Delegation of Monitoring in a Principal-Agent Relationship

Strausz, R.G.

Publication date:
1995

Delegation of Monitoring in a Principal-Agent Relationship

Roland Strausz*

Free University of Berlin

June 14, 1995

Abstract

This paper studies a principal-agent relationship with moral hazard in which the principal or the supervisor can monitor the agent's hidden action by using identical monitoring technologies. The paper shows that delegation of monitoring to the supervisor is profitable because of two effects. With delegation the principal can better regulate the incentives (incentive effect) and can commit to wage structures to which she could not commit without delegation (commitment effect). As a logical step collusion is introduced and it is shown that even with the possibility of collusion delegation is an optimal strategy.

JEL Classification: D82, L23

Keywords: Monitoring, delegation, moral hazard, collusion.

*FU Berlin, Boltzmannstr. 20, D-14195, Berlin, Germany. This paper was almost entirely written at CentER at Tilburg Universtiy, where I benefitted much from the comments and suggestions of Helmut Bester, Paul de Bijl, Jan Bouckaert, Eric van Damme and Sjaak Hurkens. Furthermore I would like to thank the participants of the ENTER Jamboree 1995 for their helpful comments. E-mail: Roland_Strausz@ccmailer.wiwiss.fu-berlin.de

1 Introduction

Standard agency theory tells us that optimal incentive schemes make use of all available information related to the agent's performance. Hart and Holmström (1987) see in this sufficient statistic result "the main predictive content of the basic agency model". They conclude that "agency relationships create a demand for monitoring". Principals are interested in monitoring, since it creates information about the agent's behavior. Tirole (1986), however, notes that if monitoring is performed by a third party (the supervisor) then new problems can arise. The supervisor has his own interests and these may be in conflict with the principal's. Tirole introduces the possibility of collusion between the supervisor and the agent and shows that it limits the scope of implementable contracts. Recent models like Laffont and Tirole (1991) and Kofmann and Lawarrée (1993) develop the idea of collusion in a principal-supervisor-agent hierarchy further.

None of these papers, however, explains why the principal needs a third player as monitor. Implicitly they assume that the supervisor is more efficient in monitoring, or that the principal does not have the time. The explanation is therefore similar to why the principal needs an agent in the first place. The question, however, is important, since the threat of collusion seems to indicate that the principal is better off performing monitoring herself. Tirole (1992) even claims that "[s]tandard sufficient statistics principles for rewarding agents do not hold in the presence of collusion". This paper explicitly addresses the question of engaging an independent supervisor.

The problem is analyzed in a simple agency setting with hidden action (moral hazard). Monitoring of the agent's action is possible and can be performed by either the principal or an independent supervisor. The paper shows that delegation of monitoring is preferred by the principal, even when there exist possibilities of collusion. There are two explanations for the result. First, the model assumes that the decision to monitor and the choice of the agent's action are taken simultaneously. We take this as the most realistic setting. The principal then has to use contracts to create two types of incentives. On the one hand, she has to induce the agent to take a high effort level. On the other hand, she must set incentives in order for monitoring to take place, since the agent will not choose a high effort level when monitoring does not occur. When the principal does not delegate monitoring she has only one contract through which she can regulate both incentives. If the principal delegates monitoring, then she has also the contract of the supervisor by which she can create incentives. The idea that in a principal-agent relationship with monitoring the principal has to create two types of incentives is not new. Khalil (1991) studies a model in which the principal can audit the agent's action ex post.[1] The paper shows that if the principal cannot commit to auditing ex ante, then the principal has to design contracts in such a way that she will have incentives to audit ex-post. Khalil claims that the model is equivalent to a "three-person scenario" with three physically distinct players: a player who designs the contract, an agent who chooses an action and a third player who decides whether to audit the agent's action. In our context this claim is not correct.

Second, it is assumed that information which results from the monitoring procedure is private. This implies that the monitor has to decide whether to make the information public or to withhold it. When the principal delegates monitoring, she cannot commit to revealing information which will hurt her. She can therefore not use a carrot and stick approach, because she will not reveal information which forces her to hand out the carrots. If the principal delegates monitoring commitment to a carrot and stick approach is possible and, as we show, optimal. Delegation has therefore also a commitment effect. We show that with delegation the principal is able to approximate the first best solution by setting wage differences which tend to infinity. It is then argued that this implies that there are large benefits from side-contracting (collusion) and that it is unrealistic to assume that under these extreme circumstances collusion will not take place. As a logical step we, therefore, introduce the possibility of costly collusion, which enables us to

[1] Note that we assume that the agent's action and the monitoring decision are chosen simultaneously rather than sequentially. In this respect it is illuminating to stress the difference between auditing, which typically occurs after an action has been taken, and monitoring which occurs while an action is taken or a task is being performed.

parameterize the degree of commitment. It is shown that even without the commitment effect delegation is still profitable due to the incentive effect mentioned above.

The rest of the paper is organized as follows. The next section introduces the model. Section 3 analyzes the game when the principal acts as monitor. Section 4 studies the game when the principal delegates monitoring to the supervisor. Section 5 and 6 study different aspects of collusion. Section 7 concludes.

2 The Model

The game is played by three risk neutral players: a principal, a supervisor and an agent. All players have the objective to maximize their expected payoff. Outside options of the supervisor and agent are normalized to zero. Both players are also protected by limited liability. They cannot be forced to make positive transfers to the principal.

The agent is employed by the principal's firm. He has the choice between working at a high effort level a_H and working at a low effort level a_L. The parameter a_i $(i = H, L)$ represents the disutility of effort in monetary terms. It is assumed that the agent dislikes effort, which implies that $\Delta a \equiv a_H - a_L > 0$. The effort of the agent is not observable.

As the owner of the firm the principal receives the output resulting from the agent's effort. A high output y_H results when the agent's effort is a_H. The principal receives a low output y_L when the agent's action is a_L, where $\Delta y \equiv y_H - y_L > 0$. Output occurs after the agent has exerted his effort. We assume that the principal captures the output directly and cannot produce hard evidence on the total output received. Output is therefore also not observable.

We assume that a costly monitoring technology is available. The technology is controlled by the monitor, who can either be the principal or the supervisor. If the monitor decides to monitor, he first has to pay a cost $c > 0$. After paying the cost the action of the agent is revealed with a probability λ. Note that the monitoring procedure is imperfect in the sense that not always a result is obtained. If, however, a result is obtained then the monitor has hard evidence about the agent's action. The evidence is private to the monitor, but when it is revealed, it is verifiable by a third party.

The monitoring procedure itself is not verifiable. It cannot be checked by the players whether the monitor actually paid the cost c and monitored. Therefore, if the monitor claims that nothing was observed from the monitoring process, then this can have three causes. It might be that the monitor did not monitor at all. It may have happened that the monitor did monitor, but did not get a result. Or, it might be that the monitor did monitor and got a result, but did not reveal it. The only verifiable states of the world in the model are therefore the following. State H: It is revealed that the agent's action was a_H. State L: It is revealed that the agent's action was a_L. State N: Nothing is revealed about the action of the agent.

The principal offers a contract w to the agent and a contract t to the supervisor, where a contract is a set of contingent transfers from the principal to a certain player covering all verifiable states of the world. Together with the assumption of limited liability the general form of a feasible contract is $w \equiv (w_H, w_L, w_N) \in \mathbb{R}^3_+$ and $t \equiv (t_H, t_L, t_N) \in \mathbb{R}^3_+$.

Before presenting the timing of the game we recapitulate the main assumptions of our principal-agent model. The agent's action and the resulting output are unverifiable. Costly monitoring can be performed by either the principal or the supervisor and is also unverifiable. Only the result of monitoring is verifiable, but in first instance private information to the person who monitored.

The timing of the game is as follows:

t=0: The principal assigns the monitoring technology to herself or to the supervisor.

t=1: The principal offers a contract w to the agent and a contract t to the supervisor.

t=2: The supervisor and agent decide whether to accept the contract.

t=3: The agent and the monitor play a simultaneous move game. The agent chooses the high or the low action (a_H or a_L). The monitor decides whether to monitor or not (M or N).

t=4: In the case of monitoring, nature reveals the true action to the monitor with probability λ.

t=5: If nature revealed the agent's action-decision, the monitor decides whether to make the obtained information public or to withhold it.

t=6: Payoffs are realized.

The informational structure is such that except for the monitor's and the agent's decision and the fact whether nature revealed anything in step 4 all variables and parameters are common knowledge between the players.

We look for subgame perfect equilibria of the game by taking the following procedure. First, we consider the case in which the principal has control over the monitoring technology. We call this "the game without delegation". We characterize the optimal contract and compute the maximum payoff to the principal. Second, we study the case in which the principal delegates monitoring to the supervisor and refer to this subgame as "the game with delegation". We then compare and investigate whether by delegation the principal can achieve a higher payoff.

We are interested in comparing the costs of implementing the action a_H when the principal acts as monitor and when the supervisor controls the monitoring technology. Therefore we assume that parameters are such that the principal prefers to implement the action a_H with at least some positive probability under both policies. This will be the case if the difference Δy is large enough.

As a benchmark consider the case in which the agent's action is observable and verifiable. It is obvious that in this case the principal can appropriate the entire surplus from the action a_H without the use of the costly monitoring technology. By conditioning the contracts on the observed action she can attain the first best outcome $U_P = y_H - a_H$. A similar result obtains when we assume that output is verifiable.

When effort and output are not verifiable the action a_H can only be induced when monitoring takes place. Without monitoring the only state which can occur is the state

N. The most general contract is a one-dimensional wage $w = w_N \in \mathbb{R}_+$. Consequently, the principal cannot induce the agent to take the action a_H and her maximum payoff in this case is $U_P = y_L - a_L$.

3 The Game without Delegation

In this section we analyze the game in which the principal controls the monitoring technology. We show that in this setting it is optimal for the principal to induce the agent to take the high action with probability one. When the monitoring technology is inefficient this requires that the principal has to monitor with probability one and that a rent is left to the agent. When the monitoring technology is relatively efficient, it is possible for the principal to induce the high action without full monitoring. In the optimum the agent receives his reservation wage and monitoring occurs with a probability less than one.

If the principal does not delegate the monitoring decision then the game is played by the principal and the agent only. The supervisor plays no role and we can set his contract to $t = (0, 0, 0)$. In the rest of this section the supervisor will be disregarded.

Suppose that the principal decides to monitor and that monitoring is successful. This means that the principal has the verifiable evidence that the agent's action was a_i $(i = H, L)$. In stage 5 of the game she has to decide whether to reveal the evidence. The decision directly affects her payoffs. Concealing the evidence results in a payment of w_N, while revealing the information results in a payment of w_i. The principal will therefore only reveal the evidence, if $w_N \geq w_i$.[2] This observation has an important implication. The principal cannot credibly commit to rewarding the agent for taking the action a_H by setting $w_H > w_N$. In the existing principal-agent literature with monitoring, however, it is often obtained that in this setting it is optimal for the principal to use a carrot and stick approach (e.g. Border and Sobel (1987), Mookerhjee and Png (1989)). Here such

[2]Without loss of generality we adopt the tie-breaking rule that the principal reveals her evidence when she is indifferent.

a scheme is not possible.

The following proposition shows that we can restrict our attention to contracts of the form $w_N \geq w_H$ and $w_N \geq w_L$. This implies that in stage 5 the principal will always reveal her information.

Proposition 1 *Without loss of generality we may assume that the optimal contract w satisfies the conditions $w_N \geq w_H$ and $w_N \geq w_L$ and induces full revelation of the principal's monitoring evidence.*

Proof: We prove that any payoff associated with a contract which does not specify $w_N \geq w_H$ and $w_N \geq w_L$ can also be attained by a contract which does satisfy the conditions. Let a contract $w = (w_H, w_N, w_L)$ be such that for some $i = H, L$ we have $w_i > w_N$. If the principal monitors and obtains the result a_i, she will decide not to reveal it. Instead of paying w_i the principal will pay the lower wage w_N. The payoffs associated with the contract w are therefore identical to that of the contract w' with $w'_j = \min\{w_j, w_N\}$. Note that the contract w' does satisfy the condition $w'_N \geq w'_H$ and $w'_N \geq w'_L$.

Q.E.D.

By restricting attention to contracts which specify $w_N \geq w_H$ and $w_N \geq w_L$ stage 5 of the game becomes redundant. Any result which is obtained from the monitoring process is automatically revealed. As a consequence the subgame as of stage 3 is reduced to a simultaneous move game with two actions for each player. The strategy space of the principal is $S_P = \{M, N\}$, while the strategy space of the agent is $S_A = \{a_H, a_L\}$. Let $\nu \in [0, 1]$ represent the probability that the principal monitors. Similarly, let $\mu \in [0, 1]$ denote the probability that the agent takes the action a_H, then the payoff functions are

$$\begin{aligned} U_P(w, \mu, \nu) &\equiv \mu y_H + (1-\mu) y_L - \mu\nu\lambda w_H - (1-\mu)\nu\lambda w_L - (1-\nu\lambda) w_N - \nu c \\ U_A(w, \mu, \nu) &\equiv \mu\nu\lambda w_H + (1-\mu)\nu\lambda w_L + (1-\nu\lambda) w_N - \mu a_H - (1-\mu) a_L. \end{aligned}$$

We can now identify a Nash equilibrium as a pair (μ^*, ν^*) such that

$$U_A(w, \mu^*, \nu^*) = \max_{\mu \in [0,1]} U_A(w, \mu, \nu^*)$$
$$U_P(w, \mu^*, \nu^*) = \max_{\nu \in [0,1]} U_P(w, \mu^*, \nu).$$

Let $N(w)$ be the set of Nash equilibria for a given wage contract w. The combination (w, μ, ν) is feasible if it satisfies the following constraints.

$$U_A(w, \mu, \nu) \geq 0$$
$$(\mu, \nu) \in N(w)$$
$$w_N \geq w_H, w_L$$
$$w_H, w_N, w_L \geq 0.$$

Similarly, we say that the contract w is feasible if there exists a strategy pair $(\mu, \nu) \in N(w)$ such that the combination (w, μ, ν) is feasible. A subgame perfect equilibrium outcome of the game without delegation can now be defined as a feasible outcome (w^*, μ^*, ν^*) such that there does not exist a feasible contract w' for which $U_P(w^*, \mu^*, \nu^*) < U_P(w', \mu', \nu')$ for all $(\mu', \nu') \in N(w')$. It follows that a solution (w^*, μ^*, ν^*) to the following maximization problem is a subgame perfect equilibrium outcome of the game without delegation.

$$P1: \quad \arg\max_{w,\mu,\nu} \quad U_P(w, \mu, \nu)$$
$$s.t. \quad (w, \mu, \nu) \quad \text{is feasible.}$$

In the remaining part of this section we derive a solution to P1 by examining all possible equilibria. Four types of equilibria can be distinguished: The pure equilibria, the equilibria in which both players are indifferent about their actions and the two types of equilibria in which one of the players has a strict preference, while the other player is indifferent.[3]

[3]Note that the four categories are not mutually exclusive. E.g. the pure equilibrium $\mu = \nu = 1$ may well be supported by two players who are in fact indifferent between their actions.

First consider those contracts w which induce an equilibrium in pure strategies. By assumption it is profitable to implement the action a_H with at least some positive probability. Since the agent will only take the action a_H if monitoring occurs, the only type of pure equilibrium which may be induced by an optimal contract w^* is the equilibrium $\mu = \nu = 1$.

In the pure equilibrium the payoff to the principal is $U_P = y_H - \lambda w_H - (1-\lambda)w_N - c$, while the payoff to the agent equals $U_A = \lambda w_H + (1-\lambda)w_N - a_H$. For the pure actions to constitute an equilibrium it must be that given the contract w the principal prefers monitoring given that the agent takes the action a_H. This implies that the wage structure is such that $y_H - \lambda w_H - (1-\lambda)w_N - c \geq y_H - w_N$, or equivalently

$$w_N - w_H \geq c/\lambda. \tag{1}$$

Note that this condition ensures that $w_N \geq w_H$.

Likewise, given that the principal monitors, the agent has to prefer the action a_H over a_L. This requires $\lambda w_H + (1-\lambda)w_N - a_H \geq \lambda w_L + (1-\lambda)w_N - a_L$, or equivalently

$$\lambda(w_H - w_L) \geq \Delta a. \tag{2}$$

One more condition which needs to be fulfilled is that the contract is individually rational for the agent. The individual rationality constraint is

$$\lambda w_H + (1-\lambda)w_N \geq a_H. \tag{3}$$

Lemma 1 *If the optimal contract induces a pure equilibrium, then we may assume without loss of generality that the optimal contract w satisfies the conditions $w_L = 0$ and $w_N = w_H + c/\lambda$.*

Proof: We show that the payoff, associated with any contract w which induces a pure equilibrium, can also be attained by a contract w' specifying $w'_L = 0$ and $w'_N = w'_H + c/\lambda$. Let the contract w induce the pure equilibrium $\mu = \nu = 1$, then the contract must satisfy (1) and (2). Since in the pure equilibrium $\mu = \nu = 1$ the payoffs are independent of

w_L, we may set $w_L = 0$. This preserves the equilibrium $\mu = \nu = 1$ and does not affect the payoffs. Now consider the contract $w' = (w_H + (1-\lambda)\varepsilon, w_N - \lambda\varepsilon, 0)$ with $\varepsilon = (w_N - w_H - c/\lambda) \geq 0$. This contract gives the same payoff to the agent and is feasible if the contract w is feasible. It satisfies the condition $w'_N = w'_H + c/\lambda$ and since $w'_H \geq w_H$ also constraint (2) is satisfied. Therefore the contract w' supports the pure equilibrium $\mu = \nu = 1$. Note that the contract w' gives the principal the same payoff as the original contract w.

Q.E.D.

The second type of equilibrium consists of equilibria in which both players are indifferent about their actions. Let the principal monitor with probability ν, then the agent is indifferent between his two actions when $\nu\lambda w_H + (1-\nu\lambda)w_N - a_H = \nu\lambda w_L + (1-\nu\lambda)w_N - a_L$, or

$$\nu = \frac{\Delta a}{\lambda(w_H - w_L)}. \tag{4}$$

Similarly, let the agent choose the action a_H with probability μ. The principal is willing to randomize, when her payoff from monitoring equals her payoff from not monitoring. It follows that the principal is indifferent if and only if

$$\mu = \frac{\lambda(w_N - w_L) - c}{\lambda(w_H - w_L)}. \tag{5}$$

Of course, the contract w must be such that $\mu, \nu \in [0, 1]$.

Furthermore, the individual rationality constraint of the agent is $\mu\lambda\nu w_H + (1-\mu)\lambda\nu w_L + (1-\lambda\nu)w_N \geq \mu a_H + (1-\mu)a_L$ or by equation (4),

$$\lambda\nu w_H + (1-\lambda\nu)w_N \geq a_H. \tag{6}$$

Lemma 2 *If the optimal contract w induces an equilibrium (μ, ν) in which both players are indifferent then for the optimal contract it holds that $w_L = 0$ and $w_N = w_H + c/\lambda$.*

Proof: If the optimal contract induces an equilibrium in which both players are indifferent then the principal's payoff is $U_P(w, \mu, \nu) = \mu y_H + (1-\mu)y_L - w_N$. The individual rationality constraint (6) is independent of w_L. Since $\partial\mu/\partial w_L \leq 0$, the objective function is decreasing in w_L. It is therefore optimal to set w_L as low as possible, i.e. $w_L = 0$.

By assumption we have that $U_P(w, \mu, \nu)$ is larger than $y_L - a_L$, which implies that $w_N < \mu\Delta y - a_L$. Together with the condition $w_H \leq w_N$, it follows that $w_H < \mu\Delta y \leq \Delta y$. Now rewrite the principal's payoff as

$$\begin{aligned} U_P(w) &= \mu y_H + (1-\mu) y_L - w_N \\ &= y_L + \mu(y_H - y_L) - w_N \\ &= y_L + \frac{\lambda w_N - c}{\lambda w_H}\Delta y - w_N \\ &= y_L - \frac{c}{\lambda w_H}\Delta y + w_N\left(\frac{\Delta y}{w_H} - 1\right). \end{aligned}$$

Note that $U_P(w)$ is increasing in w_N. Consequently, w_N should be set as large as possible, while still satisfying the constraints ensuring that $\mu, \nu \in [0,1]$. The optimal contract, therefore, specifies $w_N = w_H + c/\lambda$.

Q.E.D.

Two more types of equilibria need to be discussed. First, the equilibrium in which the principal has a strict preference for monitoring ($\nu = 1$), while the agent is indifferent and chooses the action a_H with a probability smaller than one ($\mu < 1$). Obviously, this equilibrium cannot be sustained by an optimal contract. By increasing the wage w_H only slightly, the agent chooses $\mu = 1$ in equilibrium and the principal's payoff increases.

The fourth type of equilibrium consists of equilibria in which the agent has a strict preference for the action a_H (i.e. $\mu = 1$), while the principal is indifferent. A contract w which induces such an equilibrium must satisfy the constraint $w_H = w_N + c/\lambda$. Note that also in this type of equilibrium the payoffs are independent of w_L and we may assume that $w_L = 0$. We therefore conclude that an optimal contract satisfies the constraints $w_L = 0$ and $w_H = w_N + c/\lambda$. We can now derive the optimal contract w^*.

Proposition 2 *i) If $\lambda > (\Delta a + c)/(a_H + c)$ then under the optimal contract (w^*, t^*) the principal monitors with a probability less than one and the agent chooses the high action*

a_H with probability one. The principal's payoff is larger than $y_H - a_H - c$. The agent does not receive a rent.

ii) If $\lambda \leq (\Delta a + c)/(a_H + c)$ then under the optimal contract (w^*, t^*) the principal monitors with a probability one and the agent chooses the high action a_H with probability one. The principal's payoff is smaller than or equal to $y_H - a_H - c$. The agent does receive a rent.[4]

Proof: For a solution (w^*, μ^*, ν^*) it holds that $w_N^* = w_H^* + c/\lambda$ and consequently $\mu^* = 1$. In the optimum the principal is therefore indifferent between her actions. It follows that her payoff $U_P = y_H - c/\lambda - w_H$ is decreasing in w_H. The principal's payoff is, therefore, bounded by the constraint $w_H \geq \Delta a/\lambda$ or the individual rationality constraint, whichever binds first given that $w_N^* = w_H^* + c/\lambda$. Note that a binding individual rationality constraint together with the condition $w_N = w_H + c/\lambda$ requires that $w_H = (\lambda a_H - c + \sqrt{4(\Delta a)c\lambda + (c - a_H\lambda)^2})/(2\lambda)$. This is larger than or equal to $\Delta a/\lambda$ if and only if $\lambda \geq (\Delta a + c)/(a_H + c)$. This proves the proposition.

Q.E.D.

The proposition states that in the subgame perfect equilibrium outcome (w^*, μ^*, ν^*) the agent chooses the action a_H with probability one. A second conclusion is that the efficiency of monitoring plays an important role. In order to induce the agent to take the action a_H the principal has to monitor. In the optimum monitoring occurs with probability one if the monitoring technology is relatively inefficient, i.e. when $\lambda < (\Delta a + c)/(a_H + c)$. In this case the payoff to the principal is less than $y_H - a_H - c$. Due to the inefficient monitoring technology the principal cannot extract the whole surplus from the relationship and must leave a rent to the agent. If the probability of successful monitoring (λ) is larger than $(\Delta a + c)/(a_H + c)$ then monitoring occurs with

[4] It can be shown that the subgame perfect equilibrium outcome is unique by noting that there exists a sequence of contracts (w_n, t_n) which induce a subgame with a unique Nash equilibrium (μ_n, ν_n) that converges to the subgame perfect equilibrium outcome (w^*, μ^*, ν^*). Any subgame perfect equilibrium must therefore be a solution to P1.

a probability less than one in equilibrium. The payoff to the principal is larger than $y_H - a_H - c$. The monitoring technology in this case is efficient enough to extract the whole surplus and the agent does not receive a rent.

4 The Game with Delegation

In this section we analyze the game in which the principal delegates monitoring to the supervisor. We show that with delegation the principal can approximate the first best solution. This is possible when the principal offers an infinite wage to the agent when the high action is observed and she sets further wages in such a way that in equilibrium the monitoring probability approaches zero. It is obvious that the principal is indeed better off delegating her monitoring decision. The result depends on the fact that extreme rewards are credible, on the risk neutrality of the players, and on the (standard) assumption that players cannot side-contract. The last assumption will be relaxed in the next section.

Since the agent will only take the action a_H if monitoring takes place, the principal has to offer a contract to the supervisor which induces him to monitor. Recall that the contract of the supervisor cannot be made contingent on monitoring itself, because monitoring is not verifiable. The principal can condition the contract only on the revealed result of monitoring. As for the agent a contract to the supervisor is a vector $t \equiv (t_H, t_N, t_L)$.[5]

In the game with delegation the payoffs to the players depend on the contract (w, t) and on the outcome of the subgame which is played by the agent and the supervisor. Given that the supervisor reports truthfully the payoff functions of the principal, the supervisor and the agent are, respectively

$$U_P(w,t,\mu,\nu) \equiv \mu y_H + (1-\mu)y_L - \mu\nu\lambda(w_H + t_H) - (1-\mu)\nu\lambda(w_L + t_L) - (1-\nu\lambda)(w_N + t_N)$$

[5]Note that the supervisor reveals his result when $t_H, t_L \geq t_N$. In the following, however, these two conditions will be ignored. The optimal contract $\hat{t}$ will be such that these two conditions are automatically satisfied.

$$U_S(w,t,\mu,\nu) \equiv \mu\nu\lambda t_H + (1-\mu)\nu\lambda t_L + (1-\nu\lambda)t_N - \nu c$$
$$U_A(w,t,\mu,\nu) \equiv \mu\nu\lambda w_H + (1-\mu)\nu\lambda w_L + (1-\nu\lambda)w_N - \mu a_H - (1-\mu)a_L,$$

where μ denotes the probability that the agent chooses the action a_H and ν represents the probability that the supervisor monitors. Then given a contract (w,t) a Nash equilibrium in the simultaneous move game is a pair $(\hat{\mu},\hat{\nu})$ such that

$$U_A(w,t,\hat{\mu},\hat{\nu}) = \max_{\mu\in[0,1]} U_A(w,t,\mu,\hat{\nu})$$
$$U_S(w,t,\hat{\mu},\hat{\nu}) = \max_{\nu\in[0,1]} U_S(w,t,\hat{\mu},\nu).$$

We write $N(w,t)$ as the set of Nash equilibria in the subgame induced by the contract (w,t). The outcome (w,t,μ,ν) is said to be feasible if it satisfies the following constraints:

$$U_A(w,t,\mu,\nu) \geq 0$$
$$U_S(w,t,\mu,\nu) \geq 0$$
$$(\mu,\nu) \in N(w,t)$$
$$w_H, w_N, w_L, t_H, t_N, t_L \geq 0.$$

Similarly, we call a contract (w,t) feasible if there exists an equilibrium $(\mu,\nu) \in N(w,t)$ such that the outcome (w,t,μ,ν) is feasible. A subgame perfect equilibrium outcome of the game is a solution $(\hat{w},\hat{t},\hat{\mu},\hat{\nu})$ to P2.

$$P2: \quad \max_{(w,t,\mu,\nu)} \quad U_P(w,t,\mu,\nu)$$
$$s.t. \quad (w,t,\mu,\nu) \quad \text{is feasible,}$$

where we will call the contract $(\hat{w},\hat{t})$ an optimal contract.

We derive the best responses of the agent and the supervisor in the simultaneous move game induced by a contract (w,t). Given that the supervisor monitors with a probability ν, the agent's best response depends on the difference in payoffs between the action a_H and a_L.

Lemma 3 *Without loss of generality we may assume that the solution* $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ *to P2 satisfies* $\hat{w}_H - \hat{w}_L = \Delta a/(\lambda\hat{\nu})$, *i.e. in equilibrium the agent is indifferent between the action* a_H *and* a_L.

Proof: Let the optimal contract $\hat{w}$ be such that $\hat{w}_H - \hat{w}_L < \Delta a/(\lambda\hat{\nu})$, then the agent's unique best response is to play a_L, i.e. $\mu = 0$. The contract $\hat{w}$ cannot be optimal by assumption. Let the optimal contract $\hat{w}$ be such that $\hat{w}_H - \hat{w}_L > \Delta a/(\lambda\hat{\nu})$, then the agent's unique best response is to play a_H, i.e. $\hat{\mu} = 1$. The strict preference for action a_H implies that $\partial\mu/\partial w_H = \partial\mu/\partial w_N = 0$. Therefore, $dU_P/dw_H = -dU_A/dw_H = -\lambda\hat{\nu}$ and $dU_P/dw_N = -dU_A/dw_N = -(1 - \lambda\hat{\nu})$. This implies that if we lower $\hat{w}_H$ and raise $\hat{w}_N$ at a proportional rate of $(1 - \lambda\hat{\nu})/(\lambda\hat{\nu})$ then both the principal's and the agent's payoff do not change. We therefore can lower $\hat{w}_H$ to $\Delta a/(\lambda\hat{\nu}) + \hat{w}_L$ and raise $\hat{w}_N$ by an appropriate amount in order to create a new contract w'. If the combination $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ is a solution to P2 then also the combination $(w', \hat{t}, \hat{\mu}, \hat{\nu})$ is a solution to P2.

Q.E.D.

Given the contract (w, t) and that the agent takes the action a_H with probability μ, the supervisor's best response depends on the difference in payoffs between monitoring and not monitoring.

Lemma 4 *Without loss of generality we may assume that the solution* $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ *to P2 satisfies* $\hat{\mu}\lambda\hat{t}_H + (1 - \hat{\mu})\lambda\hat{t}_L = c + \lambda\hat{t}_N$, *i.e. in equilibrium the supervisor is indifferent about monitoring.*

Proof: Let the optimal contract $\hat{t}$ be such that $\mu\lambda\hat{t}_H + (1 - \mu)\lambda\hat{t}_L < c + \lambda t_N$ then the supervisor has a strict preference for not monitoring. Since no monitoring induces the agent to play $\mu = 0$, the contract $(\hat{w}, \hat{t})$ cannot be optimal by assumption. Let the optimal contract $\hat{t}$ be such that $\hat{\mu}\lambda\hat{t}_H + (1 - \hat{\mu})\lambda\hat{t}_L > c + \lambda\hat{t}_N$ then the supervisor has a strict preference for monitoring ($\hat{\nu} = 1$). Note that the supervisor can only have a strict preference for monitoring if his individual rationality constraint is not binding.

Furthermore, strict preference implies that if we lower $\hat{t}_H$ and $\hat{t}_L$ only slightly then $(\hat{\mu}, \hat{\nu})$ remains a Nash equilibrium. Therefore, we can either lower $\hat{t}_H$ or $\hat{t}_L$ and create a new contract t' for which the combination $(\hat{w}, t', \hat{\mu}, \hat{\nu})$ is also feasible, while resulting in a higher payoff to the principal. The combination $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ could therefore not have been a solution to P2.

Q.E.D.

From lemma 3 and 4 it follows that the optimal contract $(\hat{w}, \hat{t})$ is a contract which induces a mixed equilibrium in the simultaneous move game. The optimal contract $(\hat{w}, \hat{t})$ is therefore such that there exists an equilibrium $(\hat{\mu}, \hat{\nu})$ in the simultaneous move game with

$$\hat{\nu} = \frac{\Delta a}{\lambda(\hat{w}_H - \hat{w}_L)} \quad \text{and} \quad \hat{\mu} = \frac{\lambda(\hat{t}_L - \hat{t}_N) - c}{\lambda(\hat{t}_L - \hat{t}_H)}. \tag{7}$$

Note that the combination $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ can only be a solution to P2 if it satisfies the individual rationality constraints for the agent and the supervisor, which are respectively,

$$\nu\mu\lambda w_H + \nu(1-\mu)\lambda w_L + (1-\lambda\nu)w_N \geq \mu a_H + (1-\mu)a_L \tag{8}$$

and

$$\nu\mu\lambda t_H + \nu(1-\mu)\lambda t_L + (1-\lambda\nu)t_N \geq \nu c. \tag{9}$$

Proposition 3 *The principal can approximate the first best payoff $y_H - a_H$ by offering a contract (w, t) which specifies a wage w_H tending to infinity and a wage w_N approaching a_L. As a result the principal can achieve a higher payoff when she delegates monitoring.*

Proof: Consider a contract which specifies $t_L = t_N = w_L = 0$, $w_N = (w_H a_L)/(w_H - \Delta a)$, $t_H = c/\lambda$ and $w_H > \Delta a/\lambda$. This contract induces a mixed equilibrium $(\mu, \nu) = (1, \Delta a/(\lambda w_H))$. The reader may check that the combination $(w, t, 1, \Delta a/(\lambda w_H)$ is feasible by noting that the individual rationality constraints are satisfied in equality. The principal's payoff is $U_P(w, t, \mu, \nu) = y_H - a_H - (\Delta ac)/(\lambda w_H)$. If we let w_H tend to infinity the principal's payoff converges to the first best outcome $y_H - a_H$.

Q.E.D.

Two conclusions can be drawn from the proposition: Delegating monitoring is profitable for the principal and there does not exist an optimal contract. We will comment on both results.

There are two reasons why the principal can achieve a higher payoff by delegating monitoring. The first reason concerns the creation of incentives. The principal has to give incentives to induce the action a_H and, at the same time, to induce monitoring. When the principal controls the monitoring technology, there is only the contract w through which the principal can regulate both incentives. By delegating the monitoring process to the supervisor, she has two sets of contracts for creating appropriate incentives. From the equilibrium condition (7) we see that with delegation the principal uses the contract t to regulate the incentives for the agent and the contract w for creating incentives to monitor. By delegating monitoring the principal decouples the two types of incentives and can regulate them more accurately.

A second reason is that in the game without delegation the principal cannot commit herself to revealing the evidence a_H when w_H is larger than w_N. As a result an optimal contract specifies $w_H \leq w_N$. This lack of commitment does not occur when monitoring is delegated to the supervisor. In the game with delegation the supervisor makes the decision concerning the revelation of evidence. His decision does not depend on the contract w, like in the game without delegation, but only on the contract t. The principal can, therefore, set w_H larger than w_N and use a carrot and stick approach to discipline the agent.

Proposition 3, however, also tells us that an optimal contract $(\hat{w}, \hat{t})$ does not exist. For any contract (w, t) with an associated payoff of $y_H - a_H - \varepsilon$ there exists a contract (w', t') which has a payoff larger than $y_H - a_H - \varepsilon$. Note, however, that the payoff $y_H - a_H$ cannot be obtained in equilibrium. It would require that the agent chooses the action a_H

with probability one, while monitoring does not occur. There does not exist a contract (w, t) which induces such an equilibrium. The equilibrium $(\mu, \nu) = (1, 0)$ can only be approximated by a contract which specifies a w_H tending to infinity.

The result is similar to Border and Sobel (1987). Border and Sobel analyze a model in which a principal tries to extract an agent's endowment, while this endowment is private information. Optimal policies in their model involve rewarding the agent for truthful reporting. They demonstrate, however, that if rewards to the agent are not bounded exogenously, an optimal policy does not exist. The principal would like to monitor with an infinitely small probability, while promising infinitely large rewards when the audit confirms the report of the agent. The high reward induces the agent to tell the truth.

The result that offering an ever higher wage w_H benefits the principal is not realistic. It prompts us to question the assumptions of the model. In the literature two modifications have been proposed. First, Border and Sobel argue that it is unrealistic to assume that the principal can promise infinite rewards. They argue that in reality resources are limited and this also holds for the principal. They therefore propose an exogenous upperbound on wages. This procedure, however, is rather ad hoc, because it creates the problem of finding a plausible upperbound. Second, Mookherjee and Png (1989) contest the plausibility of risk neutrality in the context of the model. They show that with only a slight degree of risk aversion infinitely high wages are no longer optimal.

We propose a third approach and introduce the possibility of collusion between the principal and the supervisor. The next section argues that this is a natural extension of the model. As we will see, the threat of collusion creates an endogenous upperbound on wage proposals and its ultimate effect is therefore similar to Border and Sobel's assumption of an exogenous upperbound on rewards. The extension can therefore also be seen as a more realistic interpretation of Border and Sobel's ad hoc upperbound on wages.

5 Collusion between Principal and Supervisor

Consider a contract which approximates the first best solution. This means that the wage w_H is large, while the wage w_N is close to a_L. The probability that monitoring occurs is small, but strictly positive. Now suppose that monitoring does indeed occur and that it reveals that the agent's action is a_H. If the supervisor reveals this information, then the principal has to pay the agent the wage w_H. If the principal can convince the supervisor not to reveal his information, then she only has to pay the wage w_N and saves $w_H - w_N$. For contracts which approximate the first best solution this difference is large and the principal's willingness to pay to prevent revelation is high. We say that in this case the principal has a strong incentive to collude with the supervisor to prevent revelation.

The idea of collusion is modeled as follows. After stage 5 of the game the supervisor reveals the evidence to the principal. The principal can then offer the supervisor a bribe b for not revealing the evidence. If the supervisor accepts the bribe, then collusion occurs. Collusion, however, is costly. As a rational for this assumption consider that the bribe should not be detected and has to be transferred in stealth. In accordance with Tirole (1992) we assume that this cost is proportional to the size of the bribe. Thus, the cost of bribing can be expressed by a transfer parameter $k \in (0,1)$ with the following interpretation. When the principal sends a bribe of b monetary units, the supervisor receives only an equivalent of kb monetary units. It is assumed that the value of k is common knowledge.

Note that collusion can only involve the concealment of evidence, not the artificial creation of evidence. Two types of collusion are possible. Either the evidence a_H or the evidence a_L is concealed. Since with delegation a contract approximating the first best solution involves $w_H > w_N > w_L$, the principal will typically have an interest in bribing when the evidence shows a_H. In fact when the evidence shows a_L, the principal strictly prefers this to be revealed. At first sight this may induce the supervisor to threaten not to reveal the evidence a_L, if he is not paid more than the contract specifies. Such threats, however, are not credible.

Collusion between the principal and the supervisor occurs only if it is profitable for both sides. The bribe b, therefore, has to be such that both the supervisor and the principal are willing to collude. We calculate the maximum bribe, b^{max}, the principal is willing to give and the minimal bribe, b^{min}, the supervisor is willing to accept.

By colluding the principal has to pay $w_N + t_N$ instead of $w_H + t_H$. This means that collusion net of the bribe b is profitable if and only if $w_H - w_N + t_H - t_N - b > 0$. The maximum bribe the principal is willing to pay is therefore

$$b^{max} \equiv w_H - w_N + t_H - t_N. \tag{10}$$

The supervisor accepts a bribe if this is profitable to him. Since a bribe of size b is only worth kb to him, he will accept a bribe b whenever

$$kb > b^{min} \equiv (t_H - t_N). \tag{11}$$

It follows that when $kb^{max} \leq b^{min}$ there does not exist a bribe $b \geq 0$ for which both the principal and the supervisor are willing to collude.[6] As a consequence collusion will not take place. A contract is, therefore, collusion-proof if it satisfies the collusion-proofness constraint

$$w_H \leq w_N + K(t_H - t_N), \tag{12}$$

with $K \equiv (1-k)/k$.

Proposition 4 *For any contract which is not collusion-proof the principal is weakly better off proposing a contract which is collusion-proof. There exists, therefore, an optimal contract, which is collusion-proof.*

Proof: Consider a contract (w, t) which does not satisfy the collusion-proofness constraint. Then it is common knowledge between the players that collusion occurs. All players therefore know that the state of the world H will not occur. This implies that the

[6] We assume that when the supervisor is indifferent between accepting and not accepting the bribe he does not accept the bribe b. This ensures existence of an equilibrium.

relevant wage combination when the agent takes the action a_H is always (w_N, t_N). Let b be the bribe which accompanies the collusion, then the principal is weakly better off offering the collusion-proof contract $(w', t') = (w_N, w_N, w_L, t_N + kb, t_N, t_L)$.[7] Note that if the contract (w, t) together with the bribe b is feasible, then the contract (w', t') is also feasible. The contract (w', t') results in the same payoff to the agent and supervisor.

Q.E.D.

Consider the two polar cases $k = 0$ and $k = 1$. If $k = 0$ then collusion is infinitely costly and will not occur. The model is identical to the model analyzed in the previous section, in which the supervisor controls the monitoring technology and the principal has no possibility of bribing. If $k = 1$, collusion is costless. This implies that the principal can prevent the revelation of evidence at no additional cost. Collusion and the subsequent concealment of evidence will therefore occur for any contract specifying $w_H > w_N$. With $k = 1$ the principal is no longer able to commit to revealing all evidence when $w_H > w_N$. There is no direct commitment effect from delegating monitoring to the supervisor. In this respect the case $k = 1$ resembles the version of the model in which the principal does not delegate monitoring. However, the fact that the principal can regulate the incentives for monitoring and the action a_H more effectively when delegating monitoring still remains.

A subgame perfect equilibrium outcomes of the game with collusion is a solution to the following optimization problem:

$$
\begin{aligned}
P3: \quad \max \quad & U_P(w, t, \mu, \nu) \\
s.t. \quad & (w, t, \mu, \nu) \quad \text{is feasible} \\
& (w, t) \quad \text{is collusion-proof.}
\end{aligned}
$$

A contract $(\hat{w}, \hat{t})$ for which the maximum is achieved is called an optimal contract. Before determining a solution to P3 we first prove the following proposition.

[7] The bribe kb should be added to the supervisor's wage t'_N, since it might be the bribe kb, which makes the supervisor's contract individual rational.

Proposition 5 *For any $k \in (0,1]$ the principal can achieve a strictly higher payoff by delegating monitoring than when she monitors herself.*

Proof: In the game without delegation we found that the subgame perfect outcome was (w^*, μ^*, ν^*), with $\mu^* = 1$ and $\nu^* = \Delta a/(\lambda w_H^*)$. Consider the feasible outcome (w^*, t, μ^*, ν^*) in the game with delegation, where $t = (c/\lambda, 0, 0)$. The principal's payoff associated with this outcome is equal to the subgame perfect equilibrium payoff of the principal in the game without delegation: $U_P(w^*, t, \mu^*, \nu^*) = U_P(w^*, \mu^*, \nu^*)$. Note, however, that for the contract (w^*, t) the collusion-proofness constraint is not binding. We can therefore raise w_H and decrease w_N, while keeping the agent's payoff constant. This procedure will cause $\nu = \Delta a/(\lambda w_H)$ to fall and will increase the principal's payoff. It follows that the principal's payoff is strictly higher in the game with delegation than in the game without delegation.

Q.E.D.

Proposition 5 shows that the principal can attain a higher payoff by delegating monitoring even when there exist extreme collusion possibilities ($k \to 1$). In these cases the commitment effect of delegation is very small. The fact that the principal can better regulate the incentives, however, remains. This causes delegation to be even profitable when the commitment effect does not exist.

Proposition 6 *i) If $\lambda > (\Delta a - Kc)/|a_H - Kc|$ then under the optimal contract $(\hat{w}, \hat{t})$ the supervisor monitors with a probability less than one and the agent takes the action with probability one. The principal's payoff is larger than $y_H - a_H - c$. Neither the agent nor the supervisor receives a rent.*

ii) If $\lambda \leq (\Delta a - Kc)/|a_H - Kc|$ and $K \leq \lambda/(1-\lambda)$ then under the optimal contract $(\hat{w}, \hat{t})$ the supervisor monitors with probability one and the agent takes the action with probability one. The principal's payoff is less than or equal to $y_H - a_H - c$. The agent receives a rent, while the supervisor does not.

iii) If $\lambda \leq (\Delta a - Kc)/|a_H - Kc|$ and $K > \lambda/(1-\lambda)$ then then under the optimal *contract* $(\hat{w}, \hat{t})$ *the supervisor monitors with probability one and the agent takes the action with probability one. The principal's payoff is less than or equal to* $y_H - a_H - c$. *The supervisor receives a rent, while the agent does not.*

The proof of this proposition consists of several steps and is reserved for the appendix. The proposition shows that for the optimal contract the agent chooses the action a_H with probability one. There are three types of optimal contracts. If monitoring is efficient (i.e. $\lambda > (\Delta a - Kc)/|a_H - Kc|$) then the principal can set contracts in such a way that in the simultaneous move game the supervisor monitors with a probability less than one. The individual rationality constraint of the agent and the supervisor are binding in this case.

When monitoring is less efficient, monitoring must occur with probability one. To induce the agent to choose the action a_H with probability one, the principal has to leave a rent to either the supervisor or the agent. When collusion is not very costly (i.e. $K < \lambda/(1-\lambda)$), the principal prefers to leave this rent to the agent. In the other case she leaves the rent to the supervisor. In either case the payoff to the principal is less than $y_H - a_H - c$.

Recall that a similar result was obtained when we derived the optimal contract in the game without delegation. Also there the efficiency of monitoring determined whether the principal was able to appropriate the entire surplus created by the action a_H. Note however, that the monitoring technology does not need to be as efficient as in the game without delegation in order to extract the whole surplus, since $(\Delta a - Kc)/|a_H - Kc| < (\Delta a + c)/(a_H + c)$ for all $K \geq 0$. This is due to the effect that with delegation the principal can better regulate incentives.

Proposition 7 *The principal's maximum payoff is decreasing with the cost of collusion* k.

Proof: For the solution $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ to P3 we found that $\hat{t}_H > 0$ and that the collusion-proofness constraint is binding. By decreasing the variable k the collusion-proofness constraint is relaxed, which increases the principal's payoff.

Q.E.D.

The intuition behind proposition 7 becomes clear when one notes that an alternative way of interpreting the transfer technology parameter k is to view it as the degree of commitment. If k is small then the principal can commit to handing out large rewards if it is observed that the agent took the high action. A larger k then reduces the set of contracts to which the principal can commit. In terms of commitment the proposition then states that the principal can attain a higher payoff when she is able to commit to a larger set of contracts.

6 Collusion between Agent and Supervisor

One can extend the model further by adding another possibility of collusion. By allowing the agent to collude with the supervisor the set of collusion-proof contracts is further restricted. Let the agent be able to make use of the same bribing technology as the principal. At a proportional cost k he can send bribes to the supervisor. Again we do not analyze the bargaining game, but simply assume that if there exists a surplus from colluding, collusion takes place. We do not analyze how the surplus is divided over the three players. For simplicitely assume that the supervisor can only ask one player for a collusionairy offer.[8] In this case the incentives of the principal and the agent are mutually exclusive. If the evidence shows that the agent's action was a_H then only the principal will make an offer to bribe, since for an optimal contract it will typically hold that $w_H > w_N$. Likewise, if the evidence is a_L, then only the agent benefits from the concealment of evidence.

[8]The analysis is not changed when one assumes more general bargaining situations in which the agent and principal can make alternating bids to the supervisor. The important assumption is that if given the contract (w, t) there exists an initial surplus from colluding then this leads to side-payments in equilibrium.

We derive the collusion-proofness constraint of the agent. By colluding the agent prevents the supervisor from revealing the evidence a_L. This implies that he receives the wage w_N instead of the wage w_L. The agent's willingness to pay is therefore $w_N - w_L$. To the supervisor collusion is only profitable if he receives at least a transfer of $t_L - t_N$. Collusion between the agent and the supervisor is therefore not profitable if

$$k(w_N - w_L) \leq t_L - t_N. \tag{13}$$

Equation (13) is the collusion-proofness constraint with respect to the agent. Without loss of generality we may assume that the optimal contract satisfies the collusion-proofness constraint with respect to the agent. Suppose the optimal contract $(\hat{w}, \hat{t})$ does not satisfy the constraint. Since it is in the interest of the agent and the supervisor to collude, bribing will occur. This means that instead of paying the wage combination $(\hat{w}_L, \hat{t}_L)$ the principal will always need to pay the wage $(\hat{w}_N, \hat{t}_N)$. A contract $(w', t') = (\hat{w}_H, \hat{w}_N, \hat{w}_N, \hat{t}_H, \hat{t}_N, \hat{t}_N)$ results in the same payoff to the principal. The contract (w', t') must therefore also be optimal. Note that the contract satisfies the collusion-proofness constraint.

A subgame perfect equilibrium outcome is a contract (w, t) and an equilibrium pair (μ, ν) which maximizes the following expression.

$$\begin{aligned}
\max_{w,t,\mu,\nu} \quad & U_P(w, t, \mu, \nu) \\
s.t. \quad & (w, t, \mu, \nu) \quad \text{is feasible} \\
& (w, t) \quad \text{is collusion-proof w.r.t. the principal} \\
& (w, t) \quad \text{is collusion-proof w.r.t. the agent.}
\end{aligned}$$

The previous section showed that optimality requires that the agent chooses the action a_H with probability one. In equilibrium the supervisor does therefore not observe the action a_L, when he monitors. This implies that in equilibrium the principal never needs to pay the supervisor the wage t_L. Consequently, she can set t_L as large as she wants without affecting the payoff of the players. It follows that the principal can costlessly prevent collusion between the agent and the supervisor. She offers the supervisor a large reward if he observes that the agent's action was a_L. The reward is set in such a way, that there does not exist a surplus between the agent and the supervisor when they collude, i.e. $t_L \geq k(w_N - w_L) + t_N$.

Proposition 8 *Allowing the possibility of collusion between the agent and the supervisor does not affect the maximum payoff the principal can achieve. As a consequence the principal is better off by delegating monitoring to a supervisor.*

Proof: Let $(\hat{w}, \hat{t})$ be the optimal contract without the possibility of collusion between the agent and the supervisor. In the simultaneous move game induced by the contract $(\hat{w}, \hat{t})$ the agent chooses the action a_H with probability one, i.e. $\hat{\mu} = 1$. Now consider the alternative contract $(w', t') = (\hat{w}_H, \hat{w}_N, \hat{w}_L, \hat{t}_H, \hat{t}_N, k(\hat{w}_N - \hat{w}_L) + \hat{t}_N)$. The contract (w', t') satisfies the collusion-proofness constraint of the agent. The outcome $(w', t', \hat{\mu}, \hat{\nu})$ is feasible and collusion-proof and gives all players the same payoff as the outcome $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ in the game without the possibility of collusion between the agent and the supervisor.

Q.E.D.

7 Conclusion

In this paper we analyzed a principal-agent model with the possibility of monitoring. We have shown that if the decision to monitor and the action of the agent are taken simultaneously then the principal gains by delegating monitoring. Delegation may be beneficial because of two effects. First, when the monitor and effort decision are taken simultaneously, the principal must create incentives to induce both monitoring and effort. By employing an external monitor she has an extra contract to her disposal and can better regulate incentives for monitoring and effort. We call this the incentive effect of delegation.

A second effect which may make it beneficial to delegate monitoring is commitment. If the evidence obtained by monitoring is private information and can be concealed, then the principal can commit to a larger range of contracts when she employs an independent supervisor. We parameterized the degree of commitment by introducing the possibility of collusion between the principal and the supervisor and showed that the principal's maximum payoff is indeed increasing with the degree of commitment. This second effect is the commitment effect of delegation.

As a logical extension we allowed the agent to collude with the supervisor in a similar fashion as the principal and showed that the principal can costlessly prevent such collusion. This result can be seen as an example of the first theme in Tirole (1992), which claims that "Under some condition, there is no loss in designing organizations which do not leave scope for collusion" (p.157). The form of collusion is, however, rather simple. Collusion takes place after the actions have been taken and the monitoring evidence has been obtained. When the supervisor and agent can collude before taking their actions then more sophisticated collusion schemes may emerge which will be costly to prevent.

Appendix: Proof of Proposition 6

In order to characterize a solution to P3 note that the problem is identical to problem P2, except for the addition of the collusion-proofness constraint. The collusion-proofness constraint puts a restriction on the difference between w_H and w_N. As a result proposition 3 will no longer hold and an optimal contract does indeed exist. Concerning the lemma's 3 and 4 we can say that the former lemma will still hold, while the latter does no longer need to hold.

Lemma A.1 *Without loss of generality we may assume that the solution to P3 satisfies i)* $\hat{\nu}\lambda(\hat{w}_H - \hat{w}_L) = \Delta a$*, i.e. the agent is indifferent in equilibrium between the action* a_H *and the action* a_L*. ii)* $\hat{\mu}\lambda\hat{t}_H - (1-\hat{\mu})\lambda\hat{t}_L = \lambda\hat{t}_N + c$ *or* $\hat{t}_H \geq \hat{t}_N + c/\lambda \wedge \hat{t}_L = 0$*, i.e. the supervisor is indifferent in equilibrium or has a unique best response to monitor. iii)*

$\hat{w}_L = 0 \wedge \hat{t}_N = 0$

Proof: For i) see the proof of lemma 3 and note that the procedure of reducing w_H and increasing w_N will not violate the collusion-proofness constraint. Therefore we have $\hat{\nu}\lambda(\hat{w}_H - \hat{w}_L) = \Delta a$.

For ii) consider the proof of lemma 4 and note that lowering t_H might violate the collusion-proofness constraint and this causes lemma 4 to fail. The variable t_L can be lowered without affecting the collusion-proofness constraint.

To prove iii) note first that $\partial\mu/\partial w_N = \partial\nu/\partial w_N = \partial\nu/\partial w_L = 0$ and $\partial\mu/\partial w_L \geq 0$. Consider the combination $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ with $\hat{w}_L > 0$ which is feasible and collusion-proof, then the combination $(w', \hat{t}, \hat{\mu}, \hat{\nu})$ with $w'_H = \hat{w}_H$, $w'_L = 0$ and $w'_N = \hat{w}_N + (1 - \hat{\mu})\lambda\hat{\nu}\hat{w}_L/(1 - \lambda\hat{\nu})$ is also feasible and collusion-proof, with $\mu' \geq \hat{\mu}$. Since by assumption $\partial U_P/\partial\mu|_{\hat{\mu}} \geq 0$, it follows that $U_P(w', \hat{t}, \mu', \hat{\nu}) \geq U_P(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$. Furthermore, we know that either $\hat{\mu}\lambda\hat{t}_H - (1 - \hat{\mu})\lambda\hat{t}_L = \lambda\hat{t}_N + c$ or that $\hat{t}_H \geq \hat{t}_N + c/\lambda$. In both cases the supervisor's individual rationality constraint is satisfied for all $t_N \geq 0$. Since the principal's payoff is decreasing in t_N and lowering t_N does not cause the collusion-proofness constraint to be violated, we know that a contract specifying $t_N > 0$ cannot be optimal.

Q.E.D.

From the first two parts of lemma A.1 it follows that an optimal contract does not necessarily induce an equilibrium in the simultaneous move game in which both the agent and the supervisor are indifferent about their actions in equilibrium and play a mixed strategy. The third part of the lemma tells us that for an optimal contract the agent gets punished as severely as possible when it is revealed that he took the action a_L. Furthermore, it is optimal not to pay the supervisor when he does not show any evidence.

An important observation is that the collusion-proofness constraint must be binding at the optimum.

Proposition A.1 *For the optimal contract* $(\hat{w}, \hat{t})$ *the collusion-proofness constraint is binding.*

Proof: According to lemma A.1 we have two cases to consider. Case 1: The solution to P3 is such that the supervisor has a unique best response to monitor. In this case it directly follows from the proof of lemma A.1 that the collusion-proofness constraint is binding. Case 2: The solution to P3 is such that a mixed equilibrium is played in the simultaneous move game. Consider a contract (w, t), which induces a mixed equilibrium, i.e. $\nu = \Delta a/(\lambda w_H)$. If the collusion-proofness constraint is not binding, then we can increase w_H and lower w_N. By increasing w_H and lowering w_N in such a way that the agent's payoff stays constant, the principal's payoff increases: $dU_P/dw_H = c\partial\nu/\partial w_H > 0$.[9] The contract is therefore not optimal if the collusion-proofness constraint is not binding.

Q.E.D.

Lemma A.2 *Without loss of generality, we may assume that the optimal contract* $(\hat{w}, \hat{t})$ *specifies* $\hat{t}_L < c/\lambda$ *and* $\hat{t}_H \geq c/\lambda$.

Proof: Lemma A.1 showed that a solution to P3 either satisfies $\hat{t}_H \geq c/\lambda \wedge \hat{t}_L = 0$ or $\hat{\mu}\lambda\hat{t}_H - (1-\hat{\mu})\lambda\hat{t}_L = \lambda\hat{t}_N + c$. If the latter case holds, then the supervisor is indifferent between his actions. Since $\mu \in [0, 1]$, it must be that either $t_L > c/\lambda \wedge t_H \leq c/\lambda$, or $t_L < c/\lambda \wedge t_H \geq c/\lambda$. Consider a feasible and collusion-proof outcome (w, t, μ, ν) with $t_H \leq c/\lambda$ and $t_L > c/\lambda$. Take $t'_L = 0$ and $t'_H = (c(t_L - t_H))/(\lambda t_L - c)$. Then the outcome (w, t', μ, ν), with $t' = (t'_H, t'_L, 0)$, is feasible and collusion-proof and gives the principal the same payoff.

Q.E.D.

Lemma A.3 *If the combination* $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ *with* $\hat{\nu} = 1$ *is a solution to P3 then it must hold that* $\hat{\mu} = 1$.

[9] Note that individual rationality implies that $(1 - \lambda\nu)w_N \geq a_L$. Since $a_L > 0$ it follows that the constraint $w_N \geq 0$ will not be violated as long as the contract w is individual rational.

Proof: We prove by contradiction. Suppose that the combination $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ is a solution to P3 with $\hat{\mu} < 1$ and $\hat{\nu} = 1$. First note that by assumption $\hat{\mu} = 0$ cannot be a solution to P3. Second, suppose that $0 < \hat{\mu} < 1$ then it follows from lemma A.1 and the assumption that $\Delta y > \Delta a$ that

$$\lambda \hat{w}_H < \Delta y. \tag{A.1}$$

By raising $\hat{w}_H$ and $\hat{w}_N$ by an $\varepsilon > 0$ we can create a new contract (w', t'), which is also feasible and collusion-proof. We have $(\mu', \nu') = (1, 1) \in N(w', t')$, i.e. the agent now has a strict preference for the action a_H. The difference in the principal's payoff between the contract (w', t') and the original contract $(\hat{w}, \hat{t})$ is

$$U_P(w', t', 1, 1) - U_P(\hat{w}, \hat{t}, \hat{\mu}, 1) = (1 - \hat{\mu})(\Delta y - \lambda \hat{w}_H) - \varepsilon.$$

By (A.1) and the assumption $\hat{\mu} > 0$ the first term on the right hand side is strictly positive. It follows that we can always find an ε such that the whole expression is positive. This implies that the original contract $(\hat{w}, \hat{t}, \hat{\mu}, 1)$ is not a solution to P3.

Q.E.D.

Lemma A.4 *There exists a solution $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ to P3 which satisfies $\hat{\mu} = 1$, i.e. in the optimum the agent chooses the action a_H with probability one.*

Proof: We derive a solution $(\hat{w}, \hat{t}, \hat{\mu}, \hat{\nu})$ of P3 and check that $\hat{\mu} = 1$. We know that it is optimal to set $\hat{t}_N = \hat{w}_L = 0$ and that the collusion-proofness constraint is binding. If the solution is such that the supervisor has a unique best response to monitor then it follows that $\hat{\nu} = 1$. Lemma A.3 then tells us that $\hat{\mu} = 1$. We therefore need only to investigate mixed equilibria.

By lemma A.2 we have that $\hat{t}_L < c/\lambda$ and $\hat{t}_H \geq c/\lambda$. In fact, if $\hat{t}_H \geq c/\lambda$, then it is optimal to have $\hat{t}_L = 0$. By substitution we can rewrite problem P3 as

$$\text{P3'}: \quad \max_{w_H, w_N \in \mathbb{R}^+} \quad y_L + \frac{cK}{\lambda(w_H - w_N)}(\Delta y - \Delta a) - (1 - \frac{\Delta a}{w_H})w_N - \frac{\Delta a}{\lambda w_H}c$$

$$s.t. \quad \left(1 - \frac{\Delta a}{w_H}\right) w_N \geq a_L \tag{A.2}$$

$$\lambda w_H \geq \Delta a \tag{A.3}$$

$$\lambda(w_H - w_N)/K \geq c, \tag{A.4}$$

where inequality (A.2) is the individual rationality constraint of the agent and the constraints (A.3) and (A.4) ensure that μ and ν are not greater than one.

The second order derivative with respect to w_N is

$$\frac{\partial^2 U_P}{\partial w_N^2} = \frac{2cK}{\lambda(w_H - w_N)^3},$$

which is positive, since $w_H > w_N$. There does not exist an internal maximum and the optimal w_N is a corner solution. This implies that either (A.4) binds, which directly implies that $\mu = 1$, or that (A.3) binds, which implies that $\nu = 1$ from which it also must follow that $\mu = 1$. Or it must be that (A.2) binds. Only in the latter case it does not immediately hold that $\mu = 1$.

Let the optimal $\hat{w}_N$ be such that $(1 - \Delta a/w_H)\hat{w}_N = a_L$, i.e. $\hat{w}_N = a_L w_H/(w_H - \Delta a)$. Substituting $\hat{w}_N$ into P3' we get a function $U_P(w_H)$

$$U_P(w_H) = y_L + \frac{cK(w_H - \Delta a)}{\lambda w_H(w_H - a_H)}(\Delta y - \Delta a) - a_L - \frac{\Delta a}{\lambda w_H} c,$$

with $\lambda w_H > \Delta a$ to ensure that $\nu \leq 1$ and the requirement that $cK(w_H - \Delta a) \leq \lambda w_H(w_H - a_H)$ to ensure that $\mu \leq 1$. The derivative of U_P with respect to w_H is

$$U_P'(w_H) = \frac{\Delta a c}{\lambda w_H^2} - \frac{c(\Delta y - \Delta a)K((w_H - \Delta a)^2 + a_L \Delta a)}{\lambda w_H^2 (w_H - a_H)^2}. \tag{A.5}$$

If $\hat{w}_N = a_L w_H/(w_H - \Delta a)$ then this implies that the individual rationality constraint is binding and therefore

$$\hat{U}_P = y_L - a_L + \mu(\Delta y - \Delta a) - \nu c.$$

By assumption we have that $\hat{U}_P > y_L - a_L$, which implies that $\mu(\Delta y - \Delta a) > \nu c$. Since $\mu = cK(w_H - \Delta a)/(\lambda w_H(w_H - a_H))$ and $\nu = \Delta a/(\lambda w_H)$, it follows that

$$\mu(\Delta y - \Delta a) > \nu c \quad \Rightarrow \quad cK(\Delta y - \Delta a)(w_H - \Delta a)\lambda w_H > \Delta a c(w_H - a_H)\lambda w_H$$

$$\Rightarrow \quad cK(\Delta y - \Delta a)(w_H - \Delta a)^2 > \Delta ac(w_H - a_H)^2 \qquad \text{(A.6)}$$

$$\Rightarrow \quad cK(\Delta y - \Delta a)((w_H - \Delta a)^2 + a_L \Delta a) > \Delta ac(w_H - a_H)^2$$

$$\Rightarrow \quad U_P'(w_H) < 0,$$

where (A.6) follows from the fact that $w_H - \Delta a > w_H - a_H$. We conclude that $U_P(w_H)$ is decreasing in w_H and reaches its maximum at $\Delta a/\lambda$ or where $cK(w_H - \Delta a)/(\lambda w_H(w_H - a_H)) = 1$. Note that this implies that if A.4 is binding in the optimum then also A.2 or A.3 is binding. We conclude that we must have $\hat{\mu} = 1$.

Q.E.D.

Proposition A.2 *i) If $\lambda \geq (\Delta a - Kc)/|a_H - Kc|$ then a solution to P3 is $\hat{w} = (\hat{w}_H, \hat{w}_H - Kc/\lambda, 0)$, $\hat{t} = (c/\lambda, 0, 0)$, with $\hat{w}_H = {}^1\!/_2[(a_H + Kc/\lambda) + \sqrt{(a_H + Kc/\lambda)^2 - 4K\Delta ac/\lambda}]$, $\hat{\mu} = 1$, and $\hat{\nu} = \Delta a/(\lambda \hat{w}_H) < 1$. ii) If $\lambda \leq (\Delta a - Kc)/|a_H - Kc|$ and $K \geq \lambda/(1-\lambda)$ then the solution to P3 is $\hat{w} = (\Delta a/\lambda, a_L/(1-\lambda), 0)$, $\hat{t} = (\Delta a/(K\lambda) - a_L/((1-\lambda)K), 0, 0)$, and $\hat{\mu} = \hat{\nu} = 1$. iii) If $\lambda \leq (\Delta a - Kc)/|a_H - Kc|$ and $K \leq \lambda/(1-\lambda)$ then the solution to P3 is $\hat{w} = (\Delta a/\lambda, (\Delta a - Kc)/\lambda, 0)$, $\hat{t} = (c/\lambda, 0, 0)$, and $\hat{\mu} = \hat{\nu} = 1$.*

Proof: If the optimal contract induces a simultaneous move game in which a mixed equilibrium is played then the previous proposition showed that for the optimum we must have either $\hat{w}_H = \Delta a/\lambda$ or $cK(\hat{w}_H - \Delta a) = \lambda \hat{w}_H(\hat{w}_H - a_H)$. This implies that $\hat{w}_H = \max\{\Delta a/\lambda, {}^1\!/_2[(a_H + Kc/\lambda) + \sqrt{(a_H + Kc/\lambda)^2 - 4K\Delta ac/\lambda}]\}$. When parameters are such that $\lambda \geq (\Delta a - Kc)/|a_H - Kc|$ then the first expression is smaller than the second expression. This condition distinguishes case i from case ii.

If the optimal contract is such that $\hat{t}_H \geq c/\lambda$ then the pure equilibrium $\mu = \nu = 1$ is induced in the simultaneous move game. The principal's payoff is $U_P = y_H - \Delta a - (1 - \lambda)w_N - \lambda t_H$, where $(1-\lambda)w_N \geq a_L$ and $t_H \geq c/\lambda$. Using the fact that the collusion-proofness constraint must be binding in the optimum, we can rewrite this as $U_P = y_H - \Delta a/\lambda + (1-\lambda)Kt_H - \lambda t_H$. The two constraints are $t_H \leq \Delta a/(K\lambda) - a_L/((1-\lambda)K)$ and $t_H \geq c/\lambda$. Note that the constraints can only be satisfied if $\lambda \leq (\Delta a - Kc)/|a_H - Kc|$. The principal's payoff U_P is increasing in t_H if and only if $(1-\lambda)K > \lambda$. In this case the optimum is found for $t_H = c/\lambda$. If $(1-\lambda)K < \lambda$ then the individual rationality constraint of the agent must be binding at the optimum, which requires $\hat{t}_H = \Delta a/(K\lambda) - a_L/((1-\lambda)K)$.

Q.E.D.

References

Border, K.C. and Sobel, J. (1987), "Samurai Accountant: A Theory of Auditing and Plunder", *Review of Economic Studies* 54, p. 525-540.

Hart, O. and Holmstöm, B. (1987), "The Theory of Contracts", in Bewley, T. (ed.) *Advances in Economic Theory*, 5th World Congress of the Econometric Society, Vol. II, Cambridge (Cambridge: Cambridge Univ. Press).

Khalil, F. (1991), "Commitment in Auditing", mimeo University of Washington.

Kofman, F. and Lawarrée, J. (1993), "Collusion in Hierarchical Agency", *Econometrica* 61, p. 629-656.

Mookherjee, D and Png, I. (1989), "Optimal Auditing, Insurance, and Redistribution", *Quarterly Journal of Economics* 417, p. 399-415.

Laffont, J.J. and Tirole, J. (1991) "The Politics of Government Decision-Making: A Theory of Regulatory Capture", *Quarterly Journal of Economics* 106, p. 1089-1127.

Tirole, J (1986), "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations", *Journal of Law, Economics and Organization* 2, p. 181-214.

Tirole, J (1992), "Collusion and the Theory of Organizations", in: J.J. Laffont (ed.), *Advances in Economic Theory*, 6th World Congress of the Econometric Society, Vol. II, (Cambridge: Cambridge Univ. Press).