Designing fiscal and monetary institutions in a second-best world
Beetsma, R.M.W.J.; Bovenberg, A.L.

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Designing Fiscal and Monetary Institutions in a Second-Best World

Roel M.W.J. Beetsma
DELTA (Joint Research Unit CNRS-EHESS-ENS) and University of Limburg

A. Lans Bovenberg
CentER, Tilburg University

ABSTRACT:
This paper explores how fiscal and monetary policy interact if commitment and access to lump-sum taxation are limited. We analyze how equilibrium outcomes for inflation, employment, and public spending are affected by the structural features of an economy, such as money holdings, outstanding public debt, labor-market distortions, society’s preferences, and the nature of the policy game. In a normative vein, we compare society’s welfare across various institutional settings and investigate how society should optimally adjust the preferences of policymakers.

Keywords: Central bank independence, central bank conservatism, commitment, discretion, Nash, monetary leadership, fiscal leadership, real money holdings, (non-indexed) debt, inflation, optimal preferences.

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Mailing address:
A. Lans Bovenberg
CentER
Tilburg University
P.O.Box 90153
5000 LE Tilburg
The Netherlands
Contents:

1. Introduction 1

2. The model 4

3. Centralized policymaking 6
   3.1. The first-best 7
   3.2. The second-best: commitment with limited access to lump-sum taxation 8
   3.3. Discretion 10

4. Decentralization of fiscal and monetary policy 12
   4.1. Central bank and government play Nash 12
   4.2. Monetary leadership 17
   4.3. Fiscal leadership 19

5. Conclusions 24

References 25

Appendix A: Derivation of the government budget constraint 30
B: Welfare comparison Nash commitment versus centralized discretion [eq.(4.1)] 31
C: Uniqueness of $\kappa^*$, the value of $\kappa$ for which (4.1) is met with equality 31
D: Welfare comparison Nash commitment versus Nash discretion [eq.(4.5)] 31
E: Uniqueness of $\kappa^{**}$, the value of $\kappa$ for which (4.5) is met with equality 32
F: Welfare comparison Nash discretion versus centralized discretion [eq.(4.6)] 32
G: Uniqueness of $\kappa^{***}$, the value of $\kappa$ for which (4.6) is met with equality 33
H: Welfare comparison commitment versus discretion under fiscal leadership [eq.(4.10)] 33
I: Proof that (4.10) is met with equality for a unique value of $\kappa$ 34
J: Proof that under discretion welfare loss Nash lower than with fiscal leadership if $\kappa=0$ and $d>0$ 35
K: Proof that under discretion welfare loss lower under fiscal than under monetary leadership if $\kappa=0$ and $d>0$ 35
L: Proof that under discretion monetary leadership better than fiscal leadership if $\kappa \to \infty$ 36
M: Proof that in general there exists a weight $\alpha_{\piM}$ such that fiscal leadership discretion improves upon centralized discretion, when it is not possible to distort the fiscal authority’s preferences 37
N: Proofs of inequalities in Table 2 38
1. Introduction

In analyzing the design of monetary institutions, most of the literature has focussed on the interaction between the monetary authorities and the private sector (see, e.g., Rogoff (1985) and Lohmann (1992)). How fiscal and monetary institutions interact has received less attention\(^1\), although fiscal and monetary policies are related through several channels. In particular, monetary policy impacts the public finances by generating seigniorage. Moreover, inflation surprises affect the real burden to the budget of servicing nominal debt. Finally, both tax policy and unanticipated inflation affect employment, a major objective of macroeconomic policy.

The modelling of both monetary and fiscal institutions allows one to distinguish between the concepts of, on the one hand, central bank independence and, on the other hand, conservatism of a central bank. Independence of the central bank involves the institutional setting of policymaking. A central bank is independent if, in setting monetary policy, it can pursue its own macro-economic objectives without interference by the fiscal authorities. In that case, macroeconomic policies are decentralized over autonomous monetary and fiscal authorities.\(^2,3\)

How conservative a central bank is depends on the objectives of the monetary authorities: A central bank is conservative if it attaches a high priority to price stability. Much of the literature on central bank independence fails to distinguish between the two concepts of independence and conservatism. It defines central bank independence in terms of the objectives of the monetary authorities rather than the institutional setting in which monetary policy is set.

In this paper, we extend the work of Debelle (1993) on the link between fiscal and monetary institutions. First, we provide more intuition. Second, we analyze how the level of money holdings affects the interaction between policy authorities. In modern economies with highly developed financial systems, money holdings tend to be quite small. The paper shows that a central bank that is both independent and conservative tends to be rather attractive for such economies.

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\(^2\) Grilli, Masciandaro and Tabellini (1991) distinguish between political independence, the ability of the central bank to choose its own economic policy objectives, and economic independence, the ability of the central bank to make unrestricted use of monetary policy instruments to pursue its monetary policy goals (see also Alesina and Grilli, 1992).

\(^3\) In the present paper the central bank is either completely dependent or completely independent. In practice independence is often intermediate, which might be modeled by varying degrees of bargaining power of the central bank relative to the fiscal authority. This, however, would be beyond the scope of the current paper.
As a third extension, we include nominal public debt into the model. In addition to nominal wage contracts, nominal public debt provides an additional incentive for policymakers to generate unanticipated inflation. In fact, the connection between central bank independence and the level of public debt is a major issue in designing the institutional framework of the European Monetary Union (EMU). To ensure price stability, the Maastricht Treaty not only includes provisions that must guarantee the independence of the central bank but also puts ceilings on public debt in the member countries. We find that high levels of public debt render an independent, conservative central bank more attractive. In addition to holdings of money and public debt, other structural features of the economy that affect the optimal design of monetary institutions are distortions in labor- and output markets, the priority society attaches to high levels of employment and public spending, and the nature of the policy game.

To analyze these issues, we formulate a game. The players in the private sector are a trade union concluding nominal wage contracts and bondholders investing in public debt. The policymakers in the public sector consist of a fiscal authority (or government) and a monetary authority (or central bank). Private agents act as Stackelberg leaders vis-à-vis the policy authorities. Hence, before policy is set, the private sector incorporates inflationary expectations in both nominal labor contracts and the required nominal return on public debt. All private sector agents hold the same, rational, expectations.

Both labor market distortions and distortionary output taxes reduce the level of employment below its socially optimal level. Within this setting, policymakers who lack commitment are tempted to adopt unanticipated inflation as an instrument to alleviate these distortions. In particular, inflation plays three roles. First, it helps to reduce distortionary taxation by generating seigniorage. Second, unanticipated inflation erodes the real value of outstanding nominal public debt, thereby reducing the need to raise public revenues through distortionary taxation. Third, unanticipated inflation boosts employment, thereby generating a first-order gain in welfare. In a rational expectations equilibrium, the private sector anticipates the incentives facing the policy authorities to generate unanticipated inflation. Hence, the second and third motives for inflation turn out to be self-defeating. Accordingly, only the first role of inflation, i.e. generating seigniorage, is socially useful in equilibrium.

We use the model to perform both positive and normative analysis. In a positive vein, inflation, output, and government spending are compared across various institutional regimes. In particular, we explore how the macroeconomic consequences of discretionary policies differ from policy outcomes when authorities are able to commit. Furthermore, assuming that the objectives of central and decentralized policymakers coincide, we investigate how centrally determined
macroeconomic policy differs from the case in which policymaking is decentralized with an independent central bank controlling monetary policy and the fiscal authorities determining taxes and public spending. Depending on how decentralized fiscal and monetary authorities interact, we investigate three non-cooperative games, namely Nash, Stackelberg with monetary authorities as leader, and Stackelberg with fiscal authorities as leader.

In a normative vein, we compare society’s welfare across the various regimes, assuming that the objectives of the policymakers coincide with those of society. We find that, except for the case with monetary leadership, decentralization of discretionary monetary and fiscal policy raises society’s welfare if money holdings are small, nominal public debt is large, and society attaches a high priority to price stability. These three factors may all have contributed to the tendency in recent years to delegate monetary policy to more independent central banks.

In the absence of commitment, decentralizing policy thus can raise welfare compared to centralized policymaking although, in contrast to central policy authorities, an independent central bank fails to internalize the government budget constraint. Intuitively, in a second best world with pre-existing distortions, the introduction of an additional distortion may actually enhance welfare. In particular, the distortion due to the failure to internalize the government budget constraint offsets distortions due to a lack of commitment.

Decentralization may also facilitate commitment of monetary policy. Hence, we explore how decentralization under commitment differs from centralization under discretion. In this way, we compare welfare losses due to the failure of an independent central bank to internalize the government budget constraint with the losses due to the lack of commitment of centralized authorities. Irrespective of the particular non-cooperative game between the decentralized authorities, granting independence to the central bank raises welfare if money holdings are low, nominal public debt is large, and society attaches a high priority to price stability.

Objective functions of the fiscal and monetary authorities can differ from each other and from the objectives of society. This brings us to the issue of how conservative central banks should be. Rogoff (1985) has shown that, in order to (partially) overcome commitment problems, society may find it optimal to delegate monetary policy to policy authorities that attach a higher priority to price stabilization than society does. Intuitively, distorting the preferences of the central bank offsets the distortion due to the absence of commitment. We extend Rogoff’s analysis by exploring how, in various institutional settings, society can improve welfare by distorting the preferences of the policy authorities. We find that, in the absence of commitment, it is typically optimal for the weight that the authorities determining monetary policies attach to inflation to exceed the corresponding weight of society. With monetary leadership, the central bank
internalizes the government budget constraint. Hence, the inflation weight of the central bank exceeds that of the society for two reasons: the incentive to stimulate employment and the incentive to reduce the value of nominal public debt. If the fiscal and monetary authority move simultaneously, the central bank does not internalize the government budget constraint. In that case, the central bank does not need to be as conservative as with monetary leadership. However, it still needs to be more conservative than society if money balances are small, as is the case for modern economies. Under fiscal leadership, the preferences of not only the monetary but also the fiscal authorities need to be distorted. In particular, the fiscal authority should be more inflation averse than the central bank. Alternatively, the priority that the fiscal authorities attaches to public spending should be reduced below that of society.

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3, monetary and fiscal policies are set centrally by the government. A central policymaker with unlimited access to lump-sum taxation can attain the first best, irrespective of whether or not he is able to commit monetary policy. Hence, the limited availability of lump-sum taxation is the ultimate source of the credibility problems associated with the (self-defeating) incentives to reduce the real value of nominal contracts through unanticipated inflation (see Alesina and Tabellini (1987)). In the absence of lump-sum taxation, the second-best allocation is attained when commitment of monetary policy is feasible. In the absence of both commitment and lump-sum taxation, the economy suffers from an inflation bias due to the self-defeating incentives to use unanticipated inflation as an instrument to boost employment and reduce the real burden of financing nominal public debt. Section 4 turns to an institutional setting in which monetary and fiscal policy are decentralized. Section 5 concludes the main text of the paper.

2. The model

The sole objective of unions is to achieve a target real wage rate, the logarithm of which we normalize to zero (for similar approaches, see Alesina and Tabellini (1987), Debelle (1993), Eijffinger and Schaling (1993) and Jensen (1994)). Therefore, the (log) of the nominal wage rate is set equal to the (rationally) expected (log) price level, \( \pi^e \). Nominal wage contracts are signed before policy is selected. Unions thus act as Stackelberg leaders vis-à-vis the authorities.

Output is given by \( Y = L^\eta \), where \( L \) is labor. Output is taxed at a rate \( \tau \). The representative firm selects employment so as to maximize profits \( PL^\eta(1-\tau) - WL \), where \( P \) and \( W \) denote price level and the wage rate, respectively. Hence, (log) output is given by \( y = (\eta/(1-\eta))(\pi - \pi^e - \tau + \log \eta) \), where \( \pi \) is the inflation rate and \( \pi^e \) the expected inflation rate. For
convenience, we normalize output by subtracting the constant \((\eta/(1-\eta))\log\eta\) from \(y\). Without any consequences for our main results, we set \(\eta=\frac{1}{2}\). Normalized output, \(x\), thus amounts to

\[
x = \pi-\pi^e-\tau. \tag{2.1}
\]

In addition to distortionary output taxes, we allow for other, non-tax, distortions due to, for example, union power in the labor market or monopoly power in commodity markets. Without tax distortions, \(x=0\) in a rational expectations equilibrium (where inflation is anticipated, i.e. \(\pi=\pi^e\), see (2.1)). The first-best output level, i.e. output with neither tax nor non-tax distortions, is denoted by \(\bar{x}\). Thus, \(\bar{x}>0\) measures the non-tax distortions and can be interpreted as an implicit tax on output. In fact, by offsetting the implicit output tax, an output subsidy \((\tau=-\bar{x})\) can raise output towards its first-best level \(\bar{x}\).

Society features a social welfare function that differs from the objectives of the unions, because the social welfare function accounts for the preferences of both workers and non-workers. Society’s preferences, defined over inflation, output and public spending, are represented by the following loss function,

\[
V_S = \frac{1}{2} \left[ \alpha_{\pi S} \pi^2 + (x-\bar{x})^2 + \alpha_{gS} (g-\bar{g})^2 \right], \quad \alpha_{\pi S}, \alpha_{gS} > 0. \tag{2.2}
\]

Welfare losses increase in the deviations of inflation, (log) output and government spending \((g\) is government spending as a share of non-distortionary output) from their targets (or first-best levels or ‘bliss points’).\(^4\) The target level of inflation corresponds to price stability. The non-distortionary output level, \(\bar{x}\), represents the bliss point for output. The first-best level of government spending, \(\bar{g}\), can be interpreted as the optimal share of non-distortionary output to be spent on public goods if (non-distortionary) lump-sum taxes would be available. Parameters \(\alpha_{\pi S}\) and \(\alpha_{gS}\) correspond to the weights of the inflation and government spending objectives, respectively, relative to the weight of the output objective. Only relative weights matter for the outcomes. The limiting case of \(\alpha_{gS} \to \infty\) corresponds to the situation where government spending is exogenously fixed at \(\bar{g}\).

Preferences of the fiscal and monetary authorities are given by, respectively,

\(^4\) Employment is directly related to output through the production function. Hence, instead of output, employment could have been included as an argument in the loss functions, with the target employment level corresponding to the output level in absence of any distortions.
\[ V_F = \frac{1}{2} \left[ \alpha_{\pi F} \pi^2 + (x-\bar{x})^2 + \alpha_{gF} (g-\bar{g})^2 \right], \quad \alpha_{\pi F}, \alpha_{gF} > 0. \] 

(2.3)

and

\[ V_M = \frac{1}{2} \left[ \alpha_{\pi M} \pi^2 + (x-\bar{x})^2 + \alpha_{gM} (g-\bar{g})^2 \right], \quad \alpha_{\pi M}, \alpha_{gM} > 0. \] 

(2.4)

The preference weights may differ both between the two policymakers and from the corresponding weights of the social welfare function. In contrast to Debelle (1993), who assumes that the monetary authority does not care about government spending, we allow for a potentially non-zero weight \( \alpha_{gM} \).

The government selects its policies subject to the government budget constraint\(^5\),

\[ g + (1+\rho) b + (1+\rho+\pi^c-\pi) d = \tau + \kappa\pi + \theta, \] 

(2.5)

where \( \rho \) denotes the ex-ante real interest rate, \( b \) the outstanding stock of single-period indexed government debt, \( d \) the initial real value of the single-period non-indexed government debt, \( \theta \) lump-sum tax revenue (\( b, d \) and \( \theta \) are expressed as shares of non-distortionary output), and \( \kappa \geq 0 \) the constant ratio of real money holdings and non-distortionary output.\(^6\) Both \( b \) and \( d \) are assumed to be non-negative. Accordingly, the government is a net debtor (except when \( b=d=0 \)). Bondholders are risk neutral and have access to outside investment opportunities with a net rate of return \( \rho \). Hence, the nominal interest rate on non-indexed debt should compensate investors for expected inflation so that the nominal interest rate is \( \rho + \pi^c \).

3. Centralized policymaking

This section assumes that a centralized policymaker controls all policy instruments. One can interpret this case as that of a dependent central bank or of cooperation between fiscal and monetary authorities who share the same objectives. Our analysis proceeds in three steps. We start with a benevolent, centralized policymaker who has unlimited access to lump-sum taxes. Irrespective of whether this policymaker is able to commit or not, this case yields zero welfare losses and is thus called first best.

Subsequently, we restrict lump-sum taxation so that the government has to resort to distortionary output taxes to finance its spending. If the policymaker can commit, the resulting

\(^5\) The government budget constraint is derived in appendix A.

\(^6\) Alesina and Tabellini (1987), Debelle (1993) and Jensen (1994), among others, assume that \( \kappa=1 \). However, as will become clear below, the value of \( \kappa \) plays an important role in our analysis.
equilibrium is second best. The third and final step is to consider the case with neither unlimited lump-sum taxation nor commitment. If the use of lump-sum taxation is limited, the absence of commitment results in additional welfare losses because the government is tempted to use unanticipated inflation as an indirect instrument to alleviate tax distortions (see also Alesina and Tabellini (1987)).

3.1. The first-best

Suppose, for the moment, that a benevolent policymaker can freely set lump-sum taxation (as a share of non-distortionary output) $\theta$. The dictator thus solves the Lagrangian over $\pi, \tau, g$ and $\theta$: 

$$L = \frac{1}{2} [\alpha_\pi \pi^2 + (\pi - \pi^e - \tau - \bar{x})^2 + \alpha_{\varrho g} (g - \bar{g})^2] + \lambda \left[ g + (1 + \rho)(b + d) + (\pi^e - \pi - \kappa \pi - \theta) \right], \quad (3.1)$$

where $\lambda$ is the Lagrange multiplier associated with the government budget constraint. The first-order conditions$^9$ corresponding to $\pi$, $\tau$, $g$ and $\theta$ are, respectively,

$$\alpha_\pi \pi + (\pi - \pi^e - \tau - \bar{x}) = (\kappa + d) \lambda, \quad (3.2)$$

$$-(\pi - \pi^e - \tau - \bar{x}) = \lambda, \quad (3.3)$$

$$\alpha_{\varrho g} (g - \bar{g}) = \lambda, \quad (3.4)$$

$$\lambda = 0. \quad (3.5)$$

Imposing the condition of rational expectations (i.e. $\pi^e = \pi$), we can rewrite the government budget constraint (2.5) as

$$\tilde{F} \equiv \tilde{g} + (1 + \rho)(b + d) + \bar{x} = [\tau + \bar{x}] + [\kappa \pi] + [\tilde{g} - g] + \theta. \quad (2.5')$$

We refer to $\tilde{F}$ as the (government) financing requirement. It amounts to the government spending target $\tilde{g}$, debt servicing costs, $(1 + \rho)(b + d)$, and a labor subsidy aimed at offsetting the implicit tax on output, $\bar{x}$. Nominal and real debt affect the financing requirement in the same way because all

7 We assume that the optimal lump-sum tax does not exceed production. Hence, $\theta \leq 1$.

8 The inflation rate equals the nominal money supply growth rate. Hence, the inflation rate can be taken as being under direct control of the policymaker.

9 Throughout the paper, the linear-quadratic structure of the model ensures that the optimal policy choice is the unique solution to the first-order conditions.
inflation is anticipated in equilibrium due to rational expectations. The last right-hand side of (2.5) represents the sources of finance: explicit and implicit tax revenues, \( \tau + \check{x} \), seigniorage revenues, \( \kappa \pi \), the shortfall of government spending from its target, \( \check{g} - g \), and lump-sum taxes, \( \theta \).

If the government has unlimited access to lump-sum taxes, it covers its entire financing requirement through these taxes. Accordingly, optimal policy is given by \( \pi = 0 \), \( \tau = -\check{x} \), \( g = \check{g} \) and \( \theta = \bar{F} \) (impose \( \pi^e = \pi \) on (3.2) and (3.3)). All distortions are eliminated: non-tax market failures in the output and labor markets are offset by output subsidies, prices are stable, and government spending is set at its target level. In the absence of any welfare losses, the resulting equilibrium is first best. In this case, commitment does not provide any value added over discretion. Intuitively, in a first-best world, the government has no need to use unanticipated inflation as an indirect instrument to alleviate market failures.

3.2. The second-best: commitment with limited access to lump-sum taxation

From now on, we assume that the government can use only a limited, exogenous amount of lump-sum taxes \( \theta \), which is strictly less than the government financing requirement \( \bar{F} \). Hence, the modified government financing requirement (or, in the following, simply "government financing requirement") becomes

\[
\bar{K} \equiv \bar{F} - \theta = [\tau + \check{x}] + [\kappa \pi] + [\check{g} - g].
\]

With commitment, the government minimizes (2.2) subject to (2.1), (2.5) and the rational-expectations restriction \( \pi^e = \pi \), for given \( \theta \). The Lagrangian is,

\[
\mathcal{L} = \frac{1}{2}[\alpha_{\pi^2} \pi^2 + (\tau + \check{x})^2 + \alpha_{\check{g}} (\check{g} - \check{g})^2] + \lambda [g + (1 + \rho)(b + d) - \tau - \kappa \pi - \theta].
\]

The first-order conditions for \( \pi \), \( \tau \) and \( g \), respectively, yield:

\[
\alpha_{\pi^2} \pi = \kappa \lambda, \tag{3.7}
\]
\[
\tau + \check{x} = \lambda, \tag{3.8}
\]
\[
\alpha_{\check{g}} (\check{g} - g) = \lambda. \tag{3.9}
\]

The restriction on lump-sum taxation can be interpreted as reflecting distributional concerns. The more a society cares about an equal income distribution, the less lump-sum taxes it would want to use. If inequality aversion is high enough, lump-sum taxes may actually be negative, thereby expanding the government financing requirement.
Eliminating $\lambda$, we arrive at:

$$\kappa \pi = \left( \kappa^2 / \alpha_{as} \right) [\tau + \bar{x}], \quad (3.10)$$

$$\tilde{g} - g = \left( 1 / \alpha_{gs} \right) [\tau + \bar{x}]. \quad (3.11)$$

We can combine (3.10) and (3.11) with (2.5") to obtain the equilibrium policy outcomes, which are presented in Table 1. Table 1 contains also an expression for society’s welfare loss.

If available lump-sum taxes would suffice to cover the government financing requirement (i.e., $\theta = \bar{F}$ or $\bar{K} = 0$), the first-best would be reached. Hence, welfare losses originate in the limited access to lump-sum taxation. The larger the part of the government financing requirement that cannot be financed by lump-sum taxation, $\bar{K}$, the larger welfare losses become. In particular, a higher $\bar{K}$ requires the government to levy higher distortionary taxes, thereby harming output. Moreover, inflation rises in order to raise more seigniorage, thereby causing stagflation. Finally, government spending falls below its target level in order to reduce the need for distortionary taxation and seigniorage.

Accordingly, distortions are smoothed out over the three sources of financing, because marginal welfare losses are increasing in the level of each distortion. Therefore, countries with serious labor market distortions (i.e., $\bar{x}$ is high) rely less on explicit taxes and more on seigniorage while government spending is crowded out. Also a higher stock of government debt raises the government financing requirement. This increases distortionary taxation, which hurts output through adverse supply-side effects. In addition, more government debt boosts inflation, hence causing stagflation, and decreases government spending. In a similar way, a larger public spending target yields stagflation.

In addition to the financing requirement, other structural features of the economy affect the policy outcomes. Most modern economies feature low holdings of real base money (i.e., $\kappa \to 0$), so that seigniorage is of only minor importance as a source of government finance. Indeed, if money holding are very small ($\kappa \to 0$), inflation vanishes and $[\tau + \bar{x}]$ and $[\tilde{g} - g]$ bear the entire burden of meeting the financing requirement.

Another important determinant of the equilibrium are the relative priorities society attaches to various objectives. To illustrate, an increase in weight attached to price stability, $\alpha_{as}$, reduces inflation, but also raises distortionary taxation (and lowers government spending), thereby harming employment. Hence, society faces a trade-off between output and inflation (a kind of "Phillips curve"). If the spending weight, $\alpha_{gs}$, increases, government spending rises, requiring more seigniorage (and thus higher inflation) and higher output taxes.
3.3. Discretion

If the government cannot commit, it attempts to use unanticipated inflation as an instrument to erode the real values of nominal wage and debt contracts. This in order to expand output and reduce the need for distortionary taxation. Under discretion, the government takes inflation expectations as predetermined. The Lagrangian associated with the government’s optimization problem thus becomes:

$$
\mathcal{L} = \frac{1}{2}[\alpha_{\pi}{\pi}^2 + (\pi - \pi^e - \tau - \bar{x})^2 + \alpha_{\pi}(g - \bar{g})^2] + \lambda[g + (1 + \rho)(b + d) + (\pi^e - \pi)d - \tau - \kappa\pi - \theta].
$$

In principle, the price-stability weight of the fiscal authority, $\alpha_{\pi}$, may differ from that of society, $\alpha_{\pi}$.

Inflation, taxes, and spending

The first order condition for the level of government spending is the same as under commitment (see (3.9)). The first order conditions for the inflation rate and the tax rate now become, respectively,

$$\alpha_{\pi}{\pi} + (\pi - \pi^e - \tau - \bar{x}) = (\kappa + d)\lambda, \quad (3.13)$$

and imposing $\pi^e = \pi$, we find the relation between seigniorage and output-tax revenues:

$$\kappa\pi = \left[\kappa(\kappa + d + 1)/\alpha_{\pi}\right][\tau + \bar{x}]. \quad (3.15)$$

Comparison of (3.15) with (3.10) (assuming that $\alpha_{\pi} = \alpha_{\pi}$) reveals that the share of seigniorage in government revenues is higher under discretion than under commitment because inflation appears more attractive as an instrument to reduce the real value of nominal contracts.
Substitution of (3.11) and (3.15) into (2.5″) yields the equilibrium policy outcomes, which are contained in Table 1. If preferences of the government and society coincide (α_{gF}=α_{gS} and α_{SF}=α_{SS}) and κ>0, inflation, seigniorage, and public spending are higher while taxes are lower under discretion than under commitment (see Table 2). Hence, discretion suffers from an inflation bias due to the self-defeating temptation to use unanticipated inflation to erode nominal contracts.

Welfare

The inability to precommit harms welfare. Subtracting welfare under commitment from the corresponding expression under discretion (see Table 1), one finds the value of commitment. This value rises with the financing requirement, ˜K, and the outstanding amount of nominal public debt.

Optimal preferences

Welfare losses due to the absence of commitment can be eliminated if the price-stability weight of the government can be adjusted.\footnote{See also Rogoff (1985). In Rogoff’s model welfare losses can not be eliminated entirely because of stochastic shocks. From the point of view of society’s welfare, a central bank that is more conservative than society does not respond properly to shocks. Hence, there is a trade off between credibility and flexibility. See also Lohmann (1992). However, Persson and Tabellini (1992) and Walsh (1993) argue that an optimal contract between the government and the central bank can eliminate the trade-off between credibility and flexibility.} The optimal weight is set such that the share of seigniorage relative to τ+̃x coincides with the corresponding share under commitment. Combining (3.10) and (3.15), we find for the optimal inflation weight,

\[ \alpha_{SF}^{opt} = \frac{(κ+d+1)α_{gS}}{κ}, \] \[ \text{or } \frac{\alpha_{SF}^{opt}}{α_{gS}} = 1 + \frac{(d+1)}{κ}. \] (3.16)

For \( \alpha_{SF}^{opt} \), policy outcomes coincide with those under commitment (given \( α_{gF}=α_{gS} \)). The optimal inflation weight exceeds society’s true inflation weight for two reasons: the incentive to stimulate employment by reducing real wage costs through unanticipated inflation and the incentive to reduce the real value of the outstanding nominal debt through unanticipated inflation. Both incentives are anticipated by the private sector and hence futile in equilibrium. To offset these self-defeating incentives, policymakers need to attach a higher weight to price stability. Intuitively, in a second best world with pre-existing distortions, introducing another distortion may be welfare improving.

The optimal mark-up on the true price-stability weight depends on the strength of the self-defeating roles of inflation, as measured by \( d+1 \), relative to the socially useful role, as measured by κ. Technological advances in financial markets have reduced real money holdings, κ. At the same
time, public debt ratios, \( d \), have increased due to fiscal imbalances. These two developments may explain why policymakers have become more conservative in recent years.

4. Decentralization of fiscal and monetary policy

In this section, macroeconomic policy is decentralized. The government sets the tax rate and the level of government spending, while an independent central bank selects the inflation rate by controlling monetary policy. We explore three policy games: a Nash game in which policymakers simultaneously select policies and two Stackelberg games in which either the fiscal or monetary authorities act as leader.

4.1. Central bank and government play Nash

Commitment

The central bank selects the inflation rate to minimize (2.4) subject to (2.1) and \( \pi^e = \pi \), taking as given the instruments controlled by the fiscal authorities (i.e. the tax rate and government spending). Similarly, the fiscal authority sets its instruments by minimizing (2.3) subject to (2.1) and (2.5), taking the inflation rate as given. Table 1 provides the expressions for the policy outcomes and society’s welfare loss.

Inflation, taxes, and public spending

Compared with centralized policymaking under commitment (i.e. the second best, see section 3.2), inflation and government spending are lower while taxes are higher. In fact, the Nash equilibrium yields the zero inflation rule studied by Jensen (1994). Intuitively, with decentralized policymaking, the central bank does not internalize the government budget constraint and thus fails to account for the social value of seigniorage.

The welfare losses due to this distortion rise with the size of the base of the inflation tax as indicated by the inverse of velocity, \( \kappa \). The second best is approached in modern economies with small holdings of base money (i.e. \( \kappa \to 0 \)) as the seigniorage motive for inflation vanishes.
Welfare

Decentralization is likely to facilitate commitment because the central banker can be held directly responsible for deviations from a given rule for inflation. Hence, we provide a welfare comparison with centralized policymaking under discretion (assuming \( \alpha_{πM} = \alpha_{πF} = \alpha_{πS} \) and \( \alpha_{gF} = \alpha_{gS} \)). Welfare losses are lowest under Nash (with commitment) if and only if (see Appendix B),

\[
2 \left( \frac{\kappa}{\kappa^2 + d + 1} \right) + \frac{\kappa^2}{\alpha_{πS}(1 + \alpha_{gS}^{-1})} < 1. \tag{4.1}
\]

The welfare comparison depends on two elements. The first term at the left-hand side of (4.1), which is a decreasing function of \((d+1)/\kappa\), reflects the relative importance of the distortions under, respectively, centralized and decentralized policymaking. Decentralized policymaking is attractive if the distortion under decentralized policymaking due to the failure of the central bank to internalize the government budget constraint (indicated by \(\kappa\)) is small compared to the distortions under centralized policymaking due to the absence of commitment (indicated by the term \(d+1\)).

The objectives of society are the second determinant of the welfare comparison and affect the left-hand side of (4.1) through the second term. In particular, if society attaches a low priority to price stability (as indicated by a small weight, \(\alpha_{πS}\)) but a high priority to public spending (as indicated by a large weight, \(\alpha_{gS}\)), decentralized policymaking, which delivers price stability but reduces seigniorage revenues and hence public spending, becomes less attractive. In equilibrium, this trade off between public spending and price stability becomes less important if small real balances make inflation an insignificant source of seigniorage. Indeed, if money balances vanish (i.e. \(\kappa \to 0\)), the second term approaches zero.

An alternative interpretation of (4.1) is that it compares the benefits and costs of decentralization. The benefit is the absence of self-defeating incentives to inflate, which is especially important if society is rather inflation-averse and nominal public debt is large. The cost of decentralization amounts to the loss of seigniorage revenues, which is large if \(\kappa\) is large and society attaches a high priority to public spending and employment. The increased tendency to decentralize fiscal and monetary policy can be explained by the decreasing costs (due to a smaller base of the inflation tax as reflected in a small value for \(\kappa\)) and increasing benefits (due to large stocks of public debt and an increased aversion against inflation).

\[\text{12 The example of New Zealand is instructive in this respect. The central bank law requires the governor of the central bank and the minister of finance to establish public inflation targets. The central bank governor is personally accountable for failure to meet these targets. For more on this example, see e.g. Lohmann (1995).}\]
Under discretion, the central bank minimizes (2.4) subject to (2.1), taking as given not only the tax rate and government spending but also the expected inflation rate, $\pi^e$. The reaction function of the central bank thus amounts to:

$$\pi = \left( \frac{1}{a_M^{-1}} \right) (\pi^e + \tau + \bar{x}). \tag{4.2}$$

Higher expected inflation, higher taxes or more non-tax labor market distortions induce the monetary authority to raise inflation in order to protect employment.

The government minimizes (2.3) over $g$ and $\tau$, subject to (2.1) and (2.5), taking both $\pi$ and $\pi^e$ as given. The government’s reaction functions are

$$\tau + \bar{x} + \pi^e - \pi = \left( \frac{1}{1 + \alpha^{-1} \phi^{i}} \right) (\bar{K} - \kappa \pi + (\pi^e - \pi)(d + 1)), \tag{4.3}$$

$$\tilde{g} - g = \left( \frac{\alpha^{-1}}{1 + \alpha^{-1} \phi^{i}} \right) (\bar{K} - \kappa \pi + (\pi^e - \pi)(d + 1)). \tag{4.4}$$

To interpret these reaction functions, we rewrite the government budget constraint as $[\bar{K} + (\pi^e - \pi)(d + 1) + \kappa \pi] = [\tau + \bar{x} + \pi^e - \pi] + [\tilde{g} - g] + (\pi^e - \pi)d$. The left hand side of the equation, which appears at the right-hand sides of both (4.3) and (4.4), represents the residual government financing requirement of the fiscal authorities, i.e., the financing requirement left for the fiscal authorities after taking into account seigniorage and the impact of inflation surprises (i.e., $\pi^e - \pi$) on debt service and output. If $\alpha_{gS} \to \infty$, the entire burden of residual finance falls on taxes as government spending is fixed at $\tilde{g}$.

Expression (4.4) reveals that unanticipated inflation (i.e., $\pi > \pi^e$) raises government spending. Intuitively, surprise inflation reduces real debt service and, by eroding real wages, boosts employment, thereby reducing the need to subsidize output. This leaves more room for public spending.

Equilibrium policy outcomes, together with society’s welfare losses, are provided in Table 1. These outcomes are found by combining (4.2)-(4.4) with the government budget constraint and the requirement that expectations be rational ($\pi^e = \pi$).
Inflation, taxes, and public spending

Whether inflation under decentralized policymaking with discretion is higher than in the second best (i.e., centralized policymaking with commitment, see section 3.2) depends on real money balances. In particular, if $\kappa$ is smaller than unity, inflation is highest with decentralization (assuming that $\alpha_{\pi M}=\alpha_{\pi S}$ and $\alpha_{gF}=\alpha_{gS}$). Intuitively, under Nash discretion, inflation is used as an instrument to boost employment. Under centralized policymaking with commitment, in contrast, inflation only acts as an instrument to raise seigniorage. If money holdings are small (i.e., $\kappa$ is smaller than unity), the tax base for seigniorage is narrow. Hence, the seigniorage argument for inflation in the second best is outweighed by its role in stimulating output and employment under decentralization. If $\kappa=1$ (as, e.g., in Debelle, 1993), the seigniorage and employment incentives for inflation balance each other. Accordingly, Nash discretion coincides with the second best.

Given decentralized policymaking, we can compare policy outcomes also between commitment and discretion. As with centralized policymaking, inflation and public spending are highest while taxes are lowest under discretion (see Table 2). In contrast to the case with centralized policymaking, the incentives to reduce debt service are not relevant because an independent central bank does not internalize the government budget constraint. Accordingly, with decentralized policymaking, the inflation bias originates only in the self-defeating incentives to boost employment.

Finally, we can compare decentralized and centralized policymaking under discretion. Centralized policymaking produces the highest inflation; in contrast to an independent central bank, a central policymaker internalizes the government budget constraint and thus uses inflation not only to boost employment but also to raise seigniorage and reduce debt service. If $\kappa=d=0$, these two additional motives for inflation are absent so that decentralization produces the same outcomes as centralized policymaking.
Welfare

Decentralization under discretion generally results in additional welfare losses compared to centralization under commitment (i.e. the second best) unless $\kappa=1$. The reason is that decentralized policymaking and discretion cause two additional distortions. First, an independent central bank does not internalize the government budget constraint. Second, the loss of commitment produces self-defeating incentives for inflation. If $\kappa=1$, the two distortions exactly offset each other.

If decentralized policymakers play Nash, commitment is preferred to discretion if and only if (see Appendix D),

$$2\kappa + \frac{\kappa^2}{\alpha_{5}(1+\alpha_{8}^{-1})} < 1. \quad (4.5)$$

This inequality looks similar to (4.1) except that the first term on the left-hand side does not depend on $d$. The reason is that with decentralized policymaking the central bank does not attempt to reduce the real value of nominal debt. This in contrast to a centralized policymaker who internalizes the government budget constraint.

Unless $\kappa$ is rather small, commitment is less attractive than discretion -- even for the central bank. Intuitively, decentralization distorts policymaking because the independent central bank fails to internalize the seigniorage motive for inflation. Hence, under decentralized policymaking, discretion plays a useful social role by providing an incentive for the central bank to tolerate inflation, thereby offsetting the distortions due to the failure of the central bank to internalize the government budget constraint. Only if the contribution of inflation to seigniorage is small (because real balances and hence the base of the inflation tax is small) or society cares a lot about price stability and little about public spending, does the commitment equilibrium, which produces the lowest inflation and public spending, yield the highest welfare.

Assuming that commitment is not possible, we can compare welfare under decentralization with that under centralization. If either $\kappa$ or $d$ is positive, decentralized policymaking yields highest welfare if and only if (see Appendix F),

$$2 \left(\frac{\kappa}{\kappa + d + 2}\right) + \frac{\kappa}{\alpha_{5}(1+\alpha_{8}^{-1})} \left(\kappa - 2\left(\frac{\kappa + d + 1}{\kappa + d + 2}\right)\right) < 1. \quad (4.6)$$

Decentralization becomes more attractive with a large stock of outstanding nominal debt and a small stock of money. The reason is that decentralization removes both the self-defeating incentive to reduce the real value of nominal debt and the socially useful motive to raise seigniorage. If holdings of nominal debt are large compared to those of money, the futile motive
(reducing debt service) dominates the non-futile one (raising seigniorage). Hence, by eliminating both motives, decentralization enhances welfare.

The three inequalities (4.1), (4.5) and (4.6) can be used to find the welfare ranking contained in Table 2, where $\kappa'$, $\kappa''$ and $\kappa'''$ are the values of $\kappa$ for which (4.1), (4.5) and (4.6),\(^{14}\) respectively, are met with equality. For very small money holdings (i.e. for $\kappa<\kappa''$), decentralized policymaking with commitment yields highest welfare while losses are highest under centralized policymaking with discretion. Intuitively, with small real balances, inflation does not play any useful social role so that the welfare ranking of the various equilibria depends on their performance in ensuring price stability. For intermediate money holdings (i.e. for $\kappa''<\kappa<\kappa'''$), decentralization with discretion is best. Finally, for sufficiently large $\kappa$ ($\kappa<\kappa'''$), centralization under discretion is better than decentralized discretion, which in turn is better than Nash commitment.

**Optimal preference weights**

Rather than decentralizing policy, properly changing the preference weights of policymakers is another way to deal with the distortions due to the absence of commitment. Altering the weights of the monetary authorities may well be possible only if the central bank is independent. In practice, decentralization might make it easier to hold the central bank responsible for the resulting inflationary rate. This would provide another argument in favor of decentralization. The second best is reached when the central bank’s price-stability weight is:

$$\alpha_{\pi M}^{\text{opt}} \equiv \alpha_{\pi S} / \kappa. \quad (4.7)$$

The central bank should be more conservative than society (i.e. $\alpha_{\pi M}^{\text{opt}}$ exceeds $\alpha_{\pi S}$) if money holdings are small (i.e. $\kappa < 1$). Intuitively, with small real balances, the failure of an independent central bank to internalize the seigniorage motive for inflation is less important than the temptation to use inflation as a futile instrument to boost employment. To reduce the temptation to use unanticipated inflation, the central bank should be made more conservative.

With decentralized policymaking, the central bank does not have to be as conservative as under centralization (compare (4.7) with (3.16)) because an independent central bank does not internalize the government budget constraint and hence does not raise inflation to aid the public finances.

\(^{14}\) It can be shown that $\kappa'$ (Appendix C), $\kappa''$ (Appendix E) and $\kappa'''$ (Appendix G) are unique.
4.2. Monetary leadership

In this subsection, we consider a Stackelberg game with the central bank as the leader. The central bank is leader if it selects the inflation rate before the government chooses the tax rate and the level of public spending. This game structure is most likely if, compared to the fiscal authorities, the central bank can react more swiftly to new situations. We assume throughout this subsection that the fiscal authority and society attach the same weight to public spending (i.e. $\alpha_g^F = \alpha_g^S$).

**Commitment**

If the central bank is Stackelberg leader, it minimizes (2.4) subject to not only (2.1) and the rational expectations constriction $\pi = \pi^e$ but also the fiscal authority’s reaction functions (4.3) and (4.4). Table 1 contains the expressions for the equilibrium policy outcomes and society’s welfare loss.

If the objectives of the central bank and society coincide, the second best is reached. Unlike in the Nash game, the monetary authorities internalize the government budget constraint by taking into account the reaction functions of the fiscal authorities. Hence, the central bank accounts for the social value of inflation in generating seigniorage. Secondly, through the ability to commit, any self-defeating incentives for inflation are avoided.

If the central bank attaches a smaller weight to government spending than society does, not only are inflation and government spending lower than in the second best, but also taxes are higher (provided that $\kappa > 0$). Intuitively, a smaller spending weight implies a higher priority for price stability. Hence, the central bank reduces inflation. The resulting increase in the residual financing requirement due to a decline in seigniorage induces the fiscal authorities not only to cut spending but also to raise output taxes (see (4.3) and (4.4)).

**Discretion**

With discretion, the central bank solves the same problem as under commitment, except that it now takes $\pi^e$ as predetermined. Just as is the case with commitment, policy outcomes coincide with those under centralization if the preferences of the authorities are the same as society’s preferences (see Table 1). For the reasons discussed in Sections 3.3 and 4.1, inflation and government spending are higher and taxes lower than in the second best or the Nash case.
Since the equilibrium coincides with centralized discretion, we can use (4.6) for a welfare comparison with Nash discretion. The benefit from leadership is that the seigniorage motive for inflation is internalized. The cost is, that by internalizing the government budget constraint, the central bank is tempted to reduce the real value of nominal public debt through unanticipated inflation. While the size of the benefit depends on money holdings, nominal debt determines how large the costs are. Hence, if \( d \) is large compared to \( \kappa \), the costs dominate the benefits and the central bank loses from leadership (i.e. (4.6) is met in that case). Indeed, in a second-best world, removing one distortion (i.e. the failure to internalize the government budget constraint) may worsen another (i.e. the lack of commitment).

If the preferences of the central bank coincide with society’s, commitment always makes society better off. This contrasts with the Nash game, where the commitment equilibrium is not second best because of the failure of the central bank to internalize the government budget constraint. With monetary leadership this distortion is absent and hence discretion can not act as a way to offset this distortion.

**Optimal preference weights**

As before, appropriate distortion of the central bank’s objectives achieves the second best. Given that the central bank internalizes the government budget constraint, it is not surprising that the optimal central bank’s price stability weight is the same as the corresponding weight of the central policymaker (assuming that \( \alpha_{\pi M} = \alpha_{\pi F} = \alpha_{\pi S} \), compare (3.16)),

\[
\alpha_{\pi M}^{opt} = (\kappa + d + 1) \alpha_{\pi S} / \kappa, \text{ or } \alpha_{\pi M}^{opt} / \alpha_{\pi S} = 1 + (d + 1) / \kappa.
\] (4.8)

Instead of increasing the inflation weight of the central bank, society could also decrease the government spending weight of the central bank (assuming that \( \alpha_{\pi M} = \alpha_{\pi F} = \alpha_{\pi S} \) and \( \alpha_{\pi F} = \alpha_{\pi S} \)). The spending weight of the monetary authorities does not distort the trade-off between spending and taxes. Rather, a lower spending weight of the central bank reduces inflation. Given the resulting residual financing requirement, the fiscal authorities implement the optimal mix between spending and taxes as long as their spending weight coincides with that of society. In particular, the second best can be attained if there exists a weight \( \alpha_{gM} \geq 0 \) such that,

\[
\delta = \left( \frac{1 + \alpha_{gM} \alpha_{gS}^2}{1 + \alpha_{gS}^2} \right) = \frac{\kappa}{\kappa + d + 1}.
\] (4.9)
The existence of such a non-negative value for $\alpha_{GM}$ is more likely if money holdings are large and nominal public debt is small. In that case, the inflation bias is relatively small and hence only a slight adjustment in preferences is required.

### 4.3. Fiscal leadership

Compared to adjustments in monetary policy, changes in fiscal policy take more time due to, for example, lengthy parliamentary procedures. Therefore, once fiscal policy variables have been selected, they cannot be changed so easily before the central bank can react. The government might be able to exploit this commitment to a given policy choice as a first-mover advantage vis-à-vis the central bank.

#### Commitment

With commitment, the reaction function of the central bank is $\pi = 0$. Hence, it does not react to fiscal policy and the policy outcomes are the same as those in the Nash (commitment) case. With commitment, monetary leadership yields positive inflation while Nash and fiscal leadership produce price stability. Zero inflation, however, yields no seigniorage. Hence, under Nash and fiscal leadership, spending needs to be lower while output taxes must be higher than under monetary leadership. If the policymakers’ preferences coincide with society’s, welfare is highest under monetary leadership because the central bank internalizes the government budget constraint and hence accounts for the social value of inflation in generating seigniorage.

#### Discretion

The fiscal authority minimizes (2.3) over $\tau$ and $g$, subject to (2.1), (2.5) and the central bank’s reaction function (4.2).

### Inflation, taxes, and public spending

Assuming that the preferences of the policymakers coincide with those of society (i.e. $\alpha_{GM} = \alpha_{GF} = \alpha_{GS}$ and $\alpha_{GF} = \alpha_{GS}$), we compare the various discretionary equilibria contained in Table 1 and ranked in Table 2. First, we compare fiscal leadership with Nash. If government spending is exogenous (i.e. $\alpha_{gS} \rightarrow \infty$), the fiscal authority must set the tax rate so as to cover the residual
financing requirement (see (4.3)). Hence, it cannot use the tax rate strategically. This implies that the two equilibria coincide.

If government spending is endogenous, fiscal leadership yields higher inflation, taxes and government spending than in the Nash case (as long as $\kappa+d$ is positive). Hence, higher seigniorage is used to raise spending rather than cut taxes. The reason for this spending bias is that the fiscal authority uses higher taxes to induce the central bank to internalize the government budget constraint (see (4.2)). This strategic use of the tax instrument implies a spending bias in that the trade-off between lower taxes (or higher output) and higher spending is distorted in favor of higher spending (while under Nash, $(\bar{g}-g)/(\tau+\bar{x})$ is equal to its socially optimal value $\alpha g S^{-1}$, it is smaller than that value under fiscal leadership, namely $\alpha g S^{-1}(1+\alpha x S)/(1+\alpha x S+\kappa+d)$. The difference between Nash and fiscal leadership increases in $\kappa+d$. This factor represents the inflation motives involving the government budget constraint, namely raising seigniorage and reducing the real burden of servicing nominal debt. These are the motives the central bank fails to internalize.

We can compare fiscal leadership also with monetary leadership (which is equivalent to centralized decision making). Just as under commitment, inflation is lowest under fiscal leadership (see Appendix N.1). Intuitively, under monetary leadership, the central bank takes into account the value of inflation in relaxing the government budget constraint. With fiscal leadership, the fiscal authorities use higher taxes strategically to encourage the independent central bank to do so. However, as noted in the previous paragraph, this behavior distorts the level of public spending. The associated costs induce the fiscal authority to go less far in forcing the central bank to inflate more than in the case of a dependent central bank. Due to the strategic use of the tax instrument, taxes are higher under fiscal than under monetary leadership (see Appendix N.2).

With higher taxes and lower inflation (and thus seigniorage), fiscal leadership may produce both higher and lower public spending than monetary leadership. In particular, public spending is higher under fiscal leadership if $\kappa<1$ and lower if $\kappa>1$ (see Appendix N.3). Intuitively, with small money holdings, lower inflation under fiscal leadership produces only little loss of seigniorage. Hence, the effect of higher output taxes dominates and fiscal leadership results in higher spending.

Table 2 compares the discretionary policy outcomes for the three objectives of policy (i.e. stable prices, employment, and public spending) under the three decentralized games. Due to the failure of the central bank to internalize the government budget constraint, inflation is lowest under Nash. The lack of seigniorage revenues results in the worst spending performance. Monetary leadership, in contrast, yields the highest inflation since the central bank accounts fully for the impact of inflation on the government budget constraint. The associated seigniorage revenues allow for the lowest output taxes. As far as inflation is concerned, fiscal leadership is an intermediate
case as the government budget constraint is internalized only partially through the strategic use of higher taxes by the fiscal authorities. This strategic use implies that output taxes and spending (if $\kappa < 1$) are highest under fiscal leadership. Overall, output and employment are highest under monetary leadership, inflation is lowest under Nash, and (if $\kappa<1$) spending is highest under fiscal leadership.

Welfare

Turning to welfare analysis, we first compare welfare under discretion with that under commitment (assuming that $\alpha_{\pi M} = \alpha_{\pi F} = \alpha_{\pi S}$ and $\alpha_{g F} = \alpha_{g S}$).\(^{15}\) In general, commitment is preferable if (see Appendix H):

\[
\left(2\kappa + \frac{\kappa^2}{\alpha_{S}}\right)\left(1+\alpha_{S} + \kappa + d\right)^2 + \left(\frac{\alpha_{S} (1+\alpha_{S})}{\alpha_{g S}}\right) \left(2\kappa + \frac{\kappa^2}{\alpha_{S}} - 1 - \frac{(d+1)^2}{\alpha_{S}}\right) < 0. \tag{4.10}
\]

In line with the earlier comparison between discretion and commitment for Nash (see (4.5)), commitment is preferred if money balances are small (in particular, if $2\kappa + \kappa^2/\alpha_{S} < 1$) while discretion is more attractive if money balances are large and nominal public debt is small (in particular, if $2\kappa + \kappa^2/\alpha_{S} > 1 + (d+1)(d+1)/\alpha_{S}$).\(^{16}\) Debelle (1993) considers the special case with zero nominal public debt (i.e. $d=0$) and relatively large money holdings (i.e. $\kappa=1$). In that case, discretion is unambiguously better than commitment.

The difficulty in welfare comparisons involving fiscal leadership is that additional welfare losses arise from the government spending bias. Therefore, in comparing fiscal leadership (without commitment) with Nash (without commitment), we explore various special cases. As noted above, both equilibria coincide if government spending is exogenous or if $\kappa=d=0$. If money holdings become very small, the social value of inflation in generating seigniorage vanishes. Hence, Nash, which delivers lower inflation, is unambiguously better (provided that $d>0$, see appendix J). If money holdings are very large, in contrast, fiscal leadership yields highest welfare. Apparently, the welfare loss from the strategic use of the tax instrument is outweighed by gains due to additional seigniorage.

Finally, we compare welfare under fiscal and monetary leadership for various special cases. If $\kappa=d=0$, both equilibria coincide (and coincide with Nash). With zero money holdings and a positive stock of nominal debt, fiscal leadership yields highest welfare (see Appendix K).

\(^{15}\) If government spending is exogenous ($\alpha_{g} \rightarrow \infty$) or if money and nominal debt holdings are zero, fiscal leadership coincides with Nash, so that the welfare comparison is as in discussed earlier in (4.5).

\(^{16}\) There exists a unique $\kappa$ for which (4.10) is met (Appendix I).
Intuitively, under fiscal leadership inflation is lower and, therefore, closer to the socially optimal rate (which is zero if $\kappa=0$). The better performance on price stability dominates the welfare losses due to the spending bias. If money holdings are very large, monetary leadership is unambiguously better (see Appendix L). As noted above, fiscal leadership coincides with Nash if public spending is exogenous. In that case, therefore, (4.6) provides the welfare comparison between fiscal and monetary leadership. Hence, fiscal leadership is best with low money holdings and large stocks of outstanding nominal public debt.

**Optimal preference weights**

To attain the second best under fiscal leadership, society needs to distort the preference weights of not only the monetary but also the fiscal authorities:

$$\alpha_{\pi M} = \alpha_{\pi S}/\kappa, \quad (4.11)$$

$$\alpha_{\pi F} = (\kappa+d+1)\alpha_{\pi S}/\kappa. \quad (4.12)$$

The adjustment in the central bank’s preferences ensures that the incentive to boost employment yields the optimal amount of seigniorage. In addition, the fiscal authority’s inflation weight needs to be reduced in order to eliminate the self-defeating inflationary incentives.

The adjustment of the monetary preferences corresponds to that under Nash (compare (4.11) with (4.7)). Fiscal preferences are adjusted in the same way as under centralized policymaking (or monetary leadership, compare (4.12) with (3.16) and (4.8)). This confirms that fiscal leadership, where the fiscal authorities use the tax instrument strategically to induce the central bank to internalize the government budget constraint, is an intermediate case between Nash, where the central bank does not internalize the government budget constraint at all, and monetary leadership, where the central bank fully internalizes that constraint.

According to (4.11) and (4.12), the fiscal authority should be more conservative than the monetary authority. This may be difficult to achieve in practice. An alternative solution would be to distort the government’s preferences for spending (given $\alpha_{\pi M}=\alpha_{\pi S}$). In particular, the second-best is established if:

$$\alpha_{\pi M} = \alpha_{\pi S}/\kappa, \quad (4.13)$$

$$\alpha_{g F} = \alpha_{g S} \left[ \frac{1+\alpha_{\pi S}/\kappa}{1+\alpha_{\pi S}/\kappa+\kappa+d} \right]. \quad (4.14)$$
Hence, to eliminate the spending bias due to the strategic use of higher taxes, the fiscal authority’s public spending weight needs to be reduced. The required adjustment depends only on $\kappa+d$, representing the public-finance motives for inflation that the central bank neglects.

Institutional constraints may prevent society from distorting the objective function of the fiscal authorities. Appendix M demonstrates that, starting from centralized policymakers, society improves its welfare by delegating monetary policy to an independent policymaker with properly adjusted preferences.

5. Conclusions

This paper has explored the macroeconomic implications of alternative institutions for monetary and fiscal policymaking. It contributed to the literature on central bank independence by stressing the various interactions between an independent central bank and fiscal policy. Within such a setting, we investigated when delegating monetary policy to an independent central bank is optimal. There is no need to decentralize monetary policy if central policymakers can commit or have unlimited access to lump-sum taxation, or if the preference weights of the policymakers can be properly adjusted.

With limited lump-sum taxation, lack of commitment, and fixed policy preferences, decentralizing monetary policy may be called for. The potential benefit of decentralization is that it introduces another distortion, namely the failure of monetary policy to internalize the government budget constraint. By reducing inflation, this distortion offsets another distortion, namely the inflation bias due to the absence of commitment. Decentralization becomes more attractive if the outstanding stock of nominal debt is large (in that case, the associated temptation facing discretionary, centralized policy to reduce the real value of this stock produces a large inflation bias), money holdings are small (in that case, seigniorage and hence the failure of decentralized authorities to internalize the social value of inflation are not important), and society attaches high priority to price stability (in that case, the lower inflation bias under decentralization is valued highly). From a positive point of view, all these factors may have contributed to the recent tendency towards granting more independence to central banks.

Society could deal with the absence of both commitment and unlimited lump-sum taxation by properly distorting policymakers’ preferences away from the true preferences of society.

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17 If the independent central bank is Stackelberg leader in the game with the fiscal authorities, decentralizing policy does not help as the central bank indirectly internalizes the government budget constraint by taking into account the reaction functions of the fiscal player.
Changing preferences may be easier with decentralized policymakers. This makes decentralization even more attractive. Of the various decentralized games we considered, Stackelberg with monetary leadership yields the highest inflation bias while the Nash game produces lowest inflation. Accordingly, the optimal weight the central bank attaches to price-stability is highest under monetary leadership and lowest under Nash. As far as inflation is concerned, fiscal leadership is an intermediate case between monetary leadership and Nash. In this case, not only monetary but also fiscal preferences need to be distorted to achieve the second best.

Our analysis could be extended in a number ways. First, we assumed that real money holdings were a constant fraction of (non-distortionary) output. This seems to be a reasonable approximation for low or modest inflation rates. The arguments in favor of decentralization and commitment would be reinforced if money holdings would depend on the (expected) inflation rate. In that case, seigniorage as a share of government revenues would be lower, the larger the amount of nominal public debt and hence the stronger the incentive for a surprise inflation. Second, the introduction of stochastic shocks seems particularly important because it would allow us to explore the trade-off between credibility and flexibility (see Lohmann (1992)). In particular, distorting policy preferences, while yielding benefits in terms of enhanced credibility of low-inflation policies, would become costly in terms of stabilizing shocks, i.e. flexibility. However, an optimal contract between the government and the central bank may eliminate the trade-off between credibility and flexibility (Persson and Tabellini, 1992, and Walsh, 1993). As a third extension, we could introduce endogenous debt accumulation. In that case, society may find it optimal to change not only the intra- but also the intertemporal trade-offs of policymaking. These extensions are left for further research, however.

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Table 1: Equilibrium policies, welfare losses and optimal weights of the objective functions

<table>
<thead>
<tr>
<th>Finance source:</th>
<th>centralized policy under commitment</th>
<th>centralized policy under discretion ((\alpha_{gF} = \alpha_{gS}))</th>
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</thead>
<tbody>
<tr>
<td>(\kappa\pi)</td>
<td>(\frac{\kappa^2}{\alpha_{gS}}) (\tilde{K})</td>
<td>(\frac{\kappa(\kappa+d+1)}{\alpha_{gF}}) (\tilde{K})</td>
</tr>
<tr>
<td>(\tau + \tilde{x})</td>
<td>(\frac{1}{P}) (\tilde{K})</td>
<td>(\frac{1}{D}) (\tilde{K})</td>
</tr>
<tr>
<td>(\tilde{g} - \tilde{g})</td>
<td>(\frac{\alpha_{gS}^{-1}}{P}) (\tilde{K})</td>
<td>(\frac{\alpha_{gS}^{-1}}{D}) (\tilde{K})</td>
</tr>
<tr>
<td>Society’s welfare loss: (\frac{1}{2} \tilde{K}^2 \ast)</td>
<td>(\frac{1}{P})</td>
<td>(\frac{\alpha_{gS}}{\alpha_{gF}} \left(\frac{\kappa+d+1}{\alpha_{gF}}\right)^\frac{\gamma}{2} + \frac{1}{\alpha_{gS}}) (\frac{D^2}{2})</td>
</tr>
<tr>
<td>Optimal weights</td>
<td>N.A.</td>
<td>(\alpha_{gF} = (\kappa+d+1)\alpha_{gS}/\kappa)</td>
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Table 1: Continued

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<th>Finance source:</th>
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<th>Nash discretion ((\alpha_{gF} = \alpha_{gS}))</th>
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<td>(\kappa\pi)</td>
<td>0</td>
<td>(\frac{\kappa}{\alpha_{gS}}) (\tilde{K})</td>
</tr>
<tr>
<td>(\tau + \tilde{x})</td>
<td>(\frac{1}{1+\alpha_{gS}^{-1}}) (\tilde{K})</td>
<td>(\frac{1}{N_D}) (\tilde{K})</td>
</tr>
<tr>
<td>(\tilde{g} - \tilde{g})</td>
<td>(\frac{\alpha_{gS}^{-1}}{1+\alpha_{gS}^{-1}}) (\tilde{K})</td>
<td>(\frac{\alpha_{gS}^{-1}}{N_D}) (\tilde{K})</td>
</tr>
<tr>
<td>Society’s welfare loss: (\frac{1}{2} \tilde{K}^2 \ast)</td>
<td>(\frac{1}{1+\alpha_{gS}^{-1}})</td>
<td>(\frac{\alpha_{gS}^{-1}}{\alpha_{gS}^2+1+\alpha_{gS}^{-1}}) (\frac{\alpha_{gM}}{N_D^2})</td>
</tr>
<tr>
<td>Optimal weights</td>
<td>N.A.</td>
<td>(\alpha_{gM} = \alpha_{gS}/\kappa)</td>
</tr>
</tbody>
</table>
Table 1: Continued

<table>
<thead>
<tr>
<th>Finance source:</th>
<th>Stackelberg; CB leader; commitment ($\alpha_{gF} = \alpha_{gS}$)</th>
<th>Stackelberg; CB leader; discretion ($\alpha_{gF} = \alpha_{gS}$)</th>
<th>Stackelberg; FA leader; discretion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k\pi$</td>
<td>$\left( \frac{\kappa^2}{\alpha_{gM}} \right) \tilde{K}$</td>
<td>$\left( \frac{\kappa}{\alpha_{nM}} \right) \tilde{K}$</td>
<td></td>
</tr>
<tr>
<td>$\tau + \tilde{x}$</td>
<td>$\left( \frac{1}{M_p} \right) \tilde{K}$</td>
<td>$\left( \frac{1}{M_D} \right) \tilde{K}$</td>
<td>$\left( \frac{1}{F_D} \right) \tilde{K}$</td>
</tr>
<tr>
<td>$\tilde{g} - g$</td>
<td>$\left( \frac{\alpha_{gS}^{-1}}{M_p} \right) \tilde{K}$</td>
<td>$\left( \frac{\alpha_{gS}^{-1}}{M_D} \right) \tilde{K}$</td>
<td>$\left( \frac{\gamma}{\alpha_{gF}} \right) \tilde{K}$</td>
</tr>
<tr>
<td>Society’s welfare loss: $\frac{1}{2} \tilde{K}^2$</td>
<td>$\left( \frac{\alpha_{nS}}{\alpha_{nM}} \right) \frac{\kappa^2}{\alpha_{gS}} \frac{\kappa^2}{\alpha_{gS}} + \frac{\kappa}{\alpha_{gS}}$</td>
<td>$\left( \frac{\alpha_{nS}}{\alpha_{nM}} \right) \frac{\kappa^2}{\alpha_{gS}} \frac{\kappa^2}{\alpha_{gS}} + \frac{\kappa}{\alpha_{gS}}$</td>
<td>$\left( \frac{\alpha_{nS}}{\alpha_{nM}} \right) \frac{\kappa^2}{\alpha_{gS}} \frac{\kappa^2}{\alpha_{gS}} + \frac{\kappa}{\alpha_{gS}}$</td>
</tr>
<tr>
<td>Optimal weights</td>
<td>$\alpha_{nM} = \alpha_{nS}$, $\alpha_{gM} = \alpha_{gS}$</td>
<td>$\alpha_{nM} = (\kappa + d + 1)\alpha_{nS} / \kappa$, $\alpha_{gM} = \alpha_{gS}$</td>
<td>$\alpha_{nM} = \alpha_{nS} / \kappa$ and $\alpha_{nS} = (\kappa + d + 1)\alpha_{gS} / \kappa$ or $\alpha_{nM} = \alpha_{nS} / \kappa$ and $\alpha_{gF} = \alpha_{gS} / (1 + \alpha_{gS} / \kappa + \kappa + d)$</td>
</tr>
</tbody>
</table>

Notes: 1. N.A. = not applicable; CB = central bank; FA = fiscal authority.

2. 

\[
P = 1 + \frac{\kappa^2}{\alpha_{xS}} + \alpha_{gS}^{-1}, \quad D = 1 + \frac{\kappa(\kappa + d + 1)}{\alpha_{xF}} + \alpha_{gS}^{-1}, \quad N_D = 1 + \frac{\kappa}{\alpha_{nM}} + \alpha_{gS}^{-1}, \quad M_p = 1 + \delta \left( \frac{\kappa^2}{\alpha_{nM}} \right) + \alpha_{gS}^{-1},
\]

\[
M_D = 1 + \delta \left( \frac{\kappa(\kappa + d + 1)}{\alpha_{nM}} \right) + \alpha_{gS}^{-1}, \quad F_D = 1 + \frac{\kappa}{\alpha_{gM}} + \frac{\gamma}{\alpha_{gF}}, \quad \delta = \frac{1 + \alpha_{gM} \alpha_{gS}}{1 + \alpha_{gS}}, \quad \gamma = \frac{\alpha_{gS}}{\alpha_{nM} + \kappa + d}.
\]
Table 2: Ranking of policies, output and welfare losses across regimes
(objectives of policymakers and society coincide)

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\kappa \geq 0$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>inflation ($\pi$)</td>
<td>$0 &lt; \kappa &lt; 1$</td>
<td>$NC = FC &lt; MC = C &lt; ND &lt; FD &lt; MD = D$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = 1$</td>
<td>$NC = FC &lt; MC = C = ND &lt; FD &lt; MD = D$</td>
</tr>
<tr>
<td></td>
<td>$\kappa &gt; 1$</td>
<td>$NC = FC &lt; ND &lt; [FD ? MC = C] &lt; MD = D$</td>
</tr>
<tr>
<td>tax rate ($\tau$) = output ($x$)</td>
<td>$0 &lt; \kappa &lt; 1$</td>
<td>$MD = D &lt; ND &lt; MC = C &lt; NC = FC; ND &lt; FD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = 1$</td>
<td>$MD = D &lt; ND = MC = C &lt; [FD ? NC = FC]$</td>
</tr>
<tr>
<td></td>
<td>$\kappa &gt; 1$</td>
<td>$MD = D &lt; MC = C &lt; ND &lt; [FD ? NC = FC]$</td>
</tr>
<tr>
<td>public spending ($g$)</td>
<td>$0 &lt; \kappa &lt; 1$</td>
<td>$NC = FC &lt; MC = C &lt; ND &lt; D = MD &lt; FD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = 1$</td>
<td>$NC = FC &lt; MC = C = ND &lt; D = MD = FD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa &gt; 1$</td>
<td>$NC = FC &lt; ND &lt; [FD ? MC = C] &lt; D = MD$</td>
</tr>
<tr>
<td>welfare loss ($L$)</td>
<td>$\kappa = 0$</td>
<td>$C = MC = NC = FC &lt; ND &lt; D = MD$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \kappa &lt; \kappa^*$</td>
<td>$C = MC &lt; NC = FC &lt; ND &lt; D = MD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = \kappa^*$</td>
<td>$C = MC &lt; NC = FC = ND &lt; D = MD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa^* &lt; \kappa &lt; \kappa^**$</td>
<td>$C = MC \leq ND &lt; NC = FC &lt; D = MD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = \kappa^**$</td>
<td>$C = MC \leq ND &lt; NC = FC = D = MD$</td>
</tr>
<tr>
<td></td>
<td>$\kappa^* &lt; \kappa &lt; \kappa^***$</td>
<td>$C = MC \leq ND &lt; D = MD &lt; NC = FC$</td>
</tr>
<tr>
<td></td>
<td>$\kappa = \kappa^***$</td>
<td>$C = MC &lt; ND = D = MD &lt; NC = FC$</td>
</tr>
<tr>
<td></td>
<td>$\kappa &gt; \kappa^***$</td>
<td>$C = MC &lt; D = MD &lt; ND &lt; NC = FC$</td>
</tr>
</tbody>
</table>

Notes:
1. $C =$ centralized commitment; $D =$ centralized discretion; $NC =$ Nash commitment; $ND =$ Nash discretion; $MC =$ monetary leadership commitment; $MD =$ monetary leadership discretion; $FC =$ fiscal leadership commitment; $FD =$ fiscal leadership discretion.
2. A question mark ? indicates that it is not possible to unambiguously rank the two alternatives.
3. $\kappa^*$, $\kappa^*$ and $\kappa^***$ are the values of $\kappa$ for which (4.1), (4.5) and (4.6), respectively, are met with equality.
Appendix A: Derivation of the government budget constraint

Real money balances in period $t$ are given by $M_t/P_t=\kappa \tilde{X}$, where $\tilde{X}$ is the (constant) output level in absence of any distortions (the antilog of $\tilde{x}$). It follows immediately that $(M_t-M_{t-1})/P_t=(P_t-P_{t-1})/P_t$.

Lump-sum tax revenues are given by $\theta_t P_t \tilde{X}$. Moreover, if distortions are not too large, distortionary tax revenues can be approximated by $\tau_t P_t \tilde{X}$. In nominal terms, the government budget constraint is

$$P_t G_t + (1+r_{1t})P_t B_t + (1+r_{2t})P_t D_t = \tau_t P_t \tilde{X} + (M_t-M_{t-1}) + P_t(B_{t+1}+D_{t+1}) + \theta_t P_t \tilde{X},$$

where $G_t$ is the level of government spending. Furthermore, $B_t$ and $D_t$ are the amounts of indexed and non-indexed single-period public debt, respectively, sold at the end of the previous period against price $P_{t-1}$ and interest rates $r_{1t}$ and $r_{2t}$, respectively. Finally, $(M_t-M_{t-1})$ is the increase in the nominal money supply. Dividing left and right hand side by $P_t \tilde{X}$ gives the government budget constraint in shares of non-distortionary output:

$$g_t + (1+r_{1t})b_t + (1+r_{2t}-\pi_t)d_t = \tau_t + \kappa \pi_t + b_{t+1} + d_{t+1} + \theta_t,$$

where $\pi_t \equiv (P_t-P_{t-1})/P_t$ and where we have approximated $(1+r_{2t})P_{t-1}/P_t$ by $(1+r_{2t}-\pi_t)$. For a risk neutral investor, who has access to an outside investment opportunity against a constant net rate of return $\rho$ (the exogenous ex ante real interest rate), to be willing to invest in government debt, its expected return should at least be equal to $\rho$. Hence, given that the government tries to borrow against the lowest possible costs, $r_{1t}=\rho$ and $r_{2t}=\rho+\pi^e_t$. In other words, the interest rate on nominal debt should compensate investors for any expected inflation during the lifetime of the debt. Hence, the government budget constraint becomes,

$$g + (1+\rho)b + (1+\rho+\pi^e_t-\pi_t)d = \tau + \kappa \pi + \theta,$$

where we have suppressed the time subscript and assumed that all debt is repaid and no new debt is issued.

---

18 Most of the literature, including Alesina and Tabellini (1987) and Debelle (1993), implicitly use the approximation $X \approx \tilde{X}$. 
Appendices B-N:

**B: Welfare comparison Nash commitment versus centralized discretion [eq.(4.1)]**

Assume loss functions monetary and fiscal authorities coincide with society’s. Nash commitment is better than centralized discretion if and only if,

\[
\left(1 + \frac{\kappa (\kappa + d - 1)}{\alpha_{es}} + \frac{1}{\alpha_{gs}}\right)^2 < \left(1 + \frac{1}{\alpha_{gs}}\right) \left(\frac{(\kappa + d + 1)^2}{\alpha_{es}} + 1 + \frac{1}{\alpha_{gs}}\right)
\]

Working out the products (treating \((1 + \alpha_{gs}^{-1})\) as a single term), regrouping and canceling the common terms on the left hand side (LHS) and the right hand side (RHS) gives,

\[
2 \left(\frac{\kappa (\kappa + d + 1)}{\alpha_{es}}\right) \left(1 + \frac{1}{\alpha_{gs}}\right) \left(\frac{\kappa^2 (\kappa + d + 1)^2}{\alpha_{es}^2}\right) < \left(\frac{(\kappa + d + 1)^2}{\alpha_{es}}\right) \left(1 + \frac{1}{\alpha_{gs}}\right)
\]

Multiplying left and RHS by \(\left(\frac{(\kappa + d + 1)^2}{\alpha_{es}}\right)^{-1}\) \(\left(\frac{1}{1 + \alpha_{gs}}\right)^{-1}\) gives (4.1).

**C: Uniqueness of \(\kappa^*\), the value of \(\kappa\) for which (4.1) is met with equality**

Differentiate (4.1) with respect to \(\kappa\), to yield:

\[
\frac{2(d+1)}{(\kappa + d + 1)^2} + \frac{2\kappa}{\alpha_{es}(1 + \alpha_{gs}^{-1})} > 0.
\]

Combine this with the facts that for \(\kappa = 0\) the LHS of (4.1) is less than one, that for \(\kappa = (\alpha_{es}(1 + \alpha_{gs}^{-1}))\) the LHS of (4.1) is larger than one, and that the LHS of (4.1) is continuous, to conclude that \(\kappa^*\) is unique.
Assume that the loss functions of the monetary and fiscal authorities coincide with society’s loss function. Nash commitment is better than Nash discretion if and only if,

\[
\left(1 + \frac{\kappa}{\alpha_{xS}} + \frac{1}{\alpha_{gS}}\right)^2 < \left(1 + \frac{1}{\alpha_{xS}}\right) \left(1 + \frac{1}{\alpha_{gS}}\right).
\]

Working out the products (treating \((1+\alpha_{gS}^{-1})\) as single term), canceling common terms and multiplying left and RHS by \(\alpha_{xS}(1+\alpha_{gS}^{-1})^{-1}\) gives (4.5).

**E: Uniqueness of \(\kappa^*\), the value of \(\kappa\) for which (4.5) is met with equality**

Differentiate (4.5) with respect to \(\kappa\), to yield:

\[
2 + \frac{2\kappa}{\alpha_{xS}(1+\alpha_{gS}^{-1})} > 0.
\]

Combine this with the facts that for \(\kappa=0\) the LHS of (4.5) is less than one, that for \(\kappa=\frac{1}{2}\) the LHS of (4.5) is larger than one, and that the LHS of (4.5) is continuous, to conclude that \(\kappa^*\) is unique.

**F: Welfare comparison Nash discretion versus centralized discretion [eq.(4.6)]**

Assume that the loss functions of the monetary and fiscal authorities coincide with society’s loss function. Nash discretion is better than centralized discretion if and only if,

\[
\left(1 + \frac{\kappa}{\alpha_{xS}} + \frac{1}{\alpha_{gS}}\right)^2 \left(1 + \frac{\kappa(\kappa+d+1)}{\alpha_{xS}} + \frac{1}{\alpha_{gS}}\right)^2 < \left(1 + \frac{\kappa}{\alpha_{xS}}\right) \left(1 + \frac{1}{\alpha_{gS}}\right) \left(1 + \frac{\kappa(\kappa+d+1)}{\alpha_{xS}} + \frac{1}{\alpha_{gS}}\right) \left(1 + \frac{\kappa(\kappa+d+1)}{\alpha_{xS}}\right).
\]

Working out products (treating \((1+\alpha_{gS}^{-1})\) as a single term), canceling common terms and multiplying by \(\alpha_{xS}(1+\alpha_{gS}^{-1})^{-2}\), yields,

\[
1 + 2\kappa(\kappa+d+1) + 2 \left(\frac{\kappa(\kappa+d+1)}{\alpha_{xS}(1+\alpha_{gS}^{-1})}\right)^2 < (\kappa+d+1)^2 + 2\kappa + 2 \left(\frac{\kappa(\kappa+d+1)}{\alpha_{xS}(1+\alpha_{gS}^{-1})}\right)^2 + \kappa^2.
\]

Bringing the third and fourth term on the RHS to the LHS and the first and second term on the LHS to the RHS yields,
\[
\left( \frac{\kappa}{\alpha_{es}(1+\alpha_{eS}^{-1})} \right)^2 (\kappa+d+1) + \kappa(\kappa+d+1)^2 - 2(\kappa+d+1)^2 - \kappa < (\kappa+d+1)^2 + 2\kappa - 2\kappa(\kappa+d+1) - 1.
\]

Working out LHS and RHS, rearranging and dividing by (κ+d) yields:

\[
\left( \frac{\kappa}{\alpha_{es}(1+\alpha_{eS}^{-1})} \right)^2 - 2(\kappa+d+1) - \kappa(\kappa+d+2) < (\kappa+d+2) - 2\kappa.
\]

Bringing the final term on the RHS to the LHS and dividing by (κ+d+2) yields (4.6).

\[G: \text{Uniqueness of } \kappa^{***}, \text{ the value of } \kappa \text{ for which (4.6) is met with equality}\]

Rewrite

\[
2 \left( \frac{\kappa}{\kappa+d+2} \right) + \frac{\kappa}{\beta} \left( \kappa - 2 \left( \frac{\kappa+d+1}{\kappa+d+2} \right) \right) = 1, \text{ where } \beta = \alpha_{es}(1+\alpha_{eS}^{-1}),
\]

to yield,

\[f(\kappa) = \kappa^3 + d\kappa^2 + (\beta - 2(d+1))\kappa - \beta(d+2) = 0, \quad (G.1)\]

which is a third order polynomial equation in \(\kappa\). We will show that this equation has exactly one solution for \(\kappa\) which is non-negative.

Note that \(f(0)<0\), that \(f''(\kappa)=6\kappa+2d>0\) if \(\kappa\geq0\) and \(d>0\), and that \(f(\kappa)\) and its derivatives are continuous. Moreover, denote the solutions of \(f'(\kappa)=3\kappa^2 + 2d\kappa + (\beta - 2(d+1))=0\) by \(\kappa_1 \leq \kappa_2\) if such solutions indeed exist, in which case at least one of them is non-positive. We now have the following possibilities:

1. \(f'(\kappa)=0\) has no positive solution, hence there is no extremum for \(\kappa>0\). Combined with the facts that \(f(0)<0\) and that \(f(\kappa)\to\infty\) if \(\kappa\to\infty\) (because \(\kappa^3\) dominates), this implies a unique solution to equation (G.1).

2. \(\kappa_1 \leq 0 < \kappa_2\). Hence, at most one extremum exists for \(\kappa>0\). Combined with the facts that \(f(0)<0\) and that \(f(\kappa)\to\infty\) if \(\kappa\to\infty\) (because \(\kappa^3\) dominates), this implies that there is no extremum for \(\kappa>0\) (though there may a point of inflexion) and that equation (G.1) has unique solution.
Assume that the loss functions of the monetary and fiscal authorities coincide with society’s loss function. Commitment under fiscal leadership is better than discretion if and only if,

\[
\left(1 + \frac{\kappa}{\alpha_{gS}} + \frac{\gamma}{\alpha_{gS}}\right) < \left(1 + \frac{1}{\alpha_{gS}} + \frac{1 + \frac{\gamma}{\alpha_{gS}}}{\alpha_{gS}}\right).
\]

Work out products on both sides, subtract \(1+\gamma/\alpha_{gS}^2\) from both sides, multiply both sides by \(\alpha_{ns}\alpha_{gS}\) and bring all terms on the RHS to the LHS and rearrange, to yield,

\[
2\kappa\alpha_{gS} + \frac{\alpha_{gS}}{\alpha_{ns}} \kappa^2 - (1+\alpha_{gS}+\alpha_{ns})^2 + 2\gamma(\alpha_{xs}+\kappa) - \alpha_{xs} \gamma^2 < 0.
\]

Substitute the expression for \(\gamma\), multiply both sides by \((1+\alpha_{ns}+\kappa+d)^2\) and rearrange to yield,

\[
2\kappa\alpha_{gS} + \frac{\alpha_{gS}}{\alpha_{ns}} \kappa^2 - (1+\alpha_{gS}+\alpha_{ns})^2 + (1+\alpha_{gS}+\kappa+d)[2(1+\kappa)(\alpha_{gS}+\kappa) - (1+\alpha_{gS}+\alpha_{gs})(1+\alpha_{ns}+\kappa+d)]
\]

\[-\alpha_{ns}(1+\alpha_{ns})^2 < 0.
\]

Working out the term in square brackets, this is equivalent to

\[
2\kappa\alpha_{gS} + \frac{\alpha_{gS}}{\alpha_{ns}} \kappa^2 - (1+\alpha_{gS}+\kappa+d)^2 + (1+\alpha_{gS}+\kappa+d)(1+\alpha_{ns}+\kappa+d)(1+\alpha_{gs}+\kappa) - (1+\alpha_{gS}+\alpha_{ns})(1+\alpha_{gs}+\kappa+d)]
\]

\[-\alpha_{gs}(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d) < 0 \Leftrightarrow
\]

\[
2\kappa\alpha_{gS} + \frac{\alpha_{gS}}{\alpha_{ns}} \kappa^2 - (1+\alpha_{gS}+\kappa+d)^2 + (1+\alpha_{gS}+\kappa+d)(1+\alpha_{ns}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)(1+\alpha_{gs}+\kappa+d)] < 0.
\]
Rewriting the term between square brackets and dividing both sides by $\alpha_g S$ yields,

$$
\begin{aligned}
2\kappa \left( \frac{1}{\alpha_{xs}} \right) \kappa^2 - 1 & \left( 1 + \alpha_{xs} + \kappa + d \right)^2 + \left( \frac{(1+\alpha_{xs})}{\alpha_g S} \right) 2 \alpha_{xs} \kappa + \kappa^2 - \alpha_{xs} - (d+1)^2 < 0 \iff \\
2\kappa \left( \frac{1}{\alpha_{xs}} \right) \kappa^2 - 1 & \left( 1 + \alpha_{xs} + \kappa + d \right)^2 + \left( \frac{\alpha_{xs}(1+\alpha_{xs})}{\alpha_g S} \right) \left( \frac{2\kappa + \kappa^2 - 1 -(d+1)^2}{\alpha_{xs}} \right) < 0,
\end{aligned}
$$

which is (4.10).

**I: Proof that (4.10) is met with equality for a unique value of $\kappa$**

First note that, if $2\kappa + \kappa^2 / \alpha_{xs} = 1$, the LHS of (4.10) is negative, that, if $2\kappa + \kappa^2 / \alpha_{xs} = 1 + (d+1)^2 / \alpha_{xs}$, the LHS of (4.10) is positive, and that the LHS of (4.10) is continuous. Hence there exists at least one value for $\kappa$ for which (4.10) is met with equality.

Now differentiate the LHS of (4.10) with respect to $\kappa$, to yield,

$$
\begin{aligned}
2 + \frac{2\kappa}{\alpha_{xs}} & \left( 1 + \alpha_{xs} + \kappa + d \right)^2 + \left( \frac{2\kappa + \kappa^2}{\alpha_{xs}} - 1 \right) 2 \left( 1 + \alpha_{xs} + \kappa + d \right) + \left( \frac{2\kappa + \kappa^2}{\alpha_{xs}} - 1 \right) 2 \left( \frac{\alpha_{xs}(1+\alpha_{xs})}{\alpha_g S} \right) > \\
(1 + \alpha_{xs} + \kappa + d) & \left( 1 + \alpha_{xs} + \kappa + d \right) 2 + \left( \frac{2\kappa}{\alpha_{xs}} \right) + 2 \left( \frac{2\kappa + \kappa^2}{\alpha_{xs}} - 1 \right) > 0,
\end{aligned}
$$

which completes the proof.

**J: Proof that under discretion welfare loss Nash lower than with fiscal leadership if $\kappa=0$ and $d>0$**

Assume that the loss functions of the monetary and fiscal authorities coincide with society’s loss function. Use $\kappa=0$. Nash discretion is better than fiscal discretion if and only if,

$$
\left( \frac{1}{\alpha_{xs}} + \frac{1}{\alpha_{gs}} \right)^2 + \left( \frac{1}{\alpha_{gs}} + \frac{1}{\alpha_{gs}} \right) < \left( \frac{1}{\alpha_{xs}} + \frac{1}{\alpha_{gs}} \right)^2 + \left( \frac{1}{\alpha_{xs}} + \frac{1}{\alpha_{gs}} \right).
$$

Working out products and skipping common terms on both sides, yields,

$$
\frac{2\gamma}{\alpha_g S} + \gamma^2 + \frac{2\gamma}{\alpha_{gs}} + \frac{\gamma^2}{\alpha_{gs}} + \frac{2\gamma}{\alpha_{gs}} < \frac{\gamma^2}{\alpha_{gs}} + \frac{2\gamma}{\alpha_{gs}} + \frac{1}{\alpha_{gs}} + \frac{\gamma^2}{\alpha_{gs}} + \frac{1}{\alpha_{gs}} + \frac{1}{\alpha_{gs}}.
$$
Multiply both sides by $\alpha_{\text{gs}}\alpha_{gS}^2$ and rearrange to yield,

$$2\gamma(\alpha_{\text{gs}}\alpha_{s}+\alpha_{gS}+\alpha_{gs})+\gamma^2(1-(\alpha_{\text{gs}}\alpha_{s}+\alpha_{gs})) < \alpha_{\text{gs}}\alpha_{s}+2\alpha_{gS}+1+\alpha_{gs}.$$  

Note that if $d=0$, $\gamma=1$ (because $\kappa=0$, by assumption), and, hence, the LHS equals the RHS. Because $\gamma$ decreases with an increase in $d$, the LHS is decreasing in $d$ and, hence, LHS<RHS if $d>0$, which completes the proof.

**K: Proof that under discretion welfare loss lower under fiscal than under monetary leadership if $\kappa=0$ and $d>0$**

Assume loss functions monetary and fiscal authorities coincide with society’s. Use $\kappa=0$. Under discretion, fiscal leadership is better than monetary leadership if and only if,

$$\left\{\frac{1}{\alpha_{gs}}+\frac{\gamma^2}{\alpha_{gS}}\right\}^{2}\left[1+\frac{1}{\alpha_{gS}}\right] < \left\{\frac{1}{\alpha_{gs}}\right\}^{2}\left[\left(\frac{d}{\alpha_{gs}}\right)^2 + \frac{1}{\alpha_{gS}}\right].$$

Work out products, cancel common terms on both sides, multiply both sides by $\alpha_{gs}\alpha_{gS}^2$ and bring the terms on the RHS to the LHS, to yield,

$$\gamma^2[\alpha_{gs}\alpha_{gS}+\alpha_{gs}-(d+1)^2]-2\gamma[\alpha_{gs}\alpha_{gS}+\alpha_{gs}+\alpha_{gs}(d+1)^2]+$$

$$[\alpha_{gs}\alpha_{gS}+\alpha_{gs}+2\alpha_{gS}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2] < 0.$$  

Substitute the expression for $\gamma$ (using $\kappa=0$) and multiply both sides by $(1+\alpha_{gs}+d)^2$ to yield,

$$(1+\alpha_{gs})^2[\alpha_{gs}\alpha_{gS}+\alpha_{gs}-(d+1)^2]-2(1+\alpha_{gs})(1+\alpha_{gs}+d)[\alpha_{gs}\alpha_{gS}+\alpha_{gs}+\alpha_{gs}(d+1)^2]+$$

$$(1+\alpha_{gs})^2[\alpha_{gs}\alpha_{gS}+\alpha_{gs}+2\alpha_{gS}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2] < 0 \Leftrightarrow$$

$$(1+\alpha_{gs})^2[\alpha_{gs}\alpha_{gS}+\alpha_{gs}-(d+1)^2]-2(1+\alpha_{gs})d+2(1+\alpha_{gs})^2][\alpha_{gs}\alpha_{gS}+\alpha_{gs}+\alpha_{gs}(d+1)^2]+$$

$$[(1+\alpha_{gs})^2+2(1+\alpha_{gs})d+d^2][\alpha_{gs}\alpha_{gS}+\alpha_{gs}+2\alpha_{gS}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2] < 0.$$  

A number of terms cancel, so that,

$$-(1+\alpha_{gs})^2(d+1)^2-2(1+\alpha_{gs})d[\alpha_{gs}(d+1)^2]-2(1+\alpha_{gs})^2\alpha_{gs}(d+1)^2+(1+\alpha_{gs})^2[2\alpha_{gs}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2]$$

$$2(1+\alpha_{gs})d[2\alpha_{gs}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2]+d^2[\alpha_{gs}\alpha_{gS}+\alpha_{gs}+2\alpha_{gS}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2]<0$$

$$(1+\alpha_{gs})^2[2\alpha_{gs}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2]-(1+\alpha_{gs})^2(d+1)^2-2(1+\alpha_{gs})^2\alpha_{gs}(d+1)^2+$$

$$2(1+\alpha_{gs})d[2\alpha_{gs}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2]+d^2[\alpha_{gs}\alpha_{gS}+\alpha_{gs}+2\alpha_{gS}+1+\alpha_{gs}^2-\alpha_{gs}(d+1)^2] < 0.$$
If \( d=0 \), the LHS is zero. Therefore, let us evaluate the derivative of the LHS with respect to \( d \), which can be rewritten as:

\[
-2(1+\alpha_{ns})\alpha_{xs}(2\alpha_{gs}+1+\alpha_{gs}^2)+2d(1+\alpha_{gs})(-\alpha_{ns}-\alpha_{xs}^2-2\alpha_{ns}\alpha_{gs}-\alpha_{gs}\alpha_{xs})-
\]

\[
2(1+\alpha_{ns})\alpha_{gs}(1+\alpha_{gs})(d+1)^2-4(1+\alpha_{ns})\alpha_{gs}(1+\alpha_{gs})d(d+1)-2\alpha_{gs}^2d(d+1)^2-2\alpha_{gs}d^2(d+1)<0.
\]

Hence, if \( \kappa=0 \), then, irrespective of the value of \( d>0 \), fiscal leadership discretion is better than monetary leadership discretion.

**L: Proof that under discretion monetary leadership better than fiscal leadership if \( \kappa \to \infty \)**

Assume that the loss functions of the monetary and fiscal authorities coincide with society’s loss function. Under discretion, the ratio of welfare losses under fiscal and monetary leadership can be written as,

\[
\left(\frac{1+\gamma^2}{\alpha_{xs}}\right)^{1+\gamma} \left(\frac{1+(d+1)}{\kappa}\right)^{\frac{1}{\alpha_{gs}}} \left(\frac{1+\alpha_{gs}^{-1}}{\alpha_{xs}}\right) > 1,
\]

If \( \kappa \to \infty \), this converges to,

\[
\left(\frac{1+\gamma^2}{\alpha_{xs}}\right)^{1+\gamma} > 1,
\]

which completes the proof.

**M: Proof that in general there exists a weight \( \alpha_{sM} \) such that fiscal leadership discretion improves upon centralized discretion, when it is not possible to distort the fiscal authority’s preferences**

Assume that the preferences of the fiscal authority coincide with society’s preferences and that \( \alpha_{sM}=\alpha_{gs} \). Note that if \( \alpha_{sM}=\alpha_{gs}(1+\kappa+d) \), welfare losses under fiscal leadership discretion coincide with welfare losses under centralized discretion (because \( \gamma=1 \) for this value of \( \alpha_{sM} \)). Hence, if the
derivative of the expression for welfare losses under fiscal leadership discretion with respect to $\alpha_{\pi M}$ is not equal to zero, when evaluated at $\alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d)$, there exists an $\alpha_{\pi M}$ for which fiscal leadership improves upon centralization.

First, note that, when evaluated at $\alpha_{\pi M} = \alpha_{\pi S}/(1+\kappa+d)$, $
\frac{\partial \gamma}{\partial \alpha_{\pi M}} = \frac{-(1+\kappa+d)\left(1 + \frac{(1+\kappa+d)^2}{\alpha_{\pi S}}\right)}{(1+\kappa+d) + \left(\frac{\alpha_{\pi S}}{1+\kappa+d}\right)^2}
$

The derivative of welfare losses under fiscal leadership discretion with respect to $\alpha_{\pi M}$ and evaluated at $\alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d)$ is not equal to zero if and only if,

\[
\left(\frac{2\alpha_{\pi S} + 2\gamma}{\alpha_{\pi M}}\right)^3 \frac{\partial \gamma}{\partial \alpha_{\pi M}} \left(1 + \frac{\kappa}{\alpha_{\pi M}} + \frac{\gamma}{\alpha_{\pi S}}\right)^2 - 2\left(\frac{\kappa}{\alpha_{\pi M}} + \frac{1}{\alpha_{\pi S}}\right) \frac{\partial \gamma}{\partial \alpha_{\pi M}} \left(1 + \frac{\gamma^2}{\alpha_{\pi S}}\right) \neq 0,
\]

evaluated at $\alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d)$.

Dividing by 2 and using that $\gamma=1$ if $\alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d)$, this is equivalent to,

\[
\left(\frac{\alpha_{\pi S}}{\alpha_{\pi M}}\right)^3 \frac{\partial \gamma}{\partial \alpha_{\pi M}} \left(1 + \frac{\kappa}{\alpha_{\pi M}} + \frac{1}{\alpha_{\pi S}}\right)^2 - \left(\frac{\kappa}{\alpha_{\pi M}} - \frac{1}{\alpha_{\pi S}}\right) \frac{\partial \gamma}{\partial \alpha_{\pi M}} \left(1 + \frac{1}{\alpha_{\pi S}}\right) \neq 0,
\]

evaluated at $\alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d)$.

Working out the products, removing terms which cancel and multiplying by $\alpha_{\pi S}^3\alpha_{\pi M}^3$ this can be rewritten as,

\[
-(\kappa \alpha_{\pi S} - \alpha_{\pi S} + \alpha_{\pi M} \alpha_{\pi S} \gamma + \kappa \alpha_{\pi M} \alpha_{\pi S}) \left(\frac{\partial \gamma}{\partial \alpha_{\pi M}}\right) \neq 0, \text{ evaluated at } \alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d) \Leftrightarrow \n\left(\kappa \alpha_{\pi M} - \alpha_{\pi S}\right) \left(1 + \alpha_{\pi S} + \alpha_{\pi M} \right) \frac{\partial \gamma}{\partial \alpha_{\pi M}} \neq 0, \text{ evaluated at } \alpha_{\pi M}=\alpha_{\pi S}/(1+\kappa+d).
\]

Note that the first term on the LHS is negative, while the second term in general depends on d. Hence, in general, fiscal leadership discretion can improve upon centralized discretion.
N: Proofs of inequalities in Table 2

N.1: Inflation ($\kappa>0$)

$\pi^C <(=)(>)\pi^ND$ $\Leftrightarrow$ $\kappa <(=)(>)1$: The (in)equality $\pi^C <(=)(>)\pi^ND$ can almost immediately be reduced to $(\kappa-1)(1+\alpha_{gS}^{-1}) <(=)(>)0$, which completes the proof.

$\pi^ND <\pi^FD$: Comparison of both expressions shows that this is fulfilled if and only if $\gamma<1$, which is always the case if policymakers' preferences coincide with society's.

$\pi^FD <\pi^MD$: This is equivalent to

$$
(\kappa+d+1) \left(1 + \frac{\kappa}{\alpha_{xs}} + \frac{\gamma}{\alpha_{gS}}\right) > \frac{\kappa(\kappa+d+1)}{\alpha_{xs}} + \alpha_{gS}^{-1} \Leftrightarrow
$$

$$
(\kappa+d+1) \left(1 + \frac{\gamma}{\alpha_{gS}}\right) > 1 + \alpha_{gS}^{-1} \Leftrightarrow
$$

$$
(\kappa+d) + (\kappa+d+1) \left(\frac{\gamma}{\alpha_{gS}}\right) > \alpha_{gS}^{-1} \Leftrightarrow
$$

$$
(\kappa+d)(1+\alpha_{xs}+\kappa d)\alpha_{gS} + (\kappa+d+1)(1+\alpha_{xs}) > 1+\alpha_{xs} + \kappa + d \Leftrightarrow
$$

$$
(\kappa+d)(1+\alpha_{xs} + \kappa d)\alpha_{gS} + \alpha_{xs} > 0,
$$

which completes the proof.

$\pi^C <\pi^{MD}$: This is equivalent to

$$
\kappa \left(1 + \frac{\kappa(\kappa+d+1)}{\alpha_{xs}} + \alpha_{gS}^{-1}\right) < (\kappa+d+1) \left(1 + \frac{\kappa^2}{\alpha_{xs}} + \alpha_{gS}^{-1}\right) \Leftrightarrow
$$

$$
\kappa(1+\alpha_{gS}^{-1}) < (\kappa+d+1)(1+\alpha_{gS}^{-1}) \Leftrightarrow
$$

$$
0 < (d+1)(1+\alpha_{gS}^{-1}),
$$

which completes the proof.
\( \pi^C < \pi^{FD}, \text{ if } \kappa \leq 1: \) One has,
\[
\pi^C < \pi^{FD} \iff \\
\kappa(1 + \gamma \alpha_{s \delta}^{-1}) < (1 + \alpha_{s \delta}^{-1}) \iff \\
\kappa\left(1 + \alpha_{s \delta}^{-1}(1 + \alpha_{s \delta} + \kappa + d)ight) < (1 + \alpha_{s \delta}^{-1})(1 + \alpha_{s \delta} + \kappa + d) \iff \\
(\kappa - 1 - \alpha_{s \delta}^{-1})(1 + \alpha_{s \delta} + \kappa + d) + \kappa \alpha_{s \delta}^{-1}(1 + \alpha_{s \delta}) < 0 \iff \\
(\kappa - 1)(1 + \alpha_{s \delta} + \kappa + d) - \alpha_{s \delta}^{-1}(1 + \alpha_{s \delta} + \kappa + d) + \kappa \alpha_{s \delta}^{-1}(1 + \alpha_{s \delta}) < 0 \iff \\
(\kappa - 1)(1 + \alpha_{s \delta} + \kappa + d) + \alpha_{s \delta}^{-1}(\kappa - 1)(1 + \alpha_{s \delta}) < \alpha_{s \delta}^{-1}(\kappa + d) \iff \\
(\kappa - 1)(1 + \alpha_{s \delta} + \kappa + d) + (\kappa + d) < \alpha_{s \delta}^{-1}(\kappa + d),
\]
which is fulfilled if \( \kappa \leq 1. \)

\( \text{N.2: Taxation (} \kappa > 0) \)

\( \tau^{MD} < \tau^{ND}: \) This follows immediately from the fact that \( \kappa(\kappa + d + 1)/\alpha_{s \delta} > \kappa/\alpha_{s \delta}. \)

\( \tau^C < \tau^{NC}: \) This follows immediately from the fact that \( \kappa^2/\alpha_{s \delta} > 0. \)

\( \tau^{ND} < (\approx)(>) \tau^C \iff \kappa < (\approx)(>) 1: \) This follows from the fact that \( \kappa^2/\alpha_{s \delta} < (\approx)(>) \kappa/\alpha_{s \delta} \iff \kappa < (\approx)(>) 1. \)

\( \tau^{ND} < \tau^{FD}: \) This follows immediately from the fact that \( \gamma < 1 \) if policymakers’ preferences coincide with society’s preferences.

\( \tau^{ND} < \tau^{NC}: \) This follows immediately from the fact that \( \kappa/\alpha_{s \delta} > 0. \)

\( \tau^C < \tau^{FD} \text{ if } \kappa \geq 1: \) We have that \( \tau^C < \tau^{FD} \iff \)
\[
1 + \frac{\kappa^2}{\alpha_{s \delta}} > \alpha_{s \delta}^{-1} \iff 1 + \frac{\kappa}{\alpha_{s \delta}} + \frac{\gamma}{\alpha_{s \delta}} \iff \\
\kappa(\kappa - 1)\alpha_{s \delta} + \left( \frac{\kappa + d}{1 + \alpha_{s \delta} + \kappa + d} \right) \alpha_{s \delta} > 0
\]
which is fulfilled for \( \kappa \geq 1. \)
Most of the rankings follow immediately from the rankings derived for tax policy. Only some of the rankings which involve fiscal leadership with discretion have to be established.

\[ g^{FD} < (=) (\geq) g^{MD} \iff \kappa > (=) (\leq) 1 : \]

We have that \( g^{FD} < (=) (\geq) g^{MD} \iff \]

\[
1 + \frac{\kappa}{\alpha_{x5}} < (\geq) \gamma \cdot \left( 1 + \frac{\kappa(\kappa + d + 1)}{\alpha_{x5}} \right)
\]

\[
(1 + \alpha_{x5}) \left( \alpha_{x5} + \kappa(\kappa + d + 1) \right) > (\geq) \gamma \cdot \left( (\alpha_{x5} + \kappa)(1 + \alpha_{x5} + \kappa + 1) \right)
\]

\[
\kappa(\kappa + d + 1) + \alpha_{x5}(\kappa(\kappa + d) + \kappa(1 + \alpha_{x5} + \kappa + d)) \iff
\]

\[
\alpha_{x5}(\kappa - 1)(\kappa + d) > (\geq) 0 \iff \kappa > (\geq) 0 ,
\]

which completes the proof.

\[ g^{ND} < g^{FD} : \]

We have that \( g^{ND} < g^{FD} \iff \]

\[
1 + \frac{\kappa}{\alpha_{x5}} > \gamma \cdot \left( 1 + \frac{\kappa}{\alpha_{x5}} \right)
\]

which is always fulfilled if policymakers’ preferences coincide with society’s.

\[ N.4: \text{Welfare} \]

Welfare rankings can be found immediately by combining equations (4.1), (4.5) and (4.6) for the various values of \( \kappa \).