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# Equilibrium Selection in Team Games <sup>\*</sup>

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## Abstract

It is shown that in team games, i.e. in games in which all players have the same payoff function, the risk-dominant equilibrium may differ from the Pareto dominant one.

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# 1 Introduction

The general theory of equilibrium selection that has been proposed in Harsanyi and Selten (1988) invokes two completely different selection criteria: risk dominance and payoff dominance. The first is based on *individual* rationality, while the second incorporates *collective* rationality. The latter criterion captures the idea that if one equilibrium  $E_1$  yields all players in the game uniformly higher payoffs than the equilibrium  $E_2$  does, then rational players are more tempted to play the former. The first criterion captures the idea that, in a situation where players do not yet know which of the two equilibria,  $E_1$  or  $E_2$ , will be chosen, players will lean towards that equilibrium that appears less risky in the situation at hand. For the case of  $2 \times 2$  games, Harsanyi and Selten give an axiomatic characterization of their risk-dominance relation. In the special case of a symmetric  $2 \times 2$  game, the risk-dominance relation can be easily characterized:  $E_1$  risk dominates  $E_2$  if and only if each player finds it optimal to play according to  $E_1$  if he expects the other to play in accordance with  $E_1$  with a probability of at least  $1/2$ .

The criteria of payoff dominance and of risk dominance may yield conflicting recommendations, and in such cases Harsanyi and Selten give precedence to payoff dominance. An example of such a conflict is illustrated in the stag hunt game from Figure 1 which has been adapted from Aumann (1990). (Also see Harsanyi and Selten (1988, Sect. 10.12)). Each player has two strategies, a safe one and a risky one, and if both play their risky strategy, the unique Pareto efficient outcome results. However, playing this strategy is very risky: If one player chooses it while the other player chooses the safe strategy, then the payoff to the first is only zero. In contrast, the safe strategy guarantees a payoff of 7, and it might even yield more. In this stag hunt game  $(R, R)$  is the payoff dominant equilibrium, while  $(S, S)$  is the risk-dominant equilibrium. (Indeed each player finds it optimal to play  $S$  as long as his opponent does not choose  $R$  with a probability more than  $7/8$ .)

	$R$	$S$
$R$	9,9	0,8
$S$	8,0	7,7

Figure 1: stag hunt

The payoff dominance requirement is based on collective rationality, i.e. on the assumption that rational individuals will cooperate in pursuing their common interests if the conditions of the game permit them to do so. Harsanyi and Selten argue that risk dominance is only important in those cases where there is some uncertainty about which equilibrium “should” be selected. If one equilibrium gives all players a strictly higher payoff than any other equilibrium (and if this equilibrium satisfies all other desirable properties that the selection theory imposes) such uncertainty will not exist – each player can be reasonably certain that all other players will opt for this equilibrium – and this makes risk-dominance comparisons irrelevant. It is this argument that leads Harsanyi and Selten to give precedence to payoff dominance.

Yet, relying on collective rationality is somewhat unsatisfactory. For one, it implies that the final theory is not ordinal, that is, two games with the same best reply structure need not have the same solutions. For example, the game from Figure 1 is best-reply-equivalent to one in which the off-diagonal payoffs are zero and in which the payoffs to  $(R, R)$  and  $(S, S)$  are  $(1,1)$  and  $(7,7)$  respectively, and, in the latter, payoff dominance selects  $(7,7)$  as the outcome. Secondly, one feels that it should be possible to obtain collective rationality as an outcome of individual rationality: If one equilibrium is uniformly better than another, then the players’ individual deliberations should bring them to play this equilibrium. The reason that this does not happen in a game as that from Figure 1 – at least if one views the risk-dominant equilibrium as the outcome of the individual deliberation process – is that indeed a player is not sufficiently certain *ex ante* that his “partner” will play the risky equilibrium. In fact, he cannot be sure of this exactly because of the fact that his partner cannot be sure that *he* will play it: one only needs a “grain of doubt” in order for it to be the unique rationalizable strategy to play safe. (See Carlsson and Van Damme (1993ab), and, for an informal argument to that extent, Schelling (1960, Chapter 9).)

The above raises the question of whether, in games in which players can indeed be quite certain that the opponents will play the payoff dominant equilibrium, or at least in games in which this equilibrium is uniquely focal, risk-dominance considerations will induce players to play this equilibrium. This note aims to address this issue, and it

provides a negative answer. We consider *team games*, that is, games in which all players have the same payoff function. Any maximum of this function is trivially an equilibrium and one might argue that, in those cases in which the maximum is unique, this maximum provides the unique focal equilibrium of the game. In other words, under those situations the conditions are most favorable for individual rationality to be in agreement with collective rationality. We provide examples to illustrate that, even in these cases, risk-dominance considerations do not necessarily lead to the playing of the unique payoff dominant equilibrium. Specifically, we show that the modified Harsanyi/Selten theory, that does not invoke payoff comparisons, may select a Pareto dominated equilibrium in a team game.<sup>1</sup>

## 2 Notation and Definitions

In this section we introduce notation, and define team games and the risk dominance relation. Readers already familiar with these concepts can immediately turn to Section 3.

Let  $\Gamma = \langle A_i, u_i \rangle_{i \in I}$  be a strategic form game.  $I$  is the player set,  $A_i$  is the set of pure strategies of player  $i$  and  $u_i : A \rightarrow \mathbb{R}$  is the payoff function of this player ( $A = \prod_{i \in I} A_i$ ). We write  $S_i$  for the set of mixed strategies of player  $i$ ,  $S = \prod_{i \in I} S_i$  for the set of mixed strategy profiles and  $u_i(s)$  for the expected payoff to player  $i$  for when  $s \in S$  is played. A strategy profile  $s^*$  is a (Nash) *equilibrium* of  $\Gamma$  if no player can improve his payoff by a unilateral change in strategy, i.e.

$$u_i(s^*) = \max_{s_i \in S_i} u_i(s^*_{-i}, s_i), \quad (2.1)$$

where  $(s^*_{-i}, s_i)$  is shorthand notation for  $(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*)$ . We write  $E(\Gamma)$  for

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<sup>1</sup>We will not spell out the full details of the Harsanyi and Selten (1988) theory. The reader is referred to the flowchart on p. 222 of their book for a quick overview of the theory. In the examples we take certain shortcuts through this flowchart. We leave it to the reader to prove that these shortcuts are justified.

the set of equilibria of  $\Gamma$ . The (linear) *tracing procedure* is a map  $T$  from  $S$  into  $E(\Gamma)$ , hence, it converts each mixed strategy profile into an equilibrium of the game. Formally the map  $T$  is associated with a homotopy. For  $s \in S$  and a homotopy parameter  $t \in [0, 1]$  write  $\Gamma^{t,s}$  for the game  $\langle A_i, u_i^{t,s} \rangle_{i \in I}$  in which the payoff function of player  $i$  is given by

$$u_i^{t,s}(a) = tu_i(a) + (1-t)u_i(s \setminus a_i). \quad (2.2)$$

Hence, for  $t = 1$  we have the original game, while for  $t = 0$  each player faces a (trivial) one-person problem. In nondegenerate cases, the game  $\Gamma^{0,s}$  contains exactly one equilibrium  $e(0, s)$  and this will remain an equilibrium of  $\Gamma^{t,s}$  as long as  $t$  is sufficiently small. Now it can be shown that the equilibrium graph

$$E = \{(t, \bar{s}) : t \in [0, 1], \bar{s} \in E(\Gamma^{t,s})\} \quad (2.3)$$

contains a unique distinguished curve  $\{e(t, s) : t \in [0, 1]\}$  that connects  $e(0, s)$  with an equilibrium  $e(1, s)$  of  $\Gamma$ . (See Harsanyi and Selten (1988) and Schanuel et al. (1992) for the technical details. In particular, the latter paper points out how  $e(1, s)$  can be found by applying the logarithmic tracing procedure.) The endpoint  $e(1, s)$  of this path is the linear trace  $T(s)$  of  $s$ . This tracing map  $T$  is used to define the risk-dominance relation.

Imagine that the players are uncertain about which of two equilibria,  $s^*$  or  $s^{**}$ , should be considered as the solution of the game. Player  $i$  assumes that his opponents already know it and he himself attaches probability  $z_i$  to the solution being  $s^*$  and the complementary probability  $1 - z_i$  to the solution being  $s^{**}$ . Obviously, in this case he will play his best response against the correlated strategy  $z_i s_{-i}^* + (1 - z_i) s_{-i}^{**}$  of his opponents. Assume that if player  $i$  has multiple best responses, he plays each of them with equal probability and denote the resulting centroid best reply by  $b_i(z_i; s^*, s^{**})$ . Now, an opponent  $j$  of  $i$  does not know  $i$ 's beliefs  $z_i$ . Assume that, according to the principle of insufficient reason, such a player considers  $z_i$  to be uniformly distributed on  $[0, 1]$ .

Clearly, an opponent will then predict  $i$  to play the mixed strategy

$$s_i(s^*, s^{**}) = \int_0^1 b_i(z_i; s^*, s^{**}) dz_i. \quad (2.4)$$

Let  $s(s^*, s^{**})$  be the mixed strategy vector determined by (2.4), the so called bicentric prior associated with  $s^*$  and  $s^{**}$ . We now say that:

- (i)  $s^*$  risk dominates  $s^{**}$  if  $T(s(s^*, s^{**})) = s^*$ ,
- (ii)  $s^{**}$  risk dominates  $s^*$  if  $T(s(s^*, s^{**})) = s^{**}$ , and
- (iii) there is no dominance relation between  $s^*$  and  $s^{**}$  if  $T(s(s^*, s^{**})) \notin \{s^*, s^{**}\}$ .

We note that the risk-dominance relation need not be transitive, nor complete. We also note that a simple characterization of this relation can be given for 2-person  $2 \times 2$  games with two strict equilibria, say  $s^*$  and  $s^{**}$ . Write  $z_i^*$  for the critical belief of player  $i$  where he is indifferent between both pure strategies, that is,  $b_i(z_i^*; s^*, s^{**}) = (1/2, 1/2)$ . Then,  $s^*$  risk dominates  $s^{**}$  if and only if  $z_1^* + z_2^* < 1$ .

The Harsanyi/Selten solution of a game is found by applying an iterative elimination procedure. Starting from an initial candidate set (consisting of all so called primitive equilibria), candidates that are payoff dominated or risk dominated are successively eliminated until exactly one candidate is left. We will consider the modification of that theory that only invokes risk dominance. In both our examples the initial candidate set will simply be the set of all pure equilibria of the game. It will be clear from the above definition that it can easily happen that there is no risk-dominance relation between pure equilibria, say  $s^*$  and  $s^{**}$ , and that both are “maximally stable”. In this case Harsanyi and Selten propose to replace the pair by one substitute equilibrium, viz. by the equilibrium  $s^{***}$  that results when the tracing procedure is applied to the mixed strategy in which each player  $i$  chooses  $1/2s_i^* + 1/2s_i^{**}$ . (Note that this, so called centroid strategy, typically differs from the bicentric prior as determined by (2.4).) Hence, in case of a deadlock with two equally strong candidates  $s^*$  and  $s^{**}$ , these equilibria are eliminated from the initial candidate set. They are replaced by the equilibrium  $s^{***}$  and

the process is restarted with this new candidate set. As in both our examples, a single equilibrium remains after at most one substitution step has been performed, there is no need to go into further details of the process.

We conclude this section by giving the definition of a *team game*.  $\Gamma = \langle A_i, u_i \rangle_{i \in I}$  is said to be a team game if  $u_i = u_j$  for  $i, j \in I$ , hence, all players always have the same payoff. Writing  $u$  for this common payoff function, we say that the team game is *generic* if there exists a unique  $a^* \in A$  at which  $u$  attains its maximum. Attention will be confined to symmetric games, i.e. the payoff to a player depends only on which actions are chosen and not on the identities of the players choosing them. Because of this symmetry we can confine ourselves to analyzing the situation from the standpoint of player 1. In the next two sections we investigate risk dominance in generic symmetric team games and compute the associated (modified) solutions.

### 3 A Two-Person Example

The discussion in this section is based on the 2-person game from Figure 2. (The right hand side of the picture displays the best reply structure associated with this game.)

	<i>T</i>	<i>M</i>	<i>B</i>
<i>T</i>	3	0	2
<i>M</i>	0	$3 - \epsilon$	2
<i>B</i>	2	2	$2 + \epsilon$

Figure 2: A Symmetric Two-Person Team Game ( $0 < \epsilon < 1$ )

The game has three (pure) equilibria with payoffs 3,  $3 - \epsilon$  and  $2 + \epsilon$ . We first investigate the risk dominance relationship between *T* and *B*. Note that when a player is uncertain whether the opponent will play *T* or *B*, but is certain that this player will not choose *M*, then this player will never be tempted to choose *M* since *M* is never a best response against a mixture  $zT + (1 - z)B$ . This shows that *M* is irrelevant for the risk-dominance relationship between *T* and *B* and that this relationship can be determined simply in



the  $2 \times 2$  reduced game spanned by  $T$  and  $B$ . Now, as is shown by the RHS of Figure 2, the bicentric prior relevant for this risk-dominance relation assigns almost all weight to  $T$ , hence, the tracing path starts at  $T$  and it stays there:  $T$  risk dominates  $B$ . A similar argument establishes that  $M$  risk dominates  $B$ .

It remains to investigate the risk-dominance relation between  $T$  and  $M$ . As is shown by the RHS of Figure 2 the bicentric prior relevant for the comparison of these equilibria, is approximately equal to  $(1/3, 1/3, 1/3)$ . It follows that the unique equilibrium of the game  $\Gamma^{0,s}$  is  $(B, B)$ . Since  $(B, B)$  is a strict equilibrium of  $\Gamma^{1,s}$  it is a strict equilibrium of  $\Gamma^{t,s}$  for all  $t \in [0, 1]$ , hence, the distinguished curve in the graph  $E$  is constantly equal to  $(B, B)$ . Therefore

$$T(s(T, M)) = B \tag{3.1}$$

and there is no risk-dominance relation between  $T$  and  $M$ . Hence, there is a deadlock: Both  $T$  and  $M$  dominate  $B$ , but  $T$  and  $M$  are equally strong. To resolve the deadlock we apply the substitution step, hence, we start the tracing procedure with the prior  $1/2T + 1/2M$ . Again, as the RHS of Figure 2 makes clear, the unique best response against this prior is  $B$ , so that the tracing path starts at  $B$  and remains there. The substitution set eliminates the pair  $\{T, M\}$  and replaces it with the equilibrium  $B$ . Hence, if we modify the theory of Harsanyi and Selten, by not imposing the payoff dominance requirement, then the equilibrium  $B$  is selected in the game of Figure 2. Individual rationality, as incorporated into risk dominance, does not lead to collectively efficient outcomes, not even in generic symmetric team games.

At the intuitive level, one may explain the phenomenon as follows. Risk considerations favor the selection of equilibria that give "reasonably good" payoffs against a set of diffuse priors: A player does not know what the others will do and he investigates what action gives good outcomes no matter what the others do. These considerations favor actions that have large stability sets, i.e. that are best responses against many mixed strategies of the opponents. In Figure 2,  $B$  is such a good and safe strategy. In fact, we could make the stability set of  $B$  to cover almost the entire strategy simplex without losing the

fact that  $T$  is the unique payoff dominant equilibrium: just replace 2 by  $3-2\epsilon$  everywhere in the payoff matrix. By increasing the payoff associated to  $B$ , one makes  $B$  more attractive, hence, at the same time  $T$  is made less attractive. What this makes clear is that there is nothing special about team games. Either one assumes that the logic of common payoffs and collective rationality is so strong that players do not have any doubt to start with about what to play, or one allows for prior doubt and then one does not see how the common payoff assumption helps to reduce it.

The doubt concerning what equilibrium to play may, for example, arise out of slight payoff uncertainty as in Carlsson and Van Damme (1993a,b). One may imagine that it is common knowledge among the players that they are playing a team game, but each player may have a tiny bit of private information about what the actual payoffs are. If the uncertainty is small, then, if the actual game is generic, the strategy combination that attains the maximum will be mutually known to a high degree. However, around a nongeneric game the latter will not hold. As Carlsson and Van Damme show, such nongeneric games exert an influence on “far removed” generic games: since players choose safe strategies in non-generic games, they are forced to choose safe strategies also in generic games, in order to avoid coordination failures.

To illustrate this argument, consider the modification from Figure 2 as in Figure 3. This is a nongeneric game. As Schelling (1960, Appendix C) already argued, the unique focal equilibrium in this game is  $B$ : If players cannot communicate, then, if they aim to coordinate on  $T$  or  $M$ , they will actually succeed only with 50% probability and, hence, it is better to play  $B$  ( $5/2 > 1/2 \cdot 3 + 1/2 \cdot 0$ ). Now, consider a game that is close to the one from Figure 3, but that is generic and that has a unique maximum associated with  $T$ . Should a player play  $T$ ? Well, if he is not exactly sure that he observed the correct payoffs the answer is: maybe not. In that case, the actual payoffs may be such that the maximum is at  $M$ , or that his opponent thinks that the maximum is there. In such a case, it might still be better to choose  $B$  and thereby avoid the coordination problem.

	$T$	$M$	$B$
$T$	3	0	2
$M$	0	3	2
$B$	2	2	$5/2$

Figure 3: A Coordination Game

To conclude this section, we give one individualistic rationality argument that, for 2-person team games, distinguishes the payoff maximal equilibrium from any other pure equilibrium. Is it true that in such games, the payoff maximal equilibrium *pairwise risk dominates* any other pure equilibrium. Specifically, if  $s^*$  is the Pareto dominant equilibrium and  $s^{**}$  is any other pure equilibrium, then in the  $2 \times 2$  game in which the players only have  $\{s^*, s^{**}\}$  available,  $s^*$  risk dominates  $s^{**}$ . Hence, the Pareto dominant equilibrium may be said to be the pairwise risk-dominant one. A proof is simple and uses the alternative characterization of risk dominance for  $2 \times 2$  games given in the previous section: Since the sum of the off-diagonal payoffs is smaller than the sum of the diagonal (equilibrium) payoffs, the sum of the players' critical probabilities for switching away from the payoff maximal equilibrium is less than one. An illustration is provided by the reduced games associated with the equilibrium  $T$  from Figure 2, which are displayed in Figure 4. In the game on the LHS,  $T$  dominates  $M$ , while in the game on the RHS,  $T$  dominates  $B$ .

	$T$	$M$		$T$	$B$
$T$	3	0		3	2
$M$	0	$3-\epsilon$		0	$2-\epsilon$

Figure 4: Reduced Games Associated with Figure 2

It should be noted that, in general, the concept of pairwise risk dominance captures the overall risk situation rather badly, see Carlsson and Van Damme (1993b). This is also evident from the LHS of Figure 4: when a player believes that his opponent may play  $T$  or  $M$ , then he has an incentive to play  $B$ , however,  $B$  is not present in the reduced game. In this respect it is also interesting to refer to the relationship between risk dominance and the stochastic stability of equilibria in an evolutionary context (Kandori et al. (1993), Young (1993)). In symmetric  $2 \times 2$  games only the risk-dominant equilibrium is stochastically stable, but Peyton Young already provided an example of a  $3 \times 3$  game in which the stochastically stable equilibrium differs from the pairwise risk-dominant one. Recently, Kandori and Rob (1993) have shown that for 2 player symmetric games that satisfy the Total Bandwagon Property (TBP) and the Monotone Share Property (MSP) only the pairwise risk-dominant equilibrium is stochastically stable. (TBP says that any best response against a mixture is an element of the mixture; MSP says that if a pure strategy is eliminated, the shares of all other pure strategies increase in the completely mixed equilibrium; the game from Figure 2 violates the Total Bandwagon Property.)

## 4 A Three-Person Example

The above two-person example is somewhat unsatisfactory since the solution process involves using the tie-breaking procedure and the latter might be considered ad hoc. The aim of this section is to provide a three-person team game that has a non-payoff maximal equilibrium that strictly risk dominates any other pure equilibrium. In fact, the example has the additional (desirable) feature that the details of the tracing procedure do not matter: since the game is symmetric, the risk-dominant solution is determined directly from the bicentric prior (2.4). Finally, the example shows that for 3-player games the Pareto dominant equilibrium need not be even pairwise risk dominant. The example in question is the game given in Figure 5. (Here  $x$  is a real number in the interval  $[0, 1]$ , player 1 chooses a row, 2 a column and 3 a matrix. Obviously, the payoff dominant equilibrium is  $L$  if  $x > 1/3$ , while it is  $R$  if  $x < 1/3$ ).

	$L$	$R$
$L$	$3x$	$2x$
$R$	$2x$	$x$
	$L$	

	$L$	$R$
$L$	$2x$	$x$
$R$	$x$	$1$
	$R$	

Figure 5: The Three-Person Team Game  $\Gamma(x)$  ( $0 < x < 1$ )

It is easily seen that  $\Gamma(x)$  has two strict Nash equilibria, viz.  $(L, L, L)$  and  $(R, R, R)$ . To determine the risk-dominance relationship between these two equilibria, we compute the bicentric prior as in (2.4). If the players 2 and 3 play  $(L, L)$  with probability  $z$  and  $(R, R)$  with probability  $1 - z$ , then player 1 strictly prefers to play  $L$  if and only if

$$zx > (1 - z)(1 - x),$$

or, equivalently,

$$z > 1 - x. \tag{4.1}$$

Applying the principle of insufficient reason, the players 2 and 3 attach a probability  $x$  to the inequality (4.1) being satisfied. Hence, the prior beliefs of the players 2 and 3 are described by player 1's mixed strategy

$$xL + (1 - x)R \tag{4.2}$$

Since the game  $\Gamma(x)$  is symmetric with respect to the players, the mixed strategy (4.2) actually is the prior of each player  $i \in \{1, 2, 3\}$ . To determine the risk-dominance

relationship between  $(L, L, L)$  and  $(R, R, R)$ , we have to determine the best response of player  $i$  when his opponents  $j$  and  $k$  independently randomize according to (4.2). The reader easily verifies that  $L$  is the unique best response if and only if

$$x(1 - (1 - x)^2) > (1 - x)^3$$

or, equivalently,

$$x^2 - 3x + 1 < 0 \tag{4.3}$$

Hence, if  $x \in \left[\frac{3-\sqrt{5}}{2}, 1\right]$  then  $(L, L, L)$  is the risk-dominant equilibrium of the game. Now  $\frac{3-\sqrt{5}}{2} > 1/3$ , so that, there exists a range where the risk-dominant solution is  $(R, R, R)$ , even though the payoff dominant equilibrium is  $(L, L, L)$ .

We conclude this section by noting that also the equilibrium selection theory that has recently been proposed in Harsanyi (1995) and that involves a multilateral risk comparison of equilibria that is not based on the tracing procedure, does not always select the payoff dominant equilibrium in team games. Harsanyi proposes to select that equilibrium that has the largest stability region. As was already indicated in Section 3, the payoff dominant equilibrium may have a rather small stability region. Also for the example discussed in this section, the reader may verify that the equilibrium with the largest stability region, need not be payoff dominant. (Harsanyi defines the stability region of a strategy as the set of correlated strategies of the opponents against which the strategy is a best response, hence, the stability region of  $L$  is the set in the three-dimensional unit simplex where  $p(R, R) \leq x$ . He does not just take the Lebesgue measure of this set but first applies a transformation of the simplex.)

## 5 Conclusion

From one point of view one might argue that a generic team game is simple to play: Firstly, the payoffs of the players coincide so that there is no conflict of interest; sec-

only, the payoff function admits a unique maximum so that there is no risk of confusion. Hence, one might say that the unique payoff dominant equilibrium is the unique focal point: Players might view the game just as a one-person decision problem and solve it accordingly. These arguments are even more compelling for symmetric games. Upon closer inspection it turns out, however, that the above argument is not entirely convincing: The Pareto dominant focal point is not robust and this is reflected in the fact that risk-dominance considerations need not select it. Hence, even in symmetric team games, collective rationality need not be implied by individual rationality. With Harsanyi and Selten (1988, p. 359) we may conclude that “if one feels that payoff dominance is an essential aspect of game theoretic rationality, then one must *explicitly* incorporate it into one’s concept of rationality.”

Nevertheless, there is a sense in which team games are different from games in which the players’ payoff functions do not coincide. Aumann (1990) argued that, in the stag hunt game of Figure 1, even communication cannot help to bring about the equilibrium  $(R, R)$  in case players are convinced *a priori* that  $(S, S)$  is the solution of the game without communication. When communication is possible each player will always (i.e. no matter how he intends to play) suggest the other to play  $R$  since he can only benefit by having the other do so. Consequently, no new information is revealed by communication, hence, communication cannot influence the outcome. One may argue that things are somewhat different if it is common knowledge that the game is a team game: In this case a player has no incentive whatever to suggest an outcome different from the payoff maximal one. It remains to be investigated whether risk dominance selects the payoff dominant equilibrium if one, or more, rounds of preplay communication are added to the game.

At a somewhat more abstract level one may raise the question of *why* payoff dominance and risk dominance may be in conflict even in team games. I conjecture that this is because of the ordinality property of the risk-dominance concept. Hence, the conjecture is that there exist two team games with the same best reply correspondence that have different payoff-dominant equilibria. Thus far, I have not been able to formally prove this conjecture.

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