

Tilburg University

Risk, Inside Money, and the Real Economy

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Publication date:
2021

Document Version
Early version, also known as pre-print

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

van Buggenum, H. (2021). *Risk, Inside Money, and the Real Economy*. (CentER Discussion Paper; Vol. 2021-020). CentER, Center for Economic Research.

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No. 2021-020

RISK, INSIDE MONEY, AND THE REAL ECONOMY

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19 July 2021

ISSN 0924-7815
ISSN 2213-9532

Risk, Inside Money, and the Real Economy*

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July 19, 2021

Abstract

In modern economies, most money takes the form of inside money; deposits created by commercial banks to fund credit extension. Because inside money is used as a payment instrument, doubts about the risks associated with credit extension can affect aggregate outcomes. This paper constructs and analyzes a model of risky credit extension, inside money creation, and monetary exchange. When credit extension is sufficiently risky, a positive probability of bank default arises and this affects the return characteristics of inside money. Depositors then demand a risk premium for holding inside money, which drives a wedge between bankers' funding costs and the social benefits of money creation. This wedge negatively affects credit extension, output, and welfare. A government can restore efficiency by swapping risky inside money for risk-free forms of government debt.

Keywords: inside and outside money, risk, policy, investment, new monetarism.

JEL Codes: E41, E43, E51, E52, E63.

*This paper is based on Chapter 3 of my PhD dissertation, entitled *Banks and Financial Markets in Microfounded Models of Money*, at Tilburg University. I would like to thank Lukas Altermatt, David Andolfatto, Aleksander Berentsen, Jeffrey Campbell, Fabio Canetg, Reyer Gerlagh, Pedro Gomis-Porqueras, Morgane Richard, Florian Sniekers, Burak Uras, Ping Wang, and Randall Wright for valuable discussions and comments. I also thank seminar participants at the Tilburg University Econ Workshop, the 2018 and 2019 Bernoulli Center for Economics Workshop on Monetary Economics, the 2018 Dutch Economists Day, the 2019 RGS Doctoral Conference, and the 2019 ENTER Jamboree Conference.

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1 Introduction

Inside money is issued by private intermediaries in the form of liabilities. Most inside money takes the form of deposits created by commercial banks when they extend credit to the private sector. Rough estimates suggest that in developed economies, inside money constitutes approximately 80% to 90% of the broad money supply.¹ The remaining fraction of money is called outside money and mostly takes the form of government debt. Despite the relevance of inside money, the consensus among most economists during the Great Moderation was that inside money creation just reflects, and not affects, economic conditions.² The experience of the 2007 financial crisis however demonstrated that the risks associated with inside money creation, for example the perceived possibility of bank default, can cause a severe recession.

The current paper analyzes how concerns associated with risky credit extension can affect economic efficiency through the fact that money creation and credit extension are two sides of the same coin. The paper develops a model that unifies risky credit extension, inside money creation by banks, and an explicit role for monetary exchange. In the literature such a model has not been considered before, while unifying the three features mentioned above is important for understanding how credit conditions, through the creation of money and monetary exchange, affect economic performance.

I uncover that the possibility of bank default gives rise to a channel through which aggregate credit risk affects output and welfare. Specifically, when the risk embedded in credit extension is large so that it cannot be absorbed by bankers' equity in bad states of the world, the bankers in my model are only partially compensated for the social benefits of inside money creation. The intuition for this result is that the bankers cannot commit to re-capitalize their banks in case their equity is fully depleted. This means that when credit extension is sufficiently risky, the depositors demand a risk premium for funding banks. Due to the wedge between bankers' funding costs and the social benefits of money creation, the model economy experiences a credit crunch which reduces money creation, output, and welfare. When in all states of the world bankers' equity remains positive, the wedge disappears and the supply of money and credit are socially optimal.

The welfare implications of aggregate risk give rise to an active role for policy: By injecting capital into banks or by purchasing risky assets from banks, a government can restore efficiency when there are concerns associated with the possibility of bank default. Being financed by issuing risk-free government debt, these policies work because the government is effectively swapping risky inside money for risk-free outside money. The

¹For example, the 2007 level for the Euro Area was approximately 80%.

²Woodford (2008), for example, discusses some arguments in favor of assigning a role for monetary aggregates in making decisions on monetary policy. Woodford (2008) argues that none of these arguments provide a compelling reason to assign an important role for monetary aggregates in the conduct of monetary policy. McCallum (2008) refines the discussion of Woodford (2008).

government thus takes over the risk of credit extension that cannot be born by the bankers themselves and this has fiscal implications; taxation becomes contingent on the return earned by risky credit extension so that risk is transferred from the banking system to the tax payers.

The theoretical results outlined above formalize the concerns that gave rise to the policies deployed during the 2007 financial crisis and, at least partially, explain why these policies have been effective. Early policy interventions during the onset of the crisis dealt with individual financial institutions that had already failed. As the crisis deepened the perceptions of the overall default risk in the financial sector however rose, leading to concerns about a potential collapse of credit extension and money creation. This convinced policymakers of the need for a systematic approach towards insolvency risk.³

In the United States, such an approach was implemented by means of the Troubled-Asset Relief Program (TARP). TARP included, among other measures, investment in bank capital through the Capital Purchase Program and debt security purchases through the Public-Private Investment Program and the Securities Purchase Program. Evidence from the TARP literature shows that these policies have led to more credit extension and less systematic risk in the banking sector, matching the model's theoretical predictions.⁴

The theoretical results of the current paper arise in a modified version of the workhorse model of monetary exchange developed by Lagos and Wright (2005). The bankers in the model have access to a decreasing returns-to-scale (DRS) investment opportunity, representing credit extension, that is subject to undiversifiable systematic risk. The bankers finance investment by issuing inside money to the non-bankers and they also create equity by contributing own resources to investment. Because of commitment frictions, the bankers cannot be forced to contribute additional resources when equity becomes negative. This feature captures banks' limited liability and implies that inside money can be risky due to the possibility of bank default. The non-bankers use money to trade goods in a market where exchange is *quid pro quo*. Just before the non-bankers start trading in this market, news is released regarding the return on bankers' investment projects, which potentially affects non-bankers' valuation of inside money. After trade has taken place and the returns from investment have materialized, the bankers redeem inside money. If the bankers' returns from investment are sufficiently small, bank equity is fully eroded and the holders of inside money, that are the non-bankers, face losses.

Because the bankers are able to absorb some risk from investment with their equity, depending on the parameters, the model's equilibrium exhibits either risk-free inside money or risky inside money. The latter type of equilibrium is at the core of the analysis

³Among others, Calomiris and Khan (2015) point out that Secretary of the US Treasury Henry Paulson and FED Chairman Ben Bernanke testified numerous times together before Congress in mid-to late-September 2008 in favor measures to prevent a systemic collapse of the banking sector.

⁴See Section 2 for a short discussion of the findings from the TARP literature.

and represents a situation in which the risk associated with credit extension implies a strictly positive probability of a systemic solvency crisis. There are at least three reasons, discussed below, for why an equilibrium characterized by risky inside money arises.

First, risky inside money can arise when the aggregate risk associated with investment increases or when bankers' leverage increases. Though the former may sound as a tautology, only if credit extension is sufficiently large bank equity is unable to absorb increased risk. An increase in bankers' leverage occurs when the DRS property of investment vanishes, eliminating rents earned by the bankers, or when the fraction of bankers in the economy shrinks, so that every banker needs to finance a greater amount of credit extension with the same amount of equity.

Second, risky inside money may arise because of a drop in aggregate investment productivity. If investment productivity is large, the bankers find it attractive to extend much credit and as a result the supply of money is large. This can imply that the non-bankers end up with idle money balances that they only use as a savings vehicle. The non-bankers are therefore willing to swap these idle balances for illiquid securities. Effectively, the bankers can then finance themselves by issuing a combination of risk-free money and risky illiquid debt securities. When investment productivity reduces and hence the supply of inside money contracts, there comes a point at which idle money balances disappear. Liquidity premia which reflect the marginal value of money as a payment instrument then arise, refraining the bankers from issuing illiquid debt securities since issuing inside money becomes a cheaper source of funding.

Third, risky inside money can arise following an increased demand for money from the non-bankers. This increased demand can be the result of an increased importance of money as a payment instrument or a reduction in the supply of outside money. The increased demand for money drives down bankers' funding cost so that they increase credit extension and leverage. Increasing government debt is therefore not only attractive because it fosters monetary exchange through an increased supply of outside money, but also because it reduces the reliance of an economy on potentially risky inside money. Hence, the current paper also indicates that policies aiming to reduce government debt accumulated during the financial crisis, may have negative consequences.

The remainder of this paper develops as follows. Section 2 briefly discusses the related literature. Section 3 introduces the model setup, and defines the notion of equilibrium and welfare. Section 4 analyzes risk in an economy without government. Section 5 analyzes the effects of outside money and characterizes policies that increase economic efficiency. Section 6 concludes the analysis. Lemmas and propositions that do not immediately follow from statements in the main body of the paper, are proven in Appendix C.

2 Related Literature

On the conceptual level, the current paper draws heavily from the new monetarist literature. To introduce inside money creation in the model of Lagos and Wright (2005) I build upon work by Williamson (2012) and, following Altermatt (2017), I assume that each banker has access to a DRS investment project similar to that in Aruoba and Wright (2003), and Lagos and Rocheteau (2008). To let risk play meaningful role, I use insights from Andolfatto, Berentsen, and Waller (2014) and Dang, Gorton, Holmström, and Ordoñez (2017): Just before monetary exchange takes place, news is realized affecting the value of (inside) money. My approach differs from the above mentioned papers in three fundamental ways.

First, I assume that goods cannot be produced when monetary exchange takes place. Instead, agents trade money for goods produced in the past; inventories. Existing studies of risky inside money, for example by Andolfatto et al. (2014) and Dang et al. (2017), obtain results based on consumption smoothing. In my model, this mechanism is not operational; goods that are sold in exchange for money were produced in the past and this implies that in equilibrium, consumption levels are independent of the realized aggregate state of the economy. Nevertheless, the model re-confirms findings of Andolfatto et al. (2014) and Dang et al. (2017): If we remove the model feature which makes risky money relevant, namely that the non-bankers learn about the returns on the bankers' investment projects just before they trade money for goods, social welfare improves.

Second, I deviate substantially from Andolfatto et al. (2014). These authors focus on exogenously given outside money, while I focus on endogenous inside money issued to finance risky investment. This allows for a study of how risk affects the level of investment, something that is infeasible in the setup of Andolfatto et al. (2014).

Third, contrasting existing models with banks like that of Williamson (2012) and Altermatt (2017), in my setup agents become bankers randomly and this feature allows the risk characteristics of inside money to arise endogenously. Because the agents learn whether they are a banker only after the production of resources has taken place, the bankers own a limited amount of resources that they can invest without resorting to external finance. In this sense, bank capital arises endogenously in the model.

My approach also relates to papers on stabilization policies in monetary economies, for instance Berentsen and Waller (2011, 2015) and Draack (2018). They study how short term changes in policy rates or money supply can improve welfare in a monetary economy that faces aggregate shocks. However, only risk-free outside money is considered. By introducing risky inside money, my approach complements the aforementioned papers by identifying stabilization policies to make money risk-free.

Since inside money is backed by productive investment projects in my model, my results align with papers on liquid real assets. Examples include Geromichalos, Licari, and

Suárez-Lledó (2007), Lagos and Rocheteau (2008), Andolfatto, Berentsen, and Waller (2016), and Geromichalos and Herrenbrueck (2016). All these papers predict that real assets earn a liquidity premium when they overcome trading frictions. Moreover, if these real assets reflect scalable investment opportunities, over-investment occurs. With investment risk present in my model, I can show how over-investment depends on aggregate risk.

Due to the risk-bearing role of bankers, the current paper is also related to the literature studying assets in models with heterogeneous risk appetites. Examples include Genaioli, Shleifer, and Vishny (2012), Barro, Fernández-Villaverde, Levintal, and Mollerus (2017), and Caballero and Farhi (2018). Central in this literature is the securitization process of claims on risky endowments into risk-free and risky assets. The models of this literature explain how a shortage of risk-free assets drives down the risk-free interest rate. My model replicates this finding and in addition, demonstrates that too much risk can be welfare reducing due to the role for assets in payment.

In other branches of the literature, to the best of my knowledge, Benigno and Robatto (2019) are the first and only to study the endogenous risk characteristics of money in a model with a role for assets as payment instruments. The authors incorporate a risky Lucas tree asset, as in Lucas (1978), that backs money in a cash-in-advance economy à la Lucas and Stokey (1987). Risky money is shown to be welfare reducing, but no production or investment decisions take place in the model. Also, their approach relies critically on information frictions that make risky assets less suitable as money. Such frictions are not present in my model.

Finally, the current paper relates to an extensive literature on TARP. Calomiris and Khan (2015) assess TARP according to a broad variety of criteria. In line with the predictions of the current paper, they find that the announcement of TARP policies led to a significant reduction in the TED spread, that is the difference between the bank-to-bank overnight lending rate (LIBOR) and the Treasury bill rate, which is a proxy for the perceived riskiness of the banking system. Empirical evidence on the effectiveness of TARP in stimulating credit extension by banks is mixed. On the one hand, as predicted by this paper, Taliaferro (2009), Li (2013), Berger and Roman (2015), and Ng, Vasvari, and Wittenberg-Moerman (2016) document that TARP banks increased loan supply. On the other hand, Egly and Mollick (2013) find no significant increase in lending activity by banks that participated in the TARP Capital Purchase Program. Evidence provided by Berger and Roman (2017) suggests a positive effect of TARP on real economic conditions. Hoshi and Kashyap (2010) argue that policies similar to TARP were deployed in Japan during the 1990s, but with mixed effects on the real economy.

3 The Model

Time is discrete and continues forever. A unit measure of infinitely lived agents and a government populate the economy. During each time period $t \geq 0$, three markets convene sequentially; first a *centralized market* (CM), then an *inside money market* (MM), and finally a *decentralized market* (DM). During period t , the aggregate state of the economy is $j_t \in \{h, l\}$, and states h and l are referred to as, respectively, *high* and *low*. States are independently and identically distributed across time. Specifically, in each period t , the high state occurs with probability ρ and low state with probability $1 - \rho$. Just before DM t convenes, agents learn j_{t+1} ; the aggregate state of the economy in period $t + 1$.

There are three tradable objects available in the economy. One of them is a perishable real good referred to as *general good*. The other two are *H-money* and *L-money*, which are perfectly divisible payment instruments with a state-contingent real return:

- *H-money*: When issued during period t , this asset delivers one unit of general good to the bearer in CM $t + 1$ if $j_{t+1} = h$ and nothing if $j_{t+1} = l$.
- *L-money*: When issued during period t , this asset delivers one unit of general good to the bearer in CM $t + 1$ if $j_{t+1} = l$ and nothing if $j_{t+1} = h$.

After maturing in CM $t + 1$, the physical objects or record-keeping entries representing H- and L-money are destroyed. H- and L-money issued in period t can therefore be used to settle transactions only in period t . General goods function as numeraire. Prices of H- and L-money issued in period t are denoted ϕ_t^h and ϕ_t^l in CM t , $\tilde{\phi}_t^h$ and $\tilde{\phi}_t^l$ in MM t , and $\bar{\phi}_t^h$ and $\bar{\phi}_t^l$ in DM t . In DM t , agents are informed about j_{t+1} and therefore know whether H-money or L-money will be redeemed in CM $t + 1$. We can therefore define

$$\hat{\phi}_t^{j_{t+1}} = \begin{cases} \bar{\phi}^h & \text{if } j_{t+1} = h \\ \bar{\phi}^l & \text{if } j_{t+1} = l \end{cases}$$

as the effective DM t price of a claim to one unit of CM $t + 1$ general good. Henceforth, I shall therefore consider $\hat{\phi}_t^{j_{t+1}}$ as the relevant DM price.

In each period t , agents can be of three different types: *bankers*, *buyers*, and *sellers*. Collectively, the buyers and the sellers constitute the *non-bankers*. Types are independently and identically distributed across time and across aggregate states, and are allocated to the agents randomly in the following way: At the beginning of the MM agents learn whether they are bankers or non-bankers and at the beginning of the DM the non-bankers learn whether they are buyers or sellers. The measures of bankers, buyers, and sellers are deterministic and given by $(1 - \psi)$, $\psi\pi$, and $\psi(1 - \pi)$, respectively.

Agents can produce and consume general goods in the CM. Production of general goods in the MM and DM is infeasible, but all agents have access to two storage tech-

nologies. The first storage technology allows agents to transfer general goods from the CM to the MM at a one-to-one rate. The second storage technology allows agents to transfer general goods from the MM to the DM, also at a one-to-one rate. Transferring general goods from DM t to CM $t + 1$ is impossible.

Preferences of agent $i \in [0, 1]$ are described by the function

$$\mathcal{U}_i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t [y_{i,t} + \theta_{i,t} u(\hat{c}_{i,t})],$$

where $y_{i,t}$ denotes net consumption of general goods in CM t , $\hat{c}_{i,t}$ denotes consumption of general goods in DM t , and $\theta_{i,t} = 1$ if agent i is a buyer during period t and $\theta_{i,t} = 0$ otherwise. Function $u : \mathbb{R}_+ \mapsto \mathbb{R}$ is twice continuously differentiable and satisfies the properties $u' > 0$, $u'' < 0$, $\lim_{c \rightarrow 0} u'(c) = \infty$, and $\lim_{c \rightarrow \infty} u'(c) = 0$. Finally, $\beta \in (0, 1)$ is the subjective discount factor.

In the MM, the bankers have access to an investment opportunity. This is a shortcut to an environment in which some agents, based on their preferences, specialize in extending credit to entrepreneurs (not modeled explicitly) who need to be monitored. When monitoring an entrepreneur inhibits participation in the DM, e.g. because monitoring requires time and effort, then one can choose u such that the agents who know that they will not become buyers, i.e. the bankers, specialize in extending credit to entrepreneurs.

If a banker devotes $\tilde{k}_{r,t}$ units of general good to investment in MM t , then he or she obtains $a_{t+1} \tilde{k}_{r,t}^\alpha$ units of general goods in CM $t + 1$, dependent on the aggregate state of the economy:⁵

$$a_{t+1} = \begin{cases} a^h & \text{if } j_{t+1} = h \\ a^l & \text{if } j_{t+1} = l \end{cases}, \quad a^h \geq a^l, \quad \text{and} \quad \alpha \in (0, 1).$$

Importantly, the fact that a_{t+1} depends on the aggregate state of the economy implies that bankers' investment projects are subject to systematic risk. This risk cannot be diversified away by means of pooling resources into a single institution.

Agents in the economy are characterized by limited commitment to their obligations against other agents, rendering unsecured credit arrangements infeasible. Nevertheless, issuing liabilities is feasible for the bankers. They can pledge the returns from their investment projects as collateral for the liabilities that they issue. Because of limited record-keeping, agents cannot borrow against investment projects that are to be under-

⁵Think, for example, of a large measure of entrepreneurs. Each entrepreneur has no resources but has access to an investment project generating $a_{t+1} k_t^\alpha$ units of general goods. When the bankers can make take-it-or-leave offers to the entrepreneurs, this implies a return from investment $a_{t+1} k_t^\alpha$ for the bankers. One can also assume that investment returns are given by $af(k)$, with f satisfying the usual properties. Using $f(k) = k^\alpha$ simplifies the analysis and also makes it easy to understand when risky inside money arises.

taken in the future. Therefore, non-bankers cannot obtain credit or issue liabilities.

Government is active only during the CM and the MM. In the CM, government has the power to levy lump-sum taxes. Moreover, it can decide to issue (or purchase) H-money and L-money in both the CM and the MM. Let m_t^h and m_t^l denote the amount of outside H- and L-money, respectively, in circulation at the end of CM t . Similarly, \tilde{m}_t^h and \tilde{m}_t^l denote the amount of outside money in circulation at the end of MM t . In CM t , lump-sum taxes are set such that government satisfies its flow budget constraint:

$$\phi_t^h m_t^h + \phi_t^l m_t^l + \tau_t^{j_t} = \tilde{m}_{t-1}^{j_t}, \quad (1)$$

where $\tau_t^{j_t}$ denotes lump-sum taxes and j_t the state of the economy. In MM t the government's budget constraint is given by:

$$\tilde{\phi}_t^h \tilde{m}_t^h + \tilde{\phi}_t^l \tilde{m}_t^l = \tilde{\phi}_t^h m_t^h + \tilde{\phi}_t^l m_t^l. \quad (2)$$

As I shall focus on stationary equilibria, I drop the time indices and define a stationary policy as a tuple $\langle \tau^h, \tau^l, m^h, m^l, \tilde{m}^h, \tilde{m}^l \rangle$ that satisfies Equations (1) and (2).

To conclude the model setup, the quasi-linear utility structure implies that expected social welfare can be written as:

$$\mathcal{W} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\pi \psi u(\hat{c}_{b,t}) + (1 - \psi) \beta \mathbb{E}[a] \tilde{k}_{r,t}^\alpha - q_t \right], \quad \mathbb{E}[a] \equiv \rho a^h + (1 - \rho) a^l$$

where $\hat{c}_{b,t}$ is DM consumption by the buyers, $\tilde{k}_{r,t}$ is investment by the bankers, and q_t is the net production of general goods in the CM. Technological constraints faced by the economy imply that $q_t \geq \pi \psi \hat{c}_{b,t} + (1 - \psi) \tilde{k}_{r,t}$. In short, social welfare is generated by producing general goods in CM, and using these goods to let the buyers consume in the DM and to let the bankers invest in the MM. It follows that first-best allocations satisfy:

$$u'(\hat{c}_{b,t}^*) = 1, \quad \tilde{k}_{r,t}^* = (\alpha \beta \mathbb{E}[a])^{\frac{1}{1-\alpha}}, \quad \text{and} \quad q_t^* = \psi \pi \hat{c}_{b,t}^* + (1 - \psi) \tilde{k}_{r,t}^*.$$

3.1 The Optimal Decisions of Agents

To save on notation, I drop time subscripts and index variables that belong to the next period with superscript $+$ and variables that belong to the previous period with superscript $-$. Most importantly, $j^+ \in \{h, l\}$ denotes the next period's aggregate state, which is revealed already in the DM. To guide the discussion, Figure 1 describes the typical optimal behavior of agents in the different sub-markets for a baseline economy with only inside money.

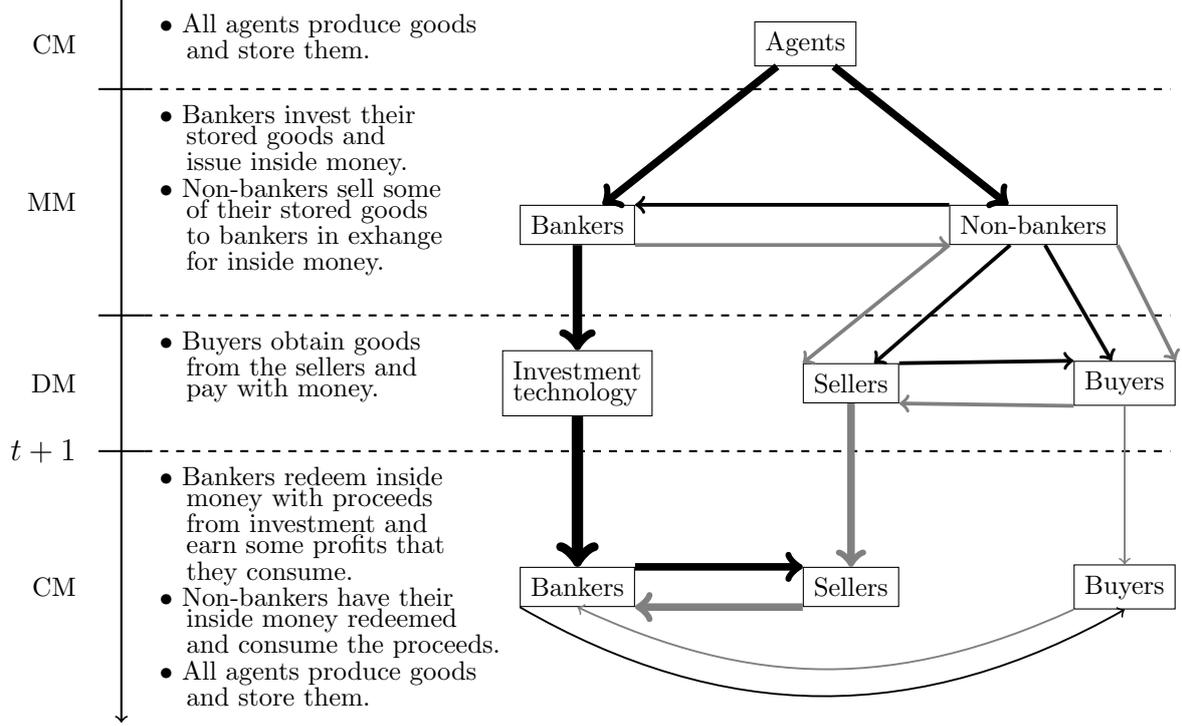


Figure 1: Description of the typical optimal behavior of agents in a baseline environment with only inside money. In black are the flows of goods and in gray are the flows of inside money.

3.1.1 Optimal Decisions in the DM

In the DM agents learn the next period's aggregate state j^+ . Agents in the DM are of three types: buyers (indexed with b), sellers (indexed with s), and bankers (indexed with r). For these three types of agents, let $V(\hat{k}_i, \hat{z}_i^h, \hat{z}_i^l; j^+)$ denote the state-contingent value of entering the next CM with \hat{k}_i units of general good devoted to investment, \hat{z}_i^h units of maturing H-money, and \hat{z}_i^l units of maturing L-money. Here, because types are i.i.d. distributed over time and across agents, I conjecture that the value function V is independent of the agent's current type. Also, I conjecture that $\partial V / \partial \hat{k}_i = \alpha a^{j^+} \hat{k}_i^{\alpha-1}$, $\partial V / \partial \hat{z}_i^j = 1$ if $j^+ = j$, and $\partial V / \partial \hat{z}_i^j = 0$ if $j^+ \neq j$. Finally, because the non-bankers randomly become buyers and sellers in the DM, I conjecture that they enter the DM with the same holdings of general goods q_{nr} , H-money \tilde{z}_{nr} , and L-money \tilde{z}_{nr}^l , where nr is the subscript for non-bankers. Similarly, the bankers enter the DM with \tilde{k}_r units of general good devoted to investment, an inventory \tilde{q}_r of general goods that can be sold in the DM, net H-money holdings \tilde{z}_r^h , and net L-money holdings \tilde{z}_r^l .

Buyers: Buyers value consumption in the DM according to the flow utility function u . The typical behavior of a buyer in the DM is therefore that he or she acquires general goods, offered for sale by the sellers, by means of giving up money. Specifically, the DM

problem of a buyer implies the following value function of entering the DM:

$$\begin{aligned}
W_b(\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+) &= \max_{\hat{c}_b^{j^+}, \hat{z}_b^h, \hat{z}_b^l} \left\{ u\left(\hat{c}_b^{j^+}\right) + \beta V(0, \hat{z}_b^h, \hat{z}_b^l; j^+) \right\} \\
\text{s.t.} \quad \hat{c}_b^{j^+} - \tilde{q}_{nr} &\leq \hat{\phi}^h(\tilde{z}_{nr}^h - \hat{z}_b^h)\mathbb{I}_{j^+=h} + \hat{\phi}^l(\tilde{z}_{nr}^l - \hat{z}_b^l)\mathbb{I}_{j^+=l}, \\
\hat{c}_b^{j^+} &\geq 0, \quad \hat{z}_b^h \geq 0, \quad \text{and} \quad \hat{z}_b^l \geq 0.
\end{aligned}$$

The second and third constraint follow from the fact that credit is not feasible for the buyers, and thus represent cash-in-advance constraints. Notice that when the high (low) state is announced, the choice of L-money (resp. H-money) carried out of the DM is irrelevant to the buyer as this type of money is not redeemed in the next CM. Therefore, the choices for \hat{z}_b^h and \hat{z}_b^l do not need to be conditioned on the announced state j^+ .

The solution to the problem of the buyer implies that consumption satisfies

$$c_b^{j^+} = \begin{cases} \tilde{q}_{nr} + \hat{\phi}^{j^+} \tilde{z}_{nr}^{j^+} & \text{if } u'(\tilde{q}_{nr} + \hat{\phi}^{j^+} \tilde{z}_{nr}^{j^+}) > \beta / \hat{\phi}^{j^+} \\ \beta / \hat{\phi}^{j^+} & \text{if } u'(\tilde{q}_{nr} + \hat{\phi}^{j^+} \tilde{z}_{nr}^{j^+}) \leq \beta / \hat{\phi}^{j^+} \end{cases}.$$

In words, in the DM the buyer compares the marginal utility of consumption to the opportunity cost of consumption. The latter is given by $\beta / \hat{\phi}^{j^+}$; the foregone return on money. A buyer increases consumption until marginal utility equals opportunity cost, or until all money has been spend and the constraint $\tilde{z}_{nr}^{j^+} \geq 0$ binds.

To conclude, the envelope theorem tells us that $\partial W_b / \partial \tilde{q}_{nr} = u'(\hat{c}_b^{j^+})$, $\partial W_b / \partial \tilde{z}_{nr}^j = \hat{\phi}^j u'(\hat{c}_b^{j^+})$ if $j = j^+$ and $\partial W_b / \partial \tilde{z}_{nr}^j = 0$ if $j \neq j^+$.

Sellers: In the DM, the sellers do not gain utility from the consumption of general goods. The typical behavior of sellers in the DM is therefore that they inelastically supply their inventories of general goods. Money holdings carried into the next CM therefore become

$$\hat{z}_s^h = \tilde{z}_{nr}^h + \tilde{q}_{nr} / \hat{\phi}^h \quad \text{and} \quad \hat{z}_s^l = \tilde{z}_{nr}^l + \tilde{q}_{nr} / \hat{\phi}^l. \quad (3)$$

When the high (low) state is announced, the choice of L-money (resp. H-money) carried out of the DM is irrelevant to the seller as this type of money will not be redeemed in the next CM. Therefore, the choices for \hat{z}_s^h and \hat{z}_s^l do not need to be conditioned on the announced state j^+ .

The value function of entering the DM for the seller becomes

$$W_s(\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+) = \beta V(0, \hat{z}_s^h, \hat{z}_s^l; j^+),$$

which, given the properties of V , \hat{z}_s^h , and \hat{z}_s^l , can be written as

$$W_s(\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+) = \tilde{q}_{nr}\beta/\hat{\phi}^{j^+} + \beta V(0, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+).$$

Bankers: Just like the sellers, in the DM the bankers do not gain utility from the consumption of general goods. They will therefore inelastically supply their inventories of general goods, although in general equilibrium it turns out that the bankers enter the DM without such inventories. Capital investment by the bankers cannot be liquidated prematurely, which implies that general goods devoted to investment and money holdings carried into the next CM satisfy

$$\hat{z}_r^h = \tilde{z}_r^h + \tilde{q}_r/\hat{\phi}^h, \quad \hat{z}_r^l = \tilde{z}_r^l + \tilde{q}_r/\hat{\phi}^l, \quad \text{and} \quad \hat{k}_r = \tilde{k}_r. \quad (4)$$

Just like for the sellers, when the high (low) state is announced, the choice of L-money (resp. H-money) carried out of the DM is irrelevant to the banker. Again, the choices for \hat{z}_r^h and \hat{z}_r^l do not need to be conditioned on the announced state j^+ .

The value function of entering the DM for the banker becomes

$$W_r(\tilde{k}_r, \tilde{q}_r, \tilde{z}_r^h, \tilde{z}_r^l; j^+) = \beta V(\hat{k}_r, \hat{z}_r^h, \hat{z}_r^l; j^+),$$

which, given the properties of V , \hat{z}_r^h , and \hat{z}_r^l , can be written as

$$W_r(\tilde{k}_r, \tilde{q}_r, \tilde{z}_r^h, \tilde{z}_r^l; j^+) = \tilde{q}_r\beta/\hat{\phi}^{j^+} + \beta V(\tilde{k}_r, \tilde{z}_r^h, \tilde{z}_r^l; j^+).$$

3.1.2 Optimal Decisions in the MM

In the MM agents are of two types: bankers and non-bankers. Because these types are revealed just before the MM convenes, I conjecture that both types enter the MM with the same inventory of general goods q , H-money z^h , and L-money z^l .

Non-bankers: In the MM, the non-bankers must decide how to invest their wealth. They can invest in inventories \tilde{q}_{nr} , meaning that they can carry goods to the DM, and can invest in H- and L-money $(\tilde{z}_{nr}^h, \tilde{z}_{nr}^l)$. The typical behavior of the non-bankers is that they sell some of their general goods to the bankers in exchange for inside money. The non-bankers can then use that money to acquire additional general goods in the DM in case they become buyers. For a non-banker, the value function of entering the MM is

$$\begin{aligned} \Omega_{nr}(q, z^h, z^l) &= \max_{\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l} \mathbb{E} [\pi W_b(\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+) + (1 - \pi)W_s(\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+)] \\ \text{s.t.} \quad &\tilde{\phi}^h (\tilde{z}_{nr}^h - z^h) + \tilde{\phi}^l (\tilde{z}_{nr}^l - z^l) \leq q - \tilde{q}_{nr}, \\ &\tilde{q}_{nr} \geq 0, \quad \tilde{z}_{nr}^h \geq 0, \quad \text{and} \quad \tilde{z}_{nr}^l \geq 0. \end{aligned}$$

The expectations operator accounts for the fact that in the MM, j^+ is still unknown. Moreover, a non-banker takes into account the fact that he or she will become a buyer (seller) in the DM with probability π (resp. $1 - \pi$).

Let the Lagrange multipliers associated with the non-bankers' constraints be denoted with λ_{nr} , μ_{nr}^q , $\rho\mu_{nr}^h$, and $(1 - \rho)\mu_{nr}^l$. The FOCs of the non-bankers problem are then given by:

$$\begin{aligned}\tilde{q}_{nr} : & \quad \lambda_{nr} = \mathbb{E} \left[\pi u' \left(\hat{c}_b^{j^+} \right) + \beta(1 - \pi) / \hat{\phi}^{j^+} \right] + \mu_{nr}^q, \\ \tilde{z}_{nr}^h : & \quad \tilde{\phi}^h \lambda_{nr} = \rho \left[\pi \hat{\phi}^h u' \left(\hat{c}_b^h \right) + \beta(1 - \pi) + \mu_{nr}^h \right], \\ \tilde{z}_{nr}^l : & \quad \tilde{\phi}^l \lambda_{nr} = (1 - \rho) \left[\pi \hat{\phi}^l u' \left(\hat{c}_b^l \right) + \beta(1 - \pi) + \mu_{nr}^l \right].\end{aligned}$$

These FOCs verify the conjecture that all the non-bankers leave the MM with identical amounts of general goods and money, conditional on the conjecture that all agents enter the MM with identical amounts of general goods and money.

To conclude, the envelope theorem implies $\partial\Omega_{nr}/\partial q = \lambda_{nr}$, $\partial\Omega_{nr}/\partial z^h = \tilde{\phi}^h \lambda_{nr}$ and $\partial\Omega_{nr}/\partial z^l = \tilde{\phi}^l \lambda_{nr}$.

Bankers: Compared to the non-bankers, the bankers can devote general goods \tilde{k}_r to a scalable, risky investment project and they can also acquire funds for investment by issuing inside money. The typical behavior of bankers in the MM is that they invest their initial inventories of general goods into their investment opportunity, representing equity finance. Moreover, the bankers attract external funding by issuing money to the non-bankers in exchange for general goods, which the bankers in turn devote to their investment projects.

The value function of entering the MM as a banker is given by

$$\begin{aligned}\Omega_r(q, z^h, z^l) &= \max_{\tilde{k}_r, \tilde{q}_r, \tilde{z}_r^h, \tilde{z}_r^l} \mathbb{E} \left[W_r(\tilde{k}_r, \tilde{q}_r, \tilde{z}_r^h, \tilde{z}_r^l; j^+) \right] \\ \text{s.t.} & \quad \tilde{k}_r + \tilde{\phi}^h (\tilde{z}_r^h - z^h) + \tilde{\phi}^l (\tilde{z}_r^l - z^l) \leq q - \tilde{q}_r, \\ & \quad \tilde{k}_r \geq 0, \quad \tilde{q}_r \geq 0, \quad \tilde{z}_r^h + a^h \tilde{k}_r^\alpha \geq 0, \quad \text{and} \quad \tilde{z}_r^l + a^l \tilde{k}_r^\alpha \geq 0.\end{aligned}$$

The last two constraints represent the money creating ability of bankers: They can issue H- and L-money but are limited in doing so by the returns on their investment projects. The constraints thus imply limited liability for the bankers in the sense that if the issued money cannot be paid back with the returns from the banker's assets alone, then the banker cannot be forced ex-post to re-capitalize its bank.

Let the Lagrange multipliers associated with the bankers' constraints be denoted with λ_r , μ_r^k , μ_r^q , $\beta\rho\mu_r^h$, and $\beta(1 - \rho)\mu_r^l$. The FOCs for the bankers' problem can be combined

The second and third constraint imply that agents cannot issue money in the CM, which follows from the fact that agents cannot pledge returns from investment projects not yet undertaken.

Let ς , v^q , v^h , and v^l denote the Lagrange multipliers of the agents' constraints. The FOCs for the agents' decisions are then given by

$$\begin{aligned} y_{-i}^j &: & 1 &= \varsigma, \\ q &: & 1 &= \psi \lambda_{nr} + (1 - \psi) \lambda_r + v^q, \\ z^h &: & \phi^h &= (1 - v^q) \tilde{\phi}^h + v^h, \\ z^l &: & \phi^l &= (1 - v^q) \tilde{\phi}^l + v^l. \end{aligned}$$

From the fact the FOCs are independent of agents' previous types, investment decisions, and money holdings $(\hat{k}_{i-}, \hat{z}_{i-}^h, \hat{z}_{i-}^l)$, it follows that, in line with earlier conjectures, agents carry identical amounts of general goods, identical amounts of H-money, and identical amounts of L-money out of the CM.

To conclude, from the envelope theorem it follows that $\partial V / \partial \hat{k}_{i-} = \alpha a^j \hat{k}_{i-}^{\alpha-1}$, $\partial V / \partial \hat{z}_{i-}^{j'} = 1$ if $j = j'$, and $\partial V / \partial \hat{z}_{i-}^{j'} = 0$ if $j \neq j'$, confirming earlier conjectures about V . Observe that in equilibrium, v^q must be zero as otherwise the marginal productivity of investment and the marginal utility of DM consumption become infinitely large. Also, without loss of generality assume that agents are willing to carry money from the CM to the MM so that $v^h = v^l = 0$. In what follows, I will therefore replace $\tilde{\phi}^h$ with ϕ^h and $\tilde{\phi}^l$ with ϕ^l .

3.2 Equilibrium

We can now formally define and analyze stationary equilibria.

Definition 1. *Given a policy $\langle \tau^l, \tau^h, m^h, m^l, \tilde{m}^h, \tilde{m}^l \rangle$ that satisfies Equations (1) and (2), a stationary equilibrium is a collection of prices $\{\phi^h, \phi^l, \hat{\phi}^h, \hat{\phi}^l\}$ and allocations $\{q, y_b^h, y_b^l, y_s^h, y_s^l, y_r^h, y_r^l, z^h, z^l\}$, $\{\tilde{k}_r, \tilde{q}_{nr}, \tilde{q}_r, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l, \tilde{z}_r^h, \tilde{z}_r^l\}$, $\{\hat{k}_r, \hat{c}_b^h, \hat{c}_b^l, \hat{z}_b^h, \hat{z}_b^l, \hat{z}_s^h, \hat{z}_s^l, \hat{z}_r^h, \hat{z}_r^l\}$, such that:*

1. *Agents maximize utility:*

- (a) $\{y_{i-}^j, q, z^h, z^l\}$ attains $V(\tilde{k}_r \mathbb{I}_{i-}=r, \hat{z}_{i-}^h, \hat{z}_{i-}^l; j)$ for all $i-, j \in \{b, s, r\} \times \{h, l\}$.
- (b) $\{\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l\}$ attains $\Omega_{nr}(q, z^h, z^l)$ and $\{\tilde{k}_r, \tilde{q}_r, \tilde{z}_r^h, \tilde{z}_r^l\}$ attains $\Omega_r(q, z^h, z^l)$.
- (c) $\{\hat{c}_b^{j+}, \hat{z}_b^h, \hat{z}_b^l\}$ attains $W_b(\tilde{q}_{nr}, \tilde{z}_{nr}^h, \tilde{z}_{nr}^l; j^+)$ for all $j^+ \in \{h, l\}$, $\{\hat{z}_s^h, \hat{z}_s^l\}$ satisfies Equation (3), and $\{\hat{k}_r, \hat{z}_r^h, \hat{z}_r^l\}$ satisfies Equation (4).

2. *Goods markets clear:*

- (a) *For the CM $q + \psi[\pi y_b^j + (1 - \pi) y_s^j] + (1 - \psi) y_r^j = (1 - \psi) a^j \hat{k}_r^{1-\alpha}$ for all $j \in \{h, l\}$.*

(b) For the MM $\psi\tilde{q}_{nr} + (1 - \psi)(\tilde{q}_r + \tilde{k}_r) = q$.

(c) For the DM $\pi\psi c_b^{j+} = \psi\tilde{q}_{nr} + (1 - \psi)\tilde{q}_r$ for all $j^+ \in \{h, l\}$.

3. Asset markets clear:

(a) For the CM $z^j = m^j$ for all $j \in \{h, l\}$.

(b) For the MM $\psi\tilde{z}_{nr}^j + (1 - \psi)\tilde{z}_r^j = \tilde{m}^j$ for all $j \in \{h, l\}$.

(c) For the DM $\psi[\pi\hat{z}_b^j + (1 - \pi)\hat{z}_s^j] + (1 - \psi)\hat{z}_r = \tilde{m}^j$ for all $j \in \{h, l\}$.

The focus of this paper will be on solving for the equilibrium values of q , \tilde{k}_r , and \hat{c}_b^{j+} , since they matter for welfare. To ease on the notation, I henceforth refer to \tilde{k}_r as k .

3.2.1 Partial Equilibrium in the DM

The partial DM equilibrium can be of two types. First, money may be plentiful, meaning that the non-negativity constraints for money carried out of the DM are slack for the buyers. Second, money may be scarce, meaning that the non-negativity constraints for money carried out of the DM bind for the buyers.

Type i: Plentiful money. The price of money is determined by equating the marginal benefit of consumption to the marginal cost of consumption. Because sellers and bankers supply their inventories of general goods inelastically, aggregate consumption by buyers must equal the total amount of general good available in the DM. The equilibrium price thus solves $u'((\psi\tilde{q}_{nr} + (1 - \psi)\tilde{q}_r)/(\pi\psi)) = \beta/\hat{\phi}^{j+}$. It follows that this price is independent of the announced state j^+ , so define

$$\hat{\phi} \equiv \beta/u'((\psi\tilde{q}_{nr} + (1 - \psi)\tilde{q}_r)/(\pi\psi)).$$

For a Type i equilibrium to exist, the value of money held by the buyers must be large enough to purchase all general goods supplied by the sellers and bankers; $\tilde{z}_{nr}^{j+} \geq \bar{z}$, where

$$\bar{z} = \frac{\psi(1 - \pi)\tilde{q}_{nr} + (1 - \psi)\tilde{q}_r}{\psi\pi\hat{\phi}}.$$

Type ii: Scarce money. The price of money is such that the value of money held by the buyers is exactly large enough to purchase aggregate inventories of general goods.

$$\hat{\phi}^{j+} = \frac{\psi(1 - \pi)\tilde{q}_{nr} + (1 - \psi)\tilde{q}_r}{\psi\pi\tilde{z}_{nr}^{j+}}.$$

To ensure existence, the marginal benefit of consumption must exceed the marginal cost of consumption for the buyers. This holds whenever $\tilde{z}_{nr}^{j+} < \bar{z}$.

It follows that the partial equilibrium in the DM is unique, with the price of money potentially dependent on the announced state of the economy and with buyers' consumption predetermined by inventories of general goods. To ease on the notation, I henceforth refer to $\hat{c}_b^{j^+}$ as c_b and find that

$$\hat{\phi}^{j^+} = \max \left\{ \frac{\psi(1-\pi)\tilde{q}_{nr} + (1-\psi)\tilde{q}_r}{\psi\pi\tilde{z}_{nr}^{j^+}}, \frac{\beta}{u'((\psi\tilde{q}_{nr} + (1-\psi)\tilde{q}_r)/(\psi\pi))} \right\}, \quad (5)$$

$$c_b = \frac{\psi\tilde{q}_{nr} + (1-\psi)\tilde{q}_r}{\psi\pi}. \quad (6)$$

Figure 2 illustrates how prices are determined in the DM. As an example, suppose that it is announced that the low state will realize ($j^+ = l$) so that we need to price L-money.⁶ If the non-bankers enter the DM with relatively little L-money, then money will be scarce. The buyers will sell all their L-money, and the sellers and bankers will sell all their holdings of general good. The price of L-money adapts so that the real value of L-money held by the buyers equals the supply of goods by the sellers and bankers. The latter is demonstrated in Panel 2b; if money is scarce then the real value of money is independent of the amount of money and determined by the amount of available general goods. As a result, the DM price of money is decreasing in the supply of L-money when L-money is scarce, which can be seen in Panel 2a.

If the non-bankers enter the DM with relatively much L-money then money will become plentiful. Basically, we can no longer have an equilibrium in which the buyers spend all their L-money holdings as this would drive down the DM price of L-money to a level where the opportunity cost of consumption would exceed the marginal benefit of consumption. Because the marginal benefit of consumption is predetermined, meaning that it is only a function of fundamentals and the total amount of general goods carried into the DM, we obtain a lower bound on the DM price of L-money. At this lower bound, the opportunity cost of consumption, $\beta/\hat{\phi}^l$, equals the marginal benefit of consumption, $\beta/\hat{\phi}$. We see this back in Figure 2.

Finally, notice how a scarcity of money drives up the DM prices of money, which is unattractive for the bankers and sellers. If they carried some general goods into the DM, these agents are effectively forced to sell these goods at a low price and this drives down DM consumption in equilibrium.

3.2.2 Partial Equilibrium in the Money Market

From the analysis of the individual decision making problems we know that the total amount of general goods carried into the MM must be positive; $q > 0$. Moreover, bankers will invest a strictly positive amount into their investment project ($k > 0$) and we also

⁶By assumption, H-money has a price of zero in the DM when it is announced that the low state will realize.

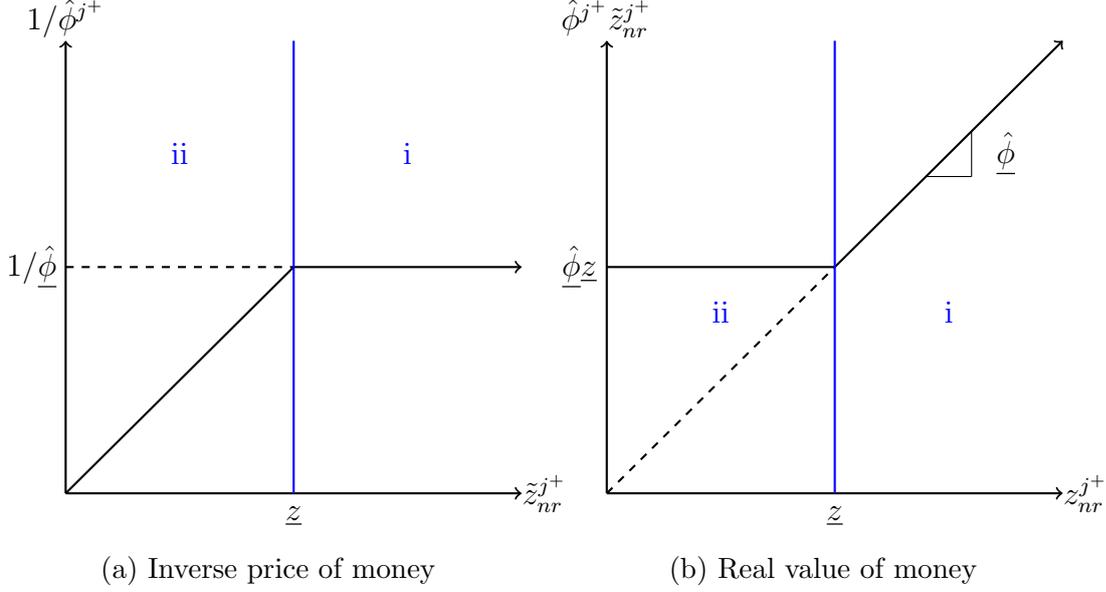


Figure 2: The inverse price of money and real value of money in the DM. Here, j^+ denotes the state of the economy in the next period, which is announced at the beginning of the DM.

know that consumption by buyers in the DM must be strictly positive. From the latter $\psi \tilde{q}_{nr} + (1 - \psi) \tilde{q}_r > 0$ follows. Finally, given that we study a monetary equilibrium we must have $\mu_{nr}^h = \mu_{nr}^l = 0$ to ensure that non-bankers are willing to leave the MM with money. Given that general goods must be carried into the DM, we also need to consider μ_{nr}^q and μ_r^q .

Lemma 1. $\mu_{nr}^q = 0$ and $\phi^h / \hat{\phi}^h + \phi^l / \hat{\phi}^l = 1$.

The first part of Lemma 1 follows from the fact that carrying general goods from the MM to the DM is more attractive for a non-banker than for a banker. The second part of Lemma 1 follows from the law-of-one-price (LOOP): Carrying $1/\hat{\phi}^h$ units of H-money plus $1/\hat{\phi}^l$ units of L-money into the DM buys one unit of general good in the DM with certainty. The MM price of this portfolio thus equals the MM price of general goods.

From the proof of Lemma 1 it follows as well that μ_r^q is strictly positive if and only if $\mu_r^h + \mu_r^l$ is strictly positive. In words, when the banker exhausts his or her capacity to create H-money and/or L-money then he or she is not willing to carry general goods to the DM. Here we can again apply the LOOP: If the banker carries general goods to the DM, then he or she can decide to sell x of these goods in the MM and obtain $x/\hat{\phi}^h$ units of H-money plus $x/\hat{\phi}^l$ units of L-money, giving him or her the same fundamental return as the general goods sold. When constraints regarding issuance of H-money and/or L-money bind, the banker however does not only value additional money for its fundamental return, but also because the additional money relaxes these issuance constraints.

Lemma 2. $\mu_r^q > 0 \Leftrightarrow \rho/\phi^h \neq (1 - \rho)/\phi^l$, $\mu_r^h > 0 \Leftrightarrow \rho/\phi^h < (1 - \rho)/\phi^l$, and $\mu_r^l > 0 \Leftrightarrow \rho/\phi^h > (1 - \rho)/\phi^l$

Lemma 2 tells us that bankers exhaust their capacity to issue H-money or L-money when the expected return on carrying H-money does not equal the expected return on carrying L-money. Observe that the former equals ρ/ϕ^h and the latter $(1 - \rho)/\phi^l$. As an example, suppose that the expected return of holding H-money is larger than the expected return of holding L-money. The banker then issues as much L-money as possible, even if this would give him or her more resources than he or she needs to finance investment, as he or she can always use the proceeds to purchase H-money and earn a profit (in expectation).

Also, observe that the banker never exhausts his or her capacity to issue H-money and to issue L-money simultaneously. Given the level of investment that is optimal from the perspective of the banker, this strategy would return more general goods than needed for investment. Some general goods must then be carried to the DM by the banker, which cannot be optimal as the banker is better off selling these general goods to alleviate the binding money issuance constraints.

Lemma 3. $\hat{\phi}^h > \hat{\phi}^l \Leftrightarrow \rho/\phi^h < (1 - \rho)/\phi^l$, $\hat{\phi}^h = \hat{\phi}^l \Leftrightarrow \rho/\phi^h = (1 - \rho)/\phi^l$, and $\hat{\phi}^h < \hat{\phi}^l \Leftrightarrow \rho/\phi^h > (1 - \rho)/\phi^l$. If $\rho/\phi^h > (1 - \rho)/\phi^l$ then in the DM money must be scarce when the low state is announced; $\tilde{z}_{nr}^l < \bar{z}$. If $\rho/\phi^h < (1 - \rho)/\phi^l$ then in the DM money must be scarce when the high state is announced; $\tilde{z}_{nr}^h < \bar{z}$.

Lemma 3 implies that the fundamental return of holding money is related to the scarcity of money. Specifically, if for example non-bankers carry less L-money into the DM than H-money, and money is scarce in the DM when the low state is announced, then L-money will earn a lower fundamental return than H-money. Because L-money is scarcer than H-money, Equation (5) implies that L-money has a higher DM price than H-money. Anticipating this relatively high DM price of L-money, non-bankers accept a lower fundamental return on L-money.

We are now in a position to determine the equilibrium types that can occur in the MM, taking into account which equilibrium type will occur in the DM:

- Type I: An equilibrium in which $\rho/\phi^h = (1 - \rho)/\phi^l$ and money is always plentiful in the DM regardless of the announced state. In this equilibrium $\underline{\hat{\phi}} = \hat{\phi}^h = \hat{\phi}^l$.
- Type II: An equilibrium in which $\rho/\phi^h = (1 - \rho)/\phi^l$ and money is always scarce in the DM regardless of the announced state. In this equilibrium $\underline{\hat{\phi}} < \hat{\phi}^h = \hat{\phi}^l$.
- Type IIIa: An equilibrium in which $\rho/\phi^h > (1 - \rho)/\phi^l$, money is plentiful in the DM when the high state is announced, and scarce when the low state is announced. In this equilibrium $\underline{\hat{\phi}} = \hat{\phi}^h < \hat{\phi}^l$.

- Type IIIb: An equilibrium in which $\rho/\phi^h > (1 - \rho)/\phi^l$ and money is always scarce in the DM. In this equilibrium $\underline{\hat{\phi}} < \hat{\phi}^h < \hat{\phi}^l$.
- Type IVa: An equilibrium in which $\rho/\phi^h < (1 - \rho)/\phi^l$, money is plentiful in the DM when the low state is announced, and scarce when the high state is announced. In this equilibrium $\underline{\hat{\phi}} = \hat{\phi}^l < \hat{\phi}^h$.
- Type IVb: An equilibrium in which $\rho/\phi^h < (1 - \rho)/\phi^l$ and money is always scarce in the DM. In this equilibrium $\underline{\hat{\phi}} < \hat{\phi}^l < \hat{\phi}^h$.

Lemma 4. $(1 - \psi)k \geq (1 - \psi\pi)q - \psi\pi(\phi^h m^h + \phi^l m^l)$, with equality in partial equilibria of the Type II, IIIb, and IVb.

Lemma 4 follows from the fact that real wealth of non-bankers in the MM is given by $q + \phi^h m^h + \phi^l m^l$; general goods carried into the MM plus the real value of outside money carried into the MM. Using Lemma 1 it can then be shown that in the DM buyers can never consume more than this quantity. In this sense, the MM wealth of a non-banker represents his purchasing power in the DM.

At this point, one could solve for the partial equilibrium in the MM given a certain amount of general goods and outside money available. Because in stationary equilibrium agents can perfectly anticipate which partial equilibrium type will realize in the MM, it is more efficient to solve for equilibrium in the CM and MM jointly. Doing so, I will distinguish between an economy with only inside money and an economy with both inside and outside money.

3.3 Definition of Risky Money

Because the concepts of risk-free and risky money are at the core of this paper, I conclude this section with:

Definition 2. An equilibrium with risk-free money has $\tilde{z}_{nr}^h = \tilde{z}_{nr}^l$ or $\min\{\tilde{z}_{nr}^h, \tilde{z}_{nr}^l\} \geq \bar{z}$.

The first characterization of risk-free money is straightforward: If the non-bankers hold as much H-money as L-money, then the future real value of their money holdings is certain. The second characterization is more subtle as it allows the future real value of the non-bankers' money holdings to depend on the state of the economy. Nevertheless, from Section 3.2.1 we know that the non-bankers spend at most \bar{z} in terms of real money balances in the DM. When non-bankers hold more H-money and L-money than what they spend in the DM, idle money balances arise so that at the margin, non-bankers will value money only for its fundamental return and not for the liquidity services that it provides. As will turn out to be the case, holding money then does not command a liquidity premium. Therefore, non-bankers are willing to hold idle money balances $\tilde{z}_{nr}^j - \bar{z}$

as illiquid securities that cannot be spent in the DM and banks are willing to issue these securities. This allows the remaining (inside) money to be risk-free.

4 The Economy With Only Inside Money

Consider an economy without any government intervention, meaning that only inside money is available. Because inside money is always in zero net supply within the private sector, we have $\psi \tilde{z}_{nr}^h + (1 - \psi) \tilde{z}_r^h = 0$ and $\psi \tilde{z}_{nr}^l + (1 - \psi) \tilde{z}_r^l = 0$, which we can use to rule out two equilibrium types:

Lemma 5. *The equilibrium cannot be of Type IVa or IVb.*

Lemma 5 implies that we cannot have an equilibrium in which the expected return on H-money falls short of that on L-money. To rationalize such a situation would require that non-bankers carry less H-money out of the MM than L-money, meaning that in the DM H-money is scarcer than L-money. The bankers must then provide less H-money than L-money, but are not willing to do so as they can issue more H-money than L-money and they prefer issuing H-money (that they can sell at a relatively high price) over issuing L-money (that they can sell at a relatively low price). It remains to consider equilibria of Type I, II, IIIa, and IIIb.

Type I: Money is always plentiful in the DM. Because $\hat{\phi}^h = \hat{\phi}^l$, define $\phi \equiv \hat{\phi}^h$. Given that $\rho/\phi^h = (1 - \rho)/\phi^l$, it follows from Lemma 1 that $\phi^h = \rho\phi$ and $\phi^l = (1 - \rho)\phi$. From Equation (5) we obtain $u'(c_b) = \beta/\phi$. From the MM FOCs of the bankers and non-bankers, where $\mu_{nr}^q = 0$ by Lemma 1 and $\mu_r^h = \mu_r^l = 0$ by Lemma 2, we find that $k^{1-\alpha} = \alpha\mathbb{E}[a]\phi$ and $\lambda_{nr} = \lambda_r = \beta/\phi$. Using this in the CM FOCs, in which $\nu_q = 0$, it follows that $\beta = \phi$. We thus have $k = k^*$, $c_b = c_b^*$, and $q = q^*$; first-best allocations are attained. Intuitively, money does not earn a liquidity premium in the equilibrium of the Type I, as money is always plentiful in the DM. This implies that consumption and investment attain their first-best levels, since the fundamental return earned by money equals agents their rate of time preference.

In Appendix C.7, I derive that a Type I equilibrium exists if and only if

$$(1 - \psi)k^* \geq (1 - \psi\pi)c_b^* \quad \text{and} \quad (1 - \psi)a^l k^*/(\alpha\mathbb{E}[a]) \geq \psi(1 - \pi)c_b^*. \quad (7)$$

These conditions can be found by supposing that bankers do not carry general goods to the DM. We can therefore assume without loss of generality that the bankers only devote resources to investment and do not carry general goods into the DM. The conditions imply that we obtain first-best allocations when the average return on investment is sufficiently high and when the return on investment in the low aggregate state is not too

small. Intuitively these conditions ensure that bankers' optimal amount of investment leads to enough inside money creation so that money is, regardless of the state of the economy, always plentiful in the DM.

Type II: Money is risk-free and always scarce in the DM. Because $\hat{\phi}^h = \hat{\phi}^l$, we can continue to use $\phi \equiv \hat{\phi}^h$, and $\phi^h = \rho\phi$ and $\phi^l = (1 - \rho)\phi$ will still hold. In addition, from Equation (5) we learn that $\tilde{z}_{nr}^h = \tilde{z}_{nr}^l$. This makes sense; money is scarce and the DM prices of money are independent of the announced state so that the amount of L-money held by non-bankers must equal the amount of H-money held by non-bankers. Because inside money is in zero net supply, we must also have $\tilde{z}_r^h = \tilde{z}_r^l$. To save on notation, I drop the superscripts for these variables.

To determine ϕ , observe that Lemma 4 implies that $(1 - \psi)k = (1 - \psi\pi)q$, or equivalently $(1 - \psi)k = (1 - \psi\pi)c_b$. From the FOCs of the bankers, where $\mu_r^h = \mu_r^l = 0$ by Lemma 2, we know that $\lambda_r = \beta/\phi$ and $k^{1-\alpha} = \alpha\mathbb{E}[a]\phi$. From the MM FOCs of the non-bankers, where $\mu_{nr}^q = 0$ by Lemma 1, we find that $\lambda_{nr} = \pi u'(c_b) + \beta(1 - \pi)/\phi$. Substituting λ_{nr} and λ_r in the CM FOCs, where $\nu_q = 0$, we obtain that $\psi\pi u'(c_b) = 1 - (1 - \psi\pi)\beta/\phi$. The equilibrium price in the equilibrium of the Type II thus uniquely solves

$$(1 - \psi)(\alpha\mathbb{E}[a]\phi)^{\frac{1}{1-\alpha}} = (1 - \psi\pi)u'^{-1}\left(\frac{1 - (1 - \psi\pi)\beta/\phi}{\psi\pi}\right). \quad (8)$$

to equilibrate supply of (LHS) and demand for (RHS) inside money. Intuitively, the demand for money is decreasing in ϕ since higher ϕ imply a higher liquidity premium commanded by holding money. The supply of money is determined by bankers' investment and is increasing in ϕ since higher ϕ imply a lower funding cost for the bankers.

In Appendix C.8 I show that an equilibrium of Type II exists if and only if

$$(1 - \psi)k^* < (1 - \psi\pi)c_b^* \quad \text{and} \quad a^l \geq \alpha \frac{\psi(1 - \pi)}{1 - \psi\pi} \mathbb{E}[a]. \quad (9)$$

These conditions imply that the average return on investment is relatively low and that the return on investment in the low state is relatively high. Intuitively, these conditions imply that absent of a liquidity premium earned by money, bankers' investment leads to too little money creation to have plentiful money in the DM. Nevertheless, the amount of risk embedded in investment is sufficiently low so that it can be fully absorbed by bankers equity.

Because of the scarcity of money in the current equilibrium type, we have that $\phi > \beta$; money commands a liquidity premium to reflect the marginal value of money as a payment instrument in the DM. In turn this implies that $c_b < c_b^*$ and $k > k^*$. Because money is scarce in the DM, non-bankers anticipate that general goods will be relatively cheap in the DM, making it unattractive to carry these goods into the DM. Bankers however face

low funding cost so that they invest more than the first-best level. Finally, one can show that $q < q^*$, meaning that general goods production is inefficiently low.

Type IIIa: money is risky, plentiful in the DM when the high state is announced but scarce in the DM when the low state is announced. To solve for this equilibrium, I exploit MM equilibrium relationships (C.1)-(C.3) between ϕ^h , $\hat{\phi}^h$, ϕ^l , $\hat{\phi}^l$, and c_b that can be found in Appendix C.5. Because money is plentiful when the high state is announced we have $\hat{\phi}^h u'(c_b) = \beta$ and $\lambda_{nr} \phi^h = \beta\rho$. As we also have that $\lambda_r \phi^h = \beta\rho$, it follows that $\phi_h = \beta\rho$. H-money therefore does not command a liquidity premium, which makes sense as money is plentiful in the DM when the high state is announced. Because bankers exhaust their full capacity to create L-money ($\mu_r^l > 0$) and because bankers do not want to carry general goods to the DM ($\mu_r^g > 0$), Equation (5) implies that DM clearance for the low state requires $\hat{\phi}^l (1 - \psi) a^l k^\alpha = \psi(1 - \pi) c_b$. We can then use Equations (C.1) and (C.3), and the banker's MM FOC for k to obtain that ϕ^l uniquely solves

$$\begin{aligned} & \psi(1 - \pi) u'^{-1} \left(\frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi(1 - \rho) + \rho[1 - (1 - \pi)(1 - \rho)\beta/\phi^l]} \right) \\ &= \phi^l \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi} + 1 \right) a^l (1 - \psi) (\alpha\rho a^h \beta + \alpha a^l \phi^l)^{\frac{\alpha}{1-\alpha}}, \quad (10) \end{aligned}$$

where the LHS represents non-bankers' money demand to finance DM consumption and the RHS represents bankers' money supply to finance investment. The former is decreasing in ϕ^l and the latter is increasing in ϕ^l , as a higher value for ϕ^l represents a higher liquidity premium for L-money and a lower funding cost for bankers.

To ensure existence, we need to check that money is plentiful in the DM when the high state is announced but scarce when the low state is announced. The following two conditions are obtained as a result:

$$(1 - \psi) \frac{a^l}{\alpha \mathbb{E}[a]} k^* < \psi(1 - \pi) c_b^*, \quad (11)$$

$$\left[\alpha\beta \left(\rho a^h + \frac{(1 - \rho)a^l}{\varphi} \right) \right]^{\frac{1}{1-\alpha}} \geq \frac{1 - \psi\pi}{1 - \psi} u'^{-1} \left(\frac{1 - (1 - \pi)\varphi}{\pi(1 - \rho) + \rho[1 - (1 - \pi)\varphi]} \right), \quad (12)$$

where

$$\varphi \equiv \frac{1 + \frac{1-\rho}{\rho} \left(1 - \frac{\alpha\psi(1-\pi)}{1-\psi\pi} \right) \pi}{1 + \left(\frac{\alpha\psi(1-\pi)}{1-\psi\pi} \frac{a^h}{a^l} - 1 \right) \pi}. \quad (13)$$

For the details of this derivation, see Appendix C.9.

Intuitively, these conditions impose that investment productivity in the low state is relatively low and that investment productivity in the high state is relatively high. This

ensures that money is scarce in the low state and plentiful in the high state. Because holding L-money commands a liquidity premium, it follows that investment by the bankers is above the first-best level ($k > k^*$) since they can attract a cheap source of funding by issuing L-money. Also, DM consumption by the buyers is below the first-best level because, anticipating the low price for goods in the DM when the low state is announced, the non-bankers limit the amount of goods they carry into the DM; $c_b < c_b^*$. Finally, one can also show that $(\phi^h + \phi^l) [\rho \tilde{z}_{nr}^h + (1 - \rho) \tilde{z}_{nr}^l] > \phi^h \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l$, meaning that the expected return on money issued by the bankers exceeds the return on risk-free money; a risk premium arises to reflect the risky return earned by inside money.

Type IIIb: money is risky and always scarce in the DM. We again consider Equations (C.1)-(C.3) from Appendix C.5 to solve for the equilibrium. Observe that we must have both $(1 - \psi)k = (1 - \psi\pi)c_b$, which clears the MM, and $(1 - \psi)\hat{\phi}^l a^l k^\alpha = \psi(1 - \pi)c_b$, which clears the low state DM. They can be combined to obtain $(1 - \psi\pi)\hat{\phi}^l a^l k^\alpha = \psi(1 - \pi)k$. Substituting out $\hat{\phi}^l$ by using Equation (C.3) and substituting out k using the bankers' MM FOCs we obtain:

$$\phi^l = \phi^h \frac{\rho}{1 - \rho} \frac{1 - (1 - \psi)\beta\rho/\phi^h + \left(\frac{\alpha\psi(1-\pi)}{1-\psi\pi} \frac{a^h}{a^l} - 1\right) [1 - (1 - \psi\pi)\beta\rho/\phi^h]}{1 - (1 - \psi)\beta\rho/\phi^h + \frac{1-\rho}{\rho} \left(1 - \frac{\alpha\psi(1-\pi)}{1-\psi\pi}\right) [1 - (1 - \psi\pi)\beta\rho/\phi^h]}. \quad (14)$$

Because we must have $\rho/\phi^h > (1 - \rho)/\phi^l$ in an equilibrium of the Type IIIb, we can assume that the RHS of Equation (14) is increasing in ϕ^h . Let $\phi^l = f(\phi^h)$ represent Equation (14). We can use f in another equilibrium relationship, namely $(1 - \psi)k = (1 - \psi\pi)c_b$. Substituting we obtain

$$u'^{-1} \left(\left[\frac{\psi\pi\rho}{1 - (1 - \psi\pi)\rho\beta/\phi^h} + \frac{\psi\pi(1 - \rho)}{1 - (1 - \psi)\beta\rho/\phi^h - \psi(1 - \pi)(1 - \rho)\beta/f(\phi^h)} \right]^{-1} \right) = \frac{1 - \psi}{1 - \psi\pi} [\alpha a^h \phi^h + \alpha a^l f(\phi^h)]^{\frac{1}{1-\alpha}}. \quad (15)$$

The LHS of this equation can be interpreted as the non-bankers demand for money and the RHS can be interpreted as the bankers' supply of money. The latter is increasing in ϕ^h and the former decreasing. Intuitively, a higher value for ϕ^h implies a higher liquidity premium associated with holding both H- and L-money (recall that $\phi^l = f(\phi^h)$ with $f' > 0$), which reduces the demand for money and increases the supply of money.

In Appendix C.10 I show that existence of a Type IIIb requires $\rho/\phi^h > (1 - \rho)/\phi^l$ and $\beta\rho/\phi^h < 1$. The first condition ensures that L-money earns a lower expected return than H-money, and the second condition ensures that money is always scarce in the DM.

With φ as defined in Equation (13), these conditions reduce to:

$$a^l < \alpha \frac{\psi(1-\pi)}{1-\psi\pi} \mathbb{E}[a], \quad (16)$$

$$\left[\alpha\beta \left(\rho a^h + \frac{(1-\rho)a^l}{\varphi} \right) \right]^{\frac{1}{1-\alpha}} < \frac{1-\psi\pi}{1-\psi} u'^{-1} \left(\frac{1-(1-\pi)\varphi}{\pi(1-\rho) + \rho[1-(1-\pi)\varphi]} \right). \quad (17)$$

Intuitively, these conditions imply that the average productivity of investment is low and that the productivity of investment in the low state is low relative to the average productivity of investment. Hence, money is always scarce in the DM and because the risk embedded in investment is relatively large, the bankers cannot fully absorb this risk with their equity. As a result, inside money becomes risky.

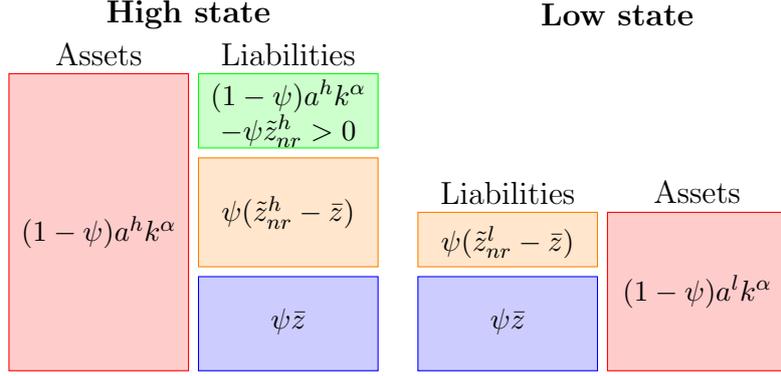
In the equilibrium of the Type IIIb, holding H-money and L-money commands a liquidity premium. Therefore, anticipating low prices for goods in the DM, the non-bankers limit the amount of goods carried to the DM so that $c_b < c_b^*$. The bankers however face low funding cost and therefore over-invest; $k > k^*$. In addition one can show that $q < q^*$. Finally, $(\phi^h + \phi^l) [\rho z_{nr}^h + (1-\rho)z_{nr}^l] > \phi^h z_{nr}^h + \phi^l z_{nr}^l$ holds; just like in an equilibrium of the Type IIIb and because of similar reasons, money issued by bankers earns a risk premium.

Comparing the various existence conditions above, we find that they mutually exclude each other, that an equilibrium always exist, that the equilibrium is unique, and that we can characterize when risky inside money arises:

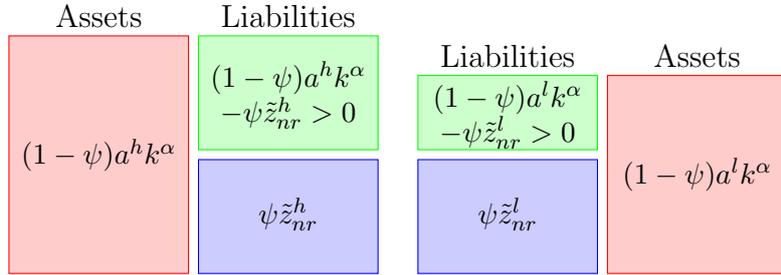
Proposition 1. *In an economy without outside money, there exists a unique stationary equilibrium. Money is risk-free when the equilibrium is of the Type I or II. First-best allocations are achieved when the equilibrium is of the Type I.* Q.E.D.

Figure 3 illustrates how bankers' balance sheets look like for the various equilibrium types. Notice that an equilibrium with risky inside money resembles a situation in which, when the low state realizes, bankers equity is fully eroded and inside money holders face losses. This represents a state of the world in which the banking system faces a systemic solvency crisis and anticipating that such a state occurs with strictly positive probability, the non-bankers demand a risk premium for holding inside money.

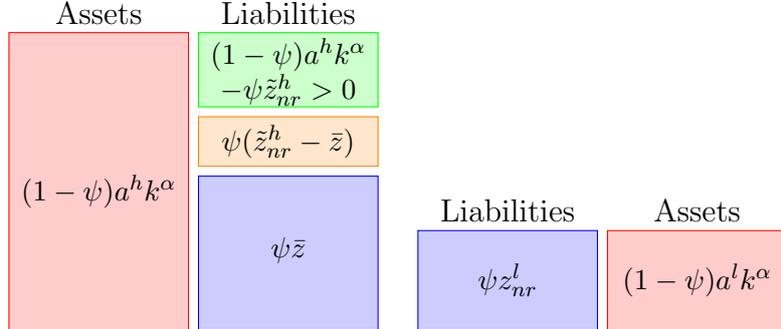
Figure 4 illustrates how investment productivity and investment risk determine which equilibrium type arises. In this figure, investment productivity is captured by the first-best amount of aggregate investment; $(1-\psi)k^*$. Investment risk is captured by the quotient $a^l/\mathbb{E}[a]$; the lower $a^l/\mathbb{E}[a]$, the higher is investment risk. From Figure 4 it follows that risk-free money requires either (i) high investment productivity when investment risk is relatively large, or (ii) little investment risk when investment productivity is relatively low.



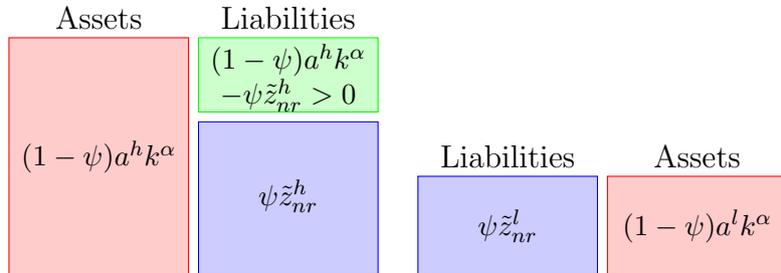
(a) Balance sheets of bankers in an equilibrium of Type I. Drawn under the assumption that net worth of bankers fully eroded in the low state.



(b) Balance sheets of bankers in an equilibrium of Type II.



(c) Balance sheets of bankers in an equilibrium of Type IIIa.



(d) Balance sheets of bankers in an equilibrium of Type IIIb.

Figure 3: State-dependent balance sheets of bankers. Assets are shown in red and net worth is shown in green. In orange $\psi \max\{z_{nr}^j - \bar{z}, 0\}$ is shown and in blue $\psi \min\{z_{nr}^j, \bar{z}\}$.

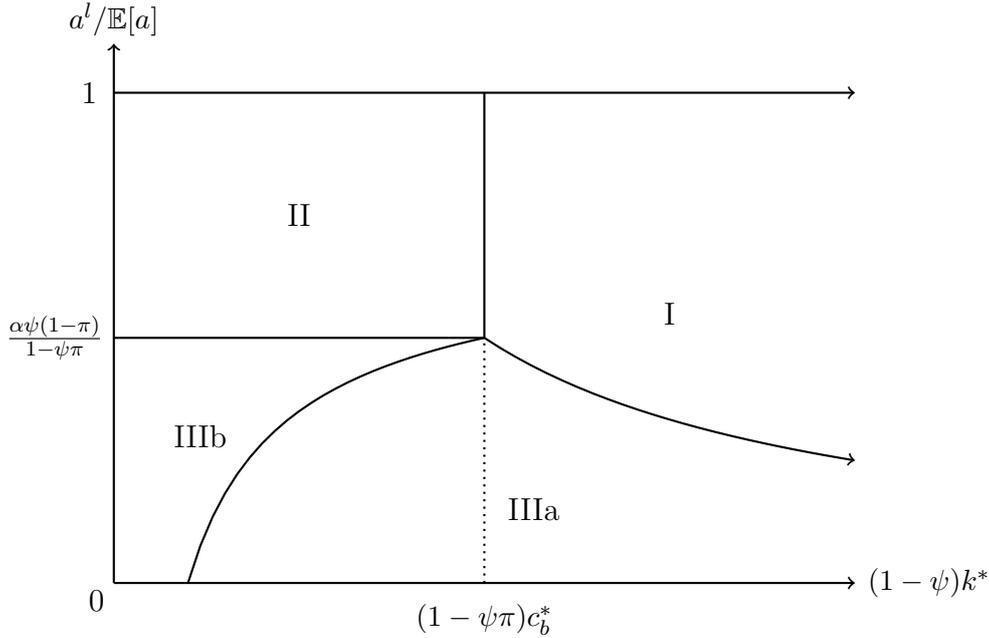


Figure 4: The occurrence of different equilibrium types. The horizontal axis captures the first-best level of aggregate investment. The vertical axis captures the risk-embedded of investment, measured by the ratio $a^l/\mathbb{E}[a]$. The shape of the line that separates regions IIIa and IIIb depends on the parameter specifications, but is always upward sloping and always intersects the horizontal axis to the right of the origin.

Regarding (i), keeping investment risk constant, the higher is investment productivity the more investment takes place and the more L-money and H-money is issued. In Figure 3, this implies that the bankers' balance sheets and the amount of inside money (captured by the blue boxes) gradually expand. It therefore becomes more likely that money is always plentiful in the DM, meaning that $\tilde{z}_{nr}^h \geq \bar{z}$ and $\tilde{z}_{nr}^l \geq \bar{z}$, so that we are in an equilibrium of the Type I. Despite the fact that bankers' equity may be unable to absorb all investment risk in an equilibrium of the Type I, this risk can be packaged into illiquid securities issued to the non-bankers (the orange boxes in Figure 3a that capture the difference between the liabilities issued by the bank and the amount of money needed to have plentiful money in the DM), leaving sufficient amounts of inside money to ensure that money remains plentiful in the DM (see Section 3.3 for the discussion on these illiquid debt securities).

Regarding (ii), if investment productivity is relatively low, then first-best allocations cannot be achieved by the inside money economy. Money is scarce and the bankers finance investment by issuing money as it is a cheap source of funding compared to issuing illiquid securities (meaning that the orange boxes disappear in Figures 3b and 3d). To ensure that money remains risk-free, the risk embedded in investment should not be too large compared to bankers' equity. Bankers equity is governed by the rents earned by investment, captured by α , and the fraction ψ of bankers in the economy. The effects

of changing these exogenous variables will be explained in the next subsection, which also contains a more detailed description of how the bankers' balance sheets change with investment productivity and investment risk.

Before proceeding, observe that when an economy is away from first-best allocations it is always characterized by over-investment and under-consumption; $k > k^*$ and $c_b < c_b^*$. These properties are in line with new monetarist models in which real investment backs money, e.g. Lagos and Rocheteau (2008) and Andolfatto et al. (2016).

4.1 When Does Risky Inside Money Arise?

Consider an economy that is initially characterized by risk-free inside money, meaning that it is in an equilibrium of either Type I or II. Because of the reasons discussed below, the economy can become characterized by risky inside money.

An increase in investment risk: If there is a sufficiently large increase in investment risk, then risk-free money is no longer feasible. If money was initially scarce, meaning that we were in an equilibrium of the Type II, then the increased risk can no longer be borne by bankers' equity. Specifically, the green bar in the RHS balance sheet of Figure 3b, representing bankers' equity in the low aggregate state, reduces as investment risk increases, until it vanishes completely and we end up in the equilibrium of the Type IIIb, with the bankers' balance sheets given by Figure 3d. If money was initially plentiful, meaning that the economy was in an equilibrium of the Type I, then the increase in investment risk impairs bankers' ability to create L-money because of the collateral constraints. As a result, L-money will become scarce and holding it will command a liquidity premium. Also, the risk from investment can no longer be fully transferred into illiquid securities. Specifically, the orange bar in the RHS balance sheet of Figure 3a, representing the low state value of illiquid debt securities issued by the bankers, reduces as investment risk increases. At some point the box vanishes completely and we end up in the equilibrium of the Type IIIa, with the bankers' balance sheets given by Figure 3c.

A reduction in investment productivity: If the economy is initially characterized by risk-free money, then a reduction in investment productivity can push the economy into an equilibrium of the Type III. Looking at Figure 5, this would correspond to an economy that is originally represented by point A, experiences a reduction in k^* , and then ends up in point B. Notice that the economy ends up in an equilibrium with risky inside money only if investment risk is large enough, because otherwise it would end up in an equilibrium of the Type I or II.⁷ In Figure 5, this means that point A must belong

⁷For example when moving from point C to D in Figure 5. Then, the bankers' balance sheets in Figure 3a gradually contract and the orange boxes, representing the state-contingent value of illiquid debt, shrink. At some point, these orange boxes completely disappear and the balance sheet becomes

to region Ib, or in mathematical terms it means that initially we must have the following two conditions satisfied:

$$(1 - \psi) \frac{a^l}{\alpha \mathbb{E}[a]} k^* \geq \psi(1 - \pi) c_b^* \quad \text{and} \quad a^l < \frac{\alpha \psi (1 - \pi)}{1 - \psi \pi} \mathbb{E}[a].$$

As explained before, if investment productivity is high then though bankers cannot finance themselves with risk-free debt, risk-free money is still feasible as the risk embedded in investment can be packaged into illiquid securities. However, when investment productivity declines, money becomes scarce and liquidity premia arise. Bankers therefore prefer to issue money rather than illiquid securities. Because investment risk is too large to be borne by the bankers' equity, the issued money becomes risky. In Figure 3a, this implies that the bankers' balance sheets shrink so that the orange boxes, representing state-contingent value of illiquid debt securities issued by the bankers, gradually shrink until they disappear and the bankers' balance sheets are given by Figure 3c.

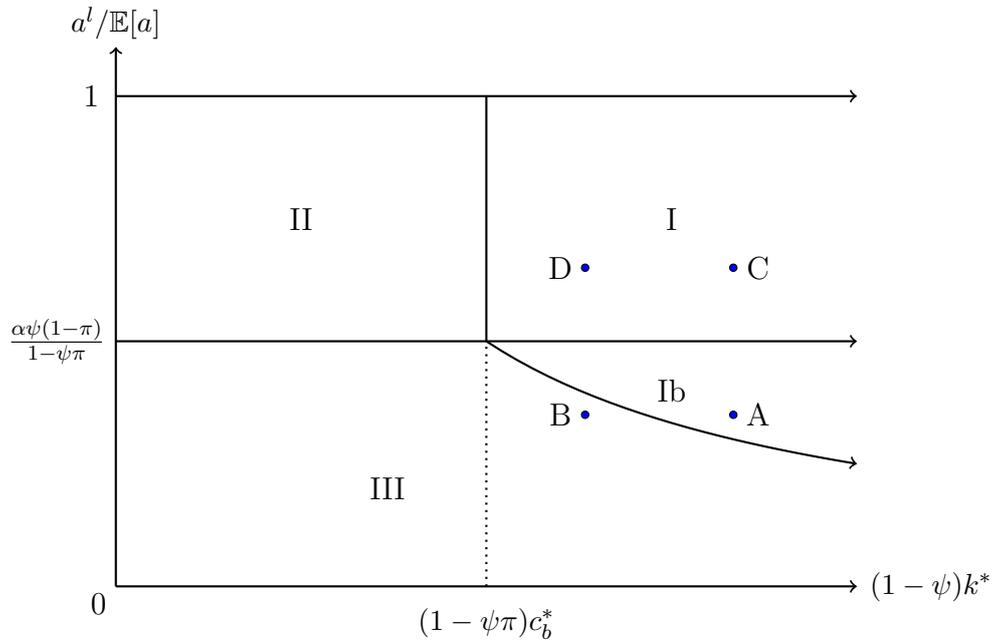


Figure 5: The effect of an reduction in investment productivity. The economy moves from point A to point B, thus from an equilibrium with risk-free inside money to an equilibrium with risky inside money.

An increased demand for money: When the first-best level of buyers' DM consumption increases, more money is needed to sustain first-best allocations. Figure 6 illustrates how the partitioning of the parameter space changes for an increase in c_b^* . The figure shows how it becomes more difficult to sustain first-best allocations, and if investment given by Figure 3b. The state-contingent value of equity is then always strictly positive, so that inside money remains risk-free.

risk is large enough, that the economy may become characterized by risky inside money. Returns from investment in the low state are then too low to ensure that bankers create plentiful money. As a result L-money becomes scarce and investment risk can no longer be fully packaged into illiquid securities.

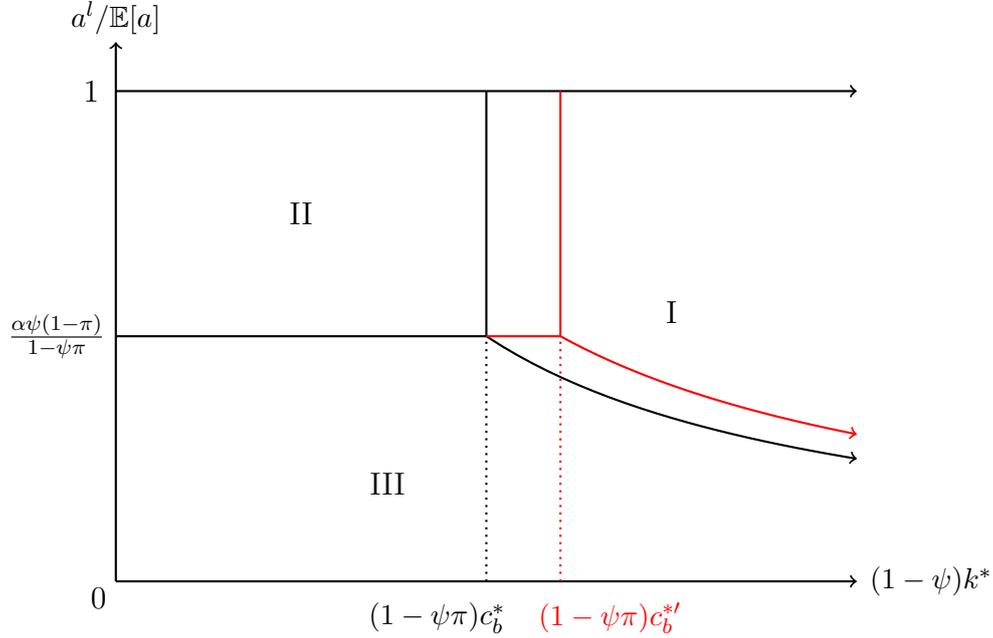


Figure 6: The effect of a higher first-best level of DM consumption.

Reduced rents from investment: Because of the DRS property, the marginal return from investment exceeds the average return from investment. The difference between these two represents a rent earned by the bankers, and this rent can be used to bear risk; if the low state realizes bankers give up some rents to redeem inside money in exchange for higher profits when the high state realizes. In this way, the risk from investment is fully borne by the bankers and not by the holders of inside money. As α increases, the marginal return from investment approaches the average return. As a result, rents decrease and it becomes more difficult to let the bankers bear the risk from investment. Figure 7 illustrates how the partitioning of the parameter space changes when α increases. From the figure we see that achieving risk-free money, i.e. an equilibrium of the Type I or II, will require lower investment risk. In fact, the model is well-behaved in the limit when $\alpha \rightarrow 1$, i.e. when the DRS property vanishes (see Appendix B).

A reduced fraction of bankers: Finally consider a reduction in the fraction of bankers in the economy, meaning that parameter ψ increases. To ensure that the capacity of a risk-free economy to attain first-best allocations does not change, suppose that $(1 - \psi)k^*$ and $\psi\pi c_b^*$ are unaffected by means of changing k^* and π . In this way, though there are less bankers and more non-bankers, the first-best aggregate amount of consumption and

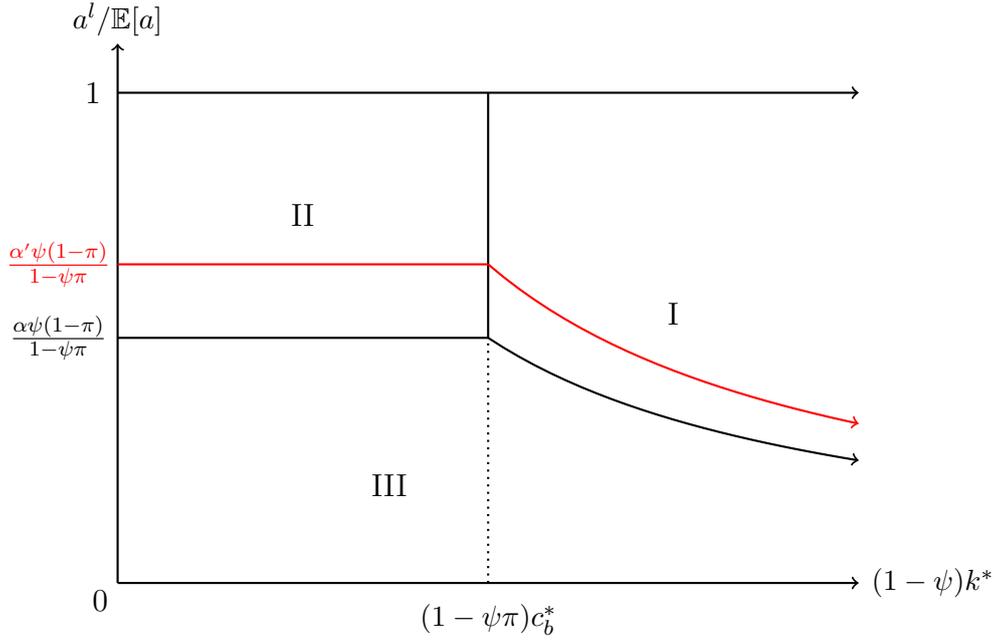


Figure 7: The effect of an increase in α , that means reduced rents from investment by bankers.

investment are left unchanged. Figure 8 shows how the partitioning of the parameter space changes for such a reduction in the fraction of bankers.

From Figure 8, we see that the effects are similar to the effects of an increase in α . The reason for this result is again due to the ability of the bankers to bear risk. Recall that in the MM, the bankers already have some capital in the form of general goods carried into the MM. This capital can act as a buffer; when the realized return from investment is low, the bankers give up their return on capital to redeem inside money. As the fraction of bankers reduces, the amount of investment undertaken by an individual banker in comparison to the general goods that he or she carried into the MM increases, representing an increase in leverage. This makes it more difficult for the banker to bear all risk.

4.2 The Effects of Risky Inside Money

When no news regarding the future state is announced just before DM trade takes place, or equivalently when there is opacity about bankers' return on investment, the quasi-linear utility structure implies that only the future expected value of money matters. Prices and allocations for q , k , and c_b would then become equivalent to those in an economy with $a^l = \mathbb{E}[a]$. To evaluate the effects of risky inside money, I therefore compare two economies, labeled \mathcal{A} and \mathcal{B} , which differ only in the distribution of state-contingent investment productivity. Specifically, $\mathbb{E}[a_{\mathcal{A}}] = \mathbb{E}[a_{\mathcal{B}}]$ and $a^l_{\mathcal{A}} < a^l_{\mathcal{B}} = \mathbb{E}[a_{\mathcal{B}}]$ so that economy \mathcal{A} has risky investment and economy \mathcal{B} has risk-free investment.

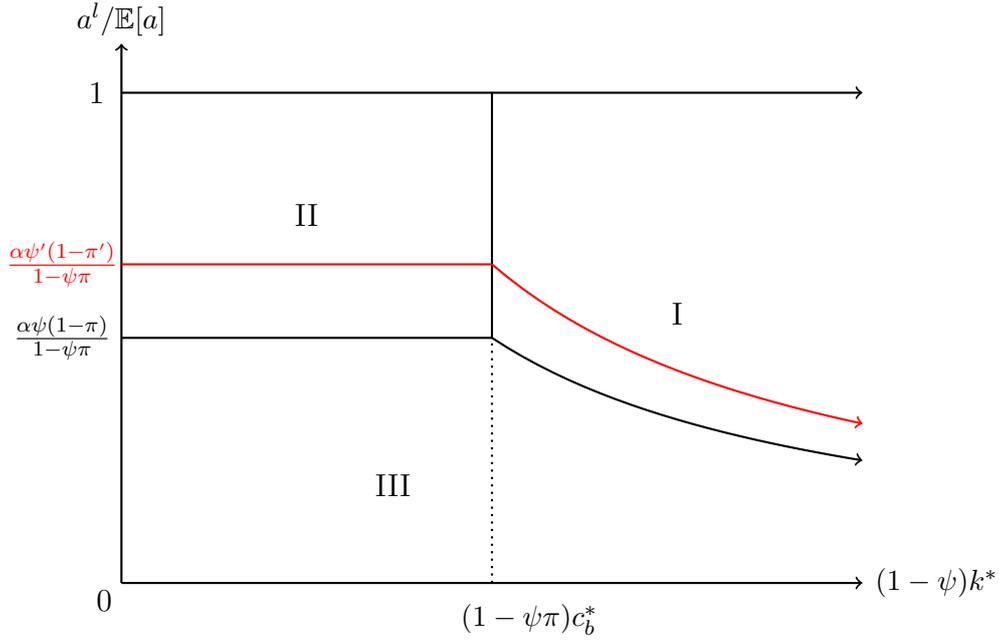


Figure 8: The effect of an increase in ψ and a reduction in π , keeping $\psi\pi$ constant.

Proposition 2. *If economy \mathcal{A} is characterized by an equilibrium of the Type I or II then $\mathcal{W}_A = \mathcal{W}_B$. If economy \mathcal{A} is characterized by an equilibrium of the Type IIIa or IIIb then $\mathcal{W}_A < \mathcal{W}_B$. Therefore, opacity about the return on bankers' assets is optimal.*

Proposition 2 implies that given investment risk, the presence of risky inside money reduces welfare. The rationale for this important result depends on how the economy would look like if investment is risk-free. If investment productivity is high relative to optimal DM consumption, then an economy without investment risk is characterized by first-best allocations. If however investment risk is large enough, bankers' binding collateral constraints restrain inside money creation so that first-best allocations cannot be attained. Anticipating a scarcity of money, it becomes too unattractive for the non-bankers to carry general goods to DM, while it becomes too attractive for the bankers to invest.

Recall from Lemma 4 that in the market economy, investment supports DM consumption through inside money creation; $k(1-\psi) \geq c_b(1-\psi\pi)$ must hold. Therefore, consider as second-best welfare

$$\begin{aligned} \widehat{\mathcal{W}}(1-\beta) &= \max_{q, c_b, k} \{ \psi\pi u(c_b) + (1-\psi)\beta\mathbb{E}[a]k^\alpha - q \} \\ \text{s.t.} \quad & (1-\psi)k + \psi\pi c_b \leq q \quad \text{and} \quad (1-\psi)k \geq (1-\psi\pi)c_b. \end{aligned}$$

The first constraint always binds, as available resources should be used. Letting $\lambda(1-\psi)$ denote the Lagrange multiplier associated with the second constraint, second-best

allocations satisfy

$$1 = \alpha\beta\mathbb{E}[a]\hat{k}^{\alpha-1} + \lambda, \quad \lambda = \frac{\psi\pi}{1-\psi\pi}[u'(\hat{c}_b) - 1], \quad \text{and} \quad \hat{c}_b = \min \left\{ \frac{(1-\psi)}{1-\psi\pi}\hat{k}, c_b^* \right\}.$$

Note that λ can be interpreted as the social externalities associated with investment. If and only if investment productivity is low relative to first-best DM consumption, meaning that $k^*(1-\psi) < c_b^*(1-\psi\pi)$, then $\lambda > 0$ and the second-best allocations deviate from the first-best. Specifically, there will be under-consumption and over-investment.

In a market economy with little investment risk, so that money is risk-free, the second-best allocations are attained. First, we know that in such an economy the constraint $k(1-\psi) = c_b(1-\psi\pi)$ binds. Second, we know that $1 = \psi\pi u'(c_b) + (1-\psi\pi)\beta/\phi$ and $\alpha\beta\mathbb{E}[a]k^{\alpha-1} = \beta/\phi$. In this sense, the externalities associated with investment are correctly incorporated in bankers' funding costs.

In an economy with too much investment risk, such that money is risky, markets however fail to attain second-best allocations because of one of the following two reasons. First, it may be the case that the economy with risky money has an equilibrium in which the constraint $k(1-\psi) \geq c_b(1-\psi\pi)$ is slack. In such an economy, there is too much over-investment and under-consumption than necessary to attain second-best allocations. This will only occur for an equilibrium of the Type IIIa.

Second, if the constraint $k(1-\psi) \geq c_b(1-\psi\pi)$ binds, then markets fail to incorporate externalities from investment in bankers' funding costs. As I demonstrate in the proof of Proposition 2, the marginal funding costs faced by the bankers are too high compared to the social benefits of investment. The latter depend on the marginal benefit of consumption, which in turn is governed by the expected prices at which general goods can be sold in the DM, i.e. $\mathbb{E}[\beta/\hat{\phi}^{j+}]$. Equation (5) shows that this term is proportional to the expected future value of money, that is $\mathbb{E}[z_{nr}^{j+}]/c_b$. Because c_b is again linearly related to investment, the marginal benefits of DM consumption thus depend on the expected return on inside money, or equivalently bankers' average funding costs. Investment however depends on bankers' marginal funding cost. Because bankers face a binding collateral constraint for L-money issuance, these marginal funding costs are driven above the average funding costs. This effect is therefore directly related to the DRS property of investment, or equivalently the ability of bankers to earn rents:

Corollary 1. *When economy \mathcal{A} is characterized by an equilibrium of the Type IIIb, then $\lim_{\alpha \rightarrow 1} \mathcal{W}_{\mathcal{A}} = \lim_{\alpha \rightarrow 1} \mathcal{W}_{\mathcal{B}}$.*

Proposition 2 re-confirms the findings of Andolfatto et al. (2014) and Dang et al. (2017) as opacity about banks' balance sheets is optimal. It must be stressed that a novel mechanism gives rise to this finding in the current paper. Andolfatto et al. (2014) and Dang et al. (2017) find that opacity is welfare improving because it removes consumption

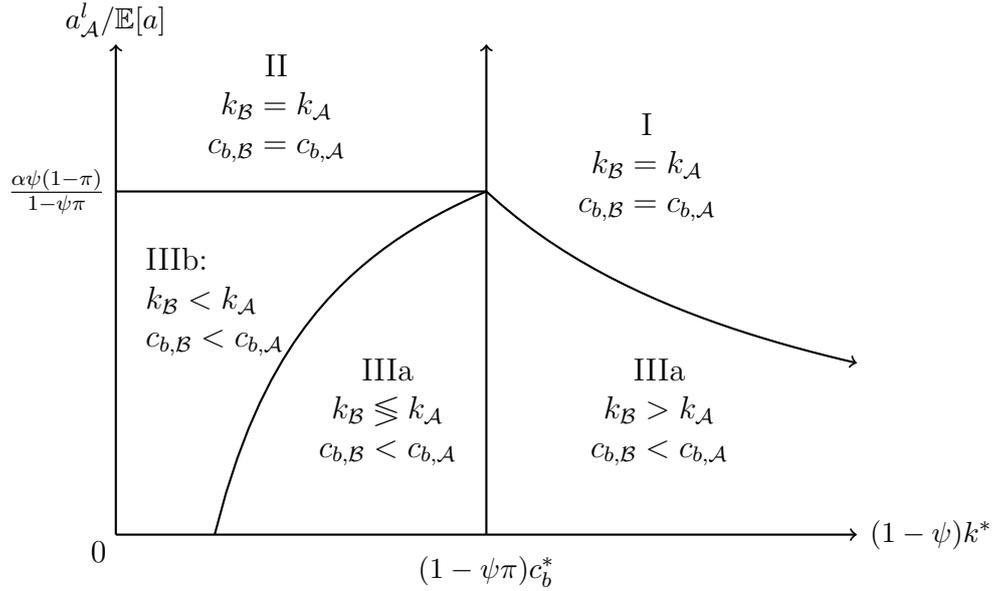


Figure 9: The effects of risky inside money on equilibrium investment and DM consumption for an economy characterized by risky inside money. Allocations denoted with a subscript \mathcal{A} are for the economy with risky inside money, allocations denoted with a subscript \mathcal{B} are for an economy with risk-free investment but that is otherwise similar.

volatility. In my approach consumption volatility plays no role as DM consumption is deterministic regardless of whether money is risky or not (see Equation (6)). Instead, the current paper argues that those who issue inside money, should be appropriately compensated for the social benefits associated with money issuance. This is infeasible in the market economy when investment risk becomes too large to be borne by bankers' equity.

Proposition 3. *If economy \mathcal{A} is in an equilibrium of the Type I or II, then $k_{\mathcal{A}} = k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} = c_{b,\mathcal{B}}$. If economy \mathcal{A} is in an equilibrium of the Type IIIb, then $k_{\mathcal{A}} < k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$. If economy \mathcal{A} is in an equilibrium of the Type IIIa and $(1-\psi)k^* \geq (1-\psi\pi)c_b^*$ holds, then $k_{\mathcal{A}} > k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$. Finally, if economy \mathcal{A} is in an equilibrium of the Type IIIa and $(1-\psi)k^* < (1-\psi\pi)c_b^*$, then $c_{b,\mathcal{A}} > c_{b,\mathcal{B}}$ and depending on the exact parameter specifications we have can have both $k_{\mathcal{A}} \geq k_{\mathcal{B}}$ and $k_{\mathcal{A}} \leq k_{\mathcal{B}}$.*

Figure 9 illustrates Proposition 3. It shows how the real effects of risky inside money depend on investment productivity, captured by $(1-\psi)k^*$. If productivity is high, having risky inside money implies more investment by the bankers and less consumption by the buyers. When inside money becomes risky, it must be the case that money becomes scarce. As a result, the economy moves from a situation without liquidity premia to a situation with liquidity premia. These premia make it unattractive to carry general goods to the DM for the non-bankers, and more attractive to invest for the bankers. Hence, DM consumption decreases while investment increases.

On the other hand, when investment productivity is low we see that risky inside money reduces both DM consumption and investment. Here, we can build upon the insights from Proposition 2: When investment productivity is low, an economy with risky inside money is in an equilibrium of the Type IIIb while an economy with risk-free money is in an equilibrium of the Type II. In these equilibria, $(1 - \psi)k = (1 - \psi\pi)c_b$. Also, in the equilibrium with risky inside money, in contrast to an equilibrium with risk-free money, the social benefits of money creation are not fully incorporated in the bankers' marginal funding cost. As a result, risky inside money has a negative effect on investment and because investment is needed to support DM consumption, risky inside money also has a negative effect on DM consumption. Because the underrepresentation of the social benefits from money in the bankers' funding costs relate to the ability of the bankers' to earn rents from investment, it follows that

Corollary 2. *When economy \mathcal{A} is characterized by an equilibrium of the Type IIIb, then $\lim_{\alpha \rightarrow 1} k_{\mathcal{A}} = \lim_{\alpha \rightarrow 1} k_{\mathcal{B}}$ and $\lim_{\alpha \rightarrow 1} c_{b,\mathcal{A}} = \lim_{\alpha \rightarrow 1} c_{b,\mathcal{B}}$.*

Observe that when economy \mathcal{A} is in an equilibrium of the Type IIIa and economy \mathcal{B} is in an equilibrium of the Type II, then the effect of risky inside money on investment depends on the exact parameter specifications. DM consumption in economy \mathcal{A} will however always be lower than DM consumption in economy \mathcal{B} . Here, there are two opposing forces at work. On the one hand, the negative effect of risky inside money on DM consumption reduces the need for investment as follows from the property $(1 - \psi)k \geq (1 - \psi\pi)c_b$. On the other hand, in economy \mathcal{A} money becomes plentiful in the DM when the high state is announced, which implies that constraint $(1 - \psi)k \geq (1 - \psi\pi)c_b$ becomes slack. That means, economy \mathcal{A} invests too much than what is needed to facilitate second-best DM consumption.

Proposition 4. *If economy \mathcal{A} is in an equilibrium of the Type I or II, then $\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l = \phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l$. If economy \mathcal{A} is in an equilibrium of the Type IIIa or IIIb, then $\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l > \phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l$.*

Proposition 4 states that when economy \mathcal{A} has risky money, the return on risk-free money, $[\phi^h + \phi^l]^{-1}$, is lower than in economy \mathcal{B} . This property could be thought of as a flight-into-safety; non-bankers worry about the risk associated with holding risky inside money and therefore prefer to hold risk-free forms of money. Because risk-free money is not available, the return on risk-free money reduces to attain an equilibrium. Proposition 4 is in line with results in the literature on the optimal securitization of risky endowments.⁸ Models from this literature predict that increased risk and a scarcity of risk-free assets drive down risk-free interest rates. My approach demonstrates that this result holds true in an environment with assets used as payment instruments, created endogenously by the bankers to finance risky investment.

⁸Examples of papers in this literature include Barro et al. (2017), Gennaioli et al. (2012), and Caballero and Farhi (2018).

5 The Economy With Inside and Outside Money

To understand how outside money affects the existence of risky money, consider an environment in which outside money is in positive supply and risk-free; $m^h = \tilde{m}^h = m^l = \tilde{m}^l > 0$. To ease the notation, let $m \equiv m^h = m^l$ denote the stationary supply of risk-free outside money. Inside money remains in zero net supply so $\psi \tilde{z}_{nr}^h + (1 - \psi) \tilde{z}_r^h = m$ and $\psi \tilde{z}_{nr}^l + (1 - \psi) \tilde{z}_r^l = m$.

Lemma 6. *The partial equilibrium in the MM cannot be of type IVa or IVb.*

Because outside money is risk-free, the fact that bankers can create more H-money than L-money again rules out an equilibrium in which H-money is scarcer than L-money. It remains to consider the remaining four equilibrium types:

Type I: Money is always plentiful in the DM. Previous insights imply that first-best allocations are again achieved in an equilibrium of Type I. In Appendix C.15, I show that the equilibrium of Type I exists if and only if

$$(1 - \psi)k^* + \beta m \geq (1 - \psi\pi)c_b^* \quad \text{and} \quad (1 - \psi)\frac{a^l}{\alpha \mathbb{E}[a]}k^* + \beta m \geq \psi(1 - \pi)c_b^*. \quad (18)$$

These conditions can again be found by assuming that bankers do not carry general goods into the DM. The presence of risk-free outside money now increases the likelihood of being in an equilibrium of the Type I. Intuitively, this is because there is now more money available to sustain the first-best level of DM consumption.

Type II: Money is risk-free and always scarce in the DM. Because $\hat{\phi}^h = \hat{\phi}^l$, we can continue to use ϕ , and $\phi^h = \rho\phi$ and $\phi^l = (1 - \rho)\phi$ will still hold. In addition Equation (5) still implies that $\tilde{z}_{nr}^h = \tilde{z}_{nr}^l$, and because inside money is in zero net supply and outside money is risk-free, we must also have $\tilde{z}_r^h = \tilde{z}_r^l$. For notational convenience we can therefore remove the superscripts associated with these variables. To determine ϕ observe that Lemma 4 implies that $(1 - \psi)k = (1 - \psi\pi)q - \phi m$, or equivalently $(1 - \psi)k + \phi m = (1 - \psi\pi)c_b$. From the first-order conditions of the bankers we know that $k^{1-\alpha} = \alpha \mathbb{E}[a]\phi$. From previous insights we also know that in the equilibrium of the Type II, $\psi\pi u'(c_b) = 1 - (1 - \psi\pi)\beta/\phi$. The equilibrium value for ϕ thus uniquely solves:

$$(1 - \psi)(\alpha \mathbb{E}[a]\phi)^{\frac{1}{1-\alpha}} + \phi m = (1 - \psi\pi)u'^{-1}\left(\frac{1 - (1 - \psi\pi)\beta/\phi}{\psi\pi}\right), \quad (19)$$

where the real supply of money (LHS) now consists of inside money, governed by investment, and outside money. As in the economy with only inside money, the real supply of money is increasing in ϕ .

In Appendix C.16, I show that the equilibrium of Type II exists if and only if:

$$(1 - \psi)k^* + \beta m < (1 - \psi\pi)c_b^* \quad \text{and} \quad a^l \geq \alpha \frac{\psi(1 - \pi) - \phi m (\alpha \mathbb{E}[a]\phi)^{\frac{1}{\alpha-1}}}{1 - \psi\pi} \mathbb{E}[a]. \quad (20)$$

It can be verified that a higher supply of outside money will lead to a lower equilibrium price of money. Therefore, as the supply of outside money increases the first existence conditions becomes stricter while the second existence condition is relaxed.

Type IIIa: Money is risky, plentiful in the DM when the high state is announced but scarce in the DM when the low state is announced. In this equilibrium type, we again use that Equation (5) implies $\hat{\phi}^l \tilde{z}_{nr}^l = (1 - \pi)c_b$. Using that $\tilde{z}_r^l = -a^l k^\alpha$, $k^{1-\alpha} = \alpha [\rho\beta a^h + \phi^l a^l]$, $\psi \tilde{z}_{nr}^j + (1 - \psi)\tilde{z}_r^j = m$ for all $j \in \{l, h\}$, and Equations (C.1) and (C.3), we obtain the equilibrium relationship

$$\begin{aligned} & \psi(1 - \pi)u'^{-1} \left(\frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi(1 - \rho) + \rho[1 - (1 - \pi)(1 - \rho)\beta/\phi^l]} \right) \\ &= \phi^l \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi} + 1 \right) \left[a^l(1 - \psi) (\alpha\rho a^h\beta + \alpha a^l \phi^l)^{\frac{\alpha}{1-\alpha}} + m \right], \end{aligned} \quad (21)$$

where the LHS (decreasing in ϕ^l) represents money demand and the RHS (increasing in ϕ^l) represents money supply.

In Appendix C.17, I show that existence requires $\beta(1 - \rho)\phi^l < 1$ and $(1 - \psi)k \geq (1 - \psi\pi)c_b - \phi m$, boiling down to

$$(1 - \psi) \frac{a^l}{\alpha \mathbb{E}[a]} k^* + \beta m < \psi(1 - \pi)c_b^* \quad (22)$$

and

$$\begin{aligned} & \psi(1 - \pi)u'^{-1} \left(\frac{1 - (1 - \pi)(1 - \rho)\beta/\phi'}{\pi(1 - \rho) + \rho[1 - (1 - \pi)(1 - \rho)\beta/\phi']} \right) \geq \\ & \phi' \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi'}{\pi} + 1 \right) \left[a^l(1 - \psi) (\alpha\rho a^h\beta + \alpha a^l \phi')^{\frac{\alpha}{1-\alpha}} + m \right], \end{aligned} \quad (23)$$

where ϕ' uniquely solves

$$\begin{aligned} & (1 - \psi) (\alpha\beta\rho a^h + \alpha a^l \phi')^{\frac{1}{1-\alpha}} + (\rho\beta + \phi')m = \\ & (1 - \psi\pi)u'^{-1} \left(\frac{1 - (1 - \pi)(1 - \rho)\beta/\phi'}{\pi(1 - \rho) + \rho[1 - (1 - \pi)(1 - \rho)\beta/\phi']} \right). \end{aligned} \quad (24)$$

Type IIIb: Money is risky and always scarce in the DM. Again consider Equations (C.1)-(C.3) to solve for the equilibrium. Because money is always scarce in the DM,

we must have that $(1 - \psi)k + (\phi^h + \phi^l)m = (1 - \psi\pi)c_b$. Because the expected return on H-money exceeds the expected return on L-money, bankers exhaust their capacity to create L-money. Therefore, we have $\tilde{z}_r^l = -a^l k^\alpha$. We can use Equation (5) to obtain that $\psi(1 - \pi)c_b = \hat{\phi}^l [a^l(1 - \psi)k^\alpha + m]$ must hold. We can therefore use Equations (C.1) and (C.3) to obtain the following system:

$$u'^{-1} \left(\left[\frac{\psi\pi\rho}{1 - (1 - \psi\pi)\rho\beta/\phi^h} + \frac{\psi\pi(1 - \rho)}{1 - (1 - \psi)\rho\beta/\phi^h - \psi(1 - \pi)(1 - \rho)\beta/\phi^l} \right]^{-1} \right) = \frac{(1 - \psi)(\alpha a^h \phi^h + \alpha a^l \phi^l)^{\frac{1}{1-\alpha}} + (\phi^h + \phi^l)m}{1 - \psi\pi} \quad (25)$$

$$= \frac{\phi^l}{1 - \rho} \left[1 + \frac{\rho\psi(1 - \pi)(\rho\beta/\phi^h - (1 - \rho)\beta/\phi^l)}{1 - (1 - \psi\pi)\rho\beta/\phi^h} \right] \times \frac{a^l(1 - \psi)(\alpha a^h \phi^h + \alpha a^l \phi^l)^{\frac{\alpha}{1-\alpha}} + m}{\psi(1 - \pi)}. \quad (26)$$

Existence of an equilibrium of Type IIIb requires that money is always scarce in the DM and that $\rho/\phi^h > (1 - \rho)/\phi^l$. Together, these conditions imply we must have $\beta > \rho/\phi^h > (1 - \rho)/\phi^l$. In Appendix C.18, I show that necessary conditions for existence are:

$$(1 - \psi)k^* < (1 - \psi\pi)c_b^* - \beta m, \quad (27)$$

$$a^l < \alpha \frac{\psi(1 - \pi) - \phi m (\alpha \mathbb{E}[a]\phi)^{\frac{1}{\alpha-1}}}{1 - \psi\pi} \mathbb{E}[a],$$

where ϕ solves Equation (19). Sufficient conditions for existence are given by Equation (27) and:

$$a^l < \alpha \frac{\psi(1 - \pi) - \phi m (\alpha \mathbb{E}[a]\phi)^{\frac{1}{\alpha-1}}}{1 - \psi\pi} \mathbb{E}[a], \quad (28)$$

$$\psi(1 - \pi)u'^{-1} \left(\frac{1 - (1 - \pi)(1 - \rho)\beta/\phi'}{\pi(1 - \rho) + \rho[1 - (1 - \pi)(1 - \rho)\beta/\phi']} \right) < \phi' \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi'}{\pi} + 1 \right) \left[a^l(1 - \psi) (\alpha \rho a^h \beta + \alpha a^l \phi')^{\frac{\alpha}{1-\alpha}} + m \right], \quad (29)$$

where ϕ solves Equation (19) and ϕ' solves Equation (24).

From the above information we can conclude that parameters and policy pin down whether the economy is in an equilibrium of the Type I, II or III. Because the system of equations for determining equilibrium prices in an equilibrium of the Type IIIb is highly non-linear, I do not show that we always have a unique equilibrium or that it is uniquely determined whether an equilibrium of the Type III must be of the subtype a or

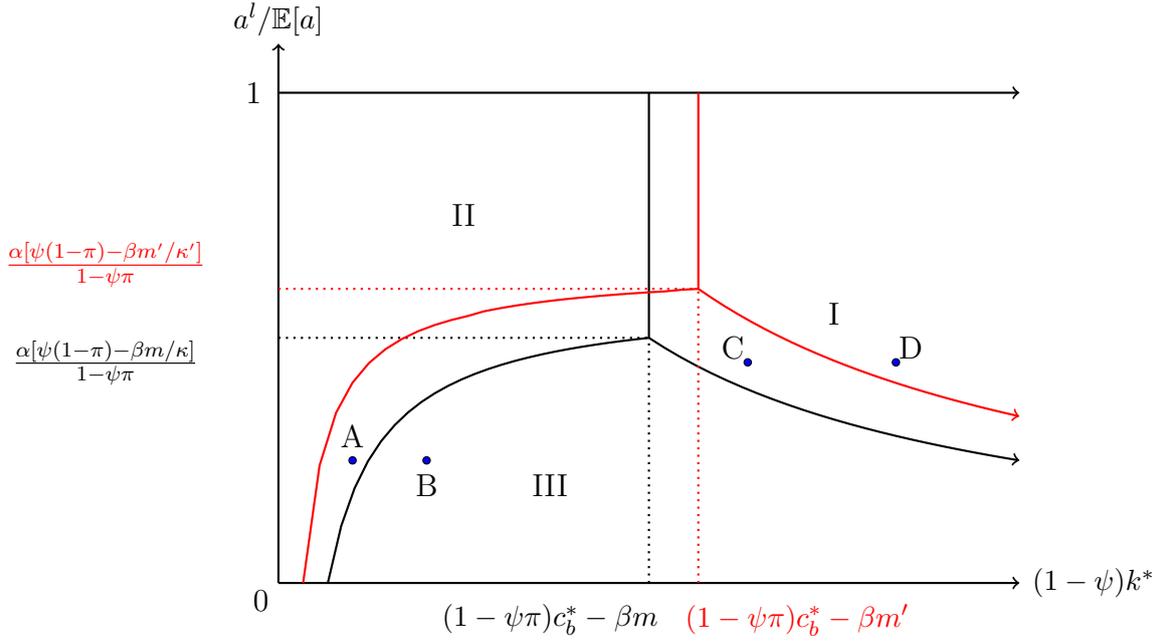


Figure 10: The effect of an reduction in the supply of outside money. The initial supply of outside money and the new supply of outside money are denoted with m and m' respectively, and $(1 - \psi)\kappa \equiv (1 - \psi\pi)c_b^* - \beta m$ and $(1 - \psi)\kappa' \equiv (1 - \psi\pi)c_b^* - \beta m'$. The parametrization is such that $\beta m < \psi(1 - \psi)c_b^*$

b. Showing uniqueness is also not necessary for the results that follow.

Proposition 5. *In an economy without outside money there exists a stationary equilibrium. The equilibrium is unique and characterized by risk-free money when an equilibrium of the Type I or II exists. Money is risky when an equilibrium of the Type IIIa or IIIb exists. First-best allocations are achieved when an equilibrium of the Type I exists. Q.E.D.*

5.1 When Does Risky Inside Money Arise?

To understand how the presence of risk-free outside money affects the possibility of having an equilibrium with risky money, consider Figure 10. It illustrates Proposition 5 with two different levels for the supply of outside money $m' < m$, under the assumption that $\beta m < \psi(1 - \pi)c_b^*$ as otherwise an equilibrium always has risk-free money. Compared to the economy with only inside money, there are two new effects that can push an economy into an equilibrium with risky money.

Increased investment productivity: If an economy is characterized by relatively much investment risk and a relatively low investment productivity, then an increase in investment productivity may push the economy into an equilibrium with risky money. In Figure 10, this is illustrated by an economy that moves from point A to point B . This channel was not present in an economy with only inside money and is driven by the

effect of outside money on bankers' equity. Because all agents are symmetric when they enter the MM, an increased supply of outside money implies that agents enter the MM with a higher level of real wealth. If an agent turns out to be a banker, this real wealth contributes directly to his equity. If investment productivity is very low compared to the supply of outside money, bankers therefore issue only small amounts of inside money, reducing bankers' leverage and increasing bankers' ability to bear investment risk. It may even be the case that there is no inside money issuance, which occurs when bankers enter the MM with more real wealth than they need to finance investment.

If investment productivity increases, the banker however finds it optimal to also increase investment and as a result he will become more reliant on inside money issuance. As a result leverage increases, reducing the capacity of a banker to bear risk and opening the door for risky inside money. The latter only occurs if the initial equilibrium is of the Type II; money is risk-free but scarce. If money were plentiful and investment productivity would increase (e.g. a move from point C to D in Figure 10) then money remains risk-free because all risk can be packaged into illiquid securities.

A reduced supply of outside money: Looking at Figure 10, we see that an equilibrium with risk-free money becomes less likely if m declines. The reason for this result is twofold. First, if investment risk increases money can remain risk-free as long as investment productivity is sufficiently high because risk embedded in investment can then be put into illiquid securities as money is plentiful. With less outside money an economy needs to rely more on investment by bankers as a source of money creation. Given a particular level of risk, a smaller supply of outside money therefore requires greater first-best levels of aggregate investment to ensure that money remains plentiful. In the figure, this mechanism is represented by a rightward shift in the line separating regions I and III.

Second, if investment productivity is relatively low then money is scarce and risk cannot be packaged into illiquid securities. As a result, investment risk can only be borne by the equity of bankers and inside money holders. A reduction in the supply of outside money reduces bankers' equity, increasing leverage and opening the door for risky inside money. In Figure 10, the effect of a reduction in the supply of outside money on the leverage of banks is captured by an upward shift of the line separating regions II and III.

Figures that illustrate how the partitioning of the parameter space looks like for the other experiments that were considered in Section 4.1, can be found in Appendix A. The intuition for what these figures show is very similar to the intuition for the figures in Section 4.1.

5.2 The Real Effects of Risky Money

Consider again economies, \mathcal{A} and \mathcal{B} , with $\mathbb{E}[a_{\mathcal{A}}] = \mathbb{E}[a_{\mathcal{B}}]$ but $a_{\mathcal{A}}^l < a_{\mathcal{B}}^l = \mathbb{E}[a_{\mathcal{B}}]$. Because we now have outside money in the model, assume that $(\phi_{\mathcal{A}}^l + \phi_{\mathcal{A}}^h)m_{\mathcal{A}} = (\phi_{\mathcal{B}}^l + \phi_{\mathcal{B}}^h)m_{\mathcal{B}}$, so that the real value of outside money issued in the CM is the same in both economies.

Proposition 6. *Let policy be such that $(\phi_{\mathcal{A}}^l + \phi_{\mathcal{A}}^h)m_{\mathcal{A}} = (\phi_{\mathcal{B}}^l + \phi_{\mathcal{B}}^h)m_{\mathcal{B}}$. If economy \mathcal{A} is characterized by an equilibrium of the Type I or II then $\mathcal{W}_{\mathcal{A}} = \mathcal{W}_{\mathcal{B}}$. If economy \mathcal{A} is characterized by an equilibrium of the Type IIIa or IIIb then $\mathcal{W}_{\mathcal{A}} < \mathcal{W}_{\mathcal{B}}$.*

The intuition for Proposition 6 is similar to that for the economy without outside money, but nevertheless there is a slight difference when economy \mathcal{B} is characterized by an equilibrium of the Type IIIb. Then, the presence of risk-free outside money increases the wedge between the marginal funding cost faced by bankers and the average return on money. The reason is that because inside money is risky, it bears a risk premium. As a result, the expected return on inside money exceeds the expected return on outside money and thus it also exceeds the expected return on the total amount of money held by the non-bankers.

Regarding the effects of risky money on real allocations and the return on risk-free money, Propositions 3 and 4 remain valid given the assumption $(\phi_{\mathcal{A}}^l + \phi_{\mathcal{A}}^h)m_{\mathcal{A}} = (\phi_{\mathcal{B}}^l + \phi_{\mathcal{B}}^h)m_{\mathcal{B}}$. This property is immediate from the proofs of these Propositions, which also imply:

Corollary 3. *If economy \mathcal{A} is characterized by risky inside money, then lump-sum taxes in economy \mathcal{A} are lower than in economy \mathcal{B} : $\tau_{\mathcal{A}} < \tau_{\mathcal{B}}$.*

Corollary 3 demonstrates the fiscal consequences of risky inside money. Because the presence of risky inside money makes holding risk-free outside money more attractive, the equilibrium return earned by holding outside money reduces (see Proposition 4). This reduces interest expenses for the government, so that it can reduce taxation.

5.3 Stabilization Policies

Consider two stabilization policies to alleviate the negative consequences of risky-inside money; (i) injecting capital into banks and (ii) buying risky assets from banks, both financed by issuing risk-free outside money. In the current framework, these two interventions can be studied by letting government change the composition of outside money during the MM to ensure that the non-bankers can hold risk-free money. That means, \tilde{m}^h and \tilde{m}^l are chosen such that $\tilde{m}^l = \tilde{m}^h + (1 - \psi)(\tilde{z}_r^l - \tilde{z}_r^h)$.

Asset Purchases: In the model, asset purchases are equivalent to government purchasing H-money and/or L-money from the bankers. The reason is that these instruments

represent claims on the output of bankers' risky investment projects. Effectively, the bankers thus sell risky money to the government and get risk-free outside money in return. The bankers sell this risk-free outside money to non-bankers in return for general goods. Though the stock of outside money held by agents is risk-free, the net position of the government in terms of H-money and L-money is risky; $\tilde{m}^h < \tilde{m}^l$. The reason is that government now owns some H-money issued by the bankers. Effectively, the government has taken abundant H-money out of circulation and has injected scarce L-money into the economy.

Capital Injections: Government buys shares in banks and pays for these shares by issuing risk-free debt. In the model, this is equivalent to government buying H-money from the bankers and giving the bankers risk-free outside money in return. H-money is a state-contingent asset that is repaid by the bankers when the high state materializes, that means when the bankers' assets generate high returns. Because investment risk is high, returns from these assets are too low to leave something for equity holders (the bankers and the government) when the low state materializes, which is why the government is not purchasing L-money when it injects capital. The risk-free outside money that is issued by the government can then be kept on banker's balance sheets and be used to provide non-bankers with higher returns in case the low state is realized. Effectively, government has taken H-money out of circulation by purchasing it from the bankers in the form of equity, paid for with L-money. Again, we obtain $\tilde{m}^h < \tilde{m}^l$.

Proposition 7. *Given a policy that satisfies $m^h = m^l = m$ and $\tilde{m}^l = \tilde{m}^h + (1 - \psi)(\tilde{z}_r^l - \tilde{z}_r^h)$, there exists a unique equilibrium and this equilibrium is characterized by risk-free money. Prices in this equilibrium are given by $\rho\phi^h = (1 - \rho)\phi^l = \hat{\phi}^h = \hat{\phi}^l = \phi \geq \beta$. Here, ϕ satisfies $\phi m + (1 - \psi)[\alpha\mathbb{E}[a]\phi]^{\frac{1}{1-\alpha}} \geq (1 - \psi\pi)u'^{-1}((1 - (1 - \psi\pi)\beta/\phi)/(\psi\pi))$, with equality if $\phi > \beta$.*

The chosen government policy ensures that non-bankers hold risk-free money but it also implies state-contingent taxation as government issues more L-money than H-money. This represents the fact tax payers' money is put at risk. To ensure that the real indebtedness of government does not change, consider that the CM value of outside money $(\phi^h + \phi^l)m$ remains unchanged when conducting stabilization policies. We can then compare two economies, \mathcal{A} and \mathcal{B} that are characterized by the same parameters but different government policy. Specifically, $(\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l)m_{\mathcal{A}} = (\phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l)m_{\mathcal{B}}$, $\tilde{m}_{\mathcal{A}}^l = \tilde{m}_{\mathcal{A}}^h + (1 - \psi)(\tilde{z}_{r,\mathcal{A}}^l - \tilde{z}_{r,\mathcal{A}}^h)$, and $\tilde{m}_{\mathcal{B}}^h = \tilde{m}_{\mathcal{B}}^l$.

Proposition 8. *If economy \mathcal{B} is characterized by an equilibrium of the Type I or II then $\mathcal{W}_{\mathcal{A}} = \mathcal{W}_{\mathcal{B}}$, $k_{\mathcal{A}} = k_{\mathcal{B}}$, $c_{b,\mathcal{A}} = c_{b,\mathcal{B}}$, $\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l = \phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l$, and $\tau_{\mathcal{A}} = \tau_{\mathcal{B}}$. If economy \mathcal{B} is characterized by an equilibrium of the Type IIIa or IIIb then $\mathcal{W}_{\mathcal{A}} > \mathcal{W}_{\mathcal{B}}$, $c_{b,\mathcal{A}} > c_{b,\mathcal{B}}$,*

$\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l < \phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l$, and $\mathbb{E}[\tau_{\mathcal{A}}] \geq \tau_{\mathcal{B}}$. If economy \mathcal{B} is characterized by an equilibrium of the Type IIIb, then $k_{\mathcal{A}} > k_{\mathcal{B}}$, if economy \mathcal{A} is characterized by an equilibrium of the Type I then $k_{\mathcal{A}} < k_{\mathcal{B}}$. In other cases $k_{\mathcal{A}} \geq k_{\mathcal{B}}$ or $k_{\mathcal{A}} \leq k_{\mathcal{B}}$ depends on the exact parametrization of the model.

Proposition 8 demonstrates that the stabilization policy implements second-best allocations given the constraint $(1 - \psi)k + \Delta \geq (1 - \psi\pi)c_b$, where Δ is the CM value of outside money that is pinned down by government policy. Hence, the negative effects of risky inside money are effectively undone by the policy intervention. As follows from Corollary 3, the policy intervention has indirect fiscal consequences that operate through prices. Specifically, not only does the stabilization policy make taxation state dependent, it also increases the average level of taxation across states. This is due to an increased return on risk-free money following the implementation of the stabilization policy.

The stabilization policy does not necessary implement first-best allocations. If government debt and the first-best level of aggregate investment are relatively large, i.e. $(1 - \psi)k^* + \Delta \geq (1 - \psi\pi)c_b^*$, enough money is available to sustain first-best allocations when stabilization policies are implemented. If not, the economy ends up in an equilibrium with risk-free but scarce money. To also achieve the first-best allocations, the supply of outside money must be increased until $(1 - \psi)k^* + \Delta \geq (1 - \psi\pi)c_b$ is satisfied.

6 Conclusion

This paper shows how economies can become subject to risky money and what the consequences of risky inside money for welfare and allocations are. Reasons that can push an economy into an equilibrium with risky inside money all affect the capacity of bankers to absorb risk either by themselves, or the capacity to create plentiful inside money. Most importantly, I uncover a novel mechanism through which risky money negatively affects welfare. Specifically, if inside money is risky the positive externalities associated with money cannot be incorporated properly in the funding costs that inside money issuers face. This finding suggests a role for the kind of government interventions that we have seen during the 2007 financial crisis, which bring the economy back into an equilibrium with risk-free money. Nevertheless, such policies may not be enough to achieve first-best allocations as money may remain scarce. A shortage of money is always harmful for an economy, regardless of whether money is risky or not. Optimal government policies should therefore be aimed at supplying as much risk-free money as possible. Only then first-best allocations can be achieved. A fruitful area of further research therefore considers limits to taxation, which inhibit outside money issuance by government.

A Supplementary Figures

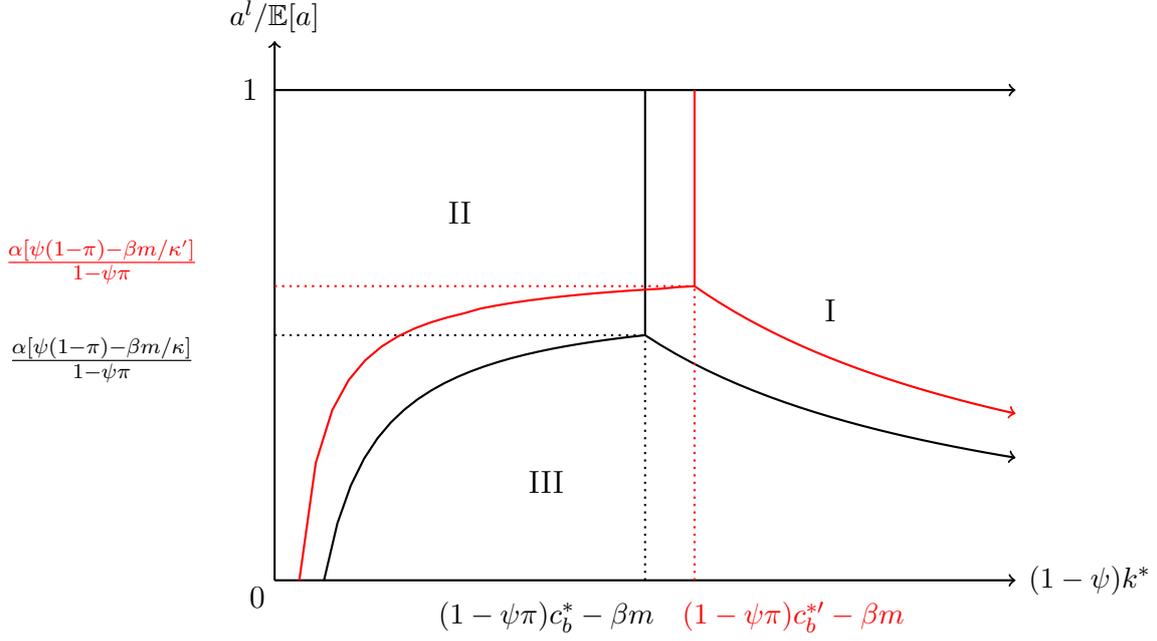


Figure 11: The effect of a higher first-best level of DM consumption, keeping the amount of risk-free outside money, m , fixed. Here, $(1-\psi)\kappa \equiv (1-\psi\pi)c_b^* - \beta m$ and $(1-\psi)\kappa' \equiv (1-\psi\pi)c_b^{*'} - \beta m$. Drawn under the assumption that $\beta m < \psi(1-\pi)c_b^*$ (otherwise a type III equilibrium cannot exist for the initial parametrization).

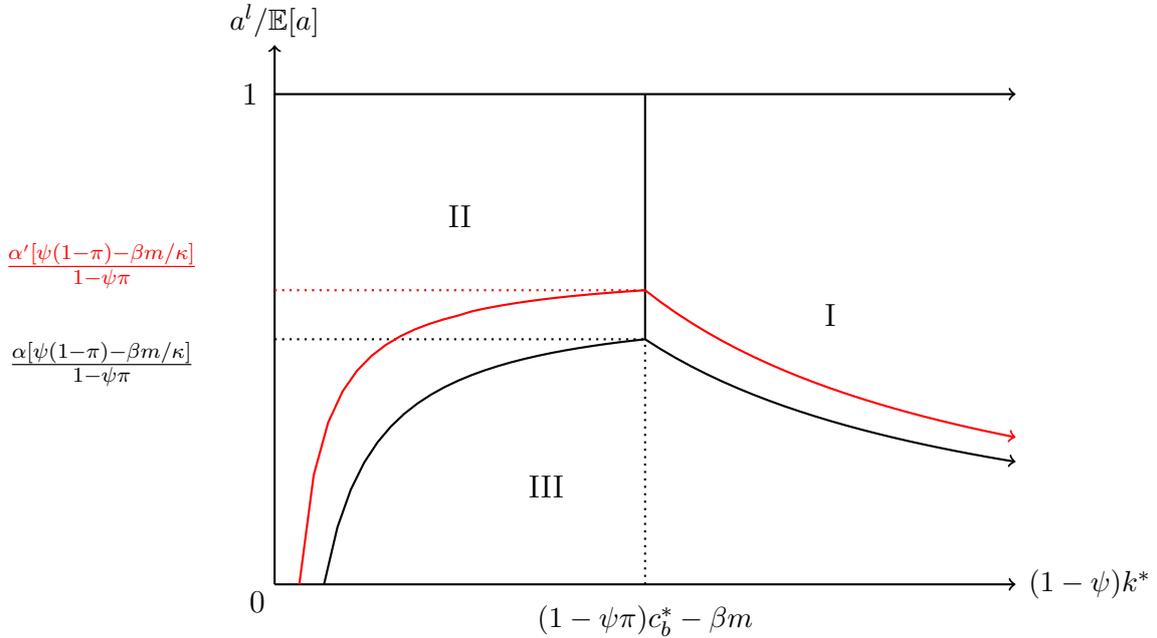


Figure 12: The effect of a decline in α . The initial supply level is denoted α and the new level α' , and $(1-\psi)\kappa \equiv (1-\psi\pi)c_b^* - \beta m$. Drawn under the assumption that $\beta m < \psi(1-\pi)c_b^*$ (otherwise a type III equilibrium cannot exist).

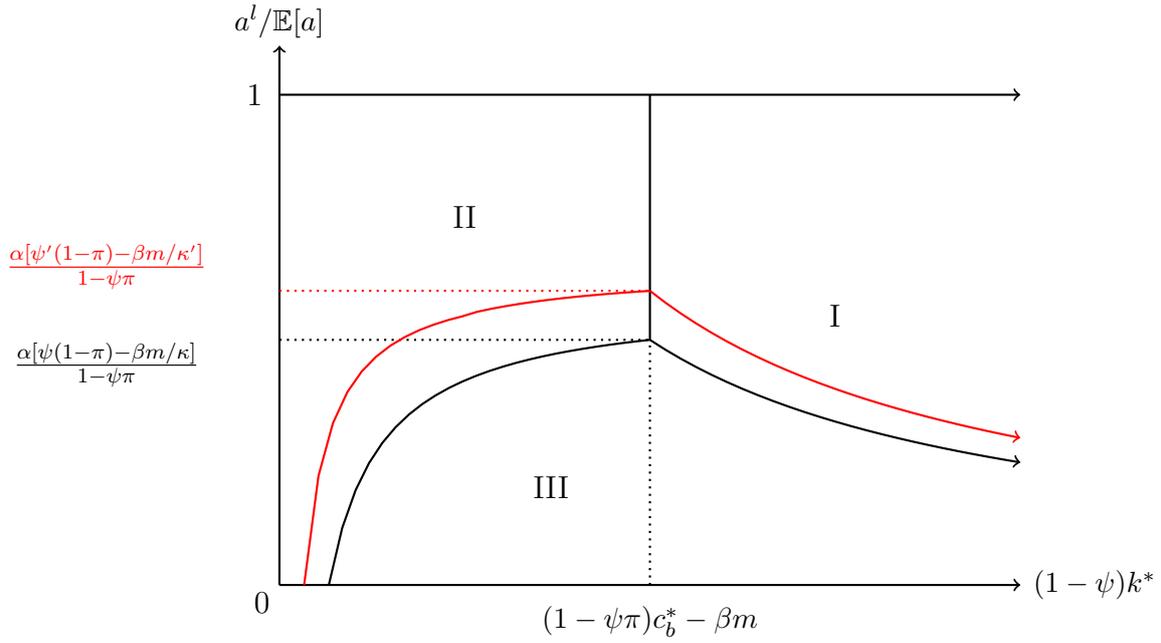


Figure 13: The effect of an increase in ψ and reduction of π such that $\psi\pi$ remains unchanged. Here, $(1-\psi)\kappa \equiv (1-\psi\pi)c_b^* - \beta m$ and $(1-\psi')\kappa' \equiv (1-\psi\pi)c_b^* - \beta m$. Drawn under the assumption that $\beta m < \psi(1-\pi)c_b^*$ (otherwise a type III equilibrium cannot exist for the initial parametrization).

B Equilibrium Allocations in the CRS Limit

This appendix shortly considers how the equilibrium allocations behave when $\alpha \rightarrow 1$ and the supply of outside money is risk-free as in Section 5. The type of equilibrium that emerges in the limit then depends on the productivity of investment. Specifically, because the marginal product of capital investment approaches $\mathbb{E}[a]$, we find that

$$\lim_{\alpha \rightarrow 1} k^* = \begin{cases} 0 & \text{if } \beta \mathbb{E}[a] \leq 1 \\ \infty & \text{if } \beta \mathbb{E}[a] > 1 \end{cases}.$$

Hence, the relevant case is one in which $\beta \mathbb{E}[a] \leq 1$ as otherwise the first-best level of investment would go to infinity and (inside) money would always be plentiful.

Type I: Money is always plentiful in the DM. From Equation (18), it immediately follows that in the limit this equilibrium exists if and only if

$$\beta m \geq (1 - \psi\pi)c_b^*.$$

Prices again satisfy $\phi^h = \rho\phi$, $\phi^l = (1 - \rho)\phi$ and $\beta/\phi = u'(c_b)$. From Equation (C.1) we obtain $u'(c_b) = 1$, so that consumption is at the first-best level, and $\beta = \phi$. Also capital investment is at the first best level and thus equal to zero, i.e. $k = k^* = 0$.

Type II: Money is risk-free and always scarce in the DM. From the first-order condition of bankers, we know that in an equilibrium with capital investment, $k^{1-\alpha} = \alpha \mathbb{E}[a]\phi$. In the limit, we therefore have $\phi = 1/\mathbb{E}[a]$ if $k > 0$ and $\phi < 1/\mathbb{E}[a]$ if $k = 0$. With a strictly positive amount of capital investment, consumption becomes

$$c_b = u'^{-1} \left(\frac{1 - (1 - \psi\pi)\beta \mathbb{E}[a]}{\psi\pi} \right)$$

and because $(1 - \psi)k + \phi m = (1 - \psi\pi)u(c_b)$, it follows that

$$k = \frac{(1 - \psi\pi)u'^{-1} \left(\frac{1 - (1 - \psi\pi)\beta \mathbb{E}[a]}{\psi\pi} \right) - m/\mathbb{E}[a]}{1 - \psi\pi}$$

which is indeed positive when

$$m < \mathbb{E}[a](1 - \psi\pi)u'^{-1} \left(\frac{1 - (1 - \psi\pi)\beta \mathbb{E}[a]}{\psi\pi} \right).$$

With $\phi < 1/\mathbb{E}[a]$, there is no inside money issuance and investment. Because $\phi m = (1 - \psi\pi)u'(c_b)$, the price of money then satisfies

$$\phi m = (1 - \psi\pi)u'^{-1} \left(\frac{1 - (1 - \psi\pi)\beta/\phi}{\psi\pi} \right).$$

It follows that $\phi \leq \mathbb{E}[a]$ if and only if

$$m \geq \mathbb{E}[a](1 - \psi\pi)u'^{-1} \left(\frac{1 - (1 - \psi\pi)\beta\mathbb{E}[a]}{\psi\pi} \right).$$

Combining Equations (19) and (20) with the fact that in the limit $k^* = 0$, we find that the existence condition for the Type II equilibrium becomes

$$\beta m < (1 - \psi\pi)c_b^* \quad \text{and} \quad a^l \geq \frac{\psi(1 - \pi) - \frac{(1 - \psi)m}{(1 - \psi\pi)\mathbb{E}[a]u'^{-1}\left(\frac{1 - (1 - \psi\pi)\beta/\phi}{\psi\pi}\right) - m}}{1 - \psi\pi}.$$

Type IIIa: Money is risky, plentiful in the DM when the high state is announced but scarce in the DM when the low state is announced. Clearly, this equilibrium can only exist when $k > 0$. Otherwise, there would be no inside money issuance and because outside money is risk-free, all money held by the non-bankers is risk-free.

Because the optimal level of capital investment satisfies $k^{1-\alpha} = \alpha[\rho\beta a^h + \phi^l a^l]$, we find that in the limit

$$\rho\beta a^h + \phi^l a^l = 1,$$

which thus gives us that $\phi^l = (1 - \rho\beta a^h)/a^l$. Moreover, the equilibrium relationship in Equation (21), i.e. $\psi(1 - \pi)c_b = \hat{\phi}^l(a^l(1 - \psi)k + m)$, implies that

$$\begin{aligned} c_b &= u'^{-1} \left(\frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi(1 - \rho) + \rho[1 - (1 - \pi)(1 - \rho)\beta/\phi^l]} \right) \\ \psi(1 - \pi)c_b &= \phi^l \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi} + 1 \right) [a^l(1 - \psi)k + m] \end{aligned}$$

which give us c_b and k .

Appendix C.17 implies that existence requires $\beta(1 - \rho)\phi^l < 1$ and $(1 - \psi)k \geq (1 - \psi\pi)c_b - \phi m$. These conditions boil down to checking that

$$\begin{aligned} \psi(1 - \psi)c_b &\geq \phi^l \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi} + 1 \right) \\ &\quad \times \max \{ a^l[(1 - \psi\pi)c_b - (\rho\beta + \phi^l)m] + m, m \}, \end{aligned}$$

which requires at least that $\beta m < (1 - \psi\pi)c_b^*$.

Type IIIb: Money is risky and always scarce in the DM. Again, this equilibrium can only exist when $k > 0$. Because the optimal level of capital investment satisfies $k^{1-\alpha} = \alpha[\phi^h a^h + \phi^l a^l]$, we find that in the limit with $k > 0$

$$\phi^h a^h + \phi^l a^l = 1.$$

This implies that $\phi^h = g(\phi^l)$, where g is an increasing function. Because the equilibrium requires $(1 - \pi\psi)c_b = (1 - \psi)k + [\phi^h + \phi^l]m$ and $\psi(1 - \pi)c_b = \hat{\phi}^l [a^l(1 - \psi)k + m]$, we obtain the following system of equations in ϕ^h and k

$$\begin{aligned} u'^{-1} & \left(\left[\frac{\psi\pi\rho}{1 - (1 - \psi\pi)\rho\beta/\phi^h} + \frac{\psi\pi(1 - \rho)}{1 - (1 - \psi)\rho\beta/\phi^h - \psi(1 - \pi)(1 - \rho)\beta/g(\phi^h)} \right]^{-1} \right) \\ & = \frac{(1 - \psi)k + [\phi^h + g(\phi^h)]m}{1 - \psi\pi} \\ & = \frac{\phi^l}{1 - \rho} \left[1 + \frac{\rho\psi(1 - \pi)[\rho\beta/\phi^h - (1 - \rho)\beta/g(\phi^h)]}{1 - (1 - \psi\pi)\rho\beta/\phi^h} \right] \frac{a^l(1 - \psi)k + m}{\psi(1 - \pi)}. \end{aligned}$$

This system is highly non-linear, so it is again difficult to show whether the equilibrium will be unique. However, when $m = 0$ the solution is simpler since we know from the analysis in Section that there is also an equilibrium relationship $\phi^l = f(\phi^l)$, where ϕ^l is an increasing function.

Necessary conditions for existence are given by

$$\beta m < (1 - \psi\pi)c_b^* \quad \text{and} \quad a^l < \frac{\psi(1 - \pi) - \frac{(1 - \psi)m}{(1 - \psi\pi)\mathbb{E}[a]u'^{-1}\left(\frac{1 - (1 - \psi\pi)\beta/\phi}{\psi\pi}\right) - m}}{1 - \psi\pi} \quad (\text{B.1})$$

and sufficient conditions for existence are (B.1) together with

$$\begin{aligned} \psi(1 - \psi)c_b & < \phi^l \left(\frac{\rho}{1 - \rho} \frac{1 - (1 - \pi)(1 - \rho)\beta/\phi^l}{\pi} + 1 \right) \\ & \times \max \{ a^l[(1 - \psi\pi)c_b - (\rho\beta + \phi^l)m] + m, m \}. \end{aligned}$$

C Proofs and Derivations

C.1 Proof of Lemma 1

Consider that $\mu_{nr}^q > 0$. Given that $\mu_{nr}^h = 0$ and $\mu_{nr}^l = 0$, we can combine the FOCs of the non-bankers to obtain:

$$\left[\pi u'(c_b) + (1 - \pi) \mathbb{E} \left[\beta / \hat{\phi}^{j^+} \right] + \mu_{nr}^q \right] \left(\frac{\phi^h}{\hat{\phi}^h} + \frac{\phi^l}{\hat{\phi}^l} \right) = \pi u'(c_b) + (1 - \pi) \mathbb{E} \left[\beta / \hat{\phi}^{j^+} \right].$$

With $\mu_{nr}^q > 0$ we must have $\phi^h / \hat{\phi}^h + \phi^l / \hat{\phi}^l < 1$. From the FOCs of the banker we can obtain

$$\left(\mathbb{E} \left[\beta / \hat{\phi}^{j^+} \right] + \mu_r^q \right) \left(\frac{\phi^h}{\hat{\phi}^h} + \frac{\phi^l}{\hat{\phi}^l} \right) = \mathbb{E}[\beta / \hat{\phi}^{j^+}] + \frac{\rho \beta \mu_r^h}{\hat{\phi}^h} + \frac{(1 - \rho) \beta \mu_r^l}{\hat{\phi}^l}.$$

Since $\mu_{nr}^q > 0$ we must have $\mu_r^q = 0$ as otherwise no goods are carried into the DM. But then we obtain a contradiction when $\phi^h / \hat{\phi}^h + \phi^l / \hat{\phi}^l < 1$. Q.E.D.

C.2 Proof of Lemma 2

First, demonstrate that $\mu_r^h \mu_r^l = 0$ by means of a contradiction. If $\mu_r^h \mu_r^l > 0$, then for sure $\mu_r^q > 0$. Also, $\tilde{z}_r^h = -a^h k^\alpha$ and $\tilde{z}_r^l = -a^l k^\alpha$. Given that the banker exhausts his budget constraint, it follows that $k = q + \phi^h m^h + \phi^l m^l + \phi^h a^h k^\alpha + \phi^l a^l k^\alpha$. Also, from the banker's FOCs we know that $k^{1-\alpha} = \alpha (a^h \phi^h + a^l \phi^l)$. Dividing both sides of the previous equation with k then implies $qk^{-1} < 0$, which contradicts the fact that $q > 0$ and $k > 0$.

Next, observe that it follows from the banker's FOCs that $\frac{1+\mu_r^h}{1+\mu_r^l} = \frac{(1-\rho)/\phi^l}{\rho/\phi^h}$. Given that $\mu_r^h \mu_r^l = 0$ it directly follows that $\mu_r^h > 0 \Leftrightarrow \rho/\phi^h < (1-\rho)/\phi^l$, and $\mu_r^l > 0 \Leftrightarrow \rho/\phi^h > (1-\rho)/\phi^l$. Given that $\mu_r^q > 0 \Leftrightarrow \mu_r^h + \mu_r^l > 0$, $\mu_r^q > 0 \Leftrightarrow \rho/\phi^h \neq (1-\rho)/\phi^l$ follows directly. Q.E.D.

C.3 Proof of Lemma 3

Follows directly from the FOCs of the non-bankers when $\mu_{nr}^h = \mu_{nr}^l = 0$. Q.E.D.

C.4 Proof of Lemma 4

To clear the DM we must have $\psi \pi (\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h) \geq q - (1 - \psi)k$ and $\psi \pi (q_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l) \geq q - (1 - \psi)k$. Multiply the equations with $\phi^h / \hat{\phi}^h$ and $\phi^l / \hat{\phi}^l$ respectively and add them up to obtain that $\psi \pi (\phi^h \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l) = (\phi^h / \hat{\phi}^h + \phi^l / \hat{\phi}^l) [q - (1 - \psi)k - \psi \pi \tilde{q}_{nr}]$. From Lemma 1 we know that $(\phi^h / \hat{\phi}^h + \phi^l / \hat{\phi}^l) = 1$ so we obtain $\psi \pi (\tilde{q}_{nr} + \phi^h \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l) = q - (1 - \psi)k$. The budget constraint for non-bankers in the MM is given by $\tilde{q}_{nr} + \phi^h \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l \leq q + \phi^h m^h + \phi^l m^l$. As the non-bankers exhaust their budget constraint it follows that $(1 - \psi)k \geq (1 - \psi \pi)q - \psi \pi (\phi^h m^h + \phi^l m^l)$

In a partial equilibrium of type II, IIIb, and IVb, money is always scarce in the DM. Therefore $\psi\pi(q_{nr} + \hat{\phi}^h z_{nr}^h) = q - (1 - \psi)k$ and $\psi\pi(q_{nr} + \hat{\phi}^l z_{nr}^l) = q - (1 - \psi)k$. It follows that $(1 - \psi)k = (1 - \psi\pi)q - \psi\pi(\phi^h m^h + \phi^l m^l)$. Q.E.D.

C.5 MM Equilibrium Relationships

Using the MM FOCs and the Lemmas derived above, the following relationships arise:

$$\frac{1}{u'(c_b)} = \frac{\psi\pi\rho}{1 - (1 - \psi) \max\{\rho\beta/\phi^h, (1 - \rho)\beta/\phi^l\} + \psi(1 - \pi)\rho\beta/\phi^h} + \frac{\psi\pi(1 - \rho)}{1 - (1 - \psi) \max\{\rho\beta/\phi^h, (1 - \rho)\beta/\phi^l\} + \psi(1 - \pi)(1 - \rho)\beta/\phi^l}, \quad (\text{C.1})$$

$$\frac{\hat{\phi}^h}{\phi^h} = 1 + \frac{1 - \rho}{\rho} \frac{1 - (1 - \psi) \max\{\rho\beta/\phi^h, (1 - \rho)\beta/\phi^l\} - \psi(1 - \pi)\rho\beta/\phi^h}{1 - (1 - \psi) \max\{\rho\beta/\phi^h, (1 - \rho)\beta/\phi^l\} - \psi(1 - \pi)(1 - \rho)\beta/\phi^l}, \quad (\text{C.2})$$

$$\frac{\hat{\phi}^l}{\phi^l} = 1 + \frac{\rho}{1 - \rho} \frac{1 - (1 - \psi) \max\{\rho\beta/\phi^h, (1 - \rho)\beta/\phi^l\} - \psi(1 - \pi)(1 - \rho)\beta/\phi^l}{1 - (1 - \psi) \max\{\rho\beta/\phi^h, (1 - \rho)\beta/\phi^l\} - \psi(1 - \pi)\rho\beta/\phi^h}. \quad (\text{C.3})$$

C.6 Proof of Lemma 5

In an equilibrium of type IVa or IVb we must always have $\hat{\phi}^l < \hat{\phi}^h$. Evaluating Equation (5), it follows that we must have $\tilde{z}_{nr}^l > \tilde{z}_{nr}^h$. Also, from Lemma 2 we know that $\mu_r^h > 0$ so we can use that $\tilde{z}_r^h = -a^h k^\alpha$. Because H-money is in zero net supply, it follows that $\tilde{z}_{nr}^h = \frac{1 - \psi}{\psi} a^h k^\alpha$. Using $\tilde{z}_{nr}^l > \tilde{z}_{nr}^h$ and the fact that L-money is in zero net supply it follows that $\tilde{z}_r^l < -a^h k^\alpha \leq -a^l k^\alpha$, which violates the bankers' constraint regarding L-money issuance; $\tilde{z}_r^l + a^l k^\alpha \geq 0$. Q.E.D.

C.7 Derivation of Equation (7)

To ensure existence of an equilibrium of type I the following conditions need to be satisfied:

$$\begin{aligned} 0 \leq -\tilde{z}_r^l &\leq a^l k^\alpha && (\tilde{z}_{nr}^l > 0, \tilde{z}_r^l + a^l k^\alpha \geq 0) \\ 0 \leq -\tilde{z}_r^h &\leq a^h k^\alpha && (\tilde{z}_{nr}^h > 0, \tilde{z}_r^h + a^h k^\alpha \geq 0) \\ 0 \leq q + \frac{1 - \psi}{\psi} (\rho\phi\tilde{z}_r^h + (1 - \rho)\phi\tilde{z}_r^l) &&& (\tilde{q}_{nr} \geq 0) \\ 0 \leq q + \frac{1 - \psi}{\psi} (\rho\phi\tilde{z}_r^h + (1 - \rho)\phi\tilde{z}_r^l) &&& (\tilde{q}_{nr} + \phi z_{nr}^h \geq c_b) \\ &\quad - \frac{1 - \psi}{\psi} \phi\tilde{z}_r^h - \frac{q - (1 - \psi)k}{\psi\pi} && \\ 0 \leq q + \frac{1 - \psi}{\psi} (\rho\phi\tilde{z}_r^h + (1 - \rho)\phi\tilde{z}_r^l) &&& (\tilde{q}_{nr} + \phi z_{nr}^l \geq c_b) \\ &\quad - \frac{1 - \psi}{\psi} \phi\tilde{z}_r^l - \frac{q - (1 - \psi)k}{\psi\pi} && \\ 0 \leq q - k - (\rho\phi\tilde{z}_r^h + (1 - \rho)\phi\tilde{z}_r^l) &&& (\tilde{q}_r \geq 0) \end{aligned}$$

Sufficient and necessary conditions for these relationships to hold are given by Equation (7). Q.E.D.

C.8 Derivation of Equation (9)

To ensure existence of the equilibrium of Type II we need that the following conditions hold for $q = c_b$.

$$\begin{aligned}
0 &\leq -\tilde{z}_r \leq a^l k^\alpha && (\tilde{z}_{nr} \geq 0, \tilde{z}_r + a^l k^\alpha \geq 0) \\
0 &\leq -\tilde{z}_r \leq a^h k^\alpha && (\tilde{z}_{nr} \geq 0, \tilde{z}_r + a^h k^\alpha \geq 0) \\
0 &\leq q + \frac{1-\psi}{\psi} \phi \tilde{z}_r && (\tilde{q}_{nr} \geq 0) \\
0 &\leq q - k - \phi \tilde{z}_r && (\tilde{q}_r \geq 0) \\
\beta &< \phi u'(c_b) && (\text{scarce money in DM})
\end{aligned}$$

Using Equation (C.1), it follows that $\beta < \phi u'(c_b)$ if and only if $\beta < \phi$. Because the LHS of Equation (8) is increasing in ϕ and the RHS of Equation (8) is decreasing in ϕ , it follows that $\beta < \phi$ if and only if $(1-\psi)k^* < (1-\psi\pi)c_b^*$. This gives the first part of Equation (9). Then, combine the conditions for $\tilde{q}_r \geq 0$ and $\tilde{z}_r + a^l k^\alpha$ to obtain that $k - q \leq a^l k^\alpha \phi$. Using that $c_b = q$, $(1-\psi)k = (1-\psi\pi)c_b$, and $k^{1-\alpha} = \alpha \mathbb{E}[a] \phi$ this reduces to $a^l(1-\psi\pi) \geq \alpha \mathbb{E}[a] \psi(1-\pi)$. Existence thus requires Equation (9) to hold. Assuming that $q_r = 0$, one can then show that Equation (9) being satisfied is also sufficient for existence. Q.E.D.

C.9 Derivation of Equations (11) and (12)

To ensure existence of the Equilibrium of Type IIIa the following conditions need to be satisfied:

$$\begin{aligned}
0 &\leq -\tilde{z}_r^l \leq a^l k^\alpha && (\tilde{z}_{nr}^l > 0, \tilde{z}_r^l + a^l k^\alpha \geq 0) \\
0 &\leq -\tilde{z}_r^h \leq a^h k^\alpha && (\tilde{z}_{nr}^h > 0, \tilde{z}_r^h + a^h k^\alpha \geq 0) \\
0 &\leq q + \frac{1-\psi}{\psi} (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) && (\tilde{q}_{nr} \geq 0) \\
0 &\leq q + \frac{1-\psi}{\psi} (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) - \frac{1-\psi}{\psi} \hat{\phi}^h \tilde{z}_r^h - \frac{q - (1-\psi)k}{\psi\pi} && (\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h \geq c_b) \\
0 &\leq q + \frac{1-\psi}{\psi} (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) - \frac{1-\psi}{\psi} \hat{\phi}^l \tilde{z}_r^l - \frac{q - (1-\psi)k}{\psi\pi} && (\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l \geq c_b) \\
0 &\leq q - k - (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) && (\tilde{q}_r \geq 0), \\
\beta &< \hat{\phi}^l u'(c_b) && (\tilde{z}_{nr}^l < \bar{z})
\end{aligned}$$

where $q = (1 - \psi)k + \psi\pi c_b$. Because of the equilibrium properties, $\tilde{q}_r \geq 0$, $\tilde{q}_{nr} \geq 0$, $\tilde{z}_{nr}^l > 0$, $\tilde{z}_r^h + a^l k^\alpha \geq 0$ and $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l \geq c_b$ are satisfied automatically.

Using Equations (C.1)- (C.3), it follows that $\beta < \hat{\phi}^l u'(c_b)$ iff $\beta(1 - \rho) < \phi^l$ when $\beta\rho = \phi^h$. Because the LHS of Equation (10) is decreasing in ϕ^l and the RHS of Equation (10) is increasing in ϕ^l , it follows that $\beta(1 - \rho) < \phi^l$ if and only if Equation (11) holds.

Next, we need to ensure that $\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h \geq c_b$ is satisfied. Using Lemma 1 and $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l = c_b$, this is the same as showing $\tilde{q}_{nr} + (\phi^l \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l) \geq c_b$. Then, since $\tilde{q}_r = 0$ we have $q_{nr} = \pi c_b$. Using that $q = (1 - \psi)k + \psi\pi c_b$, $q_{nr} + (\phi^l \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l) \geq c_b$ is the same as $(1 - \psi\pi)c_b \leq (1 - \psi)k$. Using this in Equation (10) it thus suffices to check $\hat{\phi}^l a^l k^\alpha (1 - \psi\pi) < (1 - \psi)k$. Using Equation (C.3) and the FOC for k , one can verify that $\hat{\phi}^l a^l k^\alpha (1 - \psi\pi) \leq (1 - \psi)k$ when $\beta(1 - \rho)/\phi^l \geq \varphi$ and $\beta\rho/\phi^h = 1$, where φ is defined in Equation (13). Because the LHS of Equation (10) is decreasing in ϕ^l and the RHS of Equation (10) is increasing in ϕ^l , this holds whenever Equation (12) is satisfied.

Finally, $\tilde{z}_{nr}^h > \tilde{z}_{nr}^l$ always holds, so that $\tilde{z}_{nr}^h > 0$ holds. Also, $\tilde{z}_r^h + a^h k^\alpha > 0$ holds as the bankers never exhaust their capacity to create H-money and L-money simultaneously.

Q.E.D.

C.10 Derivation of Equations (16) and (17)

For the equilibrium of Type IIIb to exist we need that for $q = (1 - \psi)k + \psi\pi c_b$,

$$\begin{aligned}
0 &\leq -\tilde{z}_r^l \leq a^l k^\alpha && (\tilde{z}_{nr}^l > 0, \tilde{z}_r^l + a^l k^\alpha \geq 0) \\
0 &\leq -\tilde{z}_r^h \leq a^h k^\alpha && (\tilde{z}_{nr}^h > 0, \tilde{z}_r^h + a^h k^\alpha \geq 0) \\
0 &\leq q + \frac{1 - \psi}{\psi} (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) && (\tilde{q}_{nr} \geq 0) \\
0 &\leq q + \frac{1 - \psi}{\psi} (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) - \frac{1 - \psi}{\psi} \hat{\phi}^h \tilde{z}_r^h - \frac{q - (1 - \psi)k}{\psi\pi} && (\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h \geq c_b) \\
0 &\leq q + \frac{1 - \psi}{\psi} (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) - \frac{1 - \psi}{\psi} \hat{\phi}^l \tilde{z}_r^l - \frac{q - (1 - \psi)k}{\psi\pi} && (\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l \geq c_b) \\
0 &\leq q - k - (\phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l) && (\tilde{q}_r \geq 0) \\
\beta &< \hat{\phi}^h u'(c_b) && (\tilde{z}_{nr}^h < \bar{z}) \\
\beta &< \hat{\phi}^l u'(c_b) && (\tilde{z}_{nr}^l < \bar{z}) \\
1 &< \frac{\rho}{1 - \rho} \frac{\phi^l}{\phi^h} && (\text{High return on H-money})
\end{aligned}$$

Because of the equilibrium properties, $\tilde{z}_{nr}^h > \tilde{z}_{nr}^l > 0$, $\tilde{z}_r^h + a^h k^\alpha \geq 0$, $\tilde{z}_r^l + a^l k^\alpha \geq 0$, $\tilde{q}_{nr} \geq 0$, $\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h \geq c_b$, $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l \geq c_b$, and $\tilde{q}_r \geq 0$ are satisfied automatically.

To ensure that $(1 - \rho)\phi^h < \rho\phi^l$, consider Equation (14). From it, we can deduce that $(1 - \rho)\phi^h < \rho\phi^l$ if and only if $a_l(1 - \psi\pi) < \alpha\mathbb{E}[a]\psi(1 - \pi)$. This gives Equation (16). Because $(1 - \rho)\phi^h < \rho\phi^l$, Equations (C.2) and (C.3) imply that $\hat{\phi}^l > \hat{\phi}^h$. To check that

$\beta < \hat{\phi}^h u'(c_b)$ and $\beta < \hat{\phi}^l u'(c_b)$, it is sufficient to check that $\beta < \hat{\phi}^h u'(c_b)$, or equivalently $\rho\beta < \phi^h$. Because the LHS of Equation (15) is increasing in ϕ^h and the RHS is decreasing in ϕ^h , $\rho\beta < \phi^h$ if and only if Equation (17) holds. Q.E.D.

C.11 Proof of Lemma 6

In an equilibrium of type IVa or IVb we must always have $\hat{\phi}^l < \hat{\phi}^h$. Evaluating Equation (5) it follows that we must have $\tilde{z}_{nr}^l > \tilde{z}_{nr}^h$. Also, from Lemma 2 we know that $\mu_r^h > 0$ so we can use that $\tilde{z}_r^h = -a^h k^\alpha$. Because $\psi \tilde{z}_{nr}^h + (1 - \psi) \tilde{z}_r^h = m$ it follows that $\tilde{z}_{nr}^h = \frac{(1 - \psi)a^h k^\alpha + m}{\psi}$. Using $\tilde{z}_{nr}^l > \tilde{z}_{nr}^h$ and $\psi \tilde{z}_{nr}^l + (1 - \psi) \tilde{z}_r^l = m$ it follows that $\tilde{z}_r^l < -a^h k^\alpha \leq -a^l k^\alpha$, which violates the bankers' constraint for L-money issuance: $\tilde{z}_r^l + a^l k^\alpha \geq 0$. Q.E.D.

C.12 Proof of Propositions 2 and 6

By assumption $(\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l) m_{\mathcal{A}} = (\phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l) m_{\mathcal{B}}$, so denote these quantities with Δ . Observe that for Proposition 2 we have $\Delta = 0$ and that for Proposition 6 we have $\Delta > 0$. Here I combine the proofs of both propositions by supposing that $\Delta \geq 0$.

If economy \mathcal{A} is characterized by an equilibrium of Type I, then so is economy \mathcal{B} . Both economies attain first-best allocations so $\mathcal{W}_{\mathcal{A}} = \mathcal{W}_{\mathcal{B}}$. If economy \mathcal{A} is characterized by an equilibrium of Type II, then so is economy \mathcal{B} . Because only $\mathbb{E}[a]$ matters for equilibrium investment and DM consumption in an equilibrium of Type II, we have $k_{\mathcal{A}} = k_{\mathcal{B}}$, $c_{b,\mathcal{A}} = c_{b,\mathcal{B}}$ and $q_{\mathcal{A}} = q_{\mathcal{B}}$. It follows that $\mathcal{W}_{\mathcal{A}} = \mathcal{W}_{\mathcal{B}}$.

It remains to consider a situation in which economy \mathcal{A} is characterized by an equilibrium of Type IIIa or IIIb. Suppose first that economy \mathcal{B} is characterized by first-best allocations, i.e. an equilibrium of Type I. It then follows that $\mathcal{W}_{\mathcal{A}} < \mathcal{W}_{\mathcal{B}}$, as economy \mathcal{A} does not attain first-best allocations. Suppose second that economy \mathcal{B} is characterized by an equilibrium of Type II. It must then be the case that $(1 - \psi)k^* + \Delta \leq (1 - \psi\pi)c_b^*$, and note that k^* , Δ , and c_b^* are the same for economy \mathcal{A} and \mathcal{B} . In the market economy, the condition $(1 - \psi)k + \Delta \geq (1 - \psi\pi)c_b$ has to be satisfied. Welfare in a market economy must therefore satisfy $\mathcal{W} \leq \widehat{\mathcal{W}}$, where $\widehat{\mathcal{W}}$ is given by

$$\begin{aligned} \widehat{\mathcal{W}}(1 - \beta) &\leq \max_{q, c_b, k} [\psi\pi u(c_b) + (1 - \psi)\beta\mathbb{E}[a]k^\alpha - q] \\ \text{s.t.} \quad &(1 - \psi)k + \psi\pi c_b \leq q \quad \text{and} \quad (1 - \psi)k + \Delta \geq (1 - \psi\pi)c_b. \end{aligned}$$

If $(1 - \psi)k^* + \Delta \geq (1 - \psi\pi)c_b^*$, then the above program is solved by the first best allocations. If not, then the program is solved when $\psi\pi[u'(c_b) - 1] = (1 - \psi\pi)[1 - \alpha\beta\mathbb{E}[a]k^{\alpha-1}]$, $(1 - \psi)k + \psi\pi c_b = q$, and $(1 - \psi)k + \Delta = (1 - \psi\pi)c_b$.

For economy \mathcal{B} , one can verify that $(1 - \psi)k_{\mathcal{B}} + \psi\pi c_{b,\mathcal{B}} = q_{\mathcal{B}}$, and $(1 - \psi)k_{\mathcal{B}} + \Delta = (1 - \psi\pi)c_{\mathcal{B}}$. Moreover, the MM FOCs for bankers in this economy imply that

$\alpha\beta\mathbb{E}[a]k_{\mathcal{B}}^{\alpha-1} = \beta/\phi$, and the CM FOCs imply that $1 = \psi\pi u'(c_b) + (1 - \psi\pi)\beta/\phi$. It follows that $\psi\pi [u'(c_{b,\mathcal{B}}) - 1] = (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{B}}^{\alpha-1}]$ and hence we have $\mathcal{W}_{\mathcal{B}} = \widehat{\mathcal{W}}$.

To complete the proof we must now rule out that $\mathcal{W}_{\mathcal{A}} = \widehat{\mathcal{W}}$. If $\mathcal{W}_{\mathcal{A}} = \widehat{\mathcal{W}}$ holds, then we must have that $(1 - \psi)k_{\mathcal{A}} + \psi\pi c_{b,\mathcal{A}} = q_{\mathcal{A}}$ and $(1 - \psi)k_{\mathcal{A}} + \Delta = (1 - \psi\pi)c_{\mathcal{A}}$ hold. Without loss, suppose that economy \mathcal{A} is characterized by an equilibrium of Type IIIb, as $(1 - \psi)k + \Delta = (1 - \psi\pi)c_b$ only holds in an equilibrium of Type IIIa when Equation (23) holds with equality (for $\Delta = 0$ Equation (23) holds with equality if and only if Equation (12) holds with equality).

Then, using the MM FOCs of the household and the banker, together with $1 = \psi\lambda_{nr} + (1 - \psi)\lambda_r$, we obtain that $\psi\pi[u'(c_b) - 1] = (1 - \psi\pi)(1 - \lambda_r) + \psi(1 - \pi)\mu_r^q$ and $(1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{B}}^{\alpha-1}] = (1 - \psi\pi)[(1 - \lambda_r) + \alpha a^l \beta k^{\alpha-1}(1 - \rho)\mu_r^l]$. Therefore, we must have $\psi(1 - \pi)\mu_r^q = \mu_r^l(1 - \psi\pi)(1 - \rho)\alpha\beta a^l k^{\alpha-1}$.

From the FOCs of the banker we know that $\lambda_r = \mathbb{E}[\beta/\hat{\phi}^{j+}] + \mu_r^q$, $\lambda_r\phi^h = \beta\rho(1 + \mu_r^h)$ and $\lambda_r\phi^l = \beta(1 - \rho)(1 + \mu_r^l)$. The second and third equation can be combined to obtain $\lambda_r(\phi^h/\hat{\phi}^h + \phi^l/\hat{\phi}^l) = \mathbb{E}[\beta/\hat{\phi}^{j+}] + \beta\rho\mu_r^h/\hat{\phi}^h + \beta(1 - \rho)\mu_r^l/\hat{\phi}^l$. Apply Lemma 1 and combine with $\lambda_r = \mathbb{E}[\beta/\hat{\phi}^{j+}] + \mu_r^q$ to obtain that $\mu_r^q = \beta\rho\mu_r^h/\hat{\phi}^h + \beta(1 - \rho)\mu_r^l/\hat{\phi}^l$. In an equilibrium of type IIIb we have $\mu_r^l > 0$ and $\mu_r^h = 0$, so $\mu_r^q = \beta(1 - \rho)\mu_r^l/\hat{\phi}^l$. Therefore, $\psi(1 - \pi)\mu_r^q = \mu_r^l(1 - \psi\pi)(1 - \rho)\alpha\beta a^l k^{\alpha-1}$ if and only if $1/\hat{\phi}^l = \alpha a^l k^{\alpha-1} \frac{1 - \psi\pi}{\psi(1 - \pi)}$. Notice that in the limit as $\alpha \rightarrow 1$, this condition becomes $1/\hat{\phi}^l = a^l \frac{1 - \psi\pi}{\psi(1 - \pi)}$.

In an economy with an equilibrium of Type IIIb, clearance of the DM requires $\hat{\phi}^l[(1 - \psi)a^l k^{\alpha} + \Delta/(\phi^h + \phi^l)] = \frac{\psi(1 - \pi)}{1 - \psi\pi} [(1 - \psi)k + \Delta]$. Moreover, since $\hat{\phi}^l > \hat{\phi}^h$ we need that $\hat{\phi}^l > \phi^h + \phi^l$. To see why, consider a contradiction: $\hat{\phi}^l \leq \phi^h + \phi^l$. Because we must have $\mu_{nr}^h = \mu_{nr}^l = \mu_{nr}^q = 0$, we have $\lambda_{nr} = \pi v'(c_b) + (1 - \pi)\mathbb{E}[\beta/\hat{\phi}^{j+}]$, $\lambda_{nr}\phi^h = \rho[\hat{\phi}^h\pi v'(c_b) + (1 - \pi)\beta]$ and $\lambda_{nr}\phi^l = (1 - \rho)[\hat{\phi}^l\pi v'(c_b) + (1 - \pi)\beta]$. Therefore $\lambda_{nr}(\phi^h + \phi^l) = \mathbb{E}[\hat{\phi}^{j+}] [\lambda_{nr} - (1 - \pi)\mathbb{E}[\beta/\hat{\phi}^{j+}]] + \beta(1 - \pi)$. If $\hat{\phi}^h < \hat{\phi}^l \leq \phi^h + \phi^l$ it follows that we need $1/\mathbb{E}[\hat{\phi}^{j+}] \geq \mathbb{E}[1/\phi^{j+}]$, which according to Jensen's inequality only holds when $\hat{\phi}^h = \hat{\phi}^l$. We thus obtain a contradiction. Using that $\hat{\phi}^l > \phi^h + \phi^l$ and $\Delta \geq 0$, it follows from $\hat{\phi}^l [(1 - \psi)a^l k^{\alpha} + \Delta/(\phi^h + \phi^l)] = \frac{\psi(1 - \pi)}{1 - \psi\pi} [(1 - \psi)k + \Delta]$ that $1/\hat{\phi}^l \geq a^l k^{\alpha-1} \frac{1 - \psi\pi}{\psi(1 - \pi)}$. Notice however that in the limit as $\alpha \rightarrow 1$, we find that $1/\hat{\phi}^l = \frac{\psi(1 - \pi)}{1 - \psi\pi} \frac{(1 - \psi)a^l k + \Delta/(\phi^h + \phi^l)}{(1 - \psi)k + \Delta}$. Moreover, since the optimal level of capital investment by bankers satisfies $k^{1 - \alpha} = \alpha[\phi^h a^h + \phi^l a^l]$, in the limit as $\alpha \rightarrow 1$ we have that $1 = \phi^h a^h + \phi^l a^l$ which in turn implies that $a^l(\phi^h + \phi^l) < 1$. Therefore $\lim_{\alpha \rightarrow 1} 1/\hat{\phi}^l \geq a^l \frac{1 - \psi\pi}{\psi(1 - \pi)}$, with equality if and only if $\Delta \rightarrow 0$.

Concluding $1/\hat{\phi}^l \geq \alpha a^l k^{\alpha-1} \frac{1 - \psi\pi}{\psi(1 - \pi)}$, so $\psi\pi [u'(c_{b,\mathcal{A}}) - 1] > (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{A}}^{\alpha-1}]$ and $\mathcal{W}_{\mathcal{A}} < \widehat{\mathcal{W}} = \mathcal{W}_{\mathcal{B}}$. In a Type IIIb equilibrium, we have that $\lim_{\alpha \rightarrow 1, \Delta \rightarrow 0} \mathcal{W}_{\mathcal{A}} = \widehat{\mathcal{W}}$. Q.E.D.

C.13 Proof of Proposition 3

If economy \mathcal{A} is characterized by an equilibrium of Type I or II, then it follows from the proof of Proposition 2 that $k_{\mathcal{A}} = k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} = c_{b,\mathcal{B}}$.

Next, consider that economy \mathcal{A} is characterized by an equilibrium of Type IIIb. From the proof of Proposition 2 we then know that $k_{\mathcal{A}}(1 - \psi) = c_{b,\mathcal{A}}(1 - \psi\pi)$ and $k_{\mathcal{B}}(1 - \psi) = c_{b,\mathcal{B}}(1 - \psi\pi)$. Therefore, consider the function

$$g(k) = \psi\pi \left[u' \left(\frac{1 - \psi}{1 - \psi\pi} k \right) - 1 \right] - (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k^{\alpha-1}].$$

From the proof of Proposition 2 we know $\psi\pi [u'(c_{b,\mathcal{B}}) - 1] = (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{B}}^{\alpha-1}]$ and $\psi\pi [u'(c_{b,\mathcal{A}}) - 1] > (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{A}}^{\alpha-1}]$. It follows that $g(k_{\mathcal{A}}) > g(k_{\mathcal{B}})$. Because $g'(k) < 0$, we obtain $k_{\mathcal{A}} < k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$.

Finally, consider that economy \mathcal{A} is in an equilibrium of Type IIIa so that $k_{\mathcal{A}} > k^*$ and $c_{b,\mathcal{A}} < c_b^*$. If $(1 - \psi)k^* \geq (1 - \psi\pi)c_b^*$ (note that c_b^* and k^* are the same for economies \mathcal{A} and \mathcal{B}), then economy \mathcal{B} is in an equilibrium of Type I so $k_{\mathcal{B}} = k^*$ and $c_{b,\mathcal{B}} = c_b^*$. It remains to consider a situation in which $(1 - \psi)k^* < (1 - \psi\pi)c_b^*$. Consider Figure 14. Economy \mathcal{A} is represented by a point somewhere in region IIIa2. If economy \mathcal{A} is represented by a point on the blue line⁹ that separates regions IIIb and IIIa2, then we know from the previous paragraph that $k_{\mathcal{A}} < k_{\mathcal{B}}$. If economy \mathcal{A} is represented by a point on the red line that separates regions IIIa1 and IIIa2, then $(1 - \psi)k^* = (1 - \psi\pi)c_b^*$ so we know that $k_{\mathcal{A}} > k_{\mathcal{B}}$. Because the equilibrium value for k is continuous in the model parameters, it follows that on any horizontal line that connects the blue and red line in Figure 14, there exists points where $k_{\mathcal{A}} > k_{\mathcal{B}}$, $k_{\mathcal{A}} = k_{\mathcal{B}}$, and $k_{\mathcal{A}} < k_{\mathcal{B}}$.

It remains to consider $c_{b,\mathcal{A}}$ vs. $c_{b,\mathcal{B}}$ when economy \mathcal{A} is in an equilibrium of Type III and $(1 - \psi)k^* < (1 - \psi\pi)c_b^*$. Consider Figure 14 and pick an arbitrary point in region IIIa2 that represents economy \mathcal{B} . Draw a vertical line through this point and let economy \mathcal{C} be represented by the point where the vertical line and the blue line intersect. Without loss, we can assume that economy \mathcal{C} only differs from \mathcal{A} and \mathcal{B} in terms of the distribution of a . Specifically, $a_{\mathcal{A}}^l < a_{\mathcal{C}}^l < a_{\mathcal{B}}^l = \mathbb{E}[a_{\mathcal{B}}]$, and $\mathbb{E}[a_{\mathcal{A}}] = \mathbb{E}[a_{\mathcal{B}}] = \mathbb{E}[a_{\mathcal{C}}]$. We have already proven that $c_{b,\mathcal{C}} < c_{b,\mathcal{B}}$ is true (see the analysis when economy \mathcal{A} was characterized by an equilibrium of Type IIIb.) Suppose that $c_{b,\mathcal{C}} < c_{b,\mathcal{A}}$. Because economy \mathcal{A} and \mathcal{C} only differ in terms of a , it follows from Equation (C.1) that $c_{b,\mathcal{C}} < c_{b,\mathcal{A}}$ if and only if $\phi_{\mathcal{A}}^l < \phi_{\mathcal{C}}^l$ (both economies have $\phi_{\mathcal{A}}^h = \phi_{\mathcal{C}}^h = \beta\rho$). From Equation (C.3) it follows that $\phi_{\mathcal{A}}^l < \phi_{\mathcal{C}}^l$ if and only if $\hat{\phi}_{\mathcal{A}}^l < \hat{\phi}_{\mathcal{C}}^l$. For any economy in a Type III equilibrium we have $a^l \hat{\phi}^l k^{\alpha} (1 - \psi) = (1 - \pi)\psi c_b$, so we must also have $a_{\mathcal{A}}^l \hat{\phi}_{\mathcal{A}}^l k_{\mathcal{A}}^{\alpha} > a_{\mathcal{C}}^l \hat{\phi}_{\mathcal{C}}^l k_{\mathcal{C}}^{\alpha}$. Given $a_{\mathcal{A}}^l < a_{\mathcal{C}}^l$ and $\hat{\phi}_{\mathcal{A}}^l < \hat{\phi}_{\mathcal{C}}^l$, we must have $k_{\mathcal{A}} > k_{\mathcal{C}}$. Using the bankers' FOC for k and $\phi_{\mathcal{A}}^h = \phi_{\mathcal{C}}^h = \beta\rho$, $k_{\mathcal{A}} > k_{\mathcal{C}}$ if and only if

⁹The exact shape of this line is unimportant for the proof. It can be shown that it has a positive slope and intersects the x-axis to the right of the origin.

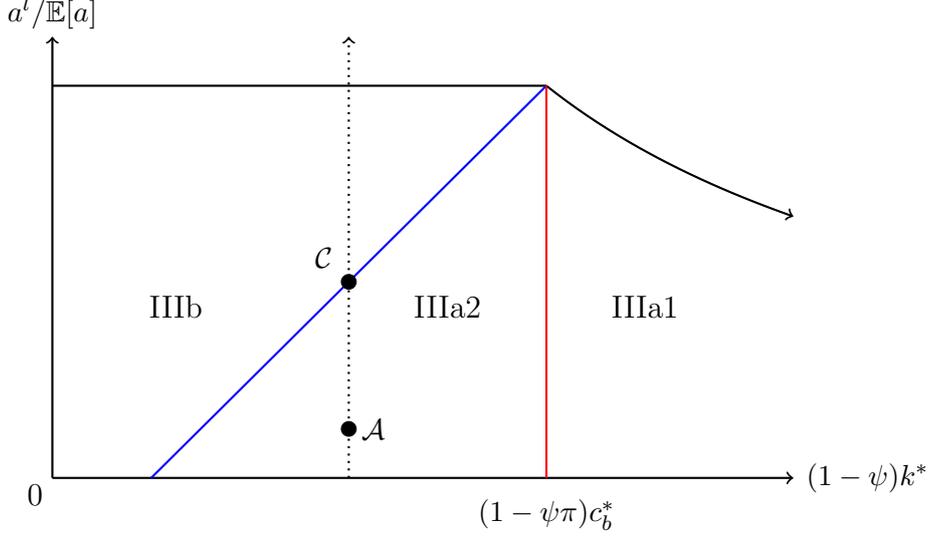


Figure 14

$\beta\mathbb{E}[a_{\mathcal{A}}] + a_{\mathcal{A}}^l(\phi_{\mathcal{A}}^l - \beta(1 - \rho)) > \beta\mathbb{E}[a_{\mathcal{C}}] + a_{\mathcal{C}}^l(\phi_{\mathcal{C}}^l - \beta(1 - \rho))$. Since $\phi^l > \beta(1 - \rho)$ for any economy with an equilibrium of type IIIa, we obtain a contradiction. It follows that we must have $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$. Q.E.D.

C.13.1 Proof of Proposition 3 With Risk-Free Outside Money

If economy \mathcal{A} is characterized by an equilibrium of Type I or II, then it follows from the proof of Proposition 6 that $k_{\mathcal{A}} = k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} = c_{b,\mathcal{B}}$.

Next, consider that economy \mathcal{A} is characterized by an equilibrium of type IIIb. From the proof of Proposition 6 we then know that $k_{\mathcal{A}}(1 - \psi) + \Delta = c_{b,\mathcal{A}}(1 - \psi\pi)$ and $k_{\mathcal{B}}(1 - \psi) + \Delta = c_{b,\mathcal{B}}(1 - \psi\pi)$. Therefore, consider the function

$$g(k) = \psi\pi \left[u' \left(\frac{(1 - \psi)k + \Delta}{1 - \psi\pi} \right) - 1 \right] - (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k^{\alpha-1}].$$

From the proof of Proposition 6 we know $\psi\pi [u'(c_{b,\mathcal{B}}) - 1] = (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{B}}^{\alpha-1}]$ and $\psi\pi [u'(c_{b,\mathcal{A}}) - 1] > (1 - \psi\pi) [1 - \alpha\beta\mathbb{E}[a]k_{\mathcal{A}}^{\alpha-1}]$. It follows that $g(k_{\mathcal{A}}) > g(k_{\mathcal{B}})$. Because $g'(k) < 0$, we obtain $k_{\mathcal{A}} < k_{\mathcal{B}}$ and $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$.

Finally, consider that economy \mathcal{A} is in an equilibrium of Type IIIa so that $k_{\mathcal{A}} > k^*$ and $c_{b,\mathcal{A}} < c_b^*$. If $(1 - \psi)k^* + \Delta \geq (1 - \psi\pi)c_b^*$ (c_b^* and k^* are the same for economies \mathcal{A} and \mathcal{B}), then economy \mathcal{B} is in an equilibrium of type I so $k_{\mathcal{B}} = k^*$ and $c_{b,\mathcal{B}} = c_b^*$. It remains to consider a situation in which $(1 - \psi)k^* + \Delta < (1 - \psi\pi)c_b^*$. When the equilibrium is such that Equation (23) binds, we have $k_{\mathcal{A}}(1 - \psi) + \Delta = c_{b,\mathcal{A}}(1 - \psi\pi)$ and the analysis in the preceding paragraph applies, so $k_{\mathcal{A}} < k_{\mathcal{B}}$. Because k is continuous in the model parameters as long as prices are determined according to Equation (21) (which pins down prices for an equilibrium of Type IIIa), there must also exist parameters for which we

have an equilibrium of Type IIIa with $k_{\mathcal{A}} > k_{\mathcal{B}}$ and $(1 - \psi)k^* + \Delta < (1 - \psi\pi)c_b^*$. This follows from what we have already shown: if $(1 - \psi)k^* + \Delta \geq (1 - \psi\pi)c_b^*$ and economy \mathcal{A} is in an equilibrium of Type IIIa then $k_{\mathcal{A}} > k_{\mathcal{B}}$.

It remains to consider $c_{b,\mathcal{A}}$ vs. $c_{b,\mathcal{B}}$ when economy \mathcal{A} is in an equilibrium of Type IIIa and $(1 - \psi)k^* < (1 - \psi\pi)c_b^*$. For \mathcal{A} we must have that Equations (22) and (23) hold. If Equation (23) binds, then we have $k_{\mathcal{A}}(1 - \psi) + \Delta = c_{b,\mathcal{A}}(1 - \psi\pi)$ and we already know that $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$. Therefore, suppose that Equation (23) does not bind. Because for $a_l = \mathbb{E}[a]$ an equilibrium of type IIIa can never exist, it follows that there exist an economy \mathcal{C} , with parameters that differ from economies \mathcal{A} and \mathcal{B} only in terms of a , for which Equation (22) holds and Equation (23) binds. Specifically $a_{\mathcal{A}}^l < a_{\mathcal{C}}^l < a_{\mathcal{B}}^l = \mathbb{E}[a_{\mathcal{B}}]$, $\mathbb{E}[a_{\mathcal{A}}] = \mathbb{E}[a_{\mathcal{B}}] = \mathbb{E}[a_{\mathcal{C}}]$, and $(\phi_{\mathcal{C}}^h + \phi_{\mathcal{C}}^l)m = \Delta$. Because Equation (23) binds for economy \mathcal{C} we know that $c_{b,\mathcal{C}} < c_{b,\mathcal{B}}$. Using Equation (21) one can then show $\phi_{\mathcal{C}}^h < \phi_{\mathcal{A}}^l$, so that Equation (C.1) implies $c_{b,\mathcal{A}} < c_{b,\mathcal{C}}$. Q.E.D.

C.14 Proof of Proposition 4

By assumption we have in an economy with outside and inside money that $(\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l)m_{\mathcal{A}} = (\phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l)m_{\mathcal{B}}$, so denote these quantities with Δ and observe that in an economy with only inside money we have $\Delta = 0$. To demonstrate that the Proposition also holds true for an economy with outside and inside money, I use $\Delta \geq 0$.

If economy \mathcal{A} is characterized by an equilibrium of Type I or II, then it follows trivially that economy \mathcal{A} is characterized by the same prices and real allocations as economy \mathcal{B} so $\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l = \phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l$.

Consider that economy \mathcal{A} is characterized by an equilibrium of Type IIIa or IIIb. Suppose first that $k^*(1 - \psi) + \Delta \geq (1 - \psi\pi)c_b^*$. Then we know that economy \mathcal{B} is in an equilibrium of Type I, so $\phi_{\mathcal{B}}^h + \phi_{\mathcal{B}}^l = \beta$. Economy \mathcal{A} is characterized by an equilibrium of Type IIIa or IIIb. Hence $\rho\beta \leq \phi_{\mathcal{A}}^h$ and $\beta(1 - \rho) < \phi_{\mathcal{A}}^l$, so $\phi_{\mathcal{A}}^h + \phi_{\mathcal{A}}^l > \beta$ follows.

Suppose second that $k^*(1 - \psi) + \Delta < (1 - \psi\pi)c_b^*$. From Proposition 3 we know that $c_{b,\mathcal{A}} < c_{b,\mathcal{B}}$. Equation (C.1) pins down c_b as a function of ϕ^h and ϕ^l , so we can consider isoclines for c_b and $\phi^h + \phi^l$ in the (χ, φ) -space, where $\chi \equiv \beta\rho/\phi^h$ and $\varphi \equiv \beta(1 - \rho)/\phi^l$. The slope of an isocline for c_b in this space satisfies:

$$\frac{\partial \varphi}{\partial \chi} = \begin{cases} -\frac{\rho}{1-\rho} \frac{1-\psi\pi}{\psi(1-\pi)} \left(1 + \frac{\psi(1-\pi)(\chi-\varphi)}{1-(1-\psi\pi)\chi}\right)^2 - \frac{1-\psi}{\psi(1-\pi)} & \text{if } \varphi < \chi \\ -\left[\frac{1-\rho}{\rho} \frac{1-\psi\pi}{\psi(1-\pi)} \left(1 + \frac{\psi(1-\pi)(\varphi-\chi)}{1-(1-\psi\pi)\varphi}\right)^2 + \frac{1-\psi}{\psi(1-\pi)}\right]^{-1} & \text{if } \varphi > \chi \end{cases}.$$

If $\varphi = \chi$ then the slope is undetermined, that means there is a kink in the isocline. From the derivatives it follows that this isocline is concave. The slope of an isocline for $\phi^h + \phi^l$

satisfies:

$$\frac{\partial \varphi}{\partial \chi} = -\frac{\rho}{1-\rho} \left(\frac{\varphi}{\chi} \right)^2.$$

It follows that this isocline is convex.

Construct two sets: $A = \{(\chi, \varphi) \in (0, 1]^2 : c_b \leq c_{b,B}\}$ and $B = \{(\chi, \varphi) \in (0, 1]^2 : \phi^h + \phi^l < \phi_B^h + \phi_B^l\}$. $A' = \{(\chi, \varphi) \in (0, 1]^2 : c_b = c_{b,B}\}$ belongs to the graph of a concave function (the isocline for $c_{b,B}$) and $B' = \{(\chi, \varphi) \in (0, 1]^2 : \phi^h + \phi^l = \phi_B^h + \phi_B^l\}$ belongs to the graph of a convex function (the isocline for $\phi_B^h + \phi_B^l$). Therefore $A \cap B = \{(\beta\rho/\phi_B^h, \beta(1-\rho)/\phi_B^l)\}$. Because $c_{b,A} < c_{b,B}$ it follows that $\{(\beta\rho/\phi_A^h, \beta(1-\rho)/\phi_A^l)\} \in A$ but $\{(\beta\rho/\phi_A^h, \beta(1-\rho)/\phi_A^l)\} \notin B$. Hence $\phi_A^l + \phi_A^h > \phi_B^l + \phi_B^h$. Q.E.D.

C.15 Derivation of Equation (18)

To ensure existence of the equilibrium of Type I the following conditions need to be satisfied for $q = (1-\psi)k + \psi\pi c_b$, $k = k^*$ and $c_b = c_b^*$:

$$\begin{aligned} 0 &\leq -\tilde{z}_r^l \leq a^l k^\alpha && (\tilde{z}_{nr}^l > 0, \tilde{z}_r^l + a^l k^\alpha \geq 0) \\ 0 &\leq -\tilde{z}_r^h \leq a^h k^\alpha && (\tilde{z}_{nr}^h > 0, \tilde{z}_r^h + a^h k^\alpha \geq 0) \\ 0 &\leq q + \frac{1-\psi}{\psi} (\rho\phi\tilde{z}_r^h + (1-\rho)\phi\tilde{z}_r^l - \phi m) && (\tilde{q}_{nr} \geq 0) \\ 0 &\leq q + \frac{1-\psi}{\psi} (\rho\phi\tilde{z}_r^h + (1-\rho)\phi\tilde{z}_r^l - \phi m) + \frac{\phi m}{\psi} && (\tilde{q}_{nr} + \phi\tilde{z}_{nr}^h \geq c_b) \\ &\quad - \frac{1-\psi}{\psi} \phi\tilde{z}_r^h - \frac{q - (1-\psi)k}{\psi\pi} \\ 0 &\leq q + \frac{1-\psi}{\psi} (\rho\phi\tilde{z}_r^h + (1-\rho)\phi\tilde{z}_r^l - \phi m) + \frac{\phi m}{\psi} && (\tilde{q}_{nr} + \phi\tilde{z}_{nr}^l \geq c_b) \\ &\quad - \frac{1-\psi}{\psi} \phi\tilde{z}_r^l - \frac{q - (1-\psi)k}{\psi\pi} \\ 0 &\leq q - k + \phi m - (\rho\phi\tilde{z}_r^h + (1-\rho)\phi\tilde{z}_r^l) && (\tilde{q}_r \geq 0) \end{aligned}$$

Sufficient and necessary conditions for these relationships to hold are Equation (18).

Q.E.D.

C.16 Derivation of Equation (20)

To ensure existence of the equilibrium of Type II we need that the following conditions hold for $q = (1-\psi)k + \psi\pi c_b$ and $c_b = \phi m + q$.

$$\begin{aligned} -m &\leq -(1-\psi)\tilde{z}_r \leq (1-\psi)a^l k^\alpha && (\tilde{z}_{nr} > 0, \tilde{z}_r + a^l k^\alpha \geq 0) \\ 0 &\leq q + \frac{1-\psi}{\psi} (\phi z_r - \phi m) && (\tilde{q}_{nr} \geq 0) \end{aligned}$$

$$\begin{aligned}
0 &\leq q + \phi m - k - \phi \tilde{z}_r & (\tilde{q}_r \geq 0) \\
\beta &< \phi u'(c_b) & (\tilde{z}_{nr} < \bar{z})
\end{aligned}$$

Using Equation (C.1) we have that $\beta < \phi u'(c_b)$ iff $\phi > \beta$. Because the LHS of Equation (19) is increasing in ϕ and the RHS of Equation (19) is decreasing in ϕ we have $\phi > \beta$ if and only if the first part of Equation (20) holds.

Together $\tilde{q}_r \geq 0$ and $\tilde{z}_r + a^l k^\alpha \geq 0$ imply that $a^l k^\alpha \geq k - q - \phi m$ must hold. Combing $q = (1 - \psi)k + \psi\pi c_b$ and $c_b = q + \phi m$, we obtain $(1 - \psi\pi)q = (1 - \psi)k + \psi\pi\phi m$. Therefore $a^l k^\alpha \geq k - q - \phi m$ if and only if $a^l k^\alpha(1 - \psi\pi) \geq \psi(1 - \pi)k - \phi m$. Use the bankers' FOC for k to obtain that this is equivalent to the second part of Equation (20). Hence, having Equation (20) satisfied is necessary for existence. Supposing that $q_r = 0$ on can then show that having Equation(20) satisfied is also sufficient for existence. Q.E.D.

C.17 Derivation of Equations (22) and (23)

To have existence of the equilibrium of Type IIIa the following conditions must hold for $q = (1 - \psi)k + \psi\pi c_b$, $\psi\pi c_b = q - (1 - \psi)k$, and $\beta\rho = \phi^h$:

$$\begin{aligned}
-m &\leq -\tilde{z}_r^l \leq a^l k^\alpha & (\tilde{z}_{nr}^l > 0, \tilde{z}_r^l + a^l k^\alpha \geq 0) \\
-m &\leq -\tilde{z}_r^h \leq a^h k^\alpha & (\tilde{z}_{nr}^h > 0, \tilde{z}_r^h + a^h k^\alpha \geq 0) \\
0 &\leq q + \frac{1 - \psi}{\psi} [(\tilde{z}_r^h - m)\phi^h + (\tilde{z}_r^l - m)\phi^l] & (\tilde{q}_{nr} \geq 0) \\
0 &\leq q + \frac{1 - \psi}{\psi} [(\tilde{z}_r^h - m)\phi^h + (\tilde{z}_r^l - m)\phi^l] & (\tilde{q}_{nr} + \phi\tilde{z}_{nr}^h \geq c_b) \\
&\quad + \frac{\hat{\phi}^h m}{\psi} - \frac{1 - \psi}{\psi} \hat{\phi}^h \tilde{z}_r^h - c_b \\
0 &\leq q + \frac{1 - \psi}{\psi} [(\tilde{z}_r^h - m)\phi^h + (\tilde{z}_r^l - m)\phi^l] & (\tilde{q}_{nr} + \phi\tilde{z}_{nr}^l \geq c_b) \\
&\quad + \frac{\hat{\phi}^l m}{\psi} - \frac{1 - \psi}{\psi} \hat{\phi}^l \tilde{z}_r^l - c_b \\
0 &\leq q - k - \phi^h(\tilde{z}_r^h - m) - \phi^l(\tilde{z}_r^l - m) & (\tilde{q}_r \geq 0) \\
\beta &\leq \hat{\phi}^l u'(c_b) & (\tilde{z}_{nr}^l < \bar{z})
\end{aligned}$$

Because of the equilibrium properties, $\tilde{z}_r^h + a^l k^\alpha \geq 0$, $\tilde{z}_{nr}^l \geq 0$, $\tilde{q}_{nr} \geq 0$, $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l \geq c_b$ and $\tilde{q}_r \geq 0$ are satisfied automatically. Using Equations (C.1) and (C.3) it follows that $\beta < \hat{\phi}^l u'(c_b)$ if and only if $\beta(1 - \rho) < \phi^l$. Because the LHS of Equation (21) is decreasing in ϕ^l and the RHS of Equation (21) is increasing in ϕ^l , it follows that $\beta(1 - \rho) < \phi^l$ if and only if Equation (22) holds.

Next, we need to ensure that $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^h \geq c_b$ is satisfied. Using Lemma 1 and $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l = c_b$, this is the same as showing $\tilde{q}_{nr} + \phi^h \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l \geq c_b$. Then, since $\tilde{q}_r = 0$

we have $\tilde{q}_{nr} = \pi c_b$. Using that $q = (1 - \psi)k + \psi\pi c_b$, $\tilde{q}_{nr} + \phi^h \tilde{z}_{nr}^h + \phi^l \tilde{z}_{nr}^l \geq c_b$ if and only if $(1 - \psi)k + \phi m \geq (1 - \psi\pi)c_b$. With $\beta\rho = \phi^h$, the bankers FOCs imply that k is increasing in ϕ^l and Equation (C.1) implies that c_b is decreasing in ϕ^l . Because the LHS of Equation (21) is decreasing in ϕ^l and the RHS of Equation (21) is increasing in ϕ^l , it follows that $(1 - \psi)k + \phi m \geq (1 - \psi\pi)c_b$ if and only if Equation (23) holds.

Finally, $\tilde{z}_{nr}^h \geq \tilde{z}_{nr}^l$ always holds, so that $\tilde{z}_{nr}^h \geq 0$ holds. Also, $\tilde{z}_r^h + a^h k^\alpha > 0$ holds as the bankers never exhaust their capacity to create H-money and L-money simultaneously.

Q.E.D.

C.18 Derivation of Equations (27)-(29)

To ensure existence of an equilibrium of Type IIIb, the following conditions need to be satisfied $q = (1 - \psi)k + \psi\pi c_b$, $\psi\pi c_b = q - (1 - \psi)k$, and $c_b = (\phi^h + \phi^l)m + q$:

$$\begin{aligned}
-m &\leq -\tilde{z}_r^l \leq a^l k^\alpha && (\tilde{z}_{nr}^l > 0, \tilde{z}_r^l + a^l k^\alpha \geq 0) \\
-m &\leq -\tilde{z}_r^h \leq a^h k^\alpha && (\tilde{z}_{nr}^h > 0, \tilde{z}_r^h + a^h k^\alpha \geq 0) \\
0 &\leq q + \frac{1 - \psi}{\psi} [(\tilde{z}_r^h - m)\phi^h + (\tilde{z}_r^l - m)\phi^l] && (\tilde{q}_{nr} \geq 0) \\
0 &\leq q + \frac{1 - \psi}{\psi} [(\tilde{z}_r^h - m)\phi^h + (\tilde{z}_r^l - m)\phi^l] && (\tilde{q}_{nr} + \phi \tilde{z}_{nr}^h \geq c_b) \\
&+ \frac{\hat{\phi}^h m}{\psi} - \frac{1 - \psi}{\psi} \hat{\phi}^h \tilde{z}_r^h - c_b \\
0 &\leq q + \frac{1 - \psi}{\psi} [(\tilde{z}_r^h - m)\phi^h + (\tilde{z}_r^l - m)\phi^l] && (\tilde{q}_{nr} + \phi \tilde{z}_{nr}^l \geq c_b) \\
&+ \frac{\hat{\phi}^l m}{\psi} - \frac{1 - \psi}{\psi} \hat{\phi}^l \tilde{z}_r^l - c_b \\
0 &\leq q - k - \phi^h(\tilde{z}_r^h - m) - \phi^l(\tilde{z}_r^l - m) && (\tilde{q}_r \geq 0) \\
\beta &< \hat{\phi}^l u'(c_b) && (\tilde{z}_{nr}^h < \bar{z}) \\
\beta &< \hat{\phi}^l u'(c_b) && (\tilde{z}_{nr}^l < \bar{z}) \\
0 &\leq \rho\beta/\phi^h - (1 - \rho)\beta/\phi^l
\end{aligned}$$

Because of the equilibrium properties, $\tilde{z}_r^h + a^l k^\alpha \geq 0$, $\tilde{z}_{nr}^l \geq 0$, $\tilde{q}_{nr} \geq 0$, $\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h \geq c_b$, $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l \geq c_b$ and $\tilde{q}_r \geq 0$ are satisfied automatically.

To have $\beta < \hat{\phi}^l u'(c_b)$ and $\beta < \hat{\phi}^h u'(c_b)$, it follows from Equations (C.1)-(C.3) that $\beta\rho < \phi^h$ and $\beta(1 - \rho) < \phi^l$. According to Equation (C.1), c_b is decreasing in ϕ^h and ϕ^l , so that when $\beta\rho < \phi^h$ and $\beta(1 - \rho) < \phi^l$ we must have $c_b < c_b^*$. The bankers' FOCs show that k is increasing in ϕ^h and ϕ^l , therefore $k > k^*$ when $\beta\rho < \phi^h$ and $\beta(1 - \rho) < \phi^l$. Because in equilibrium we have $(1 - \psi\pi)c_b = (1 - \psi)k + (\phi^h + \phi^l)m$, $(1 - \psi\pi)c_b^* > (1 - \psi)k^* + \beta m$ must hold, giving us Equation (27) as a necessary condition.

From Equations (C.2) and (C.3) it follows that $\rho\beta/\phi^h > (1 - \rho)\beta/\phi^l$ if and only

if $\hat{\phi}^l > \hat{\phi}^h$. Because $\tilde{q}_{nr} + \hat{\phi}^h \tilde{z}_{nr}^h = c_b$, $\tilde{q}_{nr} + \hat{\phi}^l \tilde{z}_{nr}^l = c_b$, we need $\tilde{z}_{nr}^h > \tilde{z}_{nr}^l$. Since $\psi \tilde{z}_{nr}^h + (1 - \psi) \tilde{z}_r^h = m$ and $\psi \tilde{z}_{nr}^l + (1 - \psi) \tilde{z}_r^l = m$, $\tilde{z}_{nr}^h > \tilde{z}_{nr}^l$ if and only if $-\tilde{z}_r^h > -\tilde{z}_r^l$. Consider that in the MM a banker deviates from the equilibrium allocations and chooses $(\tilde{q}_r, k, \tilde{z}_r^{h'}, \tilde{z}_r^{l'})$ instead of $(\tilde{q}_r, k, \tilde{z}_r^h, \tilde{z}_r^l)$ to solve his MM maximization problem. Let $\tilde{z}_r^{h'} = \tilde{z}_r^{l'} \equiv \tilde{z}_r'$ and $(\phi^h + \phi^l) \tilde{z}_r' = \phi^h \tilde{z}_r^h + \phi^l \tilde{z}_r^l$. Because $\rho\beta/\phi^h > (1 - \rho)\beta/\phi^l$, we have $\tilde{z}_r' > \rho\tilde{z}_r^h + (1 - \rho)\tilde{z}_r^l$, making the banker strictly better off. Therefore, $(0, k, \tilde{z}_r^{h'}, \tilde{z}_r^{l'})$ cannot satisfy the constraints of the banker. Because $k \geq 0$, $q_{nr} = 0$, and $z_r' + a^h k^\alpha \geq 0$ hold, we must have that $z_r' + a^l k^\alpha < 0$. Using that $q = (1 - \psi)k + \psi\pi c_b$, $c_b = q + (\phi^h + \phi^l)m$, and $k^{1-\alpha} < \alpha\mathbb{E}[a](\phi^h + \phi^l)$ (this follows from $\rho\beta/\phi^h > (1 - \rho)\beta/\phi^l$), we obtain $\tilde{z}_r' + a^l k^\alpha < 0$ only if $a_l(1 - \psi\pi) < \psi(1 - \pi) - (\phi^h + \phi^l)m/k$.

For this to be true, Equation (27) must hold. To demonstrate why, consider that Equation (27) does not hold. Then $a_l(1 - \psi\pi) < \psi(1 - \pi) - (\phi^h + \phi^l)m/k$ only if $(\phi^h + \phi^l)m/k < \phi m/\tilde{k}$, where ϕ solves Equation (19) and $\tilde{k} = (\alpha\mathbb{E}[a]\phi)^{\frac{1}{1-\alpha}}$. Suppose first that $k \leq \tilde{k}$. Let \tilde{c}_b be such that $\psi\pi u'(\tilde{c}_b) = 1 - (1 - \psi\pi)\beta/\phi$, so that $(1 - \psi\pi)\tilde{c}_b = (1 - \psi)\tilde{k}$. It follows that $k \leq \tilde{k}$ if and only if $c_b \leq \tilde{c}_b$. Define χ and φ as in the proof of Proposition 4 and construct two sets: $A = \{(\chi, \varphi) \in (0, 1]^2 : c_b \leq \tilde{c}_b\}$ and $B = \{(\chi, \varphi) \in (0, 1]^2 : \phi^h + \phi^l < \phi\}$. Equilibrium prices must be represented by a point in set A, with property $\chi > \varphi$. Then, from the proof of Proposition 4 we learn that $A \cap B = \{(\beta/\phi, \beta/\phi)\}$, so it follows that $\phi^h + \phi^l > \phi$. We obtain that $(\phi^h + \phi^l)m/k < \phi m/\tilde{k}$ cannot hold. It remains to consider $k > \tilde{k}$, which can only hold when $\phi^h + \phi^l > \phi$. Let k' be the level of k observed in a risk-free version of the economy with a supply of outside money m' such that $m'(\phi^{h'} + \phi^{l'}) = m(\phi^h + \phi^l)$. It follows that $k' < \tilde{k}$. In addition we can apply the proof of Propositions 3 and 6 to derive that $k < k'$. We thus obtain a contradiction.

To show sufficiency of Equations (27), (28) and (29), define the implicit functions f and g . f determines ϕ^l as function of ϕ^h such that Equation (25) holds. g determines ϕ^l as a function of ϕ^h such that Equation (26) holds. Both f and g are continuous. Equations (27), (28) imply that $(1 - \rho)\beta < (1 - \rho)\phi = f(\rho\phi) < g(\rho\phi)$. Equations (27) and (29) imply that $f(\rho\beta) > g(\rho\beta) = \phi'$. Because $f' < 0$ and $\phi > \beta$, it follows that there must exist a pair (ϕ^h, ϕ^l) with properties $1 > \beta\rho/\phi^h > \beta\rho/\phi^l$ that jointly satisfies the equilibrium Equations (25)-(25). Applying Equations (C.1), (C.2) and (C.3), for this pair, $\beta < \hat{\phi}^l u'(c_b)$ and $\beta < \hat{\phi}^h u'(c_b)$ hold. It can be verified as well that $\tilde{z}_r^h + a^h k^\alpha > 0$ and $\tilde{z}_{nr}^h > \tilde{z}_{nr}^l$, so that $\tilde{z}_{nr}^h > 0$ and thus all existence conditions are verified. Q.E.D.

C.19 Proof of Proposition 7

MM clearance requires $\psi \tilde{z}_{nr}^h + (1 - \psi) \tilde{z}_r^h = \tilde{m}^h$ and $\psi \tilde{z}_{nr}^l + (1 - \psi) \tilde{z}_r^l = \tilde{m}^l$. Government policy then implies that $\tilde{z}_{nr}^h = \tilde{z}_{nr}^l$. From Equation (5) it follows that $\hat{\phi}^h = \hat{\phi}^l$, so the equilibrium must be of Type I or II. In these equilibria $k(1 - \psi) + \phi m \geq (1 - \psi\pi)c_b$, with equality if $\phi > \beta$. Substituting the banker's FOC for k and Equation (C.1), we

obtain $\phi m + (1 - \psi) [\alpha \mathbb{E}[a] \phi]^{\frac{1}{1-\alpha}} \geq (1 - \psi \pi) u'^{-1}((1 - (1 - \psi \pi) \beta / \phi) / (\psi \pi))$, with equality if $\phi > \beta$. Q.E.D.

C.20 Proof of Proposition 8

Follows directly from combining (proofs of) Propositions 3, 4, 6, and 7. Q.E.D.

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