Output Stabilization in EMU
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Output Stabilization in EMU: is there a Case for an EFTS?

Bas van Aarle $^1$ and Svend-Erik Hougaard Jensen$^2$

Abstract

Macroeconomic performance in the Economic and Monetary Union (EMU) will be impaired if macroeconomic shocks are largely asymmetric, fiscal policy flexibility is limited, goods markets adjust sluggishly, labour mobility is low and automatic stabilization from federal taxes and government spending is low like in the EU currently. This paper addresses the question whether a system of fiscal transfers to stabilize differences in national business cycles can improve the overall macroeconomic performance in the monetary union.

JEL-code: E32,E52,E61,E62
Keywords: Monetary and Fiscal Policy, EMU, ECB

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Introduction

Economic and Monetary Union (EMU) is likely to induce static and dynamic efficiency gains for the participating countries. The EC Commission (1990) studied in detail the possible gains from EMU in its "One Market, One Money" study. On the other hand, in the EMU, when completed, national policymakers are limited in their ability to actively stabilize output fluctuations in their economies induced by macroeconomic shocks. A monetary union implies that individual monetary policy is replaced by the common monetary policy of the ECB. Common monetary policy can be directed to stabilize symmetric, that is, EMU-wide shocks but can not be directed at stabilizing asymmetric, that is, country specific shocks. In particular, EMU implies the loss of the possibility to adjust the nominal exchange rate in case members experience asymmetric shocks. Loss of the exchange rate instrument as a shock absorber is potentially harmful if the countries that join the EMU do not constitute an 'optimal currency area'.

Countries are less likely to form an optimal currency area if macroeconomic shocks are predominantly asymmetric, nominal rigidities prevail in the short run, labour mobility is low and automatic stabilization from federal taxes and government spending is low. A considerable empirical literature on the degree to which macroeconomic shocks in the EU are predominantly symmetric or asymmetric, exists currently. Studies by Weber (1991), Bayoumi and Eichengreen (1993), Bayoumi and Prasad (1995) and Christodoulakis, Dimelis and Kollintzas (1995) suggest that asymmetric shocks are fairly important in most countries of the European Union (EU). Decressin and Fatas (1995) analyse labour mobility in the US and the EU and find evidence that labour mobility is considerably smaller in the EU than in the US. Finally, a number of studies analysed the automatic stabilization from federal taxes and government spending. Sachs and Sala-i-Martin (1991), von Hagen (1991) and Goodhart and Smith (1993) show that this automatic stabilization is considerable in mature federations like the U.S. and Canada. Because of the current small size of the federal EU budget (about 1.3% of EU GDP), such automatic stabilization from federal taxes and government spending is currently fairly small in the EU. The significance of asymmetric shocks, limited labour mobility and low automatic stabilization from federal taxes and government expenditures cast doubts whether the EU is actually an optimal currency area.

EMU, therefore, is likely to shoulder the national fiscal authorities with a higher adjustment
burden from stabilizing output fluctuations. The scope for national fiscal policies to stabilize business cycle fluctuations will be limited in the EMU, however, if rigid fiscal stringency and convergence criteria are imposed. Moreover, the increasing integration of goods, labour and capital markets makes tax bases more mobile w.r.t. increases in the tax rates and increases the spillovers in case of uncoordinated fiscal policies.

Hughes Hallett and Vines (1993) compare economic performance of EMU with a regime of floating exchange rates, using simulation tools. It is pointed out that an EMU, compared to flexible exchange rates, performs poorly when hit by asymmetric shocks, featuring an independent ECB and low fiscal policy flexibility and lacking fiscal coordination. Hougaard Jensen and Jensen (1995) analyse the costs in terms of foregone output from complying with the convergence criteria of the Maastricht Treaty for a small EU country.

This paper focuses on stabilization policies in an EMU confronted with symmetric and asymmetric shocks. We consider a system of fiscal transfers that aims at stabilizing asymmetric shocks in the EMU. In case of asymmetric shocks, such a transfer system might act as a substitute for the shock-absorbing capacity of exchange rate realignments like in the EMS and for the automatic stabilizing role of federal government spending and taxation that occurs in mature fiscal federations. It is shown how such a European System of Fiscal Transfers (EFTS) can increase macroeconomic efficiency in the monetary union by providing the participating countries cushion against asymmetric shocks. Such a transfer system was advocated by van der Ploeg (1991) who suggested to supplement the Maastricht Treaty on EMU with an EFTS aiming at counteracting the consequences of asymmetric macroeconomic shocks in a monetary union. While some work has been developed on such an EFTS, notably by Italianer and van Heukelen (1993) and von Hagen and Hammond (1995), the full implications of such a system have remained largely unexplored.

Section 2 introduces a stylized two-country EMU model that will serve as our analytical device. A system of fiscal transfers that aims at stabilizing differences in business cycle fluctuations in the EU is proposed. Section 3 solves the model and derives its analytical properties. In section 4 numerical simulation of the model is used to study the dynamic adjustment in the EMU after symmetric and asymmetric output shocks.
5.2 A two-Country EMU model

Consider an EMU that consists of two countries. Assume that the economies of both countries is given as follows,

<table>
<thead>
<tr>
<th>Table 1</th>
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</thead>
<tbody>
<tr>
<td>A two-Country EMU Model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1a) ( y(t) = \alpha c(t) - \gamma r(t) + \sigma y^*(t) + \eta s(t) + u(t) )</td>
<td>(1b) ( y^<em>(t) = -\alpha c(t) - \gamma r^</em>(t) + \sigma y(t) + \eta s^<em>(t) + u^</em>(t) )</td>
</tr>
<tr>
<td>(2a) ( m(t) - p^c(t) = \kappa y(t) - \lambda i(t) )</td>
<td>(2b) ( m^<em>(t) - p^c^</em>(t) = \kappa y^<em>(t) - \lambda i^</em>(t) )</td>
</tr>
<tr>
<td>(3a) ( c(t) = p^c(t) - p^c(t) )</td>
<td>(3b) ( c(t) = p^c(t) - p^c(t) )</td>
</tr>
<tr>
<td>(4a) ( i(t) = r(t) + p^c )</td>
<td>(4b) ( i^<em>(t) = r^</em>(t) + p^c^* )</td>
</tr>
<tr>
<td>(5a) ( p^c(t) = \mu p(t) + (1 - \mu)p^c^*(t) )</td>
<td>(5b) ( p^c^<em>(t) = \mu p^</em>(t) + (1 - \mu)p(t) )</td>
</tr>
<tr>
<td>(6a) ( s(t) = f(t) + n(t) )</td>
<td>(6b) ( s^<em>(t) = f^</em>(t) - n(t) )</td>
</tr>
</tbody>
</table>

where \( y(t) \) denotes real output at time \( t \), \( c(t) \) competitiveness of country 1 vis-à-vis country 2, \( m(t) \) nominal money balances, \( p(t) \) the domestic output price level, \( p^c(t) \) the consumer price level and \( w(t) \) the nominal wage. \( i(t) \) and \( r(t) \) are the nominal and real interest rates. \( s(t) \) is an index of the fiscal stance. The variables of the second country are indicated with an asterisk. Variables are in logarithms and defined as deviations from their long term non inflationary, balanced growth path. The policy reaction functions that are given by (9)-(11) and explained later on, close the model. To solve the model later on in an insightful manner it is assumed

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2 See OECD (1995) for recent estimates of the NAIRU in the EU. Jaeger and Parkinson (1994) investigate the presence of “hysteresis” in unemployment due to interaction between the NAIRU and cyclical unemployment. The European unemployment is to a considerable extent a structural unemployment problem rather than the cyclical unemployment that we consider here. See EC Commission (1995) for the breakdown of European unemployment into structural and cyclical parts. The OECD (1994) Jobs Study analyzes the unemployment problem in the OECD countries in great detail.
that the countries do not differ w.r.t. the structural parameters that feature in (1)-(9).

(1) gives aggregate demand as a function of competitiveness, the real interest rate, foreign output, fiscal policy and demand shocks that hit the economy. \( \nu(t) \) is an aggregate demand shock that hits the economy: it impacts the economy at \( t=0 \), after which it decays exponentially: \( \nu(t) = \nu(0) e^{-\rho t} \). \( \rho \) measures how long the shock impacts upon the economy. If \( \rho \) equals zero, the shock is permanent while if \( \rho \) is positive, the shock has some persistence but dies out gradually. In the limiting case where \( \rho \) goes to infinity, the shock dies out instantaneously.

The demand for the common currency is given by (2). The EU-wide nominal interest rate, \( i(t) \), clears the European money market. The supply of the common currency is controlled by the ECB. Monetary policy of the ECB and its impact on macroeconomic performance is considered in the next section. (3) defines the competitiveness of country 1 vis-à-vis country 2 as the difference of output prices\(^3\). (4) and (5) define the nominal interest rate and the consumer price level which is a weighted average of the price level of domestic and foreign goods. The fiscal stance (6) is defined as the real fiscal deficit, \( f(t) \), plus the net real fiscal transfer from country 2 to country 1, \( n(t) \), as a consequence of the EFTS.

(7) and (8) describe the price and wage formation process in the presence of staggered price and wage contracts of the type introduced by Taylor (1980a) and Calvo (1983). In case of staggered contracts only a fraction, \( \delta \), of all nominal contracts are renewed in a period. The average length of a contract, therefore, equals \( 1/\delta \). (7) determines the price level as a weighted average of all outstanding wage contracts, implying that the price level is a predetermined variable and the economy displays Keynesian price-stickiness in the short run. Differentiating w.r.t. time gives the rate of inflation as a function of the real wage\(^4\). Wages in (8) equal the weighted

\(^3\) With flexible exchange rates competitiveness equals the real exchange rate which is defined as \( c = e(t) + p^*(t) - p(t) \) in which \( e(t) \) denotes the nominal exchange rate between country 1 and 2. Normalising, the final conversion rate, \( E(t) \), to 1 implies that the real exchange rate, \( c(t) = p^*(t) - p(t) \), since \( e(t) = \ln(E(t)) = 0 \), in that case. The choice of the final conversion rates of national currencies to the new common currency is by no means a trivial issue in practice, as the analysis of Giovannini (1992) shows.

\(^4\) Differentiating (7) and (8) w.r.t. time gives:

\[
\dot{p} = \delta ( \int_0^\tau w(t) e^{-\delta(\tau-t)} dt - \dot{w}(t) ) = \delta ( w(t) - \dot{p}(t) )
\]

\[
\dot{w} = \delta ( \int_0^\tau (p(t) - \dot{p}(t) - \dot{y}(t)) e^{-\delta(\tau-t)} dt - p(t) - \dot{y}(t) ) = \delta ( w(t) - \dot{p}(t) - \dot{y}(t) )
\]
average of expected future prices and excess demand. Nominal wages are forward looking and changes instantaneously upon the arrival of new information. $E_t$ denotes the expectations operator. Since in this analysis output shocks are deterministic rather than stochastic, the assumption of rational expectations of the wage setters implies perfect foresight and we can disregard the expectations operator.

Staggered wage and price setting gives rise to nominal rigidities in the economy and unemployment persistence if the economy is hit by a negative output shock. From a policy perspective, the nominal rigidities create a Phillips curve alike trade-off between price and output stability, in the short run. $\delta$ reflects the institutional rigidities that limit the adjustment of prices in goods and labour markets. A higher value of $\delta$ implies that a higher fraction of outstanding wage contract renews instantaneously. In that case, the higher price and wage flexibility dampens business cycle fluctuations. Note, finally, that the forward-looking character of nominal wages implies that (anticipated) macroeconomic policies affect the behaviour of wages by altering the expectations on future prices and excess demand.

The analysis abstains from modelling the interaction of the EU and the rest of the world explicitly and treats the EU as a more or less closed economy. To some extent this simplification might be defended by the empirical observation that in case of most EU countries intra-EU trade dominates extra-EU trade to a large extent. The impact of changes in the world economy on the EU might take the form of an output shock $\nu_i(t)$ in this analysis. A decline in competitiveness of the EU economy in world trade then might be represented as a negative output shock. See the EC Commission (1990), Chapter 6 and Kenen (1993) on the external dimensions and implications of EMU.

Throughout the analysis we will assume that the structural model parameters coincide

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5 See Taylor (1980b) for an empirical analysis of staggered contracts and the output inflation trade-off.

6 The EC Commission (1990) recognizes the importance of a higher degree of price and wage flexibility: "(..) it is worth mentioning here that wage-price flexibility remains the basic adjustment channel as a substitute for the nominal exchange rate. (..) However, in so far as nominal rigidities hamper market adjustments, fiscal policy measures can alleviate temporary country-specific disequilibria. This is indeed the traditional role of fiscal policy as a tool for stabilization." (p.102)

7 See van Aarle (1996) for an empirical analysis of intra-EU trade and foreign direct investment. It is found that in 1980 55.70% of total exports of the EU 12 countries has other countries in the EU 12 as destination. In 1994 this number had increased to 63.34%, indicating at increasing integration of goods markets in the EU.
in both countries and that the countries are of equal size, simplifying assumptions that are useful in deriving macroeconomic equilibrium of the two-country EMU\(^8\). We assume, furthermore, that national fiscal policies are aimed at stabilizing national business cycle fluctuations, apart from an exogenous component, \(f\):

\[
\begin{align*}
  f(t) &= f - \chi (y(t) - \bar{y}) \\
  f^*(t) &= f^* - \chi (y^*(t) - \bar{y}^*)
\end{align*}
\]  

(9) implies that the fiscal deficit is increased if the economy is in recession and decreased if output is above the targeted level of output, \(\bar{y}\), viz. \(\bar{y}^*\). Fiscal policies affect the economy because of their influence on aggregate demand (1).

Consider next the introduction of a system of fiscal transfers (EFTS) that is designed to alleviate the adjustment burden of asymmetric shocks in a monetary union. Its basic working is summarized as follows by van der Ploeg (1991), p. 144:

"The task of the EFTS is to make exchange-rate changes unnecessary by transferring income from one country to another country when there are such shifts in preferences [i.e. asymmetric shocks]. In practice, the scheme operates by transferring income from individuals of one nation to individuals of another nation and replaces, to a certain extent, the national unemployment compensation schemes. One could envisage a Community-wide tax, which in itself would act as an automatic stabiliser, whose proceeds are used to finance a Community-wide unemployment compensation scheme. It is crucial that such a version of the EFTS is budget-neutral. (...) It is a pity that the Delors report does not contain any recommendations for the establishment of a EFTS, because without it regional imbalances induced by shifts in preferences may persist."

von Hagen and Hammond (1995) construct an insurance based system of fiscal transfers and investigate a number of desirable properties of such a system. The stabilization scheme

\(^8\) If both countries differ in their structural parameters the adjustment processes in both economies will differ even when hit by the same symmetric macroeconomic shock. In that case it becomes difficult, if not impossible, to determine how much of the outcomes are due to the fact that countries differ in their structural parameters and how much is due to the symmetric and asymmetric macroeconomic shocks themselves.
considered in this paper is slightly different from the system of von Hagen and Hammond (1995): it acts as an automatic output stabilizer rather than directly offsetting asymmetric shocks like in von Hagen and Hammond\(^9\). More specifically, it automatically transfers resources from countries experiencing a boom to countries that suffer from recession:

\[ n(t) = \xi (y^*(t) - y(t)) \] (10)

The transfer system aims at stabilizing the divergence in the business cycles of both countries and does not affect the business cycle of the aggregate EU economy that is only affected by ECB monetary policy and average fiscal policy in the EU\(^{10}\). (10) has the considerable advantage for the policymakers that there is no need to estimate the actual size of the asymmetric macroeconomic shocks. A disadvantage in practice could be that countries might have a strategic incentive to reduce their own stabilization effort in the knowledge that this lack of activism - implying a low value of \(\chi\) - will be partly compensated by higher fiscal transfers from the EFTS. In our analysis we will abstract from such strategic considerations.

### 5.3 EMU Averages and Differences

To solve the model we use the Aoki factorization of the variables into averages and differences. Assuming both countries to be of equal size, averages of a variable are defined as \(x_A(t) = \frac{1}{2}(x(t) + x^*(t))\) and differences as \(x_D(t) = x(t) - x^*(t)\). The individual country variables, consequently, are defined as \(x(t) = x_A(t) + \frac{1}{2}x_D(t)\) and \(x^*(t) = x_A(t) - \frac{1}{2}x_D(t)\). This method has the advantage that dynamics are decomposed into two independent dynamic sub-systems: an average system that gives the dynamics of the aggregate EU economy and a difference system that describes differences between the two countries. Thus, the average system is particularly suited to study

\(^9\) In the context of our two-country EMU model, the transfer system of von Hagen and Hammond might be defined as: \(n(t) = -\xi v_D(t)\), implying that a negative asymmetric supply shock in country 1, i.e. \(v_D(t)<0\), will be counteracted by an automatic fiscal transfer from country 2 to country 1.

\(^{10}\) Note that the transfer system aims at stabilization rather than redistribution since output is defined in terms of deviations from a long-run balanced growth equilibrium. Redistribution essentially aims at reducing disparities in long-run equilibrium output (per capita). On allocational and redistributive dimensions of fiscal transfers in the EU see in particular the EC Commission (1993).
macroeconomic variables at the level of the aggregate EU economy, whereas the difference system is suited to address issues of macroeconomic convergence between both parts of the EU. Moreover, it has the nice property that macroeconomic shocks are decomposed into the part that is completely symmetric, the shocks found in the average system, and a part that is completely asymmetric, the shocks found in the difference system.

The (average) money supply in the monetary union, \( m_A(t) \), is controlled by the European Central Bank. The transmission channel of ECB monetary policy is the EU wide interest rate: expansionary monetary policy induces a lower nominal interest rate to restore money market equilibrium (2). Since prices are sticky in the short run, the decrease of the nominal interest rate implies a decrease in the real interest rate which boosts output (1) in the short run. We assume that monetary policy of the ECB is also set according to a linear feedback rule:

\[
    m_A(t) = \bar{m}_A - \zeta (y_A(t) - \bar{y}_A)
\]

where \( \bar{m}_A \) denotes some exogenous, possibly zero money supply target. (11) implies that the money supply is expanded if output is below the targeted level of output, \( \bar{y}_A \). The resulting decrease in nominal and real interest rates boosts output in the short run.

\( \zeta \) could be interpreted as a measure of conservativeness and independence of the ECB: a value of \( \zeta \) of zero would amount to an ultra conservative/independent ECB that only sticks to its monetary target under all circumstances. A more conservative ECB implies that the fiscal authorities, however, are facing a higher adjustment burden if symmetric shocks hit the EU economy. A combination of a conservative ECB and strict fiscal stringency criteria, implying low values of \( \zeta \) and \( \chi \) leaves the EU economy without effective automatic stabilization if symmetric shocks occur. This will lead to suboptimal average economic performance, in particular if nominal rigidities are considerable in the short run. Prevalence of the price stability objective in ECB monetary policy is proclaimed in the Maastricht Treaty although monetary policies are allowed to foster real economic performance, as long as it does not interfere with the price stability objective. A strict interpretation of the Maastricht Treaty would amount to a zero or at least small value of the feedback parameters \( \zeta \) and \( \chi \). Given the stabilizing impact of ECB monetary policy on EU aggregate output, at least in the short run because of the nominal rigidities, it cannot be ruled out that in practice the ECB will pursue policies that are geared to stabilize
to some extent fluctuations of the real aggregate EU economy. In that case $\zeta$ is larger than zero"11.

We can distinguish two alternative institutional settings of EMU: (i) if $\zeta=\chi=\xi=0$, there is no automatic stabilization from ECB monetary policy, national fiscal policies or an EFTS. This regime might be interpreted as the regime envisaged in the Maastricht Treaty, stressing the dedication of the ECB to price stability, the need for fiscal stringency and the absence of fiscal bail-out of individual countries by fiscal transfers. (ii) a regime with $\zeta, \chi, \xi>0$ that recognizes the inefficiencies that can arise in a monetary union with the aforementioned policy assignment if macroeconomic shocks are predominantly asymmetric, labour mobility, price and wage flexibility are low and if there is little automatic stabilization from federal expenditures and taxation. Table 2 provides the average and difference systems that can be derived from (1)-(11).

Table 2

Average and Difference Systems of the two-Country EMU Model

<table>
<thead>
<tr>
<th>Average system</th>
<th>Difference system</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1c) $y_A(t) - \gamma r(t) + \sigma y_A(t) + \eta s_A(t) + v_A(t)$</td>
<td>(1d) $y_D(t) = 2a c(t) - \gamma r_D(t) - \sigma y_D(t) + \eta s_D(t) + v_D(t)$</td>
</tr>
<tr>
<td>(2c) $m(t) - p_A(t) - \kappa y_A(t) - \lambda i(t)$</td>
<td>(2d) $m_D(t) - p_D(t) - \kappa y_D(t)$</td>
</tr>
<tr>
<td>(4c) $i(t) = r_A(t) - k_A$</td>
<td>(4d) $r_D(t) = r^c$</td>
</tr>
<tr>
<td>(5c) $p_A(t) = p_A(t)$</td>
<td>(5d) $p_D(t) = (2\mu - 1)p_D(t)$</td>
</tr>
<tr>
<td>(6c) $s_A(t) = f_A(t)$</td>
<td>(6d) $s_D(t) = f_D(t) + z(t)$</td>
</tr>
<tr>
<td>(7c) $p_A(t) = \delta \int w_A(t)e^{-\delta t - \omega}dt$</td>
<td>(7d) $p_D(t) = \delta \int w_D(t)e^{-\delta t - \omega}dt$</td>
</tr>
<tr>
<td>$- \dot{p}_A = \delta(w_A(t) - p_A(t))$</td>
<td>$- \dot{p}_D = \delta(w_D(t) - p_D(t))$</td>
</tr>
<tr>
<td>(8c) $w_A(t) = \delta E\int(p_A(t) + \psi y_A(t))e^{-\delta t - \omega}dt$</td>
<td>(8d) $w_D(t) = \delta E\int(p_D(t) + \psi y_D(t))e^{-\delta t - \omega}dt$</td>
</tr>
<tr>
<td>$- \dot{w}_A = \delta(w_A(t) - p_A(t) - v y_A(t))$</td>
<td>$- \dot{w}_D = \delta(w_D(t) - p_D(t) - v y_D(t))$</td>
</tr>
<tr>
<td>(9c) $f_A(t) = f_A - \chi(y_A(t) - \bar{y}_A)$</td>
<td>(9d) $f_D(t) = f_D - \chi(y_D(t) - \bar{y}_D)$</td>
</tr>
<tr>
<td>(11) $m_A(t) = \bar{m}_A - \zeta(y_A(t) - \bar{y}_A)$</td>
<td>(10) $n(t) = -\zeta y_D(t)$</td>
</tr>
</tbody>
</table>

11 While certainly interesting because of the interdependencies and externalities in the EMU, the current analysis does not consider the strategic interaction that might occur between the ECB and national fiscal authorities. It is clear that coordination of fiscal policies by both countries or even coordination of monetary and fiscal policies can be helpful in alleviating the adjustment from symmetric and asymmetric shocks in the EMU. Indeed, the EC Commission (1990) considers coordination, with autonomy and discipline the three main components of the budgetary regime in the EMU (p. 113).
The average and difference systems provide a good insight into the working of the EU aggregate economy and factors that create divergences between the two different countries. Intra-EU imports transmit fluctuations in output, as given by (1), in one part of the EU to the other part with a propensity $\sigma$. Intra-EU trade, therefore, tends to amplify output fluctuations in the average system and to reduce output fluctuations in the difference system. The relative competitiveness of country 1 relative to country 2, as defined in (3) does not affect average output but has a strong distributional impact. Similarly, the EFTS, defined in (10), does not affect average fiscal stances in (6) but impacts on the difference in fiscal stance and by that on difference output.

From (1)-(7) and (9)-(11) we can write output in the average and difference systems as functions of the price level, wages, structural and policy parameters and output shocks:

$$y_A(t) = \frac{1}{\Delta_A} \left\{ -\left( \frac{\gamma}{\lambda} + \gamma \delta \right) p_A(t) + \gamma \delta w_A(t) + \frac{\gamma \xi}{\lambda} \bar{m}_A + \left( \frac{\gamma \eta}{\lambda} \bar{\chi} + \eta \bar{r} \right)_A + v_A(t) \right\}$$

$$y_D(t) = \frac{1}{\Delta_D} \left\{ -\left( 2\alpha + \gamma \delta (2\mu - 1) \right) p_D(t) + \gamma \delta (2\mu - 1) w_D(t) + \eta \left( f_D + \chi \bar{y}_D \right) + v_D(t) \right\}$$

where $\Delta_A = 1 + \gamma (\kappa + \zeta) / \lambda - \sigma + \eta \bar{\chi}$ and $\Delta_D = 1 + \sigma + \eta (\bar{\chi} + 2 \bar{\xi})$. Substituting (12) into (8) gives two dynamic systems of the price level and the wage rate, the average and difference system:

$$\begin{bmatrix} \dot{p}_i \\ \dot{w}_i \end{bmatrix} = A_i \begin{bmatrix} p_i(t) \\ w_i(t) \end{bmatrix} + b_i + u_i(t) \quad i \in \{A,D\}$$

$A_i$, $b_i$, $u_i(t)$ are found in the Appendix A in (A.6) and (A.11).

The dynamic systems in (13) contain a backward-looking variable, the price level $p_i(t)$ and a forward-looking variable, the wage rate, $w_i(t)$. Saddlepoint stability, therefore, requires $A$ to have a positive and a negative eigenvalue. A necessary and sufficient condition for (13) to display saddlepoint stability is that the determinant of $A_i$, which equals the product of the eigenvalues, is negative. We can write the adjustment of the dynamic system towards its

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12 The eigenvalues are given by: $-\frac{1}{2} \left[ \text{Trace}(A_i) + \sqrt{(\text{Trace}(A_i))^2 - 4 \text{Det}(A_i)} \right]$ where $\text{Trace}(A_i) = -\nu \delta^2 / \Delta_\nu$,

$\text{Trace}(A_D) = -\nu \delta^2 (2\mu - 1) / \Delta_D$, $\text{Det}(A_D) = -\nu \delta^2 \Delta_\nu$ and $\text{Det}(A_\nu) = -2\nu \delta^2 / \Delta_\nu$. The stable, negative eigenvalue, $h_i$, measures the adjustment speed towards steady state: the adjustment speed, $-h_i$, of the dynamic systems is defined as the absolute value of the stable eigenvalue of $A$. The adjustment of the system towards steady-state is monotonic if the discriminant $D_i = \text{Trace}(A_i)^2 - 4 \text{Det}(A_i)$ is positive and oscillatory if it is negative.
steady-state A in Figure 1 as:

\[
p_i(t) = (p_i(0) - p_i(\infty)) e^{h_i t} + p_i(\infty)
\]

\[
w_i(t) = (w_i(0) - w_i(\infty)) e^{h_i t} + w_i(\infty) \quad i \in \{A, D\}
\]

where \( h_i \) equals the stable eigenvalue of \( A_i \). Steady-state average prices and wages equal:

\[
P_A(\infty) = w_A(\infty) = \bar{m} + \frac{\lambda}{\gamma} \left( \frac{\delta \zeta}{\lambda} + \eta \chi \right) \bar{y}_A + \eta \bar{f}_A + v_A(0) e^{-\rho_A^*}
\]

\[
P_D(\infty) = w_D(\infty) = -\frac{1}{2\alpha} \left[ \eta \left( \bar{f}_A + \chi \bar{y}_D \right) + v_D(0) e^{-\rho_D^*} \right]
\]

The initial value of the price level is given because the price level is a predetermined variable. The initial value of the forward-looking nominal wage follows from the condition that wages adjust at \( t=0 \) such as to place the dynamic system on its unique saddlepoint stable trajectory SS. Appendix A derives the unique initial values \( w_i(0) \) that satisfy this condition. It is shown that the unique initial level of the average and difference wage consistent with long-run convergence to the steady-state equals:

\[
w_A(0) = \frac{z_A + \delta}{\Delta_A} P_A(0) + \frac{\nu \delta}{\Delta_A} \left( \frac{\gamma}{\lambda} \bar{m} + \left( \frac{\delta \zeta}{\lambda} + \eta \chi \right) \bar{y}_A + \eta \bar{f}_A + \frac{v_A(0)}{z_A + \rho_A} \right)
\]

\[
w_D(0) = \frac{z_D + \Delta}{\delta} P_D(0) + \frac{\nu \delta}{\Delta_D} \left( \eta \chi \bar{y}_D + \eta \bar{f}_D + \frac{v_D(0)}{z_D + \rho_D} \right)
\]

in which \( z_i \) denotes the unstable eigenvalue of \( A_i \). Figure 1 gives the phase diagram that pictures the dynamics of the dynamics average and difference systems (13).
The forward-looking wage rate displays a similar "overshooting" property as the exchange rate in the Dornbusch (1976) model: the overshooting of the wage rate allows the labour market to clear in the short run in the presence of price rigidities. An increase at $t=0$ of the policy targets $\bar{f}_i$ or $\bar{y}_i$ e.g. or an unanticipated permanent positive output shock $\nu_i(0)$ imply that the $\dot{w}_i=0$ locus shifts upward to the new $\dot{w}_i'=0$ locus in Figure 2. The wage rate jumps upwards from A to B on the new stable branch of the dynamic system. From B the system gradually converges to the new steady-state $A'$. The initial wage rate overshoots its new long-run equilibrium value and during the adjustment process, wage rates decline and prices increase.
Because of the nominal rigidities in the short run implied by the staggered price and wage contracts, the average system displays Keynesian features during the adjustment phase: a negative symmetric shock induces a drop of output which, however, can be counteracted by automatic stabilization from monetary and fiscal policy. In the long run, on the other hand, the model displays neoclassical features with full adjustment of price and wages to ensure that output returns at its long run (non-inflationary) equilibrium level, which itself is independent of monetary and fiscal policy. The case for active fiscal stabilization in the EMU of symmetric shocks, becomes more pressing, if the symmetric shock becomes more prolonged, i.e. if $\rho$ gets smaller and if the degree of price rigidity in the short run increases, i.e $\delta$ is smaller.

Asymmetric shocks induce a similar adjustment in wages, prices and output in the difference system. Also the need to stabilize asymmetric shocks gets more pressing if the shocks have considerable persistence and if price stickiness is high, i.e. if $\rho$ and $\delta$ are small. Both fiscal flexibility and the EFTS enable to smooth output fluctuations of output in the difference system when EMU is hit by asymmetric shocks. Fiscal flexibility, though, causes higher steady-state differences in the fiscal deficit whereas EFTS fosters fiscal convergence in the long run as it partly redistributes the adjustment burden from asymmetric shocks from the 'unfortunate' to the 'fortunate part' of the EU.

We summarize these possibilities of automatic stabilization of the average and difference systems in the following proposition:
Proposition 1
(a) Fiscal and monetary flexibility smoothen fluctuations in average output induced by symmetric shocks. (b) Fiscal flexibility and the EFTS smoothen fluctuations in difference output induced by asymmetric shocks. (c) In that case the EFTS contributes to smoothen differences in the fiscal deficit whereas fiscal flexibility increases the divergence in fiscal deficits.

Proof:
A higher degree of fiscal or monetary flexibility, i.e. a higher value of $\chi$ or $\zeta$, increases the value of $z_A$, the unstable eigenvalue of the average system, if

$$\frac{v_y \delta^2}{\Delta_A} + 2 \frac{v_y \delta^2}{\lambda \Delta_A} + \frac{4}{\lambda} > 0$$

and increases $\Delta_A$. This implies that the initial jump in the average wage rate after a symmetric shock, $\upsilon_A(0)$, that is necessary to bring the economy on the new saddlepath is smaller. This jump is derived in the appendix as (A.10). From (12) it follows that it also reduces the initial decline in average output. This results in (a). A higher degree of fiscal flexibility or stabilization from the EFTS, i.e. a higher value of $\chi$ or $\xi$, increases the value of $z_D$, the unstable eigenvalue, of the difference system if

$$\frac{(v_y \delta^2(2\mu-1))^2}{\Delta_D} + 8 \frac{v_y \delta^2}{\mu \Delta_D} + \frac{2v_y \delta^2(2\mu-1)}{\Delta_D} \frac{4}{\mu} > 0.$$ 

and increases $\Delta_D$. This implies that the initial jump in the difference wage rate, $w_D(0)$, after an asymmetric shock, $\upsilon_D(0)$, that is necessary to bring the economy on the new saddlepath is smaller. This jump is derived in the appendix as (A.15). From (12) it follows that it also reduces the initial decline in difference output. This results in (b). Introducing the EFTS increases the degree of automatic stabilization of asymmetric shocks. This implies a reduction of the adjustment burden of national fiscal policies in stabilizing the national economy when hit by an asymmetric shock. Therefore the EFTS fosters the convergence of fiscal deficits. This feature is provided by (c).

Monetary and fiscal flexibility contribute to stabilize the average system when hit by symmetric shocks. In that case the fiscal deficit and the EU money supply set by the ECB simply mirror average output fluctuations: average deficits and money supply are set higher if initial output is lower. A higher degree of fiscal flexibility reduces output fluctuations in the difference
system, however at the cost of higher divergences in the fiscal deficit. The EFTS also enables
to stabilize output fluctuations in the difference systems induced by asymmetric shocks but
has an additional advantage of stabilizing the fluctuations in fiscal deficits in the difference
system.

5.4 Numerical Simulations: Symmetric and Asymmetric Shocks in the EMU

Having characterised the analytical solution of adjustment of the average and difference systems,
we turn to a simulation example to study the macroeconomic adjustment induced by symmetric
and asymmetric shocks that hit the EMU. A distinction is made between a regime (EMU 1)
without fiscal flexibility and without an EFTS ($\chi=0$ and $\xi=0$), a regime (EMU 2) with fiscal
flexibility but without an EFTS ($\chi=0.5$ and $\xi=0$) and finally a regime (EMU 3) without fiscal
flexibility but with an EFTS ($\chi=0$ and $\xi=0.25$). Monetary policy of the ECB is assumed not
to be influenced by considerations to stabilize output in any of the three regimes ($\zeta=0$), as we
are not focusing on monetary policy in the EMU. Table 3 gives the structural and policy parameters
and the resulting initial equilibrium values of the simulation.

<table>
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<tr>
<th>(i)</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$\gamma$</td>
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<tr>
<td>$\delta$</td>
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<td>$\kappa$</td>
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<tr>
<td>$\lambda$</td>
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<tr>
<td>$\mu$</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

In the initial steady-state that results with this set of structural and policy parameters, all variables
are equal to zero: $p^*_A(t)=w_A(t)=y_A(t)=m_A(t)=0$ and $p^*_D(t)=w_D(t)=y_D(t)=f_D(t)=n(t)=0$.

Figure 3 shows the adjustment induced by a symmetric negative output shock of 1%
in the 2-country EMU at $t=0$, i.e. $v_A(0)=-0.01$. $\rho$ takes the value of 0.1, implying that the shock
has considerable persistence. Average output in the EMU jumps down at the impact of the
symmetric negative demand shock. The nominal wage jumps down to the level consistent with long-run convergence to the new steady-state. After the initial decrease in output, output recovers gradually due to the initial decline in real wages. In the second EMU regime, the stabilizing force from fiscal policy also contributes to stabilization of output in the short run. The adjustment of prices is very small in this example, reflecting the nominal rigidities in the economy: with $\delta$ being equal to 0.33, contracts are renewed only every 3 year, causing a high inertia in the prices. Wages and output adjust again to their long run equilibrium of zero which is approached eventually. Monetary policy does not change of course, since we assumed $\bar{m}_a$ and $\zeta$ to be equal to 0.

Because of our symmetry assumption, a symmetric shock has no impact on the difference system. Therefore, outcomes under a regime with an EFTS but no fiscal flexibility (EMU 3) equal outcomes under the EMU 1 regime without any macroeconomic stabilizers, if a symmetric output shock hits EMU. The solid lines of the EMU 1 regime coincide, therefore, in all cases with the dashed lines of the EMU 3 regime. Also we note that, because of the symmetries that we have imposed, adjustment of macroeconomic variables in both country 1 and 2 is identical to the adjustment in the average system as given in Figure 3.
Figure 3
Adjustment after a Symmetric Output Shock ($v_A(0) = -0.01, \rho = 0.1$)
Figure 4 shows the adjustment induced by an asymmetric negative output shock of 1\% in the 2-country EMU, i.e. $\nu_D(0)=-0.01$. $\rho$ takes again a value of 0.1. Like in the average system, the wage rate jumps down at the impact of the output shock and gradually recovers along the saddle-point stable path. The decline in real wages contributes to the gradual recovery of output. The output divergence in the EMU after an asymmetric shock can be stabilized by allowing fiscal flexibility as in the EMU 2 regime, or by establishing an EFTS as in the EMU 3 regime (or introducing both of course). Since we choose $\chi=2\xi$ the effective degree of stabilization of output differences is the same in both regimes. An asymmetric shock has no impact on the average system. The dotted lines of the EMU 2 regime coincide, therefore, with the dashed lines of the EMU 3 regime, except in case of the fiscal deficit and the EFTS. Because of the symmetries that we have imposed and the definition of differences of variables, adjustment of macroeconomic variables in country 1 identical to the adjustment in the difference system in Figure 4, whereas adjustment in country 2 exactly mirrors adjustment in country 1: its economy booms and wages jump upward rather than downward, in the EMU 2 regime it runs fiscal surplus and in the EMU 3 regime it provide a fiscal transfer to country 1.

The EFTS regime not only features output stabilization but also has the additional advantage that differences in national fiscal deficits are stabilized as compared to the regime with fiscal flexibility only, which leads to divergence of fiscal deficits in the EMU. Since fiscal convergence is stressed in the Maastricht criteria as a fundamental requirement to achieve nominal and real convergence in the EMU, this seems to be characteristic of the EFTS to prefer such a transfer system over fiscal flexibility if the EMU faces asymmetric shocks. As the demand shock dies out gradually, the transfers through the EFTS also gradually decline.
Figure 4
Adjustment after an Asymmetric Output Shock ($u_D(0)=-0.01$, $\rho=0.1$)
Conclusion

If the countries participating in the EMU do not constitute an optimal currency area, macroeconomic performance in the EMU can be seriously affected if it is hit by symmetric and in particular asymmetric macroeconomic shocks. This paper studied flexibility of national fiscal policies, monetary policy of the ECB and a European Federal Transfer System (EFTS) as stabilizing devices in an EMU hit by symmetric and asymmetric shocks, featuring nominal rigidities, low labour mobility and little automatic stabilization from federal taxes and government spending. The EFTS was defined as an intra-EMU business cycle stabilizing device and did not serve in principle any redistributional or allocational purpose.

With the aid of an analytical model and simulation analysis, this paper showed how fiscal flexibility stabilizes the output fluctuations induced by symmetric and asymmetric shocks. The EFTS not only stabilizes output fluctuations induced by asymmetric shocks but also avoids the divergence in fiscal deficits that fiscal flexibility creates during the adjustment phase. Therefore, the EFTS contributes to fiscal convergence in the EMU. Fiscal convergence is seen by many economists and politicians as a necessary prerequisite for long-run sustainability of the EMU as witnesses the prominent place of the fiscal stringency criteria in the Maastricht Treaty.

Fiscal flexibility and/or the EFTS are able to substitute to a certain extent for the loss of the asymmetric shock absorbing capacity of exchange rate adjustment that EMU implies. The EFTS to some extent could replicate the degree of automatic stabilization of macroeconomic shocks by federal transfers that is seen in more mature fiscal federations like the US. From this perspective its implementation in an EMU that does not yet constitute an optimal currency area and a mature fiscal federation, deserves serious political consideration.
Appendix A  Initial jump in wages

The \{p_i(t), w_i(t)\} systems that describe the dynamics of the average and difference system are linear dynamic systems of the form,

\[
\dot{x}_i = A_i x_i(t) + b_i + v_i(t) \quad x_i(t) = \begin{bmatrix} p_i(t) \\ w_i(t) \\ b_i \\ v_i(t) \end{bmatrix} \quad i \in \{A, D\} \tag{A.1}
\]

The average and difference system display saddlepoint stability if the number of predetermined variables equals the number of stable eigenvalues and the number of non-predetermined variables equals the number of unstable eigenvalues (see e.g. Buiter (1984)). In both cases the price level \( p_i(t) \) is a predetermined variable whereas the wage rate \( w_i(t) \) is forward looking. A necessary and sufficient condition for both systems to display saddlepoint stability is that the determinant of \( A_i \) is negative. Given the initial value \( x(0) \), the stable eigenvalue, \( \lambda \), and the steady state, \( x(\infty) \), the solution to (A.1) reads:

\[
x_i(t) = (x_i(0) - x_i(\infty)) e^{\lambda i} + x_i(0) e^{\lambda} + x_i(\infty)(1 - e^{\lambda i}) \tag{A.2}
\]

The steady-state of the system, \( x(\infty) \), is found by solving:

\[
x_i(\infty) = -A_i^{-1}(b_i + v_i(0)e^{-\rho_i}) \tag{A.3}
\]

If the dynamic systems are saddlepoint stable, the forward looking variables, i.e. \( w_i(t) \), will take unique initial values, \( w_i(0) \), such that the system is placed on its unique converging dynamic trajectory, given the initial price level \( p_i(0) \). The initial value of the forward-looking variables, \( w_i(0) \), is found by applying the method proposed by Judd (1982):

- to find an analytical solution of the dynamic system, take the Laplace transform \( L[\cdot, s] \) of the dynamic system, \( \dot{x} = Ax(t) + b_i + v_i(t) \):

\[
L[\dot{x}_i, s] = A_i L[x_i, s] + L[b_i, s] + L[v_i, s] \tag{A.4}
\]

- use the fact that \( L[\dot{x}_i, s] = sL[x_i, s] - x_i(0) \). Equating both expressions for \( L[\dot{x}_i, s] \) yields:

\[
L[x_i, s] = (sI - A_i)^{-1} [L[b_i, s] - L[v_i, s] - x_i(0)] \tag{A.5}
\]

- impose the condition that \( w_i(0) \) adjusts in such a manner that saddlepoint stability of the dynamic system is ensured, given the initial price level, \( p_i(0) \). This implies that \( L[b_i, z_i] + L[v_i, z_i] + x_i(0) = 0 \), where \( s = z_i \) with \( z_i \) being the unstable eigenvalue of \( A_i \). Imposing this convergence requirement gives the unique initial wage rate, \( w_i(0) \), consistent with long-run convergence of the dynamic system to its steady-state. If we assume e.g. that the exogenous shock decays exponentially, i.e. \( v_i(t) = v_i(0)e^{-\rho_i t} \), \( x_i(0) \) is found easily since the Laplace transform of the constant \( b_i \) is defined as \( b_i/z_i \) and the Laplace transform of the exponentially dying impulse \( v_i(t) \) as \( v_i(0)/(z_i + \rho) \).

\[13\] The Laplace transform of \( f(t) \), \( F(s) \) is defined as: \[ F(s) = \int_{0}^{\infty} f(t)e^{-st} \, dt. \]
The average system is given by:

\[
\begin{pmatrix}
\hat{p}_A \\
\hat{w}_A
\end{pmatrix} =
\begin{pmatrix}
-\delta & \delta \\
-\delta(1 - \frac{\gamma \lambda}{\Delta_A}) & \delta(1 - \frac{\gamma \delta}{\Delta_A})
\end{pmatrix}
\begin{pmatrix}
\hat{p}_A(t) \\
\hat{w}_A(t)
\end{pmatrix} + \begin{pmatrix}
0 \\
\frac{\gamma \delta}{\Delta_A} \hat{m}_A - \frac{\Delta_A}{\lambda} - \eta \chi
\end{pmatrix} - \begin{pmatrix}
-\delta & 0 \\
-\delta & -\sigma\eta
\end{pmatrix} \begin{pmatrix}
\hat{p}_A(0) e^{-\delta t} \\
\hat{w}_A(0) e^{-\sigma t}
\end{pmatrix}
\] (A.6)

The inverse of the matrix \(A_A\) equals:

\[
A_A^{-1} = \frac{1}{|A_A|} \begin{pmatrix}
\delta(1 - \frac{\gamma \delta}{\Delta_A}) & -\delta \\
\frac{v \gamma \Delta_A}{\lambda} & -\delta(1 - \frac{\gamma \delta}{\Delta_A})
\end{pmatrix}
\] (A.7)

with \(\Delta_A \equiv 1 + \gamma(\chi + \xi)/\lambda - \sigma + \eta \chi\). The determinant of \(A_A\), \(|A_A|\), which equals \(-\delta^2 \gamma / \lambda \Delta_A\), is equal to the product of the eigenvalues. A necessary and sufficient condition for the average system to display saddlepoint stability, therefore, is that \(\Delta_A > 0\). The eigenvalues of the average system equal:

\[
h_A = \frac{1}{2} \left( \frac{\gamma \delta^2}{\Delta_A} + \sqrt{\left( \frac{\gamma \delta^2}{\Delta_A} \right)^2 + 4 \left( \frac{\gamma \delta^2}{\lambda \Delta_A} \right)} \right)
\]

\[
z_A = \frac{1}{2} \left( \frac{\gamma \delta^2}{\Delta_A} - \sqrt{\left( \frac{\gamma \delta^2}{\lambda \Delta_A} \right)^2 + 4 \left( \frac{\gamma \delta^2}{\lambda \Delta_A} \right)} \right)
\] (A.8)

To determine the initial state of the average system, take the Laplace transform of (A.6) and use (A.5) to write

\[
\begin{pmatrix}
L(p_A(s)) \\
L(w_A(s))
\end{pmatrix} = \frac{1}{|sI - A_A|} \begin{pmatrix}
s + \delta & -\delta \\
\frac{v \gamma \lambda}{\Delta_A} & s + \delta(1 - \frac{\gamma \delta}{\Delta_A})
\end{pmatrix}
\begin{pmatrix}
\hat{p}_A(s) + L(\frac{\gamma \delta}{\Delta_A} \hat{v}_A(0) e^{-\delta t}, \hat{w}_A(0)) \\
L(\hat{v}_A(s)) + L(\frac{\gamma \delta}{\Delta_A} \hat{v}_A(0) e^{-\delta t}, \hat{w}_A(0))
\end{pmatrix}
\] (A.9)

where \(\hat{\theta}_A = \frac{\gamma \delta}{\lambda A} \hat{m}_A + \frac{\delta \xi}{\lambda} + \eta \chi \hat{y}_A + \eta \hat{y}_A\). Taking \(z_A\) for \(s\) and using the Laplace transforms for a constant and an exponentially decaying impulse, implying \(L(\hat{\theta}_A, z_A) = \frac{\hat{\theta}_A}{z_A}, L(\hat{v}_A(0) e^{-\delta t}, z_A) = \frac{\hat{v}_A(0)}{z_A + \hat{v}_A}\), the convergence condition requires

\[
(\hat{z}_A + \delta)\hat{p}_A(0) - \delta \hat{v}_A(0) = 0
\]

\[
\hat{w}_A(0) = \frac{\hat{z}_A + \delta}{\delta} \frac{\hat{v}_A(0) - \hat{v}_A(0)}{z_A + \hat{v}_A}
\] (A.10)

for a solution to (A.6) to remain bounded.
The difference system is given by:

\[
\begin{bmatrix}
\frac{\partial}{\partial t} p_D(t) \\
\frac{\partial}{\partial t} w_D(t)
\end{bmatrix} =
\begin{bmatrix}
-\delta \\
-\delta (1 - \frac{\nu(2a+\gamma\delta(2\mu-1))}{\Delta_D})
\end{bmatrix}
\begin{bmatrix}
\delta \\
\delta (1 - \frac{\nu\gamma\delta(2\mu-1)}{\Delta_D})
\end{bmatrix}
\begin{bmatrix}
p_D(0) \\
w_D(0)
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\frac{\nu}{\Delta_D} \eta(\chi_{D}\bar{f}_D)
\end{bmatrix}
\begin{bmatrix}
0 \\
\frac{\nu}{\Delta_D} v_D(0)e^{-\rho_D t}
\end{bmatrix}
\]

The inverse of the matrix \(A_D\) equals:

\[
A_D^{-1} = \frac{1}{|A_D|} \begin{bmatrix}
\delta(1 - \frac{\nu\gamma\delta(2\mu-1)}{\Delta_D}) & -\delta \\
-\delta(1 - \frac{\nu(2a+\gamma\delta(2\mu-1))}{\Delta_D}) & -\delta
\end{bmatrix}
\]

with \(\Delta_D = 1 + \sigma + \eta(\chi + 2\xi)\). The determinant of \(A_D\), \(|A_D|\), which equals \(-\delta^2\nu^2/\Delta_D\), is equal to the product of the eigenvalues. The necessary and sufficient condition for the difference system to display saddlepoint stability, \(|A_D| < 0\), therefore, is satisfied in all cases. The eigenvalues of the difference system equal:

\[
h_D = -\frac{1}{2} \left( \frac{\nu\gamma\delta^2(2\mu-1)}{\Delta_D} + \sqrt{\left( \frac{\nu\gamma\delta^2(2\mu-1)}{\Delta_D} \right)^2 + 4\left( \frac{\nu^2\alpha\gamma^2}{\Delta_D} \right)} \right)
\]

\[
z_D = -\frac{1}{2} \left( \frac{\nu\gamma\delta^2(2\mu-1)}{\Delta_D} - \sqrt{\left( \frac{\nu\gamma\delta^2(2\mu-1)}{\Delta_D} \right)^2 + 4\left( \frac{\nu^2\alpha\gamma^2}{\Delta_D} \right)} \right)
\]

To determine the initial state of the difference system, take the Laplace transform of (A.11) and use (A.5) to write,

\[
\begin{bmatrix}
L(p_D(s)) \\
L(w_D(s))
\end{bmatrix} = \frac{1}{s|A_D|} \begin{bmatrix}
\delta(1 - \frac{\nu(2a+\gamma\delta(2\mu-1))}{\Delta_D}) & \delta(1 - \frac{\nu\gamma\delta(2\mu-1)}{\Delta_D}) \\
\delta(1 - \frac{\nu\gamma\delta(2\mu-1)}{\Delta_D}) & \delta(1 - \frac{\nu(2a+\gamma\delta(2\mu-1))}{\Delta_D})
\end{bmatrix}
\begin{bmatrix}
L(p_D(0)) + L(\frac{\nu}{\Delta_D} v_D(0)e^{-\rho_D s}) + w_D(0) \\
L(w_D(s))
\end{bmatrix}
\]

where \(\bar{b}_D = -\frac{\nu}{\Delta_D}[\eta(\chi_{D}\bar{f}_D)]\). Taking \(z_D\) for \(s\) and using the Laplace transforms for a constant and an exponentially decaying impulse, the convergence condition requires that

\[
(z_D + \delta) p_D(0) \neq \delta \frac{\bar{b}_D}{z_D} \frac{\nu}{\Delta_D} \frac{v_D(0)}{z_D + \delta} w_D(0) = 0
\]

for a solution to (A.11) to remain bounded.
References


