A Comment on Holmlund & Linden's "Job Matching, Temporary Public Employment, and Unemployment"

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by

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Abstract

Holmlund & Lindén (1993) analyse the usage of relief employment using a matching model with three stocks – regular employment, relief employment, and unemployment. In their model, the usage of relief jobs has both a direct effect on unemployment (i.e., the placement effect) as well as an indirect wage effect on unemployment. The former relationship is negative. The latter effect refers to the change in wage setting which is in response to changes in how relief jobs are used. Holmlund & Lindén argue that the only usage of relief jobs which unequivocally reduces unemployment is the policy of directing relief jobs at the flow *out* of regular employment. They show that by using relief jobs in this way, the negative direct effect on unemployment is reinforced by a negative indirect wage effect on unemployment. In this paper, we argue that Holmlund & Lindén's concept of workers' fall-back position is mis-specified since it fails to take account of the wage bargaining undertaken by those in relief employment. And it is this mis-specification which drives their result. Once due consideration is made for those in relief employment, their result no longer holds.

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1 Introduction

In the post-war period, unemployment has been an important issue for many countries. With the sharp rises in unemployment in the 1980s, and its persistence, this issue has become even more central in political discussion. Why is it that some countries experience obscene levels of unemployment whilst, at the same time, other countries seem to avoid the adverse effects of unemployment?

One country which has been the subject of much debate and analysis is Sweden. Why was it that in the 1980s, when much of Europe was experiencing mass unemployment, Sweden maintained an unemployment rate[1] which peaked at a mere 3.5% in 1983? The UK, for example, had an unemployment rate which peaked in the same year at a staggering 12.4%! What was the cause of these huge differences? Could lessons be learnt from the Swedish experience? If so, how could policy be changed to alleviate the problem of unemployment?

Whilst there are many differences between the Swedish and British labour markets (see Miller(1991) for a more-detailed discussion), one difference which was often focused on was Sweden's extensive usage of relief jobs. Was Sweden's low unemployment record due to her unremitting application of relief jobs to the problem of unemployment? Or were other forces in motion?

Holmlund & Lindén (H-L) provide a framework which attempts to answer the question of whether the usage of relief jobs is guaranteed to reduce unemployment, or whether their usage induces

[1] The unemployment rates referred to here are standardised unemployment rates, as used by the OECD.

indirect mechanisms into play which have deleterious effects regarding unemployment. Central to their analysis, they argue – quite convincingly – that the usage of relief jobs has both a directeffect on unemployment (by taking someone from unemployment into a relief job; or by preventing a worker who has lost their job from becoming unemployed), as well as an indirect wage effect on unemployment. The latter effect refers to the likelihood that by introducing relief jobs into the economy, the wage-setting mechanism is likely to be disturbed. And this disturbance in wage setting affects unemployment. If this indirect effect on unemployment is positive, and stronger than the direct effect of relief-job usage, then we would see a perverse situation where the usage of relief jobs actually increases unemployment!

When we talk about relief jobs we refer to temporary jobs which the government creates to alleviate unemployment. The idea of lowering the unemployment level by placing unemployed workers in temporary public employment is not a new idea. Indeed, in the late 1920s, politicians such as Lloyd George and Mosley , and economists such as Keynes, were already propagating such ideas. Temporary public employment programmes have been used in various types of political systems. Dictatorial countries such as Hitler's Germany and Mussolini's Italy have used them, as well as quasi-democratic countries such as Wilson's America. However, the country most noted for using relief jobs has been Sweden in the post-war epoch.

In H-L's model, relief jobs can be used in two different ways. Firstly, relief jobs can be directed at those already in unemployment; and, secondly, relief jobs can be given to those workers flowing from regular employment, i.e. workers being laid-off from regular jobs. In their model, the usage of relief jobs has both a direct effect on unemployment (i.e. the placement effect) as well as an indirect wage effect on unemployment. The former relationship is negative. The latter effect refers to the change in wage setting which is in response to changes in how relief jobs are used. Wage setting is assumed to be the result of a Nash bargain between individual workers and firms. Since the usage of relief jobs affects the average difference in value between regular employment and the worker's outside option, the wage will be affected. If this effect is positive, then the indirect wage effect on unemployment will be positive, and vice versa.

According to H-L, the only usage of relief jobs which has both a negative direct effect on unemployment *and* a negative indirect wage effect on unemployment is the policy of directing relief jobs at the outflow from regular employment. Thus the policy of directing relief jobs at those flowing out of regular employment has an unequivocally negative effect on unemployment. The policy of directing relief jobs at the unemployed, on the other hand, has a negative direct effect on unemployment but a positive indirect effect on unemployment due to reducing the average difference in value between regular employment and unemployment. Thus this use of relief jobs encourages more-aggressive wage bargaining by workers. However, the specification of the average worker's fall-back position which H-L use is incorrect[2] since it fails to take account of those in relief jobs - workers who are also involved in wage bargaining. We show that once due consideration is made for those in relief employment, the policy of directing relief jobs at the flow out of regular employment also has an ambiguous wage effect on unemployment and thus an ambiguous effect on unemployment.

[2] Their concept of the average fall-back position of workers is a special case of the measure of average fall-back position which we propose in this paper. Their concept is correct only if either relief workers do not search and/or if there are no workers in relief employment.

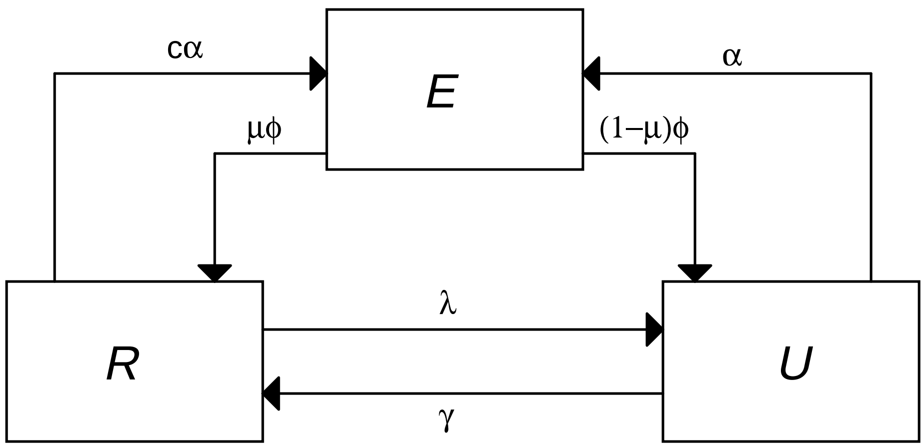
2 The model

H-L's model is a matching model where the search process is summarised by an aggregate matching function $H = h(S,V)$, where S is the number of searchers and V is the number of vacancies. H is increasing in both its arguments and exhibits constant returns to scale. The number of searchers is given as the number of unemployed plus the effective number of searchers in relief jobs, i.e. $S \equiv U + cR$, where U and R are the number of unemployed and the number of relief workers, respectively. Search effectiveness is captured by the parameter c, where $0 \leq c \leq 1$. Since relief workers are employed in full-time jobs, they search less intensively than the unemployed. We further assume that those in regular employment (E) do not search.

There are L individuals in the exogenously given labour force. The rates of regular employment, relief employment, and unemployment, are given as $e \equiv E/L$, $r \equiv R/L$, and $u \equiv U/L$, respectively. Since the labour force comprises of those in regular employment, those in relief employment, and those in unemployment, we have the following identity: $1 \equiv e + r + u$. The vacancy rate is given as $v \equiv V/L$. The variable $\theta \equiv V/S$ represents labour market tightness and $q \equiv H/V$ represents the rate at which vacant jobs are filled. By the constant returns to scale assumption, we have $q(\theta) \equiv h(S/V, 1) = h(1/\theta, 1)$, where $q'(\theta) < 0$. The flow of new hires into regular jobs is given as $H = \alpha S$, where $\alpha = (H/V)(V/S) = q(\theta)\theta$. We see that $\alpha'(\theta) > 0$ since $\alpha = \theta h(1/\theta, 1)$; thus $\alpha = h(1, \theta)$, where $h(\cdot)$ is an increasing function.

Regular job offers arrive according to a Poisson process, with arrival rates that are exogenous to the individual worker. The arrival rate for an unemployed worker is α, whilst for a relief worker it is $c\alpha$. Therefore, the arrival rate in general is higher for an unemployed searcher than for a relief worker, since those in unemployment search more intensively than those in relief jobs. The unemployed may also exit to relief jobs. Relief job offers arrive according to a Poisson process with parameter γ. There is an exogenously given rate ϕ at which regular jobs break up, and a government-determined rate λ at which relief jobs expire. A worker separated from a regular job can enter into a relief job with probability μ, or enter directly into unemployment with probability $(1-\mu)$. Relief jobs are considered to be temporary and we thus assume that the rate at which relief jobs expire is strictly greater than the separation rate for regular jobs, i.e. $\lambda > \phi$. Figure 1 illustrates the model:

Figure 1



In Figure 1, the boxes *E*, *R*, and *U*, refer to the stocks of regular employment, relief employment, and unemployment, respectively, whilst the arrows represent the flows between the stocks. In the steady state, the numbers flowing into a given stock equal the numbers flowing out of the said stock. With that in mind, we write down the following steady-state equations for regular employment and relief employment:

[1] $\phi(1-u-r)=\alpha(\theta)(u+cr)$

[2] $[c\alpha(\theta)+\lambda]r=\gamma u+\mu\phi(1-u-r)$

(From the identity $1\equiv e+r+u$, we let $e\equiv 1-u-r$ in the equations.)

Equations [1] and [2] determine u and r, given θ. In order to determine θ, we have to consider the determination of vacancies and wages. All regular jobs are equally productive. For the firm, an occupied job has an expected value of J_{o_r} if the worker entered from relief employment, and J_{o_u} if the worker entered from unemployment. The expected value of a vacant job is J_v. The discount rate is denoted by δ, y is the constant marginal product, w_{c_r} is the wage cost to the firm of a worker who entered the job from a relief job, w_{c_u} is the wage cost to the firm of a worker who entered the job from unemployment, and k is the cost of maintaining a vacancy. The wage rates are related to the wage cost via the identity $w_{c_{r,u}}\equiv w_{r,u}(1+t)$, where t is the payroll tax. J_{o_r}, J_{o_u}, and J_v satisfy the following equations:

[3] $\delta J_{o_r}=y-w_{c_r}+\phi\left(J_v-J_{o_r}\right)$

[4] $\delta J_{o_u}=y-w_{c_u}+\phi\left(J_v-J_{o_u}\right)$

[5] $\delta J_v=-k+q_r\left(J_{o_r}-J_v\right)+q_{ur}\left(J_{o_u}-J_v\right)$

where $q_r=\alpha\dfrac{cr}{v}$ and $q_u=\alpha\dfrac{u}{v}$.

A job occupied by a worker who entered the wage bargain from a relief job yields a per-period surplus of $y - w_{c_r}$ and is turned into a vacant job at the rate ϕ; worker separations from this job are associated with a capital loss of $J_v - J_{o_r}$. A job occupied by a worker who entered the wage bargain from unemployment on the other hand yields a per-period surplus of $y - w_{c_u}$ and is turned into a vacant job at the rate ϕ; worker separations from this job are associated with a capital loss of $J_v - J_{o_u}$. The cost of a vacancy per period is *k*, and vacancies become occupied at the rate q_r by workers from relief jobs and q_u by workers from unemployment. Vacancies are kept open as long as their yield is positive. In equilibrium, $J_v = 0$. The value of a job occupied by a worker who entered from a relief job is obtained from [3] as $J_{o_r} = \left(y - w_{c_r}\right) / (\delta + \phi)$, whilst the value of a job occupied by a worker who entered from unemployment is obtained from [4] as $J_{o_u} = \left(y - w_{c_u}\right) / (\delta + \phi)$. Substituting into [5] yields

[6] $$\frac{y - w_c}{\delta + \phi} = \frac{k}{q(\theta)}$$

where $w_c \equiv \left(u w_{c_u} + c r w_{c_r}\right) / (u + cr)$ is the average wage cost in the economy.

This is the average zero-profit condition for firms. The left-hand side is the average present value of profits per worker, whilst the right-hand side is the expected present value of the firm's hiring cost. Labour market tightness influences decisions on vacancies by affecting hiring costs; the tighter the labour market, the costlier it is to hire due to the longer duration of vacancies.

Wages are determined by a Nash bargain. The firm's disagreement point is the value of a vacant job, whilst the worker's threat point is the value of either being unemployed or the value of being in a relief job. Since the wage bargains are undertaken between individual firms and individual workers, there are essentially two types of Nash bargain in the economy. This is in stark contrast to H-L who completely ignore the wage bargaining undertaken by relief workers, concentrating solely on bargaining between unemployed workers and firms.

We let Λ_{e_r} and Λ_{e_u} denote expected discounted lifetime income for workers in regular employment who have entered their job from relief work and workers who have entered their job from unemployment, respectively. Λ_u denotes the value of unemployment, whilst Λ_r denotes the value of being in a relief job. The value functions can be written as

[7] $$\delta\Lambda_{e_{r_i}} = w_{r_i} + (1-\mu)\phi\left(\Lambda_u - \Lambda_{e_{r_i}}\right) + \mu\phi\left(\Lambda_r - \Lambda_{e_{r_i}}\right)$$

[8] $$\delta\Lambda_{e_{u_i}} = w_{u_i} + (1-\mu)\phi\left(\Lambda_u - \Lambda_{e_{u_i}}\right) + \mu\phi\left(\Lambda_r - \Lambda_{e_{u_i}}\right)$$

[9] $$\delta\Lambda_r = \rho_r w + c\alpha\left(\Lambda_{e_r} - \Lambda_r\right) + \lambda\left(\Lambda_u - \Lambda_r\right)$$

[10] $$\delta\Lambda_u = \rho_u w + \alpha\left(\Lambda_{e_u} - \Lambda_u\right) + \gamma\left(\Lambda_r - \Lambda_u\right)$$

where $\Lambda_{e_r} = (1/\delta)\left[w_r + (1-\mu)\phi\left(\Lambda_u - \Lambda_{e_r}\right) + \mu\phi\left(\Lambda_r - \Lambda_{e_r}\right)\right]$ is the value to the worker of a job anywhere in the economy which was entered from relief employment, whilst $\Lambda_{e_u} = (1/\delta)\left[w_u + (1-\mu)\phi\left(\Lambda_u - \Lambda_{e_u}\right) + \mu\phi\left(\Lambda_r - \Lambda_{e_u}\right)\right]$ is the value of a job anywhere in the economy which was entered from unemployment. w_r and w_u refer to the wages of workers in regular employment who entered the job from relief employment and unemployment respectively, whilst w refers to the average wage in the economy. A worker employed by firm i receives the wage rate w_{r_i} if they entered the job from relief employment, and w_{u_i} if they entered the job from unemployment. All workers are separated from their job at the rate ϕ. A worker exiting from their job has probability μ of entering a relief job and probability $(1-\mu)$ of entering unemployment. Pay in relief employment is linked to the average wage in the economy via the replacement ratio ρ_r, whilst unemployment benefits are linked to the average wage in the economy via the replacement ratio ρ_u.

The Nash bargain between a relief worker and the firm solves the following:

$$\max_{w_{r_i}} \Omega(w_{r_i}) \equiv \left[\Lambda_{e_{r_i}}(w_{r_i}) - \Lambda_r\right]^{\beta}\left[J_{o_{r_i}}(w_{r_i}) - J_v\right]^{1-\beta}$$

where $0 < \beta < 1$.

The outcome of the Nash bargain is a wage equation of the form

$$[11] \qquad w_{c_r} = y - \left[\frac{1-\beta}{\beta}\right](1+t)(\delta+\phi)\left(\Lambda_{e_r} - \Lambda_r\right)$$

where the equilibrium conditions $w_{r_i} = w_r$ and $J_v = 0$ are imposed. For these workers, the outside option, should the wage bargain not result in employment, is the value of relief employment, i.e. Λ_r. As can be seen, any policy that reduces the difference in value between working and the outside option, $\Lambda_{e_r} - \Lambda_r$, will increase w_{c_r}

The Nash bargain between an unemployed worker and the firm solves the following:

$$\max_{w_{u_i}} \Omega(w_{u_i}) \equiv \left[\Lambda_{e_{u_i}}(w_{u_i}) - \Lambda_u\right]^{\beta}\left[J_{o_{u_i}}(w_{u_i}) - J_v\right]^{1-\beta}$$

where $0 < \beta < 1$.

The outcome of the Nash bargain is a wage equation of the form

$$[12] \qquad w_{c_u} = y - \left[\frac{1-\beta}{\beta}\right](1+t)(\delta+\phi)(\Lambda_{e_u} - \Lambda_u)$$

where the equilibrium conditions $w_{u_i} = w_u$ and $J_v = 0$ are imposed. For these workers, the outside option, should the wage bargain not result in employment, is the value of unemployment, i.e. Λ_u. Again, any policy that reduces the difference in value between working and the outside option, $\Lambda_{e_u} - \Lambda_u$, will increase the wage cost w_{c_u}.

We can combine these two wage equations to gain a wage equation for the average wage in the economy. By weighting w_r by the proportion of searchers in relief employment and w_u by the proportion of searchers in unemployment, we gain the following average wage equation:

$$[13] \qquad w_c = \frac{uw_{c_u} + crw_{c_r}}{u+cr} = y - \left[\frac{1-\beta}{\beta}\right](\delta+\phi)(1+t)(\Lambda_e - \Lambda_{fb})$$

where

$$[14] \qquad \Lambda_e = \frac{u\Lambda_{e_u} + cr\Lambda_{e_r}}{u+cr} \quad \text{and}$$

$$[15] \qquad \Lambda_{fb} = \frac{u\Lambda_u + cr\Lambda_r}{u+cr}.$$

Λ_e refers to the average value of being in employment, whilst Λ_{fb} is simply the average outside option available to workers involved in wage bargaining.

From [1], [2], [7], [8], [9], [10], [14], and [15], we are able to gain an explicit expression for the various differences in value between labour-market states in terms of the average wage rate, the discount rate, the replacement ratios, and the transition rates:

[16] $$\Lambda_e - \Lambda_u = \left[(\delta + \lambda + c\alpha + \gamma)(1-\rho_u) + (\mu\phi - \gamma)(\rho_r - \rho_u)\right] w\Delta^{-1}$$

[17] $$\Lambda_r - \Lambda_u = \left[-(1-c)\alpha(1-\rho_u) + (\delta + \phi + \alpha)(\rho_r - \rho_u)\right] w\Delta^{-1}$$

[18] $$\Lambda_e - \Lambda_r = \left[(\delta + \lambda + \alpha + \gamma)(1-\rho_u) - \{\delta + \alpha + \gamma + (1-\mu)\phi\}(\rho_r - \rho_u)\right] w\Delta^{-1}$$

[19] $$\Lambda_e - \Lambda_{fb} \equiv g(\cdot)w \qquad \text{(see Appendix)}$$

where $\Delta \equiv (\delta + c\alpha + \lambda)(\delta + \phi + \alpha) + \gamma(\delta + \phi + c\alpha) + (1-c)\alpha\mu\phi$.

By substituting [19] into [13] we obtain a relationship between the average wage and tightness. Invoking [6] together with [13] and [19], we can write the equilibrium average wage equation as

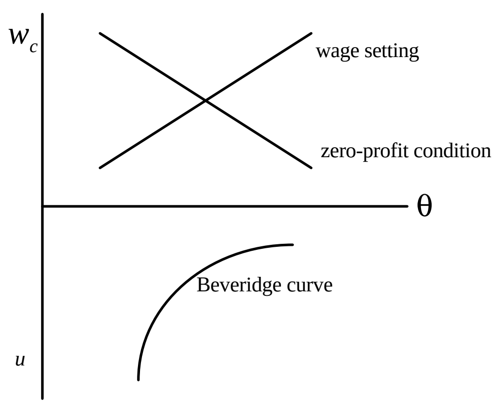
[20] $$w_c = \frac{\beta k}{(1-\beta)q(\theta)g[\alpha(\theta);\ldots]}$$

This equation determines the wage cost, given tightness. By expressing $\Lambda_e - \Lambda_{fb}$ as the average wage multiplied by $g(\cdot)$, we are able to gain an explicit expression for w_c in terms of β, k, $q(\theta)$, ρ_r, ρ_u, and the flow parameters of the model (see Appendix for the explicit expression). Note that α is the only flow parameter which is a function of θ.

The complete model is given by the wage equation [20], the zero-profit condition [6], and the two steady-state equations [1] and [2]. [6] and [20] determine θ and w_c. By substituting θ into [1] and [2], we can determine *u* and *r*.

The model is illustrated in Figure 2. The top half shows the zero-profit condition [6] and the wage equation [20] in $(w_c, \theta)-$space. The bottom half illustrates the relationship between unemployment and tightness, obtained by substituting out *r* from [1] and [2]. H-L refer to this curve as the “Beveridge curve”.

Figure 2.



From Figure 2 it can be seen that the wage-setting curve slopes upwards. We can see from [20] that θ appears in the denominator in both *q* and the value difference. As θ increases, both *q* and the value difference fall; the result being that the average wage cost increases. The zero-profit condition on the other hand is negatively sloped. The reason for this is that the higher the average wage, the lower must be labour market tightness in order to yield zero profits (see Page 5).

3 Comparative statics

The direct effect on unemployment of a change in either γ or μ is $u_\gamma < 0$ and $u_\mu < 0$, respectively. In Figure 2, this would be seen as an upward shift of the Beveridge curve. Thus for a given value of θ, unemployment will be lower. However, a change in a policy parameter will also have an indirect wage effect on unemployment. This effect will alter the value of θ. As $u_\theta < 0$, we would ideally like θ to increase as a result of the change in the policy parameter. Thus in Figure 2, we would prefer to see the wage curve shift downwards, resulting in an increase in θ. From [20], we see that any policy that increases g reduces the wage cost. This has the effect of increasing θ. Therefore, we see that a change in a policy parameter influences the wage (and θ) according to the following relationship:

$$\text{sign }(-1)\frac{\partial\theta}{\partial x} = \text{sign }\frac{\partial w_c}{\partial x} = \text{ sign }(-1)\frac{\partial g}{\partial x}$$

where x is a policy parameter. Policies which have a negative indirect effect on unemployment reinforce the direct negative effect on unemployment. Differentiating g with respect to γ and with respect to μ gives us

$$\frac{\partial g}{\partial\gamma} < 0 \qquad \text{and} \qquad \frac{\partial g}{\partial\mu} \gtreqless 0 \qquad \text{(see Appendix)}$$

The wage is thus increased by intensified hiring of unemployed workers into relief jobs. The wage effect of a policy that targets relief jobs at unemployment inflow μ, on the other hand, has an ambiguous wage effect. It is not certain that the indirect wage effect of directing relief jobs at the flow out of regular employment will reinforce the direct effect. This is in sharp contrast to H-L who, by failing to take into account the fact that relief workers are also wage bargainers, gain a negative wage effect which reinforces the negative direct effect on unemployment. Table 1 summarises these results:

Table 1

The Comparative Statics of Relief Jobs

	w_c	θ	Direct effect on unemployment	Indirect effect on unemployment	Total effect on unemployment
μ	?	?	–	?	?
γ	+	–	–	+	?

4 A Numerical Example

In this section, we undertake a numerical simulation, using the same parameter values as used by H-L in their simulation. However, instead of analysing what happens when γ is changed, we analyse the results of a change in μ . Table 2 shows what happens:

Table 2

	$u(\%)$	$r(\%)$	$e(\%)$	$v(\%)$	θ	D_v	D_r	D_u	w_c
Base run	5.00	0.00	95.00	1.96	0.39	3.9	–	10.0	100.0
$c = 0$	4.50	9.66	85.84	1.77	0.39	3.9	21.4	10.0	99.99
$c = 0.5$	2.75	5.28	91.98	1.77	0.33	3.7	10.9	11.1	100.16
$c = 1$	2.24	3.87	93.88	1.68	0.27	3.4	7.9	12.4	100.32

In the base run, no labour market programmes exist. In the following three rows, $\mu = 1$. The remaining parameter values for each simulation are: $\lambda = 1/150$; $\gamma = 0$; $\phi = 0.00075$; $\delta = 0.05/365$; $\beta = 0.546$; $\rho_r = 1$; $\rho_u = 0.5$. The cost of maintaining a vacancy *k*, is defined as $k = w_c$. The probability of filling a job is defined as $q = 0.025 * \theta^{-0.4}$. D_v, D_r, and D_u refer to the average duration in weeks of vacancies, relief jobs, and unemployment, respectively. The wage cost is normalised to 100 in the base run.

As can be seen, the usage of relief jobs reduces the rate of unemployment in each simulation. When $c = 0$, i.e. when H-L's concept of fall-back position holds, we see that the wage is indeed reduced as argued by H-L. However, when we have either $c = 0.5$ or $c = 1$, i.e. when H-L's definition of fall-back position no longer holds, we see that this is no longer the case: the wage increases in both examples.

5 Conclusion

H-L argue that relief jobs should be directed at the flow out of regular employment since this is the only policy which has a negative indirect wage effect on unemployment which reinforces the negative direct effect. As a result, unemployment is unequivocally reduced. This result stems from their incorrect specification of fall-back position which only includes the value of being unemployed. When due account is taken of those in relief jobs, we see that the policy of targeting relief jobs at the flow out of employment also has ambiguous effects on unemployment.

6 Bibliography

Holmlund B. & J. Lindén (1993), "Job Matching, Temporary Public Employment, and Equilibrium Unemployment.", *Journal of Public Economics*, Vol. 51, pp. 329-343.

Miller J. (1994), "Do Labour Market Programmes Necessarily Crowd Out Regular Employment? – A Matching Model Analysis", Mimeo, Tilburg University.

Miller J. (1991), "To What Extent Have Active Labour Market Policies in Sweden Reduced the NAIRU?", Unpublished undergraduate dissertation, Cambridge University.

7 Appendix

Let $$u = \frac{\phi\left[c\alpha(1-\mu)+\lambda\right]}{(1-c)\alpha\mu\phi+\alpha(c\alpha+c\gamma+\lambda)+\phi(c\alpha+\gamma+\lambda)} \equiv \frac{A}{V}$$

and $$r = \frac{\phi(\gamma+\mu\alpha)}{(1-c)\alpha\mu\phi+\alpha(c\alpha+c\gamma+\lambda)+\phi(c\alpha+\gamma+\lambda)} \equiv \frac{B}{V}$$

Thus

$$\Lambda_e - \Lambda_{fb} = \frac{cr\left(\Lambda_{e_r} - \Lambda_r\right) + u\left(\Lambda_{e_u} - \Lambda_u\right)}{cr+u} \equiv \frac{cB\left(\Lambda_{e_r} - \Lambda_r\right) + A\left(\Lambda_{e_u} - \Lambda_u\right)}{cB+A}$$

$$\Rightarrow \qquad \Lambda_e - \Lambda_{fb} = \left[\frac{c(\gamma+\mu\alpha)P + \left[c\alpha(1-\mu)+\lambda\right]Q}{S\left[c\alpha+c\gamma+\lambda\right]}\right]w \equiv g(\cdot)w$$

where $P \equiv \left[(\delta+\lambda+\alpha+\gamma)\left(1-\rho_u\right) - \left\{\delta+\alpha+\gamma+(1-\mu)\phi\right\}\left(\rho_r - \rho_u\right)\right]$

$Q \equiv \left[(\delta+\lambda+c\alpha+\gamma)\left(1-\rho_u\right) + (\mu\phi-\gamma)\left(\rho_r - \rho_u\right)\right]$

and $S \equiv \left[(\delta+c\alpha+\lambda)(\delta+\phi+\alpha)+\gamma(\delta+\phi+c\alpha)+(1-c)\alpha\mu\phi\right]$

Differentiating $g(\cdot)$ with respect to γ gives the following:

$$\frac{\partial g(\cdot)}{\partial\gamma} = \frac{\left(\Lambda_r - \Lambda_u\right)}{\Delta V(cB+A)^2 w} \cdot M$$

$$M \equiv -c\Delta(c\alpha+\phi)\phi\left[c\alpha(1-\mu)+\lambda\right]B + cVB\left[cB+A\right]\left[(1-\mu)\phi-\lambda\right] - AV^2(\delta+\lambda+c\alpha+\mu\phi) < 0$$

$$\Rightarrow \qquad \boxed{\frac{\partial g(\cdot)}{\partial\gamma} < 0}$$

Differentiating $g(\cdot)$ with respect to μ gives $\frac{\partial g(\cdot)}{\partial \mu} = \frac{(\Lambda_r - \Lambda_u)}{\Delta(c\alpha + c\gamma + \lambda)w} \cdot N$

$$N \equiv \phi\left[(\gamma + \mu\alpha)(\alpha + \delta + \gamma + \lambda) + \{c\alpha(1 - \mu) + \lambda\}(c\alpha + \delta + \gamma + \lambda)\right] - c\alpha$$

$$\Rightarrow \qquad \boxed{\frac{\partial g(\cdot)}{\partial \mu} \underset{<}{\geq} 0}$$