WHY MONEY TALKS AND WEALTH WHISPERS:
MONETARY UNCERTAINTY AND MYSTIQUE

Sylvester C.W. Eijffinger*,
Marco Hoeberichts**
and
Eric Schaling***

* CentER for Economic Research, Tilburg University; College of Europe
and Humboldt University of Berlin;

** CentER for Economic Research, Tilburg University, P.O. Box 90153, 5000LE Tilburg,
The Netherlands;

*** Monetary Assessment and Strategy Division, Bank of England

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Abstract
This paper analyzes the effect of monetary uncertainty on the inflationary bias and the variance of output and inflation. Monetary policy uncertainty is modeled as a shock to the central banker’s preference for inflation stabilization relative to output stabilization that cannot be observed by the public. We find that the mean and variance of inflation increase with the variance of this preference shock. However, unlike other studies, we find that monetary uncertainty may very well have a positive effect on output stabilization and therefore also on society’s welfare.

Keywords: credibility, flexibility, uncertainty

JEL Code: E52, E58

1 The authors thank Erik Canton for helpful comments.
1 Introduction

In writing about central bankers Milton Friedman once said: “From revealed preference, I suspect that by far and away the two most important variables in their loss function are avoiding accountability on the one hand and achieve public prestige on the other”. More recently, Alan Greenspan, ‘the delphic oracle of global financial markets’, speaking to Congress said: “If I’ve made myself too clear, you must have misunderstood me.”

Hence, an important feature of central bank institutions is their adherence to secrecy. The latter is generally believed to be unjustifiable on moral grounds. For instance, Mayer (1987, p.16) argues that: “If governments derive their legitimacy from the consent of the governed, this should be informed consent”. And Goodfriend (1986) reports on an intriguing case study of how a graduate law student, David R. Merrill, under the auspices of the Freedom of Information Act of 1966 filed a freedom of information request with the Board of Governors of the Federal Reserve System over its failure to disclose its minutes.

This paper investigates society’s incentives to design monetary institutions that allow for central bank secrecy. We find that the main reason for central bank secrecy is its potentially beneficial effect on stabilization policy. Unlike Lewis (1991), this result does not require the social planner’s future preferences for policy objectives to change over time with changing economic circumstances. More specifically, optimal central bank secrecy involves trading-off the harmful effects of uncertainty about monetary policy and the associated higher expected inflation, versus the potentially beneficial effects on the stabilization of output. The latter trade-off depends on the severity of society’s time-consistency problem vis-à-vis its need for stabilization policy. The analysis predicts that if the credibility problem is large relative to the need for flexibility, optimal central bank institutions will be very open and transparent and vice versa. This reverses an earlier result by Garfinkel and Oh (1995). Moreover it explains why high credibility such as the Bundesbank and the future European Central Bank (ECB), can afford to be relatively closed, and why low credibility institutions such as the Reserve Bank of New Zealand and Banco d’España need to be very open and need to publish e.g. inflation and monetary policy reports of some kind, in addition to standard bank bulletins.

The paper is organized into three remaining sections followed by four appendices. In section 2 we present the model. Section 3 contains the derivation of the relationship between central bank secrecy, i.e. monetary policy uncertainty, and social welfare. In section 4 we look at three different cases of the effects of monetary policy uncertainty. Our conclusions are given in section 5. The
appendices provide the derivations of the policy outcomes, a generalization of the model and the optimal degree of central bank secrecy.

2 The Model

In what follows, we model monetary policy uncertainty as a monetary policy game with uncertainty about the agent’s inflation stabilization preferences. We assume that there are two types of actors, wage-setters and the central bank. Wage-setters unilaterally choose the nominal wage every period, and the central bank controls monetary policy. The sequence of events is as follows. In the first stage, wage-setters sign one period nominal wage contracts [Gray(1976), Fischer (1977b)]. Wage-setters know the domestic monetary regime on average but there are random shocks to central bank preferences that cannot be observed at the time wage contracts are signed. However, they know the variance of the shocks and take this information into account in forming their expectations. In the third stage stochastic shocks to productivity realize. Similarly these shocks cannot be observed at the time contracts are negotiated. As will be shown below, the uncertainty associated with the second and third stages of the game is, respectively, of the multiplicative [Brainard (1967)] kind and the additive kind. In the fourth stage, the central bank observes the value of the productivity shock and, given its own preferences, reacts to the productivity shocks accordingly. In the fifth and final stage, output is determined by competitive firms. This timing of events is summarized in Figure 2.1.

Figure 2.1: The timing of events

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
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<th>5</th>
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<tbody>
<tr>
<td>Nominal wage contracts signed</td>
<td>Shocks to CB preferences realize</td>
<td>Productivity shocks realize</td>
<td>CB sets monetary policy</td>
<td>Output determined</td>
</tr>
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</table>

2.1 Uncertainty about inflation stabilization preferences

Adopting the specification commonly used in the literature, output is described by a reduced-form Lucas supply function:

\[ y = y^* + b(\pi - \pi^e) + \varepsilon \]  \hspace{1cm} (2.1)
where \( y \) is (the log of) output, \( y^* \) is the natural rate of output; inflation is denoted by \( \pi \), nominal wage contracts signed at time \( t-1 \) are proxied by the expected inflation rate \( \pi^e \), \( \varepsilon \) is a white noise shock to productivity with zero mean and variance \( \sigma^2 \) and \( b \) is the slope of the Phillips curve. The principal’s (society’s) loss function is given by

\[
S = \alpha \pi^2 + (y - ky^*)^2
\]  

(2.2)

where \( \alpha > 0 \) is society’s relative weight of inflation stabilization relative to output stabilization. We assume that \( k > 1 \) so that the desired level of output is above the natural level.

The agent’s (central bank’s) loss function is

\[
L = a_1 \pi^2 + (y - ky^*)^2
\]  

(2.3)

Note that in both loss functions the target rate of inflation is normalized to 0. Equation (2.4) specifies the stochastic behavior of the parameter \( a_t \),

\[
a_t = \bar{a} - x_t \quad \text{with} \quad \text{Var}[x_t] = \sigma^2_x \quad \text{and} \quad E_{t-1}[x_t] = 0
\]  

(2.4)

The wage setters expect the central bank’s preference for inflation stabilization to be \( \bar{a} \). However, at any particular point in time because of the shock \( x_t \), the central bank may be overly “conservative” or advocate too loose a monetary stance (be too “liberal”). We see the variance of the agent’s inflation stabilization preferences \( \sigma^2_x \) as monetary policy uncertainty. In the limiting case that \( \sigma^2_x = 0 \) (and \( a_t = \bar{a} \)), we are back in the original Rogoff (1985) model. Importantly, the distribution of \( x_t \) must be chosen in such a way that \( a_t \) is always positive. A negative preference weight on inflation implies a central banker who actually loves inflation and therefore will set an infinite rate of inflation. If the wage setters know that there is a positive chance of having a central banker who loves inflation, this probability, however small, will count heavily in the formation of inflationary expectations. Although there is no need to specify the distribution of \( x_t \), one could think of it to be uniformly distributed on the \([-c, c]\) interval with \( c < \bar{a} \). A normal distribution clearly doesn’t work here, because of its infinite support we can never exclude the chance that a large shock occurs and \( a_t \) becomes negative.

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2 Cukierman and Meltzer (1986) introduce persistence and dynamize their problem by letting \( a_t = \alpha - e_t \), where \( e_t = \rho e_{t-1} + x_t \) and \( 0 < \rho < 1 \). For analytical tractability we set \( \rho = 0 \).
2.2 Time-Consistent Equilibrium

For simplicity, and with no loss of generality, we assume that the control variable of the central bank is inflation. Substituting (2.1) into (2.2), the first-order conditions for a minimum indicate

\[ \pi = \frac{b}{a + b} (b\pi^e + z - \varepsilon) \tag{2.5} \]

where \( z = (k-1)y^* \). Solving for rational expectations yields

\[ \pi^e = F(.) \frac{b}{a} - z \tag{2.6} \]

where

\[ F(.) = \frac{\tilde{a}(\tilde{a} + b^2)^2 + \sigma_x^2}{\tilde{a}(\tilde{a} + b^2)^2 - b^2 \sigma_x^2} \tag{2.7} \]

This function \( F(.) \) is shown graphically in Figure 2.2 with \( b = 1 \). In order to ensure that \( F(.) > 0 \), we require that \( \sigma_x^2 < \frac{(\tilde{a} + b^2)^2}{b^2} \cdot \) Note that for \( x_t \) uniformly distributed on the interval \([-c, c] \), \( c < \tilde{a} \), this condition is always satisfied. The exact specification of \( F \) depends on the assumptions that one makes. Details can be found in Appendix B.

Solving for the same discretionary solution for expected inflation when there is no uncertainty surrounding the central bank’s preferences emphasizes the importance of (2.6). For this latter case, it is straightforward to derive

\[ \pi^e = \frac{b}{a} - z \tag{2.8} \]

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3 See Appendix A for details on the derivations in this Section.
Equations (2.6) and (2.8) indicate that inflation expectations are proportional to the output bias, $z$, a familiar conclusion in this literature. However (2.6) differs in a significant way from (2.8), as a result of the different information sets that the agents are assumed to possess. Equation (2.6) reflects the fact that agents have had to ‘guess’ about the effect of stochastic preferences on the inflation rate. And assuming that the expectations of agents “are the predictions implied by the model itself, contingent on the information economic agents are assumed to have” [Fischer (1977a)], expectations are rational. The real problem here, as we show in Appendix A, is that this involves taking expectations in the presence of nonlinearities.

From (2.7), it is clear that, if the variance is not too large, $F(.) > 0$. Moreover, as can be seen in Figure 2.2, in the limiting case that $\sigma_{\xi}^2 = 0$ and therefore $a_i = \bar{a}$, then $F(.) = 1$, and (2.6) effectively collapses to the discretionary case given by (2.8).

Using the previous results, it is straightforward to write the final solutions for output and inflation, under the case of uncertainty about inflation stabilization preferences. Respectively,
\[ \pi_t = \frac{b}{a_t + b^2} [1 + \frac{b^2}{a} F(.)] \rho - \frac{b}{a_t + b^2} \epsilon_t. \]  
\hspace{1cm} (2.9) 

And similarly for output, the solution is

\[ y_t = y^* + b(\pi - \pi^*) + \epsilon_t. \]  
\hspace{1cm} (2.10)

Figure 2.3 shows the central bank’s reaction to a preference shock \( x \) with the parameters \( \bar{a} = 1, b = 1, \rho = 0.03 \) and the productivity shock \( \epsilon = 0 \). It is clear that with uncertain preferences, expected inflation is higher than in the discretionary case without uncertainty. In that case, the central bank would set an inflation rate of 3%. Note that this result does not depend on a particular distribution of the disturbances to the agent’s preferences. Hence our result is general and follows from Jensen’s inequality\(^4\). The same picture can be drawn for the central bank’s reaction to a given productivity shock \( \epsilon \).

\[ \text{Figure 2.3: The central bank’s reaction to a preference shock } x \]

An important point arising from (2.9) is that although preference uncertainty exacerbates the existing inflation bias problem, it cannot generate an inflation bias on its own. This fairly intuitive point is clearly a function of the multiplicative nature of the problem.

\(^4\) Let \( a_t \) be a random variable with mean \( \bar{a} \), and let \( g(a_t) = \frac{b}{a_t + b^2} \) be a convex function; then \( E[g(a_t)] \geq g(\bar{a}) \).
3 The Welfare Loss and Monetary Policy Uncertainty

Using the policy outcomes that we derived in the previous section, we can now turn to the welfare analysis. Previous studies, in particular Nolan and Schaling (1996) and Lossani, Natale and Tirelli (1996) have argued that uncertainty about the policymaker’s preferences leads unambiguously to a social welfare loss. We find, however, that this is not necessarily the case. Monetary policy uncertainty may improve output stabilization and as a result the welfare loss may well be lower.

In order to assess the effect of monetary policy uncertainty on welfare, we decompose society’s expected loss in the inflationary bias, the variance of inflation, the output bias and the variance of output.

\[
E\{L\} = \alpha E[\pi^2_t] + E[(y - ky^*)^2] = \alpha(\pi^2 + Var[\pi_t]) + (k-1)y^*^2 + Var[y_t]
\]

(3.1)

Straightforward comparative statics demonstrate that rising uncertainty does increase the inflationary bias. That is

\[
\frac{\partial \pi^e}{\partial \sigma} = \frac{\partial F(.,.)}{\partial \sigma} \frac{b}{a} \frac{\partial \sigma}{\partial z} > 0
\]

(3.2)

because

\[
\frac{\partial F(.,.)}{\partial \sigma} = \frac{-a(a+b^2)^3}{[a(a+b^2)^2 - b^2\sigma_x^2]^2} > 0
\]

(3.3)

The interpretation of this expression, which is unambiguously positive, follows on from our remarks above. To the extent that increasing uncertainty threatens agents’ real wages, they will build in an inflation rate hedge into their nominal contracts. Similarly,

\[
\frac{\partial^2 F(.,.)}{(\partial \sigma)^2} = \frac{2b^2a(a+b^2)^3}{[a(a+b^2)^2 - b^2\sigma_x^2]^3} > 0
\]

(3.4)

as long as \(a(a+b^2)^2 - b^2\sigma_x^2 > 0\), which is exactly the condition we imposed on the variance of the shocks to preferences in the context of equation (2.7). Thus, the inflation penalty becomes steeper the higher monetary policy uncertainty.

From equation (3.1) we derive the following proposition:
**PROPOSITION 1:** The greater monetary policy uncertainty (the higher $\sigma_x^2$), the higher expected inflation.

Proposition 1 shows that monetary policy uncertainty leads to higher inflationary expectations. As is shown in Appendix C the variance of inflation can be found to be

$$Var(\pi) = \frac{b^2 \sigma_\epsilon^2}{(a + b^2)^2} + \left[ b z \left( \frac{b^2}{a} F + 1 \right) \right]^2 \frac{\sigma_x^2}{(a + b^2)^4}$$

(3.5)

The first part of this equation does not depend on monetary policy uncertainty. In the second part, apart from $\sigma_x^2$ itself, also $F$ depends on the degree of monetary policy uncertainty. As $F(.)$ is a positive function of $\sigma_x^2$, it is clear that the variance of inflation increases with monetary policy uncertainty.

Therefore, $\frac{\partial Var(\pi)}{\partial \sigma_x^2} > 0$.

**PROPOSITION 2:** The greater monetary policy uncertainty (the higher $\sigma_x^2$), the higher the variance of inflation.

This result is very intuitive. Introducing a shock to the parameter that determines inflation stabilization yields a more volatile rate of inflation.

To complete the welfare analysis, we now turn to the variance of output, which is the most interesting part. Output variance can easily be calculated from (2.10) and (3.5)

$$Var(y) = \frac{a^2}{(a + b^2)^2} \sigma_\epsilon^2 + \frac{b^2 \sigma_x^2}{(a + b^2)^2} \left\{ b z \left( \frac{b^2}{a} F + 1 \right) \right\}^2 - 2(\bar{a} + b^2) \sigma_x^2$$

(3.6)

Again, the first term does not depend on monetary policy uncertainty, but on the variance of productivity shocks $\sigma_\epsilon^2$. However, the second term depends on monetary policy uncertainty through $\sigma_x^2$ and $F(.)$. Interestingly, the second part of the equation can be negative if the variance of productivity shocks $\sigma_\epsilon^2$ is very high and the principal’s output target $z$ rather low. This means that monetary policy uncertainty can have a stabilizing effect on output. The expression for the variance of output also shows that the positive stabilizing effect of monetary policy uncertainty depends on the variance of output.
shocks. If the variance of output shocks is large enough, uncertainty about the central bank’s inflation stabilization preferences leads to a lower variance of output. The reason for this counterintuitive effect is that the monetary authority’s expected reaction to an output shock is stronger, the larger monetary policy uncertainty. So, in effect an increase in monetary policy uncertainty leads to less conservative behavior of the central bank.

The overall effect on welfare is not easily determined. Looking at equations (2.6), (3.5) and (3.6), it is clear that the negative welfare effects of monetary policy uncertainty are increasing in the principal’s ambitious output target $z$ and independent from the variance of productivity shocks $\sigma_\epsilon^2$.

For the positive output stabilization effect exactly the opposite holds. So, if the ratio $z/\sigma_\epsilon^2$ is small (and, therefore, the credibility problem small relative to the flexibility problem), society may benefit from uncertainty about the policymaker’s preferences.

4 Optimal Monetary Uncertainty

As was explained in the previous section monetary policy uncertainty may be beneficial for society’s welfare because the variance of output may be reduced. This implies that the trade off between credibility and flexibility is influenced by uncertainty concerning the agent’s preferences. In this Section we will determine the optimal level of monetary policy uncertainty in terms of social welfare. Society’s expected loss is given by the following expression

$$E(L) = z^2 + \Pi + \Gamma = z^2 + \frac{\alpha b^2 z^2}{a^2} F^2 + \frac{(\alpha b^2 + a^2)}{(a + b^2)^2} \sigma_\epsilon^2 + \frac{\sigma_\epsilon^2 b^2}{(a + b^2)^3} \left[ \frac{z^2 (\alpha + b^2) (\frac{b^2}{a} F + 1)^2}{(a + b)^2} - 2 \sigma_\epsilon^2 \right] \quad (4.1)$$
PROPOSITION 3: The optimal degree of monetary uncertainty $\sigma^2 x$ has an upper bound $\bar{\sigma}_x^2$

$\left( \sigma^2 x \leq \bar{\sigma}_x^2 \right)$. This upper bound goes up if the credibility/flexibility ratio $z/\sigma^2 \epsilon$ becomes smaller.

Table 1 illustrates the effect of monetary policy uncertainty $\sigma^2 x$ on social loss $S$. In this example we have chosen $b = 1$ and $\alpha = 1$. This country has a rather high variance of productivity shocks and a moderately ambitious output target. Its credibility problem is small relative to its flexibility problem ($z/\sigma^2 \epsilon$ is small) so we expect that monetary uncertainty might be beneficial. In the first column we have set $\sigma^2 x = 0$ and chosen the central banker’s expected inflation aversion equal to society’s ($\bar{\alpha} = \alpha$). Then,
in the second column, we minimize social loss $S$, using $\sigma_x^2$ as the instrument, keeping all other variables the same. Finally, in the third column we use both $\sigma_x^2$ and $\sigma_x^2$ as instruments, using the boundary condition that $\sigma_x^2 \leq \bar{a}^2/3$.

Table 1: The effect of monetary uncertainty on society’s loss

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<th>0.025</th>
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<tr>
<td>$z$</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_e^2$</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_x^2$</td>
<td>0</td>
<td>0.28</td>
<td>1.09</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>1</td>
<td>1</td>
<td>1.81</td>
</tr>
<tr>
<td>$E{\pi}$%</td>
<td>2.50</td>
<td>2.88</td>
<td>1.70</td>
</tr>
<tr>
<td>$\text{Var}{\pi}$%</td>
<td>0.125</td>
<td>0.130</td>
<td>0.066</td>
</tr>
<tr>
<td>$\text{Var}{y}$%</td>
<td>0.125</td>
<td>0.095</td>
<td>0.161</td>
</tr>
<tr>
<td>$S$%</td>
<td>0.3125</td>
<td>0.3079</td>
<td>0.2567</td>
</tr>
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</table>

We see that monetary uncertainty leads to a substantial decrease in the social loss. The positive effect on output stabilization is dominating the negative effects on the mean and variance of inflation. If we use both instruments to minimize the social loss ($\sigma_x^2$ and $\sigma_x^2$, with boundary condition), an even larger welfare gain can be reached.

5 Conclusions

In this paper we have analyzed the effects of monetary policy uncertainty on expected inflation and the variance of inflation and output. We find that uncertainty about the policymaker’s preferences leads to a higher inflationary bias and a higher variance of inflation. However, unlike previous studies, we find that the effect on output stabilization may well be positive and the same is true for overall social welfare. This is due to the fact that the average response of the policymaker to an output shock is stronger than in the absence of preference shocks. In an example it was shown that
the gains can be very substantial if the country’s flexibility problem is big relative to its credibility problem.

Our results also imply that if the credibility problem can be solved or reduced by implementing a Walsh contract or an inflation target, a country may still gain from appointing a conservative central banker with uncertain preferences.
Appendix A: The Derivation Of Equation (2.6)

Taking expectations across expression (2.5):

\[
E(\pi) = E\left( \frac{1}{a_i + b^2} \right) b(b\pi^e + z) \tag{A.1}
\]

This expression requires us to take the expected value of ratios of the inverse of a random variable. This can be achieved through a Taylor series expansion.

Our problem is to expand \( \phi(z) = x / y \) about the respective means. Assuming that the first two moments of \( E(x/y) \) exist then we can write down the expression:

\[
E\left( \frac{X}{Y} \right) \approx \frac{\mu_X}{\mu_y} - \frac{1}{\mu_y^2} \text{cov}[X,Y] + \frac{\mu_X}{\mu_y^3} \text{var}[Y] \tag{A.2}
\]

This is the (second-order) approximation used in the paper, therefore we can write

\[
E\left( \frac{1}{a_i + b^2} \right) \approx \frac{(\tilde{a} + b^2)^2 + \sigma_x^2}{(\tilde{a} + b^2)^3} \tag{A.3}
\]

So substituting (A.3) in (A.1) gives

\[
\pi^e = (b\tilde{z} + b^2\pi^e) \frac{[(\tilde{a} + b^2)^2 + \sigma_x^2]}{(\tilde{a} + b^2)^3} \tag{A.4}
\]

And rearranging gives

\[
\pi^e = \frac{b\tilde{z}}{\tilde{a}} \left( \frac{\tilde{a}(\tilde{a} + b^2)^2 + \sigma_x^2}{\tilde{a}(\tilde{a} + b^2)^2 - b^2\sigma_x^2} \right) \tag{A.5}
\]

which is expression (2.6) in the text. It is clear that as \( \sigma_x^2 \to 0 \) the first part of the expression on the right hand side collapses to unity, and (A.5) is equivalent to (2.8).
**Appendix B: The General Case**

If we replace the central bank’s loss function (2.3) with a more general loss function, the expected rate of inflation changes in the following way.

\[ L = a_i \pi_i^2 + q_i (y - ky^*)^2 \]  \hspace{1cm} (B.1)

\[ a_i = \bar{a} - x_i \quad \text{with} \quad \text{Var}[x_i] = \sigma_x^2 \text{ and } E_{t-1}[x_i] = 0 \]

\[ q_i = \bar{q} - v_i \quad \text{with} \quad \text{Var}[v_i] = \sigma_v^2, \ E_{t-1}[v_i] = 0 \text{ and } E_{t-1}[x_i v_i] = \sigma_{xv} \]

Using the derivations from Appendix A we obtain an expression for the inflationary expectations.

\[ \pi^e = \left( \frac{\bar{q}(\bar{a} + b^2 \bar{q})^2 + R}{a(\bar{a} + b^2 \bar{q})^2 - b^2 R} \right) \cdot b z \quad \text{with} \quad R \equiv \bar{q} \sigma_x^2 - \bar{a} b^2 \sigma_v^2 - (\bar{a} - \bar{q} b^2) \sigma_{xv} \]

We choose \( \bar{q} = 1 \) and \( \sigma_v^2 = \sigma_{xv} = 0 \) so that \( R = \sigma_x^2 > 0 \) and monetary policy uncertainty leads to higher inflationary expectations.

**Appendix C: The Variance of Inflation**

\[ \pi_i = \frac{b}{a_i + b^2} \left[ 1 + \frac{b^2}{a} F(.) \right] z - \frac{b}{a_i + b^2} \varepsilon_i \]

Using a Taylor expansion, the variance of \( X/Y \) can be approximated by

\[ \text{Var} \left\{ \frac{X}{Y} \right\} \approx \left( \frac{\mu_x}{\mu_y} \right)^2 \left( \frac{\text{Var}(X)}{\mu_x^2} + \frac{\text{Var}(Y)}{\mu_y^2} - \frac{2 \text{Cov}(X,Y)}{\mu_x \mu_y} \right) \]

With \( X \equiv b \left[ \frac{b^2}{a} F(.) \right] z + \varepsilon_i \) and \( Y \equiv a_i + b \) we find that

\[ \text{Var} \{ \pi \} = b^2 \sigma_{\varepsilon}^2 \left( \frac{1}{(a + b^2)^2} + \left[ b z \left( \frac{b^2}{a} + 1 \right) + \frac{\sigma_x^2}{(a + b^2)^4} \right] \right) \]
Appendix D: Optimal Central Bank Secrecy

1. The determination of $\sigma^2_x$

As said before it is crucial to find out where $\Gamma$ is minimized. From the first-order condition at this point its first derivative will be zero. This first-order condition can be written as

$$\frac{\partial \Gamma}{\partial \sigma_x^2} = 0 \iff \sigma_x^2 = G(\sigma_x^2) \tag{D.1}$$

where

$$G(\sigma_x^2) \equiv \frac{2\sigma_x^2 (\overline{a} (\overline{a} + b^2)^2 - b^2 \sigma_x^2)^3}{b^2 z^2 (\overline{a} + b^2)^3 (b^2 + \alpha)} - \frac{\overline{a} (\overline{a} + b^2)^2}{b^2} \tag{D.2}$$

If the credibility-flexibility ratio $\frac{z^2}{\sigma_x^2} \leq \frac{2\overline{a}^2}{(\overline{a} + b^2)^2 (b^2 + \alpha)}$ a solution for $\sigma^2_x$ exists and is unique. To see this we use Brouwer’s fixed point theorem. If the credibility-flexibility ratio is not too large

$$G(0) = \frac{\overline{a} (\overline{a} + b^2)^2 (2\sigma_x^2 \overline{a}^2 - (\overline{a} + b^2)^2 (b^2 + \alpha))}{b^2 z^2 (b^2 + \alpha)} \geq 0 \text{ and } G\left(\frac{\overline{a} (\overline{a} + b^2)^2}{b^2}\right) = -\frac{\overline{a} (\overline{a} + b^2)^2}{b^2} < 0.$$

Together with $G$ being a monotonically decreasing function of $\sigma_x^2$ we are now ready to prove $0 \leq \sigma_x^2 \leq G(0)$.

**Proof:** The lefthand side of equation (D.1) is a $45^\circ$ line through the origin. This line must intersect the function $G$ at one and only one point that is bounded between 0 and $G(0)$.

2. Demonstration that $\frac{\partial \sigma^2_x}{\partial \left(\frac{\sigma_x^2}{z^2}\right)} > 0$.

From (D.2) it follows that $\frac{\partial G(\sigma_x^2)}{\partial \left(\frac{\sigma_x^2}{z^2}\right)} > 0$. This implies that if the credibility-flexibility ratio goes down, the function $G(\sigma_x^2)$ shifts upward. As a consequence, the equilibrium value of $\sigma_x^2$ increases.
References


