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Abstract.
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Key-words: international trade, endogenous growth, unemployment, efficiency wages, managerial fixed costs, relative productivity

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Productivity and Unemployment in a Two-Country Model with Endogenous Growth

by Anton van Schaik and Henri de Groot

Relative to the United States, most European countries have high rates of unemployment and low levels of productivity in manufacturing. To relate these issues, we develop a leader-follower model with endogenous growth and dual labour markets, stressing the role of high-tech and high-wage sectors in trade between countries. The model shows a negative relation between unemployment and growth. The steady state relative productivity level and the corresponding rates of unemployment depend on the relative level of fixed costs in the high-tech sectors of both countries. Downsizing of firms in the leader country raises the worldwide rate of unemployment, whereas downsizing of firms in the follower country enlarges the productivity trap.

1. Introduction

The evidence on comparable productivity levels of the advanced industrialized countries shows a lack of convergence in manufacturing. Broadberry (1996) found that, starting from the beginning of the second industrial revolution ‘productivity patterns in manufacturing can most usefully be characterized as local rather than global convergence'. Van Ark (1996) reports convergence of productivity levels in manufacturing relative to the United States over the post World War II period till the end of the 1970s, divergence in the first half of the 1980s, and stabilization afterwards. Figure 1 shows the development of relative productivity for the German manufacturing sector, which can serve as an example for many European countries. The pattern is one of convergence in the 1950s, 1960s and 1970s, divergence in the 1980s, and stabilization of relative productivity in the 1990s. Bernard and Jones (1996, p.1237), exploring the OECD International Sectoral Database, concluded that ‘manufacturing does not display the pattern of convergence in labour and technological productivity found in other sectors'. Other evidence for the 1990s (*cf.* Pilat,

1996) likewise gives no indications of a renewed period of rising levels of labour productivity in manufacturing relative to the United States. At the same time, the European rates of unemployment seem to be stabilizing at high levels, relative to the 1960s and early 1970s (and relative to the United States).

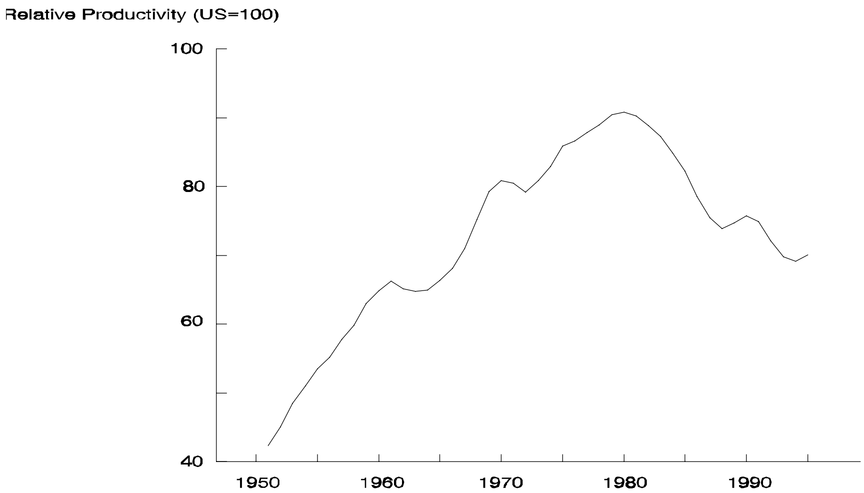

Figure 1. Relative Productivity of Germany (3-year moving average)

Source: Van Ark (1996), Deutsche Bundesbank, and Federal Reserve System

It is the purpose of this paper to relate these issues - persistently high rates of unemployment and persistently low levels of productivity in manufacturing in a follower country relative to the leader country - by developing a general equilibrium model focusing on the linkages between trade, growth, and unemployment. Central to the model is the leader-follower dichotomy, embodied in a two-country two-sector model with endogenous growth. Each country has a high-tech high-wage sector producing differentiated and tradeable goods, and a traditional low-wage sector, producing a homogeneous non-tradeable good. High-tech firms compete under monopolistic competition, leaving a potential rent which can be shared with the workers (resulting in a non-competitive wage differential). The core of the model is an engine of endogenous growth, operating only in the high-tech sector. Innovation is seen in relation to R&D expenditure which, by construction, has the capacity to take technological knowledge forward in only small steps. This idea has been materialized here by an engine of growth driven by intentional R&D in both countries. The follower country receives a productivity bonus in the form of a capacity to imitate the leader. This feature captures the idea that technological change is the joint outcome of learning activities and intentional activities directed at innovation (*cf.* Fagerberg, 1994). Technological change is thus not a free lunch, neither for the leader nor for the follower country. Follower countries may have a particular advantage in R&D as they can learn from the leader, but an intentional investment has to be made for technological progress to occur.

In order to highlight the relation between unemployment and productivity levels across countries, labour market institutions have to play a role in the model. Up till now, the relation between endogenous growth and unemployment has not received much attention in the growth literature. Most of the studies address the issue from a matching perspective. Equilibrium unemployment results from frictions in the process of matching the unemployed with vacancies posted by firms. The most comprehensive study in this field was conducted by Aghion and Howitt (1994). In their model, economic growth results in creative destruction and accompanying lay-off of workers. Bean and Crafts (1995) considered the relation between growth and unemployment from a different angle. In their model, a crucial role is played by the so-called ‘hold-up' problem. The essence of this problem is that firms have an incentive to underinvest in growth promoting activities in the presence of trade unions and in the absence of binding contracts. The labour market of our model deviates from these studies, because our focus is on distortions in the (effective) supply of labour, whereas the above-mentioned papers focus on distortions in demand for labour. Furthermore, we model unemployment as resulting from efficiency wage considerations playing a role in one sector only. This allows us to address the problem of unemployment in the context of a dual labour market, characterized by persistent non-competitive wage differentials.

The paper successively discusses the constituent parts of the model. We deliberately restrict ourselves to the presentation of the equations constituting the final model. The complete model is presented in the Appendix. The relation between trade and growth is described in Section 2. This part of the model is self-contained by assuming that R&D expenditures are determined by some rule of thumb. The labour market is added to the trade block of the model in Section 3. This part of the model is also self-contained by assuming that the rate of interest is exogenously given. Section 4 describes the steady-state characteristics of the fully fledged model with financial markets determining the rate of interest and thus the amount of investment in R&D. In Section 5, the model is used to shed some more light on the empirical issues raised above. Conclusions are presented in Section 6.

2. Growth and Trade

2.1 Growth

The economy consists of two countries, the leader country (indexed 1), starting with a (historically inherited) relatively high level of labour productivity, and the follower country (indexed 2), which lags behind the leader in terms of labour productivity. In each country, a number of high-tech firms is operating. Monopolistic competition prevails among these firms. Each firm produces a unique brand of high-tech tradeable goods, but has only a small share in the world market, so that competition is monopolistically à la Chamberlin. There is only one homogeneous factor of production, labour, which is immobile between countries. The growth and trade block of the model is summarized in Table 1 (equations without a country index hold for each country separately).

< Insert Table 1 around here >

Abstracting from issues of the dynamics of market structure (firms with different productivity levels), we assume symmetry among high-tech firms within a country so that high-tech goods produced in one country can simply be represented as a composite good. Although the assumption of symmetry of the equilibrium is restrictive, such an equilibrium is easy to analyze and conveys the basic message of the model (see also Peretto, 1996). Households have a taste for variety. Preferences over varieties are also uniform across countries and are given by a CES-function (*cf*. Dixit and Stiglitz, 1977), with an elasticity of substitution equal to $\varepsilon(>1)$. World demand for any variety produced in the one country relative to world demand for any variety produced in the other country depends on the relative price of these varieties (*i.e.*, the terms of trade). We only need one relation (equation 6) to capture trade between the two countries, revealing a negative relation between the market share of a country in the world market for high-tech goods and the terms of trade.

The representative high-tech firm of a country produces x units of output, using direct labour L_x that has labour productivity h and efficiency e (equation 1). The efficiency of a high-tech worker is a variable that can be affected by the wage setting behaviour of the firm (section 3.1 elaborates on this). The variable h, on the other hand, depends on international forces and the number of workers employed in the Research and Development lab of the firm. Firm-specific knowledge can (by assumption) only be accumulated through firm- and product-specific R&D. Current research therefore builds on past R&D experience as measured by firm-specific knowledge h. Firms employ labour L_r in their research departments to improve the production process and the quality of the product (if we interpret x as measured in quality units, both forms of innovation can be modelled as increases in h). There are constant returns in R&D with respect to the level of knowledge capital, which means that a constant rate of growth can be sustained by allocating a fixed amount of labour to the research labs (equation 2). Firms also have to incur a traditional managerial fixed cost before being able to start production. This exogenous managerial fixed cost (F) is expressed in efficiency units. Each high-tech firm thus has to employ an amount of labour equal to $L_f(\equiv F/e)$ before being able to produce. High-tech firms maximize their present discounted profits, which results in a fixed mark-up over labour costs (equation 3), and a wage rate that is chosen in such a way that it minimizes labour costs per efficiency unit (i.e., the wage rate satisfies the Solow condition and implies a constant equilibrium level of effort e; see section 3.1). The price equation shows that real wages increase with labour productivity. The mark-up is inversely related to the elasticity of substitution between any two high-tech goods. The closer these goods form substitutes, i.e., the higher ε is, the less market power firms have, and the lower the mark-up they can put on labour costs.

The growth and trade block of the model is self-contained if we assume that firms employ a fixed proportion $\beta(>0)$ of the number of production workers in R&D activities (equation 4). Implicitly, we thus assume that firms employ research labour according to some rule of thumb. Section 4 will elaborate on the determination of investments in R&D. The size of the firm is determined by the process of free entry and exit of firms, resulting in zero excess profits (equation 5). The Zero Profit Condition can be written as follows (combining the equations 1, 3, 4 and 5)

$$L_x = \frac{L_f}{\frac{\varepsilon}{\varepsilon - 1} - (1+\beta)} . \qquad \text{(ZPC)}$$

Production labour and thus high-tech output is positive if the mark-up over wage costs exceeds the factor 1+β. This can be understood since making profits requires firms to be able to spread their (managerial) fixed costs $w_T L_f$ and quasi-fixed costs $w_T L_r$ over a relatively large output. In other words, with large fixed costs and close substitution of consumption goods, firms need to produce a lot in order to break even. The implication is that the size of the firm is positively related to the R&D intensity β. We initially assume $\beta^1=\beta^2$, so that the high-tech firms in the two countries only differ in size due to differences in fixed costs L_f.

The returns from firm-specific R&D depend on the productivity of the R&D labs. They are defined in the following way:

$$z^1 \equiv \xi, \quad z^2 \equiv \xi f(d), \quad d \equiv \frac{h^2}{h^1} .$$

The parameter ξ measures the productivity of R&D in both countries. Firms in the follower country also receive a *productivity bonus* as they can copy already existing technologies. They benefit from general knowledge that spills over from the leader country. To illustrate the process of trade and growth, the specification $f(d)=d^{-\mu}$ will suffice. The productivity bonus is positively related to the intensity of international knowledge spillovers μ, and negatively to the productivity ratio d.

2.2 Trade

The relative productivity level d (initially smaller than one) is central in our two-country model. This ratio will increase as long as the follower country grows faster than the leader country. The relative rate of growth can be deduced from equation 2, which reads

(applying this equation to each country separately and then dividing)

$$\frac{\dot{h}^2/h^2}{\dot{h}^1/h^1} = \frac{z^2 e^2 L_r^2}{z^1 e^1 L_r^1} = \frac{d^{-\mu} e^2 L_f^2}{e^1 L_f^1}.$$

The last term is obtained after substituting the zero profit condition (using equation 4). The growth rate of country 1 depends on the number of researchers (measured in efficiency units) and the productivity of R&D and is constant (given constancy of the R&D intensity β over time). The growth rate of country 2 also depends on the productivity bonus. The relative rate of growth therefore depends on the strength of spillovers, the initial productivity ratio and the relative input of research labour. It is clear that the relative input of research labour can ultimately be traced back to differences in managerial fixed costs and effort levels in the high-tech sectors. If the initial relative rate of growth exceeds one, the follower country catches up with the leader country. The process of catching up will come to an end if both countries grow at the same rate. The associated Steady State Locus is (combining all the equations in Table 1)

$$w = kd^{\frac{\varepsilon-1-\mu}{\varepsilon}}, \qquad \text{(SSL)}$$

where w ($\equiv w^2/w^1$) is the wage ratio, k ($\equiv e^2/e^1$) is the relative effort level and d ($\equiv h^2/h^1$) is the relative productivity level. The relative effort level will be the topic of discussion in the next section.

To investigate the process of trade and growth, the Steady State Locus should be confronted with the Temporary Equilibrium Locus, which reads (combining the equations 1 and 3-6)

$$w = k^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{L_f^1}{L_f^2}\right)^{\frac{1}{\varepsilon}} d^{\frac{\varepsilon-1}{\varepsilon}}. \qquad \text{(TEL)}$$

Behind the dynamics of the model is the mechanism of international trade. It is important to realize that, according to mark-up pricing, the terms of trade of the follower country depends positively on the wage ratio w and negatively on the productivity ratio d. The follower country initially faces a lower productivity level (d<1) and lower wages (w<1) than the leader country. However, the lower wage ratio cannot fully compensate for the difference in labour productivity; consequently, the price of the high-tech good of the follower country is higher than the price of the high-tech good of the leader country. The follower country will experience an increase in market share in the world market for high-tech goods if it is able to increase the productivity ratio and thus to lower the terms of trade. This will always be the case if $\mu > \varepsilon - 1$. As Figure 2 reveals, in this case, the Steady State Locus slopes downwards, so that the model is stable.[1] A relatively high value of μ implies more international knowledge spillovers, so that there is more room for price decreases in the follower country. A relatively low value of ε indicates that the (bundle of) product varieties supplied by the follower country are substantially differentiated from those supplied by the leader country, so that the follower country can relatively easily compete on the world market for high-tech goods (despite the lower productivity and higher prices as compared to the leader country).

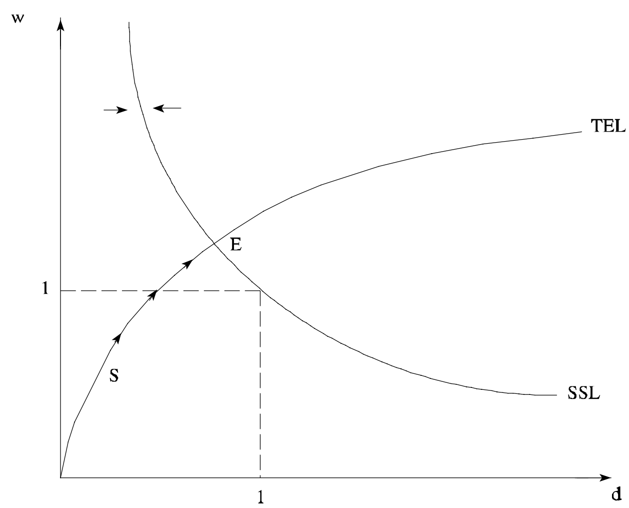

Figure 2. Trade and Growth

The diagram depicts the dynamic evolution of two countries that differ in two respects. Country 2 initially lags behind country 1. Furthermore, high-tech firms in country 2 are characterized by lower fixed costs ($F^2 < F^1$). The development of the productivity ratio is driven by two factors. It is positively affected by ‘backwardness', and negatively by the relatively small amount of research labour (measured in efficiency units) in country 2. In

[1] The SSL slopes upwards if $\varepsilon - 1 > \mu$. A relatively large value of ε implies small differences in varieties, so that there is much competition by prices. In this case, the model is stable if TEL intersects SSL from below.

the diagram, the positive ‘backwardness' effect dominates initially, resulting in catching up. Ultimately, a steady state is reached in which the positive ‘backwardness' effect is exactly offset by the negative ‘relative research labour' effect. Starting from some initial productivity ratio (point S), the economy will move along the temporary equilibrium locus towards the point where SSL and TEL intersect (point E). The steady-state relative productivity level can be written as

$$d^* = \left(\frac{e^2 L_f^2}{e^1 L_f^1} \right)^{\frac{1}{\mu}} = \left(\frac{F^2}{F^1} \right)^{\frac{1}{\mu}} .$$

The diagram exhibits a productivity trap ($d^*<1$). According to our model, the productivity trap can be traced back to persisting higher fixed costs of the high-tech sector in the leader country relative to fixed costs in the follower country. (The assumption of high fixed costs in country 1, relative to fixed costs in country 2, is justified by empirical observations on differences between the US and Europe; see Section 5.) In the steady state the two countries will grow at the same pace, but at different levels of productivity. Wages in the follower country are higher ($w^*>1$) and productivity is lower ($d^*<1$) than in the leader country. The outcome for relative productivity is independent of the total level of output and employment. The dichotomy between the process of trade and growth and the establishment of labour market equilibrium depends on the assumption that investment in R&D is determined according to some rule of thumb. This assumption will be relaxed in section 4. First, we turn to the determination of the allocation of labour.

3. The Allocation of Labour

3.1 Employment

Each country has two sectors, a traditional non-tradeables sector and a high-tech tradeables sector. The non-tradeables sector is included for various reasons. First of all, it is important from an empirical point of view, as sheltered sectors constitute a large part of the economy (this point is forcefully made in, for example, Obstfeld and Rogoff, 1996).

Secondly, we want to stress the coincidence of imperfect competition in product markets and wage differentials in labour markets (*cf*. OECD Jobs Study, 1994). We therefore need two distinct sectors that make up the economy. Tradeability of traditional goods has to be excluded as this makes full specialization possible. The allocation of labour is described in Table 2.

< Insert Table 2 around here >

The non-tradeables sector has constant labour productivity A ($=y/L_N$). L_N stands for the number of workers employed in this sector and y is the production of non-tradeables. In the remainder, we assume unitary labour productivity in both countries (equation 7). We thereby implicitly assume full and instantaneous convergence in this sector. This reflects the idea that relatively basic techniques are used in the sheltered sector of the economy, the technology of which diffuses easily across advanced countries. Comparative advantage and returns to specialization play no role in this sector, in contrast to the high-tech tradeables sector (see also Bernard and Jones (1996) for an empirical justification of this assumption). Under perfect competition, the price of a traditional good equals labour costs (equation 8). The high-tech tradeables sector has already been discussed in section 2. Each firm employs direct and indirect labour (L_x+L_r+L_f), which has efficiency e. The efficiency of a high-tech worker is a variable that can be affected by the wage setting behaviour of the firm. Following Akerlof (1982), we assume that the efficiency of an employee in the high-tech sector depends on the wage he earns, related to the wage of an employee in the traditional sector. Firms will pay higher wages as long as the increase in benefits related to the increase in efficiency more than offsets the increase in costs in the form of a higher wage bill. This trade off results in the well-known Solow condition (see Appendix). Given our specification of the efficiency wage relation (equation 12), this implies that the relative wage (equation 13) and the level of effort are constant in equilibrium. The introduction of the efficiency wage relation leads to high-tech employees receiving a non-competitive wage differential ($w_T/w_N>1$).

Consumption in a country consists of the traditional good of that country along with all varieties of the high-tech good. Households have Cobb-Douglas preferences over the two types of goods (a fixed fraction of consumption expenditures σ is spent on high-tech goods). Preferences between traditional and high-tech goods are assumed to be identical across countries. For each country, it can then be derived that goods market equilibrium is described by a fixed ratio between the consumption of traditional goods and the consumption of high-tech goods. We assume that financial capital is immobile across countries. The absence of international capital flows implies that trade must be balanced at every moment in time. Trade balance requires that the value of output of high-tech goods be equal to the value of consumption of high-tech goods. Goods market equilibrium is then described by equation 9, which reads (after substituting the equations 1, 2, 7 and 8)

$$L_N = \frac{1-\sigma}{\sigma}\frac{\varepsilon}{\varepsilon-1}\frac{w_T}{w_N}nL_x .$$

Notice that the relative wage is fixed by efficiency wage considerations (equation 13) and that $0<\sigma<1$ and $\varepsilon>1$. Thus, for a given number of high-tech firms n, there is a fixed positive relation between employment in the non-tradeables sector and the number of production workers in the high-tech sector. What this equation basically says is that, for the circular flow to be in equilibrium, an increase in the number of high-tech production workers must be accompanied by an increase in employment in the traditional sector.

Employment in the high-tech sector is described by equation 10. Using the Zero Profit Condition (and equation 4), this equation can be written as

$$L_T = \frac{\varepsilon}{\varepsilon-1}nL_x .$$

Employment in the high-tech sector exceeds the number of production workers by the extent of the mark-up. This is intuitively clear as a large mark-up implies that firms can afford relatively high fixed costs. The allocation of labour now follows from the solution of the number of production workers. To get a quick glance at the characteristics of the equilibrium, unemployment is left out of consideration (U=0). In this case, the labour market is in equilibrium if total labour L (which is fixed) is fully allocated over both sectors. After substituting the equations for L_N and L_T into the equation for labour market equilibrium (equation 11), the result is

$$n|_{U=0}L_x = \frac{L}{\frac{\varepsilon}{\varepsilon-1}\left(1 + \frac{1-\sigma}{\sigma}\frac{w_T}{w_N}\right)} .$$

The full employment equilibrium number of high-tech production workers is fixed, the implication being that the sectoral allocation of labour is also fixed. In the full employment equilibrium the allocation of labour only depends on the parameters ε and σ and the (rigid) relative wage. The full employment equilibrium number of high-tech firms now follows (using the zero profit condition) as

$$n|_{U=0} = \frac{\frac{\varepsilon}{\varepsilon-1} - (1+\beta)}{\frac{\varepsilon}{\varepsilon-1}\left(1 + \frac{1-\sigma}{\sigma}\frac{w_T}{w_N}\right)} \frac{L}{L_f} .$$

This equation shows that the number of high-tech firms (i.e., product variety) crucially depends on the ratio between L and L_f. A decrease in the supply of labour L reduces the number of firms. However, this does not affect the allocation of labour over sectors. A decrease in L is proportionally spread over sectors because the size of the high-tech firms is fixed by the process of entry and exit of firms (given constancy of the R&D intensity β).

3.2 Unemployment

We now introduce unemployment (U>0) by explicitly modelling the flows on the labour market. We have seen that labour gets a 'state-specific' payment (w_N or w_T when employed in one of the two sectors). Unemployed people receive a real unemployment compensation equal to b, expressed in terms of the non-tradeable good. Consequently, the nominal compensation equals bw_N. This compensation is paid out of lump-sum taxes on labour income. It is assumed that the net real wage earned in the non-tradeables sector is higher than the real unemployment compensation, so that the unemployment compensation b must be sufficiently smaller than one. In principle, each worker is striving for the highest possible pay-off. Hence, all workers would like to be employed in the high-tech sector. The number of jobs in this sector is, however, limited. We assume that at some exogenous rate δ jobs in the high-tech sector become available. Upon being laid off, a worker faces two options. He can either decide to take a job in the traditional sector (these jobs are freely available), or he can join the pool of unemployed. In determining his optimal strategy, the worker has to take the following factors into consideration: (i) unemployment benefits are lower than the salaries in the traditional sector (b<1), and (ii) the inflow rate into the high-tech sector from the traditional sector αq is lower than from unemployment q. The process of weighing the two options that laid off high-tech workers face results in an endogenously determined probability, η, of entering one of the two states (i.e., the state of unemployment or traditional sector employment). The outcome for this probability is such that, ex-ante, laid off workers (who are distributed randomly) are indifferent between the two options they face. The resulting unemployment is both voluntary and involuntary. The unemployed are willing to work in the high-tech sector at the going wage, while they refuse jobs at the going wage in the traditional sector.[2]

[2] Labour market equilibrium is derived by introducing three value functions. These functions (V_j) indicate the present discounted value for a worker of being in state j (=T, N, U for being employed in the tradeables sector, non-tradeables sector and being unemployed, respectively)

$$rV_T = w_T + \delta\eta(V_N - V_T) + \delta(1-\eta)(V_U - V_T)\ ,\quad rV_N = w_N + \alpha q(V_T - V_N)\ ,\quad rV_U = bw_N + q(V_T - V_U)$$

Equilibrium requires V_N=V_U. Flow equilibrium on the labour market (i.e., a constant allocation of labour) requires $\delta\eta L_T$=$\alpha q L_N$ and $\delta(1-\eta)L_T$=qU. We refer to the Appendix for a more detailed description and derivation of labour market equilibrium.

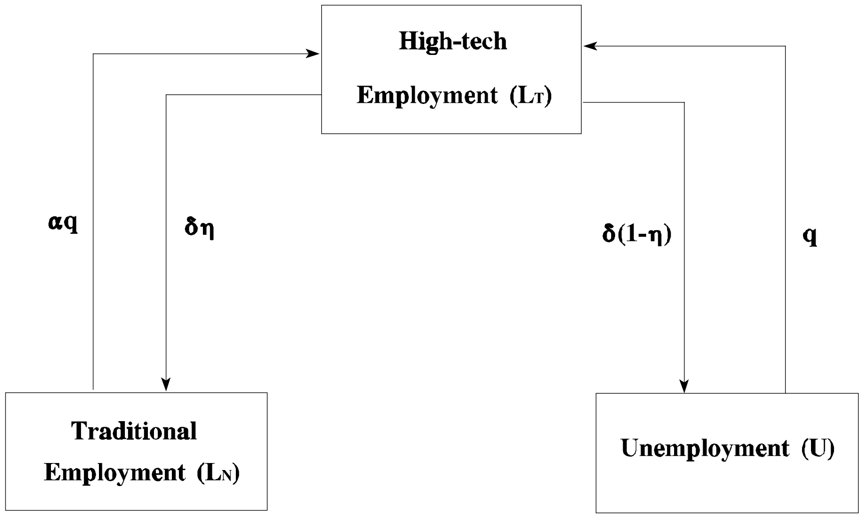

Figure 3. Labour Market Flows

Figure 3 presents a stylized interpretation of the labour market flows. The assumption that the unemployed have a higher inflow rate into the high-tech sector than workers in the traditional sector ($\alpha<1$) is important and often used as a simple and useful working hypothesis in the literature on unemployment in dual labour markets. Though not formally modelled, this may reflect that it is harder to find a high-tech job when employed in the secondary sector than when unemployed. Alternatively, people may vary in their aversion to working in the secondary sector. This simplified account of the labour market captures some crucial perceptions of labour market equilibrium (*cf*. Layard, Nickell and Jackman, 1991).

Formally, flow-equilibrium in the labour market is determined by three value functions (denoting the present discounted value of expected income streams of a worker in the traditional sector, an unemployed labourer, and a worker in the high-tech sector, respectively) and two flow-equilibrium conditions, guaranteeing a constant allocation of labour over the three states (*cf*. De Groot, 1996; see also the Appendix). In equilibrium, it is required that the value of a job in the traditional sector equals the value of being unemployed. This captures the idea that a laid-off high-tech worker is indifferent between accepting a job in the traditional sector and being unemployed. Workers discount their income at a rate of interest r (to be specified in the next section) as they can freely save and borrow in the financial market at this rate. The equilibrium number of unemployed can be derived as

$$U = \frac{1}{1-\alpha}\left[\left(\frac{\delta}{q}+\alpha\right)L_T - \alpha L\right] \quad \text{with} \quad q = \frac{(1-b)(\delta+r)}{\frac{w_T}{w_N}(1-\alpha)-(1-\alpha b)},$$

where q represents the outflow rate out of unemployment (which equals the inflow rate into the high-tech sector for the unemployed). The rate of unemployment is positively related to the total number of high-tech workers. This can be understood as follows. As more high-tech jobs become available, the number of high-tech jobs opening up as a result of lay-off increases. For a given outflow rate from the pool of unemployed, this increases the attractiveness of waiting for a high-tech job as an unemployed job seeker. The unemployment rate will rise accordingly. The outflow rate is *positively* related to the rate of interest. This results from the fact that a lower rate of interest decreases the importance attached to current payments. This increases the attractiveness of waiting for a high-wage job in the pool of unemployed and consequently lowers the outflow rate out of unemployment.

We are now able to determine the allocation of labour over employment and the state of unemployment (conditional on the rate of interest r and the R&D intensity β). The allocation of labour follows from the solution of the number of high-tech firms, which reads as follows

$$n = \frac{\frac{\varepsilon}{\varepsilon-1} - (1+\beta)}{\frac{\varepsilon}{\varepsilon-1}\left[1 + \frac{\delta\left[\frac{w_T}{w_N}(1-\alpha)-(1-\alpha b)\right]}{(1-b)(\delta+r)} + \frac{1-\sigma}{\sigma}\frac{w_T}{w_N}(1-\alpha)\right]}\frac{L}{L_f}.$$

This result is analogous to the previously derived equation for the number of firms in the case of full employment (as can be verified by putting α=δ=0). A decrease in the rate of interest will decrease the number of firms. As the size of a firm is fixed (given constancy of the R&D intensity β), employment in both sectors will decrease so that unemployment will rise. The model thus reveals a *negative* relation between unemployment and the rate of interest (conditional on the R&D intensity β). This can be explained as follows. A lower rate of interest increases the importance attached to future payments. As becoming unemployed improves the chance of acquiring a high-wage job in the near future, this becomes more attractive. Thus more people opt for unemployment than for a low-wage job in the traditional sector (which decreases the outflow rate out of unemployment). This reduces the effective supply of labour L-U, which spreads proportionally over sectors (both L_T and L_N decrease). The markets become 'thinner', so that there is less room for high-tech firms to spread fixed costs over output. As a consequence, some firms will have to exit.

If we assume (as is the case in the savings-investment equilibrium in our model; see section 4.1) that the rate of interest is positively related to the rate of growth, the model shows that catching up with the leader country is accompanied by rising rates of unemployment in the follower country. We refer to the example in Figure 2. Starting from a relatively low productivity level in point S, the follower country converges to the productivity trap E. The productivity bonus gradually decreases in size, so that the follower country faces a slowdown in the rate of growth (and the rate of interest). The decrease in the rate of interest decreases the outflow rate out of unemployment, because a larger number of people opt for unemployment than for a job in the low-wage sector of the economy. In point E, the follower country arrives at a steady-state equilibrium, where the allocation of labour does not change any more. In the next section, we will endogenize the rate of interest (and consequently the R&D intensity β), so that we can investigate the relation between the equilibrium relative productivity level and the equilibrium rate of unemployment. Attention will be restricted to a steady-state analysis.

4. The Steady State

4.1 Savings and Investment

Financial markets determine the relation between the rate of growth and the rate of interest. We recall that it is assumed that financial capital is immobile between countries. The absence of international capital flows implies that trade is balanced. This requires the value of output of high-tech goods to be equal to the value of consumption of high-tech goods. The value of consumption is therefore equal to the value of output of both sectors

(yp_N+nxp_T). Households receive wage income (w_NL_N+w_TL_T) and dividends. Dividends are paid from the difference between the value of output of high-tech goods (xp_x) and production costs ($nw_T(L_x+L_f)$). Households save and high-tech firms invest (nw_TL_r). Savings are the difference between households' income and the value of consumption. Noting that yp_N=w_NL_N, we can easily check that the circular flow is in equilibrium if savings equal investment.

< Insert Table 3 around here >

To model savings and investment behaviour, we follow Smulders and Van de Klundert (1995). Households maximize intertemporal utility, which yields the familiar Ramsey rule (equation 15). This equation relates the growth of consumption to the determinants of the consumption-savings decision. It shows that the rate of growth is high if the return on savings (r) is large relative to the subjective discount rate (θ), and if households are willing to substitute intertemporally (1/ρ is high). High-tech firms determine their optimal research effort, which leads to the planned rate of growth (equation 14). Confrontation of the required rate of growth (the Ramsey rule) with the planned rate of growth yields the steady-state interest rate and rate of growth[3]

$$r = \frac{(\varepsilon-1)[\sigma(\rho-1)\, zeL_f - \theta]}{\sigma(\rho-1)-(\varepsilon-1)} \;, \quad g = \frac{(\varepsilon-1)zeL_f - \theta}{\sigma(\rho-1)-(\varepsilon-1)} \,.$$

The steady-state rate of growth does not depend on the sectoral allocation of labour. What matters for growth are consumers' preferences, the productivity of the research labs (which for the follower country embodies the productivity bonus) and the level of fixed

[3] Stability of the steady state equilibrium with positive growth requires the following parameter restrictions: $\sigma(\rho-1)>\varepsilon-1$ and $(\varepsilon-1)zeL_f>\theta$.

costs (measured in efficiency units).[4] The model is completed by determining the steady-state R&D intensity β, which equals the present discounted value of the growth rate (equation 18).

4.2 Comparative Static Analysis

We define a (global) steady state as a situation in which both the rates of growth and the relative productivity level are constant. The steady-state relative productivity level is then determined as

$$d^* = \left(\frac{e^2 L_f^2}{e^1 L_f^1} \right)^{\frac{1}{\mu}} .$$

This result is already known from the growth and trade block of the model (section 2): persisting differences in productivity levels between countries can ultimately be traced back to persisting differences in relative management costs (measured in efficiency units) of the high-tech sectors (under the assumption of equal preference parameters in both countries).

We are now able to derive comparative static results. Table 4 shows the effects of an increase in the fixed cost requirement in both countries (F and thus L_f increases). An increase in the fixed costs of the high-tech firms in the leader country will lead to exit of firms. The remaining firms are bigger in size and can afford larger R&D labs so that productivity growth is raised. As a consequence, the relative productivity level decreases. In the leader country, more firms leave the market than in the follower country. This is

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4 The independence of the rate of growth from the scale of the economy as represented by L is important in the light of the ongoing discussion on the relevance of scale effects that characterize so many of the endogenous growth models (e.g., Romer (1990), Grossman and Helpman (1991)). We refer here to Jones (1995a) for an extensive discussion on this issue. Based on time-series evidence, Jones (1995b) draws the conclusion that the prediction of ’scale effects’ is inconsistent with empirical evidence for industrialized countries. He therefore proposes a modified version of the Romer model in which the long-run growth rate is crucially dependent on the rate of population growth. The model in this paper presents an alternative way to overcome the ’problem’ of scale effects (see Smulders and Van de Klundert, 1995). A difference between this model and the Jones model is that parameters influencing savings behaviour like the intertemporal elasticity of substitution and the subjective discount rate determine the rate of growth in our model, whereas they play no role in the Jones model.

caused by the fact that firms in the follower country benefit from the increase in the productivity bonus (*d* is lower), so that they have to invest less in R&D (in country 2, the increase in L_r is smaller than in country 1). At the same time, the higher rate of interest decreases unemployment in both countries (more people opt for a job in the traditional sector than for unemployment). Growth and unemployment are thus negatively related. In the leader country the higher rate of growth is accompanied by a higher level of output of the tradeables sector, whereas in the follower country output of tradeables is lower.

< Insert Table 4 around here >

An increase in the fixed costs of the high-tech firms in the follower country has no effects on worldwide growth and interest rates, so that the allocation of labour over sectors and unemployment does not change. The follower country has no impact on the rate of growth, because there are no knowledge spillovers from the follower to the leader country. An increase in fixed costs leads to exit of firms, so that there is more room to invest in R&D. This raises the relative productivity level. Thus, to keep the same growth rate as in country 1, firms in country 2 also have to invest more in R&D, because the productivity bonus decreases.

Labour market policies leave the long-run rate of growth and relative productivity unaffected in our model. This result is due to the assumption of free entry and exit in the high-tech sector.[5] Unemployment compensations do, however, affect the allocation of labour and the level of production. More concretely, lowering unemployment benefits increases the outflow rate out of unemployment, as the costs of waiting for a high-wage job are increased. More people are willing to accept a job in the traditional sector, so production of non-tradeable goods increases. The increased purchasing power of consumers in the economy will positively affect the profitability of high-tech firms and induce firms to enter this sector. As the size of an individual firm is fixed, high-tech employment will increase. Equilibrium unemployment will have fallen.

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[5] Solving the model under the alternative assumption that the number of high-tech firms is exogenously given (leaving potential excess profits) results in a negative relation between, for example, unemployment benefits and growth. This is illustrated in a simplified version of the current model (with exogenous R&D intensity, a fixed number of firms, and no intertemporal considerations of the workers in their decision how to allocate when laid off in the high-tech sector (De Groot and Van Schaik, 1997)).

5. The Productivity Trap

The theoretical model developed in this paper can serve as a useful framework to address the connection between productivity and unemployment. In the current section, we will show how parts of a rich, descriptive body of literature can be reconciled with some simple, basic ideas present in our model. This exercise can deepen our understanding of some (stylized) empirical facts.

Following World War II, for more than 30 years, many European countries experienced high rates of growth of industrial productivity, strongly exceeding the US performance. This development was parallelled by historically unprecedented low rates of unemployment in the 1950s and 1960s, whereas the 1970s were characterized by an upsurge in European unemployment rates. Important elements in explaining these developments are a huge potential to catch up with the US in the early post war period, along with the establishment of a generous European welfare system, starting in the late 1960s (*cf.* De Groot and Van Schaik, 1997). Since the late 1980s, European productivity levels relative to the US have been stabilizing at levels lower than those reached at the end of the 1970s, whereas unemployment has been structurally higher. How can these two features be reconciled?

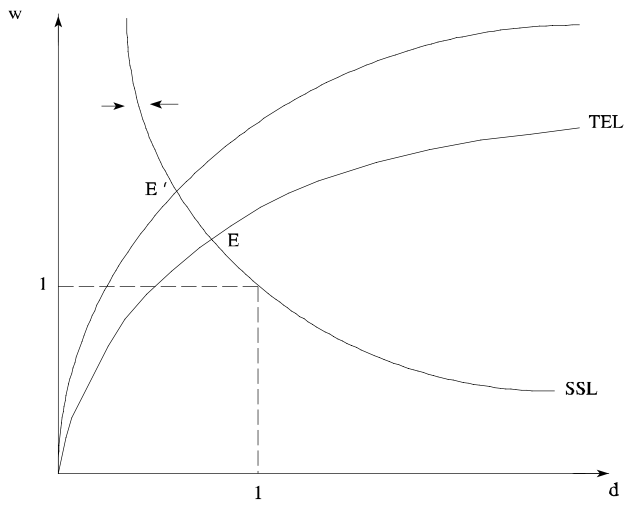

Figure 4. A Change in the Productivity Trap

The stabilization of unemployment rates (at relatively high levels in Europe and relatively low levels in the US) is indicative of the arrival of a period of steady growth. Two important phenomena point in this direction. First, there is the stabilization of relative productivity levels. Secondly, relative wages in manufacturing are stabilizing at levels corresponding to these relative productivity levels. Nominal wages in the European industries (more specifically, in Germany and France) are significantly above those in the US.[6] Figure 4 crudely mimics the empirical evidence, with point E representing the situation at the end of the 1970s, and point E' representing the situation in the 1990s. The remainder of this section will focus on (i) why European countries lag behind the US, and (ii) why the productivity gap was enlarged in the 1980s (i.e., in terms of the diagram, what caused the upward shift of the temporary equilibrium locus).

We have seen that persisting differences in productivity levels between countries can be traced back to persisting differences in relative management costs (measured in efficiency units) of the growth-generating high-tech, high-wage sectors of the economy. The relative productivity level will decrease if relative management costs decrease. In addition, as has become clear from the comparative static analysis in the previous section, the effects on the equilibrium rate of unemployment will depend on the causes of the change in the equilibrium productivity ratio. A decrease in the management costs of the leader country will increase unemployment in both countries, whereas a decrease in the management costs of follower country 2 leaves unemployment unaffected.

Traditional fixed costs are higher in the US industries than in Europe. Gordon (1994) shows that the ratio of managers/administrators to clerical, service and production workers in the US is about four times as large in the US than in Germany or France. In addition, non-competitive wage differentials are more pronounced in the US than in European countries (*cf.* Hartog, Van Opstal and Teulings, 1997). The difference in management costs results, according to our model, in relatively large firms in the US. This allows them to spend relatively much on R&D, resulting in a structural lead in terms of

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[6] This follows from calculations of hourly wages in manufacturing, using OECD National Accounts (at current exchange rates). Nominal wages in France and Germany in 1980 were already higher than those in the US (point E in Figure 3). Relative wages in these countries dropped below 1 in the early 1980s, in order to recover in the second half of the 1980s and to exceed those of the US again. In the early 1990s, relative wages exceeded those of the late 1970s (corresponding to point E' in Figure 4). Other sources confirm this development of relative wages (*cf.* Leamer, 1996).

productivity over Europe.

The literature has paid a great deal of attention to the causes of the structural difference in fixed (management) costs between Europe and the US. Most US firms grew mature in a tradition of mass production (*cf.* Nelson and Wright, 1992). This is in strong contrast with Europe. Broadberry (1996, p. 340) summarizes the literature on this point by arguing that ‘The greater prevalence of mass production in the USA can be explained by both demand and supply factors. On the demand side, standardization in the USA was facilitated by the existence of a large, homogeneous home market in the USA compared with fragmented national markets stratified by class differences in Europe, coupled with greater reliance on differentiated export markets. On the supply side, resource-intensive American machinery could not be adopted on the same scale in Europe, where resource costs were considerably higher.'

According to Lazonick (1991), US firms could afford a high fixed cost strategy. In Lazonick’s theory, fixed costs play a central role. Fixed costs are a problem for the firms as they face uncertainty on both the demand and the supply side. On the other hand, fixed costs are a strategic variable. By investing in a low fixed cost strategy, the organization is certain that unit costs are relatively insensitive to the scale of production. In contrast to this adaptive strategy, a firm can choose an innovative strategy. Such a strategy is characterized by strongly decreasing unit costs when production expands. An innovative strategy may result in product or process innovation. However, there is uncertainty involved as the high fixed costs have to be transformed into low unit costs. According to Lazonick, investments in specialized facilities for research, development, and marketing, and in managerial bureaucracy play an important role. Following the world wide recession in the early 1980s, European firms adopted a more adaptive strategy, hampering innovations in production, organization, and marketing. Competing on the world market consequently became more and more difficult, in a period in which, in addition, transportation and communication costs decreased dramatically. A radical strategic change was needed to survive. A process of ‘downsizing' started, which characterizes most of the strategic policies pursued by firms operating on the world market.

The emphasis on cost reduction and improving efficiency started a process of reducing fixed costs, resulting into reductions of the size of the firm (downsizing). This induces entry of new firms and reduces each firm’s market share, which results in smaller

R&D departments. In our model, the reduction in number of managers (L_f) and research workers (L_r) together induces an equal decline in the number of production workers (L_x). Consequently the so-called bureaucratic burden (which can be defined as L_f/L_x) increases (see Appendix). This is in accordance with empirical evidence (*cf*. Gordon, 1996). Not only European but also US firms are adopting strategies aimed at cost reduction and efficiency improvements (*cf*. McKinsey, 1996). The lower relative productivity levels in the 1990s compared with the late 1970s point to a stronger reduction in fixed costs in Europe than in the US (according to our model). Downsizing in Europe, in our model, results in an increase of the productivity gap with the US. This reduces the market share of European firms in the world market. Reduction in fixed costs in the leader country retards economic growth, resulting in a world-wide increase in unemployment. Hence, part of the European unemployment problem can be traced back to strategic behaviour of US firms aimed at reducing fixed costs.[7]

6. Conclusion

The performance of the European and the US economies is significantly different in terms of productivity and unemployment. We argued in this paper that in part these differences can be traced back to institutional differences. More concretely, differences between countries in fixed costs of firms competing on the world market for high-tech goods were shown to be important determinants of growth and relative productivity levels. A positive relation between these costs and R&D intensities was established. Downsizing by means of reducing these fixed costs thus negatively affects economic growth.

Another important characteristic of our model is the negative relation between

[7] A characteristic of our model is that the world rate of growth is ultimately determined by the leader country. This is due to our assumption of one-sided knowledge spill-overs. Relaxation of this assumption would alter this result, but not affect the general tendencies sketched in this section. Another consideration is that by the assumption of free entry in the high-tech sector, labour market institutions do not affect the rate of growth (and the relative productivity level). We are convinced that institutional differences play an important role in explaining the empirical tendencies that are central in this paper. Our main focus in this paper is, however, on differences in the strategies pursued by individual firms as reflected in the fixed costs. In order not to complicate the analysis more than necessary, we abstain from an analysis of the model under blocked entry. We refer to De Groot and Van Schaik (1997) for an analysis that focuses on labour market institutions and their effect on growth and the productivity gap.

growth and unemployment. The unemployed face a trade off between accepting a job in the low wage (sheltered) sector and remaining unemployed, enhancing the probability of finding a high-wage job in the open sector of the economy in the near future. A lower rate of growth (and the accompanied lower rate of interest) increases unemployment as it increases the value attached to future payments. Waiting as an unemployed person for a high wage job becomes more attractive.

This paper did not focus on differences in labour market institutions between Europe and the US. However, most analyses of the European unemployment problem (*cf.* OECD Jobs Study, 1994) focus on these differences. Notwithstanding the importance of labour market institutions, the processes of trade and growth create their own dynamics, which may have important consequences for the labour market. These processes have to be taken into account when studying the European unemployment problem. This paper is an attempt to integrate these issues within a consistent framework. If downsizing remains an important trend in a globalizing world economy, the limited possibilities for firms to engage in 'in house' R&D will further retard economic growth. The associated reduction in interest rates reduces the costs of waiting as an unemployed person for attractive jobs, so that a further upward pressure on unemployment will result. Although, in our simple model, downsizing in Europe has no impact on the world wide rate of growth, it will increase the productivity gap with the US. Summarizing, the disappointing developments on European labour markets since the late 1970s can in part be traced back to developments in the US. Downsizing seems to have played a crucial role both in retarding worldwide economic growth and increasing the productivity gap between Europe and the US.

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Table 1. Growth and Trade

$$x = heL_x \tag{1}$$

$$\dot{h} = zheL_r \tag{2}$$

$$p_T = \frac{\varepsilon}{\varepsilon - 1}\frac{w_T}{he} \tag{3}$$

$$L_r = \beta L_x \tag{4}$$

$$xp_T = w_T(L_x + L_r + L_f) \tag{5}$$

$$\frac{x^2}{x^1} = \left(\frac{p_T^2}{p_T^1}\right)^{-\varepsilon} \tag{6}$$

Endogenous variables[8]

L_x, L_r, $\dot{h}$, x, p_T, w_T

Explanation of symbols

β	= R&D intensity;
ε	= price elasticity of high-tech goods (ε>1);
e	= effort level;
h	= labour productivity high-tech sector;
L_f	= fixed labour high-tech firm;
L_r	= research labour high-tech firm;
L_x	= production labour high-tech firm;
p_T	= price high-tech good;
x	= output high-tech firm;
w_T	= nominal wage rate high-tech sector;
z	= productivity of R&D.

[8] The system consists of 11 equations and 12 unknowns. Given initial values for h^1 and h^2, the system can be solved for all real variables and relative prices. The characteristics of the steady state, in which h^2/h^1 is a constant (and endogenously determined), are discussed in section 4.2.

Table 2. The Allocation of Labour.

$$y = L_N \tag{7}$$

$$p_N = w_N \tag{8}$$

$$\frac{1-\sigma}{\sigma} = \frac{y p_N}{n x p_T} \tag{9}$$

$$L_T = n(L_x + L_r + L_f) \tag{10}$$

$$L = L_T + L_N + U \tag{11}$$

$$e = -a + c\left(\frac{w_T}{w_N}\right)^{\gamma} \tag{12}$$

$$\frac{w_T}{w_N} = \left(\frac{a}{(1-\gamma)c}\right)^{\frac{1}{\gamma}} \tag{13}$$

Endogenous variables[9]

y, n, e, p_N, w_N, L_N, L_T, U

Explanation of symbols

a, c, γ = parameters efficiency wage relation;
σ = share of consumption expenditures spent on high-tech goods ($0<\sigma<1$);
L = total labour supply (exogenous);
L_N = employment non-tradeables sector;
L_T = employment tradeables sector;
n = number of high-tech firms;
U = unemployment;
w_N = nominal wage rate non-tradeables sector.

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9 For each country the system consists of 7 equations and 8 unknowns. Assuming U=0, the system can be solved in terms of the ratio between the total amount of labour L and the number of production workers L_x. The number of production workers (size of the firm) is determined by the zero profit condition (given constancy of the R&D intensity β.) Assuming U>0, an additional equation is needed, representing flow equilibrium on the labour market. Imposing the parameter restriction $0<\gamma<1$ and $a>(1-\gamma)c>0$ yields an equilibrium relative wage larger than one.

Table 3. The Steady State.

$$z^1 \equiv \xi \;, \quad z^2 \equiv \xi d^{-\mu} \;, \quad d \equiv \frac{h^2}{h^1} \;, \quad g \equiv \frac{\dot{h}}{h}$$

$$r = (\varepsilon - 1)\,(g + zeL_f) \tag{14}$$

$$g = \frac{r - \theta}{\sigma(\rho - 1)} \tag{15}$$

$$\beta = \frac{g}{r} \tag{16}$$

Endogenous variables

$$g,\ r,\ \beta$$

Explanation of symbols

1/ρ	= intertemporal elasticity of substitution ($\rho>1$);
θ	= subjective discount rate;
g	= rate of growth;
r	= rate of interest.

Table 4. Comparative static results

<table>
<tr><th rowspan="2"></th><th colspan="4">Effects of increase in</th></tr>
<tr><th colspan="2">F^1</th><th colspan="2">F^2</th></tr>
<tr><td>Country</td><td>1</td><td>2</td><td>1</td><td>2</td></tr>
<tr><td>r rate of interest</td><td>+</td><td>+</td><td></td><td></td></tr>
<tr><td>g rate of growth</td><td>+</td><td>+</td><td></td><td></td></tr>
<tr><td>β R&D intensity</td><td>+</td><td>+</td><td></td><td></td></tr>
<tr><td>L_r research labour</td><td>++</td><td>+</td><td></td><td>+</td></tr>
<tr><td>L_x production labour</td><td>++</td><td>+</td><td></td><td>+</td></tr>
<tr><td>L_f fixed labour</td><td>+</td><td></td><td></td><td>+</td></tr>
<tr><td>L_T/n firm size</td><td>++</td><td>+</td><td></td><td>+</td></tr>
<tr><td>q outflow rate out of unemployment</td><td>+</td><td>+</td><td></td><td></td></tr>
<tr><td>n number of firms</td><td>--</td><td>-</td><td></td><td>-</td></tr>
<tr><td>L_T employment tradeables</td><td>+</td><td>+</td><td></td><td></td></tr>
<tr><td>L_N employment non-tradeables</td><td>+</td><td>+</td><td></td><td></td></tr>
<tr><td>w_T/w_N wage differential</td><td></td><td></td><td></td><td></td></tr>
<tr><td>e effort</td><td></td><td></td><td></td><td></td></tr>
<tr><td>U unemployment</td><td>-</td><td>-</td><td></td><td></td></tr>
<tr><td>d productivity ratio</td><td colspan="2">-</td><td colspan="2">+</td></tr>
<tr><td>$f(d)$ productivity bonus</td><td colspan="2">+</td><td colspan="2">-</td></tr>
<tr><td>p terms of trade</td><td colspan="2">+</td><td colspan="2">-</td></tr>
<tr><td>w wage ratio</td><td colspan="2">+</td><td colspan="2">-</td></tr>
<tr><td>x output per firm</td><td>+</td><td>-</td><td></td><td>+</td></tr>
<tr><td>nx total output tradeables</td><td>+</td><td>-</td><td></td><td>+</td></tr>
</table>

Note: Effects which are equal for both countries are denoted by a single - or +. A double sign (-- or ++) indicates that the effects are larger in size for the country in question than for the other country.

Appendix

This Appendix will in turn describe consumer behaviour, producer behaviour, and labour market equilibrium in the two countries. Then the steady-state solution of the model will be derived. Where it leads to no confusion, country indices (j=1, 2) and time indices are omitted (equations hold for both countries in this case). Unless otherwise stated, symbols are defined in the main text and in Tables 1-3.

Consumer behaviour.

We assume that consumers maximize their intertemporal utility U_0 in three steps. In the first step, they decide how to divide total income among savings on the one hand and consumption on the other. In the second step they decide how to divide total consumption expenditures among high-tech goods on the one hand and traditional goods on the other. In the final step they decide how to divide their expenditures on the high-tech goods among the n^1+n^2 varieties of this good that are available to them. In the *first* step the consumer maximizes

$$U_0 = \int_0^\infty \frac{C_t^{1-\rho} - 1}{1-\rho} e^{-\theta t} dt \,, \tag{A.1}$$

C being a composite good, subject to

$$\dot{A}_t = r_t A_t + I_t - C_t P_{Ct} \,, \tag{A.2}$$

where CP_C is total expenditures on consumption goods and I is total wage income in the economy. The current value Hamiltonian corresponding to this problem equals

$$H = \frac{C_t^{1-\rho} - 1}{1-\rho} + \lambda_t (r_t A_t + I_t - C_t P_{Ct}) \,. \tag{A.3}$$

The First Order Conditions (FOC) corresponding to this problem are

$$\frac{\partial H}{\partial C_t} = C_t^{-\rho} - \lambda_t P_{Ct} = 0 \,, \tag{A.4}$$

and

$$\dot{\lambda}_t = \theta\lambda_t - \frac{\partial H}{\partial A_t} = \lambda_t (\theta - r_t) \,. \tag{A.5}$$

From the FOC (A.4) we derive

$$\frac{\dot{\lambda}_t}{\lambda_t} = -\rho \frac{\dot{C}_t}{C_t} - \frac{\dot{P}_{Ct}}{P_{Ct}} \,. \tag{A.6}$$

Combining (A.5) and (A.6), these equations yields

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\rho}\left[r_t - \frac{\dot{P}_{Ct}}{P_{Ct}} - \theta\right], \tag{A.7}$$

which is the familiar Ramsey rule. The *second* step in the optimization-procedure is maximizing

$$C = X^{\sigma} y^{1-\sigma}, \quad 0 < \sigma < 1, \tag{A.8}$$

subject to

$$X P_X + y p_N = C P_C. \tag{A.9}$$

Here XP_X represents expenditures on the differentiated high-tech good. This yields the following Lagrangian

$$L = X^{\sigma} y^{1-\sigma} - \nu\,(X P_X + y p_N - C P_C), \tag{A.10}$$

where ν is the shadow price associated with the budget constraint. The corresponding FOC are

$$\frac{\partial L}{\partial X} = \sigma X^{\sigma-1} y^{1-\sigma} - \nu P_X = 0, \tag{A.11}$$

and

$$\frac{\partial L}{\partial y} = (1-\sigma)\, X^{\sigma} y^{-\sigma} - \nu\, p_N = 0. \tag{A.12}$$

Eliminating ν results in

$$XP_X = \frac{\sigma}{1-\sigma}\, y p_N \quad \text{or} \quad \frac{y p_N}{1-\sigma} = CP_C. \tag{A.13}$$

This equation tells us that a fixed fraction 1-σ of total consumption expenditure CP_C is spent on traditional goods and a fixed fraction σ is spent on high-tech goods. Combining these equations with the Cobb-Douglas preference function gives an expression for the price index P_C

$$\begin{gathered} c = \frac{y}{1-\sigma}\frac{p_N}{P_C} = \left(\frac{\sigma}{1-\sigma} y \frac{p_N}{P_X}\right)^{\sigma} y^{1-\sigma} \quad \text{so} \quad \left(\frac{p_N}{1-\sigma}\right)\frac{1}{P_C} = \left(\frac{\sigma}{P_X}\right)^{\sigma}\left(\frac{p_N}{1-\sigma}\right)^{\sigma} \quad \text{so} \\ P_C = \left(\frac{P_X}{\sigma}\right)^{\sigma}\left(\frac{p_N}{1-\sigma}\right)^{1-\sigma} \end{gathered} \tag{A.14}$$

The *third* step is maximizing

$$X = \left[\sum_{i=1}^{n^1+n^2} c_i^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \quad \varepsilon > 1, \tag{A.15}$$

subject to

$$\sum_{i=1}^{n^1+n^2} c_i p_{Ti} = P_X X, \tag{A.16}$$

gives the following Lagrangian

$$L = \left[\sum_{i=1}^{n^1+n^2} c_i^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} - \iota\left[\sum_{i=1}^{n_1+n_2} c_i p_{Ti} - P_X X\right], \tag{A.17}$$

where ι represents the shadow-price associated with the budget constraint, and c_i is the consumed quantity of variety i of the high-tech good. The FOC to this problem is

$$\frac{\partial L}{\partial c_i} = X^{1/\varepsilon} c_i^{-1/\varepsilon} - \lambda p_{Ti} = 0 \quad \forall i = 1, ..., n^1 + n^2. \tag{A.18}$$

We now assume symmetry of goods produced *within* a country. From the FOC it then follows that

$$\frac{c_1^1}{c_2^1} = \left(\frac{p_T^1}{p_T^2}\right)^{-\varepsilon} \quad \text{and} \quad \frac{c_1^2}{c_2^2} = \left(\frac{p_T^1}{p_T^2}\right)^{-\varepsilon}, \tag{A.19}$$

where c_j^k is the consumed quantity in country k of a good produced in country j. We can thus derive

$$\begin{aligned} X^1 &= \left[\sum_{j=1}^{2} n^j \left(c_j^1\right)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} = \left[n^1\left(c_1^1\right)^{(\varepsilon-1)/\varepsilon} + n^2\left(c_2^1\right)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} = \\ &\quad c_1^1\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}. \end{aligned} \tag{A.20}$$

The respective consumed quantities are thus

$$c_1^1 = \frac{X^1}{\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}, \quad c_2^1 = \left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon} \frac{X^1}{\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}. \tag{A.21}$$

Rewriting the budget constraint as $n^1 c_1^1 p_T^1 + n^2 c_2^1 p_T^2 = P_X^1 X^1$ and combining with the above expressions yields the solution for P_X^1:

$$P_X^1 = \frac{n^1 p_T^1 + n^2 p_T^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon}}{\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}. \tag{A.22}$$

Similarly, we derive for country 2:

$$c_1^2 = \frac{X^2}{\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}, \quad c_2^2 = \left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon}\frac{X^2}{\left[n^1 + n^2\left(\frac{p_T^1}{p_t^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}. \tag{A.23}$$

We now invoke the constraint that the produced quantity of a good from country 1 (x^1) equals the consumed quantity ($c_1^1 + c_1^2$) so we get

$$c_1^1 + c_1^2 = x^1 = \frac{X^1 + X^2}{\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}, \tag{A.24}$$

and

$$c_2^1 + c_2^2 = x^2 = \left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon}\frac{X^1 + X^2}{\left[n^1 + n^2\left(\frac{p_T^1}{p_T^2}\right)^{\varepsilon-1}\right]^{\varepsilon/(\varepsilon-1)}}, \tag{A.25}$$

so we can derive

$$\frac{x^1}{x^2} = \left(\frac{p_T^1}{p_T^2}\right)^{-\varepsilon}. \tag{A.26}$$

Furthermore, we can derive

$$x^1 = \frac{X^1\boldsymbol{P}_X^1 + X^2\boldsymbol{P}_X^2}{n^1\left(p_T^1\right)^{1-\varepsilon} + n^2\left(p_T^2\right)^{1-\varepsilon}}\left(p_T^1\right)^{-\varepsilon} \quad \text{and} \quad x^2 = \frac{X^1\boldsymbol{P}_X^1 + X^2\boldsymbol{P}_X^2}{n^1\left(p_T^1\right)^{1-\varepsilon} + n^2\left(p_T^2\right)^{1-\varepsilon}}\left(p_T^2\right)^{-\varepsilon}. \tag{A.27}$$

Producer behaviour

High-tech firms compete monopolistically. Each firm, producing a unique brand of the high-tech good, is assumed to maximize its present discounted value

$$\max_{L_{ri}, L_{xi}, w_{Ti}} \int_0^\infty \left[x_{it} p_{Tit} - (L_{xit} + L_{rit} + L_{fi}) w_{Ti}\right] e^{-rt} dt, \quad \text{subject to} \tag{A.28}$$

$$x_i = h_i e_i L_{xi}, \tag{A.29}$$

$$e_i = -a + c \left[\frac{w_{Ti}}{w_N}\right]^\gamma, \tag{A.30}$$

$$\dot{h}_i = z_i e_i h_i L_{ri}, \tag{A.31}$$

$$x_i = \Omega_i p_{Ti}^{-\varepsilon}, \tag{A.32}$$

$$F_i = L_{fi} e_i. \tag{A.33}$$

The ‘current value' Hamiltonian corresponding to this optimization problem is

$$H = x_{it} p_{Tit} - \left(L_{xit} + L_{rit} + \frac{F_i}{e_i}\right) w_{Ti} + p_{hit}\, \xi\, e_i h_i L_{ri}, \tag{A.34}$$

where p_{hi} is the shadow price of the level of technology h_i. The FOC of this maximization problem are

$$\begin{aligned}\frac{\partial H}{\partial w_{Ti}} &= \frac{\partial x_i}{\partial e_i}\frac{\partial e_i}{\partial w_{Ti}} p_{xi} + \frac{\partial p_{Ti}}{\partial T_i}\frac{\partial T_i}{\partial e_i}\frac{\partial e_i}{\partial w_{Ti}} x_i - \left(L_{xi} + L_{ri} + \frac{F_i}{e_i}\right) + \frac{\partial e_i}{\partial w_{Ti}}\frac{F_i w_{Ti}}{e_i^2} + \frac{\partial e_i}{\partial w_{Ti}} p_{hi} z_i h_i L_{ri} \\ &= h_i L_{xi}\frac{\partial e_i}{\partial w_{Ti}} p_{Ti}\frac{\varepsilon - 1}{\varepsilon} - \left(L_{xi} + L_{ri} + \frac{F_i}{e_i}\right) + \frac{\partial e_i}{\partial w_{Ti}}\frac{F_i w_{Ti}}{e_i^2} + \frac{\partial e_i}{\partial w_{Ti}} p_{hi} z_i h_i L_{ri} = 0,\end{aligned} \tag{A.35}$$

$$\frac{\partial H}{\partial L_{xi}} = \frac{\partial x_i}{\partial L_{xi}} p_{Ti} + x_i \frac{\partial p_{Ti}}{\partial x_i}\frac{\partial x_i}{\partial L_{xi}} - w_{Ti} = e_i h_i p_{Ti}\frac{\varepsilon - 1}{\varepsilon} - w_{Ti} = 0, \tag{A.36}$$

$$\frac{\partial H}{\partial L_{ri}} = -w_{Ti} + p_{hi} z_i e_i h_i = 0, \quad \text{and} \tag{A.37}$$

$$r p_{hi} = \dot{p}_{hi} + \frac{\partial H}{\partial h_i} = \dot{p}_{hi} + \frac{\partial x_i}{\partial h_i} p_{Ti} + \frac{\partial p_{Ti}}{\partial x_i}\frac{\partial x_i}{\partial h_i} x_i + p_{hi} z_i e_i L_{ri}. \tag{A.38}$$

We now invoke the symmetry assumption. From equation (A.36) it follows that firms engage in mark-up pricing

$$p_T = \frac{\varepsilon}{\varepsilon - 1}\frac{w_T}{he} . \tag{A.39}$$

Equation (A.37) yields the optimal R&D input

$$w_T = p_h zeh . \tag{A.40}$$

Equation (A.38) is the dynamic equation governing the allocation of high-tech labour over time. Using equation (A.36) and (A.37) and rewriting yields

$$r = zeL_r + eL_x \frac{p_T}{p_h}\frac{\varepsilon - 1}{\varepsilon} + \frac{\dot{p}_h}{p_h} . \tag{A.41}$$

Finally, substituting equations (A.36) and (A.37) in equation (A.35) we get the Solow conditon

$$\frac{\partial e}{\partial w_T}\frac{w_T}{e} = 1 . \tag{A.42}$$

Using the Solow condition and the efficiency wage relation (A.30), we derive the equilibrium relative wage and the equilibrium level of effort as

$$\frac{w_T}{w_N} = \left(\frac{a}{(1-\gamma)c}\right)^{\frac{1}{\gamma}} \quad \text{and} \quad e = \frac{a\gamma}{1-\gamma} . \tag{A.43}$$

The traditional sector exhibits unitary labour productivity and perfect competition prevails

$$y = L_N , \tag{A.44}$$

$$p_N = w_N . \tag{A.45}$$

Labour market equilibrium

We introduce three value functions (Bellman equations), V_N, V_U, and V_T, denoting the present discounted value of expected income streams of a worker in the traditional sector, an unemployed person, and a worker in the high-tech sector, respectively. The worker in the traditional sector enjoys a wage rate of w_N from working and he expects in unit time to get a job in the high-tech sector with probability αq, which gives him a surplus of V_T-V_N over his current position. V_N thus satisfies

$$rV_N = w_N + \alpha q(V_T - V_N) , \tag{A.46}$$

where rV_N, in a perfect capital market, is the valuation put on having a job in the traditional sector. Similarly, we derive

$$rV_U = bw_N + q(V_T - V_U) , \tag{A.47}$$

and

$$rV_T \ = \ w_x + \delta\eta\left(V_N - V_T\right) + \delta(1-\eta)\left(V_U - V_T\right). \tag{A.48}$$

For equilibrium to hold, we require throughout that the value of a job in the traditional sector equals the value of being unemployed

$$V_N \ = \ V_U\,. \tag{A.49}$$

We impose two flow-equilibrium conditions, guaranteeing a constant allocation of labour over the three states

$$\delta\eta L_T \ = \ \alpha q L_N\,, \quad \text{and} \tag{A.50}$$

$$\delta(1-\eta)L_T \ = \ qU\,, \quad \text{where} \tag{A.51}$$

$$L_T \ = \ n(L_x + L_r + L_f)\,. \tag{A.52}$$

Finally, we impose a stock-equilibrium condition

$$L \ = \ L_T + L_N + U\,, \tag{A.53}$$

so total labour supply (L) is either employed in one of the two sectors or unemployed. Using the equations (A.46)-(A.53), we can derive the outflow rate out of unemployment and the number of unemployed as a function of r, w_T, w_N, and the parameters b, α, and δ

$$U \ = \ \frac{1}{1-\alpha}\left[\left(\frac{\delta}{q} + \alpha\right)L_T - \alpha L\right] \quad \text{and} \quad q \ = \ \frac{(1-b)(\delta + r)}{\frac{w_T}{w_N}(1-\alpha) - (1-\alpha b)}\,. \tag{A.54}$$

The steady-state solution of the model

The model is solved under the assumption that excess profits are zero

$$\pi \ = \ xp_T - (L_x + L_r + L_f)w_T \ = \ 0\,.$$

Using the mark-up relation (A.39) and the production function of high-tech firms (A.29), this can be rewritten as

$$\frac{\varepsilon}{\varepsilon - 1} \ = \ \frac{L_x + L_r + L_f}{L_x} \tag{A.56}$$

A steady state is a situation in which relative productivity, the growth rates in the two countries, the allocation of labour and the number of high-tech firms are constant. In the steady state it holds that

$$g^1 \equiv g^2 \equiv g \equiv \frac{\dot{h}}{h} = \frac{\dot{x}}{x} = \frac{\dot{X}}{X} \quad \text{and} \quad \frac{\dot{y}}{y} = 0. \tag{A.57}$$

In addition, from equations (A.8) and (A.14), it can be derived that the steady-state circular flow is characterized by

$$g = \frac{1}{\sigma}\left(\frac{\dot{p}_N}{p_N} - \frac{\dot{P}_C}{P_C}\right) = \frac{1}{\sigma}\frac{\dot{C}}{C}. \tag{A.58}$$

Using these expressions, and taking the price of the non-tradeable good in country 1 as numeraire (p_N^1=1) we can rewrite the (steady-state) Ramsey rule (A.7) as

$$g = \frac{r - \theta}{\sigma(\rho - 1)}. \tag{A.59}$$

By the choice of the numeraire, the wages in both countries are constant in the steady state. So both the price of high-tech goods (p_T) and the shadow price of investment in R&D (p_h) decline at the rate g. Using this in combination with the rate of growth (A.31), and combining (A.39)-(A.41) we can derive $L_x=r/\xi e$, and $L_r=g/\xi e$. Substituting these expressions into the zero-profit condition (A.56) and rewriting yields the planned rate of growth as

$$r = (\varepsilon - 1)(g + zeL_f). \tag{A.60}$$

Confronting the Ramsey rule with the planned rate of growth gives the steady-state solution for the savings-investment equilibrium according to which[10]

$$g = \frac{(\varepsilon-1)zeL_f - \theta}{\sigma(\rho-1)-(\varepsilon-1)} \quad \text{and} \quad r = \frac{(\varepsilon-1)\left[\sigma(\rho-1)zeL_f - \theta\right]}{\sigma(\rho-1)-(\varepsilon-1)}. \tag{A.61}$$

As relative productivity is constant in the steady state (implying $g^1=g^2$), we can derive the steady-state relative productivity level d^* as

$$d^* = \left(\frac{e^2 L_f^2}{e^1 L_f^1}\right)^{\frac{1}{\mu}}. \tag{A.62}$$

Finally, we have to determine the steady-state allocation of labour. The size of high-tech firms immediately follows from the zero-profit condition (A.56) and the solution of the equilibrium interest rate (A.61)

$$L_x + L_r + L_f = \frac{\varepsilon}{\varepsilon-1}L_x = \frac{\varepsilon}{\varepsilon-1}\frac{r}{\xi e} = \frac{\varepsilon\left[\frac{\sigma(\rho-1)zL_f}{\xi} - \frac{\theta}{\xi e}\right]}{\sigma(\rho-1)-(\varepsilon-1)}. \tag{A.63}$$

Using (A.13), (A.29), (A.39), (A.44), (A.45), and $L_x=r/\xi e$, we can derive

[10] Stability of the model requires $\sigma(\rho\text{-}1)>\varepsilon\text{-}1$. An economically meaningful steady state equilibrium is characterized by positive growth and interest rates for which it should hold that $(\varepsilon\text{-}1)zeL_f>\theta$.

$$L_N = y = \frac{XP_X}{p_N}\frac{1-\sigma}{\sigma} = \frac{nxp_T}{p_N}\frac{1-\sigma}{\sigma} = \frac{nr}{\xi e}\frac{w_T}{w_N}\frac{\varepsilon}{\varepsilon-1}\frac{1-\sigma}{\sigma}. \tag{A.64}$$

Substituting (A.52), (A.54), and (A.64) into (A.53), and using the solution for the firm size (A.63), we can solve for the equilibrium number of high-tech firms

$$n = \frac{\frac{\varepsilon}{\varepsilon-1} - (1+\beta)}{\frac{\varepsilon}{\varepsilon-1}\left[1 + \frac{\delta\left[\frac{w_T}{w_N}(1-\alpha) - (1-\alpha b)\right]}{(1-b)(\delta+r)} + \frac{1-\sigma}{\sigma}\frac{w_T}{w_N}(1-\alpha)\right]}\frac{L}{L_f}, \tag{A.65}$$

where $\beta \equiv L_r/L_x = g/r$ is derived using (A.61), w_T/w_N is given by (A.43), and the equilibrium interest rate by (A.61). From this solution for n, we can solve for the equilibrium levels of employment L_T and L_N, and for the number of unemployed U.

The ratio L_f/L_x ('the bureaucratic burden') equals

$$\frac{L_f}{L_x} = \frac{F/e}{r/\xi e} = \frac{\xi F}{r} = \frac{\xi F[\sigma(\rho-1)-(\varepsilon-1)]}{(\varepsilon-1)[\sigma(\rho-1)zF-\theta]}. \tag{A.66}$$

It immediately follows that $\partial(L_f/L_x)/\partial F<0$.