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Transfers to sustain dynamic core-theoretic cooperation in international stock pollutant control

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Abstract

For international environmental agreements aiming at world efficiency in the presence of transboundary flow pollution, it is known that, in a static context, efficiency and stability in the sense of the core of a cooperative game can be achieved using appropriately defined transfers between the countries involved. However, for accumulating pollutants, such as CO_2 in the atmosphere, a dynamic analysis is required.

This paper provides a transfer scheme for which a core property is proved analytically in a dynamic (closed-loop) game theoretic context. The characteristic function of the cooperative dynamic game yielding this result is discussed and an algorithm to compute the transfers numerically is presented and tested on an example. The transfers are also compared with an open-loop formulation of the model.

JEL classification: C73; D62; F42; H4; Q3

Keywords: Transfrontier pollution; Stock pollutant; Dynamic cooperative games; Coalitions; Core solution

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1. Introduction

This paper presents a cooperative game theoretic analysis of the economics of international agreements on transnational pollution control, when the environmental damage arises from stock pollutants that accumulate (and possibly decay). The issues raised by the necessity of cooperation amongst the countries involved, if a social optimum is to be achieved, have already been addressed in the literature in terms of game theory concepts; see e.g. Mäler (1989). However, most of these contributions have only dealt with static (one shot) games, which are only suitable for flow pollution models. With stock pollutants the problem acquires an intertemporal dimension. In this case, dynamic game theory is a more appropriate tool of analysis, as is done in, e.g. van der Ploeg and de Zeeuw (1992), Kaitala et al. (1992), Hoel (1992), Tahvonen (1994), Petrosjan and Zaccour (1995).

With exception of the last paper, these contributions leave aside the issue of the *voluntary* implementation of the international optimum. This is an important drawback since no supranational authority can be called upon to impose the optimum in a context where the countries' interest in cooperation diverges strongly between one another, and especially if some countries loose when the social optimum is implemented. In view of ensuring such implementation, it has often been suggested that financial transfers between the countries involved would provide incentives towards cooperation. [1]

This property, understood in the sense of the core of a cooperative game, has in effect been demonstrated by Chander and Tulkens (1995, 1997), who propose a particular transfer scheme based on parameters reflecting the relative intensities of the countries' environmental preferences.

This result, established for flow pollutants only (that is, in a static game model), has been extended by Germain et al. (1998a) to the larger context of open-loop dynamic games. But this extension suffers from the fact that it implies the restrictive assumption that negotiations take place once and for all and bear on emissions trajectories that extend until the end of the planning horizon. Agreements are thus binding all along. The aim of the present paper is precisely to relax this assumption: *here*, *cooperation is negotiated at each period*, i.e. players reevaluate at each time the interest of cooperation, taking account of the current stock of pollutant. This approach is one of closed-loop (or feedback) dynamic games.

In this context, financial transfers are considered again. They are now formulated in such a way that they induce cooperation in the core-theoretic sense at each period. The model thus yields a sequence of cooperative international agreements. This result is obtained under more general convexity conditions than in the earlier attempt by Germain et al. (1998b) that was restricted to linear damage cost functions.

Other approaches to the cooperation issue concentrate on the stable number of signatories by considering the trade-off for an individual country between joining some

[1] In a two-countries framework, Petrosjan and Zaccour (1995) use financial transfers to obtain a cooperative solution as a Nash equilibrium. However, being limited to two countries, they cannot deal with the issue of coalitions, which is one of our aims here. Kverndokk (1994) deals both with coalitions and financial transfers for the greenhouse gas problem, but he resorts neither to dynamic games, nor to cooperative ones.

coalition and staying out of that coalition (see Carraro and Siniscalco, 1993; Barrett, 1994). A critical comparison between these approaches and the one of this paper is given in Tulkens (1998). On the other hand, one must emphasize that this paper does not intend to describe the way actual international negotiations are conducted (e.g. the Kyoto protocol). It chooses instead a normative approach, identifying transfers that make agreements less likely to be broken. For an alternative approach in this sense, see Bahn et al. (1998).

The structure of the paper is as follows. Section 2 presents the international stock pollutant model and characterizes the international optimum as well as the closed-loop Nash non-cooperative equilibrium. In Section 3, financial transfers are formulated so as to ensure that each country is not worse off when it participates, either in the finite or in the infinite horizon case. In Section 4, transfers are further specified so as to achieve stability in the core-theoretic sense, followed by a discussion of the characteristic function of dynamic cooperative games yielding that result.

Section 5 describes an algorithm that solves the model developed in Sections 3 and 4, and presents numerical results obtained by application of the algorithm to a simple climatic change model with three regions, including a comparison with the open-loop case. Section 6 is a conclusion.

2. International optimality and non-cooperative equilibrium

Our economic model is written in discrete time. Consider n countries indexed by $i \in \mathcal{N} = \{1,2,\dots,n\}$ and some planning period $\mathcal{T} = \{1,2,\dots,T\}$ (T, the planning horizon, is a positive integer, possibly infinite). In each country, pollution is entailed by economic activity: let $\mathbf{E}_t = (E_{1t},\dots,E_{nt})'$ denote the vector of the different countries' emissions of a certain pollutant at time t. These emissions spread uniformly in the atmosphere and contribute to a stock of pollutant S according to the equation

$$S_t = [1-\delta]S_{t-1} + \sum_{i=1}^{n} E_{it}, \tag{1}$$

where the initial stock of pollutant S_0 is given and where δ is the pollutant's natural rate of degradation ($0 < \delta < 1$). This model describes, for example, the basics of the climate change problem where the flows of emissions of greenhouse gases accumulate into a stock, which only gradually assimilates and which is the cause of the climate change.

This stock of pollutant causes damages to each country's environment. For country i ($i \in \mathcal{N}$), these damages during period t are measured in monetary terms by the function $D_i(S_t)$, where D_i is supposed to be a differentiable, increasing and convex function of the current stock S_t ($D_i' > 0$, $D_i'' \geqslant 0$). As described in (1), this current stock is a function of the inherited stock S_{t-1} and of the current emissions $\mathbf{E}_t$. The only way to control the stock of pollutant is through the control of emissions.[2] More precisely, each country i can reduce its own emissions, and the cost of doing this is described

[2] That is reducing pollution is done through the reduction of emissions, and not through the cleaning of the environment.

by a differentiable, decreasing and strictly convex function $C_i(E_i)$ ($C_i' < 0, C_i'' > 0$). $C_i(E_i)$ measures the total costs incurred by country i from limiting its emissions to E_i. The convexity of the function reflects the intuitive idea that the marginal cost of reducing emissions is higher for lower levels of emissions.

From Eq. (1), it is clear that the damages to the environment of country i will depend on the emissions of all countries. In what follows, we will consider two different modes of behaviour of the countries. A first one assumes that they behave in an internationally optimal way, i.e. that each of them takes account of the impact of its pollution not only on itself but on all other countries as well. In this case, countries *jointly* choose at each period their emission levels in order to minimize total discounted costs, i.e. for each $t \in \mathscr{T}$, they solve the following problem:

$$\min_{\{\mathbf{E}_s\}_{s\in\{t,\dots,T\}}} \left\{ \sum_{s=t}^{T} \sum_{i=1}^{n} \beta^s [C_i(E_{is}) + D_i(S_s)] \right\}$$

subject to the constraints (1) and $E_{it} \geqslant 0$ ($\forall t \in \mathscr{T}, \forall i \in \mathscr{N}$), β is the discount factor ($0 < \beta \leqslant 1$). According to Bellman's principle of optimality, the solution can be found by solving the dynamic programming equations

$$W(T, S_{T-1}) = \min_{\mathbf{E}_T} \left\{ \sum_{i=1}^{n} [C_i(E_{iT}) + D_i(S_T)] \right\}, \tag{2}$$

$$W(t, S_{t-1}) = \min_{\mathbf{E}_t} \left\{ \sum_{i=1}^{n} [C_i(E_{it}) + D_i(S_t)] + \beta W(t+1, S_t) \right\},$$

$$t = 1, 2, \dots, T-1 \tag{3}$$

subject to the constraints (1) and $E_{it} \geqslant 0$ ($\forall t \in \mathscr{T}, \forall i \in \mathscr{N}$). In (2) and (3), W is called the *value function*. As the total costs of all countries are minimized, the resulting trajectories of emissions and stock constitute the *international optimum*. The convexity of the functions C_i and D_i ($\forall i \in \mathscr{N}$) suffices to guarantee that the minimum exists and is unique.

In an alternative mode of behaviour, one may assume that countries behave non-cooperatively in the sense of a Nash equilibrium, where each of them minimizes at each period only its own discounted costs, taking as given the emissions of the other countries. Formally, at each $t \in \mathscr{T}$, each country $i \in \mathscr{N}$ solves the following problem:

$$\min_{\{E_{is}\}_{s\in\{t,\dots,T\}}} \left\{ \sum_{s=t}^{T} \beta^s [C_i(E_{is}) + D_i(S_s)] \right\},$$

subject to the constraints (1), $E_{it} \geqslant 0$ and $E_{jt}\, j \neq i$ given. Within the framework of dynamic programming this leads to the value functions

$$N_i(T, S_{T-1}) = \min_{E_{iT}} \{[C_i(E_{iT}) + D_i(S_T)]\}, \tag{4}$$

$$N_i(t, S_{t-1}) = \min_{E_{it}} \{[C_i(E_{it}) + D_i(S_t)] + \beta N_i(t+1, S_t)\}, \quad t = 1, 2, \dots, T-1 \tag{5}$$

under the constraints (1) and $E_{it} \geqslant 0$ $\forall t \in \mathscr{T}$. The convexity of the functions C_i and D_i suffices to guarantee that the Nash equilibrium exists and is unique.

The family of trajectories of emissions thus determined for each i in N, together with the resulting stock, constitute a *non-cooperative feedback Nash equilibrium* (Başar and Olsder, 1995).[3] The value functions N_i characterize the costs-to-go at this equilibrium for each country i. The emissions are functions of the current stock of pollutant and the equilibrium has the property of *Markov perfectness* (Maskin and Tirole, 1988), in the sense that it remains an equilibrium if the game is restarted at any intermediate year t from any value of the stock of pollutant.

At the international optimum, and contrary to what happens at the Nash equilibrium, each country takes account of the impact of its pollution on the environment of all other countries. Therefore, from a collective point of view, the international optimum is better than the feedback Nash equilibrium. Nothing ensures, however, that this is also true at the individual level. Indeed, countries being different, it is possible that some country at sometime t is better off at the non-cooperative equilibrium than at the optimum, so that cooperation is not profitable for this country (at least at time t). The same can occur for subsets of countries—i.e. coalitions—in the sense that, by limiting cooperation to such coalitions, the members of the latter could be better off than at the international optimum. The aim of this paper is to show that there exist financial transfers between countries that can make each one of them interested in achieving the international optimum at all periods t (individual rationality), and are such that, in addition, no sub-group of countries has ever an incentive to form a coalition and enact an optimum for itself only (coalitional rationality). We are thus aiming at a *cooperative* international optimum.

To make the above argument precise, it is important to state explicitly what the fallback position is for each country when no transfers occurs. At each time t one could take the non-cooperative feedback Nash equilibrium from t onwards as such point of reference, and determine the transfers accordingly. However, one should not neglect the fact that countries know that later on, thanks to the cooperative transfers to which they will have access, they will be better off than at the non-cooperative Nash equilibrium. Hence, a better point of reference at time t is: *non-cooperation at time t, followed by cooperation afterwards.*[4]

This is a rational expectations argument, that we introduce formally in Section 3 to deal with individual rationality, and that we use again in Section 4 to deal with coalitional rationality in the framework of a dynamic cooperative game.

3. Transfers to sustain individual rationality at the international optimum

3.1. Transfers at final time T

Let us start by determining which transfers yield gains for all countries when they cooperate in the last period, for any level of the stock of pollutant S inherited from the

[3] See Mäler and de Zeeuw (1998) and van der Ploeg and de Zeeuw (1992) for applications of such an equilibrium to acid rains and climate change, respectively.

[4] This idea was already put forward in Houba and de Zeeuw (1995) in the framework of a dynamic bargaining problem.

past. In the non-cooperative equilibrium the countries are supposed to solve problem (4). Country i's total cost is then

$$N_i(T,S) = C_i(E_{iT}^N) + D_i(S_T^N), \quad i \in \mathcal{N}, \tag{6}$$

where E_{iT}^N denotes the emission equilibrium level and S_T^N denotes the resulting stock of pollutant given by

$$S_T^N = [1-\delta]S + \sum_{i=1}^{n} E_{iT}^N. \tag{7}$$

If countries cooperate, they jointly solve problem (2). Country i's total cost is

$$W_i(T,S) = C_i(E_{iT}^*) + D_i(S_T^*), \tag{8}$$

where E_{iT}^* is the optimal emission level and S_T^* is the optimal stock of pollutant, given by

$$S_T^* = [1-\delta]S + \sum_{i=1}^{n} E_{iT}^*. \tag{9}$$

By definition of the optimum, one verifies that

$$W(T,S) \triangleq \sum_{i=1}^{n} W_i(T,S) \leqslant \sum_{i=1}^{n} N_i(T,S) \triangleq N(T,S). \tag{10}$$

The difference between the two sides of this inequality measures the *ecological surplus* resulting from international cooperation.

However, (10) is not sufficient to ensure cooperation. Indeed, if $\exists i \in \mathcal{N}$ such that $W_i(T,S) > N_i(T,S)$, then country i will not cooperate without financial compensation for the higher cost it incurs. Since dynamic programming reduces the choice of emissions to one period at the time, one can use the transfers formula proposed by Chander and Tulkens (1997) in a static framework. Let

$$\theta_i(T,S) = -[W_i(T,S) - N_i(T,S)] + \mu_{i,T}[W(T,S) - N(T,S)] \tag{11}$$

be the transfer (<0 if received, >0 if paid) to country i at time T, where $\mu_{i,T} \in]0,1[$, $\forall i \in \mathcal{N}$ and $\sum_{i=1}^{n} \mu_{i,T} = 1$. Then country i's total cost *including* transfers becomes

$$\tilde{W}_i(T,S) = W_i(T,S) + \theta_i(T,S). \tag{12}$$

By construction, the budget of the transfers defined by (11) is balanced (i.e. $\sum_{i=1}^{n} \theta_i(T,S) = 0$). Since

$$\tilde{W}_i(T,S) - N_i(T,S) = \mu_{i,T}[W(T,S) - N(T,S)] \leqslant 0, \quad \forall i \in \mathcal{N} \tag{13}$$

cooperation with transfers is *individually rational* at time T, in the sense that each country has interest to participate whatever the inherited stock of pollutant S.[5]

[5] The fact that $\mu_{i,T}$ cannot be equal to 0 ensures that country i will benefit from cooperation if $W(T,S) < N(T,S)$. The fact that $\mu_{i,T}$ cannot be equal to 1 excludes that country i monopolizes all the gains of cooperation.

3.2. Transfers at earlier periods

Countries know that, whatever they do at $T-1$, financial transfers exist (defined by (11)) that make the international optimum at T preferable for each of them with respect to the non-cooperative equilibrium. Let us assume that these transfers induce cooperation,[6] *and that countries therefore expect*, *at* $T-1$, *that they will cooperate in period* T. This is the rational expectations assumption announced at the end of Section 2. The problem we wish to consider now is whether under this assumption transfers can be designed that make the countries interested to cooperate at $T-1$ as well.

In the absence of cooperation at $T-1$, each country i minimizes its own discounted total costs over the two periods $T-1$ and T, expecting cooperation and transfers in T. Thus, given the emissions of the other countries, country i solves problem (5) for $t=T-1$ with N_i under the *min* operator replaced by $\tilde{W}_i$. This leads to

$$V_i(T-1,S)=\min_{E_{i,T-1}}\{[C_i(E_{i,T-1})+D_i(S_{T-1})]+\beta\tilde{W}_i(T,S_{T-1})\},\quad i\in\mathcal{N} \tag{14}$$

under the constraints (1) and $E_{i,T-1}\geqslant 0$, $\forall i\in\mathcal{N}$.[7] This yields an equilibrium characterized at time $T-1$ by emission levels $\mathbf{E}^V_{T-1}$ as functions of the initial stock S at time $T-1$. The value function

$$V_i(T-1,S)=C_i(E^V_{i,T-1})+D_i(S^V_{T-1})+\beta\tilde{W}_i(T,S^V_{T-1}),\quad i\in\mathcal{N}, \tag{15}$$

where

$$S^V_{T-1}=[1-\delta]S+\sum_{i=1}^{n}E^V_{i,T-1} \tag{16}$$

denotes country i's discounted equilibrium costs. We will call this equilibrium the *fallback non-cooperative equilibrium* at time $T-1$.

In the case where all countries cooperate, they solve problem (3) for $t=T-1$. Optimal levels of emissions and of the resulting stock of pollutant are denoted by $\mathbf{E}^*_{T-1}$ and S^*_{T-1}, respectively. Both are functions of S. This yields

$$W_i(T-1,S)=C_i(E^*_{i,T-1})+D_i(S^*_{T-1})+\beta\tilde{W}_i(T,S^*_{T-1}) \tag{17}$$

which is country i's part in the optimal total discounted costs, *taking into account the transfers and the resulting cooperation expected in T*.

As in period T (see (10)), one verifies that

$$W(T-1,S)\triangleq\sum_{i=1}^{n}W_i(T-1,S)\leqslant\sum_{i=1}^{n}V_i(T-1,S)\triangleq V(T-1,S). \tag{18}$$

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[6] Note that, following Chander and Tulkens (1997, Section 5), one could indeed obtain the cooperative optimum with transfers as an equilibrium, called *ratio-equilibrium*.

[7] Since the value functions $\tilde{W}_i(T,S_{T-1})$ contain transfers that sum up to zero, convexity of the cost functions C_i and D_i does not ensure that the objectives in (14) are convex, unlike what happens for T. In Section 5, we present a numerical algorithm that allows to check the second-order conditions at each period.

$V(T-1,S) - W(T-1,S)$ measures the ecological surplus induced by extending cooperation to period $T-1$, with respect to the alternative scenario where cooperation is limited to T. However, (18) is again not sufficient to induce cooperation at time $T-1$. If $\exists i \in \mathcal{N}$ such that $W_i(T-1,S) > V_i(T-1,S)$, then country i will not want to extend cooperation to period $T-1$ without financial compensation.

To induce country i to participate in $T-1$, we proceed as in period T. Let

$$\theta_i(T-1,S) = -[W_i(T-1,S) - V_i(T-1,S)] \\ + \mu_{i,T-1}[W(T-1,S) - V(T-1,S)] \tag{19}$$

be the transfer paid or received by country i at time $T-1$, where $\mu_{i,T-1} \in]0,1[, \forall i \in \mathcal{N}$ and $\sum_{i=1}^{n} \mu_{i,T-1} = 1$. Then country i's total cost *including* transfers becomes

$$\tilde{W}_i(T-1,S) = W_i(T-1,S) + \theta_i(T-1,S). \tag{20}$$

By construction, the budget of the transfers defined by (19) is balanced (i.e. $\sum_{i=1}^{n} \theta_i(T-1,S) = 0$). Furthermore, it is easy to see that these transfers make cooperation individually rational at time $T-1$, whatever the inherited stock of pollutant S.

The preceding analysis can be repeated for all earlier periods. The final result will be that the countries cooperate in each period. This determines the emission levels in each period and also the trajectory of the stock of pollutant, given its initial value S_0. In turn this trajectory determines the values of the functions V_i, W_i and $\tilde{W}_i$, and therefore also the values of the transfers θ_i. In Section 4 it will be shown that these transfers yield an interesting property from the perspective of cooperative game theory for specific values of μ_i.

In the infinite horizon case, the backward reasoning considered above applies no more. However, we can consider the stationary solution by taking advantage of the fact that the cost functions C_i and D_i as well as the sharing parameters μ_i do not depend directly on time. The functional forms of the solutions thus only vary in time through the varying stock of pollutant S. The structure of the problem is then the same as in the finite horizon case.

4. Transfers to sustain coalitional rationality at the international optimum

The only condition so far on the μ_i's appearing in (11) or (19) is to take values between 0 and 1 and to sum up to 1. The aim of this section is to utilize the degrees of freedom left to obtain the property of "coalitional rationality" suggested by the core concept in cooperative game theory. We achieve this goal by extending to dynamic games the results obtained by Chander and Tulkens (1995, 1997) in a static framework.

4.1. Associating a dynamic cooperative game with the economic model

In general, a cooperative[8] game (with transferable utility) is defined as a pair $[\mathcal{N}, w(\cdot)]$, where $\mathcal{N} = \{1,\dots,n\}$ is the set of players (i.e. the n countries), and $w(\cdot)$ is the characteristic function of the game, that is, a function that associates to every subset (i.e. coalition) of $\mathcal{N}$ a real number called the *worth of the coalition*. In the present dynamic context, we consider such a game *at each point in time* of our economic model and we therefore specify a characteristic function $w_t(\cdot, S_t)$, at each t, where S_t is the inherited stock of pollutant at that time. We then consider the sequence over time of such cooperative games $[\mathcal{N}, w_t(\cdot, S_t)]$, $t = 1,\dots,T$; this sequence defines the dynamic cooperative game that we[9] associate with our economic model.

4.2. Specifying the components of the dynamic game at each t

For each of the games $[\mathcal{N}, w_t(\cdot, S_t)]$ to be fully specified, the characteristic functions $w_t(\cdot, S_t)$ should further state: (i) what the strategies are that the players (countries) have access to, individually and as coalitions; (ii) which strategy they will actually choose; and (iii) the worth each coalition may expect from using its chosen strategy. As all this will be defined for a given time t, we drop from now on the time index until further notice, letting all emissions and stock variables be those of time t, except for $\bar{S}$ which denotes the stock inherited from the past, S_{t-1}.

As to (i), we assume that the strategy space for each country i is the interval of possible emission levels E_i at time t, i.e. $[0,\infty[$, and for each coalition $U \subseteq \mathcal{N}$ the product of $|U|$ of these intervals, where $|U|$ denotes the number of countries in the coalition U. As to (ii), when some coalition $U \subseteq \mathcal{N}$ forms (including singletons), we consider that the vector $\mathbf{E}^U$ of the strategies adopted by all players is as follows:

(a) The emissions of the coalition members are the vector $\{E_i^U\colon i \in U\}$ which is the solution of the optimization problem

$$\min_{\{E_i\}_{i\in U}} \sum_{i\in U} [C_i(E_i) + D_i(S) + \beta \tilde{W}_i(S)] \quad \text{s.t. } S = [1-\delta](\bar{S}) + \sum_{i=1}^{n} E_i, \tag{21}$$

where $\forall j \in \mathcal{N} \setminus U$, $E_j = E_j^U$ as defined by (b), and $\tilde{W}_i(S)$ is defined as in (20).

(b) The emissions E_j^U, $j \in \mathcal{N} \setminus U$, of the countries outside of the coalition, are the solutions that simultaneously solve the optimization problems

$$\min_{E_j} C_j(E_j) + D_j(S) + \beta \tilde{W}_j(S), \quad j \in \mathcal{N} \setminus U, \quad \text{s.t. } S = [1-\delta]\bar{S} + \sum_{i=1}^{n} E_i, \tag{22}$$

where $\forall i \in U$, $E_i = E_i^U$ as defined by (a) and $\tilde{W}_j(S)$ is also as in (20).

[8] See, e.g. Osborne and Rubinstein (1994, p. 857). These authors use the more recent—and actually more appropriate—alternative terminology of “coalitional game”.

[9] Other concepts of dynamic cooperative games are discussed in Section 4.5.

There are two assumptions present in this specification. The first one is the "γ-assumption", [10] according to which if a coalition forms, its members together minimize the sum of their discounted total costs (item (a)) while each country outside of it reacts by minimizing its own individual discounted total cost, that is, by turning to a natural fallback position (item (b)). The vector $\mathbf{E}^U$ is thus of the nature of a Nash equilibrium of a non-cooperative game where the players are the coalition U and the individual countries outside the coalition taken as singletons. [11] Our second assumption is the rational expectations assumption, introduced in the preceding section; it is expressed here by the inclusion of $\tilde{W}_i(S)$ and $\tilde{W}_j(S)$ in the objective functions of (21) and (22), respectively, to be minimized.

Finally, and on the basis of the above, the worth of any coalition U reads as

$$w(U;S) = \sum_{i \in U} [C_i(E_i^U) + D_i(S^U) + \beta \tilde{W}_i(S^U)], \tag{23}$$

where $S^U = [1-\delta]\bar{S} + \sum_{i=1}^n E_i^U$. Note that $w(\mathcal{N};S)$ is equal to $W(S)$ as defined in (17), with $t = T-1$, i.e. to the optimal total cost for all countries together. The explicit form of the characteristic function of the games $[\mathcal{N}, w_t(\cdot, S_t)]$ is thus (23).

4.3. Imputations at each t

For any game $[\mathcal{N}, w(\cdot;S)]$ so specified, any n-dimensional vector of which the components sum up to $w(\mathcal{N};S)$ is called an *imputation* of the game. An imputation can thus be seen as a way of sharing the optimal total cost between the players.

The vector $(W_1(S),\ldots,W_n(S))$, where $W_i(S)$ is defined as in (17), is an example of such an imputation, in which each country bears its own abatement and damage costs as induced by the optimal strategy $\{\mathbf{E}^*\}$. However, the possibility of financial transfers between the countries implies that (an infinite number of) other imputations exist, associated with the same strategy. Indeed, all vectors $(\tilde{W}_1(S),\ldots,\tilde{W}_n(S))$ defined like (20), with $\sum_i \theta_i(S) = 0$, are imputations as well.

A *solution* of the game is an imputation that satisfies certain properties. Among the imputations defined as (20), the ones that satisfy

$$\sum_{i \in U} \tilde{W}_i(S) \leqslant w(U;S) \quad \forall U \subseteq \mathcal{N} \tag{24}$$

are said *to belong to the γ-core* of the game. In words, the core is the set of imputations with the property that each possible coalition bears a fraction of the total costs $W(S)$ less than or equal to $w(U;S)$, i.e. the least cost that this coalition can guarantee for itself. Imputations belonging to the core of the game defined above are therefore called "rational in the sense of coalitions" since the members of any coalition would suffer a total cost higher than the one at the optimum with transfers.

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[10] Borrowed from the concept of "γ-characteristic function" introduced by Chander and Tulkens (1995, 1997).

[11] This equilibrium is called by Chander and Tulkens a *partial agreement Nash equilibrium with respect to the coalition U*. The authors show that such an equilibrium exists and is unique under assumptions similar to those of our economic model.

4.4. Transfers inducing γ-core imputations at each t

Recall from Section 3 that $\mathbf{E}^*$ was defined as the vector of optimal emission levels and S^* as the optimal stock level, that $\mathbf{E}^V$ was the vector of emission levels and S^V the stock characterizing the fallback non-cooperative equilibrium, and that $(W_1(S),\dots,W_n(S))$ and $(V_1(S),\dots,V_n(S))$ were, respectively, the vectors of discounted total costs at the optimum and at the fallback non-cooperative equilibrium (cf. (17) and (15)), with $W(S)$ and $V(S)$ the sum of these vectors' components. Recall also that all variables mentioned here are defined at time t.

For the game $[\mathcal{N}, w(\cdot;S)]$, consider now the imputation $(\tilde{W}_1(S),\dots,\tilde{W}_n(S))$ defined by

$$\tilde{W}_i(S) = W_i(S) + \tilde{\theta}_i(S) \quad \forall i \in \mathcal{N}, \tag{25}$$

where $\tilde{\theta}_i(S)$ is a transfer of the form

$$\tilde{\theta}_i(S) = -[C_i(E_i^*) - C_i(E_i^V)] + \tilde{\mu}_i(S^*) \sum_{j=1}^{n} [C_j(E_j^*) - C_j(E_j^V)] \tag{26}$$

with $S^* = (1-\delta)S + \sum_i E_i^*$ and where

$$\tilde{\mu}_i(S^*) = \frac{F_i'(S^*)}{\sum_{j=1}^{n} F_j'(S^*)} \tag{27}$$

with

$$F_i(S^*) = D_i(S^*) + \beta \tilde{W}_i(S^*). \tag{28}$$

We can now state the following.

Theorem. *The imputation (25) belongs to the γ-core of the game $[\mathcal{N}, w(\cdot;S)]$, if one of the following two conditions is satisfied*:

(i) *$\forall i \in \mathcal{N}$, D_i is linear, or*
(ii) *$\forall i \in \mathcal{N}$, $\tilde{W}_i$ is monotonically increasing and convex and, $\forall U \subseteq \mathcal{N}$,*

$$\sum_{i \in U} [C_i(E_i^U) - C_i(E_i^V)] \geqslant 0, \tag{29}$$

where $\mathbf{E}^U$ is the vector of emission levels at the partial agreement Nash equilibrium with respect to coalition U.

Proof. Case (i) linear damage costs—is proven in Germain et al. (1998b). The proof of case (ii) is given in Germain et al. (1998c) available from the authors upon request.[12] □

[12] Notice that the transfers (26) are formulated differently than in (11) or (19). The first part of the proof of case (ii) shows that the former with (27) and (28) are equivalent to the latter.

4.5. Cooperative international optimum and alternative cooperative dynamic games

Turning back now to the cooperative dynamic game defined in Section 4.1, i.e. the sequence of games $[\mathcal{N}, w_t(\cdot, S_t)]$, $t = 1, \ldots, T$ associated with the economic model of Section 2, consider the internationally optimal trajectories of emissions and stock defined there *together* with, at each time t, the transfers defined by (26)–(28).

These optimal trajectories, combined with γ-core-theoretic transfers are the solution we propose for the cooperative dynamic game. We call this combination the *cooperative international optimum*. As we have argued above, this optimum is cooperative in a rational expectations sense.

Recently, Kranich et al. (2001) have proposed three other core concepts for cooperative dynamic games, called, respectively, the "classical", the "weak sequential" and the "strong sequential" cores. Each one of these core concepts derives from a different form of the characteristic function, the differences residing in the assumptions made on how "deviations" by coalitions may (or may not) occur over time. Our characteristic function (and hence our core concept) is actually a fourth one, in its dynamic aspects (independently of its γ nature). It is based on the assumption that if deviations occur at time t, they do not continue afterwards because from time $t+1$ on, cooperation with our transfers is more beneficial for all than non-cooperation. This rational expectations attitude assumed to be adopted by the players at time t, was expressed summarily above as "noncooperation (i.e. deviation) at time t, followed by cooperation afterwards". Our characteristic function is thus a "rational expectations characteristic function", and hence our core concept a "rational expectations core".

Referring to the formulation proposed in another paper by Filar and Petrosjan (2000), it seems to us that the economic model to which our dynamic games are associated implies (through the role played by the stock externality) that, in the notation of these authors (their Eq. (2.1)), the function v at time $t+1$ is determined by the solution retained at time t, that is, along the cooperative international optimum we have defined, by the Pareto optimal stock prevailing at t. We thus present an application of the model of dynamic games of these authors.

4.6. Two further comments

The transfers (26) used in the theorem are the sum of two components. The first term in the right-hand side of (26) is either (if > 0) an amount received by country i equal to the increase of its abatement costs due to its cooperating behaviour during this period, or (if < 0) a payment made by country i equal to its savings in abatement costs, in the case where cooperation allows for an increase in this country's emissions. The second term (always < 0) is country i's contribution to cover the total cost of the aggregate abatement effort of the cooperating countries.

Other remarks concern condition (ii). Notice that it is a sufficient, not a necessary one. It requires, with (29), that if a coalition forms, the total of the abatement costs of its members are larger than what this total would be at the fallback non-cooperative equilibrium. How likely are such coalitions? In order for a coalition U

to form—irrespective to its power to oppose (25)—it should at least satisfy

$$\sum_{i\in U}[C_i(E_i^U)-C_i(E_i^V)]+\sum_{i\in U}[F_i(S^U)-F_i(S^V)]\leqslant 0 \tag{30}$$

since otherwise its members are all better off at the fallback non-cooperative equilibrium. In (30) the magnitudes in the bracketed differences under the second summation sign are always negative, because S^U is smaller than S^V. Condition (ii) thus states that a coalition U that forms can only oppose (25) if in addition the first sum in (30) is negative, i.e. if the aggregate abatement costs of its members are lower than what they are at the fallback non-cooperative equilibrium. [13] However, one may think that the existence of such coalitions is unlikely because, as it is shown in the proof of the theorem, the sum of the emissions of the members of any coalition is always lower than what it is at the fallback non-cooperative equilibrium.

5. Computability issues

The aim of this section is to describe an original numerical algorithm that calculates the value functions and the transfers when the cost functions C_i and D_i are convex. An original algorithm is indeed needed because in the backward induction with the complex transfer parameters, the problem is not analytically tractable, contrary to number of linear-quadratic problems in the literature (such as in Mäler and de Zeeuw, 1998, for example). It is written for finite horizon problems, which is appropriate for most practical applications. The emissions and stock trajectories associated with the international optimum can be calculated by traditional nonlinear programming techniques. The transfers are more difficult to calculate, because they make use of values calculated at the fallback non-cooperative equilibrium, so that one must proceed by backward induction. The basic idea is to construct explicit approximations for the surfaces $\tilde{W}_i(t,S_{t-1})$, $i\in\mathcal{N}$, as polynomial functions of S_{t-1}, by using classical regression.

5.1. Statement of the algorithm

The algorithm consists of several steps:

Step 1: The algorithm starts by solving the optimization problem (2) associated with the international optimum. The first-order conditions are

$$C_i'(E_{it}^*)+\sum_{\tau=t}^{T}\beta^{\tau-t}[1-\delta]^{\tau-t}\sum_{j=1}^{n}D_j'(S_\tau^*)=0\quad \forall i\in\mathcal{N},\forall t\in\mathcal{T} \tag{31}$$

and

$$S_t^*=[1-\delta]S_{t-1}^*+\sum_{i=1}^{n}E_{it}^*\quad \forall t\in\mathcal{T}, \tag{32}$$

[13] In a static framework, Chander and Tulkens (1997) propose a condition on the marginal damages that ensures that all members of a coalition decrease their emissions w.r.t. the non-cooperative equilibrium. Adapted to our framework, this condition is that $\forall U\subset\mathcal{N}$, $U\neq\mathcal{N}$, $|U|\geqslant 2$, $\sum_{i\in U}F_i'(S^*)\geqslant F_j'(S^V)$, $j\in U$. This condition guarantees (29).

where C_i' and D_j' are the derivatives of functions C_i and D_j, respectively. Solving this $T(n+1)$ equations yields the optimal values of the emissions and of the stock of pollutant for all periods.

Step 2(a): To calculate the transfers, one must first solve the dynamic programming problems associated with the fallback non-cooperative equilibrium. The algorithm proceeds backwards, starting from the last period. At the final time T, given the inherited stock of pollutant S, the fallback non-cooperative equilibrium (which coincides with the Nash equilibrium at the last period) has to satisfy the conditions

$$C_i'(E_{iT}^N) + D_i'\left([1-\delta]S + \sum_{j=1}^{n} E_{jT}^N\right) = 0, \quad i \in \mathcal{N}. \tag{33}$$

Once this system of equations has been solved for the variables E_{jT}^N, and knowing by step 1 the optimal emissions at time T, it is possible to calculate the transfers $\theta_i(S,T)$ using (26) (with $\tilde{\mu}_i(S_T^*) = D_i'(S_T^*)/(\sum_{j=1}^n D_j'(S_T^*))$) as well as the value functions $\tilde{W}_i(S,T)$ using (25).

Step 2(a) is done for a set Ω_T of given values of the inherited stock of pollutant S. This set is chosen to be representative of the interval of possible values of S. This interval is bounded below by the value of S that would be obtained with zero emissions during periods $1,2,\ldots,T-1$, given S_0, and bounded above by the value of S that would be obtained with maximum emissions during the same period.

Step 2(b): We now assume that total costs with transfers at time T have the following polynomial form:

$$\tilde{W}_i(T,S) = k_{iT,m}S^m + k_{iT,m-1}S^{m-1} + \cdots + k_{iT,0}, \quad i \in \mathcal{N}, \tag{34}$$

where m is the order of the polynomial chosen so that the fit is good enough. To identify the parameters $k_{iT,m}, k_{iT,m-1}, \ldots, k_{iT,0}$, we use an ordinary least square regression method implemented in the software Matlab, so that the functions $\tilde{W}_i(T,S)$ are now approximated with *explicit analytical functions* of S.

Steps 2(a) and (b) are then repeated for period $T-1$. Given (14) and the inherited stock of pollutant S, first-order conditions associated to the fallback non-cooperative equilibrium are

$$C_i'(E_{it}^V) + D_i'\left([1-\delta]S + \sum_{j=1}^{n} E_{j,T-1}^V\right) + \beta \tilde{W}_i'\left(\left[[1-\delta]S + \sum_{j=1}^{n} E_{j,T-1}^V\right], T\right) = 0, \quad i \in \mathcal{N}. \tag{35}$$

Once this system of equations has been solved for the $E_{j,T-1}^V$, knowing the optimal emissions at time $T-1$ (by step 1) and the value functions $\tilde{W}_i(S,T)$ (by application of steps 2(a) and (b) for time T), it is possible to calculate the transfers $\theta_i(S,T-1)$ using (26)–(28) and the value functions $\tilde{W}_i(S,T-1)$ using (25). This is done for a set Ω_{T-1} of given values of the inherited stock of pollutant S, so that by regression of

Table 1
Optimal abatement rates (%) and atmospheric temperature changes (°C)

t	η_1	η_2	η_3	ΔT
1995	14.29	30.45	22.49	0.87
2005	14.47	28.74	23.08	1.00
2015	14.67	27.46	23.61	1.15
2025	14.94	26.50	24.09	1.29
2035	15.19	25.81	24.54	1.44
2045	15.37	25.31	24.96	1.59
2055	15.51	24.95	25.34	1.74
2065	15.63	24.70	25.68	1.89
2075	15.74	24.53	25.99	2.03
2085	15.83	24.42	26.28	2.18
2095	15.91	24.34	26.53	2.32
2105	15.98	24.29	26.76	2.47

the calculated values of the value functions $\tilde{W}_i(S, T-1)$ on Ω_{T-1}, the value functions $\tilde{W}_i(S, T-1)$ can be approximated with explicit analytical functions of S.

The algorithm continues backwards by repeating steps 2(a) and (b) until period 1.

Step 3: Once the value functions $\tilde{W}_i$ are known as functions of S for all times of the planning period $\mathscr{T} = \{1, \ldots, T\}$, the algorithm calculates the actual values of these value functions and of the transfers for all countries all along the optimal trajectory calculated at step 1.

5.2. A numerical example

In the following, we show some numerical results obtained by application of the algorithm to a simple climatic change model with three regions and 30 periods (originally considered by Germain and van Ypersele (1999) in a six regions framework). The three regions considered are Industrialized Countries (IC),[14] China and Rest of the World (RW). Periods are decades, starting with decade 1990–2000. Details of the equations and values of the parameters are given in Appendix A. The polynomials used are of degree three.

First, by regressing the value functions $\tilde{W}_i$ on a large set of possible values of the inherited stock of pollutant S, we were able to verify that the objective function of each country is well behaved and yields existence and uniqueness of the fallback non-cooperative equilibrium at each period.

We show the results for a planning horizon extending over 11 periods (i.e. until the end of the 21st century), but to avoid boundary problems, computations were made until $T = 30$.[15] Table 1 shows the evolution of the optimal abatement rates of emissions η_i (i.e. the reduction of emissions induced by the optimal solution w.r.t. the BAU scenario, see Appendix A) and atmospheric temperature (which is itself a

[14] The region IC groups Europe, US, Japan and Former Soviet Union.

[15] As t approaches T, results become meaningless because one ignores environmental damages after T.

Table 2
Individual pay-off differences between fallback equilibrium (V_i) and optimal trajectories without (W_i) and with ($\tilde{W}_i$) transfers (θ_i) between the regions $i = 1$–3 (in billion 1990 US$)

t	$V_1 - W_1$	$V_2 - W_2$	$V_3 - W_3$	θ_1	θ_2	θ_3	$V_1 - \tilde{W}_1$	$V_2 - \tilde{W}_2$	$V_3 - \tilde{W}_3$
1995	41.63	−12.76	51.65	10.34	−17.25	6.91	31.30	4.48	44.73
2005	41.26	−11.44	51.17	11.09	−16.13	5.04	30.17	4.69	46.13
2015	40.42	−10.30	50.58	11.56	−15.16	3.60	28.86	4.86	46.98
2025	39.48	−9.34	49.67	11.97	−14.34	2.37	27.51	5.00	47.30
2035	38.44	−8.54	48.50	12.28	−13.64	1.36	26.17	5.09	47.14
2045	37.15	−7.86	47.20	12.35	−13.01	0.66	24.81	5.15	46.54
2055	35.62	−7.27	45.72	12.22	−12.42	0.20	23.41	5.15	45.52
2065	33.90	−6.75	44.04	11.93	−11.86	−0.07	21.97	5.11	44.11
2075	32.03	−6.28	42.16	11.51	−11.31	−0.20	20.52	5.03	42.36
2085	30.04	−5.86	40.09	10.98	−10.75	−0.23	19.05	4.90	40.32
2095	27.97	−5.45	37.83	10.38	−10.18	−0.20	17.59	4.72	38.04
2105	25.85	−5.06	35.43	9.72	−9.57	−0.14	16.14	4.51	35.57

function of the stock of CO_2 in the atmosphere, see Appendix A). The evolutions of the abatement rates are contrasting: increasing slowly for IC (indexed as 1 in the table) and RW (indexed as 3), decreasing slowly for China (indexed as 2). Despite the control of emissions, atmospheric temperature continues to increase monotonically during the whole century. The increase of temperature amounts to around 2.5°C at the end of the 21st century compared to its preindustrial level.

The first three columns of Table 2 give the differences between the pay-offs of the three regions along the international optimum (without current transfers) and at the fallback non-cooperative equilibrium, ($V_i - W_i$). Positive (negative) values mean that the region is better (worse) off at the optimum. While IC and RW are better off in each period, China is worse off in every period. Thus, the international optimum does not satisfy individual rationality.

The following three columns of Table 2 give the transfers θ_i received or paid by each region to make the international optimum trajectory a cooperative one. A negative (positive) value means that the transfer is received (paid). China receives at each period a transfer that induces this country to cooperate. The principal donor is IC, but RW is also a donor until the middle of the century. After 2060 RW also becomes a receiver, but to a far less extent than China.

That these transfers are sufficient to induce cooperation from all regions at all periods is shown by the last three columns of Table 2, which give for each region the difference between its pay-off at the cooperative optimum (i.e. with current transfers) and at the fallback non-cooperative equilibrium ($V_i - \tilde{W}_i$). All values are positive which means that all regions (in particular China) win from cooperation at all periods. In other words, taking into account that cooperation prevails after t, all regions are induced to extend cooperation to current period t thanks to current transfers at period t.

Table 3 reports on differences between the pay-offs obtained by coalitions along the international optimum (with and without transfers) and at the partial agreement Nash equilibria w.r.t. these coalitions (as defined in Section 4.1). The purpose is to check

Table 3
Coalition pay-off differences between partial agreement Nash equilibria w.r.t. coalitions ($V_{(\cdot)}$) and optimal trajectories without ($W_{(\cdot)}$) and with ($\tilde{W}_{(\cdot)}$) transfers (In billion 1990 US$. Coalitions: I,R: IC and RW; C,R: China and RW; I,C: IC and China.)

t	$V_{C,R}-W_{C,R}$	$V_{C,R}-\tilde{W}_{C,R}$	$V_{I,R}-W_{I,R}$	$V_{I,R}-\tilde{W}_{I,R}$	$V_{I,C}-W_{I,C}$	$V_{I,C}-\tilde{W}_{I,C}$
1995	27.75	38.08	68.69	51.45	21.32	28.23
2005	28.27	39.36	66.88	50.75	23.18	28.22
2015	28.56	40.12	65.26	50.10	24.24	27.84
2025	28.39	40.36	63.80	49.47	24.85	27.23
2035	27.89	40.17	62.42	48.78	25.11	26.47
2045	27.24	39.58	60.93	47.93	24.91	25.57
2055	26.407	38.62	59.26	46.84	24.34	24.55
2065	25.39	37.31	57.35	45.49	23.47	23.40
2075	24.21	35.72	55.17	43.85	22.36	22.16
2085	22.90	33.88	52.70	41.95	21.08	20.85
2095	21.47	31.85	49.97	39.79	19.67	19.47
2105	19.97	29.69	46.99	37.42	18.20	18.06

coalitional rationality. It turns out that, according to Columns 1, 3 and 5, no coalition can do better by itself than what it obtains at the international optimum. Coalitional rationality is thus satisfied without transfers. But this property is particular to this example and parameter values: indeed, it is not satisfied in other similar cases, as for example in Eyckmans and Tulkens (1999), where coalitional rationality only obtains with financial transfers.

5.3. Comparing open-loop and close-loop models

It is interesting to compare the negotiation process described in this paper with the open-loop case, where the countries negotiate once and for all at the beginning of the planning period. Germain et al. (1998a) have defined the following transfers that ensure that all countries are interested to cooperate in the open-loop framework:

$$\Theta_i = -\sum_{t=1}^{T} \alpha^t [C_{it}(E_{it}^*) - C_{it}(E_{it}^{\mathrm{o}})] + \sum_{t=1}^{T} \alpha^t \frac{D'_{i,t+1}(S_{t+1}^*)}{\sum_{j=1}^{n} D'_{j,t+1}(S_{t+1}^*)} \sum_{j=1}^{n} [C_{jt}(E_{jt}^*) - C_{jt}(E_{jt}^{\mathrm{o}})], \tag{36}$$

where E_{it}^{o} $(i \in \mathcal{N}, t \in \mathcal{T})$ is the emission of country i at time t characterizing the *non-cooperative open-loop Nash equilibrium.*[16] Table 4 summarizes the results.

Columns 1 and 2 give the total discounted costs for each region at the optimum ($W_i(S_0)$) and at the non-cooperative open-loop Nash equilibrium ($O_i(S_0)$). It is clear that China has no interest to cooperate without financial compensation. Column 3 gives

[16] Not to be confused with the *feedback* Nash equilibrium defined in Section 3.

Table 4
Comparison with the open-loop case

i	$W_i(S_0)$	$O_i(S_0)$	$\Theta_i(S_0)$	$W_i(S_0)+\Theta_i(S_0)$	$\sum_t \theta_i(t,S_{t-1})$
IC	10352.0	10839.4	284.4	10636.4	202.3
CHINA	2321.5	2159.0	−240.1	2081.5	−222.1
RW	20078.9	20627.6	−44.3	20034.6	19.9

the transfers $\Theta_i(S_0)$ calculated by formula (36). With these transfers, it appears from the comparison of columns 2 and 4 that all regions gain from cooperation. Finally, column 5 gives the sum of transfers given or received by the three regions all along the planning period in the closed-loop case. Comparison of columns 3 and 5 shows that although of the same magnitude, the transfers given by IC and RW to compensate China are different in the closed-loop framework than what they are in the open-loop case.

The closed-loop case in general leads to different results than the open-loop case. The reason is that the closed-loop case allows for more strategic behaviour because the countries observe the stock of pollution at each point in time and condition their emissions on these observations. When a country considers to increase emissions, this country will argue that it leads to a higher stock and thus to less emissions by the other countries, which then partly offsets the increase. Therefore, in equilibrium, emissions will be higher in the closed-loop case than in the open-loop case.

In the model of this paper two effects influence the transfers when compared to the open-loop case. On the one hand, the closed-loop case leads to more emissions so that more transfers are needed. On the other hand, since the construct with a fallback equilibrium assumes cooperation in the future, it brings the outcome closer to the full-cooperative one, so that less transfers are needed.

One cannot say a priori whether the closed-loop case leads to higher or lower transfers than the open-loop case. It is clear that when the solutions coincide, the transfers are the same. This occurs when the damage function is linear, as shown in Germain et al. (1998b), because in that case emissions do not depend on the stock.

6. Conclusion

This paper develops transfer schemes that yield both individual and coalitional rationality for the design of international agreements that seek to achieve global optimality in stock pollutant control problems, in case these agreements can in principle be negotiated at each point in time. The paper also provides an algorithm to calculate the transfers, whose functioning is illustrated by a numerical example applied to the climate change problem in the context of a partition in three regions of the world.

Our approach presents nevertheless some limits, that invite to further research. First, the transfer scheme is supposed to guarantee the cooperative outcome simply because it makes cooperation beneficial for all countries. It would be interesting to drop this hypothesis and try to sustain the cooperative outcome as a non-cooperative equilibrium. Another track of research would be to study what happens when, for one reason or another, a country deviates from the international agreement at a certain period. Technical work could also be done on the algorithm. Its main strengths are its relative simplicity and its ability to solve problems that are not linear-quadratic, but it is limited to one state variable problems and several mathematical questions could be deepened (for example the question of the propagation of numerical errors due to the successive regressions).

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Appendix A. equations and parameters used in the numerical example

The model:

$$\eta_{it} = 1 - \frac{E_{it}}{\sigma_{it} Y_{it}}, \tag{A.1}$$

$$S_{t+1} - S_0 = [1-\delta][S_t - S_0] + \beta \sum_{i=1}^{n} E_{it}, \tag{A.2}$$

$$\Delta T_t = \gamma \ln\left(\frac{S_t}{S_0}\right), \tag{A.3}$$

$$C_{it}(E_{it}) = a_{i1}\eta_{it}^{a_{i2}} Y_{it} = a_{i1}\left[1 - \frac{E_{it}}{\sigma_{it} Y_{it}}\right]^{a_{i2}} Y_{it}, \tag{A.4}$$

$$D_{it}(S_t) = b_{i1}\Delta T_t^{b_{i2}} Y_{it} = b_{i1}\left[\gamma \ln\left(\frac{M_t}{M_0}\right)\right]^{b_{i2}} Y_{it}, \tag{A.5}$$

where E_{it} is the CO_2 emissions of region i at time t, σ_{it} the CO_2 emissions/output ratio (exogenous), Y_{it} the output (exogenous), η_{it} the control rate of the emissions, S_t the stock of CO_2 in the atmosphere, ΔT_t the variation of the atmospheric temperature (w.r.t. to its preindustrial level), C_{it} the abatement costs, D_{it} the damage costs.

The value of the parameters $S_0 = 590$ billion tons of carbon (preindustrial level), $\delta = 0.0833$ (rate of decay of CO_2 in the atmosphere between two periods), $\beta = 0.64$ (marginal atmospheric retention ratio of CO_2), γ=2.5/ln(2), annual discount rate=0.02.

Parameters of the cost and damage functions are as follows:

i	IC	CHI	RW
a_{i1}	0.07	0.15	0.1
a_{i2}	2.887	2.887	2.887
b_{i1}	0.01102	0.015523	0.02093
b_{i2}	1.5	1.5	1.5

Output (Y_{it}) and CO_2 emission/output ratio time series are borrowed from Germain and van Ypersele (1999).[17]

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