# What explains the difference between the futures' price and its "fair" value?

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# What Explains the Difference Between the Futures' Price and its "Fair" Value? Evidence from the European Options Exchange

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*Abstract*

This paper analyzes systematic deviations of the observed futures price from the value predicted by the simple cost-of-carry relationship. A model to explain this deviation (the basis) is presented in Chen, Cuny, and Haugen (1995, henceforth CCH). According to CCH, the basis should be negatively related to the return volatility of the underlying instrument. CCH themselves find support for their model on data for S&P 500 contracts in the USA. However, since the data used by CCH in testing their model at least to some extent was familiar to them when developing the model, there is a need for a test on data that is completely independent of their data. The purpose of our study is to report the results of such a test. The data is for stock index futures on the European Options Exchange in Amsterdam. The period covered is 1991 through 1993. Our results are consistent with the predictions of the CCH model. An increase in perceived volatility of the underlying index will cause a drop in the basis, as well as an increase in the open interest on the futures market.

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## 1. Introduction

The most popular model for the pricing of futures contracts is the cost-of-carry model[1]. The cost-of-carry model states that the futures price should equal its "fair" value. For a dividend paying underlying instrument this "fair" value is simply the prolonged value of the spot minus any dividends, that accrue before expiration, prolonged to the expiration date. In the following the term "fair" price, or "fair" futures price will be used to denote this dividend-adjusted prolonged value of the spot.

In practice, the simple cost-of-carry relationship is obscured by a number of factors. As shown by Cox, Ingersoll and Ross (1981), and Richard and Sundaresan (1981), the futures price may differ from the forward price if interest rates are stochastic. Hemler and Longstaff (1991), using a general equilibrium model of the Cox, Ingersoll and Ross (1985) type, show that this will indeed be the case if there is technological change that introduce randomness into the economy.

Trading frictions introduce additional deviations from the cost-of-carry relationship. The most obvious trading frictions are the fees and the bid-ask spread that traders have to pay to brokers, and to the Exchange. These tangible transaction costs will establish arbitrage bounds around the "fair" value. As long as observed prices are inside these bounds, an arbitrage operation, of shorting the more expensive alternative and buying the cheaper one, will not produce a risk-free profit.

Even within the arbitrage bounds the observed price should be pulled towards the fair price. Investors should simply opt for the cheaper one of the two identical alternatives, thus making the prices converge. Divergencies from the "fair" value are expected if tangible transaction costs, or liquidity, differ between the markets[2]. However, in the absence of systematic differences between the markets the divergence should be randomly distributed around zero.

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[1] This model explains the pricing of forward contracts but it is generally applied also to the pricing of futures contracts. A difference in prices is produced by the marked-to-market procedure which is used in trading futures contracts but not in forwards. There are many studies showing that this difference is marginal, see e.g. Cornell and Reinganum (1981), and French (1983).

[2] These costs include the impact on the price in response to a larger order or the need to split the order into smaller batches to be executed at different points in time.

Systematic deviations from the "fair" value may arise if investors experience substantial differences between the two markets. In the Chen, Cuny, and Haugen (1995) model, investors differ with respect to the customization value they attach to a particular investment. A high customization value for owning shares of a specific stock may arise 1) due to the need to hedge against risks produced by a large stake in non-tradable assets, 2) due to favourable private information, or 3) due to potential tax timing benefits related to capital gains taxation. Since having a customization value is specific for shares of stock but not for a futures position, investors faced with an increase in market risk tend to adjust the riskiness of their asset portfolios with the help of futures rather than by selling their stocks. This implies that the futures price will drop relative to its "fair" value, that is, the basis will decrease when market volatility goes up. Furthermore, since investors with a customization value attached to their stock-holdings respond to an increase in volatility by increasing the magnitude of their futures position, there should also be a positive relationship between the open interest in futures and the volatility of the underlying.

Empirical research tend to reject the simple cost-of-carry model in favour of more complicated alternatives. MacKinlay and Ramaswamy (1988) conclude that there are persistent deviations from the cost-of-carry model that increase in size with the time to maturity of the contract. Hemler and Longstaff (1991) find a significant relationship between the dividend adjusted spot-to-futures price-ratio and the volatility of the index. However, the sign of the relationship that they obtain for the three-month contracts is the opposite of that for one-month contracts. Furthermore, their results reveal a concave relationship between the dividend-adjusted price difference and the interest rate.

Chen, Cuny and Haugen (1995) find a robust, statistically significant, negative relationship between the basis, i.e. the difference between the "fair" value and the observed futures price, and volatility. They also find a significant relationship between open interest in futures contracts and volatility. The results are not sensitive to how volatility is measured, whether by the standard deviation implied by options prices or by the standard deviation based on past values of the underlying index.

The main purpose of this paper is to test the CCH model on an independent set of data. Furthermore, we shall briefly investigate to what extent the data seem to support the more complicated equilibrium model by Hemler and Longstaff (1991). The main data are from the European Options Exchange (EOE). The tests are performed on futures contracts[3] on the EOE Dutch Stock Index that expired during January 1991 to January 1994.

The structure of the paper is the following: the next section presents the institutional setting and the data. This is followed by a section in which the hypotheses to be tested are specified and our test results are presented. The last section summarizes the paper.

## 2. The Data

The data used in this study are from the European Options Exchange in the Netherlands. An advantage of using data for the Netherlands is the fact that there are no taxes on capital gains in the Netherlands. This eliminates one of the complications that have to be faced on most other markets. In the framework of the CCH-model the absence of capital gains taxes should reduce the differences in customization values that different investors attach to holding particular shares.

More specifically, we are using futures contracts on the Amsterdam EOE-index. This index consists of 25 of the most actively traded stocks from the Amsterdam Stock Exchange (ASE). The index is a portfolio with a given number of shares in each of the 25 companies. The numbers are adjusted in response to stock splits, stock dividends and rights issues. The index was not value weighted[4] in our sample period but the actual weight of a given company still changed in proportion to its relative market value.

Futures contracts on this index were introduced in May 18, 1987. Since then the liquidity of these contracts has been steadily increasing. To avoid problems caused by low turnover and lack of experienced traders the first four years of trading were not included in this study. Our sample covers the period from the beginning of 1991 to the end of 1993. The sample is restricted to the contracts that expired in this period, including also the series that expired in January 1994.

Closing prices are used throughout the study. Since the EOE-index contains the most

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[3] Futures contracts on the EOE Dutch Stock Index constitute the bulk of the futures contracts traded on the European Options Exchange (85% in 1991-1993).

[4] In February 1994 the weights were recalculated to correspond to the market capitalization of included stocks.

frequently traded shares on the ASE, the problems caused by stale prices in the index should be negligible on a daily, not to mention on a weekly level. The data on the EOE-index was provided by the Financial Futures Market Amsterdam which is a wholly owned subsidiary of the European Options Exchange.

The expiration day for the futures contracts on the ASE is the third Friday in the month. In each month of the year there are futures contracts that expire. Trading in the contracts that expire in January, April, July, and October starts one year in advance, whereas trading in contracts expiring in any other month starts approximately three months in advance. When a new contract is listed some time elapses before active trading commences. To avoid problems caused by lack of liquidity a few observations from the beginning of the life-time of the contract were omitted from the analysis[5].

The theoretical considerations mentioned in section 1 imply that there should be a negative relationship between the expected volatility of the underlying instrument and the basis. In measuring the expected volatility the most natural way is to use the volatility implied in prices for traded options on the EOE-index. These prices should reflect the market's expectations concerning the volatility during the remaining life-time of the futures contract. The implied standard deviations (ISDs) that we use are trading volume weighted averages for ISDs computed with the Black and Scholes (1973) formula on near-the-money call and put contracts on the EOE-index. The ISDs have been computed by the Institute for Research and Investment Services, a joint venture between Rabo Bank Nederland and the Robeco Group.

To compute the "fair" futures price the spot value of the index has to be prolonged to the expiration date of the contract, and adjusted for dividend coupons that fall off any of the included shares during the remaining time to expiration. The prolonging has been made

[5] The criterion was that the open interest should be non-zero for that day, and until the expiration of the contract, for a day to be included in our sample.

using the one-month[6] Amsterdam Inter Bank Offered Rate (AIBOR) which was obtained from Datastream.

In making the dividend adjustment we have assumed that all dividends have been known in advance by investors. Dividends that are paid out during the remaining life-time of the contract are multiplied by the number of shares in the index, and prolonged to the expiration date using the present one-month AIBOR. Finally, to obtain the "fair" value of the futures contract, these prolonged dividends are subtracted from the prolonged value of the spot.

The data-base was created by selecting the data for each Wednesday. When data for Wednesday was missing, the data for that week's Thursday has been used instead. In one case the data for Tuesday has been used for the lack of both Wednesday and Thursday data. Summary statistics for the weekly data are reported in Table I.

For this study the most interesting column in the table is the column for the "basis" measured as the logarithm of the observed price for the futures contract minus the logarithm of its "fair" value. The standard deviation for the sample mean is $0.690/\sqrt{600} = .029$ which implies that the basis was significantly negative. In other words, futures prices tended systematically to be below their "fair" values.

## 3. Results

The most important testable hypothesis produced by the CCH (1995) model is that the

[6] In principle the AIBOR should be chosen to correspond as closely as possible to the time to maturity of the contract. However, as shown in the table below the AIBOR-rates for different maturities are highly correlated even in first differences which indicates that the benefits of using AIBORs for several different maturities do not justify the increase in computational complexity.

**Table**: Correlation matrices between daily one, two and three month AIBORs during 1991-1993 in levels and in first differences.

| | AIBOR 1.. | AIBOR 2.. | AIBOR 3.. | | Δ AIB 1 | Δ AIB 2 | Δ AIB 3 |
|---|---|---|---|---|---|---|---|
| AIBOR 1m | 1 | | | ΔAIB 1 | 1 | | |
| AIBOR 2m | .999 | 1 | | ΔAIB 2 | .943 | 1 | |
| AIBOR 3m | .996 | .999 | 1 | ΔAIB 3 | .913 | .962 | 1 |

basis should be negatively related to the volatility of the underlying index. To test this hypothesis the regression no. 2 on weekly data reported in Table IV, p. 293 in the CCH (1995) paper was estimated with our data set. The results are reported in Table II below. To check for sample-sensitivity the same regressions were estimated on two subsets, the first subset containing all contracts that expired before July 1992, and the second subset containing the rest of the contracts.

The reason for including the daily return on the underlying index, both contemporaneous and with a one day lead, is to control for the impact from possible non-synchronous changes in the futures price and the value of the index itself, see e.g. MacKinlay and Ramaswamy (1988), Stoll and Whaley (1990), and Chan, Chan and Karolyi (1991).[7]

The high explanatory power and the high t-values for the whole sample and the subperiods in Table II may reflect the fact that we are using overlapping contracts, that is that the same day is included in several observations. This lack of independence between observat-ions, should mainly increase the significance of the index returns, not the ISD variable which will vary with the time to maturity of the contract. However, as an additional control the same regression was also run on data consisting exclusively of the shortest contract. The results are reported in the last two columns in Table II.

The results in Table II are very much in line with those reported in the CCH (1995) paper. The variable containing the ISD is highly significant in all regressions. Also in this case the daily returns on the index seem to have some impact on the basis. However, the results concerning the impact of the ISD are only marginally altered by whether the returns are included or not.

As expected the statistical significance of the regressions in the last two columns is much smaller due to the drop in degrees of freedom, and the fact that the dispersion in the explanatory variable is reduced by the reduced dispersion in $\tau$. However, these results still support the results in the previous columns in the table, implying that the significant negative relationship between the basis and volatility is not produced by an artificial dependence between our observations.

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[7] The basis with a one week lag was not included since it turned out to be insignificant in CCH (1995). Furthermore, convincing reasons to include it in a regression on weekly, as opposed to daily, data are lacking.

The final step in our analysis of the basis was to look at whether there is an interest rate impact, as predicted by Hemler and Longstaff (1991), in our data. To do this one has to tackle the problem of multicollinearity between the interest-rate impact and the volatility-impact. Both impacts will be related to the time to expiration of the contract. The longer the remaining time to expiration the larger will be the impact. Thus, the relevant measure for volatility as well as interest rates will be highly correlated with $\tau$, the time to expiration.

Hemler and Longstaff (1991) found a concave relationship between the basis and the interest rate. To check for possible non-linearities in our data a term containing the squared interest was included. In experiments with $r \cdot \tau$, $r^2 \cdot \tau$, $ISD \cdot \tau$, and $ISD^2 \cdot \tau$, the combination reported in Table III was the one with the highest explanatory power. The term containing the squared interest rate generally turned out to be more significant than the term containing the interest rate as such.

Table III reveals that the sign for the squared interest rate remains negative for both subperiods. The concave relationship is consistent with the Hemler and Longstaff (1991) model. However, for the second subperiod the coefficient for the interest rate effect is much lower and no longer statistically significant. This may suggest another interpretation: the low price of the futures contract compared to its "fair" value when the interest rate was high, in the beginning of our sample period, could simply reflect that investors were unable to obtain an interest rate close to the high values of the AIBOR at that time. This may be a consequence of the market attracting mainly small-scale arbitrageurs in the beginning of the period when the market was less liquid.

The other main prediction of the CCH (1995) model is a positive relationship between the open interest in futures contracts and volatility. To investigate whether this prediction is supported by our EOE-data the regressions reported in Table IV were estimated. The time to expiration $\tau$ is included following CCH to account for the cumulative increase in the open interest as the contract approaches maturity. The variable 'Contract number' is included as a crude way to capture the upward trend observed in the EOE-futures market. This trend could otherwise obscure the relationship between the variables that we are interested in.

The results reveal that the open interest tends to increase as perceived volatility goes up. The coefficient for the ISD has the predicted sign in both regressions. It receives a significant coefficient only in the first regression, though. In that regression the coefficient for the contract number-variable is also highly significant. This coefficient estimate tells us that futures contracts written on the EOE-index have increased with around 93 contracts per expiration month in our sample period, other things equal. Not surprisingly, when this strong trend is left out in the second regression, the relation between the ISD and the open interest is obscured.

**4. Conclusions**

Our results for EOE-index futures during 1991 through 1993 correspond to those obtained by Chen, Cuny and Haugen (1995) in the sense that the basis is negatively related to the volatility of the underlying index. When perceived volatility is high the futures price tend to be lower relative to its "fair" value than when the perceived volatility is low. This is consistent with the view that investors with a high customization value for shares are hedging their positions in the stock market by selling futures contracts when the perceived risk in the stock market is high.

The other main prediction of the CCH (1995) is also supported by our study. The open interest in futures contracts on the EOE-index was shown to be significantly related to the volatility of the index, once the trend-like increase in futures trading on the EOE was controlled for.

Finally, our results indicate that there is a highly significant negative concave relationship between the level of the interest rate, and the basis. A higher interest rate level will produce a lower futures price relative to its "fair" value. This result may be a simple consequence of relevant investors having been forced to accept a lower rate than the AIBOR that is being used in the present study to calculate the "fair" value. Our result would follow if the distance between this relevant rate and the AIBOR has diminished as a consequence of the downward trend in the AIBOR, and/or the upward trend in trading volume in EOE-futures in our sample period[8]. However, it may also point to a more complicated relationship between the basis and the interest rate such as the one in Hemler and Longstaff (1991).

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[8] Since the observed futures price usually has been below the fair value the strategy to profit from this "discount" would be to invest the money in a risk-free instrument and buy the futures contract. Thus the relevant rate should be the rate on the risk-free instrument available to the investor.

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Table I

## Summary Statistics

The sample period covers beginning of 1991 through the end of 1993. The number of expiration dates covered is 37. The number of observations is 600, and the number of weeks covered is 157. The basis is multiplied by 100 for ease of exposition. The ISD and AIBOR are reported in % p.a.

| | # days to | Open | Volume | Close | Fair | Basis: | Index | ISD | AIBOR |
|---|---|---|---|---|---|---|---|---|---|
| Average | 74.1 | 3334 | 593.6 | 300.0 | 301.5 | -0.508 | 298.3 | 15.11 | 8.55 |
| Standard | 66.6 | 2811 | 826.0 | 32.6 | 32.4 | 0.690 | 32.3 | 2.89 | 1.12 |
| Min. | 1 | 2 | 0 | 216.9 | 217.7 | -4.033 | 217.6 | 10.43 | 5.83 |
| Max. | 331 | 17145 | 5391 | 417.3 | 415.7 | 1.179 | 414.3 | 31.62 | 10.07 |

Table II

## Regressions of Basis on Volatility

Regressions on data for futures contracts on the EOE-index during 1991 through 1993. The EOE variables are daily returns on the EOE-index computed as logarithmic differences. T-ratios are reported in parentheses below the coefficient estimates. τ is the time to expiration in calendar days.

| | Whole sample | | Jan 91 - July 92 | | Aug 92 - Jan 94 | | Non-overlapping | |
|---|---|---|---|---|---|---|---|---|
| $EOE_t$ | 8.78 | - | 8.18 | - | 11.80 | - | 9.77 | - |
| | (4.04) | | (3.71) | | (6.30) | | (4.75) | |
| $EOE_{t+1}$ | -5.94 | - | -7.09 | - | 2.48 | - | -2.54 | - |
| | (-2.77) | | (-3.52) | | (6.30) | | (-1.23) | |
| $ISD^2.\tau.10^{-3}$ | -8.33 | -8.28 | -12.56 | -12.54 | -19.17 | -19.07 | -4.50 | -3.73 |
| | (-33.37) | (-32.47) | (-46.16) | (-43.58) | (-26.07) | (-24.52) | (-4.34) | (-3.38) |
| Constant | 0.114 | 0.106 | 0.184 | 0.173 | -0.155 | -0.145 | -0.004 | -0.017 |
| No.of observations | 600 | 600 | 282 | 282 | 318 | 318 | 157 | 157 |
| Adj. R2 | 0.653 | 0.638 | 0.885 | 0.871 | 0.692 | 0.654 | 0.192 | 0.063 |

Table III

**Regressions of Basis on Volatility and Interest Rates**

Results from regressing the basis on pooled data for futures contracts on the EOE-index during 1991 through 1993. T-ratios are reported in parentheses below the coefficient estimates. $\tau$ is the time to expiration of the contract.

| | Whole sample | Jan 91 - July 92 | Aug 92 - Jan 94 |
|---|---|---|---|
| ISD . $\tau$ | -.025 | -0.034 | -0.033 |
| | (-5.84) | (-4.97) | (-8.60) |
| $(AIBOR)^2 . \tau$ | -0.69 | -0.82 | -0.13 |
| | (-7.88) | (-5.81) | (-1.67) |
| Constant | 0.141 | 0.152 | 0.084 |
| No.of observations | 600 | 282 | 318 |
| Adj. $R^2$ | 0.718 | 0.866 | 0.663 |

Table IV

**Regressions of Open Interest on Volatility**

Regressions explaining the open interest in futures contracts on the EOE-index in 1991 - 1993. The open interest is measured in number of contracts. 'Contract number' is the chronological order of the contract, January 1991 equalling 1, and January 1994 equalling 37. T-ratios are reported in parentheses.

| | Whole sample | |
|---|---|---|
| ISD . 100 | 95.9 | 11.0 |
| | (2.82) | (0.31) |
| Time to exp. ($\tau$) | -22.1 | -19.3 |
| | (- 15.13) | (- 12.53) |
| Contract number | 92.8 | - |
| | (9.77) | |
| Constant | 1600 | 4597 |
| No.of observations | 600 | 600 |
| Adj. $R^2$ | 0.315 | 0.207 |