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OPTIMAL COMMITMENT IN AN OPEN ECONOMY:
CREDIBILITY VS. FLEXIBILITY

by

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Abstract

Using a graphical method, a new way of determining the optimal degree of central bank conservativeness is developed in this paper. Unlike Lohmann (1992) and Rogoff (1985a), we are able to express the upper and lower bounds of the interval containing the optimal degree of conservativeness in terms of the structural parameters of the model.

Next, we show that optimal central bank independence is higher, the higher the natural rate of unemployment, the greater the benefits of unanticipated inflation, the less inflation-averse society, the smaller the variance of productivity shocks, the smaller real exchange rate variability and the smaller the openness of the economy. These propositions are tested for nineteen industrial countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, New Zealand, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom and the United States) for the Bretton-Woods period and after (1960-1993). In testing the model we employ a latent variables method (LISREL) in order to distinguish between actual and optimal monetary regimes.
I. INTRODUCTION

Recently, in many countries both political and monetary authorities have shown an increasing interest in the objective of monetary stability and the position of the central bank. As pointed out by Persson and Tabellini (1993) recent policy reform, as well as historical experience, suggests two different routes to price stability.

The first way is the legislative approach, namely to create by law a very independent central bank with an unequivocal mandate to focus on price stability. Interest in this approach is motivated by the success of the Deutsche Bundesbank in maintaining one of the lowest rates of inflation for several decades. Moreover, the accepted statute of the European Central Bank is strongly influenced by the law governing the Bundesbank. Moreover, France and Spain reformed their central bank laws that made the Banque de France and the Banco de España more independent of government. Furthermore, countries in Central and Eastern Europe, such as the Czech Republic, Hungary and Poland, increased the legal independence of their central banks. Finally, in Latin America there are also tendencies toward granting more independence to the central banks in countries like Argentina, Chile, Mexico and Venezuela. Academic contributions in this area are Rogoff (1985a), Neumann (1991) and Lohmann (1992).

The second way is the targeting or contracting approach, namely to let the political principal of the central bank impose an explicit inflation target for monetary policy, and make the central bank governor explicitly accountable for his success in meeting this target. Recently, New Zealand, Canada, and the United Kingdom have made some progress on this route. Along these lines New Zealand enacted legislation that increased the independence of its Reserve Bank, whereas in the United Kingdom there is now a lively discussion of the desirability of making the Bank of England more independent. Important theoretical work on this approach is done by Walsh (1993) and Persson and Tabellini (1993).

In this paper we build on the Rogoff (1985a) model and therefore restrict the analysis to the legislative approach. Empirical work on the legislative approach [Alesina (1988, 1989), Grilli, Masciandaro and Tabellini (1991), Cukierman (1992), Eijffinger and Schaling (1992, 1993a, 1993b), De Haan and Sturm (1992), Alesina and Summers (1993)] has focused on the quantification of independence using a number of legal attributes from central bank laws. These studies focus on the positive issue of the relation between monetary regimes and economic performance. Broadly speaking, the conclusion is that the more independent the central bank, the lower the inflation rate, whilst the rate of output

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1) The authors owe a debt of gratitude to Marco Hoeberichts for his empirical support. They are also grateful to Marno Verbeek for his valuable suggestions with respect to the latent variables method.

growth is unaffected.

However, this literature does not explain the observed differences in central bank independence. For instance, no explanation is offered for the very high independence of the Bundesbank. It has often been pointed out that this independence may be explained by Germany’s underlying aversion to inflation associated with its experience of hyper-inflation in the 1920s.3)

This brings us to a key issue in the political economy of central banking: the relation between institutional design and individual and collective preferences. Here the question to be dealt with is the normative issue of how independent a central bank should be, i.e. the optimal degree of central bank independence.

An important study in this field is Cukierman (1994). Building on the seminal paper of Lohmann (1992), he wants to identify the economic and political factors that induce politicians to delegate more or less authority to the central bank. His theory predicts that central bank independence will be higher the larger the employment-motivated inflationary bias, the higher political instability and the larger the government debt.

These predictions were tested and, subsequently, rejected by De Haan and Van ’t Hag (1994) using regression analysis (OLS method). In testing Cukierman’s model, they employ measures of central bank independence that in - Rogoff’s (1985a) terminology - reflect the strength of the ‘conservative bias’ of the central bank as embodied in the law. In Cukierman’s model, following Lohmann (1992), central bank independence is defined as the cost of overriding the central bank, rather than as the degree of conservativeness. Cukierman’s (1994) theory also generates propositions about optimal regimes, whilst the legal measures describe actual monetary regimes.

In this paper we try to overcome these pitfalls. Building on the Rogoff (1985a) model, we identify central bank independence as the degree of conservativeness rather than the political cost of overriding the central bank. We extend this model to the open economy case and allow for deviations from purchasing power parity. Using a graphical method, we develop a new way of determining the optimal degree of conservativeness. As in Lohmann (1992), this degree depends on the balance between credibility and flexibility. However, unlike Rogoff and Lohmann, we are able to express the upper and lower bounds of the interval containing the optimal degree of conservativeness in terms of the structural parameters of the model.

Furthermore, we derive several propositions concerning the relation between economic and political factors and the optimal degree of central bank independence. We show that optimal central bank independence is higher, the higher the natural rate of unemployment, the greater the benefits of unanticipated inflation (the slope of the Phillips curve), the less inflation-averse society, the smaller the variance of productivity shocks, the smaller real exchange rate variability and the smaller the openness of the economy. These propositions

are tested for nineteen industrial countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, New Zealand, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom and the United States) for the Bretton-Woods period and after (1960-1993). In testing the model we employ a latent variables method (LISREL) in order to distinguish between actual and optimal monetary regimes.

The paper is organized into four remaining sections, followed by three appendices. In section II we present the theoretical model. Section III contains the derivation of the optimal degree of central bank independence. In section IV we test the model with the latent variables method. Our conclusions are given in Section V.

II.A SIMPLEx MACROMODEL

The main purpose of this section is to combine the Alogoskoufis (1994) model of wage and employment determination with an open economy variant of the Rogoff (1985a) model. We assume that there are two types of agents, wage-setters (the union) and the central bank. Wage-setters unilaterally choose the nominal wage every period, and the central bank controls monetary policy.

The sequence of events is as follows. In the first stage wage-setters sign each period nominal wage contracts [Gray (1976), Fischer (1977a)]. Wage-setters know the domestic monetary regime. They take this information into account in forming their expectations. In the second stage stochastic shocks to productivity and the real exchange rate realize. These shocks are random and cannot be observed at the time wage contracts are signed. In the third stage the central bank observes the values of the shocks and — contingent on the chosen regime — reacts to the shocks accordingly. In the fourth and final stage employment is determined by competitive firms. This timing of events is summarized in Figure 2.1.

**Figure 2.1. The sequence of events.**

<table>
<thead>
<tr>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal wage contracts signed</td>
<td>Shocks realize</td>
<td>Central bank sets monetary policy</td>
<td>Employment determined</td>
</tr>
</tbody>
</table>

We now move to the supply side of the model.
II.1. Aggregate supply

Consider the following supply block. Capital will be assumed fixed, and output is given by a short-run Cobb-Douglas production function

\[ y_t = \beta \ell_t + v_t \quad 0 < \beta < 1 \] (2.1)

where lower-case letters refer to logarithmic deviations from steady state values. Thus, \( y \) is the log of output, \( \ell \) the log of employment, and \( v \) a measure of productivity. \( \beta \) is the exponent of labour and is less than unity.

Having described the level of output, it remains to be specified how productivity evolves over time. For simplicity we assume that shocks to productivity are normally distributed with zero mean and finite variance

\[ v_t \sim N(0, \sigma^2_v) \] (2.2)

Firms determine employment by equalizing the marginal product of labour to the real wage \( w_t - p_t \). This yields the following employment function

\[ \ell_t = \frac{1}{1-\beta} (w_t - p_t - v_t) \] (2.3)

where \( w \) is the log of the nominal wage and \( p \) the log of the price level.

The nominal wage is set at the beginning of each period and remains fixed for one period. The objective of wage-setters is to stabilize the real consumer wage around a target level. Thus, wages in each period are set to minimize

\[ W_t = E_{t-1} \left[ \frac{1}{2} (w_t - p_c - \tau)^2 \right] \] (2.4)

where \( E_{t-1} \) is the operator of rational expectations, conditional on information at the end of period \( t - 1 \). \( \tau \) is the real wage target of the union. 4)

The consumer price index \( p_c \) is defined as

\[ p_c = (1-\mu)p_t + \mu(p_t^* + e_t) \] (2.5)

where \( \mu \) is the share of imports in GDP, \( e \) is the nominal exchange rate (defined as the domestic currency price of foreign exchange) and \( p_t^* \) is the foreign price level. Equation (2.5) can be rewritten as

4) Alternatively, the loss function (2.4) could be assumed quadratic in both the deviations of employment and the real wage from certain target levels. For an analysis along these lines see Schaling (1995), Chapter 7.
\[ p_c = p_t + \mu q_t \]  
\hspace{1cm} (2.6)

where \( q \) is the real exchange rate \( p^* + e - p \). Having redefined the CPI in terms of the producer’s price level (\( p \)) and the real exchange rate, it remains to specify how the nominal exchange rate is determined.

Here we are concerned with the factors that ultimately determine the degree of central bank independence, that is, with the longer-term behaviour of the exchange rate. As pointed out by Mussa (1991, p. 14), long-term relationships between movements in nominal exchange rates and the ratio of national price levels support the empirical relevance of purchasing power parity.  

Therefore, we assume the nominal exchange rate to be governed by

\[ \Delta e_t = \Delta p_t - \Delta p^*_t + w_t \hspace{1cm} w_t \sim N(0, \sigma^2_w) \]  
\hspace{1cm} (2.7)

where \( \Delta \) is the first-difference operator and \( w \) is a shock to the first difference of the real exchange rate with zero mean and finite variance \( \sigma^2_w \). We assume this shock is uncorrelated with the supply shock, i.e. \( \text{E}_{t-1} v_t w_t = 0 \). From the first-order conditions for a minimum of (2.4), the CPI-indexed nominal wage is given by

\[ w_t = \text{E}_{t-1} p_t + \tau \]  
\hspace{1cm} (2.8)

Taking account of (2.6) - (2.7) and of the fact that shocks to the real exchange rate cannot be observed at the time wage contracts are signed, we get

\[ w_t = \text{E}_{t-1} p_t + \tau \]  
\hspace{1cm} (2.9)

Substituting (2.9) in the labour demand function (2.3), yields the following relation between employment and unanticipated shocks

\[ \ell_t - \ell^* = \frac{1}{1 - \beta} (p_t - \text{E}_{t-1} p_t + \nu_t) \]  
\hspace{1cm} (2.10)

where \( \ell^* = \frac{-\tau}{1 - \beta} \). An unanticipated rise in prices \( p_t - \text{E}_{t-1} p_t \) reduces the real wage, and

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5) It is well known that this theory has, generally, provided a poor explanation of shorter term movements of exchange rates. See e.g. Dornbusch (1976, 1988).

6) In future work this assumption will be relaxed. Then the shocks to the real exchange rate will result from the difference between domestic and foreign shocks to productivity.

7) For a similar specification see Rogoff (1985b, p. 212). Note that the nature of the employment contract is such that the union agrees to supply whatever amount of labour is determined by firms in period \( t \), provided firms pay the negotiated wage.
causes firms to employ more labour. Thus, aggregate employment exhibits a transitory deviation from its equilibrium or "natural" rate $\ell^*$.  

Subtracting (2.10) from the labour force $\ell'$, using the approximation that the rate of unemployment $u = \ell' - \ell$, we get the following expression for the short-run determination of unemployment

$$u_t = \bar{u} - \frac{1}{1-\beta} (p_t - E_t \Delta p_t + v_t)$$

(2.11)

where $\bar{u} = \ell^* - \ell'$. $\bar{u}$ can be thought of as the equilibrium or "natural" rate of unemployment in this model. Thus, (2.11) is the well-known expectations augmented Phillips curve. Unemployment deviates from its equilibrium rate only to the extent that there are unanticipated shocks to inflation or productivity. Anticipated shocks to inflation, productivity and the real exchange rate are reflected in wages (equation (2.8)) and do not affect unemployment. We can now incorporate the Phillips curve into a monetary policy game. This is the subject of the next section.

II.2. Time-Consistent Equilibrium under a "Conservative" Central Banker

As stated by Rogoff (1985a, p. 1180), the adoption of central bank independence may be viewed as an institutional response to the time-consistency problem.

Suppose, for example, that through a system of rewards and punishments the central bank’s incentives are altered so that it places some direct weight on achieving a low rate of growth for a nominal variable such as the price level, nominal GNP, or the money supply. Rogoff demonstrates that society can make itself better off by selecting an agent to head the central bank who is known to place a greater weight on inflation stabilization (relative to unemployment stabilization) than is embodied in the social loss function $L_t$.

The social loss function $L$ depends on deviations of unemployment and CPI inflation from their optimal (socially desired) levels

$$L_t = \frac{1}{2} (\Delta p_t - \Delta p^*) + \frac{\chi}{2} (u_t - u^*)^2$$

(2.12)

where $0 < \chi < \infty$ and $\Delta p^*$ and $u^*$ are society’s CPI inflation and unemployment targets. The parameter $\chi$ is the relative weight of unemployment stabilization relative to inflation stabilization in the preferences of society. Normalizing $\Delta p^*$, $u^*$, $p_{t-1}$ and $q_{t-1}$ at zero and

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8) Actual employment equals its natural rate when all expectations are fulfilled. Hence, the natural rate of employment equals $\ell^*$. 

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using (2.6) we get

\[ L_t = \frac{1}{2} p_{c_t}^2 + \frac{\chi}{2} u_t^2 \]  

(2.13)

Rogoff shows that, in choosing among potential candidates, it is never optimal to choose an individual who is known to care "too little" about unemployment.

Suppose, for example that in period 1 society selects an agent to head the central bank in period t. The reputation of this individual is such that it is known that, if he is appointed to head the central bank, he will minimize the following loss function

\[ I_t = \frac{(1+\varepsilon)}{2} p_{c_t}^2 + \frac{\chi}{2} u_t^2 \quad 0 < \varepsilon < \infty \]  

(2.14)

When \( \varepsilon \) is strictly greater than zero, then this agent places a greater relative weight on CPI inflation stabilization then society does. Hence, following Eijffinger and Schaling (1993b, p. 5) we view the coefficient \( \varepsilon \) as a measure of the political independence of the central bank. The higher \( \varepsilon \) the more independent the central bank. Note that, if \( \varepsilon = 0 \), equation (2.14) reduces to the social loss function (2.13).

Thus, stochastic equilibrium is derived under the assumption that the monetary authorities attempt to minimize loss function \( I_t \), given by equation (2.14) above. Substituting the definition of the CPI (equation (2.6)) and the Phillips curve (2.11) in the loss function (2.14) yields

\[ I_t = \frac{1+\varepsilon}{2} [p_t + \mu q_t]^2 + \frac{\chi}{2} [\tilde{u} - \frac{1}{1-\beta} p_t + \frac{1}{1-\beta} E_t p_t - \frac{1}{1-\beta} v_t]^2 \]  

(2.15)

From the first-order conditions for a minimum of (2.15), i.e. \( \partial I_t / \partial p_t = 0 \), taking account of (2.7), and again using the normalization \( q_{t-1} = 0 \), we obtain the central bank’s reaction function to the union’s inflationary expectations

\[ p_t^I = \frac{(1-\beta)\chi}{(1+\varepsilon)(1-\beta)^2 - \chi} \tilde{u} + \frac{\chi}{(1+\varepsilon)(1-\beta)^2 + \chi} E_{t-1} p_t^I - \frac{1}{(1+\varepsilon)(1-\beta)^2 + \chi} [\chi v_t + \mu(1+\varepsilon)(1-\beta)^2 w_t] \]  

(2.16)

where superscript I stands for independent central bank regime. Taking expectations conditional on information at t-1 of (2.16) gives

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9) Price-level targeting and inflation-rate targeting are equivalent here, since \( p_{c_t} \) is known at the time the central bank commits itself to achieving a target for \( \Delta p_{c_t} \). Once monetary control errors are taken into account it becomes important to make the distinction between a zero inflation target and a target of price stability. See Fischer (1994, pp. 33-34).
Equation (2.17) is the reaction function of the union. The resulting GDP price level is

\[
p_i^* = \frac{\chi}{(1-\beta)(1-\epsilon)} \tilde{u} - \frac{1}{(1-\epsilon)(1-\beta)^2 - \chi} [\chi v_i + \mu (1+\epsilon)(1-\beta)^2 w_i]
\]

(2.18)

whilst the unemployment rate is given by

\[
u_i^* \equiv \tilde{u} - \frac{(1-\epsilon)(1-\beta)}{(1-\epsilon)(1-\beta)^2 - \chi} [v_i - \mu w_i]
\]

(2.19)

Note that a real depreciation increases unemployment (it acts like an adverse productivity shock). The reason for this is simple. Shocks that tend to cause real depreciations of the domestic currency are stabilized by the central bank (equation (2.16)), because of their potentially adverse effects on CPI inflation. The associated monetary tightening causes - given the level of nominal wages - a rise in real labour costs (reducing the producer's wage), thus driving down labour demand. Given the supply of labour, unemployment then increases.

### III. OPTIMAL COMMITMENT IN MONETARY POLICY: CREDIBILITY VERSUS FLEXIBILITY

#### III.1. Social Welfare under Central Bank Independence

We are now able to evaluate central bank independence from the perspective of society. To facilitate exposition in later sections, following Rogoff (1985a, pp. 1175-1176), we shall first develop a notation for evaluating the expected value of society's loss function under any arbitrary monetary policy regime "A", \( E_{\epsilon_t} L_t^A \):

\[
E_{\epsilon_t} L_t^A = \frac{1}{2} [\chi \tilde{u}^2] + \Pi^A + \Gamma^A
\]

(3.1)

where \( \Pi^A \equiv 1/2 (\hat{p}_c^A)^2 \), \( \hat{p}_c^A \) is the mean CPI in period \( t \), and

\[
\Gamma^A \equiv \frac{1}{2} E_{\epsilon_t} \left[ \chi \left( \frac{v_i}{1-\beta} + (p_i^A - E_{\epsilon_t} p_i^A)/(1-\beta)^2 + (p_{c}^A - E_{\epsilon_t} p_{c}^A)^2 \right) \right].
\]

Again, the first component of \( E_{\epsilon_t} L_t^A \), \( \frac{1}{2} [\chi \tilde{u}^2] \) is non-stochastic and invariant across monetary regimes. It represents the deadweight loss due to the labour market distortion (\( \tilde{u} > 0 \)). This loss cannot be reduced through monetary policy in a time-consistent rational

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\( ^{10} \) We derive equation (3.1) in Appendix A.
expectations equilibrium. The second term, $\Pi^A$, depends on the mean inflation rate. This term is also non-stochastic but does depend on the choice of monetary policy regime.

The final term, $\Gamma^A$, represents the stabilization component of the loss function. It measures how successfully the central bank offsets disturbances to stabilize unemployment and CPI inflation around their mean values.

By substituting the results relevant for the central bank [(2.18) and (2.19)] into society’s loss function (2.12) and taking expectations, we obtain the I regime counterpart of expression (3.1). Abstracting from the (common) deadweight loss, one gets

$$\Pi^I + \Gamma^I = \frac{\chi^2}{2[(1+\epsilon)(1-\beta)]^2} \tilde{u}^2 + \frac{\chi[(1+\epsilon)^2(1-\beta)^2 + \chi]}{2[(1-\epsilon)(1-\beta)^2 + \chi]^2} \sigma^2_v + \frac{\chi \mu^2 [(1+\epsilon)^2(1-\beta)^2]}{2[(1+\epsilon)(1-\beta)^2 + \chi]^3} \sigma^2_w \tag{3.2}$$

III.2. The Rogoff Theorem

Rogoff (1985a) showed that it is optimal for society (the principal) to select an agent to head the central bank that places a large, but finite weight on inflation. In this section we generalize this result for an open economy. The optimal degree of central bank independence $\epsilon^*$ is defined as that value of $\epsilon$ that minimizes the expected value of the loss function of society $E_{t-1}L^I_t$.

To solve for the value of $\epsilon$ that minimizes $E_{t-1}L^I_t$, differentiate (3.2) with respect to $\epsilon$

$$\frac{\partial E_{t-1}L^I_t}{\partial \epsilon} = \frac{\partial \Pi^I}{\partial \epsilon} + \frac{\partial \Gamma^I}{\partial \epsilon} \tag{3.3}$$

$$\frac{\partial \Gamma^I}{\partial \epsilon} = \frac{\chi^2 (1-\beta)^2 \epsilon [\sigma^2_v + \mu^2 \sigma^2_u]}{[(1+\epsilon)(1-\beta)^2 + \chi]^3} \tag{3.4}$$

$$\frac{\partial \Pi^I}{\partial \epsilon} = -\frac{\chi^2 \tilde{u}^2}{(1-\epsilon)^3 (1-\beta)^2} \tag{3.5}$$

We are now ready to prove:

**PROPOSITION 3.1:** With a positive natural rate of unemployment, in an open economy the optimal degree of central bank independence lies between zero and infinity (For $\tilde{u} > 0$, $0 < \epsilon^* < \infty$).

**Proof:** Note that $\epsilon > -1$ by assumption. Thus, by inspection of (3.5), $\partial \Pi^I/\partial \epsilon$ is strictly negative. Note also, by inspection of (3.4), that $\partial \Gamma^I/\partial \epsilon$ is strictly negative for

$$\frac{-(\chi + (1-\beta)^2)}{(1-\beta)^2} < \epsilon < 0$$

zero when $\epsilon = 0$ and positive for $\epsilon > 0$. 

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Therefore, $\partial E_{t-1} L_t / \partial \epsilon$ is strictly negative for $\epsilon \leq 0$. $\partial E_{t-1} L_t / \partial \epsilon$ must change from negative to positive at some sufficiently large value of $\epsilon$, since as $\epsilon$ approaches positive infinity, $\partial \Gamma / \partial \epsilon$ converges to zero at rate $\epsilon^{-2}$, whereas $\partial \Pi / \partial \epsilon$ converges to zero at rate $\epsilon^{-3}$. Consequently, $\epsilon^* < \infty^{11)}$

The intuition behind this result is the following. From (3.5) it can be seen that increasing the central bank’s commitment to inflation stabilization decreases the credibility component of the social loss function. On the other hand, from (3.4) it follows that having a more independent central bank increases the stabilization component of the loss function. Hence, optimal commitment in monetary policy involves trading off the credibility gains associated with lower average inflation versus loss of flexibility due to a distorted response to productivity and real exchange rate shocks.

III.3. The Ultimate Determinants of Central Bank Independence

Proposition (3.1) is Rogoff’s theorem. Rogoff is unable to write down a closed-form solution for $\epsilon^*$. He is also unable to derive propositions concerning the comparative static properties of this equilibrium. The following section can be seen as an extension of the Rogoff theorem.

Using a graphical method, we develop an alternative way of determining the optimal degree of central bank independence. Next, we show how this result is conditioned on the natural rate of unemployment ($\bar{u}$), society’s preferences for unemployment stabilization ($\chi$), the variance of productivity shocks ($\sigma_v^2$), the slope of the Phillips curve ($(1-\beta)\gamma$), the variance of shocks to the real exchange rate ($\sigma_w^2$) and the degree of openness ($\mu$).

By setting (3.3) equal to zero we obtain the first-order condition for a minimum of $E_{t-1} L_t$

$$0 = \frac{\partial \Pi}{\partial \epsilon} + \frac{\partial \Gamma}{\partial \epsilon} \quad (3.6)$$

Substituting (3.4) and (3.5) into (3.6), yields

$$\frac{-\chi^2 \bar{u}^2}{(1-\beta)^2 (1+\epsilon)^3} + \frac{(1-\beta)^2 \epsilon^2 [\sigma_v^2 + \mu^2 \sigma_w^2]}{[(1+\epsilon)(1-\beta)^2 + \chi]^3} = 0 \quad (3.7)$$

---

11) As pointed out by Rogoff (1985a, p. 1178), it is extremely difficult to write down a closed-form solution for $\epsilon^*$. 

11
Equation (3.7) determines $\varepsilon^*$ as an implicit function of $\chi$, $\tilde{u}$, $\sigma_v^2$, $\beta$, $\sigma_w^2$ and $\mu$. A solution for $\varepsilon^*$ always exists and is unique.

To show this we adapt a graphical method used by Cukierman (1992, pp. 170-172) in the context of a dynamic game.

Rewrite (3.7) as

$$
\varepsilon = \frac{[(1-\varepsilon)(1-\beta)^2 + \chi^3\tilde{u}^2]}{[\sigma_v^2 + \mu^2\sigma_w^2](1-\beta)^4(1+\varepsilon)^3} \equiv F(\varepsilon) \tag{3.8}
$$

The function $F(\varepsilon)$ on the right-hand side of equation (3.8) is monotonically decreasing in $\varepsilon$ that

$$
\frac{(1-\beta)^2\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_w^2]} < F(\varepsilon) < \frac{[(1-\beta)^2 + \chi^3\tilde{u}^2]}{[\sigma_v^2 + \mu^2\sigma_w^2](1-\beta)^4}
$$

We are now ready to prove:

**PROPOSITION 3.2:** $$(1-\beta)^2\tilde{u}^2 < \varepsilon^* < \frac{[(1-\beta)^2 + \chi^3\tilde{u}^2]}{[\sigma_v^2 + \mu^2\sigma_w^2](1-\beta)^4}$$

*Proof:* The left-hand side of (3.8) is a 45-degree straight line through the origin. Since $F(0) = \frac{[(1-\beta)^2 + \chi^3\tilde{u}^2]}{[\sigma_v^2 + \mu^2\sigma_w^2](1-\beta)^4}$ and $\frac{\partial F}{\partial \varepsilon} < 0$, these two functions must intersect at one and only one point. Moreover, since

$$
\frac{(1-\beta)^2\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_w^2]} < F(\varepsilon) < \frac{[(1-\beta)^2 + \chi^3\tilde{u}^2]}{[\sigma_v^2 + \mu^2\sigma_w^2](1-\beta)^4}
$$

is bounded between $\frac{(1-\beta)^2\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_w^2]}$ and $\frac{[(1-\beta)^2 + \chi^3\tilde{u}^2]}{[\sigma_v^2 + \mu^2\sigma_w^2](1-\beta)^4}$. The intersection occurs at a value of $\varepsilon$ that

Figure 3.1 illustrates the argument graphically. Clearly, a solution for $\varepsilon$ exists and is unique.

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12) These statements are demonstrated in Appendix B to this paper.
We are now ready to investigate the factors affecting the optimal degree of central bank independence. Hence, we identify economic and political factors that induce politicians to delegate more or less authority to this institution. We show that the delegation of authority to the central bank depends on the natural rate of unemployment, society’s preferences for unemployment stabilization, the variance of productivity shocks, the slope of the Phillips curve, the degree of openness of the economy and the variance of shocks to the real exchange rate. The results are derived by performing comparative static experiments with respect to various parameters on Figure 3.1. Derivations appear in Appendix B. We summarize the main results in six propositions.

**PROPOSITION 3.3**: The higher the natural rate of unemployment (the higher $\bar{u}$), the higher the optimal degree of central bank independence.

*Proof*: Appendix B shows that $\frac{\partial F}{\partial \bar{u}} > 0$, implying that when $\bar{u}$ goes up, the curve $F(\varepsilon)$ in Figure 3.1 shifts upward. As a consequence, the equilibrium value of $\varepsilon$ increases.

The intuition behind this result is the following. A higher natural rate of unemployment implies a higher time-consistent rate of inflation (See equation (2.18)) and, consequently, a higher credibility component of the social loss function. This means that society’s credibility problem is increased. Hence, with an unaltered relative weight placed on inflation versus unemployment stabilization the monetary authorities’ commitment to fight inflation is now too low.
PROPOSITION 3.4: The higher society’s preferences for unemployment stabilization relative to inflation stabilization (the higher $\chi$), the higher the optimal degree of central bank independence.

Proof: Appendix B shows that $\frac{\partial F}{\partial \chi} > 0$, implying that when $\chi$ goes up, the curve $F(\varepsilon)$ in Figure 3.1 shifts upward. Thus, the equilibrium value of $\varepsilon$ increases.

The underlying intuition is that, if society becomes more concerned with unemployment and more lax about inflation, the time-consistent inflation rate goes up (See equation (2.18)). Therefore, society’s credibility problem becomes more pressing. With an unchanged relative weight placed on inflation stabilization, the balance between credibility and flexibility needs to be adjusted in favour of increased commitment of fighting inflation.

PROPOSITION 3.5: The higher the variance of productivity shocks (the higher $\sigma_v^2$), the lower the optimal degree of central bank independence.

Proof: Appendix B shows that $\frac{\partial F}{\partial \sigma_v^2} < 0$, implying that when $\sigma_v^2$ goes up, the curve $F(\varepsilon)$ in Figure 3.1 shifts downward. Therefore, the equilibrium value of $\varepsilon$ decreases.

This result may be explained as follows. If the variance of productivity shocks increases, ceteris paribus, the economy becomes more unstable. Thus, the need for active stabilization policy increases (the $\Gamma^I$ component of the social loss function goes up). With an unaltered relative weight placed on inflation stabilization the balance between credibility and flexibility needs to be shifted towards more monetary accommodation.

PROPOSITION 3.6: If society is relatively unconcerned with inflation ($\chi > (1+\varepsilon)(1-\beta)^2$), the greater the benefits of unanticipated inflation (the higher $(1-\beta)^{-1}$), the higher the optimal degree of central bank independence.

Proof: Appendix B shows that, if $\chi > \frac{(1+\varepsilon)(1-\beta)^2}{2}$, $\frac{\partial F}{\partial (1-\beta)^{-1}} > 0$, implying that when $(1-\beta)^{-1}$ goes up, the curve $F(\varepsilon)$ shifts upward. Consequently, the equilibrium value of $\varepsilon$ increases.

The intuition behind this proposition is that, if the benefits of unanticipated inflation rise (See equation (2.11)), it becomes more tempting to inflate the economy. Therefore, society’s credibility problem gains in importance. With the same emphasis on inflation
stabilization, the balance between credibility and flexibility needs to be shifted towards increased commitment to price stability.

**PROPOSITION 3.7:** If the economy is more open to international trade (the higher $\mu$), the lower the optimal degree of central bank independence.

*Proof:* Appendix B shows that $\frac{\partial F}{\partial \mu} < 0$, implying that when $\mu$ goes up, the curve $F(\varepsilon)$ in Figure 3.1 shifts downward. Therefore, the equilibrium value of $\varepsilon$ decreases. The underlying argument is the following. If the economy becomes more open, domestic inflation and unemployment become more vulnerable to shocks to the real exchange rate. This means that if $\mu$ goes up, ex ante variability of both inflation and unemployment increase (see equations (2.18) and (2.19)). Note that this implication is consistent with empirical evidence found by Romer (1993, p. 884) that openness accounts for a substantial fraction of the variation in inflation among countries. As a consequence the need for active stabilization policy by the central bank increases. Thus, the balance between credibility and flexibility needs to be shifted towards more monetary accommodation.

**PROPOSITION 3.8:** The higher real exchange rate variability (the higher $\sigma_w^2$), the lower the optimal degree of central bank independence.

*Proof:* Appendix B shows that $\frac{\partial F}{\partial \sigma_w^2} < 0$, implying that when $\sigma_w^2$ goes up, the curve $F(\varepsilon)$ in Figure 3.1 shifts downward. Therefore, the equilibrium value of $\varepsilon$ decreases. The intuition is similar to that behind the previous proposition. If ex ante real exchange rate variability rises, both domestic inflation and unemployment will become more volatile (see equations (2.18) and (2.19)). Thus, the need for active stabilization policy increases (the $\Gamma^I$ component of the social loss function goes up). So, the balance between credibility and flexibility must be shifted towards less commitment.
Table 3.1. The ultimate determinants of central bank independence

<table>
<thead>
<tr>
<th>Economic and political factors</th>
<th>( \hat{u} )</th>
<th>( \chi )</th>
<th>( \sigma_v^2 )</th>
<th>( (1-\beta)^{-1} )</th>
<th>( \mu )</th>
<th>( \sigma_w^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural rate of unemployment</td>
<td>Society’s preferences for unemployment stabilization</td>
<td>Variance of productivity shocks</td>
<td>Gains from unanticipated inflation (slope of Phillips curve)</td>
<td>Degree of openness</td>
<td>Variance of shocks to the real exchange rate</td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial e^*}{\partial \hat{u}} &gt; 0 )</td>
<td>( \frac{\partial e^*}{\partial \chi} &gt; 0 )</td>
<td>( \frac{\partial e^*}{\partial \sigma_v^2} &lt; 0 )</td>
<td>( \frac{\partial e^*}{\partial (1-\beta)^{-1}} &gt; 0 )</td>
<td>( \frac{\partial e^*}{\partial \mu} &lt; 0 )</td>
<td>( \frac{\partial e^*}{\partial \sigma_w^2} &lt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

In order to confront these propositions with some cross-country evidence, we can now move on to the empirical evidence. This is the subject of the next section.

**IV. EMPIRICAL EVIDENCE**

In this section, the ultimate determinants of central bank independence discussed before are empirically investigated. We will use, for that purpose, the latent variables method (LISREL) to make a distinction between the optimal and actual (legal) degree of central bank independence. The reasons for this distinction are two-fold. First, the propositions derived in the former section are related to the optimal degree of central bank independence and not to the actual (legal) degree. These propositions formulate the relationship between the optimal degree and four economic and political factors in a country:
- the natural rate of unemployment (positive relation);
- society’s preferences for unemployment stabilization relative to inflation stabilization (positive relation);
- the variance of productivity shocks (negative relation);
- the slope of the Phillips curve (conditional positive relation);
- the degree of openness (negative relation); and
- the variance of shocks to the real exchange rate (negative relation).

These determinants, reflecting the economic and political structure of a country, explain theoretically the optimal degree of central bank independence in that country.

Second, there is also an identification and measurement problem. Whereas the determinants of central bank independence will change frequently during the sample period (i.e. the period 1960-1993), the actual degree - approximated by the legal indices of central bank independence - will hardly change in the same period. The stickiness of actual (legal)
central bank independence results from the fact that central bank laws are very occasionally adjusted in practice, especially in the industrial countries during the post-war period.\textsuperscript{13)} Moreover, it could be questioned whether the legal indices of central bank independence are a good measure of actual central bank independence (See also: Eijffinger and De Haan, 1995).

\textit{IV.1. The data}

As proxies for the ultimate determinants of central bank independence, we have chosen the following economic and political variables (See for a detailed account of these variables: Appendix C). For the natural rate of unemployment, the \textit{non-accelerating inflation rate of unemployment} (NAIRU) is taken from Layard, Nickell and Jackman (1991). They estimated the NAIRU for nineteen industrial countries in the period 1960-1988. The proxy for society’s preferences for unemployment stabilization relative to inflation stabilization is the number of years that a left-wing (socialist) party has been in government as a share of the total number of years (WLEFT). For, a left-wing government has a higher preference for unemployment stabilization and, thereby, the optimal degree of central bank independence increases under a left-wing government. The variance of productivity shocks is proxied by the \textit{variance of output growth} (GDP) on an annual basis (VPROD). We compute the slope of the Phillips curve, using labour’s income share in GDP.\textsuperscript{14)} Because data for labour’s income share are not available for all countries in our sample, we have taken the ratio between the \textit{compensation of employees} paid by resident producers to resident households and GDP (SLOPE).

The degree of openness is measured by the ratio between the \textit{imports of goods and services} and GDP (OPEN). The proxy for the variance of shocks to the real exchange rate is the variance of the ratio between the \textit{CPI inflation minus GDP deflator} and the degree of openness (VREER).

Therefore, the \textit{optimal} degree of central bank independence (OPCBI\textsubscript{M}) is explained by the following variables, taken in deviation from their mean (M)

\textsuperscript{13)} Very recently, some countries within the European Union - e.g. France and Spain - have made their central banks more independent from government because this is required by the Maastricht Treaty on Economic and Monetary Union. These changes of central bank laws are, however, too infrequent to be applicable for our empirical analysis of the determinants in the industrial countries.

\textsuperscript{14)} Since we use a Cobb-Douglas production function (equation (2.1)), the production elasticity of labour, $\beta$, equals labour’s income share in GDP.
OPCBI_M = a_1 [NAIRU_M] + a_2 [WLEFT_M] + a_3 [VPROD_M] +
(+) (-) (-)
a_4 [SLOPE_M] + a_5 [OPEN_M] + a_6 [VREER_M] (4.1)

The expected signs are denoted above the explanatory variables. The optimal degree of central bank independence is assumed to be a latent variable in our empirical model. Next to the observed explanatory variables measured in deviation from their mean (NAIRU_M, WLEFT_M, VPROD_M, SLOPE_M, OPEN_M and VREER_M), we need the actual (legal) degree of central bank independence as an observed variable. The actual degree of central bank independence is approximated by the legal degree, according to the four main indices of central bank independence in the literature.

The index of Alesina (AL) is a narrow measure of independence and based on Alesina (1988, 1989). The total index of political and economic independence of Grilli, Masciandaro and Tabellini (GMT) is a broad measure based on Grilli, Masciandaro and Tabellini (1991). The index of policy independence of Eijffinger and Schaling (ES) is, however, a narrow measure based on Eijffinger and Schaling (1992, 1993a) and extended by Eijffinger en Van Keulen (1994). These three legal indices have been normalized so that their values range, theoretically, from zero to one (AL_N, GMT_N and ES_N). The unweighted legal index of Cukierman (LVAU) is a very broad measure of independence and derived from Cukierman (1992). The Cukierman index is already normalized on its theoretical scale. Thus, the optimal degree of central bank independence is also normalized (OPCBI_NM) to have the same unit of measurement as the legal indices.15)

For our cross-country analysis, a set of nineteen industrial (OECD) countries is taken which are ranked - with some exceptions - by the above-mentioned indices. The sample period that we have chosen covers more than thirty years, namely the period 1960-1993 (for NAIRU and VREER: 1960-1988 and 1960-1989, respectively). The argument to choose such a long period is that it contains many political and business cycles and, thus, comprises changes of the political and economic structure affecting the optimal degree of central bank independence.

IV.2. The latent variables method

According to Bentler (1982), the essential characteristic of a latent variable is revealed by the fact that the system of linear structural equations in which the latent variable appears

---

15) As a consequence of the latent variables method (LISREL), these observed indices of central bank independence are also measured in deviation from their means: AL_NM, GMT_NM, ES_NM and LVAU_M. If all variables have an expected value zero, than their covariance equals E[x y].
cannot be manipulated so as to express this variable as a function of *measured* variables only.\(^{16}\)

Aigner, Hsiao, Kapteyn and Wansbeek (1984) state that, since 1970, there has been a resurgence of interest in econometrics in the topic of models involving latent variables. "That interest in such models had to be restimulated at all may seem surprising", in the opinion of Aigner et al., "since there can be no doubt that economic quantities frequently are measured with error and, moreover, that many applications depend on the use of observable proxies for otherwise unobservable conceptual variables" (p. 1323).

Estimation of a simultaneous equations model with latent variables can be done by means of a computer program for the analysis of covariance structures, such a LISREL (Linear Structural Relations). The idea behind LISREL is to compare a sample covariance matrix with the parametric structure imposed on it by the hypothesized model. Under normality, LISREL delivers *Full Information Maximum Likelihood* (FIML) estimates of the model parameters. Because of its general availability, LISREL is the most important tool for handling latent variables.

The specification of the latent variables model to be analyzed by LISREL is as follows.\(^{17}\)

Let \(\eta\) be the latent *dependent* variable, i.e. the latent optimal degree of central bank independence, and \(\xi\) be the latent *explanatory* variables, in our case the six ultimate determinants of central bank independence, satisfying a system of linear structural relations

\[
\eta = B \xi + \zeta, \tag{4.2}
\]

with \(B\) being the coefficient matrix and \(\zeta\) the disturbances. It is assumed that \(\eta, \xi, \text{ and } \zeta\) have zero expectations, and that \(\xi\) and \(\zeta\) are *uncorrelated*. Instead of the latent vectors \(\eta\) and \(\xi\), the vectors \(y\) and \(x\) are *observed*, such that

\[
y = \Lambda y = \eta + \gamma \tag{4.3}
\]


\(^{17}\) In order to avoid overlapping symbols between sections II and III (theoretical model) and section IV (latent variables model), our notation differs from that of the LISREL manual. Having one latent dependent variable, we use \(B\) and \(\gamma\), respectively, instead of the symbols \(\Gamma\) and \(\epsilon\) for the LISREL manual. Compare also Aigner et al. (1984, pp. 1370-1371) in this respect.
\[
x = \Lambda_x - \xi + \delta, \\
(4.4)
\]

with \(\Lambda_y\) and \(\Lambda_x\) the coefficient matrices, and \(\gamma\) and \(\delta\) the vectors of measurement errors, uncorrelated with \(\eta, \xi, \zeta\) and each other, but possibly correlated among themselves. The observed vectors \(y\) and \(x\) are measured as deviations from their means, thus, having zero expectations and a covariance equal to \(E[xy]\). This implies, of course, that \(\gamma\) and \(\delta\) have also zero expectations.

Therefore, \(y\) is a vector of observed legal indices of central bank independence (AL, GMT, ES and LVAU), normalized and measured in deviation from their means,

\[
y = \begin{bmatrix}
    \text{AL}_{\text{NM}} \\
    \text{GMT}_{\text{NM}} \\
    \text{ES}_{\text{NM}} \\
    \text{LVAU}_{\text{M}}
\end{bmatrix}
\]

and \(x\) is a vector of observed explanatory variables, being the non-accelerating inflation rate of unemployment (NAIRU), the percentage of years of a left-wing government (WLEFT), the variance of output growth (VPROD), the compensation of employees as share of GDP (SLOPE), the degree of openness (OPEN) and the variance of shocks to the real exchange rate (VREER), measured in deviation from their means

\[
x = \begin{bmatrix}
    \text{NAIRU}_{\text{M}} \\
    \text{WLEFT}_{\text{M}} \\
    \text{VPROD}_{\text{M}} \\
    \text{SLOPE}_{\text{M}} \\
    \text{OPEN}_{\text{M}} \\
    \text{VREER}_{\text{M}}
\end{bmatrix}
\]

So, equations (4.3) and (4.4) become, respectively

\[
\begin{bmatrix}
    \text{AL}_{\text{NM}} \\
    \text{GMT}_{\text{NM}} \\
    \text{ES}_{\text{NM}} \\
    \text{LVAU}_{\text{M}}
\end{bmatrix}
= \begin{bmatrix}
    \lambda_{y1} \\
    \lambda_{y2} \\
    \lambda_{y3} \\
    \lambda_{y4}
\end{bmatrix} \cdot \begin{bmatrix}
    \gamma_1 \\
    \gamma_2 \\
    \gamma_3 \\
    \gamma_4
\end{bmatrix} + \begin{bmatrix}
    \eta_1 \\
    \eta_2 \\
    \eta_3 \\
    \eta_4
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
    \text{NAIRU}_{\text{M}} \\
    \text{WLEFT}_{\text{M}} \\
    \text{VPROD}_{\text{M}} \\
    \text{SLOPE}_{\text{M}} \\
    \text{OPEN}_{\text{M}} \\
    \text{VREER}_{\text{M}}
\end{bmatrix}
= \begin{bmatrix}
    \lambda_{x1} \\
    \lambda_{x2} \\
    \lambda_{x3} \\
    \lambda_{x4} \\
    \lambda_{x5} \\
    \lambda_{x6}
\end{bmatrix} \cdot \begin{bmatrix}
    \xi_1 \\
    \xi_2 \\
    \xi_3 \\
    \xi_4 \\
    \xi_5 \\
    \xi_6
\end{bmatrix} + \begin{bmatrix}
    \delta_1 \\
    \delta_2 \\
    \delta_3 \\
    \delta_4 \\
    \delta_5 \\
    \delta_6
\end{bmatrix}
\]
Furthermore, $\Phi$ and $\Psi$ are defined as the covariance matrix of $\xi$ and the variance of $\zeta$, respectively, and $\Theta_\gamma$ and $\Theta_\delta$ as the true variance-covariance matrices of $\gamma$ and $\delta$, respectively. Then it follows from the above assumptions that the variance-covariance matrix $\Sigma$ of $[y', x']'$ is

$$
\Sigma = \begin{bmatrix}
\Lambda_y [B\Phi B' + \Psi] \Lambda'_y + \Theta_\gamma & \Lambda_y B\Phi \Lambda'_x \\
\Lambda_x \Phi B' \Lambda'_y & \Lambda_x \Phi \Lambda'_x + \Theta_\delta
\end{bmatrix}
$$

(4.7)

Assuming that the latent explanatory variables ($\xi$) equal the observed ($x$), thus $\xi = x$, then $\Theta_\delta = 0$ and $\Lambda_x = I$, and equation (4.7) simplifies to $^{18}$

$$
\Sigma = \begin{bmatrix}
\Lambda_y [B\Phi B' + \Psi] \Lambda'_y + \Theta_\gamma & \Lambda_y B\Phi \\
\Phi B' \Lambda'_y & \Phi
\end{bmatrix}
$$

(4.8)

The parameters occurring in $\Sigma$ ($\Lambda_y, B, \Phi, \Psi, \Theta_\gamma$) are estimated on the basis of the matrix $S$ of second sample moments of $x$ and $y$. In order to identify all parameters, additional restrictions on the parameters have to be imposed. Given these restrictions and the structure that equation (4.8) imposes on the data, LISREL computes FIML estimates of the parameters when $[y', x']$ is normally distributed, i.e. when the following criterion is minimized

$$
\ln |\Sigma| + \text{tr} [S\Sigma^{-1}]
$$

(4.9)

To be able to identify all parameters of the model, we have made the following two additional restrictions:

(i) $\lambda_{\gamma_1} = \lambda_{\gamma_2} = \lambda_{\gamma_3} = \lambda_{\gamma_4} = 1$, which implies that the latent optimal degree of central bank independence ($\eta$) has the same unit of measurement as the observed legal indices

$^{18}$ So, we make only a distinction between the latent optimal degree of central bank independence ($\eta$) and the observed actual degree ($y$) measured by the legal indices of central bank independence. Thus, the optimal degree of central bank independence is derived from the covariances of the four legal indices.
of central bank independence \((y)\),\(^{19}\) and

(ii) \(\Theta_\gamma\) is diagonal, which implies that the correlation between the observed legal indices of central bank independence \((y)\) is only caused by the latent optimal degree \((\eta)\).\(^{20}\)

### IV.3. The empirical results

On the basis of the restrictions given in the former section, LISREL computes Full Information Maximum Likelihood estimates of the parameters of the model. Computation with LISREL renders two different kinds of estimations. First, the relationship between the optimal degree of central bank independence \((\eta\) here renamed as OPCBI_NM) and the explanatory variables \((\text{NAIRU}_M, \text{WLEFT}_M, \text{VPROD}_M, \text{SLOPE}_M, \text{OPEN}_M\) and \(\text{VREER}_M\)), reflecting the ultimate determinants of central bank independence, is estimated.\(^{21}\) Second, by estimating this relationship and calculating the optimal degree of central bank independence for each country \(\text{OPCBI}_\text{NM}\), the comparison between the optimal degree and the legal indices of central bank independence \((\text{AL}_\text{NM}, \text{GMT}_\text{NM}, \text{ES}_\text{NM}\) and \(\text{LVAU}_M\)) can be made. Such a comparison is only possible if both the optimal degree and the legal indices are normalized on their theoretical scale and measured in deviation from their means.\(^{22}\)

Next to the differences of individual legal indices with the optimal degree, the average difference \((\text{AVDIF})\) may be calculated in the following way:

\[
\text{AVDIF} = \frac{[\text{AL}_\text{NM}] + [\text{GMT}_\text{NM}] + [\text{ES}_\text{NM}] + [\text{LVAU}_M]}{4} - [\text{OPCBI}_\text{NM}] \quad (4.10)
\]

This average difference is positive, if the average of legal indices exceeds the optimal degree, and negative, if the optimal degree exceeds the average of legal indices. A positive average difference indicates that the legal degree of central bank independence should be decreased, whereas a negative average difference that the legal degree should

---

\(^{19}\) This restriction is a consequence of the normalization of all observed legal indices and the latent optimal degree of central bank independence so that their values range, theoretically, from zero to one.

\(^{20}\) The measurement errors \((\gamma)\) in equation (4.3) are, thereby, uncorrelated.

\(^{21}\) Because all variables are measured in deviation from their mean and have, thus, zero expectations, the constant is eliminated from the model.

\(^{22}\) Note that the legal index of Cukierman (LVAU) is already normalized on its theoretical scale, i.e. in theory its lowest value is 0 and its highest value 1.
Table 4.1. Table based on estimation with all restrictions.  
Sample period 1960 - 1993 for all variables (except for NAIRU and VREER)

OPCBI_NM = -0.016 * NAIRU_M + 0.003 * WLEFT_M - 0.012 * VPROD_M + 0.374 * SLOPE_M - 0.188 * OPEN_M - 0.001 * VREER_M  
\( R^2 = 0.42 \)  
\( DF = 23 \)

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>AL_NGMT_N</th>
<th>ES_N</th>
<th>LVAU_M</th>
<th>AL_NM</th>
<th>GMT_NM</th>
<th>ES_NM</th>
<th>LVAU_M</th>
<th>OPCBI_NM</th>
<th>AL_NM</th>
<th>GMT_NM</th>
<th>ES_NM</th>
<th>LVAU_M</th>
<th>AVDIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSTRALIA</td>
<td>0.00</td>
<td>0.56</td>
<td>0.00</td>
<td>0.31</td>
<td>-0.38</td>
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<td>-0.42</td>
<td>-0.05</td>
<td>0.029</td>
<td>-0.41</td>
<td>0.04</td>
<td>-0.45</td>
<td>-0.08</td>
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<tr>
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<td>0.19</td>
<td>0.50</td>
<td>0.27</td>
<td>-0.38</td>
<td>-0.30</td>
<td>0.08</td>
<td>-0.09</td>
<td>-0.146</td>
<td>-0.24</td>
<td>-0.16</td>
<td>0.23</td>
<td>0.06</td>
</tr>
<tr>
<td>NORWAY</td>
<td>0.33</td>
<td>0.25</td>
<td>0.14</td>
<td>.</td>
<td>-0.05</td>
<td>.</td>
<td>-0.17</td>
<td>-0.22</td>
<td>0.050</td>
<td>-0.10</td>
<td>-0.22</td>
<td>-0.27</td>
<td>-0.20</td>
</tr>
<tr>
<td>SPAIN</td>
<td>0.00</td>
<td>0.31</td>
<td>0.00</td>
<td>0.21</td>
<td>-0.38</td>
<td>-0.18</td>
<td>-0.42</td>
<td>-0.15</td>
<td>-0.208</td>
<td>-0.17</td>
<td>0.03</td>
<td>-0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>SWEDEN</td>
<td>0.33</td>
<td>0.25</td>
<td>0.27</td>
<td>.</td>
<td>-0.05</td>
<td>.</td>
<td>-0.17</td>
<td>-0.09</td>
<td>0.229</td>
<td>-0.28</td>
<td>-0.40</td>
<td>-0.32</td>
<td>-0.32</td>
</tr>
<tr>
<td>SWITZERLAND</td>
<td>1.00</td>
<td>0.75</td>
<td>1.00</td>
<td>0.68</td>
<td>0.62</td>
<td>0.26</td>
<td>0.58</td>
<td>0.32</td>
<td>0.128</td>
<td>0.49</td>
<td>0.13</td>
<td>0.45</td>
<td>0.19</td>
</tr>
<tr>
<td>UK</td>
<td>0.33</td>
<td>0.38</td>
<td>0.25</td>
<td>0.31</td>
<td>-0.05</td>
<td>-0.11</td>
<td>-0.17</td>
<td>-0.05</td>
<td>0.143</td>
<td>-0.19</td>
<td>-0.26</td>
<td>-0.31</td>
<td>-0.19</td>
</tr>
<tr>
<td>US</td>
<td>0.67</td>
<td>0.75</td>
<td>0.50</td>
<td>0.51</td>
<td>0.28</td>
<td>0.26</td>
<td>0.08</td>
<td>0.15</td>
<td>0.118</td>
<td>0.17</td>
<td>0.14</td>
<td>-0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Notes:  
\(_N\) stands for normalized variables;  
\(_M\) stands for deviations of variables from their means;  
t-values in parentheses.
be increased in order to bring it closer to the optimal degree based on the ultimate determinants of central bank independence.

Table 4.1 shows the estimation results, with all restrictions imposed in the former section, for the sample period 1960-1993 (for NAIRU and VREER, the sample period 1960-1988 and 1960-1989, respectively).

From Table 4.1, it can be seen that all explanatory variables of the optimal degree, except NAIRU, have the expected sign. Only one explanatory variable (SLOPE) is significant at a 5% significance level. The other explanatory variables have relatively low t-values.

Nevertheless, we have calculated the optimal degree on the basis of the ultimate determinants for each country and the average difference between these variables. Positive average differences - of 0.20 or higher - are found for Germany and Switzerland, implying that the legal degree of central bank independence exceeds the optimal degree and that the legal degree should be decreased. Negative average differences - of 0.20 or lower - are observed for Australia, Norway, Sweden and the United Kingdom, meaning that the optimal degree exceeds the legal degree and that the legal degree should be increased.

For the other countries - Austria, Belgium, Canada, Denmark, Finland, France, Ireland, Italy, Japan, the Netherlands, New Zealand, Spain and the United States - the average differences are relatively small, indicating that there is no reason to adjust the central bank law in these countries from the perspective of the ultimate determinants. In some countries - notably France and Spain - the central bank has, recently, been made more independent from government which can be explained by another argument: a prerequisite for entering the third phase of Economic and Monetary Union in Europe is, among others, the independence of the national central banks of the participating countries.

The relatively low t-values for the explanatory variables in Table 4.1 could, probably, be attributed to the many severe restrictions imposed on the model by LISREL and the two additional restrictions made by us ($\lambda_y_1 = \lambda_y_2 = \lambda_y_3 = \lambda_y_4 = 1$ and $\Theta_\gamma$ is diagonal) to identify all parameters of the model. Relaxing some of these restrictions might improve the t-values of the explanatory variables.\(^23\)

Table 4.2 gives the empirical results, if we relax cumulatively five restrictions on the covariances, for the sample period 1960-1993 (for NAIRU and VREER: 1960-1988 and 1960-1989, respectively). All other restrictions on the model remain imposed.

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23) See in this respect: Aigner, Hsiao, Kapteyn and Wansbeek (1984, p. 1371). The relaxing of restrictions could imply that, although the latent variables method is still used, the assumptions of LISREL are not valid anymore.
Table 4.2 Table based on estimation with cumulative relaxation of restrictions
Sample period 1960 - 1993 for all variables (except for NAIRU and VREER)

<table>
<thead>
<tr>
<th>Lifted Restriction</th>
<th>Estimated Equation</th>
<th>R² and DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>no lifted restriction</td>
<td>OPCBI_NM = -0.016 * NAIRU_M + 0.003 * WLEFT_M - 0.012 * VPROD_M + 0.374 * SLOPE_M - 0.188 * OPEN_M - 0.001 * VREER_M</td>
<td>R²=0.42, DF=23</td>
</tr>
<tr>
<td>( \gamma_2, \xi_3 )</td>
<td>OPCBI_NM = -0.013 * NAIRU_M + 0.024 * WLEFT_M + 0.037 * VPROD_M + 0.651 * SLOPE_M - 0.392 * OPEN_M - 0.003 * VREER_M</td>
<td>R²=0.55, DF=22</td>
</tr>
<tr>
<td>( \gamma_2, \xi_6 )</td>
<td>OPCBI_NM = -0.010 * NAIRU_M + 0.108 * WLEFT_M + 0.013 * VPROD_M + 0.529 * SLOPE_M - 0.078 * OPEN_M - 0.000 * VREER_M</td>
<td>R²=0.35, DF=21</td>
</tr>
<tr>
<td>( \gamma_3, \gamma_1 )</td>
<td>OPCBI_NM = -0.005 * NAIRU_M + 0.119 * WLEFT_M + 0.010 * VPROD_M + 0.418 * SLOPE_M - 0.061 * OPEN_M - 0.000 * VREER_M</td>
<td>R²=0.40, DF=20</td>
</tr>
<tr>
<td>( \gamma_3, \xi_5 )</td>
<td>OPCBI_NM = -0.004 * NAIRU_M + 0.121 * WLEFT_M + 0.010 * VPROD_M + 0.421 * SLOPE_M - 0.069 * OPEN_M - 0.001 * VREER_M</td>
<td>R²=0.35, DF=18</td>
</tr>
</tbody>
</table>

Notes: _N stands for normalized variables; _M stands for deviations of variables from their means; t-values in parentheses.
Testing structural models, a univariate Lagrange Multiplier test is carried out for most elements in the model matrices that are constrained to equal constants. When the test statistic, having a $\chi^2$-distribution, has a value larger than 2.71 the restriction is rejected at a significance level of 10%. In the first regression, with all restrictions imposed, the constraint that the covariance of $\gamma_2$ and $\xi_3$ equals zero is rejected. This means that the disturbances of the GMT-index and the variance of productivity shocks (VPROD) are correlated. The test statistic has a value of 6.74, which is the highest of all restrictions. Therefore, we have lifted this restriction and tested the modified model.

Table 4.3 Average differences with cumulative relaxation of restrictions
Sample period 1960 - 1993 for all variables (except for NAIRU and VREER)

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>no lifted restriction</th>
<th>$\gamma_2$, $\xi_3$</th>
<th>$\gamma_2$, $\xi_6$</th>
<th>$\gamma_2$, $\gamma_1$</th>
<th>$\gamma_3$, $\gamma_1$</th>
<th>$\gamma_3$, $\xi_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSTRALIA</td>
<td>-0.22</td>
<td>-0.17</td>
<td>-0.16</td>
<td>-0.16</td>
<td>-0.15</td>
<td>-0.16</td>
</tr>
<tr>
<td>AUSTRIA</td>
<td>0.09</td>
<td>0.14</td>
<td>0.14</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>BELGIUM</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>CANADA</td>
<td>-0.05</td>
<td>-0.04</td>
<td>0.03</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>DENMARK</td>
<td>0.11</td>
<td>0.13</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>FINLAND</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
<tr>
<td>FRANCE</td>
<td>-0.11</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>GERMANY</td>
<td>0.39</td>
<td>0.43</td>
<td>0.45</td>
<td>0.45</td>
<td>0.46</td>
<td>0.45</td>
</tr>
<tr>
<td>IRELAND</td>
<td>0.11</td>
<td>0.13</td>
<td>0.10</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>ITALY</td>
<td>-0.08</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.07</td>
<td>-0.06</td>
<td>-0.08</td>
</tr>
<tr>
<td>JAPAN</td>
<td>0.14</td>
<td>0.06</td>
<td>0.06</td>
<td>0.09</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>NETHERLANDS</td>
<td>0.10</td>
<td>0.08</td>
<td>0.13</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>NEWZEALAND</td>
<td>-0.03</td>
<td>-0.12</td>
<td>-0.16</td>
<td>-0.13</td>
<td>-0.14</td>
<td>-0.13</td>
</tr>
<tr>
<td>NORWAY</td>
<td>-0.20</td>
<td>-0.04</td>
<td>-0.13</td>
<td>-0.14</td>
<td>-0.14</td>
<td>-0.14</td>
</tr>
<tr>
<td>SPAIN</td>
<td>-0.07</td>
<td>-0.20</td>
<td>-0.12</td>
<td>-0.16</td>
<td>-0.15</td>
<td>-0.17</td>
</tr>
<tr>
<td>SWEDEN</td>
<td>-0.33</td>
<td>-0.37</td>
<td>-0.33</td>
<td>-0.29</td>
<td>-0.30</td>
<td>-0.30</td>
</tr>
<tr>
<td>SWITZERLAND</td>
<td>0.32</td>
<td>0.12</td>
<td>0.21</td>
<td>0.25</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>UK</td>
<td>-0.24</td>
<td>-0.31</td>
<td>-0.22</td>
<td>-0.20</td>
<td>-0.20</td>
<td>-0.21</td>
</tr>
<tr>
<td>US</td>
<td>0.08</td>
<td>0.20</td>
<td>0.04</td>
<td>0.09</td>
<td>0.08</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Now the restriction on the covariance of $\gamma_2$ and $\xi_6$ to be zero has the highest test statistic. So, this restriction has been lifted implying that the disturbances of the GMT-index and
the variance of shocks to the real exchange rate (VREER) can be correlated. This process goes on until there is no restriction left with a test statistic higher than 2.71. The results of the subsequent estimations are summarized in Table 4.2. From this table, it is clear that the t-values of all explanatory variables, except NAIRU, improve considerably with the lifting of the restriction on the covariance of \( [\gamma_2, \xi_3] \) between the GMT-index and the variance of productivity shocks. The last variable becomes even significant at a 97.5% confidence level. All explanatory variables, except NAIRU and VPROD, have the expected sign. If we compare the coefficients of the explanatory variables in this table with those in Table 4.1, the estimated coefficients do not always seem robust. Therefore, we have also calculated the optimal degree of central bank independence and the average difference with the legal indices (AVDIF) for each country. Table 4.3 shows the average differences in case of a cumulative relaxation of the five restrictions on the covariances. The positive average differences for Germany and Switzerland appear to be still in place. The negative average differences for Australia, Norway, Sweden and the United Kingdom also still remain. Relatively small average differences are found for countries, like Belgium, Canada, Finland, France, Italy, Japan and the United States. Apparently, these countries have central bank laws which correspond more or less with their optimal degree based on the ultimate determinants, insofar as they are captured in our empirical model.

V. CONCLUSION

What may be concluded from the previous sections?

First, it is possible to derive propositions on the basis of our theoretical model which formulates the relationship between the optimal degree of central bank independence and six ultimate determinants in a country, namely the natural rate of employment, the society’s preferences for unemployment stabilization relative to inflation stabilization, the variance of productivity shocks, the slope of the Phillips curve, the degree of openness and the variance of shocks to the real exchange rate. These determinants, reflecting the economic and political structure of a country, refer only indirectly to the actual (legal) degree of central bank independence.

Second, to distinguish between the optimal and actual (legal) degree of central bank independence the latent variables method (LISREL) appears to be very fruitful as an empirical model. Not only enables this method us to explain the optimal degree by proxies for the ultimate determinants (NAIRU, WLEFT, VPROD, SLOPE, OPEN and VREER), but also to compare the optimal degree with the legal indices of central bank independence (AL, GMT, ES and LVAU). The latent variables method, based on nineteen industrial countries, for the sample period 1960-1993 (except for NAIRU and VREER) leads to estimations which support our theoretical model reasonably, if we relax cumulatively five
restrictions on the covariances.

Third, the comparison between the optimal degree and the legal indices of central bank independence renders some interesting results. Some countries - like Germany and Switzerland - seem to have a suboptimally high degree of central bank independence, whereas other - such as Australia, Norway, Sweden and the United Kingdom - appear to have a suboptimally low degree. For countries such as Belgium, Canada, Finland, France, Italy, Japan and the United States, it is fair to conclude that these countries have more or less an optimal degree of independence.

Finally, it should be mentioned that both our theoretical and empirical model can be extended with other economic and political determinants of central bank independence. One could, for example, extend the model with the degree of political instability and uncertainty in a country. These extensions constitute our research agenda for the future.

APPENDIX A. THE DERIVATION OF THE EXPECTED VALUE OF SOCIETY’S LOSS FUNCTION UNDER AN ARBITRARY MONETARY POLICY REGIME

In this Appendix, following Rogoff (1985a, pp. 1175—1176), we develop a notation for evaluating the expected value of society’s loss function under any arbitrary monetary policy regime "A", \( E_{t-1} L^A \) (equation (3.1) of the text). Unemployment under regime A is given by

\[
u_t^A = \bar{u} - \frac{1}{1-\beta} (\hat{p}_t^A - E_{t-1} \hat{p}_t^A + \mu_t) \tag{A.1}
\]

Squaring and taking expectations yields

\[
E_{t-1}(\nu_t^A)^2 = \bar{u}^2 + E_{t-1} \left[ \frac{\mu_t}{1-\beta} + \frac{1}{1-\beta} (\hat{p}_t^A - E_{t-1} \hat{p}_t^A) \right]^2 \tag{A.2}
\]

The CPI under regime A can be expanded as

\[
p_c^A = \hat{p}_c^A + (p_c^A - E_{t-1} p_c^A) \tag{A.3}
\]

where \( \hat{p}_c^A \) is the mean (expected) CPI in period t. Squaring and taking expectations, in turn, yields

\[
E_{t-1}(p_c^A)^2 = (\hat{p}_c^A)^2 + E_{t-1} (p_c^A - E_{t-1} p_c^A)^2 \tag{A.4}
\]

The expected value of society’s loss function under regime A is
E_{t-1}I_t^\Lambda = \frac{1}{2} E_{t-1}(p_t^\Lambda)^2 + \frac{\chi}{2} E_{t-1}(u_t^\Lambda)^2 \tag{A.5}

Substituting (A.2) and (A.4) into (A.5), one obtains equation (3.1) of the text.

APPENDIX B. DERIVATION OF THE PROPERTIES OF THE FUNCTION F(\varepsilon) IN THE FIRST-ORDER CONDITION.

(1) Demonstration that \( \frac{\partial F}{\partial \varepsilon} < 0 \).

The first derivative of F with respect to \( \varepsilon \) is given by

\[
\frac{\partial F}{\partial \varepsilon} = -3\tilde{u}\chi[(1+\varepsilon)(1-\beta)^2 + \chi] \frac{\Gamma - \chi}{[\sigma_v^2 + \mu^2\sigma_u^2](1-\beta)^4(1+\varepsilon)^4}
\]

which is negative.

(2) Demonstration that \( \frac{\partial^2 F}{\partial \varepsilon^2} > 0 \).

The second derivative of F with respect to \( \varepsilon \) is given by

\[
\frac{\partial^2 F}{\partial \varepsilon^2} = \frac{6\tilde{u}^2\chi\Gamma\Gamma - \chi}{[\sigma_v^2 + \mu^2\sigma_u^2](1-\beta)^4(1+\varepsilon)^5}
\]

where \( \Gamma \equiv (1+\varepsilon)(1-\beta)^2 + 2\chi \), (B.2) is positive.

(3) Demonstration that \( F(0) = \frac{[(1-\beta)^2 + \chi]^3\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_u^2](1-\beta)^4} \).

This can be shown by direct examination of the right-hand side of equation (3.8) at \( \varepsilon = 0 \).

(4) Demonstration that \( \frac{(1-\beta)^2\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_u^2]} < F(\varepsilon) < \frac{[(1-\beta)^2 + \chi]^3\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_u^2](1-\beta)^4} \).

Since \( F(0) = \frac{[(1-\beta)^2 + \chi]^3\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_u^2](1-\beta)^4} \),

\[
\lim_{\varepsilon \to 0} \frac{(1-\beta)^2\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_u^2]} \quad \text{and} \quad \frac{\partial F}{\partial \varepsilon} < 0, \quad F(\varepsilon) \text{ must be bounded between}
\]

\[
\frac{(1-\beta)^2\tilde{u}^2}{[\sigma_v^2 + \mu^2\sigma_u^2]} \quad \text{and} \quad F(0).
\]
(5) Demonstration that $\frac{\partial F}{\partial \tilde{u}} > 0$.

The first derivative of $F$ with respect to $\tilde{u}$ is given by

$$\frac{\partial F}{\partial \tilde{u}} = \frac{2[(1+\varepsilon)(1-\beta)^2 - \chi]^3 \tilde{u}}{[\sigma_v^2 + \mu^2 \sigma_w^2](1-\beta)^4(1+\varepsilon)^3}$$  \hspace{1cm} (B.3)

(B.3) is positive.

(6) Demonstration that $\frac{\partial F}{\partial \chi} > 0$

The first derivative of $F$ with respect to $\chi$ is given by

$$\frac{3[(1+\varepsilon)(1-\beta)^2 + \chi]^2 \tilde{u}^2}{[\sigma_v^2 + \mu^2 \sigma_w^2](1-\beta)^4(1+\varepsilon)^3}$$  \hspace{1cm} (B.4)

It can easily be checked that (B.4) is positive.

(7) Demonstration that $\frac{\partial F}{\partial \sigma_v^2} < 0$.

The first derivative of $F$ with respect to $\sigma_v^2$ is given by

$$\frac{\partial F}{\partial \sigma_v^2} = -\frac{[(1+\varepsilon)(1-\beta)^2 + \chi]^3 \tilde{u}^2}{[\sigma_v^2 + \mu^2 \sigma_w^2](1-\beta)^4(1+\varepsilon)^3}$$  \hspace{1cm} (B.5)

(B.5) is negative.

(8) Demonstration that $\frac{\partial F}{\partial (1-\beta)^{-1}} > 0$.

The first derivative of $F$ with respect to $(1-\beta)^{-1}$ is given by

$$\frac{\partial F}{\partial (1-\beta)^{-1}} = \frac{2\chi - (1+\varepsilon)(1-\beta)^2)[(1+\varepsilon)(1-\beta)^2 + \chi]^2 2\tilde{u}^2}{[\sigma_v^2 + \mu^2 \sigma_w^2](1-\beta)^4(1+\varepsilon)^3}.$$  \hspace{1cm} (B.6)
(B.6) is positive if $\chi > \frac{(1 - \varepsilon)(1 - \beta)^2}{2}$.

APPENDIX C. THE DATA

NAIRU:  R. Layard, S. Nickell and R. Jackman, 
Unemployment, Macroeconomic Performance and the Labour Market, Oxford, 
Estimates for NAIRU 1960-1988, Table 14, Chapter 9.

years that a left-wing party has been in the government, either alone or in a 
coalition)/(total # years), 1960-1993.

VPROD:  OECD Main Economic Indicators. 

- (Compensation of employees paid by resident producers/GDP)], in current 
prices. 

(Imports of goods and services/GDP), in current prices. OECD Paris, 1979, 

Variance of [(CPI inflation-GDP deflator)/OPEN], CPI inflation and GDP 
deflator calculated from private final consumption expenditure price index and 
GDP price index, respectively.
REFERENCES