Aging and Public Pensions in an Overlapping-Generations Model

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Abstract

In this paper decision making on public pensions is modeled within the framework of the well-known two-overlapping-generations general-equilibrium model with rational expectations. The model is used to analyze the effects of aging on the evolution of public pension schemes. Analytical results are derived for the long run as well as for the short run by the method of comparative statics and comparative dynamics respectively. This shows that the short-run consequences of aging depend crucially on the existing size of the PAYG-scheme.
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1 Introduction

In most developed countries, a substantial part of the elderly's income is provided for by means of public pension schemes. Generally, these public pension plans are financed by a Pay-As-You-Go (or PAYG) scheme where current pension payments are financed out of current tax payments that are collectively decided upon. These PAYG-schemes have drawn a great deal of attention recently due to the expected aging of the population and the increasing tax rates this might entail. It is of interest to consider what should be the reactions of policy makers to the demographic changes that are taking place. This issue is considered in the framework of an overlapping-generations (OLG) general-equilibrium model of economic growth where a central planner with an infinite horizon maximizes a Benthamite social welfare function. That is, the central planner maximizes a discounted infinite sum of weighted average utilities of successive generations of finitely lived consumers where the weights are determined by the size of the generations. Our paper shares the objectives of papers such as by Peters (1991) and Bovenberg et al. (1993). In deriving short-run effects of aging it is assumed by Peters (1991) that the central planner does not take the whole future into account. This is not the case in Bovenberg et al. (1993). They use an Auerbach-Kotlikoff (1987) intertemporal framework in deriving general-equilibrium responses to aging. Unlike our paper they are not able to give analytical solutions for transition paths and have to rely on numerical simulations. The effects of aging in an OLG-model can also be contrasted with the effects in the Ramsey-model of economic growth. A path-breaking and often cited paper in this area is Cutler et al. (1990). The main assumptions of the Ramsey-model are that infinitely-long living agents have perfect foresight and that the economy follows a path that maximizes discounted utility over an infinite horizon. The drawback of this structure is that intergenerational redistribution effects of aging cannot be taken into account. The point is that redistribution only fits in the Ramsey-framework if the planning problem facing the government can be decomposed into two subproblems, a standard problem of aggregate capital accumulation and a problem of

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distributing consumption optimally on each date among the generations alive then. This is not possible, however, if the size of the population size varies and the government maximizes a Benthamite social welfare function. Therefore, in that case the Calvo-Obstfeld result that "the Cass-Koopmans-Ramsey framework can be used to evaluate paths of aggregate consumption even in models where different generations co-exist"[1] does not hold.

The closed-economy general-equilibrium framework used in this paper is presented in Section 2. In Section 3 a central planner is introduced that decides on the allocation of production to consumption and investment, and the distribution of consumption between young and old individuals, resp.. As is well known such a command optimum can be replicated by a market economy accompanied by a PAYG-financed public pension scheme.

In Section 4 we investigate how a PAYG-pension system reacts to the aging of the population which can be expected to occur in the next century in most countries of the Western world. Aging can have several effects. First, everything else given, aging implies higher contribution rates. This effect has drawn the most attention and concern expressed e.g. in policy circles[2]. The policy reaction then may be to decrease the benefits. Secondly, aging implies that less saving is necessary to maintain a certain capital-labor ratio. In this paper both these effects of aging are taken into account and it will be derived under what circumstances aging will lead to an increasing or a decreasing relative importance of the PAYG-scheme. It should be stressed that the short-run effects can differ considerably from the effects in the long run. Therefore, we not only perform a comparative statics analysis in order to trace the long-run effects, but we also study the short-run consequences of aging. We derive analytical results for the short run for fairly general specifications for utility and production functions by the method of comparative dynamics. Morover, we distinguish between expected and unexpected shocks. By using an OLG-model we are able to qualify one of the central results of Cutler et al., namely that an expected aging of the population will lead to short-run gains in consumption possibilities. It will be shown that this result depends on the size of the PAYG-scheme. The same holds true for unexpected changes in the rate of population growth. As a corollary it is shown that an expected decrease in the rate of population growth may lead to the introduction of a public pension system in an economy where such a scheme previously did not exist.

The paper winds up with some concluding remarks in Section 5.

2 The Model

2.1 The Individuals

The analysis is based on the well-known two-overlapping-generations model where there is a large number of individuals living for two periods and every one is identical except for differences in age. Since the young's savings decisions are assumed to be decentralized and uncoordinated, each young individual takes aggregate savings and the tax rate as given. It is assumed that every individual born at time t works a fixed amount of time at wage w_t during the first period of his life. A part of this wage is taxed away by the government by a lump-sum tax τ_t to be transferred to the old. The remainder is used for savings for old age (s_t) and for consumption (c_t^y). So we get:

$$c_t^y = w_t - \tau_t - s_t. \tag{1}$$

[1] Calvo and Obstfeld (1988, p.163).

[2] See, e.g. Van den Noord and Herd (1993).

When old, the individual consumes the return on his savings and the transfer payment η from the government. So the consumption at time $t+1$ of an old individual born at time t (c_t^r) is equal to:

$$c_t^r = (1+r_{t+1})s_t + \eta_{t+1}. \tag{2}$$

Individuals are assumed not to be altruistic: they do not care about the levels of utility of individuals born in another time period. It is assumed that lifetime utility of an individual born at time t can be represented by the following separable utility function:

$$U_t = u(c_t^y) + \frac{1}{1+\theta}u(c_t^r), \tag{3}$$

where the felicity functions obey the Inada-conditions. The parameter θ indicates the rate of time preference. The first-order condition for maximization of lifetime utility reads:

$$u'(c_t^y) = \frac{1+r_{t+1}}{1+\theta}u'(c_t^r), \tag{4}$$

where $u'(c) \equiv \frac{du}{dc}$. This condition states that the young use savings to equate the marginal rate of substitution of consumption in the first and the second part of their lives to the rate of return on savings[3].

2.2 The Markets

We close the model by conventional equilibrium conditions on the factor markets and the goods market. Production per worker at time t is described by a standard neoclassical production function $f(k_t), (f'(k) \equiv \frac{df}{dk} > 0, f'' \equiv \frac{d^2f}{dk^2} < 0)$ where k_t is the capital-labor ratio. Abstracting from depreciation and assuming that firms act competitively and that labor supply is exogenous factor market equilibrium is described by the marginal productivity conditions:

$$f(k_t) - k_t f'(k_t) = w_t, \tag{5}$$

$$f'(k_t) = r_t. \tag{6}$$

Equilibrium on the goods market implies for our closed-economy model that investment be equal to savings or in per worker terms:

$$(1+n)k_{t+1} = s_t, \tag{7}$$

where n is the rate of population growth which is assumed to be determined exogenously[4]. Eqs. (1), (2), (5), (6) and (7) imply the following resource constraint for the economy as a whole:

$$f(k_t) + k_t = (1+n)k_{t+1} + c_t^y + \frac{1}{1+n}c_{t-1}^r, \tag{8}$$

which simply says that total supply of goods can be allocated to investment or to consumption by the currently living generations.

[3] Notice that an internal solution has been assumed. Excluding perverse solutions, the existence of such a solution will be guaranteed due to the requirement of equilibrium on the goods market to be discussed below.

[4] For expositional reasons, we initially assume n to be constant.

3 Decision making on public pensions

The government at time t is assumed to maximize a Benthamite social welfare function with a social discount rate ρ:

$$W_t = \sum_{i=t}^{\infty} U_{i-1} \left(\frac{1+n}{1+\rho} \right)^{i-t}, \quad (0 < \rho < \infty). \tag{9}$$

According to eq. (9) the government is a social planner who takes account of the welfare of all current and future generations where the welfare of a generation is measured by the utility of a representative individual of this generation weighted by the size of the generation[5].

Assuming that $n < \rho$ the first-order conditions for a command optimum that maximizes this social welfare function can easily be derived[6]:

$$u'(c^r_{t-1}) = \frac{1+\theta}{1+\rho} u'(c^y_t), \tag{10}$$

$$u'(c^y_{t-1}) = \frac{1+f'(k_t)}{1+\rho} u'(c^y_t). \tag{11}$$

Eq. (11) is a condition for optimal allocation between young and old alive at the same time and eq. (12) optimizes intertemporal allocation. Notice that combining these conditions with eq. (6) leads to the first-order condition for saving (eq. (4)). So the central planner respects the individual's saving decision. This implies that the command optimum can be realized in a market economy with intergenerational redistribution through a PAYG-scheme[7]. Abstracting from administrative costs the budget restriction of this scheme reads:

$$\eta_t = (1+n)\tau_t. \tag{12}$$

It follows directly from eqs. (11) and (12) that the steady state of the model is characterized by[8]:

$$r = \rho. \tag{13}$$

This is the well-known modified golden rule: the higher the social discount factor, the lower the capital stock per worker in the steady state will be, which is, of course accomplished by a higher transfer which has a negative effect on savings. Notice that the assumption $\rho > n$ implies that the economy is dynamically efficient in the steady state ($r > n$). In the case where $\rho \leq n$ the first-order conditions (11) and (12) do not apply. It is evident that in that case the optimal policy implies a steady state that satisfies the golden rule ($r = n$), however[9].

[5]The choice for this form of the welfare function is motivated by the fact that without this weighting the planner would raise the consumption of members of small cohorts relative to that of members of larger cohorts. For a discussion on the advantages of a Benthamite social welfare function see Blanchet and Kessler (1991).

[6]See Blanchard and Fischer (1989), section 3.1.

[7]It should further be noted that the optimal policy is (stationary and non-stationary) time consistent. See Bernheim (1989).

[8]When no time subscript is used the steady-state value is meant.

[9]As the golden rule implies that total consumption possibilities are maximized, any time path that does not lead to the golden rule is dominated according to the take-over criterion by a time path that leads to $r = n$.

4 The effects of aging

In this section we consider the effects of aging reflected by a decrease in population growth (n). The short-run as well as the long-run consequences of once-and-for-all changes in n are derived analytically.[10]. The long-run effects follow from comparing steady states, while the short-run consequences are traced by comparative dynamics[11]. Regarding the short run, the effects of unexpected and expected shocks can be distinguished. The analysis learns that the short-run effects of aging can be (but need not be) opposite to the effects in the long run. E.g., while a current or future decrease in population growth leads to losses in utility in the long run in a dynamically efficient economy, short-run gains in utility for young and old individuals are possible. The reason for this result is well known, i.e. aging decreases the future need of capital which can incite the social planner to decrease savings and thus increase current consumption. However, this consumption-stimulating effect can be undone if the PAYG-system entails relatively high transfers. It should be noted at the outset that this last effect is absent in Ramsey models.

The comparative statics as well as the comparative dynamics analysis is based on linearization of the model presented above around the steady state. We concentrate on the effects of once-and-for-all decreases in population growth (n). Let $n_t = n + \gamma h_t$ where h_t describes the time pattern of the perturbation of the steady-state value of the parameter n and γ the magnitude. It is assumed that at some time $t = 0$ a shock unexpectedly occurs (i.e. $h_0 = h_1 = \cdots = h < 0$) or is expected to occur at time $z > 0$ (i.e. $h_0 = h_1 = \cdots h_{z-1} = 0$, $h_z = h_{z+1} = \cdots = h < 0$). It will be convenient in the sequel to introduce total consumption per worker which is defined as:

$$c_t = c_t^y + \frac{c_{t-1}^r}{1+n}. \tag{14}$$

The linearized model can then be condensed to a system describing the changes in this variable and in the capital-labor ratio k_t:

$$\begin{bmatrix} \frac{\partial k_{t+1}}{\partial \gamma} \\ \frac{\partial c_{t+1}}{\partial \gamma} \end{bmatrix} = J \begin{bmatrix} \frac{\partial k_t}{\partial \gamma} \\ \frac{\partial c_t}{\partial \gamma} \end{bmatrix} + M \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}, \quad t = 0, 1, ... \tag{15}$$

where M is a matrix describing the effects of the current and next-period change in n on the state variables and J is the Jacobian matrix (see the Appendix for details). As the system in eq. (15) is saddlepoint stable it can be solved to find the initial effect of changes in n on total consumption i.e. $\frac{\partial c_0}{\partial \gamma}$ (see Blanchard and Kahn (1981)). The evolution of c and k can then be traced by using the same system again. The changes in all other variables can be derived from the changes of these state variables. In particular, we calculate the changes in consumption of the young and the old and the effect on the tax rate, respectively.

[10] Most results can directly be generalized to temporary changes in these parameters.

[11] This method is described by Judd (1982) for a continuous-time model. It can easily be generalized to discrete time, however. See Meijdam and Verhoeven (1994), who also show that the approximation of the short-run effects of a parameter change by this method is as reliable as a standard comparative-statics analysis. The method is explained in the Appendix.

The short-run effects of an unexpected decrease in n

An unexpected decrease in population growth leads to macroeconomic effects on total consumption that can be derived by solving the system in eq. (15). The effect of the change in n on total consumption per worker can be read off from eq. (v) in the Appendix, which is rewritten here for convenience[12]

$$\frac{\partial c_0}{\partial \gamma} = (1+n)(\lambda_2 - 1 - \delta)\frac{\partial k_0}{\partial \gamma} + (1-\lambda_2)k\sum_{j=1}^{\infty} h_j \lambda_2^{-j}$$
$$+\frac{c^r}{(1+n)^2}\left[\frac{\lambda_2 - 1}{\lambda_2}\sum_{j=0}^{\infty} h_j \lambda_2^{-j} - h_0\right]. \tag{16}$$

Notice from eq. (16) that the optimal response of total consumption is in part determined by future changes in the rate of population growth. This is a reflection of the forward-looking behavior of the central planner who takes account of the utility of all future generations. The initial macroeconomic effect on consumption appears to consist of several elements. Firstly, a decrease in n causes an upward jump in the capital-labor ratio, which implies a higher production and thus a higher consumption per worker. This is reflected by the first term in eq. (16), ($\frac{\partial k_0}{\partial \gamma} = -\frac{k}{1+n}h_0 > 0$). On top of this, the effect on initial consumption is in part determined by anticipation of the future effects of the change in n as reflected by the appearance of the terms $h_j(\ j > 0)$ in eq. (16). Two future effects can be discerned. The first is the *capital thickening* effect (k, see the second term in eq. (16)) [13] . This effect stems from the fact that a decrease in n implies that a certain level of saving per worker ceteris paribus leads to a higher capital-labor ratio next period. This is anticipated by immediately decreasing the saving rate, so that consumption increases. The second future effect comes from redistribution between the young and the old. This is caused by the change in the so-called dependency ratio defined as the number of pensioners per worker. This *dependency-ratio effect* ($\frac{c^r}{(1+n)^2}$) implies that total consumption per worker has to be shared with a larger number of pensioners. Although this is a redistribution effect it may influence savings and thus total consumption. Of course, in case of an unexpected decrease in n, the dependency-ratio effect not only affects future consumption but initial consumption as well. However, if the change in population growth is once-and-for-all, so that $h_j = h\ \forall j \geq 0$, then the initial dependency-ratio effect is exactly compensated by the future effect. This leads to the following lemma:

Lemma 1

In the case of an unexpected once-and-for-all change in population growth, the change in the dependency ratio does not affect initial total consumption.

This lemma can be viewed as an extension of the result in Cutler et al. (1990) that in a Ramsey model the dependency ratio does not influence steady-state savings.

It immediately follows that the initial macroeconomic effect on total consumption in the case of a once-and-for-all is shock positive. However, even though total consumption rises initially, old-age consumption as well as young-age consumption may fall in the short run if the negative dependency-ratio effect is strong enough. This appears from the change in consumption of the young and the old resulting from both macroeconomic effects and redistribution effects which follow from differentiating eq. (11) and using eq. (14):

[12] δ and λ_2 are defined in the Appendix. There it is also shown that $\delta > 0$ and $\lambda_2 - 1 - \delta > 0$.

[13] This exactly the opposite of the capital dilution effect that occurs in case of an increase in n. See e.g. Blanchet and Kessler (1991), p. 140.

$$\frac{\partial c_t^y}{\partial \gamma} = (1-\epsilon)[\frac{\partial c_t}{\partial \gamma} + \frac{c^r}{(1+n)^2} h_t], \tag{17}$$

$$\frac{\partial c_{t-1}^r}{\partial \gamma} = (1+n)\epsilon[\frac{\partial c_t}{\partial \gamma} + \frac{c^r}{(1+n)^2} h_t]. \tag{18}$$

The parameter ϵ indicates the share of the young in the change of total consumption. This share is defined as $\epsilon = \frac{\alpha^y}{\alpha^y + (1+n)\alpha^r}$, where α^y is the absolute rate of risk aversion of a young individual which is defined as $\alpha^y = \frac{-u''(c^y)}{u'(c^y)}$. Analogously, α^r stands for the risk aversion of an old individual. The equations show that the larger the absolute rate of risk aversion of a generation relative to the risk aversion of the other generation, the smaller its part in the change of total consumption[14]. Moreover, it indicates that the positive macroeconomic effect on individual consumption can be dominated by the dependency ratio effect dependent upon the level of consumption by the old in the steady state.

The fact that individual consumption may fall or rise in the short run seems in line with the results of Cutler et al. (1990) who conclude that decreases in the rate of population growth have theoretically ambiguous short-run effects on individual consumption. Their numerical experiments show that for some countries short-run gains but for others short-run decreases in individual consumption result in the case of aging. In particular, if the aging of the population is beginning to be felt immediately (as in the case of Japan) then the current generations will be worse off while if aging is less severe in the next two decades (as e.g. in the U.S.) short-run consumption gains can be obtained. However, our analysis qualifies these results as it turns out that whether consumption rises or falls heavily depends on the size of the PAYG-scheme. In particular, using eq. (16) the following proposition can easily be derived:

Proposition 1

An unexpected once-and-for-all decrease in n initially decreases individual consumption iff $\tau > (1-\lambda_1)s$ [15].

The interpretation of this proposition is straightforward: in the case of a large PAYG-system the increase in the consumption possibilities due to the decrease in savings of the young is not sufficient to provide the relatively larger number of old with the same, relatively high level of consumption as before the shock.

The effect of a change in n on the tax rate in period t is given by:

$$\frac{\partial \tau_t}{\partial \gamma} = [r\nu - 1 - r]\frac{\partial k_t}{\partial \gamma} + \epsilon\frac{\partial c_{t-1}}{\partial \gamma} - (1-\epsilon)\frac{c^r}{(1+n)^2} h_t. \tag{19}$$

[14] Notice that this implies that if the felicity function is of the CRRA type, i.e. the *relative* rate of risk aversion is constant, the generation with the largest level of individual consumption gets the largest part of the change in total consumption.

[15] $\lambda_1 < 1$ is the stable root of the linearized system.

The first term in this equation indicates the effect of a change in the capital-labor ratio. At a given interest rate an increase in the capital stock per worker implies an increase of the consumption by pensioners of $(1+r)\frac{\partial k_t}{\partial \gamma}$ thus decreasing the need for redistribution from young to the old through τ. An increase in the capital-labor ratio also decreases the interest rate, thus increasing the need for redistribution. This effect is measured by ν, which is the (positively defined) elasticity of the production function $\nu = \frac{-f''(k)k}{f'(k)}$. The larger this elasticity, the larger the effect of a change in the capital-labor ratio on the interest rate. The second term in equation (19) represents the effect of a change in total consumption on the tax rate. As a part ϵ of total consumption goes to the old, more total consumption ceteris paribus increases the tax rate. In the third term we recognize the dependency-ratio effect again. An increase in the dependency ratio ceteris paribus increases the tax rate. The total effect on the tax rate that results in the case of an unexpected change in the rate of population growth is ambiguous.

The short-run effects of an expected decrease in n

Notice first that in the case of an expected once-and-for-all change at time $t = z$ (i.e. $h_0 = ... = h_{z-1} = 0$, $h_z = h_{z+1} = ... < 0$) there is no dependency-ratio effect at time $t = 0$. As a result, from eqs. (17) and (18) it follows that $\frac{\partial c_0^y}{\partial \gamma}$ and $\frac{\partial c_{-1}^r}{\partial \gamma}$ are proportional to the change in total consumption. Moreover, the capital labor-ratio does not jump in the case of an expected change, i.e $\frac{\partial k_0}{\partial \gamma} = 0$.

The effect of an expected future decrease in n on c_0 is given by:

$$\frac{\partial c_0}{\partial \gamma} = \frac{\lambda_2 - 1}{\lambda_2}\left(\frac{c^r}{(1+n)^2} - \lambda_2 k\right)\sum_{j=z}^{\infty} h_j \lambda_2^{-j} \tag{20}$$

So the initial macroeconomic effect is determined by the anticipated dependency ratio effects and capital thickening effects[16]. The difference between the dependency-ratio effect and the capital-thickening effect depends on s and $\frac{c^r}{1+n}$ again. The following proposition can easily be derived from eq. (20):

Proposition 2

An anticipated once-and-for-all decrease in n initially decreases individual consumption iff $\tau \geq (1 - \lambda_1 + \delta)s$.

Comparing with Proposition 1 it is of interest to observe that no qualitative difference exists regarding the short-run effects. However, the difference in effects is of a quantitative nature as can be seen from the appearance of the parameter δ in Proposition 2[17]. This (positive) parameter can be interpreted as a measure for the speed at which changes in consumption and investment are transmitted to future periods and vice versa. In particular, if δ has a low value anticipated future shocks will strongly affect current values and expected and unexpected shocks will have similar results. In general, however, compared to Proposition 1, with a given level of savings, higher taxes are needed to cause individual consumption to fall in response to an expected decrease in n.

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[16] The capital thickening effect is reinforced by the unstable eigenvalue, however. This is due to the additional decrease in savings that is necessary in order to decrease the capital-labor ratio in anticipation of a future decrease in population growth

[17] The parameter δ is defined in the Appendix.

It is interesting to note that Proposition 2 qualifies the results from Cutler et al. once more: it implies that, contrary to an unexpected decrease, an expected future decrease in population growth may cause *total* consumption to *fall* initially. This is due to the overlapping-generations structure of our model. If individual consumption decreases due to the shock the generation born in the period before the shock rationally anticipates a decrease in public pension benefits due to the dependency-ratio effect when old (i.e. in the period of the shock) and reacts to this by increasing savings when young. This will in turn lead to a decrease in transfers in the period before the shock and so on. Eventually this will lead to an increase in savings and thus a decrease in total consumption at $t = 0$. Whether this negative effect on consumption is large enough to outweigh the positive effect due to the anticipated capital thickening effect depends on the size of the PAYG-scheme again. So, in countries where the aging of the population will not occur before the next century but where the PAYG-system is relatively extensive it might be necessary nevertheless to cut in intergenerational transfers.

Using the fact that the capital-labor ratio and the dependency ratio do not change initially, the following corollary can be derived from eq. (19) and Proposition 2:

Corollary 1

An anticipated once-and-for-all decrease in n initially increases the tax rate iff $\tau \leq (1 - \lambda_1 + \delta)s$.

This corollary has an interesting implication. It implies that in the absence of a PAYG-financed public pension system such a system can come about in anticipation of a decrease in the rate of population growth. This result is rather surprising as intuition says that an increasing population would give more room for the installation of a public pension system due to the low tax rates implied by high rates of population growth. The point is that an affluent capital stock at the time of the shock is anticipated by the forward-looking government which incites her to increase the tax rate simultaneously leading to a decrease of the savings by all generations living before the shock. These desinvestments lead to an increase in the consumption possibilities for old and young individuals.

The long-run effects of a decrease in n

In the long run, the capital stock per worker remains unchanged as immediately can be seen from the modified golden rule (eq.(13)). Moreover, the redistribution effects of a change in the dependency ratio cancel out and do not affect total consumption. So the only effect on total consumption per worker in the long run is the positive capital thickening effect. Consequently, c will be higher as can be seen from:

$$\frac{\partial c}{\partial \gamma} = -\frac{\partial s}{\partial \gamma} = -\frac{s}{1+n}h > 0. \tag{21}$$

As in Cutler et al. the long-run effect of aging on individual consumption is the sum of a positive capital thickening effect and a negative dependency-ratio effect. In contrast to this paper, we can easily determine the net effect as in our model[18] there is an explicit relation between these two effects. It immediately follows that the capital-thickening effect on individual consumption outweighs the dependency-ratio effect iff the part of total consumption per worker consumed by pensioners ($\frac{c^r}{1+n}$) is smaller than savings per worker. From eqs. (2), (7) and (9) it follows that this condition is equivalent to $\tau < (n - r)k$. This leads us to the following proposition:

[18] Notice that life expectancy at the time of birth is fixed.

Proposition 3

In a dynamically efficient economy ($r > n$) with a non-negative tax rate, a decrease in n decreases individual consumption in the long run.

As in the long run the capital-labor ratio does not change, it can easily be derived that the long-run tax rate is a negative function of the rate of population growth. Consequently, the effect of a decrease in population growth on the long-run transfer payment η is positive if $\tau \leq 0$. If the tax rate is positive, however, the effect on the transfer payment is ambiguous.

5 Concluding Remarks

In this paper the effects of aging on PAYG-financed public pension schemes and individual consumption has been analyzed in an overlapping-generations general-equilibrium framework. The long-run effects of aging are well-known. This knowledge is summarized in Proposition 3 of this paper. The short-run effects, however, are much less known in the literature. In particular, Cutler et al. (1990) claim these effects to be ambiguous, but, at the same time, suggest by numerical simulations that an unexpected decrease in the rate of population growth will have a negative effect on individual consumption, while if this decrease is expected to occur after a few decades, short-run gains in consumption will be obtained.

This paper has derived the short-run effects of aging analytically. The results are reflected in Propositions 1 and 2. These propositions claim that for unexpected as well for expected aging of the population the size of the PAYG-financed public pension scheme is the determining variable for the short-run effects. In particular, for an extensive public pension system both types of shocks may lead to losses in individual consumption. On the other hand, if the PAYG-taxes are relatively small, gains in consumption will be enabled by a decrease in savings. In an OLG-model these gains in consumption possibilities are partly transferred to the old through an increase in the tax rate. So, it follows that if a PAYG-scheme is non existent, it can come about because of an expected decrease in the rate of population growth. With an eye turned to reality this implies that the abundant growth in coverage and size of public pension schemes which has been manifest in the Western world the first two decades after World War II could be explained from the anticipation of the aging of the population to occur in the first decades of the next century when the baby-boom generation retires. So the growth of public pension schemes in the face of aging could have been a welfare-maximizing policy according to our model. Recent measures taken in the Western world aimed at restricting the growth rate of the taxes could then be explained from the fact that demographic changes appear to be much more dramatic than expected. In particular, given the then existing extensive PAYG-scheme, the sharp decrease in birth rates that started in the seventies in the EC-countries made a drop in benefit rates unavoidable.

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Appendix

Comparative dynamics

The method of comparative dynamics works as follows. Let h_t describe the time pattern of the perturbation of the steady-state value of the parameter n and γ the magnitude. So $n_t = n + \gamma h_t$. The effects of a marginal change in n can then be traced by differentiation of the first-order eqs. (4) and (12) and the resource constraint (8) with respect to γ. This yields the following linear system in the state variables k (capital stock per worker) and $c = c^y + \frac{c^r}{1+n}$ (total consumption per worker):

$$\begin{bmatrix} \frac{\partial k_{t+1}}{\partial \gamma} \\ \frac{\partial c_{t+1}}{\partial \gamma} \end{bmatrix} = J \begin{bmatrix} \frac{\partial k_t}{\partial \gamma} \\ \frac{\partial c_t}{\partial \gamma} \end{bmatrix} + M \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}, \quad t = 0, 1, ... \tag{i}$$

where M is a matrix describing the effects of the current and next-period change in the exogenous parameter on the state variables and J is the Jacobian matrix:

$$J = \begin{bmatrix} \frac{1+r}{1+n} & -\frac{1}{1+n} \\ -\delta(1+r) & 1+\delta \end{bmatrix} \qquad M = \begin{bmatrix} 0 & -\frac{k}{1+n} \\ \frac{c^r}{(1+n)^2} & \delta k - \frac{c^r}{(1+n)^2} \end{bmatrix} \tag{ii}$$

where $\nu = \frac{-f''(k)k}{f'(k)}$ is the (positively defined) elasticity of the production function, $\delta = \frac{\nu r}{\alpha^y k(1+r)(1+n)(1-\epsilon)} > 0$ and $\epsilon = \frac{\alpha^y}{\alpha^y + (1+n)\alpha^r}$. The parameter α^y stands for the absolute rate of risk aversion of a young individual which is defined as $-\frac{u''(c^y)}{u'(c^y)}$. Analogously, α^r indicates the risk aversion of an old individual.

This system comprizes one predetermined variable (k) and one forward-looking variable (c). The eigenvalues of the Jacobian matrix J are:

$$\lambda_1 = \frac{1}{2}[2 + \frac{r-n}{1+n} + \delta - \sqrt{(\frac{r-n}{1+n} + \delta)^2 + 4\delta}] \qquad \text{(iii)}$$

$$\lambda_2 = \frac{1}{2}[2 + \frac{r-n}{1+n} + \delta + \sqrt{(\frac{r-n}{1+n} + \delta)^2 + 4\delta}]$$

As easily can be checked, one of these eigenvalues lies inside while the other lies outside the unit circle. So the system is saddlepoint stable and can be solved to find the initial effect of changes in n on total consumption (see Blanchard and Kahn (1981)):

$$\frac{\partial c_0}{\partial \gamma} = \frac{\lambda_1 - j_{11}}{j_{12}} \left[\frac{\partial k_0}{\partial \gamma} + m_{11} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_i + m_{12} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_{i+1} \right] \qquad \text{(iv)}$$

$$- m_{21} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_i - m_{22} \sum_{i=0}^{\infty} \lambda_2^{-i-1} h_{i+1}.$$

where j_{ij} and $m_{ij} (i, j = 1, 2)$ denote elements of J and M. The evolution of both k and c can then be derived from the system equations. Given the evolution of total consumption we can calculate the time paths of consumption by the young or the old using the definition of c. The effect on the pension system then follows.

Using (ii) it is straightforward to derive the initial effect of a change in population growth[19]:

$$\frac{\partial c_0}{\partial \gamma} = \frac{\lambda_2 - 1}{\lambda_2} \left(\frac{c^r}{(1+n)^2} - \lambda_2 k \right) \sum_{j=1}^{\infty} h_j \lambda_2^{-j} \qquad \text{(v)}$$

$$-[(1+n)(\delta + 1 - \lambda_2)\frac{\partial k_0}{\partial \gamma} + \frac{1}{\lambda_2} \frac{c^r}{(1+n)^2}] h_0$$

The first term on the right hand side of this equation describes the initial effect on total consumption of an expected change in population growth while the second term indicates the additional effect in case of an unexpected decrease in n ($h_0 < 0$). The latter effect is, of course, positive[20]. The effect of an unexpected once-and-for-all decrease in n is the sum of both terms with $h_0 = h_1 = \cdots = h < 0$.

[19] We have used the fact that $(1+n)(\lambda_2 - 1) = \frac{\lambda_1 - j_{11}}{j_{12}} + \delta(1+n)$..

[20] Note that $\frac{\partial k_0}{\partial \gamma} = \frac{-k}{1+n} h_0 > 0$. Moreover, given the fact that $\frac{\lambda_1 - j_{11}}{j_{12}} = -\frac{j_{11}}{\lambda_2 - j_{11}} > 0$ it immediately follows from footnote (19) that $\delta + 1 - \lambda_2 < 0$