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van Aarle, B.; Engwerda, J.C.; Plasmans, J.E.J.

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Monetary and Fiscal Policy Interaction in the EMU: A Dynamic Game Approach

BAS VAN AARLE
LICOS, K.U. Leuven and University of Nijmegen, The Netherlands

JACOB ENGWERDA J.C.Engwerda@kub.nl
CentER Tilburg University, P.O. 90153, NL 5000LE Tilburg, The Netherlands

JOSEPH PLASMANS
University of Antwerp and Tilburg University, The Netherlands

Abstract. The interaction of monetary and fiscal policies is a crucial issue in a highly integrated economic area as the European Union. We investigate to which extent the EMU, that introduced a common monetary policy and restrictions on fiscal policy at the national level, benefits from macroeconomic policy cooperation due to the various interactions, spillovers and externalities from national macroeconomic policies. To study the effects of policy cooperation we compare the impact of three alternative policy regimes in a stylized dynamic model of the EMU: (i) non-cooperative monetary and fiscal policies, (ii) partial cooperation, and (iii) full cooperation both in symmetric and asymmetric settings where countries differ in structural characteristics, policy preferences and/or bargaining power. The paper introduces an analysis of coalitional behaviour in a dynamic setting into the literature.

Keywords: EMU, stabilisation policies, full and partial cooperation, LQ dynamic game

JEL codes: C70, E17, E58, E61, E63

1. Introduction

Two years after the start of the Economic and Monetary Union (EMU), already a considerable amount of experience has accumulated on the functioning of monetary and fiscal policy in this new framework of macroeconomic policy design in the European Union (EU). Monetary policy has been delegated to a supra-national authority, the European Central Bank (ECB), with a complex framework of objectives, policy instruments and decision making procedures. According to the Maastricht Treaty, the ECB should safeguard price stability in the EMU and – subject to the condition that it does not interfere with price stability – promote economic growth in the EMU. Its policies are therefore directed at controlling economic developments of the EMU economy as a whole rather than on individual countries. The design of fiscal policies in the EMU is complicated by the set of constraints on national fiscal policy imposed by the Stability and Growth Pact (SGP). According to the SGP, excessive deficits are to be avoided and are subject to sanctions.

In a highly integrated economic area like the EU, policy cooperation is likely to be of crucial importance because of the various interactions, spillovers and externalities from national macroeconomic policies and the complicated design and transmission of the common monetary policy of the ECB. This paper studies alternative regimes of macroeconomic policy cooperation and their effects in a dynamic model of macroeconomic adjustment in the EMU.

Macroeconomic policy cooperation has been one of the central issues in the theory and practice of macroeconomic policy design. This concerns both the coordination of macroeconomic policies within a country and the coordination of macroeconomic policies between different countries in the context of a multi-country setting where macroeconomic policy actions of one country are partly transmitted to the other countries through various channels in goods, labour and financial markets. These spillovers create the rationale for policy cooperation. Apart from full cooperation, also settings with partial cooperative coalitions, where only a subset of the players cooperates in their policies, have been studied in the literature. In those cases, in particular the effects of international monetary policy cooperation or international fiscal policy cooperation have been analysed, mostly in a static model framework.

The potential gains from cooperation are usually in the middle of the interest in studies of macroeconomic policy cooperation. Much less interest is given to the actual division of gains from policy cooperation. Typically, an egalitarian or a Nash bargaining division is assumed. In a multicountry context in general and the EMU in particular, issues of coalition formation, distributions of (voting) power, the distribution of cooperation gains and the stability of cooperative arrangements over time, however, are likely to be of crucial importance and to have a potential strong effect on policy making. In a dynamic setting these issues are even more important and complicated than in a static setting. This paper therefore extensively analyses these aspects of macroeconomic policy cooperation in the EMU, using a dynamic game approach.

Three policy regimes can be distinguished for a monetary union as the EMU:

(i) non-cooperative monetary and fiscal policies,

(ii) full cooperation,

(iii) partial cooperation.

This paper introduces into the literature an analysis of coalition formation in a dynamic setting. Coalitions between countries and between the ECB and one country are studied in case (iii). In case (ii), full coordination of all macroeconomic policies, i.e., the national fiscal policies and the monetary policy of the ECB are implemented in a cooperative framework. We also consider the effects of asymmetries in players' preferences and structural parameters of the model.

Decision making procedures, coalition formation, voting power and rent sharing inside the EU institutions have been studied in detail by, e.g., Widgrén [19], Laruelle and Widgrén [14], Hosli [11], Bindseil [4], Bindseil and Hantke [5], Sutter [17] and Levinsky and Silarsky [15]. These studies – while enabling us insights into issues of power distribution and coalition formation in communal policy formation –, however, do not consider a next step, namely the analysis of the effects of coalition formation and power distribution on economic policies.

Engwerda et al. [8] have studied the effects of non-cooperative macroeconomic policies in the EMU. They analyse macroeconomic stabilisation among three players (two countries and the ECB) in a dynamic model of the EMU. Cooperation has been analysed in Hughes Hallett and Ma [12], Acocella and Di Bartolomeo [3] and Engwerda et al. [9]. This paper extends these analysis by introducing coalition formation and studies how these coalitions affect policies and adjustment in the EMU.

The analysis is structured as follows: section 2 proposes a simple dynamic model of the EMU and formulates the dynamic stabilisation game between the monetary and fiscal policy makers in this setup. Sections 3 and 4 study, theoretically, the various equilibria of this dynamic stabilisation game and the resulting design of the common monetary policy and the national fiscal policies. Section 5 studies in detail numerical examples to obtain a deeper insight into the economic properties of the model. The appendices provide details about algorithms and calculations in the analytical part of the paper.

2. A dynamic EMU model

To study macroeconomic policy design in the EMU we use the comprehensive model of the EMU used by Engwerda et al. [9]. There we considered the scenario where the EMU consists of two symmetric, equally sized (blocks of) countries that share a common central bank, the ECB. In this paper we consider also asymmetric settings and the symmetric Engwerda et al. [9] model is interpreted as a benchmark scenario. The model ignores the external interaction of the EMU countries with the non-EMU countries and also the dynamic implications of government debt and net foreign asset accumulation. It consists of the following equations:

$$y_1(t) = \delta_1 s(t) - \gamma_1 r_1(t) + \rho_1 y_2(t) + \eta_1 f_1(t), \tag{1}$$

$$y_2(t) = -\delta_2 s(t) - \gamma_2 r_2(t) + \rho_2 y_1(t) + \eta_2 f_2(t), \tag{2}$$

$$s(t) = p_2(t) - p_1(t), \tag{3}$$

$$r_i(t) = i_E(t) - \dot{p}_i(t), \quad i \in \{1, 2\}, \tag{4}$$

$$m_i(t) - p_i(t) = \kappa_i y_i(t) - \lambda_i i_E(t), \quad i \in \{1, 2\}, \tag{5}$$

$$\dot{p}_i(t) = \xi_i y_i(t), \quad i \in \{1, 2\}, \tag{6}$$

in which y_i denotes the real output, s competitiveness of country 2 vis-à-vis country 1, r_i the real interest rate, p_i the price level, f_i the real fiscal deficit, i_E the nominal interest rate and m_i the nominal money balances of country (block) $i \in \{1, 2\}$. All variables are in logarithms, except for the interest rate which is in perunages, and denote deviations from their long term equilibrium (balanced growth path) that has been normalised to zero, for simplicity. A dot above a variable denotes its time derivative.

Equations (1), (2) represent output in the EMU countries as a function of competitiveness in intra-EMU trade, the real interest rate, the foreign output and the domestic fiscal deficit. Competitiveness is defined in (3) as the output price differential. Real interest rates are defined in (4) as the difference between the EMU wide nominal interest rate, i_E, and domestic inflation. Note that (4) imply that, temporarily, real interest rates may diverge among countries if inflation rates are different. (5) provide the demand for the common currency where it is assumed that the money market is in equilibrium. Notice that structural model (1)–(6) is a model with an integrated economy with several kinds of cross country effects. Besides the common nominal interest rate there are two other important direct cross-country spillovers that affect domestic output:

(i) the intra-EMU competitiveness channel (as measured by the elasticity δ), and

(ii) the foreign output channel (as measured by the elasticity ρ).

It is assumed that the common nominal interest rate is set by the ECB. Alternatively, we could have assumed – as in Engwerda et al. [8] – that a monetary targeting strategy is implemented by the ECB. In that scenario, the ECB controls the common money supply and the common money market is cleared by the common interest rate. Whereas related, both approaches differ to some degree in their transmission of the ECB's monetary policy. Rather than the monetary targeting approach, the interest rate targeting approach is proposed here. This seems to be somewhat closer to the policy strategy adopted by the ECB in practice. Domestic output and inflation are related through a Phillips curve type relation in (6).

The model (1)–(6) can be reduced to two output equations:

$$y_1(t) = b_1 s(t) - c_1 i_E(t) + a_1 f_1(t) + \frac{\rho_1}{k_1} a_2 f_2(t), \tag{7}$$

$$y_2(t) = -b_2 s(t) - c_2 i_E(t) + \frac{\rho_2}{k_2} a_1 f_1(t) + a_2 f_2(t), \tag{8}$$

in which

$$a_1 := \frac{\eta_1 k_2}{k_1 k_2 - \rho_1 \rho_2}, \quad a_2 := \frac{\eta_2 k_1}{k_1 k_2 - \rho_1 \rho_2}, \quad b_1 := \frac{\delta_1 k_2 - \rho_1 \delta_2}{k_1 k_2 - \rho_1 \rho_2}, \quad b_2 := \frac{\delta_2 k_1 - \rho_2 \delta_1}{k_1 k_2 - \rho_1 \rho_2},$$
$$c_1 := \frac{\gamma_1 k_2 + \rho_1 \gamma_2}{k_1 k_2 - \rho_1 \rho_2}, \quad c_2 := \frac{\gamma_2 k_1 + \rho_2 \gamma_1}{k_1 k_2 - \rho_1 \rho_2}, \quad k_1 := 1 - \gamma_1 \xi_1, \quad k_2 := 1 - \gamma_2 \xi_2.$$

The dynamics of the model are then represented by the following first-order linear differential equation with competitiveness, $s(t)$, as the scalar state variable and the national fiscal deficits, $f_i(t),\ i \in \{1, 2\}$, and the common interest rate, $i_E(t)$, as control variables:

$$\dot{s}(t) = \phi_4 s(t) - \phi_1 f_1(t) + \phi_2 f_2(t) + \phi_3 i_E(t), \quad s(0) := s_0, \tag{9}$$

in which

$$\phi_1 := \left(\xi_1 - \xi_2 \frac{\rho_2}{k_2}\right) a_1, \quad \phi_2 := \left(\xi_2 - \xi_1 \frac{\rho_1}{k_1}\right) a_2,$$
$$\phi_3 := \xi_1 c_1 - \xi_2 c_2 \quad \text{and} \quad \phi_4 := -(\xi_2 b_2 + \xi_1 b_1).$$

The initial value of the state variable, s_0, measures any initial disequilibrium in intra-EMU competitiveness. Such an initial disequilibrium in competitiveness could be the result of, e.g., differences in fiscal policies in the past or some initial disturbance in one country.

We assume that the fiscal authorities control their fiscal policy instrument so as to minimise the following quadratic loss functions that feature the countries' concern towards domestic nominal inflation, domestic real output and domestic real fiscal deficit:[1]

$$\min_{f_i} J_i = \min_{f_i} \int_0^\infty \left\{\alpha_i \dot{p}_i^2(t) + \beta_i y_i^2(t) + \chi_i f_i^2(t)\right\} \mathrm{e}^{-\theta t}\,\mathrm{d}t, \quad i \in \{1, 2\}, \tag{10}$$

in which θ denotes the rate of time preference and α_i, β_i and χ_i ($i \in \{1, 2\}$) represent preference weights that are attached to the stabilisation of inflation, output and fiscal deficits, respectively. Preference for a low fiscal deficit reflects the costs of excessive deficits such as proposed in the SGP that sanctions such excessive deficits in the EMU. Moreover, costs could also result from undesirable debt accumulation and intergenerational redistribution that high deficits imply and, in that interpretation, χ_i could also reflect the priority attached to fiscal retrenchment and consolidation.

As stipulated in the Maastricht Treaty, the ECB directs the common monetary policy at stabilising inflation and, as long as not in contradiction to inflation stabilisation, stabilising output in the aggregate EMU economy. Moreover, we will assume that the active use of monetary policy implies costs for the monetary policymaker: other things equal it would like to keep its policy instrument constant, avoiding large swings. Consequently, we assume that the ECB is confronted with the following optimisation problem:[2]

$$\begin{aligned}\min_{i_E} J_E^A = \min_{i_E} \int_0^\infty & \left\{\left(\alpha_{1E}\dot{p}_1(t) + \alpha_{2E}\dot{p}_2(t)\right)^2\right.\\ & \left. + \left(\beta_{1E}y_1(t) + \beta_{2E}y_2(t)\right)^2 + \chi_E i_E^2(t)\right\} \mathrm{e}^{-\theta t}\,\mathrm{d}t.\end{aligned} \tag{11}$$

Alternatively, we could consider a case where the ECB is governed by national interests rather than by EMU-wide objectives.[3] In that scenario, the ECB would be a coalition of the (former) national central banks that decide cooperatively on the common monetary policy that is based on individual, national interests rather than on EMU-wide

[1] Note that in a monetary union the fiscal players are assumed not to have any direct control over the nominal interest rate since this control is generally left for the common central bank (see, among others, Gros and Hefeker [10], Acocella and Di Bartolomeo [3]).

[2] α_E and β_E are the preference parameters with respect to inflation and output respectively of equation (14) in Engwerda et al. [8], where $\alpha_E := 1$ because of normalisation.

[3] See also Gros and Hefeker [10]. These authors compare, in a static framework, a specification of the ECB's cost function based on national variables with the standard one (11) as in Cukierman [6]. They discuss the welfare implications of the two specifications. Also van Aarle et al. [1] and De Grauwe [7] compare outcomes under an ECB's objective function based on aggregate and national variables, respectively.

objectives. In this scenario the monetary policy of the ECB will typically be more sensitive to individual country variables. Then the ECB seeks to minimise a loss function, which is assumed to be quadratic in the individual countries' inflation rates and outputs – rather than in EMU-wide inflation and output as in (11) – and the common interest rate:

$$\min_{i_E} J_E^N = \min_{i_E} \int_0^\infty \left\{ \alpha_{1E}\dot{p}_1^2(t) + \alpha_{2E}\dot{p}_2^2(t) \right.$$
$$\left. + \beta_{1E}y_1^2(t) + \beta_{2E}y_2^2(t) + \chi_E i_E^2(t) \right\} \mathrm{e}^{-\theta t}\,\mathrm{d}t. \tag{12}$$

Disregarding the monetary instrument, which in this paper is the interest rate rather than the money supply, the objective (11) can be seen as a generalisation of the loss function used in Engwerda et al. [8, expression (14), p. 262]. The loss function in (12) can also be interpreted as a loss function in which the ECB is a coalition of national central bankers which all have a share in the decision making proportional to the size of their economies.

Below, we will (for notational convenience) only elaborate the problem if the ECB objective is (11). In a similar way appropriate formulae can be directly obtained if (12) is used as the ECB's performance criterion.

Using (6) we can rewrite (10) and (11) as follows:

$$J_i = d_i \int_0^\infty \left\{ y_i^2(t) + \frac{\chi_i}{d_i} f_i^2(t) \right\} \mathrm{e}^{-\theta t}\,\mathrm{d}t, \quad i \in \{1,2\}, \tag{13}$$

$$J_E^A = \int_0^\infty \left\{ d_{1E}y_1^2(t) + d_{2E}y_2^2(t) + 2d_{3E}y_1(t)y_2(t) + \chi_E i_E^2(t) \right\} \mathrm{e}^{-\theta t}\,\mathrm{d}t, \tag{14}$$

where $d_i := \alpha_i\xi_i^2 + \beta_i$, $d_{iE} := \alpha_{iE}^2\xi_i^2 + \beta_{iE}^2$ with $i \in \{1,2\}$ and $d_{3E} := \alpha_{1E}\alpha_{2E}\xi_1\xi_2 + \beta_{1E}\beta_{2E}$[4].

Defining $x^{\mathrm{T}}(t) := (s(t), f_1(t), f_2(t), i_E(t))$, we can rewrite (7) and (8) as,

$$y_1(t) = \left(b_1, a_1, \frac{\rho_1}{k_1}a_2, -c_1 \right) x(t) =: m_1 x(t),$$
$$y_2(t) = \left(-b_2, \frac{\rho_2}{k_2}a_1, a_2, -c_2 \right) x(t) =: m_2 x(t).$$

Introducing e_j as the jth standard basis vector of $\mathbb{R}^4$ (i.e., $e_1 := (1\ 0\ 0\ 0)^{\mathrm{T}}$, etc.),

$$M_i := m_i^{\mathrm{T}} m_i + \frac{\chi_i}{d_i} e_{i+1}^{\mathrm{T}} e_{i+1}, \quad i \in \{1,2\}, \quad \text{and}$$
$$M_E^A := d_{1E}m_1^{\mathrm{T}}m_1 + d_{2E}m_2^{\mathrm{T}}m_2 + 2d_{3E}m_1^{\mathrm{T}}m_2 + \chi_E e_4^{\mathrm{T}} e_4$$

[4] In the case that national variables feature in the ECB objective function, as in (12), we have $d_{iE} = \alpha_{iE}\xi_i^2 + \beta_{iE}$ with $i = \{1,2\}$ and $d_{3E} = 0$.

the policy makers' loss functions (13), (14) can be written as:

$$J_i = d_i \int_0^\infty \left\{x^{\mathrm{T}}(t) M_i x(t)\right\} \mathrm{e}^{-\theta t}\, \mathrm{d}t, \quad i \in \{1, 2\}, \tag{15}$$

$$J_E^A = \int_0^\infty \left\{x^{\mathrm{T}}(t) M_E^A x(t)\right\} \mathrm{e}^{-\theta t}\, \mathrm{d}t. \tag{16}$$

Henceforth, for reasons of convenience, we assume that $\theta = 0$. If θ differs from zero, the model can be easily solved following the same procedure used in this paper after a simple transformation of variables.[5] All that changes in the ensueing results is that the parameter ϕ_4 has to be substituted by $\phi_4 - \frac{1}{2}\theta$.

3. Macroeconomic policy design in the EMU

This section studies alternative modes of policy cooperation in a monetary union as the EMU. We study macroeconomic policy design and macroeconomic adjustment in three alternative macroeconomic policy regimes:

(i) non-cooperative macroeconomic policies,

(ii) full cooperation and

(iii) partial cooperation.

The first two regimes are standard in macroeconomic policy analysis. The regimes where subgroups of players form coalitions in which they coordinate their policies but interact in a non-cooperative manner with the players that are not part of the coalition, is not dealt with usually, certainly not in a dynamic context. This is not because such cases would be less interesting or less relevant in practice, but rather because of a lack of analytical tools to analyse such cases. In regimes of partial cooperation, important questions need to be answered, like

(i) Why certain coalitions arise and others not?,

(ii) Do these coalitions display stability over time?,

(iii) How are the gains from cooperation distributed between the members of the coalitions?,

(iv) How do differences in initial conditions, economic structures and policy preferences affect outcomes in this scenario?

In this paper the issues (iii) and (iv) can be answered whereas some insight can be provided about the coalition formation issue. The stability issue will not be dealt with in this paper.

[5] That is, transforming $x(t)$ into $\mathrm{e}^{-(1/2)\theta t} x(t)$ (see Engwerda et al. [8, p. 263] for further details).

A study of all three regimes is necessary for a complete insight on macroeconomic policy design in the EMU. The regimes with either fully non-cooperative or fully cooperative policies are clearly the two extreme forms of policy formulation. Forms of partial cooperation combine elements of these opposite policy regimes as the following analysis shows. Since in this analysis the specific choice of the loss function from the ECB does not play any role, for notational convenience this dependency is dropped and we will just use, e.g., M_E instead of M_E^A and M_E^N.

3.1. *The non-cooperative case*

In the non-cooperative case players minimise their cost functions (15), (16) with respect to the dynamic law of motion (9) of the system. From appendix A.1 we find as equilibrium strategies in the non-cooperative open-loop case:

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} a_1 b_1 - \phi_1 K_1 \\ -a_2 b_2 + \phi_1 K_2 \\ -b_1 c_1 d_{1E} + b_2 c_2 d_{2E} + (b_2 c_1 - b_1 c_2) d_{3E} + \phi_3 K_3 \end{pmatrix} s(t) \\ =: H_{nc} s(t), \tag{17}$$

where the contents of matrices G and K_i ($i = 1, 2, 3$) can be obtained from the appendix. To stress the fact that we are dealing with the non-cooperative case we denote the cost of the players by $J_{i,nc}$, $i \in \{1, 2, E\}$, respectively. Furthermore, the resulting system is described by the differential equation $\dot{s}(t) = -a_{nc} s(t)$ with $s(0) = s_0$, where the adjustment speed, a_{nc}, is obtained as the positive eigenvalue of some related matrix that is defined in appendix A.1. Using the equilibrium controls (17) we obtain then the corresponding fiscal players' optimal costs:

$$J_{i,j} = d_i \int_0^\infty \{x^{\mathrm{T}}(t) M_i x(t)\}\,\mathrm{d}t = d_i \int_0^\infty \{(s\ f_1\ f_2\ i_E) M_i (s\ f_1\ f_2\ i_E)^{\mathrm{T}}\}\,\mathrm{d}t \tag{18}$$

$$= d_i \int_0^\infty \left\{ s(t) \big(1\ H_j^{\mathrm{T}}\big) M_i \begin{pmatrix} 1 \\ H_j \end{pmatrix} s(t) \right\} \mathrm{d}t \tag{19}$$

$$= d_i \big(1\ H_j^{\mathrm{T}}\big) M_i \begin{pmatrix} 1 \\ H_j \end{pmatrix} \int_0^\infty \{s^2(t)\}\,\mathrm{d}t \tag{20}$$

$$= d_i \big(1\ H_j^{\mathrm{T}}\big) M_i \begin{pmatrix} 1 \\ H_j \end{pmatrix} \int_0^\infty \{s_0^2\, \mathrm{e}^{-2a_j t}\}\,\mathrm{d}t \tag{21}$$

$$= d_i \big(1\ H_j^{\mathrm{T}}\big) M_i \begin{pmatrix} 1 \\ H_j \end{pmatrix} s_0^2 \frac{1}{2a_j}, \quad i = \{1, 2\},\ j = nc. \tag{22}$$

In the same way we find the ECB's optimal cost:

$$J_{E,j} = \big(1\ H_j^{\mathrm{T}}\big) M_E \begin{pmatrix} 1 \\ H_j \end{pmatrix} s_0^2 \frac{1}{2a_j}, \quad j = nc. \tag{23}$$

3.2. The cooperative case

In the full cooperative case players minimise a common cost function: $J^C := \tau_1 J_1 + \tau_2 J_2 + \tau_3 J_E$ subject to (9); τ_i ($i \in \{1, 2, 3\}$) equals the player i's bargaining power with $\tau_1 + \tau_2 + \tau_3 = 1$. In appendix A.2 we show that the equilibrium cooperative controls can be written as:

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} =: H_c s(t).$$

Using these controls the dynamic behaviour of this system is described as: $\dot{s}(t) = -a_c s(t)$ with $s(0) = s_0$, where a_c is again a positive eigenvalue of some matrix that is defined in appendix A.2. The corresponding optimal costs for the players are (22), (23) with $j = c$.

3.3. Cases with coalitions of policymakers

To determine the equilibrium open-loop solution for partial coalitions we will concentrate on the case that the fiscal authority of country 1 and the ECB coordinate their policies but act in a non-cooperative fashion with the fiscal authority of country 2. For this case we will use the shorthand notation coalition $(1, E)$.

To determine the equilibrium solution for the $(1, E)$ coalition we rewrite the system equation as:

$$\dot{s} = \phi_4 s + (-\phi_1 \ \phi_3) \begin{pmatrix} f_1 \\ i_E \end{pmatrix} + \phi_2 f_2, \quad s(0) = s_0,$$

and consider the performance criteria $J_{(1,E)} := \tau_1 J_1 + \tau_2 J_E$ with $\tau_1 + \tau_2 = 1$ as the aggregate performance of player 1 and the ECB, and J_2 for player 2, respectively. Next, we consider a with this $(1, E)$ coalition form corresponding permutation of $x(t)$:

$$\tilde{x}(t) := \begin{pmatrix} s(t) \\ f_1(t) \\ i_E(t) \\ f_2(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} x(t) =: P_{(1,E)} x(t)$$

(note that $P_{(1,E)}$ is an (orthogonal) permutation matrix). Now, we can, basically, use the non-cooperative algorithm to determine the equilibrium strategies (see appendix A.3)

$$\begin{pmatrix} f_1(t) \\ i_E(t) \\ f_2(t) \end{pmatrix} =: H_{(1,E)} s(t).$$

Using these equilibrium strategies, the system is described by: $\dot{s}(t) = -a_{(1,E)} s(t)$ with $s(0) = s_0$, where $a_{(1,E)}$ is again obtained as the positive eigenvalue of some matrix. The optimal costs for the players are (22), (23) with $j = (1, E)$. The partial coalitions $(2, E)$ and $(1, 2)$ are defined similarly and the equilibrium strategies are derived analogously.

3.4. Some coalition formation terminology

In order to obtain some insight into the question which coalition(s) might be realised and which is (are) less plausible, we introduce some terminology. Each of the five policy regimes outlined in the above subsections is called a *coalition form* and each group of two or more players that cooperate in a coalition form a *coalition*. We say that a certain coalition form is *supported by player i*, if player i has no incentive to deviate from this coalition form. If a coalition form has a coalition, then we say that this coalition form is *internally supported* if all players in the coalition support the coalition form. A coalition form is called *externally supported* if all players outside the coalition support the coalition form. If a coalition form is both externally and internally supported, then we will call this coalition form *sustainable*, that is, in such a coalition form no player has an incentive to deviate (leave this coalition form). Finally, we call a coalition form *unsustainable* if as well players inside as outside the coalition can improve by joining another coalition form.

Note that a coalition form which is internally supported is in principle *viable*. One reason why such a coalition form might not be realised is that, e.g., side-payments take place. Here we will ignore these issues. A similar remark holds with respect to the unsustainable coalition form. Such a coalition form is in principle not viable, this contrary to a coalition form which is partially supported (i.e., supported by not all players in the coalition). Such a coalition form might be viable, but this typically depends on what other coalition forms have to offer for all the different player(s). So, this requires a more detailed description of the negotiation process, something we will not go into here. The notions introduced above will in particular be used in the simulation study.

4. The symmetric case

In this section we consider the model described in the previous sections under the assumption of symmetry of country 1 and 2. In that case one can obtain theoretical results. The outcomes of this analysis are not only interesting on their own, but may be also helpfull in analysing the properties of the non-symmetric model. We make the following assumptions with respect to the various parameters:

$$\alpha_1 = \alpha_2, \quad \alpha_{1E} = \alpha_{2E}, \quad \beta_1 = \beta_2, \quad \beta_{1E} = \beta_{2E}, \quad \chi_1 = \chi_2, \quad \xi_1 = \xi_2,$$
$$\gamma_1 = \gamma_2, \quad \rho_1 = \rho_2, \quad \delta_1 = \delta_2, \quad \eta_1 = \eta_2, \quad \lambda_1 = \lambda_2 \quad \text{and} \quad \kappa_1 = \kappa_2.$$

Furthermore we introduce for notational convenience the following parameters:

$$a := a_1, \quad e := \frac{\rho_1}{k_1}a_2, \quad c := c_1, \quad b := b_1, \quad d := d_1, \quad d_E^A := \alpha_{1E}^2\xi_1^2 + \beta_{1E}^2,$$
$$d_E^N := \alpha_{1E}\xi_1^2 + \beta_{1E}, \quad g := \frac{\chi_1}{d}, \quad g_E^A := \frac{\chi_E}{2d_E^A} \quad \text{and} \quad g_E^N := \frac{\chi_E}{d_E^N}.$$

Then, the dynamics are given by the state equation

$$\dot{s} = \phi_4 s - \phi_1 f_1 + \phi_1 f_2, \quad s(0) = s_0.$$

Whereas the performance criteria reduce to:

$$J_i = d_i \int_0^\infty \left\{x^{\mathrm{T}}(t) M_i x(t)\right\} \mathrm{d}t, \quad i \in \{1, 2\}, \quad \text{and}$$
$$J_E^j = d_E^j \int_0^\infty \left\{x^{\mathrm{T}}(t) M_E^j x(t)\right\} \mathrm{d}t, \quad j \in \{A, N\},$$

with

$$M_1 := \begin{pmatrix} b^2 & ab & be & -bc \\ ab & a^2+g & ae & -ac \\ be & ae & e^2 & -ce \\ -bc & -ac & -ce & c^2 \end{pmatrix}, \qquad M_2 := \begin{pmatrix} b^2 & -be & -ab & bc \\ -be & e^2 & ae & -ce \\ -ab & ae & a^2+g & -ac \\ bc & -ce & -ac & c^2 \end{pmatrix},$$

$$M_E^N := \begin{pmatrix} 2b^2 & b(a-e) & -b(a-e) & 0 \\ b(a-e) & a^2+e^2 & 2ae & -c(a+e) \\ -b(a-e) & 2ae & a^2+e^2 & -c(a+e) \\ 0 & -c(a+e) & -c(a+e) & 2c^2+g_E^N \end{pmatrix}$$

and

$$M_E^A := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & (a+e)^2 & (a+e)^2 & -2c(a+e) \\ 0 & (a+e)^2 & (a+e)^2 & -2c(a+e) \\ 0 & -2c(a+e) & -2c(a+e) & 4c^2+2g_E^A \end{pmatrix}.$$

4.1. The various equilibrium strategies

In appendix B we calculate various parameters that are essential for calculating the equilibrium strategies for the non-cooperative coalition (nc); the cooperative coalition (c), with $\tau_1 = \tau_2 = \tau$ and $\tau_3 = 1 - 2\tau$, where $0 \leqslant \tau \leqslant \frac{1}{2}$; and the fiscal coalition $(1, 2)$, with $\tau_1 = \tau_2 = \frac{1}{2}$. These parameters are presented in table 1. Substitution of these parameters into the equilibrium strategies determined in appendix A yields then straightforwardly the equilibrium strategies presented below. It turns out that as well for the non-cooperative as the fiscal coalition case the strategies for the national and aggregate performance case are the same. Only for the cooperative case the strategies depend on the ECB's preference function. For that reason we discern below two cooperative cases, the national one (cN) and the aggregate one (cA). The equilibrium strategies are:

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix}_i = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} p_i s(t),$$

and the corresponding cost for the players is:

$$J_{1,i} = J_{2,i} = \frac{1}{2}\frac{d}{a_i}\left\{\left(b + p_i(a-e)\right)^2 + p_i^2 g\right\} s^2(0), \tag{24}$$

$$J_{E,i}^N = \frac{1}{2}\frac{d_E^N}{a_i}\left(b + p_i(a-e)\right)^2 s^2(0), \quad J_{E,i}^A = 0. \tag{25}$$

for $i = nc, cA, cN, (1,2)$. Here:

Table 1
Policy parameters for the symmetric case.

i	nc	cN	cA	(1,2)
u_i	$a(a-e)+g$	$\tau g + d_E^N(a-e)^2$	$\tau(g+(a-e)^2)$	$g+(a-e)^2$
a_i^2	$(\phi_4 - 2\phi_1 p_{nc})^2$	$\frac{\tau g \phi_4^2}{u_{cN}}$	$\frac{g\phi_4^2}{g+(a-e)^2}$	$\frac{g\phi_4^2}{u_{(1,2)}}$
K_i	$\frac{-2b^2 g}{u-\sqrt{u^2+8gb^2\phi_1^2}}$	$\frac{(\phi_4+a_c)u_{cN}+2d_E^N\phi_1 b(a-e)}{2\phi_1^2}$	$\frac{\phi_4\tau g + a_{cA}u_{cA}}{2\phi_1^2}$	$\frac{-gb^2}{\phi_4 g - u_{(1,2)}a_{(1,2)}}$
p_i	$\frac{\phi_1 K_{nc} - ab}{u_{nc}}$	$\frac{\phi_1 K_{cN} - d_E^N b(a-e)}{u_{cN}}$	$\frac{\phi_1 K_{cA} - \tau b(a-e)}{u_{cA}}$	$\frac{2\phi_1 K_{(1,2)} - b(a-e)}{u_{(1,2)}}$

with $u := 2\phi_4 u_{nc} + b\phi_1(3a-e)$.

If the coalition $(1, E)$ occurs (or its symmetric counterpart $(2, E)$) the EMU is directly involved in the game (i.e., the common interest rate differs in general from zero). As a consequence the theoretical formulae become much more involved. Therefore they are omitted.

4.2. *Some general conclusions*

First, we summarize some conclusions with respect to the number of equilibria that may appear in the game.

Theorem 1. For the cooperative and $(1,2)$ coalition the game has always a unique equilibrium. If $e < a$ the non-cooperative game also has a unique equilibrium. If $e \geqslant a$ the number of equilibria may vary between zero and two (see Engwerda et al. [9, table 2] for details).

We will restrict in the rest of this section to the case that $e < a$ and will assume, moreover, that as well $-\phi_4$ as a are positive. For a broad class of realistic model parameters these assumptions hold (in particular the positivity condition on $-\phi_4$ is satisfied if one chooses the discount factor θ large enough). As a consequence, the non-cooperative game has a uniquely defined equilibrium. Furthermore, unless stated otherwise, we will restrict our analysis to the non-cooperative, the cooperative and the coalition structure $(1,2)$.

Two striking things we observe from the previous section are that $f_1(t) = -f_2(t)$ and that the Central Bank does not influence the game, neither in a direct way (i.e., $i_E(t) = 0$) nor in an indirect way (i.e., via its parameters). These statements do not hold for the coalition form $(1, E)$. There, the fiscal instruments differ and the Central Bank uses its instruments actively to reach its goals. The symmetry assumptions are crucial too, if they are dropped the Central Bank gets also actively involved into the game.

Since we have explicit formulae for the various cost functions we can exploit these to derive some further general conclusions. Our first observation (see appendix C) is that the convergence speed of the resulting system satisfies some nice properties:

Lemma 2.

(i) $a_{cN} \leqslant a_{(1,2)}$.

(ii) $a_{nc} \leqslant a_{cN}$ if $3\tau \geqslant 2d_N$, where $d_N := \tau + (1-2\tau)(d_E^N/d)$.

(iii) $a_i(g)$ is an increasing function with

$$a_i(0) = 0 \quad \text{and} \quad a_i(\infty) = -\phi_4, \quad i \in \{nc, c, (1,2)\}.$$

(iv) $a_{cN} \leqslant a_{cA}$.

With respect to the performance criteria we first note that the fiscal players' cost in the coalition case, with the ECB considering an aggregate performance criterion, and the $(1,2)$ coalition coincide. In other words, the fiscal players are indifferent between these modes of play. This is most easily seen by first noting that both a_{cA} and p_{cA} are independent of τ. As a consequence, the corresponding cost for the fiscal players is in this cooperative case independent of τ too. Next substitute $\tau = \frac{1}{2}$ into the "aggregate" coalition cost function. It is easily verified that this cost function coincides with the cost for the $(1,2)$ coalition, which shows the correctness of the claim for an arbitrarily chosen τ.

Our next results concern the national performance criterion. We show, amongst others, that the Central Bank will prefer a non-cooperative above a cooperative mode of play if the cooperation parameter τ becomes large and that the fiscal players will prefer a partial coalition above a cooperative mode of play. The proof is again deferred to the appendix C. We used the notation $\mathrm{sgn}(a)$ here to denote the sign of variable a.

Lemma 3.

(i) $\mathrm{sgn}(J_{E,nc}^N - J_{E,cN}^N) = \mathrm{sgn}(a_{nc} - a_{cN})$.

(ii) $J_{i,cN} \geqslant J_{i,(1,2)},\ i \in \{1,2\}$.

From lemmas 2(ii) and 3(i) we have that if, e.g., $3\tau \geqslant 2d_N$, always $J_{E,cN}^N \geqslant J_{E,nc}^N$. A more detailed analysis shows that if $\tau = 0$, $J_{E,cN}^N < J_{E,nc}^N$, and therefore it is easily seen from the proof of lemma 2(ii) that there is always a threshold τ^* such that for all $\tau \geqslant \tau^*$, $J_{E,cN}^N \geqslant J_{E,nc}^N$ and for all $\tau < \tau^*$, $J_{E,cN}^N < J_{E,nc}^N$.

Now, consider the case that $\tau \geqslant \tau^*$. Since aggregate performance is minimized in the cooperative situation and according lemma 2(ii) the Central Bank's cost are higher in this situation than in the non-cooperative case, the cost of the fiscal players will be less in the cooperative mode of play than in the non-cooperative case. A similar reasoning shows that since $J_{i,cN} \geqslant J_{i,(1,2)},\ i \in \{1,2\}$, the cost of the Central Bank in the coalition $(1,2)$ mode of play will always be larger than in the cooperative case. Stated differently, we see that under this assumption the Central Bank will always prefer the non-cooperative mode of play, whereas the fiscal players prefer the coalition $(1, 2)$ mode of play. So, summarizing, we have:

Theorem 4. Assume that the ECB considers the national performance criterion. Then, there exists a number τ^* such that if $\tau \geqslant \tau^*$, the cooperative mode of play is unsustainable.

5. A simulation study

In this section we consider the differential game on macroeconomic stabilisation in the EMU that was set up in section 2, using simulations of a stylised example. We analyse five scenarios:

(i) a symmetric baseline case in which countries are of equal size, all structural and preference parameters are the same in both countries,

(ii) an asymmetric case where the EMU countries differ in stabilisation preferences,

(iii) an asymmetric case where countries differ in monetary policy transmission,

(iv) an asymmetric case where countries differ in bargaining powers in case they enter coalitions and

(v) an asymmetric case where countries differ in sensitivity to intra-EMU competitiveness.

In this way our analysis contributes to the important discussion about the implications for policymaking in EMU in case countries differ in their structural characteristics. Outcomes are analysed for all the five different equilibria outlined in section 3.

5.1. *Baseline: A symmetric EMU*

In the symmetric baseline case both countries are of equal size and all structural and preference parameters are the same. The following values for the structural model parameters are used:[6]

$$\gamma = 0.4, \quad \delta = 0.2, \quad \rho = 0.4, \quad \eta = 1, \quad \kappa = 1, \quad \lambda = 1 \quad \text{and} \quad \xi = 0.25.$$

The initial state of intra-EMU competitiveness equals $s_0 = 0.05$ (implying an initial disequilibrium of 5% in competitiveness between the two countries). Concerning the preference weights in the objective functions of the fiscal players, the following values have been assumed:

$$\alpha = 2, \quad \beta = 5, \quad \chi = 2.5 \quad \text{and} \quad \theta = 0.15.$$

[6] See Engwerda et al. [9] for a similar simulation set up. The parameter choices are related to those used in Turnovsky et al. [18] and Neck and Dockner [16].

In the ECB's loss function both countries are equally weighted. Furthermore, the ECB – in contrast to the fiscal players – cares more about inflation than about output stabilisation and has also an interest rate smoothing objective: $\alpha_{1E} = \alpha_{2E} = 0.8$, $\beta_{1E} = \beta_{2E} = 0.5$, and χ_E equals 2.5. Figure 1 displays the adjustment in the case the aggregate objective function (11) of the ECB is used (adjustment in the case where the national objective function (12) of the ECB is used is almost identical and therefore not displayed here).

The adjustment of intra-EMU competitiveness is given in panel (a). The adjustments of the policy variables are found in panels (b)–(d). The initial disequilibrium in intra-EMU competitiveness implies that output in country 1 is initially above the long-run equilibrium in country 1 and below the long-run equilibrium in country 2. This induces restrictive fiscal policies in country 1 and expansionary fiscal policies in country 2. The dose of these fiscal stabilisation, however, varies according the type of equilibrium. In the EMU there is a stabilisation externality: the restrictive fiscal policy implemented by country 1 to stabilise its economy, is harmful for country 2 which would benefit from an expansionary fiscal policy in country 1. Coalitions offer potentially a solution to such policy coordination problems. In the Pareto case and the fiscal cooperation case (which coincide in this symmetric case), the fiscal players internalise the spillovers from their fiscal instrument on the other country. A similar externality results from the fiscal policy in country 2. The common interest rate, panel (b), only reacts in the case of a coalition with one fiscal policy maker: in that case the common interest rate is partly targeted at the situation in the country with which the ECB has formed a coalition. This leads to a higher interest rate in case a coalition is formed with country 1 and a lower interest rate when a coalition is formed with country 2. Panels (e) and (f) display output in country 1 and 2 in the different cases. Table 2 gives the resulting welfare losses that the players incur in this example. Aggregate cost refers to the case where the ECB has (11) as its objective function, national cost to (12).

In this symmetric case we recognise from figure 1 and table 2 the features that we have derived analytically in section 4. In the case the ECB is using aggregate variables in its objective function, we know that the adjustment speed and losses in the Pareto and fiscal coalition form will coincide. Both equilibria are sustainable in this case, whereas the coalitions (1, E) and (2, E) are unsustainable. One reason for this last finding is likely to be the fact that the countries and the ECB have very different objectives. Firstly, the ECB is here oriented towards stabilisation of EMU aggregate fluctuations and, secondly, it cares more about inflation than output stabilisation. The fiscal players on the other hand are only interested in stabilisation of their own economy and attach a larger weight to output than to inflation stabilisation. The coalitions between a fiscal player and the ECB result in a slow adjustment speed, a feature that we will notice also in the other cases. This is also suggestive for the possible inefficient policies that may result when these coalitions are chosen. For the case of the loss function (12) of the ECB, we know from lemma 3 that the adjustment speed (measured by the size of the a_is) is fastest under fiscal cooperation, which is in addition an internally supported equilibrium in the sym-

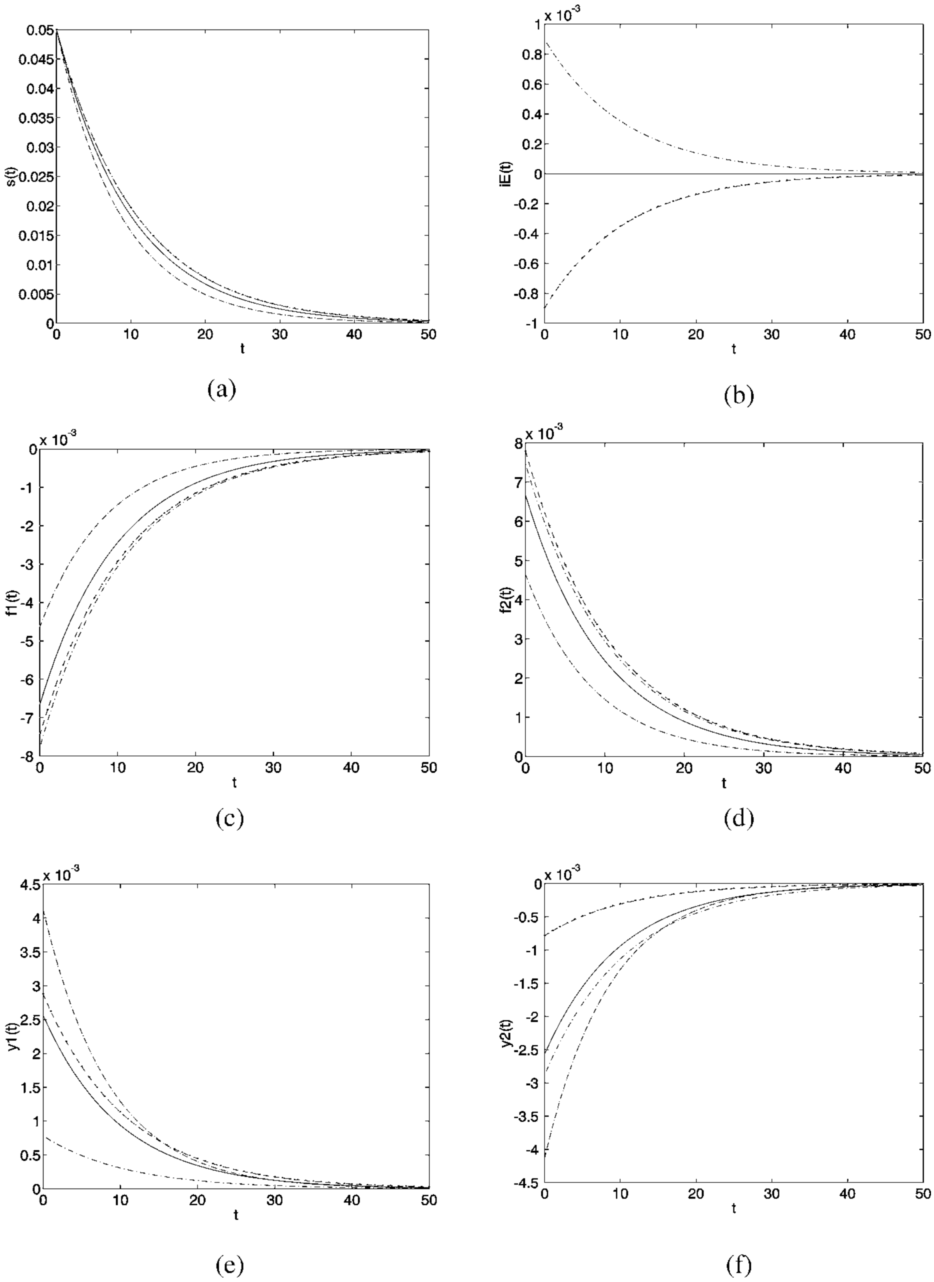

Figure 1. Symmetric baseline. Aggregate objective function ECB. – Nash, - - - Pareto, $\cdots$ $(1, 2)$, $---$ $(1, E)$, $- \cdot -$ $(2, E)$.

Table 2
Cost functions baseline (multiplied by 1,000).

	Aggregate cost					National cost				
	Nash	Pareto	(1, 2)	(1, E)	(2, E)	Nash	Pareto	(1, 2)	(1, E)	(2, E)
J_1	0.3596	0.3032	0.3032	0.4145	2.4911	0.3596	0.3040	0.3032	0.4169	2.4909
J_2	0.3596	0.3032	0.3032	2.4911	0.4145	0.3596	0.3040	0.3032	2.4909	0.4169
J_E	0	0	0	0.0088	0.0088	0.0189	0.0390	0.0423	0.0188	0.0188
a_{cl}	0.1007	0.1162	0.1162	0.0933	0.0933	0.1007	0.1143	0.1162	0.0932	0.0932

metric case. Note that both the (1, E) and (2, E) coalitions are supported by the ECB and that the Pareto coalition is unsustainable (conform theorem 4).

5.2. *Asymmetric fiscal policy preferences*

An important form of asymmetry that is likely to arise in EMU are different objectives of the fiscal authorities of the participating countries. In this second example we analyse the consequences of such asymmetries in preferences. To do so, assume that the fiscal authority of the second country has now a higher preference for output stability than country 2: $\beta_1 = 5$ and $\beta_2 = 10$.[7] In that case the following adjustment patterns result.

Optimal policies and adjustment are quite different from the baseline case and also between the two different objective functions of the ECB. With a larger desire to reduce output fluctuations, country 2 wants to use its instrument with a larger intensity. However, a larger use increases the instrument costs and implies costs also for country 1 who prefers a less active stabilisation policy of country 2. Table 3 indeed indicates that the fiscal players have larger lossses in the Nash case compared to the symmetric case. In the fiscal coalition case, country 1 is forced to share in the larger adjustment needs of country 2: it pursues a less active fiscal stabilisation policy since its fiscal surpluses also negatively affect country 2. In case the ECB participates in a coalition (i.e., in the Pareto, the (1, E) and (2, E)) and has aggregate variables in its objective function, it also shares in the increased stabilisation problem of country 2 (and the reduced problems of country 1): it sets a low interest rate which helps the stabilisation of output in country 2. This, however, at the cost of country 1 for whom this policy is counterproductive. In case national variables are featuring in the objective function of the ECB, on the other hand, the ECB sets a restrictive interest policy, this reduces inflation in country 2 but also the amount of output stabilisation in that country.

Unfortunately, coalitions are unlikely to offer a solution here. The reason is that there are no internally supported coalitions in this case: in the case (7a) is the objective function of the ECB, country 2 would prefer the Pareto case over the fiscal coalition and in case (7b) is the objective function of the ECB, country 1 would prefer the Pareto case over the fiscal coalition. In both cases, the ECB, however, does not support the Pareto policy design. Therefore, the Nash case remains the only likely outcome given that we

[7] In Engwerda et al. [8] and Engwerda [9], we experimented extensively with variations of the the χ parameter of the fiscal players which measures the preference for deficit smoothing.

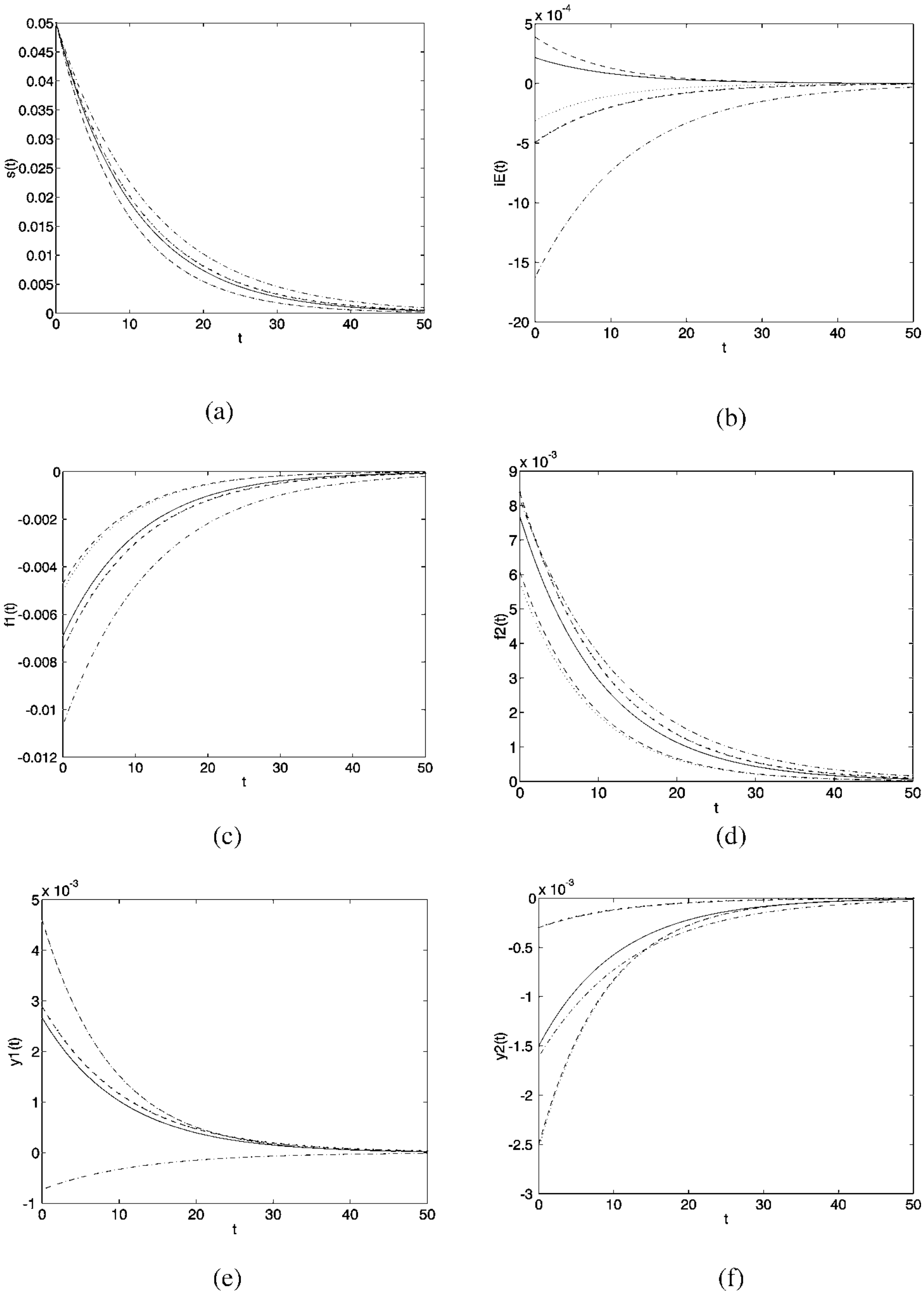

Figure 2. Asymmetric policy preferences, $\beta_1 = 5$, $\beta_2 = 10$. Aggregate objective function ECB. – Nash, - - - Pareto, $\cdots$ (1, 2), — — — (1, E), — · — (2, E).

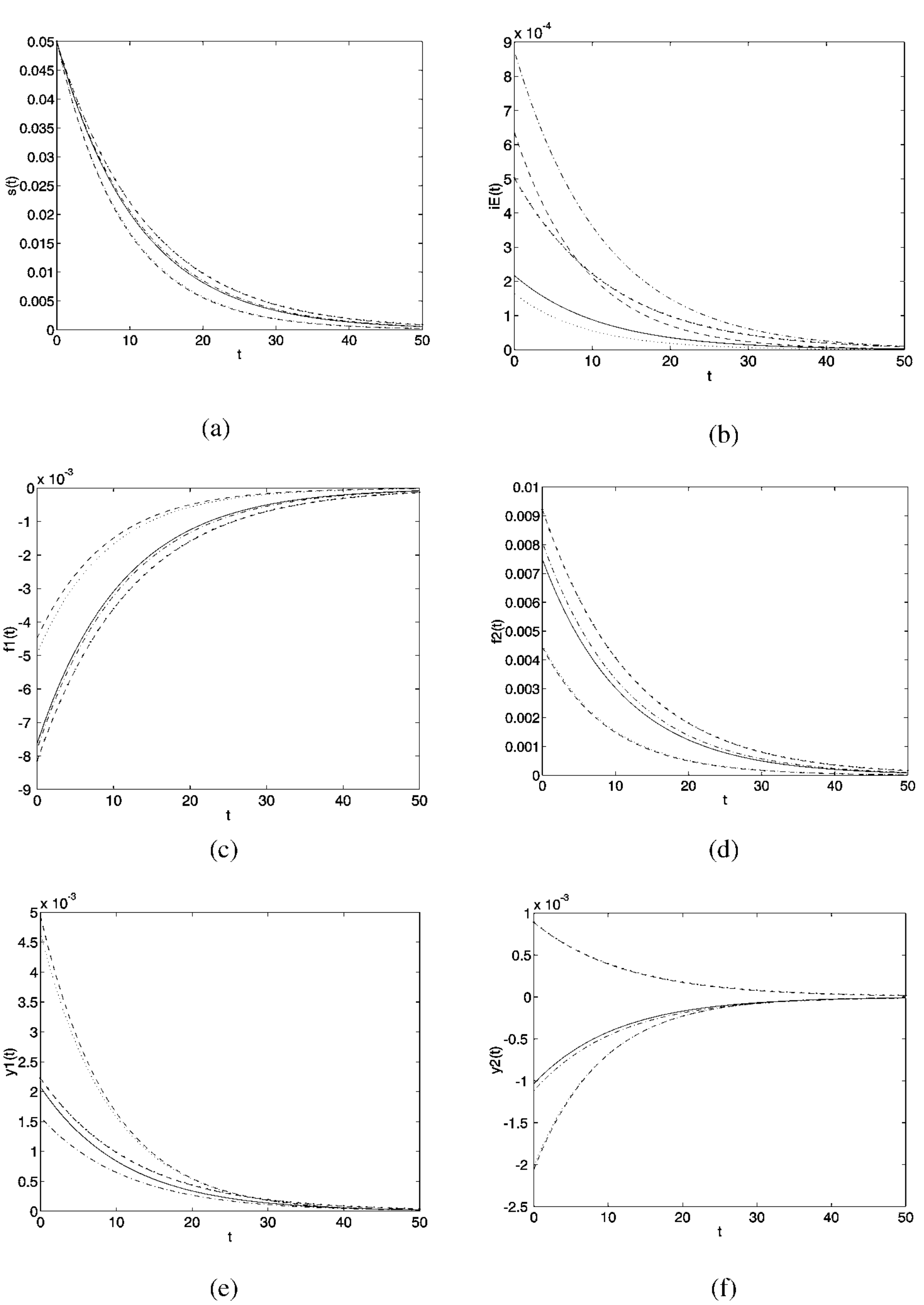

Figure 3. Asymmetric policy preferences, $\beta_1 = 5$, $\beta_2 = 10$. National objective function ECB. – Nash, - - - Pareto, $\cdots$ (1, 2), $---$ (1, E), $- \cdot -$ (2, E).

Table 3
Cost functions asymmetric fiscal policy preferences.

	Aggregate cost					National cost				
	Nash	Pareto	(1, 2)	(1, E)	(2, E)	Nash	Pareto	(1, 2)	(1, E)	(2, E)
J_1	0.4070	0.3842	0.3707	0.9028	2.5718	0.4628	0.3973	0.3981	0.4645	3.0045
J_2	0.4432	0.3343	0.3511	6.1842	0.4848	0.4175	0.2127	0.2100	5.0387	0.4169
J_E	0.0013	0.0033	0.0038	0.0257	0.0070	0.0089	0.0346	0.0399	0.0115	0.0188
a_{cl}	0.0958	0.1107	0.1105	0.0794	0.0909	0.0906	0.1084	0.1100	0.0884	0.0932

Table 4
Costs with asymmetric monetary policy transmission.

	Aggregate cost					National cost				
	Nash	Pareto	(1, 2)	(1, E)	(2, E)	Nash	Pareto	(1, 2)	(1, E)	(2, E)
J_1	0.3697	0.3437	0.3065	0.5596	3.1561	0.3759	0.3483	0.3140	0.5608	3.1560
J_2	0.3919	0.2596	0.3221	1.8148	0.3405	0.3812	0.2557	0.3084	1.8107	0.3405
J_E	0	0.0091	0	0.0217	0.0502	0.0174	0.0430	0.0430	0.0301	0.0569
a_{cl}	0.0994	0.1155	0.1165	0.0910	0.0894	0.0994	0.1135	0.1165	0.0909	0.0894

had excluded any side-payments in the coalition formation problem or any other binding arrangement to sustain coalitions of policymakers.

5.3. Asymmetric monetary policy transmission

In this example asymmetric monetary transmission is analysed: the baseline setting is again assumed, except that the first country has a smaller output semi-elasticity of the real interest rate ($\gamma_1 = 0.4$) than the second country ($\gamma_2 = 0.8$). This example is useful to illustrate the important discussion about the effects of a common monetary policy in a situation where countries differ in the transmission of monetary policy. Figures 4 displays the resulting adjustments. Since adjustment with EMU aggregate and national variables in the ECB loss function is rather similar, we depict only the first case here.

In this asymmetric setting the adjustment and policy strategies are not symmetric in both countries, although the deviations from the symmetric baseline case are not as large as in the previous example. The ECB now reacts in all strategic settings as its objective functions imply that its optimal strategy is sensitive to any asymmetry. Because the economy of country 2 is more sensitive to the common monetary policy, the monetary policy of the ECB is more directed to stabilisation in country 2, in particular when the ECB enters into a coalition with country 2. Table 4 shows the losses in this case.

We observe that there is no sustainable coalition neither for the aggregate variables nor for the national variables case. Without side payments or other institutional arrangements that would make a coalition binding, we would find the Nash case to be the likely outcome. We further observe that for both welfare loss functions the Pareto coalition form is supported by player 2, whereas player 1 supports the governments' coalition form. Furthermore we see that both other partial equilibrium forms are unsus-

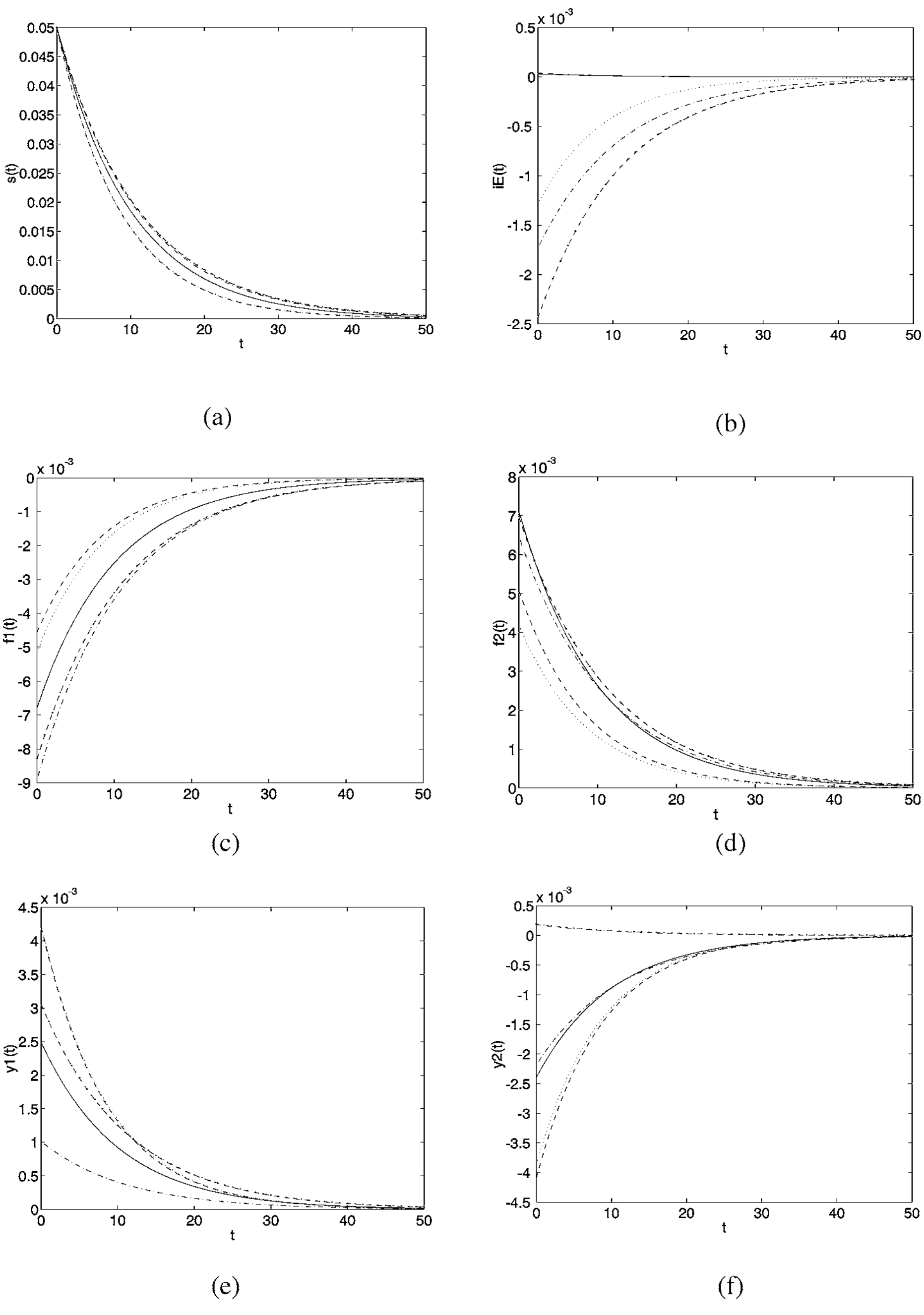

Figure 4. Asymmetric monetary transmission, $\gamma_1 = 0.4$, $\gamma_2 = 0.8$. Aggregate objective function ECB. – Nash, - - - Pareto, $\cdots$ (1, 2), $- - -$ (1, E), $- \cdot -$ (2, E).

tainable. Note that the fiscal coalition is now only second best for country 2. Because of its stronger exposure now to the monetary policy of the ECB, country 2 would definitely prefer the ECB to be included into the policy cooperation. However, the ECB is not benefiting from full cooperation, it incurs a relatively large cost by joining this coalition form (in the aggregate case).

5.4. Asymmetric bargaining powers

In cases with policy coordination, bargaining strenghts of players become important, since these will determine too how collective decision making will be influenced by the objectives of the players in the coalition. This will therefore have important consequences for macroeconomic policy formulation and adjustment in the EMU. This example, therefore, analyses the effects of different bargaining powers under cooperative policymaking in the EMU. It assumes the following scheme of bargaining powers:

$$\tau^c = \left\{\frac{3}{6}, \frac{1}{6}, \frac{2}{6}\right\}, \quad \tau^{(1,2)} = \left\{\frac{3}{4}, \frac{1}{4}\right\}, \quad \tau^{(1,E)} = \left\{\frac{3}{5}, \frac{2}{5}\right\}, \quad \tau^{(2,E)} = \left\{\frac{1}{3}, \frac{2}{3}\right\},$$

implying that in a coalition country 1 has three times as many votes as country 2 and 1,5 as many votes as the ECB, whereas the ECB has two times as many votes as country 2. This asymmetric bargaining power case leads to the following adjustment dynamics.

The Nash case is not affected by the different bargaining strengths as it implies entirely non-cooperative policy design. In the other cases, the balance of power is turning against country 2 in particular when it enters the fiscal coalition and a coalition with the ECB. In those cases, it faces a larger adjustment burden and contributing more to the stabilisation burden of the other coalition partner. In the case it acts non-cooperatively against the coalition of country 1 and the ECB, the adverse effects for country 2 are much less. In the Pareto case it is helped by an expansionary monetary policy of the ECB.

In this case player 1 supports the Pareto coalition form, since it is very powerful there. Furthermore, all coalition forms are unsustainable. Since none of the coalitions is supported by more than one player, the Nash outcome might be the ultimate outcome in this case. Therefore, comparing the results of table 5 with that of table 2, we observe that the introduction of asymmetric bargaining powers crucially changes the results of the game. The asymmetry increases the cost of the country with the smaller bargaining

Table 5
Cost with asymmetric bargaining powers.

	Aggregate cost					National cost				
	Nash	Pareto	(1, 2)	(1, E)	(2, E)	Nash	Pareto	(1, 2)	(1, E)	(2, E)
J_1	0.3596	0.1866	0.2266	0.3724	2.4267	0.3596	0.2534	0.2555	0.4663	2.4911
J_2	0.3596	0.4759	0.4546	2.6170	0.5062	0.3596	0.3672	0.3755	2.4302	0.5900
J_E	0	0.0213	0.0078	0.0222	0.0047	0.0189	0.0416	0.0486	0.0134	0.0151
a_{cl}	0.1007	0.1199	0.1215	0.0940	0.0910	0.1007	0.1135	0.1183	0.0921	0.0888

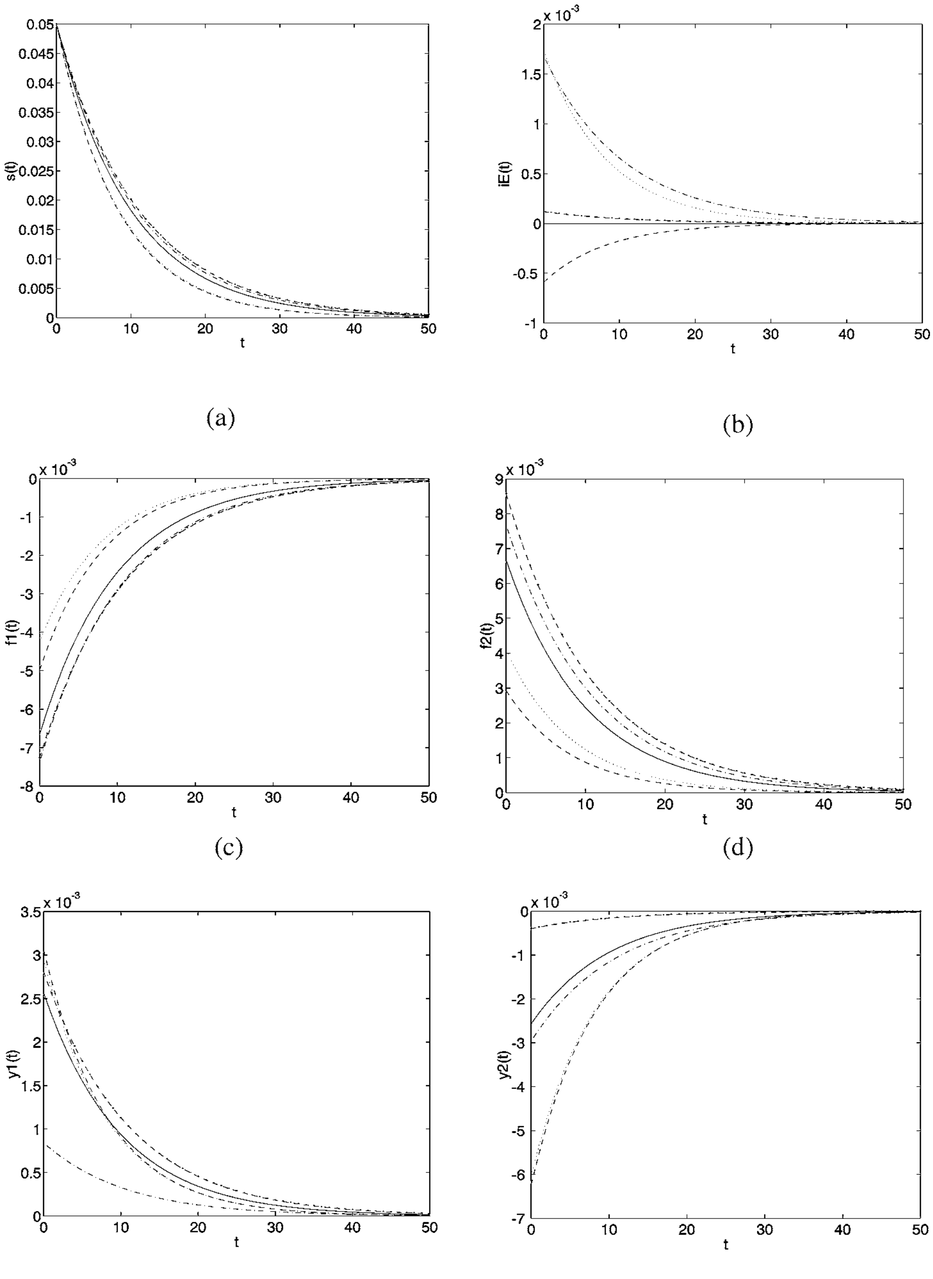

Figure 5. Asymmetric bargaining weights. Aggregate objective function ECB. – Nash, - - - Pareto, $\cdots$ (1, 2), $---$ $(1, E)$, $-\cdot-$ $(2, E)$.

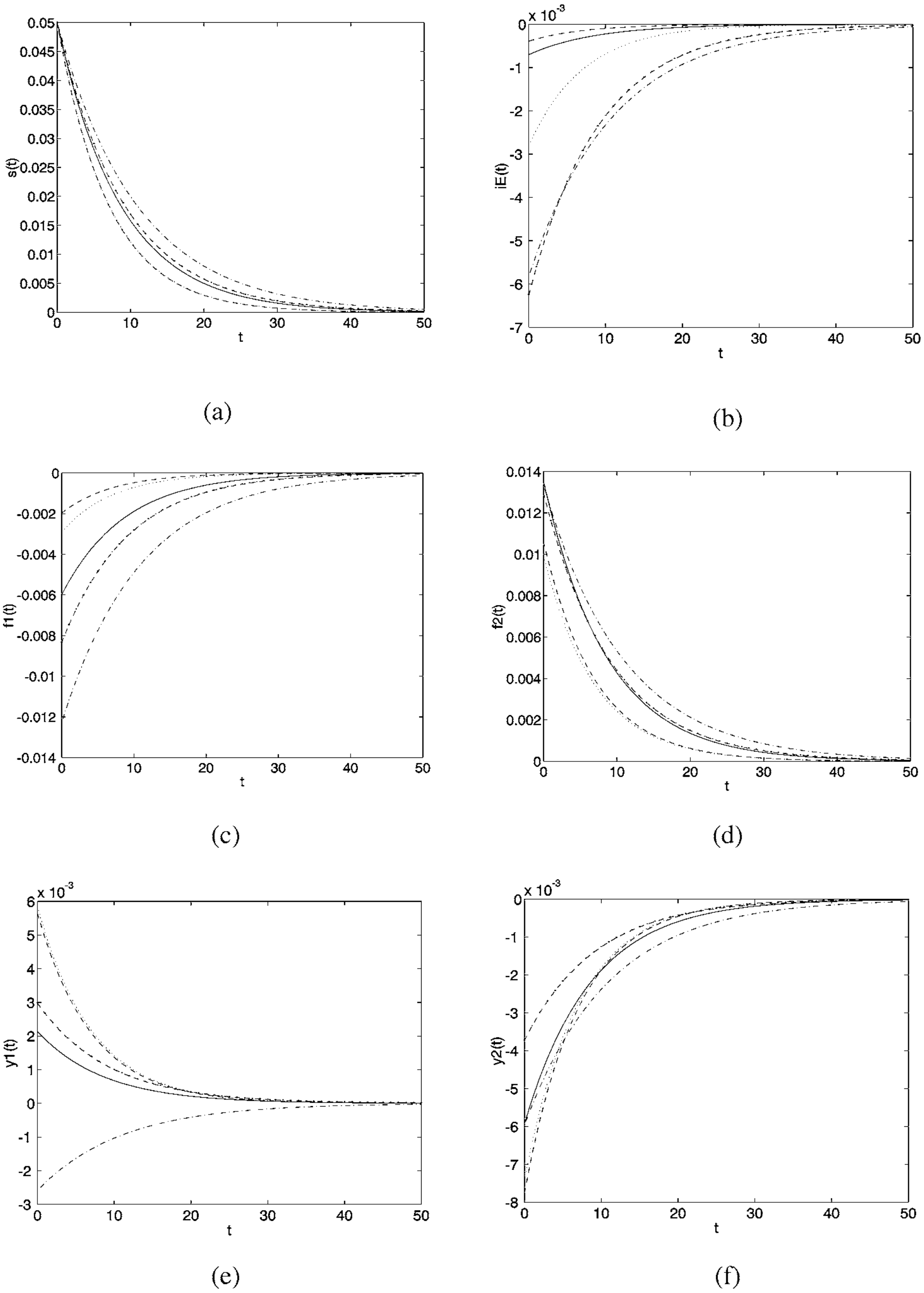

Figure 6. Asymmetric sensitivity to intra-EMU competitiveness, $\delta_1 = 0.2$, $\delta_2 = 0.4$. Aggregate objective function ECB. – Nash, - - - Pareto, $\cdots$ (1, 2), $- - -$ (1, E), $- \cdot -$ (2, E).

Table 6
Costs with asymmetric competitiveness.

	Aggregate cost					National cost				
	Nash	Pareto	$(1, 2)$	$(1, E)$	$(2, E)$	Nash	Pareto	$(1, 2)$	$(1, E)$	$(2, E)$
J_1	0.2449	0.3479	0.3050	1.1165	2.5845	0.2449	0.3347	0.3050	1.0995	2.5837
J_2	1.3824	0.9283	1.0375	8.7428	1.1306	1.3824	0.9437	1.0375	8.7341	1.1381
J_E	0.0117	0.0359	0.0029	0.2877	0.2253	0.0524	0.1202	0.0944	0.2898	0.2509
a_{cl}	0.1153	0.1412	0.1420	0.0919	0.1085	0.1153	0.1382	0.1420	0.0921	0.1083

power as its importance in a coalition is reduced, while it decreases the costs of the other country. To put it in a general way: more asymmetric bargaining powers reduce the probabilities of coalitions – and therefore of policy cooperation – as policies will be biased towards the needs of the stronger player(s), and the smaller players are less likely to stay in such "asymmetric" coalitions. This last result differs from that found in Hughes Hallett and Ma [12] in analysing the full coordination problem.

5.5. *Asymmetric degree of competitiveness*

Next, we assume that the first country's output elasticity of competitiveness is lower ($\delta_1 = 0.2$) than that of the second country ($\delta_2 = 0.4$). Such an asymmetry in the sensitiveness to competitive pressures has quite a dramatic impact as a comparison of figures 1 and 6 shows.

In this case there are no marked differences between the cases where the ECB objectives are governed by aggregate variables and where it is governed by national variables and therefore only the first case is displayed. Country 2 is now in a more disadvantaged position than in the baseline case: its higher sensitivity to the intra-EMU competitiveness variable imply that it faces a deeper recession and a higher fiscal stabilisation burden. The reduction of interest rates by the ECB that occurs in all cases is helpful to stabilise country 2 but inadequate from the perspective of country 1. Table 6 gives the losses that the players incur in this case.

In both cases no coalition form is internally supported and its emergence is therefore hard to sustain without any form of other binding element. Coalitions $(1, E)$ and $(2, E)$ are unsustainable, while in the aggregate case the coalition $(1, 2)$ is externally supported by the ECB.

6. Conclusion

Macroeconomic policy cooperation is a crucial issue in a highly integrated economic and political union such as the European Union. To study the effects of policy cooperation in a two-country model for the EMU we compared the optimal policies and the effects of five alternative policy regimes under a stylized model of the EMU:

(i) non-cooperative monetary and fiscal policies,

(ii) three partial cooperative schemes and

(iii) full cooperation.

Using numerical examples, we illustrated the sometimes complex effects that are produced by the various coalitions. We found that the sustainability of a certain type of coalition and its implications for the optimal strategies and the resulting macroeconomic adjustment, is highly sensitive to initial settings of preferences and the structural model parameters. Cooperation is often efficient for the fiscal players and, moreover, we saw that the fiscal players' cooperation (against the ECB) often leads to a Pareto improvement for them, provided that they are not very asymmetric in preferences, structural characteristics and bargaining strenght. The non-cooperative Nash equilibrium is most likely to be the outcome when countries are more asymmetric. In most simulations full cooperation does not induce a Pareto improvement for the ECB, while the governments' coalitions imply a considerable loss for the ECB compared to the non-cooperative and full cooperative cases. That implies that the Pareto form is often unsustainable. This was also shown theoretically for the symmetric case. Cases that the ECB cooperates with one government against the other, generally produces suboptimal monetary and fiscal stabilisation policies.

Finally, considering current European discussions, it is found that the ECB has a rational to pursue an institutional design that does not enforce cooperation and let to the monetary authority a high degree of independence. Therefore, the ECB will try to promote fixed rules for European policy targets. On the other hand, governments may pursue a design based on cooperation and which that leave them independent in cooperating their policies with the monetary policy of the ECB.

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Appendix A

A.1. The non-cooperative case

With $A := \phi_4,\ B_1 := -\phi_1,\ B_2 := \phi_2$ and $B_3 := \phi_3$ the system is described by

$$\dot{s}(t) = As(t) + B_1 f_1(t) + B_2 f_2(t) + B_3 i_E(t), \quad s(0) = s_0,$$

and with $x := (s\ f_1\ f_2\ i_E)$ the performance criterion of player i can be rewritten as $\int_0^\infty x(t)M_i x(t)\,\mathrm{d}t,\ i \in \{1, 2, E\}$. The non-cooperative Nash solution is found as follows (see Engwerda et al. [8] for details (note that we present here the general multivariable algorithm, whereas in our case many variables are scalar)):

1. Factorize M_i as follows

$$M_i =: \begin{pmatrix} Q_i & P_i & L_i & S_i \\ P_i^{\mathrm{T}} & R_{1i} & N_i & T_i \\ L_i^{\mathrm{T}} & N_i^{\mathrm{T}} & R_{2i} & V_i \\ S_i^{\mathrm{T}} & T_i^{\mathrm{T}} & V_i^{\mathrm{T}} & R_{3i} \end{pmatrix},$$

   where all entries are scalars.

2. Calculate

$$G := \begin{pmatrix} R_{11} & N_1 & T_1 \\ N_2^{\mathrm{T}} & R_{22} & V_2 \\ T_E^{\mathrm{T}} & V_E^{\mathrm{T}} & R_{3E} \end{pmatrix}$$

   and

$$M := \begin{pmatrix} -A & 0 & 0 & 0 \\ Q_1 & A^{\mathrm{T}} & 0 & 0 \\ Q_2 & 0 & A^{\mathrm{T}} & 0 \\ Q_E & 0 & 0 & A^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} B \\ -\widetilde{P}_1 \\ -\widetilde{P}_2 \\ -\widetilde{P}_E \end{pmatrix} G^{-1} \begin{pmatrix} P_1^{\mathrm{T}} & B_1^{\mathrm{T}} & 0 & 0 \\ L_2^{\mathrm{T}} & 0 & B_2^{\mathrm{T}} & 0 \\ S_E^{\mathrm{T}} & 0 & 0 & B_3^{\mathrm{T}} \end{pmatrix}.$$

   Here $B := (B_1\ B_2\ B_3)$ and $\widetilde{P}_i := (P_i\ L_i\ S_i),\ i \in \{1, 2, E\}$.

3. Calculate the positive eigenvalue(s) of M. If a_{nc} is a positive eigenvalue and $v =: (v_0\ v_1\ v_2\ v_3)^{\mathrm{T}}$ a corresponding eigenvector then, generically (see Engwerda et al. [8] for details), the equilibrium strategies are

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^{\mathrm{T}} + B_1^{\mathrm{T}} K_1 \\ L_2^{\mathrm{T}} + B_2^{\mathrm{T}} K_2 \\ S_E^{\mathrm{T}} + B_3^{\mathrm{T}} K_3 \end{pmatrix} s(t),$$

   where $K_i := v_i/v_0$. Using these equilibrium strategies the resulting system is described by $\dot{s}(t) = -a_{nc}s(t),\ s(0) = s_0$.

A.2. The cooperative case

To determine the cooperative strategies for this model we consider: $J^C := \tau_1 J_1 + \tau_2 J_2 + \tau_3 J_E$ with $\tau_1 + \tau_2 + \tau_3 = 1$. Introducing $\mu_1 := d_1\tau_1,\ \mu_2 := d_2\tau_2$ and $\mu_3^i := d_E^i\tau_3$, then

$$J_C^i = \int_0^\infty \left\{x^{\mathrm{T}}(t)M_C^i x(t)\right\} \mathrm{d}t,$$

where $M_C^i := \mu_1 M_1 + \mu_2 M_2 + \mu_3^i M_E^i,\ i \in \{A, N\}$.

With the notation of appendix A.1 the unique equilibrium strategies are then obtained as follows (see, e.g., Lancaster et al. [13]):

1. Factorise matrix M_C^i as

$$\begin{pmatrix} Q & S \\ S^{\mathrm{T}} & R \end{pmatrix},$$

where Q is a scalar; S a 1×3 matrix and R a 3×3 matrix.

2. Calculate the Hamiltonian matrix

$$Ham := \begin{pmatrix} -(A - BR^{-1}S^{\mathrm{T}}) & BR^{-1}B^{\mathrm{T}} \\ Q - SR^{-1}S^{\mathrm{T}} & (A - BR^{-1}S^{\mathrm{T}})^{\mathrm{T}} \end{pmatrix}.$$

3. Determine the positive eigenvalue a_c of Ham and its corresponding eigenvector $v =: (v_0\ v_1)^{\mathrm{T}}$. Calculate $K := v_1/v_0$.

Then the unique equilibrium strategies are

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = -R^{-1}\big(S^{\mathrm{T}} + B^{\mathrm{T}}K\big)s(t) =: H_c s(t),$$

and the resulting system satisfies $\dot{s}(t) = -a_c s(t),\ s(0) = s_0$.

A.3. Partial coalitions

We will elaborate the $(1, E)$ coalition. The results for the other cases are obtained similarly by considering the permutation matrix obtained from the appropriate redefined state $x(t)$.

First note that the inverse of the permutation matrix $P_{(1,E)}$ is $P_{(1,E)}^{\mathrm{T}}$ so that $x(t) = P_{(1,E)}^{\mathrm{T}}\tilde{x}(t)$. Then, with $M_i^{(1,E)} := P_{(1,E)}M_i P_{(1,E)}^{\mathrm{T}},\ i \in \{1, 2, E\}$, we find $M_{(1,E),c} := \tau_1 d_1 M_1^{(1,E)} + \tau_2 M_E^{(1,E)}$, so that

$$J_{(1,E)} = \int_0^\infty \big\{\tilde{x}^{\mathrm{T}}(t)M_{(1,E),c}\tilde{x}(t)\big\}\,\mathrm{d}t.$$

Next, introduce $B_1 := (-\phi_1\ \phi_3),\ B_2 := \phi_2$ and $B := (B_1\ B_2)$.

Then, apply step 1 and 2 described in the above appendix A.1 for the two-player case to find a corresponding matrix $M_{(1,E)}$.

Determine the positive eigenvalue(s) of $M_{(1,E)}$. If $a_{(1,E)}$ is a positive eigenvalue of this matrix and $v =: (v_0\ v_1\ v_2)$ a corresponding eigenvector, then (generically) an equilibrium strategy is

$$\begin{pmatrix} f_1(t) \\ i_E(t) \\ f_2(t) \end{pmatrix} = -G^{-1}\begin{pmatrix} P_1^{\mathrm{T}} + B_1^{\mathrm{T}}K_1 \\ L_2^{\mathrm{T}} + B_2^{\mathrm{T}}K_2 \end{pmatrix} s(t) =: H_{(1,E)}s(t),$$

where $K_i := v_i/v_0$. The resulting system is then $\dot{s}(t) = -a_{(1,E)}s(t),\ s(0) = s_0$.

Appendix B

B.1. The non-cooperative case

Note that for the determination of the optimal strategies the scaling parameters in the performance criteria are irrelevant in this case. Therefore, we assume $d_i = 1$. First, we consider the national performance criterion J_E^N.

To determine the equilibrium strategies we have to calculate the eigenvalues and eigenvectors of the corresponding matrix M (see appendix A.1). By substitution of the various parameters (with $g_E := g_E^N$) we obtain

$$G := \begin{pmatrix} a^2+g & ae & -ac \\ ae & a^2+g & -ac \\ -c(a+e) & -c(a+e) & 2c^2+g_E \end{pmatrix}.$$

Elementary calculations show that the determinant of G, det, equals $u_{1nc}u_{nc}$. Moreover, $det * G^{-1}$ equals

$$\begin{pmatrix} g(2c^2+g_E)+a(ag_E+c^2(a-e)) & a(c^2(a-e)-eg_E) & acu_{nc} \\ a(c^2(a-e)-eg_E) & g(2c^2+g_E)+a(ag_E+c^2(a-e)) & acu_{nc} \\ c(a+e)u_{nc} & c(a+e)u_{nc} & u_{nc}(a^2+ae+g) \end{pmatrix}.$$

Consequently, matrix M satisfies

$$det * M = \begin{pmatrix} -\phi_4 det - 2ab\phi_1 u_{1nc} & \phi_1^2 u_{1nc} & \phi_1^2 u_{1nc} & 0 \\ b^2 g u_{1nc} & \phi_4 det + b\phi_1 u_{2nc} & b\phi_1 g u_{3nc} & 0 \\ b^2 g u_{1nc} & b\phi_1 g u_{3nc} & \phi_4 det + b\phi_1 u_{2nc} & 0 \\ 2b^2 g u_{1nc} & b\phi_1(a-e)u_{1nc} & b\phi_1(a-e)u_{1nc} & \phi_4 det \end{pmatrix},$$

where we used the shorthand notations $u_{2nc} := a^3 g_E + ac^2 g + agg_E - e^2 ag_E - c^2 ge$ and $u_{3nc} := -c^2 e - g_E e + ac^2$. Note that $u_{2nc} + gu_{3nc} = (a-e)u_{1nc}$, a relationship which is useful in elaborating details. The structure of matrix $det * M$ is:

$$M1 = \begin{pmatrix} \tilde{a} & \tilde{c} & \tilde{c} & 0 \\ \tilde{b} & \tilde{d} & \tilde{e} & 0 \\ \tilde{b} & \tilde{e} & \tilde{d} & 0 \\ 2\tilde{b} & \tilde{f} & \tilde{f} & \tilde{g} \end{pmatrix}$$

The eigenvalues of $M1$ are

$$\tilde{g}, \quad \tilde{d}-\tilde{e}, \quad \frac{1}{2}(\tilde{a}+\tilde{d}+\tilde{e}) - \frac{1}{2}\sqrt{(\tilde{a}-\tilde{d}-\tilde{e})^2 + 8\tilde{b}\tilde{c}} \quad \text{and}$$
$$\frac{1}{2}(\tilde{a}+\tilde{d}+\tilde{e}) + \frac{1}{2}\sqrt{(\tilde{a}-\tilde{d}-\tilde{e})^2 + 8\tilde{b}\tilde{c}}.$$

Note that the eigenvector corresponding to the eigenvalue $\tilde{d}-\tilde{e}$ is $(0\ 1\ -1\ 0)^{\mathrm{T}}$, which therefore does not satisfy the additional requirements for generating an equilibrium. Furthermore, $\tilde{g} < 0$. Consequently, the game has at most two different equilibria. Given the parametric restrictions, it is easily verified that if $r := \rho_1/k_1 < 1$ (which implies that $0 < e < a$) $\tilde{a} + \tilde{d} + \tilde{e}$ is negative and

$$\lambda := \frac{1}{2}(\tilde{a} + \tilde{d} + \tilde{e}) + \frac{1}{2}\sqrt{(\tilde{a} - \tilde{d} - \tilde{e})^2 + 8\tilde{b}\tilde{c}} > 0.$$

So, under this assumption there is a unique equilibrium. A more detailed look at the eigenvalues shows that they coincide with the relevant eigenvalues (with $\mu = 1$ and $\theta = 0$) reported in Engwerda et al. [9]. So, the results obtained there apply here. In particular we have that whenever there is only one appropriate positive eigenvalue, this eigenvalue is given by λ. The corresponding eigenvector is:

$$\begin{pmatrix} -(\tilde{e} + \tilde{d} - \lambda)/\tilde{b} \\ 1 \\ 1 \\ -2(\tilde{f} - \tilde{d} - \tilde{e} + \lambda)/(\tilde{g} - \lambda) \end{pmatrix}.$$

Substitution of the corresponding parameters from M shows that

$$\begin{aligned} \lambda &:= -\frac{1}{2}b\phi_1(a+e)u_{1nc} + \frac{1}{2}\sqrt{\left(-2\phi_4 det - b\phi_1(3a-e)u_{1nc}\right)^2 + 8gb^2\phi_1^2u_{1nc}^2} \\ &= \frac{1}{2}u_{1nc}\left\{-b\phi_1(a+e) + \sqrt{\left(-2\phi_4 u_{nc} - b\phi_1(3a-e)\right)^2 + 8gb^2\phi_1^2}\right\}. \end{aligned}$$

Consequently, the eigenvalue of M we are looking for is λ/det and

$$\begin{aligned} K &:= K_1 = K_2 \\ &= \frac{2b^2g}{-2\phi_4u_{nc} - b\phi_1(3a-e) + \sqrt{\left(-2\phi_4u_{nc} - b\phi_1(3a-e)\right)^2 + 8gb^2\phi_1^2}}. \end{aligned}$$

Using this, the rest of the claims follow straightforwardly.

Next, consider the aggregate performance criterion J_E^A.

Substitution of the various parameters into appendix A.1 shows that, except for the entries (4,1), (4,2) and (4,3) which are now zero, matrix M in step (ii) of the algorithm coincides with the matrix M we determined above for the national performance case. Therefore, it is easily verified that the equilibrium strategies coincide. As a consequence, the resulting systems coincide too.

B.2. The cooperative case

First we consider again the national performance case. Let $d_N := \tau + (1 - 2\tau)(d_E^N/d)$. After substitution of the parameters we see that matrix $M_{C,N}$ (see appendix A.2) is given by

$$M_C^N = d\begin{pmatrix} 2d_Nb^2 & d_Nb(a-e) & -d_Nb(a-e) & 0 \\ d_Nb(a-e) & d_N(a^2+e^2)+\tau g & 2d_Nae & -d_Nc(a+e) \\ -d_Nb(a-e) & 2d_Nae & d_N(a^2+e^2)+\tau g & -d_Nc(a+e) \\ 0 & -d_Nc(a+e) & -d_Nc(a+e) & 2d_Nc^2 + (d_N - \tau)g_E \end{pmatrix}.$$

So, with

$$A := \phi_4, \quad B := \phi_1(-1\ 1\ 0), \quad Q := 2d_N b^2, \quad S := d_N b(a-e)(1\ -1\ 0)$$

and

$$R := \begin{pmatrix} d_N(a^2+e^2)+\tau g & 2d_N ae & -d_N c(a+e) \\ 2d_N ae & d_N(a^2+e^2)+\tau g & -d_N c(a+e) \\ -d_N c(a+e) & -d_N c(a+e) & 2d_N c^2+(d_N-\tau)g_E \end{pmatrix}$$

we can calculate the Hamiltonian of the system *Ham*. Since all entries of this matrix are scalar, it is easily verified that the positive eigenvalue equals

$$a_{cN} := \sqrt{\left(A - BR^{-1}S^{\mathrm{T}}\right)^2 + BR^{-1}B^{\mathrm{T}}\left(Q - SR^{-1}S^{\mathrm{T}}\right)}$$

and its corresponding eigenvector

$$\begin{pmatrix} BR^{-1}B^{\mathrm{T}} \\ A - BR^{-1}S^{\mathrm{T}} + a_{cN} \end{pmatrix}.$$

From this, K_{cN} immediately results. Appart from the determination of the inverse of R things can be calculated straightforwardly now. Therefore, we conclude this part of the subsection with the exposition of matrix R^{-1} (from which the verification of correctness is left to the reader). Introducing

$$u_{1cN} := (d_N-\tau)\left(\tau g g_E + (a+e)^2 d_N g_E\right) + 2\tau g d_N c^2,$$
$$u_{2cN} := d_N\left(d_N c^2(a-e)^2 - 2aeg_E(d_N-\tau)\right)$$

and the determinant of R, $det := u_{1cN}u_{cN}$, we have

$$det * R^{-1} = \begin{pmatrix} u_{1cN}+u_{2cN} & u_{2cN} & (a+e)cd_N u_{cN} \\ u_{2cN} & u_{1cN}+u_{2cN} & (a+e)cd_N u_{cN} \\ (a+e)cd_N u_{cN} & (a+e)cd_N u_{cN} & (d_N(a+e)^2+\tau g)u_{cN} \end{pmatrix}.$$

Next, we consider the aggregate performance case. Let $d_A := (1-2\tau)(d_E^A/d)$. After substitution of the parameters we see that matrix M_C^A (see appendix A.2) is given by

$$\begin{pmatrix} 2\tau b^2 & \tau b(a-e) & -\tau b(a-e) & 0 \\ \tau b(a-e) & \tau(a^2+e^2+g)+d_A(a+e)^2 & 2\tau ae + d_A(a+e)^2 & -c(a+e)(\tau+2d_A) \\ -\tau b(a-e) & 2\tau ae + d_A(a+e)^2 & \tau(a^2+e^2+g)+d_A(a+e)^2 & -c(a+e)(\tau+2d_A) \\ 0 & -c(a+e)(\tau+2d_A) & -c(a+e)(\tau+2d_A) & 2\tau c^2 + d_A(4c^2+2g_E^A) \end{pmatrix}.$$

From this similar as in the national case the matrices Q, S and R result. Introducing

$$u_{1cA} := \tau(\tau+2d_A)gc^2 + d_A g_E^A\left(\tau g + (\tau+2d_A)(a+e)^2\right),$$
$$u_{2cA} := \tau c^2(\tau+2d_A)(a-e)^2 - 2d_A g_E^A\left(2\tau ae + d_A(a+e)^2\right)$$

and $u_{cA} := \tau(g+(a-e)^2)$ the determinant of R, *det*, is $2u_{1cA}u_{cA}$ and

$$det * R^{-1} = \begin{pmatrix} u_{2cA}+2u_{1cA} & u_{2cA} & (\tau+2d_A)(a+e)cu_{cA} \\ u_{2cA} & u_{2cA}+2u_{1cA} & (\tau+2d_A)(a+e)cu_{cA} \\ (\tau+2d_A)(a+e)cu_{cA} & (\tau+2d_A)(a+e)cu_{cA} & (\tau g+(a+e)^2(\tau+2d_A))u_{cA} \end{pmatrix}.$$

From this it is easily verified (using the fact that $\phi_1 = -(\phi_4(a-e))/2b$) that the Hamiltonian *Ham* equals

$$\frac{1}{g+(a-e)^2}\begin{pmatrix} -\phi_4 g & \dfrac{\frac{1}{2}\phi_4^2(a-e)^2}{\tau b^2} \\ 2\tau b^2 g & \phi_4 g \end{pmatrix}.$$

Analogous to the national case it follows from this straightforwardly that the positive eigenvalue a_{cA} is

$$\frac{\phi_4\sqrt{g}}{\sqrt{g+(a-e)^2}} \quad \text{and} \quad K_{cA} = \tau\frac{\phi_4 g + a_{cA}((a-e)^2+g)}{2\phi_1^2}.$$

B.3. The coalition form $(1,2)$

First we consider again the national case. After substitution of the parameters we see that matrix $M_{(1,2)}$ (see appendix A.3) is given by

$$M_{(1,2),c} = \frac{1}{2}\begin{pmatrix} 2b^2 & b(a-e) & -b(a-e) & 0 \\ b(a-e) & a^2+e^2+g & 2ae & -c(a+e) \\ -b(a-e) & 2ae & a^2+e^2+g & -c(a+e) \\ 0 & -c(a+e) & -c(a+e) & 2c^2 \end{pmatrix}.$$

Consequently,

$$G := \frac{1}{2}\begin{pmatrix} a^2+e^2+g & 2ae & -c(a+e) \\ 2ae & a^2+e^2+g & -c(a+e) \\ -2c(a+e) & -2c(a+e) & 4c^2+2g_E^N \end{pmatrix}. \tag{26}$$

Using the notation

$$u_{1,(1,2)} := (a+e)^2 g_E + g\big(2c^2+g_E\big) \quad \text{and}$$
$$u_{2,(1,2)} := c^2(a-e)^2 - 2aeg_E^N,$$

elementary calculations show that the determinant of G, *det*, equals $\frac{1}{4}u_{1,(1,2)}u_{(1,2)}$. Moreover, $det * G^{-1}$ equals

$$\frac{1}{2}\begin{pmatrix} u_{1,(1,2)}+2u_{2,(1,2)} & 2u_{2,(1,2)} & \frac{1}{2}u_{(1,2)}c(a+e) \\ 2u_{2,(1,2)} & u_{1,(1,2)}+2u_{2,(1,2)} & \frac{1}{2}u_{(1,2)}c(a+e) \\ u_{(1,2)}c(a+e) & u_{(1,2)}c(a+e) & u_{(1,2)}(g+(a+e)^2) \end{pmatrix},$$

Consequently,

$$det * M_{(1,2)} = \begin{pmatrix} -\phi_4 det - \frac{1}{2}\phi_1 b(a-e)u_{1,(1,2)} & \phi_1^2 u_{1,(1,2)} & 0 \\ \frac{1}{4}b^2 g u_{1,(1,2)} & \phi_4 det + \frac{1}{2}\phi_1 b(a-e)u_{1,(1,2)} & 0 \\ \dfrac{1}{2}b^2 g u_{1,(1,2)} & \phi_1 b(a-e)u_{1,(1,2)} & \phi_4 det \end{pmatrix}.$$

The eigenvalues of $M_{(1,2)}$ are ϕ_4, $a_{(1,2)}$ and $-a_{(1,2)}$, with

$$a^2_{(1,2)} := \phi_4^2 + 4\phi_1 b \frac{\phi_4(a-e) + b\phi_1}{u_{(1,2)}} = \frac{g\phi_4^2}{u_{(1,2)}}.$$

So, there is always a unique equilibrium. The with $a_{(1,2)}$ corresponding eigenvector is

$$\left(-\frac{u_{(1,2)}\phi_4 + 2\phi_1 b(a-e) - u_{(1,2)}a_{(1,2)}}{2gb^2}, \frac{1}{2}, 1\right)^{\mathrm{T}}.$$

The rest of the conclusions follow then immediately.

Next, we consider the aggregate performance case. After substitution of the parameters in the algorithm described in appendix A.3 we see that matrix G coincides with (26) except for the last row which is multiplied by a factor two. Consequently the determinant of G, det, equals $\frac{1}{2}u_{1,(1,2)}u_{(1,2)}$. After some elementary calculations we see that $det * G^{-1}$ equals

$$\begin{pmatrix} u_{1,(1,2)} + u_{2,(1,2)} & u_{2,(1,2)} & \frac{1}{4}u_{(1,2)}c(a+e) \\ u_{2,(1,2)} & u_{1,(1,2)} + u_{2,(1,2)} & \frac{1}{4}u_{(1,2)}c(a+e) \\ u_{(1,2)}c(a+e) & u_{(1,2)}c(a+e) & \frac{1}{4}u_{(1,2)}(g + (a+e)^2) \end{pmatrix},$$

and matrix $M_{(1,2)}$ satisfies

$$det * M_{(1,2)} = \begin{pmatrix} -\phi_4 det - \phi_1 b(a-e)u_{1,(1,2)} & 2\phi_1^2 u_{1,(1,2)} & 0 \\ \frac{1}{2}b^2 g u_{1,(1,2)} & \phi_4 det + \phi_1 b(a-e)u_{1,(1,2)} & 0 \\ 0 & 0 & \phi_4 det \end{pmatrix}.$$

From this it is easily deduced that the only relevant positive eigenvalue $a_{(1,2)A}$ coincides with $a_{(1,2)}$. Moreover, it is also easily verified that the corresponding K and strategies coincide with the national case.

Appendix C

Proof of lemma 2. (i) To show that $a_{cN} \leqslant a_{(1,2)}$ we note that

$$\begin{aligned} a^2_{(1,2)} - a^2_{cN} &= \phi_4^2 \frac{g}{g + (a-e)^2} - \phi_4^2 \frac{\tau g}{\tau g + d_N(a-e)^2} \\ &= \phi_4^2 \frac{g(a-e)^2(d_N - \tau)}{(g + (a-e)^2)(\tau g + d_N(a-e)^2)} \geqslant 0. \end{aligned}$$

(ii) First note that, using the equality $\phi_4(a-e) = -2b\phi_1$, a_{nc} can be rewritten as

$$\frac{\frac{1}{2}\phi_4(a-e)(a+e) + \|\phi_4\|\sqrt{(\frac{1}{2}(a+e)(a-e) + 2g)^2 + 2g(a-e)^2}}{2(a(a-e) + g)}.$$

Consequently,

$$
\begin{aligned}
a_{cN}^2 - a_{nc}^2 &= -\phi_4^2\left\{\frac{\frac{1}{2}(a^2-e^2)^2 + 4gu_{nc}}{4u_{nc}^2} - \frac{\tau g}{u_{cN}}\right\}\\
&\quad + \phi_4^2\frac{(a^2-e^2)\sqrt{(\frac{1}{2}(a^2-e^2)+2g)^2 + 2g(a-e)^2}}{4u_{nc}^2}\\
&=: -t_1 + t_2.
\end{aligned}
$$

Since $t_2 \geqslant 0$, it is obvious that if t_1 in the above expression is negative also $a_{cN}^2 - a_{nc}^2 \geqslant 0$. Next assume that t_1 is positive. Then, $a_{cN}^2 - a_{nc}^2 \geqslant 0$ if and only if $t_2^2 - t_1^2 \geqslant 0$. Elementary calculations show that

$$
\begin{aligned}
t_2^2 - t_1^2 &= \frac{\phi_4^4 g}{u_{nc}^2 u_{cN}^2}\left\{-g(u_{cN} - \tau u_{nc})^2 + \frac{1}{4}\tau\big(a^2-e^2\big)^2 u_{cN}\right\}\\
&= \frac{\phi_4^4 g(a-e)^3}{u_{nc}^2 u_{cN}^2}\left\{-\frac{\tau^2 g}{4}(3a+e) - g d_N\big(d_N(a-e) - 2a\tau\big)\right.\\
&\qquad\left. + \frac{1}{4}\tau d_N(a-e)(a+e)^2\right\}\\
&= \frac{\phi_4^4 g(a-e)^3}{u_{nc}^2 u_{cN}^2}\left\{\frac{1}{4}g(2d_N - \tau)\big(a(3\tau - 2d_N) + (2d_N + \tau)e\big)\right.\\
&\qquad\left. + \frac{1}{4}\tau d_N(a-e)(a+e)^2\right\}.
\end{aligned}
$$

Obviously, this last expression is positive, if $3\tau - 2d_N \geqslant 0$, which concludes the proof.

(iii) For the non-cooperative case the proof can be found in Engwerda et al. [9]. The proof of the other two cases is found by straightforward differentiation.

(iv)

$$
\begin{aligned}
a_{cA}^2 - a_{cN}^2 &= \frac{g\phi_4^2}{g + (a-e)^2} - \tau\frac{g\phi_4^2}{\tau g + d_N(a-e)^2}\\
&= \frac{\phi_4^2}{g + (a-e)^2}\frac{g}{\tau g + d_N(a-e)^2}(a-e)^2(1-2\tau)\frac{d_E^N}{d} \geqslant 0. \qquad \square
\end{aligned}
$$

Proof of lemma 3. (i) From the cost functional (25) we have that

$$
J_{E,cN}^N - J_{E,nc}^N = \frac{1}{4}\frac{d_E^N s^2(0)}{a_{cN}a_{nc}}\left\{a_{nc}\big(b + p_{cN}(a-e)\big)^2 - a_{cN}\big(b + p_{nc}(a-e)\big)^2\right\}.
$$

Since, $p_i = (\phi_4 + a_i)/(2\phi_1),\ i \in \{cN, nc\}$, and $\phi_4(a-e) = -2b\phi_1$ we have

$$
\begin{aligned}
&\operatorname{sgn}\big(J_{E,cN}^N - J_{E,nc}^N\big)\\
&\quad = \operatorname{sgn}\Big(a_{nc}\big(2b\phi_1 + (\phi_4 + a_{cN})(a-e)\big)^2 - a_{cN}\big(2b\phi_1 + (\phi_4 + a_{nc})(a-e)\big)^2\Big)
\end{aligned}
$$

$$= \mathrm{sgn}\big((a_{cN} - a_{nc})a_{nc}a_{cN}(a-e)^2\big)$$
$$= \mathrm{sgn}(a_{cN} - a_{nc}).$$

(ii) From (24) we have that

$$J_{i,cN} - J_{i,(1,2)} = \frac{1}{4}\frac{ds^2(0)}{a_c a_{(1,2)}}\big\{a_{(1,2)}\big(\big(b + p_{cN}(a-e)\big)^2 + p_{cN}^2 g\big)$$
$$- a_{cN}\big(\big(b + p_{(1,2)}(a-e)\big)^2 + p_{(1,2)}^2 g\big)\big\}.$$

Using again the facts that $p_i = (\phi_4 + a_i)/(2\phi_1)$, $i \in \{cN, (1,2)\}$ and $\phi_4(a-e) = -2b\phi_1$ we have that $\mathrm{sgn}(J_{i,cN} - J_{i,(1,2)})$ can be rewritten as

$$\mathrm{sgn}\big(a_{(1,2)}\big(a_{cN}^2(a-e)^2 + (\phi_4 + a_{cN})^2 g\big) - a_{cN}\big(a_{(1,2)}^2(a-e)^2 + (\phi_4 + a_{(1,2)})^2 g\big)\big)$$
$$= \mathrm{sgn}\big(\phi_4^2 g(a_{(1,2)} - a_{cN}) + \big((a-e)^2 + g\big)\big(a_{(1,2)}a_{cN}^2 - a_{cN}a_{(1,2)}^2\big)\big)$$
$$= \mathrm{sgn}\big((a_{cN} - a_{(1,2)})\big(-\phi_4^2 g + \big((a-e)^2 + g\big)a_{(1,2)}a_{cN}\big)\big).$$

From lemma 2(i) we therefore conclude that

$$\mathrm{sgn}(J_{i,cN} - J_{i,(1,2)}) = \mathrm{sgn}\big(\phi_4^2 g - \big((a-e)^2 + g\big)a_{(1,2)}a_{cN}\big).$$

Since

$$a_{(1,2)}^2 = \phi_4^2 \frac{g}{u_{(1,2)}} \quad \text{and} \quad a_{cN}^2 = \phi_4^2 \frac{\tau g}{u_{cN}},$$

we finally have

$$\mathrm{sgn}(J_{i,cN} - J_{i,(1,2)}) = \mathrm{sgn}\left(\phi_4^4 g^2\left(1 - \frac{\tau u_{(1,2)}}{u_{cN}}\right)\right)$$
$$= \mathrm{sgn}\big((d_N - \tau)(a-e)^2\big) \geqslant 0,$$

which concludes the proof. □

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