# Catching-up and Regulation in a Two-Sector Small Open Economy

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# Catching-up and regulation in a two-sector Small Open Economy

by

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**Summary**

Emerging economies may grow fast because there is a potential for catching-up. In this paper foreign knowledge spillovers raise productivity of R&D in the exposed sector. The higher the productivity gap the more profitable investment in R&D will be. As a result labour productivity in the production department of the exposed sector rises fast. The sheltered sector realizes no productivity increases but gains because the terms of trade rise in favour of non-tradables. In the long run the rate of growth of the economy converges to the exogenous world rate. Capital mobility speeds up the process of convergence at the expense of accumulating foreign debt. Moreover, temporary shocks have long lasting effects as the economy exhibits hysteresis in case of perfect capital mobility.
Regulation in the sheltered sector induces mark-up pricing and a decline in the demand for non-tradables. The market for tradables expands and it becomes more profitable to invest in R&D. Regulation of this kind as is often practised in developing countries thus enhances economic growth. Again the results differ whether or not capital mobility is assumed. Maintaining equilibrium on the balance of trade leads to a higher long-run aggregate consumption level, which may explain the often observed reluctance to introduce free international capital mobility.

Keywords: catching-up, regulation, convergence, productivity, growth, small open economy, capital mobility, hysteresis.
JEL codes: F43, O41, L51.
ERN fields: macroeconomics, international trade.

## 1. Introduction

In neoclassical growth theory emerging economies are characterized by a relatively low capital intensity. The scarcity of capital implies a high rate of return on investment and a corresponding high rate of growth. Alternatively, one could assume that emerging economies lack the knowledge to produce at the same level as developed economies. Knowledge can of course be imitated and the relevance of international knowledge spillovers is wel documented (e.g. Coe and Helpman, 1995; Coe, Helpman and Hofmaister, 1995). However, to absorb such knowledge spillovers it may be necessary to develop a firm-specific or tacit knowledge base. Emerging economies may then be characterized by a technical disadvantage, which can be overcome by investing sufficiently in R&D. Therefore, emerging economies may grow fast because there is potential for "catching-up".

In this paper we analyse catching up in in the context of a small open economy with two sectors. The exposed or tradables sector consists of a number of specialized producers which have to compete in the international market by setting prices and investing in R&D. The economy is relatively backward because firms in the exposed sector stay behind in knowledge *vis a vis* the rest of the world. This implies two things. First, there is a potential for imitation. Second, investment in R&D in the emerging economy commands a high rate of return. Both factors contribute to a rate of growth in excess of that in the rest of the world, so that the exposed sector catches up. There is no technological change in the sheltered or non-tradables sector, but workers in this sector benefit from growth in the exposed sector as the terms of trade move in favour of non-tradables (Balassa effect).

It is assumed that the sheltered sector is subject to some form of regulation, which restricts output and increases prices. Therefore, regulation is tantamount to imperfect competition and can be conceived as a form of mark-up pricing. If output in the sheltered sector is restricted the exposed sector benefits and investment in R&D becomes more profitable. As a consequence regulation in the sheltered sector fastens growth and may for that reason be interpreted as a kind of development strategy, be it deliberate or not. Whether it is also optimal from the point of consumer welfare remains to be seen.

An important aspect of the present analysis is the prevailing regime with respect to international capital mobility. We distinguish two extreme case, i.e. balanced trade and perfect capital mobility. The results differ substantially among regimes. In case of capital mobility the emerging economy incurs debt, which has to be serviced in the long run. Policy authorities may dislike this result and opt for a restriction on the international movements of capital. Under capital mobility the economy exhibits hysteresis. Under these circumstances temporary measures will have long lasting effects. In particular, symmetric countries that are hit by asymmetric temporary shocks will not converge to the same productivity level.

It may be useful to relate our analysis to the existing literature. The engine of growth applied in the present paper is borrowed from our earlier work on endogenous growth (Smulders and Van de Klundert, 1995;  Van de Klundert en Smulders, 1997). There is a number of theoretical papers on catching-up driven by a knowledge gap but most papers assume that emerging economies learn from doing (e.g. Lucas, 1993,  Maggi, 1993; Van de Klundert and Smulders, 1996; Basu and Weil, 1996). In the present model the backward economy has a substantial potential for investing in new knowledge. In Barro and Sala-i-Martin (1995) international diffusion of knowledge requires investment by the recipient country, but there is no international trade in the model. Two-sector open economies are analysed in Turnovsky and Sen (1991, 1995). The authors discuss changes in government expenditure and supply shocks in a neoclassical world with perfect capital mobility. The Turnovsky-Sen models are less complicated than our model, because they assume perfect competition. In our analysis firms in the exposed sector compete monopolistically at home and abroad. Prices of tradables as well as prices of non-tradables have therefore to be solved.

The paper is organised as follows. In Section 2 we present the model for the dependent economy and its behavioural implications. Section 3 is devoted to an analysis of the steady state characteristics of the system. Equilibrium dynamics is discussed in Section 4, applying a linearized version of the model. Moreover, both regimes with respect to the balance of payments have to be treated separately. The dynamics in case of capital mobility turns out to be rather complicated. For this reason catching-up fenomena and regulation in the sheltered sector are discussed in more detail by presenting numerical examples in Section 5. The paper closes with some conclusions. Technicalities are delegated to a number of Appendices.

## 2. The dependent economy

### 2.1. Feasible growth

Preferences and technology are specified in Table A. Consumers trade off future consumption for present consumption according to a CRRA utility function with a measure of risk aversion $\rho$ and pure rate of time preference $\vartheta$, equation (A.1). They make a choice at every instant of time between non-tradables $C_Y$ and tradables $C_X$ according to the Cobb-Douglas specification in equation (A.2). The $X$-good in the consumption menue consists of a bundle of $N$ domestic goods and $n\bar{N}$ foreign goods. Here $n$ stands for the number of countries from which goods are imported. Variables with an upper bar relate to the outside world. The number of domestic and foreign varieties of $X$-goods is given. These goods are imperfect substitutes as shown in equation (A.3). The elasticity of substitution is constant and equals $\varepsilon > 1$.

Non-tradables are consumed or used as inputs in the production of tradables. These goods are produced by applying labour in a fixed proportion as shown in equations (A.4) and (A.5). There is no technical change in the $Y$-goods sector. Goods in the exposed sector are produced by labour and inputs from the non-tradables sector with a Cobb-Douglas technology, equation (A.6). Factor productivity in each branch of the tradables sector can be improved by employing labour for R&D, as appears from equation (A.7). Innovation builds upon a knowledge base, which is the result of in-house or firm-specific knowledge accumulated in the past ($h$) and of domestic spillovers as well as of foreign spillovers. These spillover effects are related to average knowledge levels at home ($H$) and abroad ($\bar{H}$). There are diminishing returns with respect to firm-specific knowledge ($0 < \alpha_h + \alpha_f < 1$) but constant returns with respect to the knowledge base as a whole.

Labour market equilibrium implies that the amount of labour available ($L$) equals total labour demand as expressed by the LHS of equation (A.8) Tradables are consumed at home or exported as appears from equation (A.9). Trade with the outside world is governed by alternative assumptions with respect to international capital mobility. In equations (A.10) we consider two regimes. In case of current account equilibrium exports equals imports and the LHS of (A.10) equals zero. The other case considered is perfect international capital mobility, which allows the domestic economy to accumulate foreign assets, $A$, which bear a fixed rate of interest ($r = \bar{r}$). It should be observed that prices of domestic varieties of the $X$-good are not given. Each producer of $X$-goods holds a unique position in the world economy because he or she is the sole supplier of a product variety. Therefore prices of export goods or import-competing goods can be set in the domestic economy. Prices of imported goods are of course determined abroad.

Feasible growth paths satisfy equations (A.1) - (A.10). We assume that the domestic economy is small relative to the ROW. Hence, foreign variables are determined by foreign conditions only and can be considered as exogenous variables. Preferences as well as production technology and R&D technology in the ROW are the same as in the domestic economy. In addition, it will be assumed that the ROW exhibits steady state growth with $\dot{\bar{h}}/\bar{h} \equiv \bar{g}$ constant. By choice of numeraire, prices of foreign tradables $\bar{p}_{\bar{x}i}$ are also constant in the steady state. Which path is selected by the domestic economy under these conditions depends on the behaviour of firms and consumers as well as on the regime prevailing in the international capital market.

*Table A* structure of the model

Preferences:

$$U = \int_0^\infty \frac{1}{1-\rho} C_t^{1-\rho} \exp(-\vartheta t)\, dt \tag{A.1}$$

$$\boldsymbol{C = C_X{}^{\sigma} C_Y{}^{1-\sigma}}, \qquad 0 < \sigma < 1 \tag{A.2}$$

$$C_X = \left( \sum_{i=1}^{N} c_{x_i}^{\frac{\varepsilon-1}{\varepsilon}} + \sum_{j=1}^{n\bar{N}} c_{\bar{x}_j}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > 1 \tag{A.3}$$

Technology:

$$C_Y = h_Y L_{Yc}, \tag{A.4}$$

$$\sum_{i=1}^{N} Y_{xi} = h_Y L_{Yx}, \tag{A.5}$$

$$\boldsymbol{x_i = h_i L_{xi}{}^{\gamma} Y_{xi}{}^{1-\gamma}} \tag{A.6}$$

$$\dot{h}_i = \xi \left( h_i^{1-\alpha_h-\alpha_f} H^{\alpha_h} \bar{H}^{\alpha_f} \right) L_{ri} \tag{A.7}$$

Resource constraints:

$$L_{Yc} + L_{Yx} + \sum_{i=1}^{N} \left( L_{xi} + f + L_{ri} \right) = L \tag{A.8}$$

$$c_{xi} + \bar{c}_{xi} = x_i \tag{A.9}$$

$$\left( \sum_{i=1}^{N} x_i p_{xi} - \sum_{i=1}^{N} c_{xi} p_{xi} \right) - \sum_{j=1}^{n\bar{N}} c_{\bar{x}j} \bar{p}_{\bar{x}j} = \begin{cases} 0 \\ \dot{A} - \bar{r}A \end{cases} \tag{A.10}$$

| | | | |
|---|---|---|---|
| $C$ | aggregate consumption index | $N$ | number of firms in a country |
| $C_Y$ | non-tradable consumption | $n$ | number of countries |
| $C_X$ | tradable high tech consumption index | $L$ | labour supply |
| $c_{xi}$ | consumption of high-tech good $i$ | $L_k$ | labour allocated in sector/division $k$, $k = Y, xi, ri$. |
| $Y_{xi}$ | (non-tradable) intermediates | | |
| $x_i$ | production of high-tech good $i$ | | |
| $h$ | labour productivity | $f$ | fixed labour cost |
| $H$ | average labour productivity in national high-tech sector | $A$ | Net foreign assets |

a bar denotes a foreign variable.

### 2.2 Behaviour

Consumers maximize the intemporal utility function in three stages subject to budget constraints. The three stage budgeting system is formulated in Table B. In the first stage, each consumer decides on the path of aggregate consumption over time, taking into account the accumulation of financial assets *F*.[1] The nominal interest rate *r* is exogenous in case of perfect international capital mobility. Otherwise *r* is determined endogenously by domestic savings and investment. The wage rate is denoted by *w*. Profits accruing to consumers are symbolised by Π. The second stage divides consumption over tradables and non-tradables. In the third stage, consumers decide about spending on the different varieties of the tradable good produced at home or produced abroad. The maximization procedure gives rise to the familiar Ramsey rule, equation (B.4) and demand equations for non-tradables (B.5), domestically produced tradables (B.6) and imported goods (B.7). The procedure also generates price indices for consumption (B.8) and tradables (B.9) in the domestic economy.

*Table B* Consumer behaviour

Maximization of consumer preferences (A.1), (A.2), and (A.3) subject to, respectively:

$$\dot{F}_t = r_t F_t + w_t L + \Pi - C_t P_{Ct} \qquad \text{(B.1)}$$

$$\boldsymbol{C_X P_X + C_Y P_Y = C P_C} \qquad \text{(B.2)}$$

$$C_X P_X = \sum_{i=1}^{N} c_{xi} p_{xi} + \sum_{j=1}^{n\bar{N}} c_{\bar{x}j} \bar{p}_{\bar{x}j} \qquad \text{(B.3)}$$

yields:

$$\dot{C}/C = \frac{1}{\rho}\left[ (r - \dot{P}_C / P_C) - \vartheta \right] \qquad \text{(B.4)}$$

$$\boldsymbol{C_Y P_Y = (1-\sigma)\, C P_C} \qquad \text{(B.5)}$$

$$c_{xi} = C_X \left( p_{xi} / P_X \right)^{-\varepsilon} \qquad \text{(B.6)}$$

$$c_{\bar{x}j} = C_X \left( \bar{p}_{\bar{x}j} / P_X \right)^{-\varepsilon} \qquad \text{(B.7)}$$

where

$$P_C = \left( P_X / \sigma \right)^{\sigma} \left( P_Y / (1-\sigma) \right)^{1-\sigma} \qquad \text{(B.8)}$$

$$P_X = \left( \sum_{i=1}^{N} p_{xi}^{1-\varepsilon} + \sum_{j=1}^{n\bar{N}} \bar{p}_{\bar{x}j}^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \qquad \text{(B.9)}$$

| | |
|---|---|
| $P_k$ | price (index) of good (aggregate) *k*, with *k* = *C, X, Y, xi,* $\bar{x}j$. |
| *w* | wage |
| *r* | interest rate |

[1] Financial assets include shares issued by domestic firms, domestic consumer loans and foreign assets. In the absence of capital mobility foreign assets are equal to zero.

Producer behaviour is summarized in Table C. Demand for each product variety comes from domestic and from foreign consumers as shown in equation (C.2). Producers consider total consumer demand $C_X$ and $\bar{C}_{\bar{X}}$ as well as the corresponding price indices $P_X$ and $\bar{P}_{\bar{X}}$ as given. Profit maximization therefore results in a mark-up over marginal cost which is equal to the factor $\varepsilon/(\varepsilon-1)\equiv\mu_X$, equation (C.3). The cost-minimizing factor input combination follows from equation (C.4).

The optimal R&D-strategy implies that the marginal value product of labour employed in research $p_{hi}\xi K_i$ should be equated to the marginal cost of labour ($w$), as is shown in equation (C.5). The shadow price of the knowledge base $p_{hi}$ is introduced as a Lagrangian multiplier in the maximization procedure. Firms face a trade-off with respect to investing in knowledge as appears from the no-arbitrage condition in equation (C.6). This condition says that investing a fixed amount of money in the capital market (the RHS of (C.6)) should yield the same revenue as investing that same amount of money in knowledge production. The latter raises factor productivity in commodity production and hence revenue (first term on the LHS of (C.6), it raises also the knowledge base in R&D (second term) and it yields a capital gain (last term).

*Table C* Producer behaviour.

X-sector:
$$V_i = \int_0^\infty \left[ x_{it}p_{xit} - Y_{xi}P_Y - (L_{xit} + f + L_{rit})\,w_t \right] \exp\left(-\int_0^t r(s)ds\right) dt \qquad \text{(C.1)}$$

$$x_i = C_X\left(p_{xi}/P_X\right)^{-\varepsilon} + n\bar{C}_X\left(p_{xi}/\bar{P}_X\right)^{-\varepsilon} = \Omega\ p_{xi}^{-\varepsilon} \qquad \text{(C.2)}$$

maximization of (C.1) w.r.t. $p_{xi}$, $Y_{xi}$ and $L_{ri}$, s.t. (A.6), (A.7) and (C.2) yields

$$p_{xi} = \frac{\varepsilon}{\varepsilon-1}\,\frac{1}{h_i}\left(\frac{w}{\gamma}\right)^{\gamma}\left(\frac{P_Y}{1-\gamma}\right)^{1-\gamma} \qquad \text{(C.3)}$$

$$\frac{P_Y Y_{xi}}{w L_{xi}} = \frac{1-\gamma}{\gamma} \qquad \text{(C.4)}$$

$$p_{hi} = w\ /\ \xi K_i \qquad \text{(C.5)}$$

$$p_{xi}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{x_i}{h_i} + p_{hi}\,\xi\,(1-\alpha_h-\alpha_f)(K_i/h_i)L_{ri} + \dot{p}_{hi} = r p_{hi} \qquad \text{(C.6)}$$

where $K_i \equiv h_i^{1-\alpha_h-\alpha_f}\ H^{\alpha_h}\ \bar{H}^{\alpha_f}$

Y-sector: $\boldsymbol{P_Y = \mu_Y\, w\ /\ h_Y}$ (C.7)

$\mu_Y$ mark-up rate in the Y-sector

The product in the non-tradable Y-sector is homogeneous, but regulation of competition may limit the number of competitors. Oligopolistic behaviour may then induce strategic behaviour. However, to simplify the analysis it is assumed that producers of non-tradables apply a fixed mark-up factor ($\mu_Y$) over marginal cost, equation (C.7).

Throughout the paper we assume that domestic firms are symmetric. Hence we may drop all subscripts $i$ and $j$ (for all $i$ we have $h_i$=$h$, $p_{xi}$=$p$, etc.) .

## 3. The steady state

The model can be conveniently reduced to a number of key relationships. Appendix II derives six semi-reduced forms which can be interpreted as the savings decision, investment decision, labour market equilibrium, equilibrium in the markets for tradables and for non-tradables, balance of payment equilibrium and the balance of payment regime (see table F). As shown in Appendix III, the semi-reduced model easily reveals the steady state conditions which are further discussed in this section.

*Table D* Key relationships in the steady state

Consumption decision (Ramsey):

$$r = \vartheta + (\rho\sigma + 1 - \sigma)\bar{g} = \bar{r}. \qquad \text{(D.1)}$$

Investment decision:

$$\xi\,(h/\bar{h})^{-\alpha_f} L_x/\gamma = r - (1-\alpha)\bar{g}. \qquad \text{(D.2)}$$

Labour market equilibrium:

$$\bar{g} = \xi\,(h/\bar{h})^{-\alpha_f}\left[\frac{L}{N} - f - \frac{L_{YC}}{N} - \left(\frac{1-\gamma+\gamma\mu_Y}{\mu_Y}\right)\frac{L_x}{\gamma}\right]. \qquad \text{(D.3)}$$

Equilibrium in the market for non-tradables:

$$L_{YC} = \frac{1-\sigma}{\mu_Y}\left(\frac{CP_c}{w}\right). \qquad \text{(D.4)}$$

Balance of payments equilibrium:

$$\mu_X\left(\frac{NL_x}{\gamma}\right) - \sigma\left(\frac{CP_C}{w}\right) = (\bar{r} - \bar{g})\frac{A}{w}. \qquad \text{(D.5)}$$

Balance of payments regime:

$A$=0 (balanced trade) or $A \neq 0$ (perfect capital mobility) (D.6)

In the steady state, productivity levels in the domestic tradables sector grow at the same rate as abroad, i.e. at given rate $\bar{g}$. The allocation of labour and the knowledge gap $\bar{h}/h$ are constant. The balanced growth path can be characterized as

$$\frac{\dot{h}}{h} = \frac{\dot{x}_i}{x_i} = \frac{\dot{X}}{X} = \frac{1}{\sigma}\frac{\dot{C}}{C} = \frac{1}{1-\sigma}\frac{\dot{P}_C}{P_C} = \frac{\dot{w}}{w} = \frac{\dot{A}}{A} = \bar{g}$$

The price of imported goods is set equal to unity ($\bar{p}_{\bar{x}}$=1). The price of tradables should therefore be constant in the domestic economy ($\dot{p}_x$=0).

Table D displays five key relationships that hold in the steady state. Equation (D.1) - (D.5) can be used to find the steady state solutions in $h$ and $L_x$. Substitution of (D.1) in (D.2) results in a first equation in these variables. Substitution of $CP_C/w$ according to equation (D.5) in (D.4) and substitution of the result in equation (D.3) gives the second equation in $h$ and $L_x$. From these equations we get:

$$\frac{h}{\bar{h}} = \left( \frac{\xi\beta}{\vartheta + [\sigma(\rho-1)+\alpha_h+\alpha_f+\beta]\bar{g}} \left[ \frac{L}{N} - f - \frac{A}{N}\frac{r-g}{w}\frac{1-\sigma}{\sigma\mu_Y} \right] \right)^{1/\alpha_f}, \tag{1}$$

$$L_x = \beta\gamma \left( \frac{\vartheta + [\sigma(\rho-1)+\alpha_h+\alpha_f]g}{\vartheta + [\sigma(\rho-1)+\alpha_h+\alpha_f+\beta]g} \right) \left[ \frac{L}{N} - f - \frac{A}{N}\frac{r-g}{w}\frac{1-\sigma}{\sigma\mu_Y} \right], \tag{2}$$

where

$$\beta \equiv \frac{\sigma\mu_Y}{(1-\sigma)\mu_X + \sigma(1-\gamma+\gamma\mu_Y)}.$$

In case of current account equilibrium or balanced trade the net foreign asset position equals zero ($A$ = 0). In that case the sign of the partial derivations with respect to the parameters can easily be established

$$\frac{\partial h}{\partial N} < 0 \quad \frac{\partial h}{\partial f} < 0 \quad \frac{\partial h}{\partial \xi} > 0 \quad \frac{\partial h}{\partial \vartheta} < 0 \quad \frac{\partial h}{\partial \rho} < 0$$
$$\frac{\partial h}{\partial \sigma} > 0 \quad \frac{\partial h}{\partial \alpha_h} < 0 \quad \frac{\partial h}{\partial \alpha_f} < 0 \quad \frac{\partial h}{\partial \mu_Y} > 0 \quad \frac{\partial h}{\partial \mu_X} < 0.$$

$$\frac{\partial L_x}{\partial N} < 0 \quad \frac{\partial L_x}{\partial f} < 0 \quad \frac{\partial L_x}{\partial \xi} = 0 \quad \frac{\partial L_x}{\partial \vartheta} > 0 \quad \frac{\partial L_x}{\partial \rho} > 0$$
$$\frac{\partial L_x}{\partial \sigma} > 0 \quad \frac{\partial L_x}{\partial \alpha_h} > 0 \quad \frac{\partial L_X}{\partial \alpha_f} > 0 \quad \frac{\partial L_X}{\partial \mu_Y} > 0 \quad \frac{\partial L_X}{\partial \mu_X} < 0$$

An increase in the number of firms ($N$), an increase in fixed cost ($f$) or an improvement in R&D efficiency ($\xi$) leads to a lower productivity level in the exposed sector in relation to the productivity level abroad ($\bar{h}$). The same result is obtained if the rate of time preference

($\vartheta$), the rate of risk aversion ($\rho$) or the share of income spend on tradables ($\sigma$) is raised. Spillover effects whether of domestic origin ($\alpha_h$) or of foreign origin ($\alpha_h$) have a negatieve impact on the knowledge gap. An increase in $\mu_Y$, which can be associated with more regulation in the sheltered sector, has a positive effect on the productivity level in the exposed sector. It can be concluded that regulation in the sheltered sector helps to modernize the exposed sector. On the other hand less intensive competition in the exposed sector leading to a higher mark-up factor ($\mu_X$) induces a lower productivity level.

The signs of the derivatives with respect to employment in the production of tradables ($L_x$) help explaining the results with respect to changes in the level of knowledge in the domestic economy. An increase in $N$ reduces the size of firms in the exposed sector, which makes R&D less attractive. A rise in $f$ or $\mu_X$ leads to a reduction in the production of tradables with a similar effect on R&D. A higher time preference, an increase in risk aversion or a shift in preferences towards tradables induces a higher consumption level. In this case labour is reallocated from R&D activities towards production of tradables as well as non-tradables. Higher spillover effects have a negative impact on the incentive to invest in R&D. As a result some labour in R&D laboratories becomes redundant so that a reallocation towards production departments in both sectors becomes necessary. More regulation in the sheltered sector sets labour free for production and R&D activities in the exposed sector. Finally, it should be noted that an increase in $\xi$ has no effect on the allocation of labour. The rise in productivity comes entirely from the increase in R&D efficiency.

In case of perfect capital mobility the dependent economy may accumulate foreign assets. Differentiation of equations (1) and (2) with respect to the level of real foreign assets yields:

$$\frac{\partial h}{\partial A/w} < 0 \quad \frac{\partial L_x}{\partial A/w} < 0$$

An increase in foreign assets expressed in wage units raises capital income from abroad in real terms. Higher income boosts consumption and production of non-tradables, and crowds out domestic high-tech production and R&D. Lower investment in national research results in a larger productivity gap. Alternatively, one could say that there is a trade-off between the two assets, foreign claims and firm-specific knowledge.

Whether foreign assets will be accumulated or not depends on the history of the economy. Perfect capital mobility gives rise to path-dependency, as will be shown in the next section.

## 4. Equilibrium dynamics

To study the dynamics it is convenient to linearize the model around the steady state. The key relationships in linearized form are given in Table E. Variables with a tilde relate to percentage deviations from their steady state solutions i.e. $\tilde{x}\equiv dx/x$. The variable $\tilde{a}$ is defined as the obsolute deviation of $A/\bar{h}$ from its steady state solution, i.e. $\tilde{a}\equiv da$. The linearization procedure is explained in Appendix IV.

*Table E* Key relationships in linearized form

Consumption decision (Ramsey):

$$\dot{\tilde{C}} = \frac{1}{\rho}\left[ r\tilde{r} - (1-\sigma)\dot{\tilde{h}} + \left(\frac{1-\sigma}{\varepsilon}\right)(\dot{\tilde{L}}_x + \dot{\tilde{h}})\right]. \tag{E.1}$$

Investment decision:

$$r\tilde{r} = (\zeta L_x/\gamma)[\tilde{L}_x - \alpha_f \tilde{h}] + (1-\alpha_h)\dot{\tilde{h}} - (1/\varepsilon)(\dot{\tilde{L}}_x + \dot{\tilde{h}}). \tag{E.2}$$

Labour market equilibrium:

$$\dot{\tilde{h}} = -\left(\frac{\zeta L_{YC}}{N}\right)\tilde{L}_{YC} - \left(\frac{\zeta L_x}{\gamma}\right)\left(\frac{1-\gamma+\gamma\mu_Y}{\mu_Y}\right)\tilde{L}_x + \left(\frac{\zeta L_x}{\gamma}\right)\left(\frac{1-\gamma}{\mu_Y}\right)\tilde{\mu}_Y - (\alpha_f g)\tilde{h}. \tag{E.3}$$

Equilibrium in the (world) market for tradables:

$$\tilde{p} = \frac{1-\gamma}{\varepsilon}(\tilde{\mu}_Y - \tilde{h}_Y) - \frac{1}{\varepsilon}(\tilde{h} + \tilde{L}_x). \tag{E.4}$$

Equilibrium in the market for non-tradables:

$$\tilde{L}_{YC} = \tilde{C} - \sigma\left(\tilde{h} - \frac{1}{\varepsilon}(\tilde{h}+\tilde{L}_x)\right) - \sigma\left(\gamma + \frac{1-\gamma}{\varepsilon}\right)(\tilde{\mu}_Y - \tilde{h}_Y) - \tilde{h}_Y. \tag{E.5}$$

Balance of payments equilibrium:

$$\dot{\tilde{a}} = (\bar{r}-\bar{g})\tilde{a} + \left(\frac{Nxp}{\bar{h}}\right)(1-\varepsilon)\tilde{p} - \left(\frac{\sigma CP_C}{\bar{h}}\right)\left[\tilde{C} + (1-\sigma)\left(\tilde{p} + \tilde{h} + \gamma(\tilde{\mu}_Y - \tilde{h}_Y)\right)\right]. \tag{E.6}$$

Balance of payments regime:

$$\dot{\tilde{a}} = \tilde{a} = 0 \text{ (balanced trade) or } \tilde{r} = 0 \text{ (perfect capital mobility)} \tag{E.7}$$

Note: we have assumed $s\rightarrow 0$ (small country assumption) and defined $\zeta\equiv\xi(h/\bar{h})^{-\alpha f}$, $\tilde{a}$=d($A/\bar{h}$).

### 4.1 Balanced trade

In case of equilibrium on the current account equation (E.6) can be simplified by setting $\dot{\tilde{a}} = \tilde{a} = 0$. The equations (E.1) - (E.6) can then be applied to derive a set of two linear differential equations in the level of knowledge ($h$) and production labour in the exposed sector ($L_x$). Equations (D.4) and (D.5) (which are valid also outside the steady state) can be used to arrive at

$$\frac{L_{YC}}{N} = \frac{1-\sigma}{\sigma}\,\frac{\mu_X}{\mu_Y}\,\frac{L_x}{\gamma}. \tag{3}$$

Linearization of equation (3) implies

$$\tilde{L}_{YC} = \tilde{L}_x - \tilde{\mu}_Y. \tag{4}$$

Substitution of equation (4) in equation (E.3) taking account of the definition of β results in the first differential equation:

$$\dot{\tilde{h}} = -(\alpha_f g)\tilde{h} - \left(\frac{\zeta L_x}{\beta\gamma}\right)\tilde{L}_x + \left(\frac{\zeta L_x(1-\beta\gamma)}{\beta\gamma}\right)\tilde{\mu}_Y. \tag{5}$$

Substitution of equation (4) in equation (E.5) gives an expression for $\tilde{C}$ which can be differentiated with respect to time. Applying equations (E.1) to eliminate the growth rate of consumption we arrive after some manipulation at

$$\begin{aligned}\dot{\tilde{L}}_x\left(\frac{\rho(\varepsilon-\sigma)+\sigma}{\varepsilon}\right) &= \left[\beta+\alpha_h+\sigma(\rho-1)/\mu_X\right]\left(\frac{\zeta L_x}{\beta\gamma}\right)\tilde{L}_x \\ &\quad - \alpha_f\left[\vartheta+\left(\frac{\sigma(\rho-1)}{\varepsilon}+\alpha_f\right)g\right]\tilde{h} - \left[\alpha_h+\sigma(\rho-1)/\mu_X\right]\left(\frac{\zeta L_x(1-\beta\gamma)}{\beta\gamma}\right)\tilde{\mu}_Y. \end{aligned} \tag{6}$$

From equations (5) and (6) we see that the $\dot{\tilde{h}}=0$ locus slopes downward and that the $\dot{\tilde{L}}_x=0$ locus slopes upward. Moreover, it can be easily checked that the determinant of the matrix of coefficients with respect to $\tilde{h}$ and $\tilde{L}_x$ is negative. Therefore, the model is saddlepoint stable. The phase diagram is shown in Figure 1a. The broken line represents stable arm of the saddlepath which is the upward sloping as appears from Figure 1a. For an initial value of the level of knowledge, $h(0)$, the amount of labour allocated to the production of tradables, $L_x$, jumps to the stable arm. Thereafter both variables $h$ and $L_x$ adjust towards the steady state as indicated in Figure 1a. The speed of adjustment is determined by the negative eigenvalue of the matrix of coefficients with respect to the state variables in equations (5) and (6).

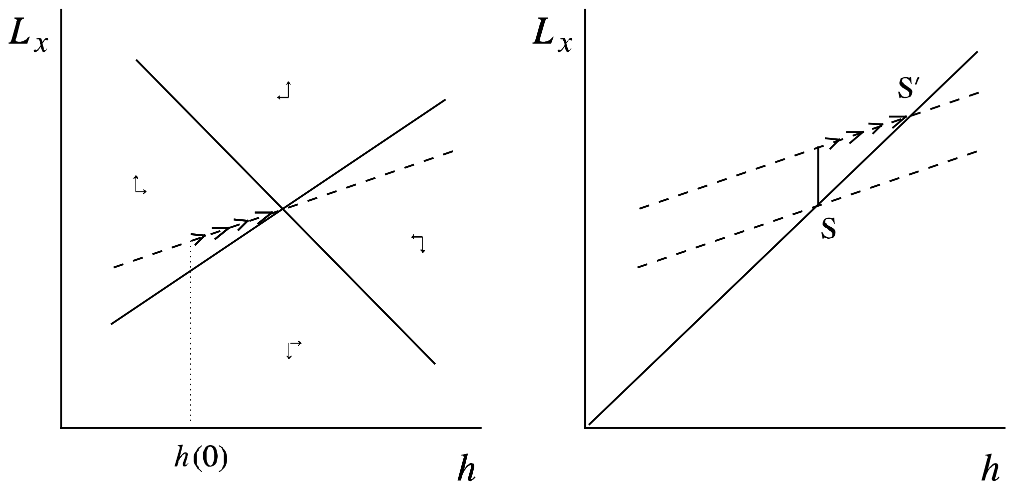


Figure 1b
Saddlepath stability

Figure 1b
More regulation sheltered sector

More regulation in the sheltered sector goes along with an increase in $\mu_Y$. This leads to an upward shift of the stable arm of the saddlepath as shown in Figure 2b. From equation (E.1) it appears that the rate of interest does not change in the long run, $\tilde{r}(\infty)=0$. Substitution of this result in equation (E.2) gives:

$$\tilde{L}_x(\infty) = \alpha_f \tilde{h}(\infty) \tag{7}$$

If the rate of interest is constant a rise in $L_x$, inducing a higher rate of return on investment in R&D, has to be compensated by an increase in $h$, which lowers productivity in the R&D department. Equation (7) leads to a proportional change of both variables along the ray OS in Figure 1b. The new steady state is found at point $S'$. On impact of the shock, $L_x$ jumps towards the new stable arm. From there on the economy gradually approaches the new long-run equilibrium. More regulation in the sheltered sector increases prices of nontradables relative to importables. Imports rise and domestic production has to rise in order to pay for these imports. Hence $L_x$ jumps up on impact. Larger sales stimulate incentives for innovation and firms increase R&D efforts. As productivity levels in the exposed sector rise faster, the price of domestic tradables declines and consumers shift their consumption further away from non-tradables. Whether it is optimal from a welfare point of view to increase $\mu_Y$ will be discussed in Section 4.

### 3.2 Perfect capital mobility

Under perfect international capital mobility the domestic rate of interest equals the foreign rate of interest which is constant. As a consequence: $\tilde{r}$=0. Substitution of this result in equations (E.1) and (E.2) along with substitution of equation (E.5) in equation (E.3) results in a system of four differential equations in $C$, $h$, $L_x$ and $a$. In the steady state $\dot{\tilde{C}} = \dot{\tilde{h}} = \dot{\tilde{L}}_x = \dot{\tilde{a}} = 0$, but that leaves us with three equations in four unknowns, because equation (E.1) with $\tilde{r}$=0 is always satisfied. This implies that the model exhibits hysteresis: we need to know the entire transition dynamics to solve for the steady state.

The information contained in equation (E.1) can be preserved by integration of this equation which yields:

$$\tilde{C} = \tilde{\Xi} - \left(\frac{1-\sigma}{\rho\varepsilon}\right)\left[ (\varepsilon-1)\tilde{h} - \tilde{L}_x \right]. \tag{8}$$

where $\Xi$ is the constant of integration, which can be interpreted as the permanent change in consumption (due to a shock that hits the economy) for given values of $\tilde{h}$ and $\tilde{L}_x$.

Equation (8) can be used to eliminate $L_x$ from the equations in Table E. The variable $\dot{\tilde{L}}_x$ can be eliminate by using equation (E.1). As shown in Appendix V, the model can be reduced to a system of differential equations in three state variables, which can be written in matrix notation as:

$$\begin{bmatrix} \dot{\tilde{h}} \\ \dot{\tilde{c}} \\ \dot{\tilde{a}} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & r-g \end{bmatrix} \begin{bmatrix} \tilde{h} \\ \tilde{c} \\ \tilde{a} \end{bmatrix} + \begin{bmatrix} a_{14} & \left(\frac{1-\sigma}{\sigma}\right)a_{14} & a_{16} \\ a_{24} & \left(\frac{1-\sigma}{\sigma}\right)a_{24} & -a_{22} \\ a_{34} & -a_{34} & a_{36} \end{bmatrix} \begin{bmatrix} \tilde{\mu}_Y \\ \tilde{h}_Y \\ \tilde{\Xi} \end{bmatrix} \tag{10}$$

As explained in Appendix V the permanent consumption effect $\Xi$ can be found along with the time paths of $h$, $C$ and $a$ by imposing solvability with respect the net foreign position and assuming $a(0) = 0$. Moreover, it is shown there that the dynamics of the model can be represented by a phase diagram in $\tilde{h}$ and $\tilde{C}$. The slope of the stable manifold depends on the size of spillovers and the elasticity of demand $\varepsilon$. For $\alpha_f$ larger (smaller) than $\varepsilon-1$ consumption and productivity change in the same (opposite) direction along the transition path. The intuition behind this result is given below. The relation between net foreign assets ($a$) and productivity ($h$) is negative. Intuitively, investment in domestic knowledge and investment in foreign assets are substitutes. The two-panel diagrams in Figures 2a and 2b depict the relations among the three state variables $\tilde{h}$, $\tilde{C}$, and $\tilde{a}$. The broken curve in both figures indicates the stable arm of the saddlepath with respect to $\tilde{h}$ and $\tilde{C}$.

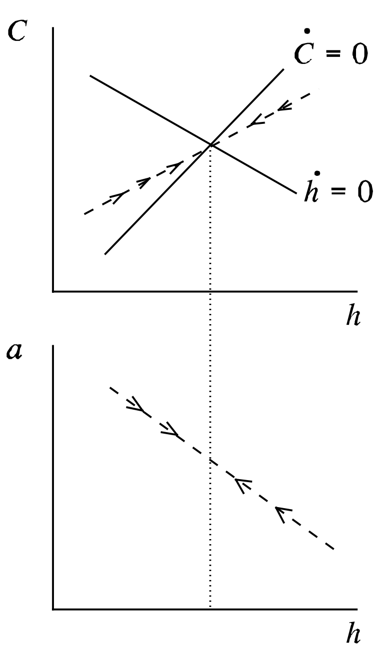


*Figure 2a*
small $\alpha_f$

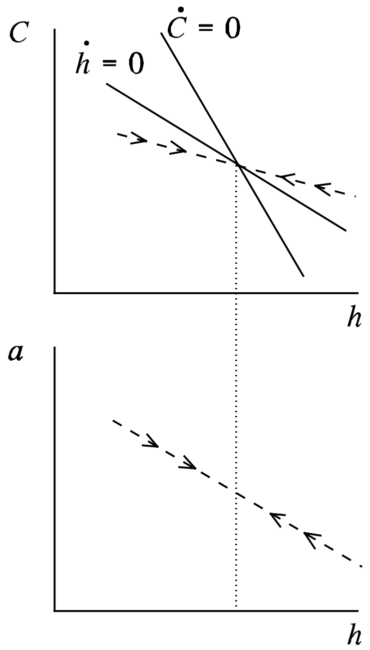


*Figure 2b*
large $\alpha_f$

In order to gain some insight in the time path of consumption, we combine equation (7) and (8), which results in:

$$\tilde{C}(0) \ - \ \tilde{C}(\infty) \ = \ \frac{1-\sigma}{\rho\,\varepsilon} \Big[ \ (\varepsilon - 1 - \alpha_f)\,\tilde{h}(\infty) \ + \ \tilde{L}_x(0) \ \Big]. \tag{9}$$

The initial jump in high tech employment ($\tilde{L}_x$) can be derived from (E.2)[2] and substituted into (9), which yields:

$$\tilde{C}(0) \ - \ \tilde{C}(\infty) \ = \ \frac{1-\sigma}{\rho} \left[ \ \eta\,\alpha_h \ + \ (1-\eta)\left( \frac{\varepsilon - 1 - \alpha_f}{\varepsilon} \right) \right] \tilde{h}(\infty), \tag{11}$$

where $\eta \equiv -\lambda/[\varepsilon(\zeta L_x/\gamma)-\lambda]$ and $\lambda<0$ is the stable root of the dynamic model.

Equation (11) shows that the timing of consumption depends crucially on the balance between the elasticity of demand for exportables ($\varepsilon$) and spillover parameters. If $\alpha_f$ is small relative to $\varepsilon-1$, consumption grows less over time, i.e. $\tilde{C}(0)>\tilde{C}(\infty)$. In other words, consumers wait to save and draw a bill on the future. The reason is that consumer

[2] Note that if the stable root of the dynamic model equals $\lambda$, for any variable $\tilde{x}$ we may write $\dot{\tilde{x}}(t) \ = \ \lambda\,[\,\tilde{x}(t) \ - \ \tilde{x}(\infty)\,]$. Using this result to eliminate $\dot{\tilde{h}}$ and $\dot{\tilde{L}}_x$ in (E.4), and subsequently substituting $\tilde{r}$=0, $\tilde{h}(0)$=0 and equation (7), we find $\tilde{L}_x(0)=\eta[\varepsilon\alpha_h-(\varepsilon-1-\alpha_f)]\tilde{h}(\infty)$, where $\eta$ is defined as in the main text.

prices tend to rise over time. Note that only prices of nontradables matter for the consumer price index, since the share of home-produced tradables in total tradables consumption of the small open economy is negligible. Prices of non-tradables depend on wage costs. Higher productivity in the exposed sector drives up wage costs for the sheltered sector. Price increases in this sector fuel consumer price inflation[3] which makes it attractive to consume now rather than in future. However, if $\alpha_f$ is large, a counterforce becomes important. Employment and production in the tradables sector will increase substantially in response to productivity changes (see (7)), which lowers the price of tradables. Consequently, wage costs fall, and the consumer price index falls. Note that the price fall in response to a productivity improvement is steeper the lower the price elasticity $\varepsilon$ is. Hence, if $\alpha_f$ is large relative to $\varepsilon$, consumption levels will rise over time (more than in the initial steady state), and if $\alpha_f$ is small relative to $\varepsilon$, consumption will rise less over time.

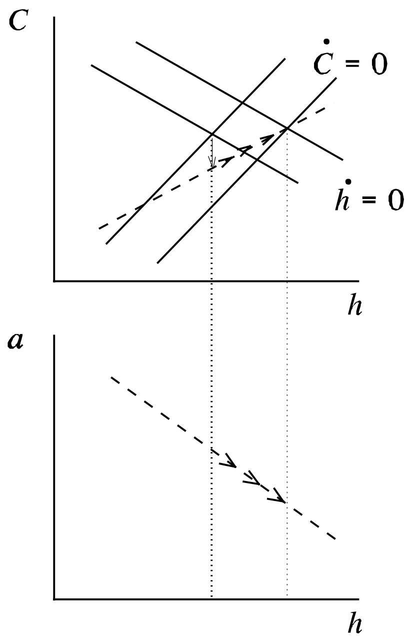


*Figure 3*
Increase in $\mu_Y$

The effects of an increase in $\mu_Y$ are illustrated in Figure 3 for the case $\alpha_f > \varepsilon - 1$. The $\dot{h} = 0$ locus shifts upward and the $\dot{C} = 0$ locus shifts downward. These shifts imply a downward movement of the stable arm of the saddlepath. On impact of the shock consumption declines, but as productivity in the exposed sector improves consumption goes up again. In the long run productivity has risen, but consumption still stays behind.

[3] This can be clearly seen by inspecting equations (AA.14)-(AA.16) in the Appendix.

The amount of foreign assets declines, because consumers incur foreign debt to smooth consumption over time.[4]

The picture shown in Figure 3 is merely an illustration, because the shift of the $\dot{\tilde{h}} = 0$ locus and the $\dot{\tilde{C}} = 0$ locus cannot be analytically determined in a meaningful way. To get a better understanding of the implications of a change in $\mu_y$ numerical examples may therefore be useful.

## 4. Numerical examples

Although in case of balanced trade the model can be solved analytically it is instructive to compare numerical results under both regimes with respect ot the balance of payments. The parameter values for the reference path are equal to:[5]

$\rho=2$, $\vartheta=0.03$, $\sigma=0.8$, $\varepsilon=2.5$, $\alpha_h=0.5$, $\alpha_f=0.4$, $\xi=0.01$, $\gamma=0.8$, $\mu_Y=1.1$, $h_Y=1$, $f=2.2$, $L=100$, $n=25$, $N=\bar{N}=8$, $\bar{g}=1.877\%$.

Because the time paths of the variables are monotonic it is sufficient to present results for the periods $t = 0$ and $t = \infty$.

### 4.1 Catching up

Catching up occurs when the level of knowledge in relation to the foreign level lies below the steady state values. In a linearized version of the model this boils down to a negative deviation of $h$ from its (future) steady state level. The outcomes in case the level of knowledge in the domestic economy lies 1% below its steady state value under balanced trade are given in Table 1.

Under balanced trade the system converges to a unique steady state. Catching-up implies a rise in the knowledge level and a temporary higher rate of growth of total factor productivity in the exposed sector. As a result of this process consumption of tradables and non-tradables rises. The initial rate of interest lies above the world level. This makes it attractive for firms to employ a large share of employment in R&D initially. As the level of knowledge approaches its steady state level the rate of interest converges towards the level prevailing in the rest of the world. In the steady state both rates of interest

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4 The stable arm of the saddle path shown in the lower panel of Figure 3 does not shift, because $\tilde{a}(0)=0$. See also Turnovsky and Sen (1991).

5 In our numerical example, the rest of the world is characterized by exactly the same parameters as the domestic economy. Hence, we linearize around a steady state with $h/\bar{h}=1$. Note that by setting $h/\bar{h}=1$ and $A=0$ in equation (1), we find $\bar{g}$. The ROW can be considered as a closed economy with endogenous growth as in Smulders and van de Klundert (1995).

should be equal because the rates of growth are then equal and consumer preference are assumed to be the same everywhere, equation (D.1).

*Table 1* Catching up ($\tilde{h}$=−1)

| Case | Balanced Trade | | Capital Mobility | |
|---|---|---|---|---|
| Period<br>Variable | t = 0 | $t \to \infty$ | $t$ = 0 | $t \to \infty$ |
| Price tradables $\tilde{p}$ | 0.440 | 0.000 | 0.550 | −0.248 |
| Productivity $\tilde{h}$ | −1.000 | 0.000 | −1.000 | 0.444 |
| Growth rate $\tilde{g}$ | 0.850 | 0.000 | 1.622 | 0.000 |
| Consumption $\tilde{C}$ | −0.562 | 0.000 | −0.386 | −0.438 |
| Tradables cons. $\tilde{C}_X$ | −0.677 | 0.000 | −0.480 | −0.397 |
| Non-tradables cons. $\tilde{C}_Y$ | −0.100 | 0.000 | −0.009 | −0.602 |
| Labour tradables $\tilde{L}_x$ | −0.100 | 0.000 | −0.374 | 0.175 |
| Rate of interest $\tilde{r}$ | 0.416 | 0.000 | 0.000 | 0.000 |
| Foreign assets $\tilde{a}$ | - | - | 0.000 | −5.102 |
| Rate of convergence (%) | 1.596 | | 2.109 | |

In case of perfect international capital mobility there is an inflow of foreign capital to equate interest rates in every period. Consumers as a group are in a position to borrow abroad in order to smooth consumption over time. The preference for current consumption implies that consumption declines over time in contrast with the case of balanced trade. In the long run consumers have to service foreign debt, incurred in the course of the transition to a steady state. It should be noted that the initial consumption of non-tradables is relatively high, but so is the consumption of tradables with a relatively higher import share. The accumulation of knowledge goes beyond the steady state level attained under balanced trade. The reason is a perfectly elastic of savings, which makes the accumulation of knowledge easier.

The steady state is not unique. If the economy starts at a level of knowledge which is 2% below the value attained in the steady state under balanced trade all results for $t = \infty$ in case of capital mobility double. Therefore, productivity levels are path-dependent and convergence of countries in terms of productivity levels is arbitrary to some extent. This result flies in the face of the standard views on convergence in neoclassical theory. Most of this literature is based on the notion of a closed economy (e.g. Barro and Sala-i-Martin, 1995). If capital mobility is assumed in a neoclassical context the economy jumps instantaneously to its unique steady state levels. Physical capital is imported so as to equate the marginal product of capital with the going rate of interest on world capital markets. Partial international mobility of capital in the sense that physical capital can be used as collateral for international borrowing, but human capital can not gives standard neoclassical results with the economy adjusting gradually to its unique long-run equilibrium (cf. Barro, Mankiw, Sala-i-Martin 1992).

Path-dependency of consumption levels is a well-known feature of small open economies. In response to adverse temporary domestic shocks, foreign borrowing allows consumers to mitigate the short-run fall in consumption, at the cost of higher debt service and lower long-run consumption. In a one-sector economy, the accumulation of capital is not affected as long as the supply of other factors of production (labour) remains the same so that the physical marginal productivity of capital is not affected (cf. Blanchard and Fisher, 1989, p.66). However, if the path-dependent change in consumption affects both tradable and non-tradable production, as in our model, the allocation of labour over the two sectors is permanently affected. Then also the incentives to accumulate capital in the different sectors of the economy are permanently affected which results in a long-run change in the domestic capital stock (see in this connection also Turnovsky and Sen, 1991 and 1995).

**4.2. Increasing regulation**

The results of a one percent permanent rise in $\mu_Y$ are presented in Table 2. An increase of the mark-up factor in the sheltered sector induces a fall in output and employment on impact of the shock. As a result R&D becomes more attractive. Under balanced trade the interest rate rises so that the necessary savings are generated. This implies that the level of consumption declines in the short run.

Productivity in the tradables sector rises over time. This makes it attractive to allocate more labour in the production department of the tradables sector. The additional products can be sold, because the rise in productivity allows for a reduction in prices. The increase in knowledge reduces the rate of return on investment and in the long run the rate of interest falls back to its original level as does the rate of growth. However, the long-run consumption level increases because of a rise in productivity in the exposed sector. Nevertheless, the measure of intertemporal welfare at $t = 0$ declines under impact of the weight attached to consumption in the short run.

In case of capital mobility consumption goes up in the short run and the economy runs a balance of trade deficit. The inflow of capital is sufficient to keep the rate of interest at the level in the outside world. Compared with the case of balanced trade there is now less need to increase the production of tradables at $t = 0$. As a consequence there is more labour available for R&D activities and the level of productivity in the exposed sector rises faster than under balanced trade as appears from the rate of convergence. The increase in foreign debt induces a burden in the long run. In the new steady state the economy must run a trade surplus to service debt. With a perfectly elastic supply of savings the rise in productivity is higher than under balanced trade. However, exported goods have to be sold at a lower price. Long-run consumption falls, but welfare increases from the vantage point $t = 0$.

*Table 2* Increasing regulation ($\tilde{\mu}_Y$=1)

| Case | Balanced Trade | | Capital Mobility | |
|---|---|---|---|---|
| Period / Variable | t = 0 | $t = \infty$ | $t = 0$ | $t = \infty$ |
| Price tradables $\tilde{p}$ | −0.021 | −0.392 | 0.071 | −0.601 |
| Productivity $\tilde{h}$ | 0.000 | 0.842 | 0.000 | 1.216 |
| Growth rate $\tilde{g}$ | 0.716 | 0.000 | 1.367 | 0.000 |
| Aggr. consumption $\tilde{C}$ | −0.124 | 0.350 | 0.024 | −0.020 |
| Tradables cons. $\tilde{C}_X$ | 0.032 | 0.603 | 0.198 | 0.268 |
| Non-tradables cons. $\tilde{C}_Y$ | −0.748 | −0.663 | −0.670 | −1.170 |
| Labour tradables $\tilde{L}_x$ | 0.252 | 0.337 | 0.022 | 0.486 |
| Rate of interest $\tilde{r}$ | 0.350 | 0.000 | 0.000 | 0.000 |
| Foreign assets $\tilde{a}$ | - | - | 0.000 | −4.298 |
| Welfare $\tilde{W}$ | 0.000 | 0.350 | 0.010 | −0.020 |
| Rate of convergence (%) | 1.596 | | 2.109 | |

***Welfare considerations***

Changes in welfare depend on the initial value of $\mu_Y$. This can be shown by a sensitivity analysis with respect to the mark-up factor in the sheltered sectors in the initial steady state. Figure 4 presents the results. On the vertical axis is the change in intertemporal welfare as a result of a small increase change in $\mu_Y$. Welfare is the present discounted value of consumption as of the period of the policy shock. The optimal value $\mu_Y$* for which the change in welfare equals zero is above unity in both balance of payment regimes.

Optimal mark-up rates are lower than in a closed economy without growth where welfare is not distorted if mark-up rates are equal across sectors. This would imply $\mu_Y{}^*=\mu_X=\varepsilon/(\varepsilon-1)=1.67$ in this example. In the open economy, a rise in $\mu_Y$ induces an expansion of the tradables sector and a loss in the terms of trade as more goods have to be sold abroad. Therefore the optimal mark-up rate in the non-tradables sector has to be lower than the mark-up rate prevailing in the tradables sector. Indeed, it can be proven that a value of $\mu_Y$ that equals $\mu_X(\varepsilon-1)/\varepsilon=1$ maximizes national welfare of the small open economy without growth by optimally trading off terms of trade gains and domestic price distortions.[6]

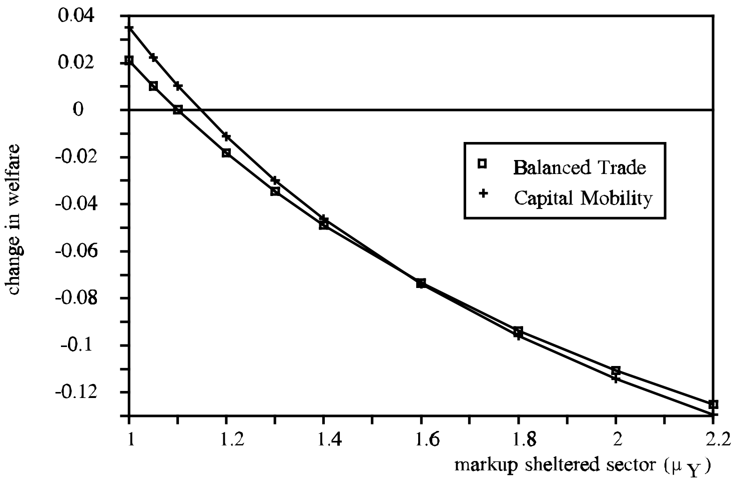


*Figure 4*
Welfare effects of regulation

Next, growth effects should be taken into account in the determination of the welfare maximizing value of $\mu_Y$. Because of knowledge spillovers, high tech firms invest too little in R&D. When $\mu_Y$ is raised, demand for high tech products increases which boosts the rate of return to innovation. Hence, higher mark-ups in the sheltered sector provide a second-best instrument to compensate the dynamic distortion in the exposed sector.[7]

Finally, the balance of payment regime plays a role. Capital mobility provides an additional reason to regulate the sheltered sector. Under capital mobility, regulating the sheltered sector is a more efficient instrument to compensate the knowledge externality than under balanced trade so that the optimal $\mu_Y$ is larger under the former regime than under the latter one. Stimulating investment is less costly when the supply of foreign savings is perfectly elastic. In contrast, when trade has to be balanced, increasing $\mu_Y$ result in temporarily high rates of interest which impose an intertemporal welfare cost. In the

[6] This result only holds if the share of domestically produced tradables is negligible in total consumption ($s \rightarrow 0$), which is the natural assumption if the country is really small and there is no "home-preference". With a larger share, we have $\mu_Y^* > 1$: production of tradables should be stimulated and their price should be lower, since prices of tradables now influence consumer prices. Intuitively, the larger $s$, the more we move to the closed economy caseand the closer $\mu_Y^*$ will be to $\mu_X$.

[7] Only domestic spillovers matter in this respect. Foreign spillovers provide a flow of knowledge like manna from heaven. From a national perspective, no policy can influence this so it is not an externality that a small open economy can internalize.

absence of national knowledge spillovers ($\alpha_h$=0), both regimes call for zero regulation ($\mu_Y$=1).[8]

Excessive regulation (policies that set $\mu_Y$ above $\mu_Y$*) bring about welfare losses. Figure 4 shows that the welfare losses are smaller in the regime of capital mobility than in the regime of balanced trade if regulation is modest. However, if the sheltered sector is heavily regulated, balanced trade performs better. First note that under capital mobility, the optimal level of regulation is higher, so that starting from modestly high $\mu_Y$ society still reaps welfare gains by regulating under capital mobility, while under balanced trade it incurs losses. However, note also that welfare is more sensitive to regulation under capital mobility than under balanced trade (compare the slope of the two curves in Figure 4). Supply of funds is perfectly elastic which allows investment to be more sensitive to changes in innovation incentives. Excess regulation creates overaccumulation of the national knowledge stock and expansion of tradables that is larger than under balanced trade. The associated larger deterioration in the terms of trade makes capital mobility perform worse.

***X-inefficiency***

Returning to regulation, it is often argued that regulation in the sheltered sector induces X-inefficiency. In our model a change in X-inefficiency can be introduced in the form of an autonomous change of labour productivity in the sheltered sector ($h_Y$). A rise in $h_Y$ leads to an immediate decline in consumption levels, but has no effect on the accumulation of knowledge. This can easily be checked in case of balanced trade. Inspection of the equations (5) and (6) reveals that there is no term in $\tilde{h}_Y$. A similar result holds in case of capital mobility as appears from numerical simulation of the system of equations in (10). Intuitively, a decline in output of non-tradables caused by a fall in labour productivity induces a proportional increase in the price of these goods. This induces a proportial decline of the demand for non-tradables. This implies that there is no need for a reallocation of labour. Consumption of tradables also declines, because producers apply less non-tradables in the production process. The welfare consequences of induced X-inefficiency, if any, have to be taken into account in a final assessment of regulation in the sheltered section.

[8] Provided again that $s \to 0$.

## 5. Conclusions

Emerging economies are able to catch up by investing in R&D. The larger the knowledge gap, the higher the rate of return and the faster growth. The international capital market can boost convergence because additional investment can be financed without inducing a rise in the interest rate. This allows consumers to take an advance on future welfare by smooting consumption over time.

Under capital mobility, temporary country-specific shocks have permanent effects on net foreign asset positions and productivity (hysteresis). Hence, countries that are symmetric in economic structure and technological and social capability may hold different net foreign asset positions which result in different total factor productivity levels. This may explain for the lack of productivity convergence in open economies (cf. Bernard and Jones 1996).

In developing countries, the sheltered sector may be operating on the basis of traditional rules and norms. As a result, the absence of perfectly competitive markets may cause mark-up rates in this sector. Deregulation would lower markup rates. However, national economic policies that deliberately abstain from reducing high markup rates in the sheltered sector of the economy may boost national productivity growth. Such a strategy may offset dynamic distortions in the economy because of knowledge spillovers, so that regulation in the sheltered sector may improve welfare.

However, when markup rates in the sheltered sector become too large, welfare losses will occur. Productivity growth in the dynamic exposed sector of the economy will be high, but at the cost of severe price distortions. Capital flows will exacerbate the overaccumulation of capital, so that welfare losses are larger under capital mobility than under balanced trade. International capital mobity aggrevate policy mistakes. National policy authorities may therefore adopt capital flow restrictions in small open economies in order to mitigate the adverse welfare effects of mistakes in regulation and other growth enhancing policies.

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## Appendix I. Solving for Price Variables

With symmetric firms within the country, all domestic firms charge the same price and consumption levels of any good from a certain country are the same. The foreign price of high tech goods is taken as the numerair.

$p_{xi} = p$ for all $i$,
$c_{xi} = c_x$ for all $i$.
$\bar{p}_{xj} = 1$ for all $j$ and all $t$.

The valueshare of home-produced high-tech goods in total high-tech consumption, $s$, can be found from (B.6) and (B.7):

$$s \equiv \frac{N c_{xi} p_{xi}}{N c_{xi} p_{xi} + n \bar{N} c_{\bar{x}j} \bar{p}_{\bar{x}j}} = \frac{1}{1 + (n\bar{N}/N) p^{\varepsilon - 1}}. \tag{AA.1}$$

The small open economy assumption implies that $N$ is negligible relative to $n\bar{N}$, so that $s \rightarrow 0$. Hence, the consumer price index $P_C$ depends on $P_Y$ and $\bar{p}$ only. We find an expression for $P_Y$ by eliminating $w$ between (C.7) and (C.5):

$$P_Y = \frac{ph}{\mu_x} \frac{(\mu_Y/h_Y)^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}. \tag{AA.2}$$

From (B.8), (AA.2), the small country assumption ($s \rightarrow 0$), and the choice of numeraire ($\bar{p}$=1 for all $t$), we find:

$$\hat{P}_C = (1-\sigma)\hat{P}_Y + \sigma[s\hat{p} + (1-s)\hat{\bar{p}}] = (1-\sigma)(\hat{h} + \hat{p}). \tag{AA.3}$$

We assume that preferences are equal and homothetic across the world. This means that any pair of goods is consumed everywhere in the same ratio as it is produced:

$$c_x / c_{\bar{x}} = \bar{c}_x / \bar{c}_{\bar{x}} = x / \bar{x} = p^{-\varepsilon}. \tag{AA.4}$$

where the last equality follows from (B.6) and (B.7).

Combining (A.6), (C.4) and (C.7), we find:

$$x = hL_x \left( \frac{1-\gamma}{\gamma} \frac{h_Y}{\mu_Y} \right)^{1-\gamma}, \tag{AA.5}$$

which holds also for foreign variables. Substitution of (AA.5) in (AA.4) gives:

$$p = \left( \frac{h}{\bar{h}} \frac{L_x}{\bar{L}_{\bar{x}}} \right)^{-1/\varepsilon} \left( \frac{\mu_Y / h_Y}{\bar{\mu}_Y / \bar{h}_Y} \right)^{(1-\gamma)/\varepsilon}, \tag{AA.6}$$

$$\hat{p} \;=\; (\bar{g} \;-\; \hat{h} \;-\; \hat{L}_x)/\varepsilon\,, \tag{AA.7}$$

where we have assumed that balanced growth prevails in the rest of the world so that $\bar{L}_x$ is constant.

**Table F. Key relationships**

Consumption decision (Ramsey):

$$\hat{C} \;=\; \frac{1}{\rho}\left[r \;-\; (1-\sigma)\hat{h} \;+\; \left(\frac{1-\sigma}{\varepsilon}\right)\left(\hat{L}_x + \hat{h} - \bar{g}\right) \;-\; \vartheta\right]. \tag{F.1}$$

Investment decision:

$$r \;=\; \xi\,(h/\bar{h})^{-\alpha_f} L_x/\gamma \;+\; (1-\alpha_h)\hat{h} \;-\; \alpha_f\bar{g} \;+\; (\bar{g}-\hat{h}-\hat{L}_x)/\varepsilon\,. \tag{F.2}$$

Labour market equilibrium:

$$\hat{h} \;=\; \xi\,(h/\bar{h})^{-\alpha_f}\left[\frac{L}{N} \;-\; f \;-\; \frac{L_{YC}}{N} \;-\; \left(\frac{1-\gamma+\gamma\mu_Y}{\mu_Y}\right)L_x/\gamma\right]. \tag{F.3}$$

Equilibrium in the (world) market for tradables:

$$p \;=\; \left(\frac{h}{\bar{h}}\;\frac{L_x}{\bar{L}_{\bar{x}}}\right)^{-1/\varepsilon}\left(\frac{\mu_Y/h_Y}{\bar{\mu}_Y/\bar{h}_Y}\right)^{(1-\gamma)/\varepsilon}. \tag{F.4}$$

Equilibrium in the market for non-tradables:

$$L_{YC} \;=\; \frac{1-\sigma}{\mu_Y}\left(\frac{CP_c}{w}\right). \tag{F.5}$$

Balance of payments equilibrium:

$$\mu_X wNL_x/\gamma \;-\; \sigma CP_c \;=\; \dot{A} \;-\; \bar{r}A\,. \tag{F.6}$$

Balance of payments regime:

$\dot{A}$=$A$=0 (balanced trade) or $r$=$\bar{r}$ (perfect capital mobility) (F.7)

Note: we assume that domestic firms are symmetric, that the rest of the world consists of identical countries, that domestic and foreign preferences and production functions symmetric, and that the foreign price is numeraire ($\bar{p}_{xi}$=1). The variable $s$ is defined as the share of home-produced high tech goods in total high tech goods consumption.

## Appendix II. Key relationships

Table F contains seven key relationships of the model in the variables $C$, $r$, $h$, $L_x$, $L_{YC}$, $p$, $P_C$, $w$, and $A$. This section of the appendix explains how to derive these semi-reduced forms. In the next sections of the appendix , we use the results (i) to characterize the steady state, and (ii) to linearize the model and study the dynamics.

We derive the Ramsey equation in (F.1) by combining (B.4), (AA.3) and (AA.7).

The equation that characterizes the investment decision is found by substitution of the appropriate equations in equation (C.6). First, divide the equation by $p_{hi}$. Eliminate $x$, $p$, and $p_{hi}$ in the first term by substituting (C.3), (AA.5) and (C.5) respectively. Eliminate $L_{ri}$ and $p_{hi}$ in the second term by substituting (A.7) and (C.5) respectively. The rate of change of the price of knowledge that appears in third term can be rewritten as $\hat{p}_h = \hat{p} + \alpha_f(\hat{h} - \bar{g})$ by differentiating (C.5), taking into account that $h$=$H$, and using (C.3) and (C.7) to eliminate $\hat{w}$. We find (F.2) when we finally substitute (AA.7) to eliminate $\hat{p}$ and introduce the definition

$$\zeta \;\equiv\; \xi K_i / h_i \;=\; \xi \left( h / \bar{h} \right)^{-\alpha_f}, \tag{AA.8}$$

where the last equality follows from the symmetry assumption.

The labour market equation in (F.3) is derived as follows. From the symmetry assumption and equations (A.5), (C.4) and (C.7), we find

$$L_{YX} \;=\; \frac{1-\gamma}{\mu_Y} \, \frac{N L_x}{\gamma}. \tag{AA.9}$$

Substitution of this result and (AA.8) into (A.8) yields (F.3).

Equation (F.4) replicates (AA.6). Equation (F.5) is found by combination of equations (B.5), (C.7) and (A.4).

In order to find the Balance of Payments equation, we substitute (B.3), (B.2) and (B.5) into (A.10), take into account symmetry, and find:

$$N x p \;-\; \sigma C P_c \;=\; \dot{A} \;-\; \bar{r} A. \tag{AA.10}$$

Eliminating $x$ and $p$, by substituting (AA.5) and (C.3) respectively, we arrive (F.6).

## Appendix III. Steady State (Table D)

The steady state is characterized (defined) by constant growth rates. Equation (F.3) implies that $L_x$, $L_{YC}$ and $\zeta \equiv \xi (h/\bar{h})^{-\alpha f}$ should be constant. It can be checked from (F.5) and (F.6) that $A/w$ should be constant. Hence, in the steady state we may write:

$$\dot{L}_x = \dot{L}_{YC} = 0, \quad \hat{h} = \hat{\bar{h}} \equiv \bar{g}, \quad \hat{A} = \hat{w}. \tag{AA.11}$$

Then it follows from (AA.7), (AA.9) and (F.3) and from (AA.3), (AA.5), (AA.2), (C.7), and (F.5) respectively that:

$$0 = \hat{p} = \hat{L}_{YX} = \hat{L}_{YC}, \tag{AA.12}$$

$$\bar{g} = \frac{1}{1-\sigma}\hat{P}_C = \hat{x} = \hat{P}_Y = \hat{w} = \frac{1}{\sigma}\hat{C}. \tag{AA.13}$$

Substituting (AA.11)-(AA.13) into the key relationships in table F, we find their steady state counterparts which are displayed in table D in the main text.

## Appendix IV. Linearization of the model (Table E)

The purpose of this section is to find the linearized version of the model in terms of the four main variables, viz. $h$, $L_x$, $c$, and $a$, and three variables of secondary interest, viz. $p$, $r$, $L_{YC}$. We introduce the variable $a \equiv A/\bar{h}\bar{p}_{xi}$ (which equals $a = A/\bar{h}$ by our choice of the numeraire) which should be interpreted as the ratio of net foreign assets to domestic assets. We choose this variable because it is a predetermined variable (note that the ratio $A/w$ which is used in section 3 is not), which allows us to exploit the condition $\tilde{a}(0)=0$.

Our linearization procedure involves taking total differentiation, where all parameters except $\mu_Y$ and $h_Y$ are considered as constants. Tilded variables are expressed as relative deviations from the initial balanced growth path, i.e. for any variable $u$, $\tilde{u} \equiv \mathrm{d}u/u$ or $\mathrm{d}u = u\tilde{u}$, so that $\mathrm{d}\hat{u} = \dot{\tilde{u}}$. An exception is $\tilde{a}$ which is defined as the absolute (rather than relative) deviation from the original growth path, i.e. $\tilde{a} \equiv \mathrm{d}a$ and $\mathrm{d}\dot{a} = \dot{\tilde{a}}$, which allows us to consider the situation in which $A=a=0$ initially (balanced trade).

Equations (E.1)-(E.4) directly follow from linearization of equations (F.1), (F.2), (F.3) and (F.4). In order to find equation (E.5), we first linearize (F.5). The resulting equation contains terms with $\tilde{P}_C - \tilde{w}$, which we want to eliminate. In order to find an expression for $\tilde{P}_C - \tilde{w}$, first note from (B.8), (B.9), the symmetry assumption, the small country assumption ($s \to 0$), and the choice of the numerair ($\bar{p}=1$) that:

$$\tilde{P}_C = (1-\sigma)\tilde{P}_Y + \sigma[s\tilde{p} + (1-s)\tilde{\bar{p}}] = (1-\sigma)\tilde{P}_Y. \tag{AA.14}$$

Linearization of (C.7) and (C.3) gives:

$$\tilde{w} = \tilde{h} + \tilde{p} - (1-\gamma)(\tilde{\mu}_Y - \tilde{h}_Y) \tag{AA.15}$$

$$\tilde{P}_Y = \tilde{w} + \tilde{\mu}_Y - \tilde{h}_Y \tag{AA.16}$$

Linearizing (F.5) and substituting (AA.14), (AA.15), (AA.16) and (E.4), we find (E.5).

In order to find a convenient Balance-of-Payments relationship, we divide (F.5) by $\bar{h}$ and rewrite the equation in terms of $a \equiv A/\bar{h}$ instead of $A$:

$$Nxp/\bar{h} \; - \; \sigma CP_C/\bar{h} \; = \; \dot{a} \; + \; \bar{g}a \; - \; \bar{r}a.$$

Linearizing this expression (recalling our definition $\tilde{a} \equiv \mathrm{d}a$), substituting (AA.14), (AA.15), and (AA.16) to eliminate $\tilde{P}_C$ and $\tilde{w}$, and substituting (E.4), we arrive at (E.6).

The six equations (E.1)-(E.6) contain seven variables (viz. $\tilde{w}$, $\tilde{r}$, $\tilde{C}$, $\tilde{L}_x$, $\tilde{h}$, $\tilde{L}_{YC}$, $\tilde{a}$). The equation that completes the system depends on which of the two Balance of Payment regimes applies. (E.7) summarizes the two regimes.

## V. Solving the dynamics under Perfect capital Mobility

*Representation of the model by three differential equations*

Substitution of (E.5) and (E.7b) into (E.1)-(E.3) and (E.6) yields a system of four differential equations in the variables $\tilde{c}$, $\tilde{L}_x$, $\tilde{h}$, and $\tilde{a}$. As explained in the main text, the dynamic system exhibits hysteresis and we need to further reduce the system. To this end we take the integral of (E.1) with $\tilde{r}$=0. It gives the expression that allows to eliminate $L_x$:

$$\tilde{L}_x \; = \; \left(\frac{\rho\varepsilon}{1-\sigma}\right)[\tilde{C} - \Xi] \; + \; (\varepsilon - 1)\tilde{h}, \qquad \text{(AA.17)}$$

where $\Xi$ is the constant of integration. Using this result to eliminate $\tilde{L}_x$ and $\dot{\tilde{L}}_x$ in (E.32), we find (for $\tilde{r}$=0):

$$\dot{\tilde{C}} \; = \; \left(\frac{\zeta L_x}{\gamma}\right)\left[\varepsilon(\tilde{C} - \Xi) \; + \; \left(\frac{(1-\sigma)(\varepsilon - 1 - \alpha_f)}{\rho}\right)\tilde{h}\right] \; - \; \frac{\alpha_h(1-\sigma)}{\rho}\dot{\tilde{h}}. \qquad \text{(AA.18)}$$

The elimination of $L_{YC}$ between (E.3) and (E.5) and substitution of (AA.17) in the resulting expression gives the expression for $\dot{\tilde{h}}$. Since we want to compare the regime of perfect capital mobility to that of balanced trade, we linearize around an initial steady state without net foreign assets. Hence we may substitute equation (3) from the main text. The resulting expression is displayed in the matrix in (10) in the main text. The coefficients are defined as:

$$a_{11} \; = \; -\left(\frac{\zeta L_x}{\gamma}\right)\left(\frac{1-\gamma+\gamma\mu_Y}{\mu_Y}\right)(\varepsilon - 1) \; - \; \alpha_f g \qquad <0$$

$$a_{12} = -\left(\frac{\zeta L_x}{\gamma}\right)\left[\left(\frac{1-\gamma+\gamma\mu_Y}{\mu_Y}\right)\left(\frac{\rho}{1-\sigma}\right)\varepsilon + \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\mu_X}{\mu_Y}\right)\left(\frac{\sigma\rho+1-\sigma}{1-\sigma}\right)\right] \qquad <0$$

$$a_{16} = \left(\frac{\zeta L_x}{\gamma}\right)\left[\left(\frac{1-\gamma+\gamma\mu_Y}{\mu_Y}\right)\left(\frac{\rho}{1-\sigma}\right)\varepsilon + \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\mu_X}{\mu_Y}\right)\left(\frac{\sigma\rho}{1-\sigma}\right)\right] \qquad >0$$

$$a_{14} = \left(\frac{\zeta L_x}{\gamma}\right)\left(\frac{(1-\sigma)\mu_X + \sigma(1-\gamma)}{\mu_Y}\right) \qquad >0$$

$$a_{21} = -\alpha_h\left(\frac{1-\sigma}{\rho}\right)a_{11} + \left(\frac{\zeta L_x}{\gamma}\right)\left(\frac{1-\sigma}{\rho}\right)(\varepsilon-1-\alpha_f) \qquad ?$$

$$a_{22} = -\alpha_h\left(\frac{1-\sigma}{\rho}\right)a_{12} + \left(\frac{\zeta L_x}{\gamma}\right)\varepsilon \qquad >0$$

$$a_{24} = -\alpha_h\left(\frac{1-\sigma}{\rho}\right)a_{14} \qquad <0$$

$$a_{31} = (Nxp/\bar{h})(\varepsilon-1) \qquad >0$$

$$a_{32} = (Nxp/\bar{h})\left[\left(\frac{\varepsilon-\sigma}{1-\sigma}\right)\rho - 1\right] \qquad >0$$

$$a_{36} = -(Nxp/\bar{h})\left(\frac{\varepsilon-\sigma}{1-\sigma}\right)\rho \qquad <0$$

$$a_{34} = -(Nxp/\bar{h})\left(\frac{(1-\sigma)\mu_X + \sigma(1-\gamma)}{\mu_X}\right) \qquad <0$$

Finally we derive a differential equation for net foreign assets *a*. Use (AA.17) to eliminate $\tilde{L}_x$ in (E.6). Under the condition of zero initial net foreign assets, this gives the third differential equation in (10).

*Solving the dynamic system*

From (10) and the initial conditions for the predetermined variables *h* and *a* we have to solve the time paths of $\tilde{h}$, $\tilde{c}$, and $\tilde{a}$, and the unknown constant Ξ.

To start with, note that the first two differential equation can be solved independently of $\tilde{a}$. In particular, we can draw a Phase diagram in a $\tilde{h}$, $\tilde{c}$ plane. Saddle-point stability applies under mild conditions (if downward sloping, the $\dot{c}$=0 locus should be less steep than the $\dot{h}$=0 locus, which is violated only for very small values of ρ). Two possibilities arise because of the ambiguous sign of $a_{21}$. If $\alpha_f$ is large, $a_{21}$ is negative, implying that the $\dot{c}$=0 locus and the stable manifold slope upward (a sufficient condition is $\alpha_f > \varepsilon - 1$). If $\alpha_f$ is small, the $\dot{c}$=0 locus and the stable manifold slope downward.

Ignoring the third differential equation, we can solve $\tilde{h}$ and $\tilde{c}$ as a function of time $t$, the unknown constant $\Xi$, and the given shocks to the system $\tilde{h}_0$, $\tilde{\mu}_Y$, and $\tilde{h}_Y$. Denoting the vector of shocks by $\tilde{z}$ and the stable (negative) root by $\lambda$, we can express the solutions as:

$$\tilde{h} = \tilde{h}(t, \Xi, \tilde{h}_0, \tilde{\mu}_Y, \tilde{h}_Y) = e^{\lambda t} \cdot \tilde{h}_0 \quad + \quad (1 - e^{\lambda t}) \cdot \tilde{h}(\infty, \Xi, \tilde{z}) \tag{AA.19}$$

$$\tilde{c} = \tilde{c}(t, \Xi, \tilde{h}_0, \tilde{\mu}_Y, \tilde{h}_Y) = e^{\lambda t} \cdot \tilde{c}(0, \Xi, \tilde{z}) \quad + \quad (1 - e^{\lambda t}) \cdot \tilde{c}(\infty, \Xi, \tilde{z}) \tag{AA.20}$$

Next we have to solve for $\tilde{a}$ and $\Xi$. To this end we use the third differential equation and two natural restrictions on the value of net foreign assets $\tilde{a}$. First, we impose the condition that the long-run value of $a$ is finite. The country has to be solvent: an exploding foreign debt ("Ponzi-game", $a \to -\infty$) should be excluded. The transversality condition rules out an infinitely large net foreign asset position ($a \to \infty$). Second, we impose that net foreign assets are a predetermined variable: $\tilde{a} = \tilde{a}_0$, where $\tilde{a}_0 = 0$.

The procedure is as follows. We substitute the solutions for $\tilde{h}$ and $\tilde{c}$ in the third differential equation. The resulting differential equation in $\tilde{a}$ and the constants $\tilde{\mu}_Y$, $\tilde{h}_Y$, and $\Xi$ can be solved. Imposing the conditions that $a$ is finite for $t \to \infty$ and that $\tilde{a}$=0 for $t$=0, we find a relation between $\tilde{h}_0$, $\tilde{a}_0$, $\tilde{\mu}_Y$, $\tilde{h}_Y$, and $\Xi$ which is the solution for $\Xi$. To simplify notation, write the third differential equation in (10) as

$$\dot{\tilde{a}} = (r - g)\tilde{a} + \tilde{x}$$

where

$$\tilde{x} \equiv a_{31}\tilde{h} + a_{32}\tilde{c} + a_{34}(\tilde{\mu}_Y - \tilde{h}_Y) + a_{36}\Xi. \tag{AA.21}$$

Substituting the solutions for $\tilde{h}$ and $\tilde{c}$, we can write:

$$\dot{\tilde{a}}(t, \Xi, \tilde{z}) = (r - g) \cdot \tilde{a}(t, \Xi, \tilde{z}) + e^{\lambda t} \cdot \left[ \tilde{x}(0, \Xi, \tilde{z}) - \tilde{x}(\infty, \Xi, \tilde{z}) \right] + \tilde{x}(\infty, \Xi, \tilde{z}). \tag{AA.22}$$

Integrating and imposing the No-Ponzi game condition, we arrive at:

$$\tilde{a}(t, \Xi; \tilde{z}) = \left[ \left( \frac{\tilde{x}(\infty, \cdot) - \tilde{x}(0, \cdot)}{r - g - \lambda} \right) - \frac{\tilde{x}(\infty, \cdot)}{r - g} \right] e^{\lambda t} + \left[ \frac{-\tilde{x}(\infty, \cdot)}{r - g} \right] (1 - e^{\lambda t}). \tag{AA.23}$$

The terms in the first brackets equals $\tilde{a}(0, \cdot)$ which should equal $\tilde{a}_0$. From this equality, $\Xi(\tilde{z})$ follows.

The lower panel in Figures 2 and 3 in the main text is found in the following way. First, find the relation between $\tilde{C}$ and $\tilde{h}$ along the stable manifold by substituting $\dot{\tilde{C}} = \lambda [\tilde{C} - \tilde{C}(\infty)]$ into the second differential equation. Next, substitute the resulting expression and the relation $\dot{\tilde{a}} = -\lambda \tilde{a}(\infty)$ in the third differential equation. This gives an expression for $\tilde{a}$ as the sum of a term multiplied by $\tilde{h}$ and a term involving constants. With $\tilde{a}_0$=0, this constant should be zero (in fact this is an alternative way to find $\Xi$). The term premultiplying $\tilde{h}$ is the slope of the stable manifold depicting the relation between $a$ and $h$. We find the following expression for this slope, which is most likely to be negative:

$$\frac{(a_{21}a_{32} - a_{22}a_{31}) + \lambda a_{31}}{(\lambda - a_{22})(\lambda - r - g)}. \tag{AA.24}$$

## VI. Welfare

Welfare at time $t$=0 [see (A.1)] can be written as:

$$W(t) = \left(\frac{1}{1-\rho}\right) C(t)^{1-\rho} \int_t^\infty e^{-R(s)} ds, \tag{AA.25}$$

where $R$ is the cumulative growth-corrected discount rate, defined as:

$$R(s) \equiv \int_0^s [\vartheta + (\rho-1)\cdot\hat{C}(\tau)] d\tau. \tag{AA.26}$$

Hence, welfare depends on the time path of consumption, captured by $C(0)$ and $\hat{C}$. Taking total differentials, solving the resulting integrals by using the fact that all variables develop monotonically with speed of adjustment $\lambda$ [see (AA.20)], and taking into account that consumption grows at rate $\sigma g$ in the steady state, we derive for welfare at $t$=0[9]:

$$\left(\frac{1}{1-\rho}\right)\frac{dW(t)}{W(t)} \equiv \tilde{W}(t) = \tilde{C}(0) + \left(\frac{\lambda}{\vartheta + (\rho-1)\sigma g - \lambda}\right)[\tilde{C}(t) - \tilde{C}(\infty)].$$

9 $\tilde{W}$ is defined as $\tilde{W} \equiv dW/[(1-\rho)W]$ so that $\tilde{W}>0$ if welfare rises [note that $W<(>)0$ if $\rho>(<)1$].