradables and the price index of aggregate consumption.

Producers maximize the value of firm over an infinite horizon. Each firm faces a downward sloped total demand function for its products as appears from eqs. (B.4) and (B.5). Profit maximization implies that firms set a mark-up over (marginal) cost equal to $\varepsilon/(\varepsilon-1)$, as in eqs. (B.8). Labour demand for R&D follows from setting marginal revenue ($\xi K p_h$) equal to marginal cost (w), eqs. (B.9). The shadow price of firm-specific knowledge p_h is introduced as a Lagrangian multiplier in the maximization procedure.[2] Firms face a trade-off with respect to investing in specific knowledge as appears from the arbitrage conditions (B.10). These conditions say that investing an amount of money equal to p_h in the capital market (the RHS of

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[1]   Alternatively, C_Y can be interpreted as leisure.

[2]   The maximization problem of firm k in country i can be represented by the following Hamiltonian $H^k = p^k(X^k;\cdot)X^k - w^i(X^k/h^k + L_R^k) + p_h^k\left[\xi(h^k)^{1-\alpha_h-\alpha_f}(\bar{h}^i)^{\alpha_h}(\bar{h}^j)^{\alpha_f}L_R^k\right]$, where $p^k(\cdot)$ is the firm's demand function, see (B.4), $X/h=L_x$ is labour employed in production, see (A.2), the term in square brackets is firm-specific knowledge accumulation $\dot{h}$, see (A.3), and p_h is the co-state variable. The firm's instruments are X^k and L_R^k and it controls state variable h^k.

Consumer behaviour

$$\dot{C}^i/C^i = r^i - \dot{p}_c^i/p_c^i - \vartheta \tag{B.1}$$

$$C_Y^i \, p_Y^i = (1-\sigma) \, C^i \, p_c^i \tag{B.2}$$

$$C_X^i \, p_X^i = \sigma \, C^i \, p_c^i \tag{B.3}$$

$$D^i = C_X^i \left(p^i/p_X \right)^{-\varepsilon} \tag{B.4}$$

$$M^i = C_X^i \left(p^j/p_X \right)^{-\varepsilon} \tag{B.5}$$

where

$$p_X = \left[(p^i)^{1-\varepsilon} + (p^j)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{B.6}$$

$$p_c^i = \left(\frac{p_X}{\sigma} \right)^{\sigma} \left(\frac{p_Y^i}{1-\sigma} \right)^{1-\sigma} \tag{B.7}$$

Producers behaviour tradables sector

$$p^i = \frac{\varepsilon}{\varepsilon - 1} \, \frac{w^i}{h^i} \tag{B.8}$$

$$p_h^i = \frac{w^i}{\xi K^i} \tag{B.9}$$

$$p^i \left(\frac{\varepsilon-1}{\varepsilon} \right) L_X^i + \xi(1-\alpha_h-\alpha_f) \left(\frac{K^i}{h^i} \right) p_h^i \, L_R^i + \dot{p}_h^i = r^i p_h^i \tag{B.10}$$

where $K^i \equiv (h^i)^{1-\alpha_h-\alpha_f} \, (\overline{h}^i)^{\alpha_h} \, (\overline{h}^j)^{\alpha_f}$

Producers behaviour non-tradables sector

$$p_Y^i = w^i \tag{B.11}$$

Balance of payments

$$X^i p^i - \sigma C^i p_c^i + r^i A^i = \dot{A}^i \tag{B.12}$$

Symbols

A	net foreign assets	p_c	price index consumption
r	nominal interest rate	p_X	world price index tradables
w	wage rate	p_h	firm's shadow price of knowledge
p	price of tradables		

All equations apply to $i=1,2$ and $j \neq i$.

B.10) should yield the same revenue as investing that same amount of money in knowledge creation. The latter raises labour productivity in the tradables sector and hence revenue in this sector (first term on the LHS of B.10), it raises also the knowledge base in R&D (second term) and it yields a capital gain (last term). In the non-tradables sector perfect competition prevails. As a consequence, the price of non-tradables equals wage cost per unit of output, eqs. (B.11).

Finally, eqs. (B.12) imply that domestic net savings are invested in net foreign assets (*A*). Domestic savings are the sum of two components. First, foreign assets increase with the difference between the revenue from the production of tradables and the amount spent on the consumption of tradables, i.e. with the trade balance. Second, interest payments on existing foreign assets add to the amount available for investing in the international capital market. It should be observed that under perfect capital mobility the rate of interest is uniform across countries ($r^1 = r^2$). At the other extreme there is the case of balaced trade or zero mobility implying $A = 0$. Both regimes with respect to the balance of payments will be analysed in the paper.

2.3. Semi-reduced model

The model exhibits constant returns to scale in the reproducible factors h^1 and h^2, see (A.3). This is the core property that allows for endogenous growth. Using $h^i = \bar{h}^i$, eqs. (A.3) can be rewritten as

$$\frac{h^{'i}}{h^i} = R^i L_R^i \ ,$$ (1)

where $R^i = \xi \left(h^i / h^j \right)^{-\alpha_f}$. (2)

Note that R&D employment L_R is a decision variable for the firm and is determined by profit maximization. The productivity of R&D employment, R^i, depends on the ratio of knowledge levels or productivity ratio. A country with relatively low productivity ($h^i/h^j<1$) benefits from relatively more international spillovers and therefore realizes a higher growth rate for a given amount of labour in R&D. This explains how backward countries may catch up. Note that h^i/h^j is given by history. Since knowledge is firm-specific, firms have to accumulate their own knowledge which takes time. Hence, even if firms are allowed to borrow in a perfect international capital market, there is no immediate adjustment of international productivity levels.

To further analyse the properties of the model, it is convenient to reduce the system to the set of key relationships which is presented in Table C. Eqs. (C.1) relate to the familiar Ramsey rules for optimal saving. Variables with a hat refer to growth rates. The arbitrage rule which governs investment decisions by firms is in the background of the reduced-forms (C.2).

Labour market equilibrium is rewritten in eqs. (C.3). The third term on the LHS represents labour allocated to R&D activities as follows from (1) and (2). According to (C.4), the market for non-tradables is in equilibrium if non-tradables production (LHS) equals demand which equals the fraction $(1-\sigma)$ of total consumption expenditures divided by the price of nontradables. The latter equals the wage which can be written in terms of the tradables price if (B.8) is taken into account. The balance of payments conditions in their reduced form are given in eqs. (C.5). The LHS represents the trade balance deflated by the wage rate w. Eqs. (A.2) and (B.8) have been substituted in (B.12) to derive this expression. Relative prices of tradables in eq. (C.6) depend on relative supply and price elasticity ε. According to eq. (C.7), accumulated assets of the one country are mirrored by accumulated debts of the other country. Eq. (C.8) directly follows from (B.8) and shows that relative real wages reflect relative productivity.

The reduced-form model consists of 16 equations in 17 unknowns, viz. C^i, h^i, L_x^i, L_Y^i, $r^i - \hat{p}^i$, p_c^i/p^i, A^i/w^i ($i = 1, 2$), s, w^1/w^2, and p^1/p^2. The additional equation that closes the model depends on the balance of payment regime. In case of perfect international capital mobility, interest rates are equal. Without capital mobility, net foreign assets are zero:

$$r^1 = r^2 \ \text{(capital mobility)} \quad or \quad A^1 = 0 \ \text{(balanced trade).} \tag{3}$$

Table C Key relationships

Ramsey rule

$$\hat{C}^i = (r^i - \hat{p}^i) - (\hat{p}_c^i - \hat{p}^i) - \vartheta \qquad i=1,2 \qquad (C.1)$$

where $\hat{p}_c^1 - \hat{p}^1 = (1-\sigma)\hat{h}^1 + \sigma(1-s)(\hat{p}^2 - \hat{p}^1)$

$$\hat{p}_c^2 - \hat{p}^2 = (1-\sigma)\hat{h}^2 + \sigma s(\hat{p}^1 - \hat{p}^2)$$

$$s = \frac{1}{1 + (p^1/p^2)^{\varepsilon-1}}$$

Investment decision

$$\xi\left(h^i/h^j\right)^{-\alpha_f} L_x^i + (1-\alpha_h)\,\hat{h}^i - \alpha_f\,\hat{h}^j = r^i - \hat{p}^i \qquad i=1,2 \qquad (C.2)$$

Labour market equilibrium

$$L_Y^i + L_x^i + \frac{\hat{h}^i}{\xi\left(h^i/h^j\right)^{-\alpha_f}} = L \qquad i=1,2 \qquad (C.3)$$

Equilibrium in the market for nontradables

$$L_Y^i = (1-\sigma)\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C^i p_c^i}{h^i p^i} \qquad i=1,2 \qquad (C.4)$$

Balance of payments

$$\frac{\varepsilon}{\varepsilon-1} L_x^i - \sigma\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C^i p_c^i}{h^i p^i} = \frac{\dot{A}^i}{w^i} - r^i\frac{A^i}{w^i} \qquad i=1,2 \qquad (C.5)$$

Equilibrium in markets for tradables

$$\frac{p^i}{p^j} = \left(\frac{h^i L_x^i}{h^j L_x^j}\right)^{-1/\varepsilon} \qquad (C.6)$$

Equilibrium in the market for foreign bonds

$$\frac{A^i}{w^i} = -\frac{A^j}{w^j}\frac{w^i}{w^j} \qquad (C.7)$$

Relative wages

$$\frac{w^i}{w^j} = \frac{p^i}{p^j} \cdot \frac{h^i}{h^j} \qquad (C.8)$$

In all equations $j \neq i$

3. Foreign debt, technological leadership and growth in the steady state.

A steady state is characterized by a fixed allocation of labour among the various activities. The productivity levels in the domestic and foreign tradables sector grow at the same rate, g. The balanced growth path can be characterized as

$$g \equiv \frac{\dot{h}}{h} = \frac{\dot{X}}{X} = \frac{1}{\sigma}\frac{\dot{C}}{C} \tag{4}$$

Moreover, by selecting tradables in country 2 as the numéraire ($p^2=1$, $\dot{p}^2=0$) we get:

$$g = \frac{1}{1-\sigma}\frac{\dot{p}_c}{p_c} = \frac{\dot{p}_Y}{p_Y} = \frac{\dot{w}}{w} = \frac{\dot{A}}{A} \tag{5}$$

We now use the key relationships in Table C and the steady state results above to examine how long-run growth g and relative productivity h^i/h^j are related to (a given level of) net foreign assets. First, we combine (C.1) and (C.2) to represent an equilibrium in the (national) capital market in which the required rate of return given by (C.1) equals the realized rate of return given by (C.2). In the steady state this can be written as:

$$R^i L_x^i = \hat{\vartheta} + (\alpha_f + \alpha_h)g \qquad i=1,2 \tag{6}$$

The higher discount rate $\hat{\vartheta}$, the higher the required rate of return and the more profitable firms' R&D projects should be for a given growth rate. This requires a larger firm size (L_x^i) so that the knowledge stock of the firm can be applied at a larger scale, or a higher productivity of R&D (R^i).

As can be seen from (6), $R^iL_x^i$ is the same in both countries in the steady state. Moreover, since their growth rates are equal, the same applies to $R^iL_R^i$, see (1) and (2). Note from (2) that R^i is lowest in the country with highest productivity. Hence, the high productivity country will have a larger fraction of its labour force engaged in both production of tradables (L_x) and research (L_R). I.e. if $h^i>h^j$, then $L_x^i>L_x^j$ and $L_R^i>L_R^j$.

Second, we combine (C.3) and (C.5) to characterize feasible growth as constrained by labour market and balance of payments. In the steady state, feasible growth can be written as:

$$g = R^i\left(L - \frac{1-\sigma}{\sigma}\hat{\vartheta}\frac{A^i}{w^i} - \frac{L_x^i}{\beta}\right) \qquad i=1,2 \tag{7}$$

where $\beta \equiv \left(\frac{\varepsilon}{\varepsilon-1}\frac{1-\sigma}{\sigma} + 1\right)^{-1}$. Feasible growth is high if labour productivity in R&D (R) is high and if employment in R&D is large, see (1). The latter requires that a small fraction of the labour force is employed in tradables production (see last term in (7)) and nontradables production. A high real foreign debt ($A/w<0$) implies a high debt burden and low consumption

so that labour in the non-tradables sector is small (see second term in brackets).

Third, we calculate relative wages from (2), (6), (C.6), and (C.8):[3]

$$\frac{w^i}{w^j} = \left(\frac{h^i}{h^j}\right)^{(\varepsilon-1-2\alpha_f)/\varepsilon} \tag{8}$$

Higher productivity positively (negatively) affects a country's relative wages if $\varepsilon-1>2\alpha_f$ ($\varepsilon-1<2\alpha_f$). Since export demand is elastic ($\varepsilon-1>0$), increases in productivity translate into higher earnings. However, large spillovers (α_f) shift part of the rents to foreign workers.

Finally, combining (6)-(8) we find:

$$\frac{\vartheta + (\beta+\alpha_f+\alpha_h)g}{\beta} =$$
$$\left(\frac{h^i}{h^j}\right)^{-\alpha_f}\left[L - \frac{1-\sigma}{\sigma}\vartheta a_w^i\right] = \left(\frac{h^i}{h^j}\right)^{\alpha_f}\left[L + \frac{1-\sigma}{\sigma}\vartheta a_w^i\left(\frac{h^i}{h^j}\right)^{(\varepsilon-1-2\alpha_f)/\varepsilon}\right], \tag{9}$$

where $a_w^i \equiv A^i/w^i$. The second equality in (9) characterizes the relationship between foreign assets a_w^i and the productivity ratio h^i/h^j. The LHS (RHS) of this equality is decreasing (increasing) in the productivity ratio. The solid lines in Figure 1 depict both sides of the equation for $a_w^i=0$. The intersection is at $h^i/h^j=1$, implying a symmetric steady state solution. The broken lines in Figure 1 represent the case where country i has a debt ($a_w^i<0$): the LHS-curve (RHS-curve) shifts up (down) relative to the case with zero net foreign assets. The result is a higher productivity ratio. This shows that net foreign assets and relative productivity are negatively related, as is depicted in Figure 2.
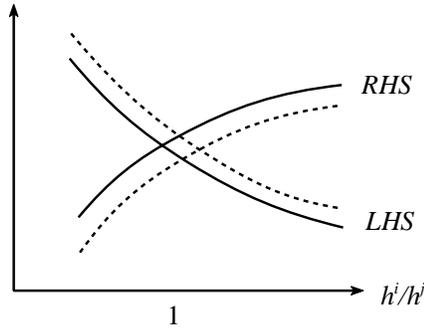

Figure 1

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[3]   Note that (2) and (6) imply $L_x^i/L_x^j = R^j/R^i = \left(h^i/h^j\right)^{2\alpha_f}$.
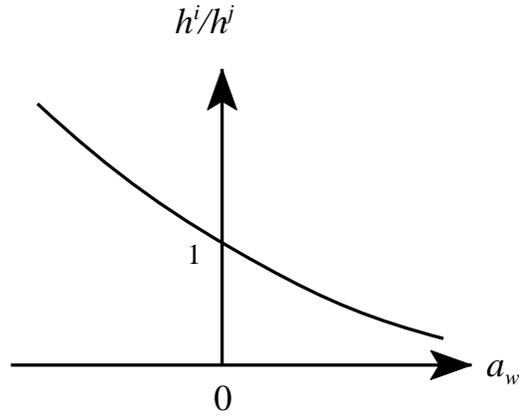
Figure 2

Under balanced trade the net foreign asset position is zero. The backward country catches up fully and in the steady state both countries attain the same productivity level in the tradables sector. The situation is different in case of capital mobility. In the steady state, debtor countries are technological leaders, while creditor countries' productivity is relatively low.

Countries that accumulate foreign debts are typically those with many investment opportunities as the result of catch up potential. As we will show in the next sections, the country that initially lags behind and gains access to the international capital market becomes indebted and leapfrogs. Intuitively, the rate of return is initially higher in the backward country. Therefore, capital flows from the leading country towards the backward country.

Leapfrogging is due to the existence of a non-tradables sector. For $\sigma=1$ the steady state level of h^i/h^j is unity regardless of the amount of foreign debt, see (9). Consider a hypothetical steady state in which country 1 has higher productivity than country 2 despite the fact that $\sigma=1$. The former would charge lower prices and employ more labour in the tradables sector than the latter. As a result, growth would be lower in country 1 than in country 2 for two reasons: $R^1 < R^2$ and $L_R^1 = L - L_x^1 < L_R^2 = L - L_x^2$. But unequal growth rates imply that the situation cannot be a steady state. With non-tradables ($\sigma<1$), the high productivity country employs a large fraction of its labor force not only in production of tradables but also in research. The country grows at the same rate as the low productivity country since higher research labour input ($L_R^1 > L_R^2$) is offset by lower research productivity ($R^1 < R^2$). More labour is available for tradables production and research since labour engaged in non-tradables production is lower. Labour is shifted from the nontradables sector to the production and research divisions of the export sector in order to be able to service foreign debt. In other

words, the debt burden accounts for lower consumption, lower demand for non-tradables and a shift of labour to the investment sector, resulting in a relatively higher level of productivity, compared to the creditor country.

The relation between growth and asset positions can also be derived from eq. (9). In case of zero foreign debt ($a_w=0$), implying $h^i/h^j=1$ the rate of growth is equal to

$$\bar{g} = \frac{\beta\xi L - \vartheta}{\alpha_h + \alpha_f + \beta} \tag{10}$$

If the tradables sector does not exist ($\sigma=\beta=1$), the growth rate reduces to

$$\bar{\bar{g}} = \frac{\xi L - \vartheta}{\alpha_h + \alpha_f + 1} \tag{11}$$

Eq. (11) implies that in absence of a tradables sector the growth rate depends positively on the productivity parameter of the R&D-division (ξ) and the common scale factor (L). The growth rate depends negatively on the pure rate of time preference (ϑ) and the significance of knowledge spillovers ($\alpha_h + \alpha_f$). Under balanced trade and the existence of a tradables sector formula (10) applies. The aggregate growth rate then depends positively on the elasticity of substitution in the tradables sectors (ε) and consumers' preferences for tradables (σ).

Under perfect capital mobility the aggregate rate of growth (g) depends also on the foreign asset position (a_w) as appears from eq. (9). If the net foreign asset position differs from zero the world rate of growth can be higher or lower than the rate of growth with $a_w=0$ depending upon the parameters of the model. It can be shown that a sufficient condition for the growth rate under capital mobility to be lower than under balanced trade is $\varepsilon-1 > 2\alpha_f$. The opposite result $g > \bar{g}$ requires $\varepsilon-1 < 2\alpha_f$ as a necessary condition.[4] In Figure 3 these results are illustrated for $\alpha_f = 0.25$ and different values of ε.[5]

Two opposing forces affect the growth rate. In the high productivity country (i.e., the debtor country), research productivity is lower but research employment is higher compared to a situation without capital flows. The growth rate falls with international capital flows if the latter force dominates, which happens if spillovers are relatively small according to the condition $\varepsilon-1>2\alpha_f$. In the steady state, the country with highest R&D activity (L_R) is the country with lowest national R&D productivity (R), viz. the debtor country. Without international spillovers, this is clearly an inefficient distribution of world R&D activity and tends to lower the growth rate.

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[4]   See appendix 1.

[5]   The other parameters used in the numerical exercise are $\vartheta=0.05$, $\sigma=0.75$, $\alpha_h = 0.5$, $\xi=0.015$ and $L=15$. With these parameters we get $g = \bar{g}$ irrespecive of a_w if $\varepsilon = 1.2$.
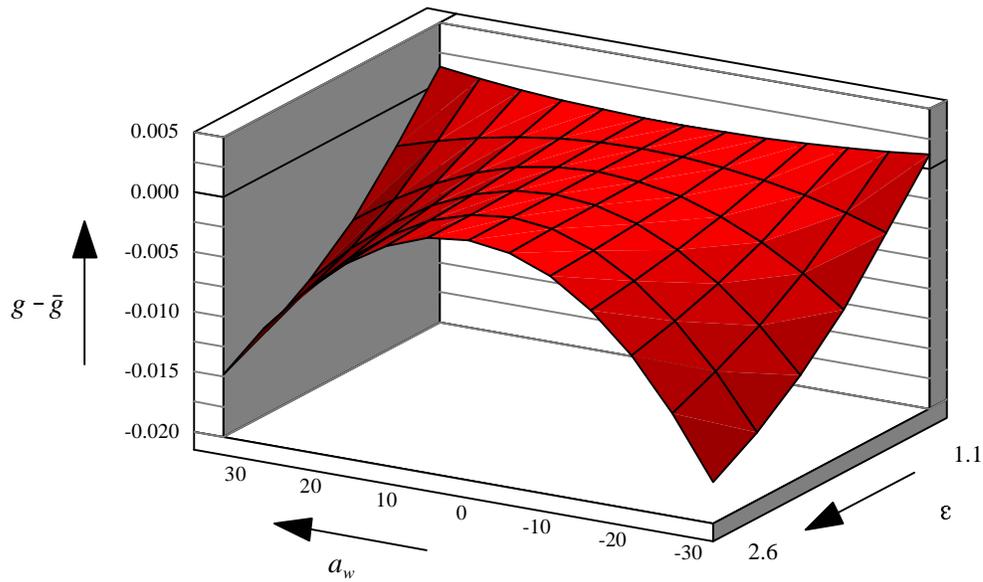
Figure 3

However, if international spillovers are important, world growth may benefit from a country investing a lot in R&D, despite the fact that *national* research productivity in this country is low, since spillovers raise *world* research productivity. Indeed, if $2\alpha_f > \varepsilon - 1$, growth may be positively related to world imbalances in financial wealth. If ε is small, the debtor country suffers from a large terms of trade decline associated with its technological lead. A large shift in labour allocation from nontradables to tradables is needed to generate sufficient export earnings to service its debt. International knowledge spillovers also have adverse effects on the debtor country's income. If α_f is high, it is difficult to improve its competitive position since any increase in national productivity also benefits foreign producers. Hence, if ε is small relative to α_f, the debtor country is forced to invest a lot in knowledge accumulation and the creditor country is able to increase nontradables consumption without adverse effects on its growth rate. Both forces lead to a higher world growth rate.

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4. On the dynamics of catching up

In the previous section we studied the relation between foreign debt, productivity and growth in the steady state, without determining the equilibrium levels of these variables separately. In this section we will show that we need to study the entire transitional dynamics the find the steady state values.

To study the dynamics of the model it is convenient to linearize around the steady state in case of symmetry ($h^1/h^2 = 1$, $a_w = 0$). The key relationships in linearized form are presented in Table D. The equations correspond with the key relationships in the original model, Table C. Variables with a tilde relate to percentage deviations from their hypothetical steady state value.[6] Tilded variables depend on the time index t which is omitted where possible. Variables without a tilde relate to the solutions in the symmetric steady state. We have rewritten the Ramsey rule in terms of $z \equiv C/h^\sigma$ because this variable is constant in the steady state. For the same reason we have substituted $a \equiv A/h$ in the equation for the balance of payments.

The table contains 12 equations. We concentrate on the solution for the following 15 variables: $\tilde{z}^i$, $\tilde{r}^i - \dot{\tilde{p}}^i$, $\tilde{g}^i$, $\tilde{L}_x^i$, $\tilde{L}_Y^i$, $\tilde{a}^i$ and p^i for $i=1,2$, and $\tilde{h}^1 - \tilde{h}^2$. To close the model we need three additional equations. First, our choice of numeraire implies $\tilde{p}^2=0$. Next, note that we solve for the productivity ratio h^i/h^j, rather than the individual productivity levels of the two countries. The reason is that the former is constant in the steady state, while the latter are not.[7] The following identity provides an additional equation:

$$\dot{\tilde{h}}^r = g\left(\tilde{g}^1 - \tilde{g}^2\right), \tag{12}$$

where $h^r \equiv h^1/h^2$, so that $\tilde{h}^r \equiv \tilde{h}^1 - \tilde{h}^2$. Finally, we use the linearized equivalents of (3):

$$\tilde{r}^1 = \tilde{r}^2 \text{ (capital mobility)} \quad or \quad \tilde{a}^1 = 0 \text{ (balanced trade)} \tag{13}$$

The system of reduced-form equation in Table D can be solved analysing country differences ($\tilde{x}^r \equiv \tilde{x}^1 - \tilde{x}^2$) and country summations ($\tilde{x}^s \equiv \tilde{x}^1 + \tilde{x}^2$) separately. The former decribe how a certain variable in country 1 deviates from that in country 2 (recall that all tilded variables are deviations from a situation in which the countries are identical). The summation variables relate to the integrated world economy and are useful as an intermediate step in calculating the variables for separate countries, because by definition: $\tilde{x}^1=(\tilde{x}^r+\tilde{x}^s)/2$ and $\tilde{x}^2=(\tilde{x}^s-\tilde{x}^r)/2$.

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[6]   Hence, $\tilde{x} \equiv dx/x$. The only exception is variable $\tilde{a}$ which relates to absolute difference from the steady state, $\tilde{a} \equiv da$, because in the symmetric equilibrium $a = 0$. Because we linearize around a steady state in which $p^1=p^2=1$ for all t, we have $d\hat{p}^i = \dot{\tilde{p}}^i$.

[7]   Once $\tilde{h}^i - \tilde{h}^j$ is solved, we can solve for $\tilde{h}^i$ by integrating $\tilde{g}^i$ over t.

Table D Key relationships in linearized model

Ramsey rule

$$\dot{\tilde{z}}^{\,i} = r\tilde{r}^{i} - \dot{\tilde{p}}^{i} - g\tilde{g}^{i} + \frac{\sigma}{2}\left(\dot{\tilde{p}}^{i} - \dot{\tilde{p}}^{j}\right) \qquad\qquad i=1,2 \qquad \text{(D.1)}$$

where $z \equiv C/h^{\sigma}$

Investment decision

$$\xi L_{x}\left[\alpha_{f}(\tilde{h}^{\,j} - \tilde{h}^{\,i}) + \tilde{L}_{x}^{i}\right] + (1-\alpha_{h})g\tilde{g}^{\,i} - \alpha_{f}g\tilde{g}^{\,j} = r\tilde{r}^{\,i} - \dot{\tilde{p}}^{\,i} \qquad\qquad i=1,2 \qquad \text{(D.2)}$$

Labour market equilibrium

$$\xi L_{Y}\,\tilde{L}_{Y}^{i} + \xi L_{x}\,\tilde{L}_{x}^{i} + g\tilde{g}^{\,i} + \alpha_{f}g\,(\tilde{h}^{\,i} - \tilde{h}^{\,j}) = 0 \qquad\qquad i=1,2 \qquad \text{(D.3)}$$

Equilibrium in the market for non-tradables

$$\tilde{L}_{Y}^{i} = \tilde{z}^{\,i} - \frac{\sigma}{2}\left(\tilde{p}^{\,i} - \tilde{p}^{\,j}\right) \qquad\qquad i=1,2 \qquad \text{(D.4)}$$

Balance of payments

$$L_{x}\left(\tilde{L}_{x}^{i} - \tilde{L}_{Y}^{i}\right) + \vartheta\tilde{a}^{\,i} = \dot{\tilde{a}}^{\,i} \qquad\qquad i=1,2 \qquad \text{(D.5)}$$

where $a \equiv A/h$

Equilibrium in market for tradables

$$\tilde{p}^{\,i} - \tilde{p}^{\,j} = -\frac{1}{\varepsilon}\,(\tilde{h}^{\,i} - \tilde{h}^{\,j} + \tilde{L}_{x}^{i} - \tilde{L}_{x}^{j}) \qquad\qquad \text{(D.6)}$$

Equilibrium in the market for foreign bonds

$$\tilde{a}^{\,i} = -\tilde{a}^{\,j} \qquad\qquad \text{(D.7)}$$

In all equations $j \neq i$.

4.1. Balanced trade

The reduced-form model in differences can be compressed to a system of two differential equation in $\tilde{h}^r$ and $\tilde{L}_x^r$. The dynamic equation for $\tilde{h}^r$ can be derived from eqs. (12), (D.3), (D.5), (D.7), and (13). The dynamic equation in $\tilde{L}_x^r$ is found from eqs. (D.1), (D.2), (D.4) and (D.6). The result can be presented in matrix notation as:[8]

$$\begin{bmatrix} \dot{\tilde{h}}^r \\ \dot{\tilde{L}}_x^r \end{bmatrix} = \begin{bmatrix} -2\alpha_f g & -\xi(L_x + L_Y) \\ -2\alpha_f(\vartheta + 2\alpha_f g) & \xi(L_x + L_Y)(\alpha_h - \alpha_f) + \xi L_x \end{bmatrix} \cdot \begin{bmatrix} \tilde{h}^r \\ \tilde{L}_x^r \end{bmatrix} \tag{14}$$

The determinant of the matrix in (14) is negative. Therefore, the system of differential equations in country differences is saddle-point stable. The corresponding phase diagram is drawn in Figure 4. As appears from eqs. (14) the $\dot{\tilde{h}}^r = 0$ locus slopes upward and the $\dot{\tilde{L}}_x^r = 0$ locus slopes downward. The stable arm of the saddle path is indicated by the broken line. If country 1 relatively backward we have $\tilde{h}^r(0) < 0$. Then the system jumps to the stable arm at $t = 0$. The backward economy employs less labour in the production of tradables than the leading country. The same applies to employment in nontradables (note that (D.5) implies $\tilde{L}_Y = \tilde{L}_x$ under balanced trade). Therefore, the backward country allocates relatively more labour in the R&D division which boosts growth. After $t=0$, the variables $\tilde{h}^r$ and $\tilde{L}_x^r$ adjust gradually along the stable arm as indicated in Figure 4. In the long run there is complete catching up, $\tilde{h}^r(\infty) = \tilde{L}_x^r(\infty) = 0$, and both countries' productivity expands at the same rate.

4.2. Capital mobility

Under perfect capital mobility eq. (13) applies, so that $\tilde{r}^r=0$. Taking differences with respect to eq. (D.1) then results in:

$$\dot{\tilde{z}}^r = -(1-\sigma)\dot{\tilde{p}}^r - \dot{\tilde{h}}^r. \tag{15}$$

Eq. (15) implies that the system of differential equations obtained from Table D by taking differences has a zero root. The model exhibits hysteresis: we need to know the entire transition dynamics to solve for the steady state.

The information contained in eq. (15) can be applied by integrating the equation to obtain:

$$\tilde{z}^r = \tilde{v} - (1-\sigma)\tilde{p}^r - \tilde{h}^r, \tag{16}$$

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[8]   We have used $\xi L_x = \vartheta + (\alpha_f + \alpha_h)g$ from (2) and (6) to simplify the element on the second row, first column of the matrix.
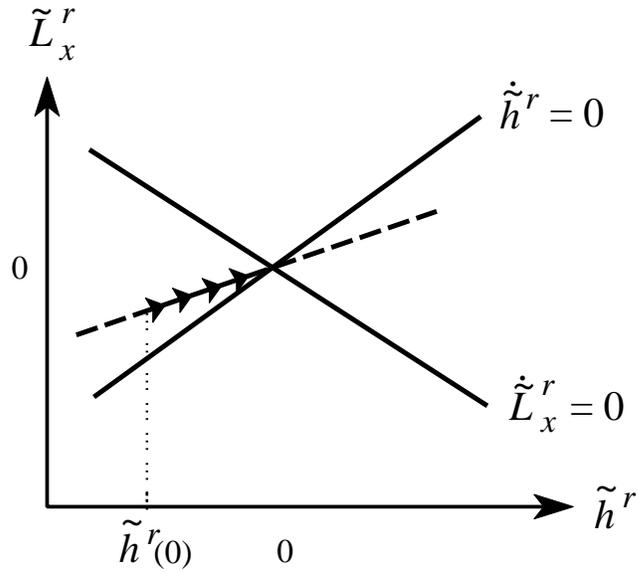
$$\tilde{L}^r_x$$

$$\dot{\tilde{h}}^r = 0$$

$$0$$

$$\dot{\tilde{L}}^r_x = 0$$

$$\tilde{h}^r$$

$$\tilde{h}^r(0) \qquad 0$$

Figure 4

where $\tilde{v}$ is a constant of integration, which can be solved from the initial conditions of the dynamic model.

The model is given by (16), (D.2)-(D.7), and (13) and can be reduced to the following system of three differential equations:

$$
\begin{bmatrix} \dot{\tilde{h}}^r \\ \dot{\tilde{p}}^r \\ \dot{\tilde{a}}^r \end{bmatrix} =
\begin{bmatrix} \mu_{11} & \mu_{12} & 0 \\ \mu_{21} & \mu_{22} & 0 \\ 0 & \mu_{32} & \vartheta \end{bmatrix}
\cdot
\begin{bmatrix} \tilde{h}^r \\ \tilde{p}^r \\ \tilde{a}^r \end{bmatrix}
-
\begin{bmatrix} \xi L_Y \\ \xi L_Y(\alpha_h - \alpha_f - 1) \\ L_x \end{bmatrix} \cdot \tilde{v}
\tag{17}
$$

where

$$\mu_{11} = \xi L_x + \xi L_Y - 2\alpha_f g$$
$$\mu_{12} = \xi \varepsilon L_x + \xi L_Y > 0$$
$$\mu_{21} = (\alpha_h - \alpha_f)\mu_{11} - \xi L_Y + 2\alpha_f (\xi L_x + g)$$
$$\mu_{22} = (\alpha_h - \alpha_f)\mu_{12} - \xi L_Y$$
$$\mu_{32} = -(\varepsilon - 1)L_x$$

The sub-system in $\tilde{h}^r$ and $\tilde{p}^r$ is self-contained and saddlepoint stable if the determinant of the relevant matrix (Δ) is negative. From eqs. (17) we find:

$$\Delta = \xi L_x \left[(\varepsilon - 1 - 2\alpha_f)\xi L_Y - 2\alpha_f\varepsilon(\xi L_x + g) \right] \tag{18}$$

It will be assumed that $\Delta < 0$. A sufficient condition for this to be true is $(\varepsilon - 1 - 2\alpha_f) < 0$. More in general, the determinant will be negative for sufficiently small values of L_Y. If the non-tradables sector is non-existent, we always have saddlepoint stability. The first two differential equations can be used to solve for $\tilde{h}^r$ and $\tilde{p}^r$ in terms of $\tilde{v}$ and $\tilde{h}^r(0)$. This yields:

$$\tilde{h}^r(t) = (1 - e^{-\lambda t})\tilde{h}^r(0) + e^{-\lambda t}\tilde{h}^r(\infty), \tag{19}$$

$$\tilde{h}^r(\infty) = -\left(\frac{\varepsilon}{-\Delta/\xi L_x}\right)\xi L_Y\tilde{v}, \tag{20}$$

$$\tilde{p}^r(t) = (1 - e^{-\lambda t})\tilde{p}^r(0) + e^{-\lambda t}\tilde{p}^r(\infty), \tag{21}$$

$$\tilde{p}^r(\infty) = \left(\frac{1 + 2\alpha_f}{-\Delta/\xi L_x}\right)\xi L_Y\tilde{v}, \tag{22}$$

$$\tilde{p}^r(0) = \frac{\lambda\tilde{p}^r(\infty) - (1 - \alpha_h + \alpha_f)\xi L_Y\tilde{v} - \mu_{21}\tilde{h}^r(0)}{\lambda + \mu_{22}}, \tag{23}$$

where λ is the absolute value of the negative root of the relevant sub-matrix. Substitution of (21) in the third differential equation in (17) yields:

$$\dot{\tilde{a}}^r(t) = \mu_{32}\left\{\tilde{p}^r(\infty) + e^{-\lambda t}\left[\tilde{p}^r(0) - \tilde{p}^r(\infty)\right]\right\} + \vartheta\tilde{a}^r - L_x\tilde{v}. \tag{24}$$

This linear differential equation in the state variable $\tilde{a}$ can be solved as

$$\tilde{a}^r(t) = \frac{-\mu_{32}\left[\tilde{p}^r(0) - \tilde{p}^r(\infty)\right]}{\lambda + \vartheta}e^{-\lambda t} + \frac{L_x\tilde{v} - \mu_{32}\tilde{p}^r(\infty)}{\vartheta} \tag{25}$$

The state variable $\tilde{a}^r$ is predetermined, so that $\tilde{a}^r(0)$ is given. Consequently, by setting $t=0$ and substituting (22) and (23), eq. (25) can be used to express $\tilde{v}$ in terms of $\tilde{a}^r(0)$ and $\tilde{h}^r(0)$.

Summarizing, we find that the long-run solution depends on $\tilde{v}$, see (20), which in turn depends on initial conditions. This clearly points out that the model exhibits hysteresis. Further implications are derived from the numerical examples in the next section.

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4.3. The integrated world economy

Next we turn to the system of summation for a symmetrical world applying eqs. (D.1) - (D.7). It is straightforward to show that the system can be reduced to a single differential equation in $\tilde{L}_x^s = \tilde{L}_x^1 + \tilde{L}_x^2$, which applies both under perfect capital mobility and under balanced trade:

$$\dot{\tilde{L}}_x^s = \xi L_x \left(\frac{\alpha_f + \alpha_h + \beta}{\beta} \right) \tilde{L}_x^s \tag{26}$$

The differential equation (26) is unstable, so that the only viable solution under perfect foresight must be: $\tilde{L}_x^s = \tilde{L}_Y^s = \tilde{z}^s = \tilde{g}^s = r\tilde{r}^s - \dot{\tilde{p}}^s = 0$. As a result, these summations are all invariant over time. There is no inertia in the closed economy, obtained by summation of country variables. In this world the steady state is attained instantaneously. As $\tilde{g}^s = 0$, the summation of knowledge levels must be equal to the summation of the initial deviations from the symmetrical steady state level: $\tilde{h}^s(t) = \tilde{h}^s(0) \ \forall t$.

The solutions for country variables ($\tilde{x}^i$) follow from combining $\tilde{x}^r$ and $\tilde{x}^s$. For instance, $\tilde{L}_x^1 = \tilde{L}_x^r/2$, $\tilde{L}_x^2 = -\tilde{L}_x^r/2$, and similar for all other stationary variables. The knowledge levels in both countries are calculated as $\tilde{h}^1 = (\tilde{h}^s(0) + \tilde{h}^r)/2$ and $\tilde{h}^2 = (\tilde{h}^s(0) - \tilde{h}^r)/2$. Both $\tilde{h}^r$ and $\tilde{h}^s$ are predetermined. In order to model country 1 as the less developed country, we impose $\tilde{h}^r(0) < 0$. In contrast, $\tilde{h}^s(0)$ can be chosen arbitrarily. The reason is that all variables of the linearized model are calculated as deviations from a steady state that is purely hypothetical. It does not make much sense, however, to analyse country deviations from the hypothetical steady state in isolation. Instead, we compare the results for individual countries in case of capital mobility with those in case of balanced trade.

5. Numerical examples

In this section we present numerical results applying to the linearized model. First, we discuss catching up in case of balanced trade and in case of capital mobility for a benchmark case. Second, we investigate the sensitivity of the results for changes in parameter values. In this connection special attention will be paid to the welfare consequences of the choice made with respect to the balance of payments regime. The parameters used in the benchmark case are equal to

$$\vartheta = 0.05, \qquad \sigma = 0.75, \qquad \varepsilon = 2.5, \qquad \alpha_h = 0.5, \qquad \alpha_f = 0.25, \qquad \xi = 0.01$$
$$L = 15, \qquad a_w = 0$$

The time paths of the variables are monotonic. It is therefore sufficient to present results for $t = 0$ and $t = \infty$, alongside with the speed of convergence for the system, λ. In all cases

considered we assume $\tilde{h}^r = -1$.

5.1. The benchmark case

The results for the benchmark case are presented from different angles in Tables 1 and 2. In Table 1 the variables in differences with respect to countries 1 and 2 are shown for the case of balanced trade as well as the case of perfect capital mobility. Table 2 gives the outcomes for the two countries ($i = 1, 2$) as a difference between the results under capital mobility and balanced trade. In both tables the hypothetical steady state to which the variables in percentage deviations relate is eliminated by construction. It is therefore possible to compare both balance of payments regimes.

Under balanced trade the model attains a symmetric steady state in the long run. This explains the zeros in the second column of Table 1. The backward country (which is country 1) catches up fully. At $t = 0$ this country starts with a productivity level that falls short of the productivity level of the leading country by 1 percent. This is shown in the first column of Table 1. Initially the rate of return is higher in the backward country. This implies that more will be invested in R&D. The growth rate at $t = 0$ is higher in country one, so that there is a clear case of catching up. The extra labour required in the R&D division comes from the production of tradables and non-tradables in the same proportion. Because country 1 is the backward nation its consumption level in the initial situation is lower than that in the leading economy. Tradables produced in country 1 are initially expensive, because of the relatively low productivity level.

Under perfect capital mobility the (nominal) rate of interest is equalized across countries which explains the zeros in the third and fourth column of Table 1.The levelling of interest rates is brought forth by an inflow of capital in country 1. As a result this country builds up foreign debt, which is reflected in the net foreign asset position of both countries in the long run ($t = \infty$). Comparing both regimes (Table 2) we see that the rate of growth is now higher in country 1 and lower in country 2. Catching up proceeds slightly faster: the rate of convergence under capital mobility equals $\lambda = 4.404\%$, whereas under balanced trade we get $\lambda = 4.308\%$. The main implication of a higher growth rate in case of capital mobility is, however, that country 1 overtakes country 2 in productivity levels. In the long run the productivity level of country 1 exceeds that of country 2 as appears from both tables.

The labour required for speeding up R&D comes from the production division of tradables and not from the non-tradables sector. The situation is now different compared with balanced trade. Foreign borrowing allows consumption smoothing in the backward country. To meet consumption demand tradables can be imported, but non-tradables have to be produced at home. In the long run the bill has to be paid. Foreign debt has to be serviced and there is a lower consumption level than under balanced trade. But then there is also less need to produce non-tradables in country 1, so that more labour is available for the production and

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export of tradables. It is the reallocation of labour from the non-tradables sector to the tradables sector in the course of time which also induces leapfrogging. During the process of adjustment towards the steady state extra labour is allocated to R&D under capital mobility compared with balanced trade.

Table 1 Catching up: country differences

Case	Balanced trade		Capital mobility	
Period Variable	$t = 0$	$t = \infty$	$t = 0$	$t = \infty$
Growth rate $\tilde{g}^r$	1.292	0.000	1.831	0.000
Consumption $\tilde{C}^r$	-0.608	0.000	-0.257	-0.386
Labour tradables $\tilde{L}_x^r$	-0.226	0.000	-0.599	0.193
Labour non-tradables $\tilde{L}_Y^r$	-0.226	0.000	0.013	-0.502
Productivity $\tilde{h}^r$	-1.000	0.000	-1.000	0.386
Rate of interest $\tilde{r}^r$	0.032	0.000	0.000	0.000
Price tradables $\tilde{p}^r$	0.491	0.000	0.640	-0.232
Foreign assets $\tilde{a}^r$	-	-	0.000	-1.042

Table 2 Catching up: regime differences

Case	Country 1		Country 2	
Period Variable	$t = 0$	$t = \infty$	$t = 0$	$t = \infty$
Growth rate $\tilde{g}^i$	0.269	0.000	-0.269	0.000
Consumption $\tilde{C}^i$	0.176	-0.193	-0.176	0.193
Labour tradables $\tilde{L}_x^i$	-0.186	0.096	0.186	-0.096
Labour non-tradables $\tilde{L}_Y^i$	0.120	-0.251	-0.120	0.251
Productivity $\tilde{h}^i$	0.000	0.193	0.000	-0.193
Rate of interest $\tilde{r}^i$	-0.246	0.000	-0.214	0.000
Price tradables $\tilde{p}^i$	0.149	-0.282	0.000	0.000
Foreign assets $\tilde{a}^i$	0.000	-0.521	0.000	0.521
Intertemporal welfare $\tilde{U}^i(0)$	0.005	-	-0.005	-

The results in Table 2 give rise to additional observations. First, it turns out that the outcomes for country 2 are the exact opposite of those of country 1 with two exceptions. The

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price of tradables does not change in country 2 because our choice of the numéraire. The nominal interest rate is another exception. That the country results are mirror images can be explained by the fact that the summations of variables are time-invariant under both balance of payments regimes as observed above. This does not hold for the nominal interest rate, because in case of capital mobility interest rates are uniform across countries.

Second, intertemporal welfare or lifetime utility at $t=0$ as defined in (A.4)[9] is higher for country 1 and lower for country 2 under capital mobility in comparison with balanced trade. This is atypical as one would expect capital mobility to be welfare improving as it opens the possibility of consumption smoothing. Moreover, capital mobility has a uniform impact on both countries by affecting the steady state growth rate as discussed in Section 3. However, both effects are of second order in the linear approximations around a symmetrical steady state. What remains is the impact of knowledge spillovers in both countries. Under capital mobility country 1 grows faster and country 2 grows slower than under balanced trade. Domestic knowledge spillovers imply underinvestment from a welfare point of view. Capital mobility mitigates this effect in country 1 but aggrevates it in country 2. Cross-country spillovers have an opposite effect. The returns to innovation undertaken by one country accrue partly to its trading partner, thereby deteriorating its competitive position. Hence, foreign spillovers result in overinvestment from the point of national welfare. Capital mobility aggrevates overinvestment in country one. Sensitivity analysis is required to see whether the result presented in Table 2 is robust or not.

5.2. The role of the non-tradables sector
The results with respect to leapfrogging crucially depend on the existence of a non-tradables sector. The sensitivity of the results for changes in σ is shown in Figures 5a and 5b. The other parameters are the same as in the benchmark case. As before, the productivity level in country 1 is one percent below that of country 2 in the initial situation. As σ increases the difference in long-run productivity levels declines as shown in the upper panel of Figure 5a. At the same time long-run debt incurred by country 1 declines, but never approaches zero. Unequal rates of return on investment in R&D induce international capital flows also in the absence of a non-tradables sector. However, the larger the non-tradables sector the more debt is accumulated in country 1 (lower panel of Figure 5a). Diminishing returns with respect to investment in R&D are less severe if labour can be reallocated from non-tradables to tradables production.

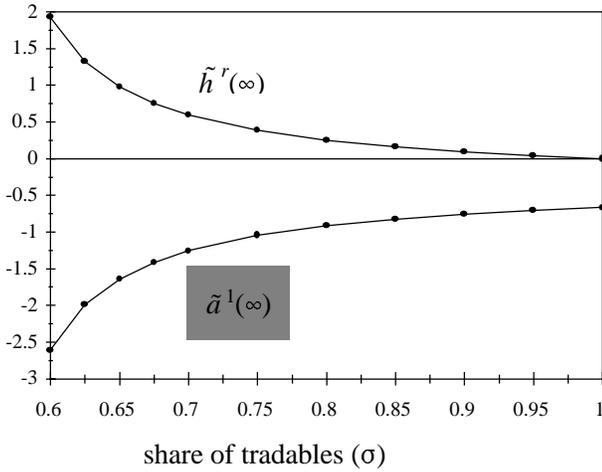[9]   See appendix 3 for the linearization procedure for intertemporal welfare.
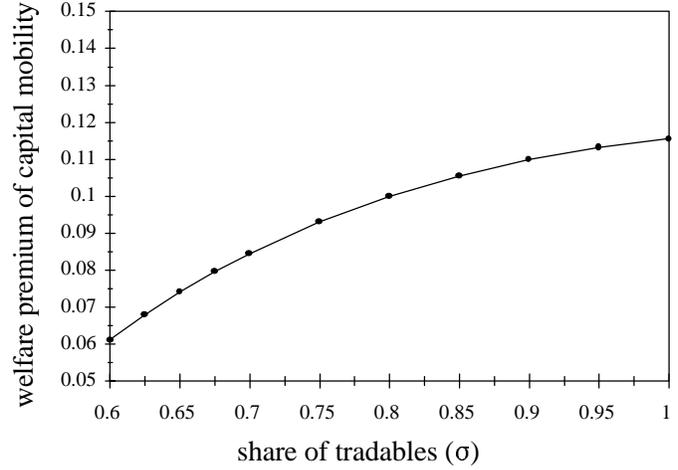
Figure 5a

Figure 5b[10]

In Figure 5b we present the difference in lifetime welfare in case of capital mobility and in case of balanced trade for country 1. (The result for country 2 is of the same magnitude but opposite in sign). The difference is positive and increasing in σ. Capital mobility is more welfare improving the larger the tradables sector, i.e. the more open the economy.

5.3. The role of spillovers

The importance of the domestic spillover parameter α_h must be seen in connection with the foreign spillover parameter α_f. In Figure 6 we vary α_h and stick to $\alpha_f=0.25$ (the benchmark value). On the vertical axis we measure again the difference in welfare between both balance-of-payments regimes for country 1. There is no non-tradables sector: $\sigma=1$. Moreover, deviating from the logarithmic sub-utility function in eqs. (A.4) we consider other cases of risk-aversion by introducing a sub-utility function with a constant relative rate of risk-aversion (ρ). This change has a substantial impact on the algebra of the model, but here we only present numerical result.

In case of a logarithmic function ($\rho=1$) intertemporal utility is higher under capital mobility as long as $\alpha_h > \alpha_f (= 0.25)$. If domestic spillovers are relatively more important than

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[10] Figure 5b plots how much intertemporal welfare in country 1, $U^1(0)$, is higher under capital mobility than under balanced trade. We calculate this as the difference between $\tilde{U}^1(0)$ under capital mobiltiy and $\tilde{U}^1(0)$ under balanced trade (see appendix 3).

foreign spillovers, underinvestment from a national perspective arises. Capital mobility stimulates investment and improves national welfare in the backward country. However, if foreign spillovers are relatively more important, country 2 gains from the rapid growth in country 1 and it is its welfare that rises.
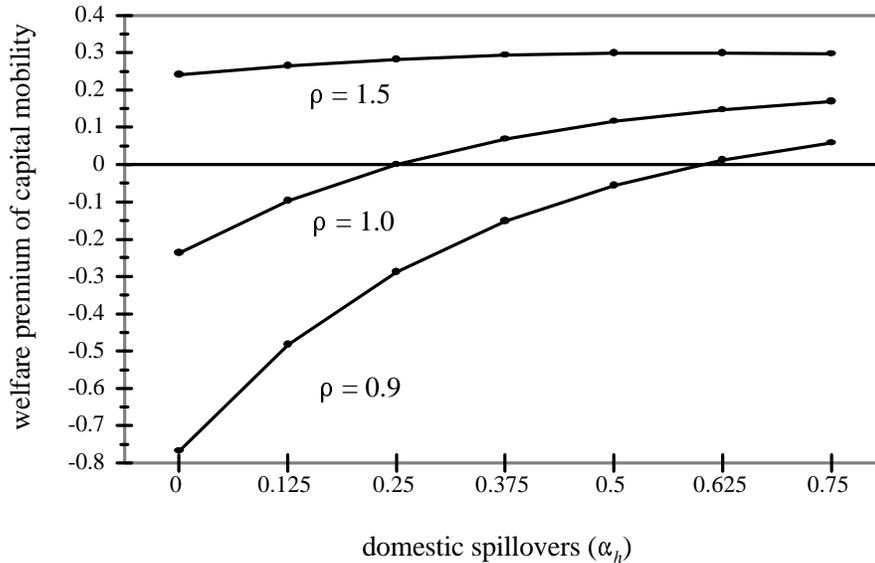

Figure 6

Different attitudes with respect to risk-aversion have a substantial impact as shown in Figure 6. If consumers are relatively risk-avers (for instance $\rho=1.5$) consumption smoothing becomes relatively important. Consumers welcome the opportunity to spread their risk very much. Capital mobility provides this opportunity. As a consequence, convergence is faster under capital mobility if risk aversion is higher.[11] This means that foreign spillovers that arise from unequal productivity levels play less of a role, which stimulates the welfare gains from

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[11]   Indeed, we find that λ increases in ρ under capital mobility (for $\alpha_h=0.5$, we find $\lambda = 8.417\%$, 8.427%, 8.461% for $\rho = 0.9$, 1.0, 1.5 respectively). However, under balanced trade, λ falls with ρ (for $\alpha_h=0.5$, we find $\lambda = 5.264\%$, 5.047%, 4.186% for $\rho = 0.9$, 1.0, 1.5 respectively). Higher risk aversion decreases in*ter*temporal substitution and therefore reduces the speed of convergence over time. Consumers prefer to smooth consumption more and national savings are lower. Under balanced trade, national investment is lower accordingly which slows down convergence between the two countries. However, under capital mobility, international capital flows are required to equalize international rates of return. With lower national savings, international capital flows are relatively larger so that the difference in national investment rates become larger, the more risk-averse investors are. This speeds up convergence between the two countries.

capital mobility for country one. The opposite result holds in case of less risk-avers consumers, as shown in Figure 6 for $\rho=0.9$.

6. Conclusions

Capital mobility speeds up international convergence of per capita output levels. In the neoclassical model there is even an immediate equalization of output levels per worker across countries. Restrictions on international mobility of financial capital or introduction of adjustment costs in the accumulation of augmentable factors of production mitigate the extreme neoclassical case. In our two-sector two-country model we stress the importance of firm-specific knowledge which must be acquired by investing in R&D. Therefore, even under perfect mobility of financial capital it takes time to fully catch up.

More important, we show that the country that is initially backward in terms of knowledge levels leapfrogs the initially leading country if non-tradables are taken into account. The backward country incurs foreign debt to smooth consumption over time. Consumption in the short run is higher at the expense of consumption later on. Non-tradables are part of total consumption. The fall in consumption of non-tradables sets free labour which can be allocated in the tradables sector to increase export production and R&D. The result is that the knowledge level in the once backward country overtakes that in the formerly leading country.

Spillovers of knowledge foster economic growth in the world economy. In our model knowledge spillovers imply constant returns on world-wide level with respect to augmentable factors of production and long-run economic growth is endogenous for the world as a whole.

The results places the convergence literature in a different perspective. Long-run growth of the global economy as well as relative cross-country productivity levels are path-dependent and depend on the initial distribution of financial wealth across countries. International financial market institutions matter not only for the speed of convergence but also for the long-run distribution of productivity. The openness of the economy, as measured by the relative size of the tradables sector, plays a key role in the processes of convergence and leapfrogging.

Appendix

1. Sufficient conditions for g declining in $|a_w|$

From (9), we derive by total differentiating:

$$\frac{dg}{da_w} \gtreqless 0 \quad \Leftrightarrow \quad \frac{\alpha_f\left[1+(h^{\,r})^{2\alpha_f+\psi}\right]}{2\alpha_f + \psi\, a_w J(h^{\,r})^{\psi}} \lesseqgtr 1$$

where

$$h^r = h^i/h^j, \quad a_w = A^i/w^i,$$

$$\psi \equiv (\varepsilon - 1 - 2\alpha_f)/\varepsilon,$$

$$J \equiv \frac{(1/\sigma - 1)\vartheta}{L + (1/\sigma - 1)\vartheta\, a_w (h^{\,r})^{\psi}} = \frac{1-\sigma}{\sigma}\vartheta (h^{\,r})^{\alpha_f}\left(\frac{\beta}{\vartheta + (\beta + \alpha_f + \alpha_h)g}\right) > 0.$$

Now we make use of the fact that h^r and a_w are negatively related and that symmetry ($h^r=1$) arises if net foreign assets are zero ($a_w=0$). If $\varepsilon - 1 > 2\alpha_f$, then $\psi > 0$ and

$$h^{\,r} = 1 \quad \wedge \quad a_w = 0 \quad \Rightarrow \quad \frac{dg}{da_w} = 0,$$

$$h^{\,r} > 1 \quad \wedge \quad a_w < 0 \quad \Rightarrow \quad \frac{dg}{da_w} < 0,$$

$$h^{\,r} < 1 \quad \wedge \quad a_w > 0 \quad \Rightarrow \quad \frac{dg}{da_w} < 0.$$

Hence, $a_w=0$ yields a maximum for g if $\varepsilon - 1 > 2\alpha_f$. $\qquad\qquad\qquad\square$

2. Some notes on the linearization procedure

Our linearization procedure involves taking total differentiation, where all parameters are considered as constants. Tilded variables are expressed as relative deviations from the initial balanced growth path, i.e. for any variable u, $\tilde{u} \equiv du/u$ or $du = u\tilde{u}$. Non-tilded variables that show up in the linearized model refer to steady state values. When linearizing the time derivatives of variables in the original model, we should make the distinction between variables that are constant in the steady state (stationay variables) and those which are not. For the latter we can easily take the total differential in terms of growth rates, e.g. $d\hat{h} = dg = g\tilde{g}$. In contrast, for the time derivative of a stationary variable, say u_s, we have $d\hat{u}_s = \dot{\tilde{u}}_s$. An exception is $\tilde{a}$ which is defined as the absolute (rather than relative) deviation from the original growth path, i.e. $\tilde{a} \equiv da$

and $d\dot{a}=\dot{\tilde{a}}$, which allows us to consider the situation in which $A=a=0$ initially (balanced trade).

Given the above procedure, (D.2), (D.3), (D.6) and (D.7) directly follow from their equivalent in Table C. This appendix explains how to derive the remaining equations.

To derive (D.1) from (C.1), first note that the LHS of (C.1) can be written as $\hat{z}+\sigma\hat{h}$. The total derivative of this expression reads:

$$d\hat{C} = \dot{\tilde{z}} + \sigma g\tilde{g}.$$

By our choice of numeraire, prices are constant in the long run. Moreover, we linearize around a symmetric steady state so that $s=0.5$. Therefore total derivative of the consumer price inflation can be written as:

$$d(\hat{p}_c^{\,i} - \hat{p}^{\,i}) = \dot{\tilde{p}}_c^{\,i} - \dot{\tilde{p}}^{\,i} = (1-\sigma)g\tilde{g}^{\,i} + (\sigma/2)(\dot{\tilde{p}}^{\,j} - \dot{\tilde{p}}^{\,i}).$$

Using the last two results, we straightforwardly find (D.1).

To derive (D.4), we use the definition of $z \equiv C/h^\sigma$ and the following expression of the consumer price index that follows from subsequent linearization and substitution of (B.7), (B.6), (B.11), and (B.8):

$$\tilde{p}_c^{\,i} - \tilde{p}^{\,i} = (1-\sigma)\tilde{h}^{\,i} + (\sigma/2)(\tilde{p}^{\,j} - \tilde{p}^{\,i}).$$

Finally we derive (D.5). Dividing (B.12) by h^i and using (A.2) to eliminate X, we find:

$$p^{\,i}L_x^{\,i} - \sigma C^{\,i}p_c^{\,i}/h^{\,i} = \left[(\hat{A}^{\,i} - \hat{p}^{\,i}) - (r^{\,i} - \hat{p}^{\,i})\right]A^{\,i}/h^{\,i}.$$

Using the definition $a \equiv A/h$, we may rewrite the RHS as $\dot{a} - (r-g)a$. Total differentiation of the result gives:

$$pL_x(\tilde{p}+\tilde{L}_x) - (\sigma Cp_c/h)(\tilde{C}+\tilde{p}_c-\tilde{h}) = \dot{\tilde{a}} - a(r\tilde{r} - g\tilde{g}) - (r-g)\tilde{a}.$$

From (C.4) we find $\tilde{C} + \tilde{p} - \tilde{h} = \tilde{L}_Y + \tilde{p}$. We linearize around a steady state characterized by balanced trade. Hence, we can set $a=0$ and $\sigma Cp_c/h=NL_x$. Furthermore, $r-g=\vartheta$, which follows from (C.1), (4) and (5). Substitution of these results yields (D.5).

3. Welfare

Welfare at time t [see (A.4)] can be written as:

$$U(t) = \ln C(t)/\vartheta + \int_t^\infty F(s)\,ds,$$

where F is the cumulative growth-corrected discount rate, defined as:

$$F(s) \equiv \int_0^s \hat{C}(\tau)\,d\tau.$$

Hence, welfare depends on the time path of consumption, captured by $C(0)$ and $\hat{C}$. Taking total differentials, solving the resulting integrals by using the fact that all variables develop monotonically with speed of adjustment λ and denoting the growth rate of consumption by g_c, we derive:

$$dU(t) \equiv \tilde{U}(t) = \frac{1}{\vartheta}\left\{\tilde{C}(t) + \frac{g_c}{\vartheta}\left[\left(\frac{\vartheta}{\lambda+\vartheta}\right)\tilde{g}_c(0) + \left(\frac{\lambda}{\lambda+\vartheta}\right)\tilde{g}_c(\infty)\right]\right\}.$$

Note that $\hat{C} = \hat{z} + \sigma\hat{h}$, so that $g_c = \sigma g$ in the steady state and

$$\tilde{g}_c(t) = \frac{\lambda\left[\tilde{z}(t) - \tilde{z}(\infty)\right]}{\sigma g} + \tilde{g}(t),$$

where the numerator of the first term on the RHS represents $\dot{\tilde{z}}(t)$.

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