Optimal conservativeness in the Rogoff (1985) model

Eijffinger, S.C.W.; Hoeberichts, M.M.; Schaling, E.

Publication date:
1995

Citation for published version (APA):
Eijffinger, S. C. W., Hoeberichts, M. M., & Schaling, E. (1995). *Optimal conservativeness in the Rogoff (1985) model: A graphical and closed-form solution*. (CentER Discussion Paper; Vol. 1995-121). Unknown Publisher.

Optimal Conservativeness in the Rogoff (1985) Model: A Graphical and Closed-Form Solution

Sylvester Eijffinger[1], Marco Hoeberichts[2], Eric Schaling[3]

November 1995

Correspondence to:
CentER for Economic Research
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands.

Abstract

In Rogoff (1985) it is shown that society can make itself better off by appointing a central banker who places an additional weight on inflation rate stabilization relative to employment stabilization. Using an envelope theorem, Rogoff shows that this additional weight must be positive and finite.

In this paper, we first use a graphical method to derive narrower boundaries for this extra weight. Then a closed-form solution is given, using Ferrari's method for solving a biquadratic equation. Finally, the comparative static properties of the optimal weight are determined by means of the graphical representation.

[1]CentER for Economic Research and Department of Economics, Tilburg University and College of Europe

[2]CentER for Economic Research and Department of Economics, Tilburg University

[3]Monetary Assessment and Strategy Division, Bank of England

1 Introduction

In this paper we develop a theoretical model, following Rogoff (1985), that provides us insight in the economic and political factors that determine the optimal degree of conservativeness of a central bank. Unlike Rogoff (1985), Fratianni and Huang (1995) and Eijffinger and Schaling (1995) we are able to derive the closed-form solution for the optimal level of conservativeness. The structure of this paper is as follows. In section 2 we present a simple macroeconomic model that corresponds with the Rogoff (1985) model as is used by Eijffinger and Schaling (1995). Then, in section 3 the optimal degree of conservativeness is derived. First, in subsection 3.1 by determining the boundaries and then in subsection 3.2 by calculating the closed-form solution. In subsection 3.3 the comparative static properties of the optimum are derived. Finally, section 4 concludes.

2 The Rogoff (1985) Model

In this section we shall present a version of the Rogoff (1985) model as it is used by Eijffinger and Schaling (1995). The model explains the relation between economic and political factors and the optimal degree of commitment or conservativeness. The degree of commitment or conservativeness is represented by the weight the central bank attaches to inflation rate stabilization relative to employment stabilization. We will derive an algebraic solution for the optimal degree of conservativeness (optimal in terms of society's loss), which is a function of the natural rate of unemployment, society's preferences for inflation rate stabilization relative to employment stabilization, the variance of productivity shocks and the benefits of unanticipated inflation (the slope of the Phillips-curve).

2.1 Timing of Events

There are two types of agents: wage setters (the union) who choose the nominal wage every period and the central bank that controls monetary policy. The timing of events is as follows. In the first stage the monetary regime is chosen by the government. In the second stage wage setters sign one-period nominal wage contracts, knowing the monetary regime and taking this information into account when forming their expectations. In the third stage random shocks to productivity realize. In the fourth stage the central bank observes the shocks and reacts according to the chosen regime. In the fifth and final stage employment is determined by competitive firms.

```
     1             2              3            4           5
-----|-------------|--------------|------------|-----------|-----
monetary      nominal wage     shocks     CB sets    employment
regime chosen contracts signed realize    monetary   determined
                                          policy
```

Figure 1: The Sequence of Events in the Rogoff model

The game between the union and the central bank will be solved as follows. First, we calculate the nominal wage, employment and, hence, the rate of unemployment as a function of the exogenous natural rate of unemployment ($\tilde{u}$), the productivity shock (μ_t), slope of the Phillips-curve ($\frac{1}{1-\beta}$) and the endogenous rate of inflation and inflationary expectations of the union. The central bank doesn't play a role yet. Then a game between the union and the central bank is played, from which the rate of inflation and the inflationary expectations of the union result as a function of the exogenous $\tilde{u}$, μ_t, χ, $\frac{1}{1-\beta}$ and ϵ, where χ measures how much society cares about unemployment relative to inflation and ϵ is the level of conservativeness of the central bank.

2.2 Aggregate Supply

As the capital stock will be assumed fixed, output is given by the following Cobb-Douglas production function:

$$Y_t = e^{\mu_t} L_t^{\beta} \bar{K}^{1-\beta},\ 0 < \beta < 1$$

Taking logarithms yields:

$$y_t = \beta l_t + (1-\beta)\bar{k} + \mu_t,\ 0 < \beta < 1 \tag{1}$$

where lower-case letters refer to natural logarithms and the subscript t denotes time. Thus, y is the log of output, l the log of employment, $\bar{k}$ the log of the fixed capital stock and μ a measure of productivity. We assume that $E_{t-1}\mu_t = 0$ and $\sigma_\mu^2 = Var(\mu_t) < \infty$, where E_{t-1} denotes expectations based on period t-1 information. Labor demand is determined by the firms setting the real wage equal to the marginal product of labor:

$$w_t - p_t = \log(\frac{\partial Y}{\partial L}) = \log\beta\ + \beta l_t + (1-\beta)\bar{k} - l_t + \mu_t$$

from which the demand for labor follows as:

$$l_t = -\frac{1}{1-\beta}(w_t - \log\beta - (1-\beta)\bar{k} - p_t - \mu_t) \tag{2}$$

where w is the log of the nominal wage and p the log of the price level. The union knows this labor demand function, but it cannot observe the values of p_t and μ_t. Therefore, it seeks to minimize $E_{t-1}(l_t - l^*)^2$, where l^* is the level of employment that would arise if the contracts could be negotiated after observing the productivity shocks. The employment target of the union, l^*, can be seen as the number of insiders. The unions set $E_{t-1}l_t = l^*$ from which the nominal wage is given by:

$$w_t = E_{t-1}p_t - (1-\beta)l^* + (1-\beta)\bar{k} + \log\beta \tag{3}$$

This is the nominal wage that the union asks in order to minimize its expected loss. Substituting (3) in the labor demand function (2), we obtain the following relation between employment and unanticipated shocks:

$$l_t = l^* + \frac{1}{1-\beta}(p_t - E_{t-1}p_t + \mu_t) \tag{4}$$

Surprise inflation decreases the real wage and causes the firms to employ more labor. Note that the capital stock, as it is assumed fixed and hence known to both the firms and the unions, does not affect employment in this model. Now, let L_s be the supply of labor. If we define the unemployment rate u as $(L_s - L)/L$, then $u \approx l_s - l$. So with equation (4) the rate of unemployment is determined by:

$$u_t = l_s - l^* - \frac{1}{1-\beta}(p_t - E_{t-1}p_t + \mu_t) \tag{5}$$

Now we define $\tilde{u} := l_s - l^*$. This is the structural component of the rate of unemployment which is determined by the employment target of the union. It does not depend on unanticipated shocks. It can be seen as the natural rate of unemployment:

$$u_t = \tilde{u} - \frac{1}{1-\beta}(p_t - E_{t-1}p_t + \mu_t) \tag{6}$$

This relation is known as the "expectations augmented Phillips curve". Unanticipated shocks to inflation or productivity make unemployment deviate from its natural rate. Anticipated shocks do not affect unemployment.

2.3 The Social Loss Function

Society dislikes inflation and unemployment. If we normalize $P_{t-1} = 1$, then the rate of inflation can be approximated by p_t. Now the social loss function is defined by:

$$S_t = \frac{1}{2}p_t^2 + \frac{\chi}{2}u_t^2 \tag{7}$$

where $0 < \chi < \infty$ is the weight of unemployment stabilization relative to inflation stabilization in society's preferences. It can be seen as the objective of the government to minimize S_t. The only instrument for the government to achieve its goal is to appoint a central banker who has the appropriate level of conservativeness. Conservativeness is defined by Rogoff (1985) as the additional weight that the central banker places on inflation stabilization (price stability) compared with society. This central banker plays a game with the union from which the level of inflation p_t and the rate of unemployment u_t result as a function of the exogenous variables and the level of conservativeness of the central bank. Note that in this setting ($p_{t-1} = 0$) the log of the price level is the same as the level of inflation. In the next subsection we will calculate society's loss for an arbitrarily conservative central banker.

2.4 Time Consistent Equilibrium under a Conservative Central Bank

Rogoff demonstrates that it is optimal for society to select an agent to head the central bank who places a greater weight on inflation stabilization than society does in its loss function. However, keep the loss function more general and allow also for a smaller (but positive) weight. So the central banker minimizes:

$$I_t = \frac{1+\epsilon}{2}p_t^2 + \frac{\chi}{2}u_t^2, \ -1 < \epsilon < \infty. \tag{8}$$

Note that if $\epsilon = 0$, I_t reduces to the social loss function. The coefficient ϵ can be seen as a measure of conservativeness of the central bank.

If we use the expression for unemployment (6), the central bank's loss function, dependent on the inflationary expectations of the union, becomes:

$$I_t = \frac{1+\epsilon}{2}p_t^2 + \frac{\chi}{2}\left(\tilde{u} - \frac{1}{1-\beta}p_t + \frac{1}{1-\beta}E_{t-1}p_t - \frac{1}{1-\beta}\mu_t\right)^2 \tag{9}$$

The central bank minimizes its loss function with respect to the price level, i.e. $\frac{\partial I_t}{\partial p_t} = 0$. After rearranging terms, the following price level, contingent on the union's inflationary expectations, results:

$$p_t = \frac{\chi(1-\beta)}{(1-\beta)^2(1+\epsilon)+\chi}\tilde{u} + \frac{\chi}{(1-\beta)^2(1+\epsilon)+\chi}(E_{t-1}p_t - \mu_t) \tag{10}$$

This is the central banker's reaction function to the union's inflationary expectations.

As the union knows the reaction function of the central bank, it will base its inflationary expectations on this expression (10). However, the union cannot observe the values of p_t and μ_t. Therefore, we take expectations conditional on information at t-1. Rearranging terms yields the following expression for the inflationary expectations of the union:

$$E_{t-1}p_t = \frac{\chi}{(1-\beta)(1+\epsilon)}\tilde{u} \tag{11}$$

As the central bank knows this rule followed by the union, it determines its optimal price level by substituting (11) into (10). The resulting price level as a function of only the exogenous variables is:

$$p_t = \frac{\chi}{(1-\beta)(1+\epsilon)}\tilde{u} - \frac{\chi}{(1-\beta)^2(1+\epsilon)+\chi}\mu_t \tag{12}$$

If we substitute (11) and (12) into equation (6), the resulting unemployment rate as a function of only the exogenous variables is:

$$u_t = \tilde{u} - \frac{(1-\beta)(1+\epsilon)}{(1-\beta)^2(1+\epsilon)+\chi}\mu_t \tag{13}$$

This is the outcome of the game between the union and the central bank for a given value of ϵ, i.e. for a given level of conservativeness. We are now able to calculate society's expected loss. If we substitute the expressions (12) for p_t and (13) for u_t in the social loss function S_t (7) we get:

$$E_{t-1}(S_t) = \frac{1}{2}E_{t-1}(p_t)^2 + \frac{\chi}{2}E_{t-1}(u_t)^2$$

$$= \frac{\chi\tilde{u}^2}{2} + \frac{\chi^2}{2(1-\beta)^2(1+\epsilon)^2}\tilde{u}^2 + \frac{\chi[(1+\epsilon)^2(1-\beta)^2+\chi]}{2[(1+\epsilon)(1-\beta)^2+\chi]^2}\sigma_\mu^2 \tag{14}$$

Now the social loss function consists of three elements. The first element is a *deadweight loss* that cannot be reduced through monetary policy. The second element represents society's *credibility component* that can be reduced by making ϵ larger. The third element is the *flexibility component* which, as a result of two opposite effects, can be reduced by making the central bank less conservative. This is the well-known trade off between credibility and flexibility.

2.5 The Optimal Degree of Central Bank Conservativeness

Now we know the expected loss of society as a function of the exogenous variables $\tilde{u}$, χ, $(1-\beta)^{-1}$ and σ_μ^2 and the degree of conservativeness of the central bank, ϵ. What kind of central banker should be appointed? Someone who shares society's preferences ($\epsilon = 0$), someone who just cares about inflation ($\epsilon \to \infty$) or someone who places a larger but finite weight on inflation stabilization ($0 < \epsilon < \infty$). The optimal value of ϵ minimizes $E_{t-1}S_t$. Then we have to determine the value of ϵ so that:

$$\frac{\partial E_{t-1}S_t(\epsilon)}{\partial \epsilon} = \frac{-\chi^2 \tilde{u}^2}{(1-\beta)^2(1+\epsilon)^3} + \frac{\chi^2 \sigma_\mu^2 \epsilon (1-\beta)^2}{[(1+\epsilon)(1-\beta)^2 + \chi]^3} = 0 \qquad (15)$$

In this first-order condition, the first term is always negative (or zero). This means that a larger ϵ reduces the credibility component of the loss function, as stated in the previous section. The second term represents the marginal cost of an increase in ϵ in terms of flexibility. This term is always positive (or zero). From this first-order condition it can easily be seen that $\epsilon = 0$ is not optimal. Perfect stabilization yields $\frac{\partial E_{t-1}S_t(0)}{\partial \epsilon} = -\frac{\chi^2 \tilde{u}^2}{(1-\beta)^2} < 0$, so that a larger ϵ will lead to a smaller expected loss. On the other hand, if ϵ becomes large and inflation is being minimized, the marginal cost of reducing ϵ is small relative to the stabilization gain. This can be seen on the first-order condition because the first component converges faster to zero than the second, so that $\frac{\partial E_{t-1}S_t(\epsilon)}{\partial \epsilon} > 0$ if $\epsilon \to \infty$. Now we know that for an optimal degree of conservativeness of the central bank, we must have $0 < \epsilon < \infty$.

In order to find the optimal ϵ, an equation of fourth degree in ϵ must be solved.

3 Solving the Rogoff (1985) model

3.1 Determining the Boundaries for the Optimal Degree of Conservativeness

Using an envelope theorem, Rogoff is only able to say that the optimal degree of conservativeness must be between zero and infinity. However, by means of a graphical method[1] we are able to derive narrower boundaries for the optimal degree of conservativeness. First, we rewrite the first-order condition (15) as:

$$\epsilon = \frac{[(1+\epsilon)(1-\beta)^2+\chi]^3\tilde{u}^2}{\sigma_\mu^2(1-\beta)^4(1+\epsilon)^3} =: F(\epsilon) \tag{16}$$

As $F(0) > 0$ and $\frac{\partial F}{\partial \epsilon} < 0$ (See Appendix B) the continuous functions $F(\epsilon)$ and $G(\epsilon) := \epsilon$ (i.e. a 45-degree line through the origin) must intersect at one and only one $\epsilon > 0$. This intersection occurs at the optimal value of ϵ. By calculating the limits of the function F for $\epsilon = 0$ and $\epsilon \to \infty$ (See Appendix B), it can be shown that the optimal ϵ^* is bounded between $\frac{\tilde{u}^2(1-\beta)^2}{\sigma_\mu^2}$ and $\frac{\tilde{u}^2[(1-\beta)^2+\chi]^3}{\sigma_\mu^2(1-\beta)^4}$. Figure 2 illustrates the argument graphically.

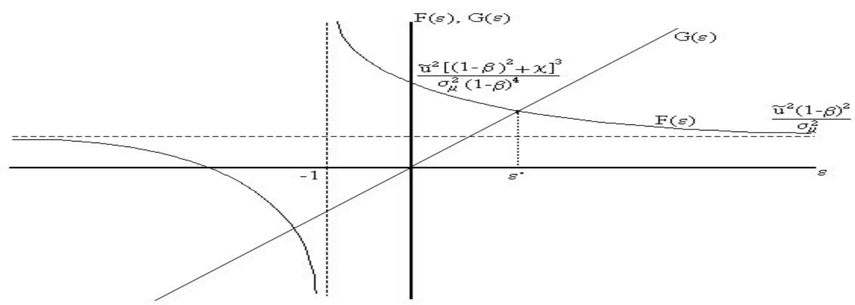

Figure 2: The Optimal Degree of Conservativeness

Clearly, another point of intersection exists. However, this occurs at $\epsilon < -1$. These points give a negative preference weight (See equation (8)) and are therefore excluded.

---

[1] See also Cukierman (1992) pp. 190-192 for an analogous application of this method.

3.2 Analytical Solution for the Optimal Degree of Conservativeness

In this section we will derive an expression for the optimal degree of conservativeness as a function of the exogenous variables. We solve the first-order condition, which is an equation of fourth degree. In order to solve an equation of fourth degree, Ferrari's method can be used. For a complete description of this method, see Uspensky (1948).

For notational convenience, we define the parametric abbreviations $\psi := \frac{\chi}{1+\epsilon}$ which is the ratio of preferences and $D := (1-\beta)^2$ which is the squared capital's share of income. Using this notation, the first-order condition (15) can be rewritten as:

$$H(\psi) = \psi^4 + 3D\psi^3 + 3D^2\psi^2 + \left(D^3 + \frac{D^2\sigma_\mu^2}{\tilde{u}^2}\right)\psi - \frac{D^2\sigma_\mu^2\chi}{\tilde{u}^2} = 0 \quad (17)$$

The boundary condition as derived in the previous section now becomes that the optimal ratio of preferences, ψ^*, is bounded between $\frac{\chi\sigma_\mu^2 D^2}{\sigma_\mu^2 D^2 + (D+\chi)^3\tilde{u}^2}$ and $\frac{\chi\sigma_\mu^2}{\sigma_\mu^2 + D\tilde{u}^2}$. Equation (17) can be solved for ψ by using Ferrari's method.

First, we write the equation as:

$$\psi^4 + 3D\psi^3 = -3D^2\psi^2 - \left(D^3 + \frac{D^2\sigma_\mu^2}{\tilde{u}^2}\right)\psi + \frac{D^2\sigma_\mu^2\chi}{\tilde{u}^2} \quad (18)$$

Adding $\frac{9D^2}{4}\psi^2 + Y(\psi^2 + \frac{3}{2}D\psi) + \frac{Y^2}{4}$ to both sides yields[2]:

$$\left(\psi^2 + \frac{3}{2}D\psi + \frac{Y}{2}\right)^2 =$$

$$\left(-\frac{3}{4}D^2 + Y\right)\psi^2 + \left(\frac{3}{2}DY - \frac{D^2\sigma_\mu^2}{\tilde{u}^2} - D^3\right)\psi + \left(\frac{1}{4}Y^2 + \frac{D^2\sigma_\mu^2\chi}{\tilde{u}^2}\right) \quad (19)$$

This expression (19) has the attractive feature that the left-hand term is a perfect square. If we are able to write the right-hand side as a perfect

[2]In general, with $f(x) := x^4 + ax^3 + bx^2 + cx + d = 0$, the term $\frac{a^2}{4} + y\left(x^2 + \frac{a}{2}x\right) + \frac{y^2}{4}$ should be added to both sides of the equation.

square as well, then we are able to solve the equation. As the auxiliary variable Y still is indeterminate so that all results hold for every Y, we want to determine Y so that the right-hand side of this equation can be written in the form $(e\psi + f)^2$. In general, $A\psi^2 + B\psi + C = (e\psi + f)^2$ if $B^2 - 4AC = 0$. Applied to the right-hand side of equation (19) this condition yields:

$$\left(\frac{3}{2}DY - \frac{D^2\sigma_\mu^2}{\tilde{u}^2} - D^3\right)^2 - 4\left(Y - \frac{3}{4}D^2\right)\left(\frac{1}{4}Y^2 + \frac{D^2\sigma_\mu^2\chi}{\tilde{u}^2}\right) = 0 \quad (20)$$

This is an equation of third degree in Y. Now we define $Z := Y - D^2$ so that $Y = Z + D^2$.[3] When we substitute this into equation (20), the square term drops out and we obtain:

$$Z^3 + \left(\frac{3D^3\sigma_\mu^2 + 4D^2\sigma_\mu^2\chi}{\tilde{u}^2}\right)Z + \frac{D^4\sigma_\mu^2}{\tilde{u}^2}\left(D + \chi - \frac{\sigma_\mu^2}{\tilde{u}^2}\right) = 0 \quad (21)$$

We solve this equation for Z in Appendix A, using Cardan's method.

Now, with $Y^* = Z^* + D^2$ we know that the right-hand side of equation (19) can be written in the form $(e\psi + f)^2$. Take $e = \sqrt{Z^* + \frac{1}{4}D^2}$ and $f = \frac{\frac{3}{2}DZ^* + \frac{1}{2}D^3 - \frac{D^2\sigma_\mu^2}{\tilde{u}^2}}{\sqrt{4Z^* + D^2}}$. Now the whole problem boils down to solving:

$$\left(\psi^2 + \frac{3D}{2}\psi + \frac{Z^* + D^2}{2}\right)^2 = \left(\psi\sqrt{Z^* + \frac{1}{4}D^2} + \frac{\frac{3}{2}DZ^* + \frac{1}{2}D^3 - \frac{D^2\sigma_\mu^2}{\tilde{u}^2}}{\sqrt{4Z^* + D^2}}\right)^2 \quad (22)$$

for ψ. This biquadratic equation can be split up into two quadratic equations with two (real and complex) solutions each:

$$\psi_{1,2}^* = -\frac{3}{4}D + \frac{1}{2}\sqrt{\frac{1}{4}D^2 + Z^*} \pm \frac{1}{2}\sqrt{\frac{1}{2}D^2 - Z^* - \frac{D^2}{\sqrt{D^2 + 4Z^*}}\left(\frac{4\sigma_\mu^2}{\tilde{u}^2} - \frac{1}{2}D\right)} \quad (23)$$

$$\psi_{3,4}^* = -\frac{3}{4}D - \frac{1}{2}\sqrt{\frac{1}{4}D^2 + Z^*} \pm \frac{1}{2}\sqrt{\frac{1}{2}D^2 - Z^* + \frac{D^2}{\sqrt{D^2 + 4Z^*}}\left(\frac{4\sigma_\mu^2}{\tilde{u}^2} - \frac{1}{2}D\right)} \quad (24)$$

[3] When $f(x) := x^3 + ax^2 + bx + c = 0$, choose $z := x - \frac{a}{3}$. After this transformation the square term drops out and $f(z) := z^3 + pz + q$ will follow.

As Figure 2 in the previous section shows, only two real solutions exist. The other two must be complex. Which are real and which are complex, is determined by the last term in the last square root: if $D > \frac{8\sigma_\mu^2}{\tilde{u}^2}$ then $\psi_{1,2}^*$ are the real solutions, otherwise $\psi_{3,4}^*$. From equation (17) and Figure 3 it can easily be seen that there is one positive and one negative solution for $H(\psi) = 0$.

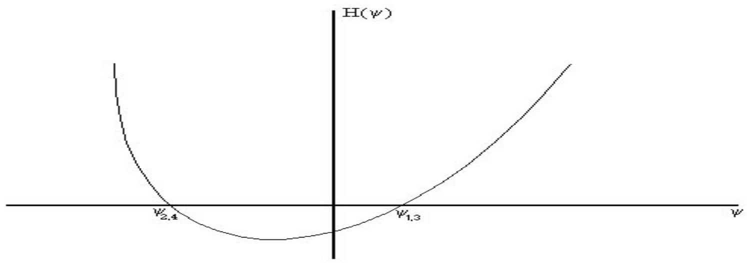

Figure 3: The Real Solutions for the Ratio of Preferences of the Central Bank

The second-order condition for a minimum that:

$$4\psi^{*3} + 9D\psi^{*2} + 6D^2\psi^* + D^3 + \frac{D^2\sigma_\mu^2}{\tilde{u}^2} > 0 \tag{25}$$

is satisfied for $\psi > 0$. Only one real candidate (ψ_1 or ψ_3) for the optimum remains, the other real root (ψ_2 or ψ_4) is maximizing the loss:

$$\psi_1^* = -\frac{3}{4}D + \frac{1}{2}\sqrt{\frac{1}{4}D^2 + Z^*} + \frac{1}{2}\sqrt{\frac{1}{2}D^2 - Z^* - \frac{D^2}{\sqrt{D^2 + 4Z^*}}\left(\frac{4\sigma_\mu^2}{\tilde{u}^2} - \frac{1}{2}D\right)} \tag{26}$$

$$\psi_3^* = -\frac{3}{4}D - \frac{1}{2}\sqrt{\frac{1}{4}D^2 + Z^*} + \frac{1}{2}\sqrt{\frac{1}{2}D^2 - Z^* + \frac{D^2}{\sqrt{D^2 + 4Z^*}}\left(\frac{4\sigma_\mu^2}{\tilde{u}^2} - \frac{1}{2}D\right)} \tag{27}$$

Which of these two is real, is determined by the last term in the last square root. If $D > \frac{8\sigma_\mu^2}{\tilde{u}^2}$ then ψ_1^* is the real solution, otherwise ψ_3^*.[4] Now,

[4] It is hard to say which of these two solutions is most likely since $\frac{\sigma_\mu^2}{\tilde{u}^2}$ is difficult to assess.

the optimal degree of central bank conservativeness, ϵ^*, can be computed with:

$$\epsilon^* = \frac{\chi}{\psi^*} - 1$$

3.3 Comparative Static Properties of the Optimal Degree

As a consequence of the complicated structure of the expression for the optimal degree of central bank conservativeness, it is extremely difficult to determine the signs of the partial derivatives of ϵ with respect to $\chi, \tilde{u}, \sigma_\mu^2$ and $\frac{1}{1-\beta}$ (the slope of the Phillips curve). However, the signs can be determined by means of the graphical representation. Using this representation, it is possible to derive the comparative static properties of the optimal value of ϵ by analyzing the properties of $F(\epsilon)$. If one of the exogenous variables $\tilde{u}$, χ, $(1-\beta)^{-1}$ or σ_μ^2 causes a rise of $F(\epsilon)$ so that $F(\epsilon) > G(\epsilon)$ for a given ϵ, then ϵ must increase in order to restore the balance. Figure 2 illustrates the argument graphically.

So if we want to know how a change in the exogenous variables affects the optimal value of ϵ, it suffices to look at the partial derivatives of $F(\epsilon)$ with respect to the exogenous variables. This leads to the following four propositions:

First, according to equation (11) a higher natural rate of unemployment leads to a higher expected level of inflation. Therefore the credibility problem gets bigger which can be reduced by a higher ϵ (See (B. 3)):

$$\frac{\partial \epsilon}{\partial \tilde{u}} > 0$$

Second, if society becomes more concerned about unemployment, according to equation (11) the expected level of inflation goes up. Therefore the credibility problem gets bigger so that ϵ must rise (See (B. 4)):

$$\frac{\partial \epsilon}{\partial \chi} > 0$$

Third, if the variance of the productivity shocks increases, the economy becomes more unstable and the need for active stabilization increases (the stabilization component in society's loss function increases), so the optimal degree of central bank conservativeness decreases (See (B. 5)):

$$\frac{\partial \epsilon}{\partial \sigma_\mu^2} < 0$$

Fourth, if the benefits of surprise inflation rise, it becomes more tempting to inflate the economy. As a consequence the expected level of inflation rises and society's credibility problem becomes more important and the optimal ϵ must rise (See (B. 6)):

$$\frac{\partial \epsilon}{\partial (1-\beta)^{-1}} > 0 \text{ if } \chi > \frac{(1+\epsilon)(1-\beta)^2}{2}$$

As can be seen in equation (6) $(1-\beta)^{-1}$ is also a multiplier for the productivity shocks μ_t. The larger this multiplier, the more unstable the economy becomes and the more need for stabilization there is. If χ becomes smaller, this effect on the stabilization component outweighs the effect on the credibility component and ϵ must fall:

$$\frac{\partial \epsilon}{\partial (1-\beta)^{-1}} < 0 \text{ if } \chi < \frac{(1+\epsilon)(1-\beta)^2}{2}$$

4 Conclusion

In this paper we presented the Rogoff (1985) model. Using a graphical method, we are able to express the upper and lower bounds of the interval containing the optimal degree of conservativeness in terms of the structural parameters of the model. Then we solve the first-order condition and find a closed-form solution for the optimal degree of conservativeness. Finally, we use a graphical representation to determine the comparative static properties of the optimum. The optimal degree of conservativeness depends positively on the natural rate of unemployment and society's preference weight for employment stabilization. It depends negatively on the variance of the productivity shocks. The effect of a rise in the slope of the Phillips curve is conditional but likely to be positive.

Appendix A: Solving of Auxiliary Variable Z

In order to solve equation (21) by Cardan's method as described in Uspensky (1948), we first have to define the following symbols:

$$p := \frac{D^2\sigma_\mu^2}{\tilde{u}^2}(3D + 4\chi) \tag{A. 1}$$

$$q := \frac{D^4\sigma_\mu^2}{\tilde{u}^2}\left(D + \chi - \frac{\sigma_\mu^2}{\tilde{u}^2}\right) \tag{A. 2}$$

$$\Lambda := 4p^3 + 27q^2 \tag{A. 3}$$

$$\sqrt{\frac{\Lambda}{108}} = \frac{D^3\sigma_\mu^2}{\tilde{u}^2}\sqrt{\frac{\sigma_\mu^2}{27\tilde{u}^2}(3D + 4\chi)^3 + \frac{D^2}{4}\left(D + \chi - \frac{\sigma_\mu^2}{\tilde{u}^2}\right)^2} \tag{A. 4}$$

$$A := -\frac{q}{2} + \sqrt{\frac{\Lambda}{108}} \tag{A. 5}$$

$$B := -\frac{q}{2} - \sqrt{\frac{\Lambda}{108}} \tag{A. 6}$$

Now a solution for $Z^* = \sqrt[3]{A} + \sqrt[3]{B}$ is:

$$Z^* = \sqrt[3]{\frac{D^4\sigma_\mu^2}{2\tilde{u}^2}\left(\frac{\sigma_\mu^2}{\tilde{u}^2} - D - \chi\right) + \frac{D^3\sigma_\mu^2}{\tilde{u}^2}\sqrt{\frac{\sigma_\mu^2}{27\tilde{u}^2}(3D + 4\chi)^3 + \frac{D^2}{4}\left(D + \chi - \frac{\sigma_\mu^2}{\tilde{u}^2}\right)^2}}$$

$$+\sqrt[3]{\frac{D^4\sigma_\mu^2}{2\tilde{u}^2}\left(\frac{\sigma_\mu^2}{\tilde{u}^2} - D - \chi\right) - \frac{D^3\sigma_\mu^2}{\tilde{u}^2}\sqrt{\frac{\sigma_\mu^2}{27\tilde{u}^2}(3D + 4\chi)^3 + \frac{D^2}{4}\left(D + \chi - \frac{\sigma_\mu^2}{\tilde{u}^2}\right)^2}} \tag{A. 7}$$

Appendix B: Derivation of the Properties of the Function $F(\epsilon)$ in the First-Order Condition

- Demonstration that $\frac{\partial F}{\partial \epsilon} < 0$.

  The first derivative of F with respect to ϵ is given by:

  $$\frac{\partial F}{\partial \epsilon} = \frac{-3\tilde{u}^2[(1+\epsilon)(1-\beta)^2+\chi]^2}{\sigma_\mu^2(1-\beta)^4(1+\epsilon)^4} \tag{B. 1}$$

  which is negative.

- Demonstration that $\frac{\partial^2 F}{\partial \epsilon^2} > 0$.

  The second derivative of F with respect to ϵ is given by:

  $$\frac{\partial^2 F}{\partial \epsilon^2} = \frac{6\tilde{u}^2\chi\Gamma[\Gamma-\chi]}{\sigma_\mu^2(1-\beta)^4(1+\epsilon)^5} \tag{B. 2}$$

  where $\Gamma \equiv (1+\epsilon)(1-\beta)^2+2\chi$, (B. 2) is positive.

- Demonstration that $F(0) = \frac{[(1-\beta)^2+\chi]^3\tilde{u}^2}{\sigma_\mu^2(1-\beta)^4}$.

  This can be shown by direct examination of the right-hand side of equation (16) at $\epsilon = 0$.

- Demonstration that $\frac{(1-\beta)^2\tilde{u}^2}{\sigma_\mu^2} < F(\epsilon) < \frac{[(1-\beta)^2+\chi]^3\tilde{u}^2}{\sigma_\mu^2(1-\beta)^4}$.

  Since

  $$F(0) = \frac{[(1-\beta)^2+\chi]^3\tilde{u}^2}{\sigma_\mu^2(1-\beta)^4}$$

  $$\lim_{\epsilon\to\infty} F(\epsilon) = \frac{(1-\beta)^2\tilde{u}^2}{\sigma_\mu^2}$$

  and $\frac{\partial F}{\partial \epsilon} < 0$, $F(\epsilon)$ must be bounded between $\frac{(1-\beta)^2\tilde{u}^2}{\sigma_\mu^2}$ and F(0).

- Demonstration that $\frac{\partial F}{\partial \tilde{u}} > 0$.

  The first derivative of F with respect to $\tilde{u}$ is given by

  $$\frac{\partial F}{\partial \tilde{u}} = \frac{2[(1+\epsilon)(1-\beta)^2+\chi]^3\tilde{u}}{\sigma_\mu^2(1-\beta)^4(1+\epsilon)^3} \tag{B. 3}$$

So (B. 3) is positive.

- Demonstration that $\frac{\partial F}{\partial \chi} > 0$

  The first derivative of F with respect to χ is given by

$$\frac{3[(1+\epsilon)(1-\beta)^2+\chi]^2\tilde{u}^2}{\sigma_\mu^2(1-\beta)^4(1+\epsilon)^3} \quad \text{(B. 4)}$$

  It can easily be checked that (B. 4) is positive.

- Demonstration that $\frac{\partial F}{\partial \sigma_\mu^2} < 0$.

  The first derivative of F with respect to σ_μ^2 is given by:

$$\frac{\partial F}{\partial \sigma_\mu^2} = \frac{-[(1+\epsilon)(1-\beta)^2+\chi]^3\tilde{u}^2}{\sigma_\mu^2(1-\beta)^4(1+\epsilon)^3} \quad \text{(B. 5)}$$

  (B. 5) is negative.

- Demonstration that $\frac{\partial F}{\partial (1-\beta)^{-1}} > 0$.

  The first derivative of F with respect to $(1-\beta)^{-1}$ is given by:

$$\frac{\partial F}{\partial (1-\beta)^{-1}} = \frac{[2\chi-(1+\epsilon)(1-\beta)^2][(1+\epsilon)(1-\beta)^2+\chi]^2 2\tilde{u}^2}{\sigma_\mu^2(1-\beta)^3(1+\epsilon)^3} \quad \text{(B. 6)}$$

  (B. 6) is positive if $\chi > \frac{(1+\epsilon)(1-\beta)^2}{2}$.

References

Cukierman, A. (1992), *'Central Bank Strategy, Credibility and Independence: Theory and Evidence'*, Cambridge: MIT Press.

Eijffinger, S. and Haan, J. De (1995), 'The Political Economy of Central Bank Independence', Discussion Paper, CentER for Economic Research, No. 9587.

Eijffinger, S. and Schaling, E. (1995), 'The Ultimate Determinants of Central Bank Independence', Forthcoming in: S. Eijffinger and H. Huizinga (eds.), *'Positive Political Economy: Theory and Evidence'*, Cambridge: Cambridge University Press.

Fratianni, M. and Huang, H. (1995), 'Central Bank Reputation and Conservativeness', Discussion Paper, London School of Economics, Financial Markets Group, No. 216.

Lohmann, S. (1992), 'Optimal Commitment in Monetary Policy: Credibility versus Flexibility', *American Economic Review*, 82, March, pp. 273-286.

Rogoff, K. (1985), 'The Optimal Degree of Commitment to an Intermediate Monetary Target', *Quarterly Journal of Economics*, 100, pp. 1169-1190.

Schaling, E. (1995), *'Institutions and Monetary Policy: Credibility, Flexibility and Central Bank Independence'*, Aldershot: Edward Elgar Publishing Limited.

Uspensky, J. (1948), *'Theory of Equations'*, New York: McGraw-Hill, pp. 83-98.