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Publication date:
2014

Document Version
Early version, also known as pre-print

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Lekniute, Z., Beetsma, R. M. W. J., & Ponds, E. H. M. (2014). *A Value-Based Approach to the Redesign of US State Pension Plans*. SSRN.

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A value-based approach to the redesign of US state pension plans*

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May 18, 2014

Abstract

We explore the financial sustainability of a typical U.S. state civil servants pension fund under the continuation of current policies and under alternative policies, such as changes in contribution, indexation and investment allocation policies. Applying the value-based asset-liability management method, we find that all participant cohorts derive substantial net benefit from the current pension contract, while all tax-paying cohorts have to make substantial contributions. The proposed adjustment measures can alleviate a substantial part of the burden on tax payers, although at the cost of the fund participants. Especially a policy of conditional inflation indexation seems promising.

Keywords: U.S. state civil servants pension funds, underfunding, value-based asset-liability management, contributions, benefits, indexation, participants, tax payers

JEL Codes: G23, H60

*We thank Hindrik Angerman, Jean-Pierre Aubry, Henk Hoek, Stephen Lerch, Allan Milligan, Joshua Rauh, Matthew Smith, Sjoerd Timmermans, Peter Vlaar, Bas Werker and ICPM Discussion Forum (Sacramento, 2013) participants for helpful and detailed discussions. We also thank ICPM for financial support. The usual disclaimer applies.

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1 Introduction

In the past decades the U.S. has witnessed a trend away from defined-benefit (DB) towards defined-contribution (DC) pension plans. However, an exception to this trend are the state pension plans, which manage the pensions of the state civil servants. Despite the aging of the population these plans still largely operate on a DB basis, although it is clear that in most cases the promises made to participants of the state pension funds cannot be honored in the absence of drastic measures to guarantee the financial sustainability of these funds. However, the pension promises made to the state employees cannot so easily be reneged upon (see Brown and Wilcox (2009)). In fact, these promises may even get priority to the states' debt holders when a state goes bankrupt. Hence, in many instances, a state's pension fund threatens the state's public finances, possibly resulting in large claims on its tax payers and, to the extent that taxes cannot be raised, in a crowding out of public services. A recent overview by Munnell *et al.* (2013) of the funding ratios (ratios of assets over liabilities) of U.S. state pension funds illustrates the urgency of the problem. For a sample of 109 state plans and 17 locally-administered plans, the paper estimates the aggregate funding ratio for 2012 at 73%. Almost a quarter of the plans have a funding ratio below 60%, while only a small fraction of 6% have a funding ratio above 100%. The funding ratios are calculated on the basis of GASB standards that prescribe that assets are reported on an actuarially-smoothed basis, while the discount rate for the liabilities is typically set at around 8 percent, reflecting the expected long-term investment return on the fund's assets. These standards have been criticized by economists (Novy-Marx and Rauh (2009) and Bader and Gold (2003)) who claim that the future streams of benefit payments should be discounted at a discount rate reflecting their riskiness. As the state pension benefits are protected under most state laws, these payments can be seen as guaranteed and so this would plead for discounting future benefits against the risk-free interest rate. Doing so would lead to a severe fall in the already-low average funding ratios of the state plans. However, a precise assessment of the riskiness of state pension promises likely remains elusive for the foreseeable future. For example, the recent default of the city of Detroit may be an instructive case of whether pensions should be seen as a contractual obligation that cannot be diminished or impaired, or whether the holders of pension rights should be treated like other creditors, so that benefit cuts cannot be ruled out. In fact, a U.S. bankruptcy judge declared in December 2013 that legally pensions can be cut (Bomey and Priddle, 2013).

In this paper we explore the financial sustainability and redistributive features of a typical U.S. state defined-benefit (DB) pension plan under unchanged fund policies and under alternative policies in terms of contributions, benefits or indexation aimed at alleviating the financial burden on the tax payer. We assume that pension promises cannot be defaulted upon. We apply the so-called method of value-based asset-liability management (value-based ALM) to address reform-induced value redistribution among the different stakeholders of the fund, i.e. the various cohorts of plan participants and tax payers. Essentially, the method involves rewriting a pension plan in terms of embedded options held by the fund's stakeholders. By

providing a market-based valuation of all cash flows associated with the pension contract, the method demonstrates the consequences of the various interventions for the different stakeholders. Policy changes are always a zero-sum game, implying that the total value of the contract to all stakeholders together is unchanged and that policy changes can only shift value among groups of stakeholders. To the best of our knowledge, our paper is the first attempt at applying value-based ALM to the study of reform-induced redistribution within U.S. state pension funds.

The value-based ALM approach has its roots in the pioneering papers of Sharpe (1976) and Sharpe and Treynor (1977) in utilizing derivative pricing to value contingent claims within pension funds. More recent applications of derivative pricing to pension plans are, among others, Blake (1998), Exley et al. (1997), Chapman et al. (2001), Ponds (2003), Bader and Gold (2007) and Hoevenaars and Ponds (2008). Our paper is closest in spirit to Biggs (2010), which also employs an option-based approach to value the market price of pension liabilities of U.S. state pension plans. However, it does not address reform plans and the associated redistribution effects. Value-based ALM has also been employed in the Dutch discussion on the redesign of the second-pillar pension funds offering defined-benefit plans. In particular, the CPB Netherlands Bureau for Economic Policy Analysis (2012) has applied it to investigate the generational fairness of various pension reform plans.

Our paper is complementary to related contributions studying the degree of underfunding of U.S. state pension plans, the causes of underfunding and the measures that could be taken to solve plan deficits. Novy-Marx and Rauh (2011) conduct a careful analysis of the value of the pension promises made by U.S. state pension funds under different assumptions about the riskiness of state pension plans and their seniority relative to state debt in the case of default. Several papers explore the extent of underfunding and the question what the appropriate target of the funding ratio should be (compare D’Arcy et al. (1999), Bohn (2011), Lucas and Zeldes (2009)). Brown et al. (2011) put forward the U.S. academic and public discussions that assume that adequate funding requires 100% funding of the liabilities.

Regarding the causes of the underfunding of state pension plans, some papers point at the implications of what may be called the ‘accounting game’. The accountability horizon of pension fund management, politicians and public sector union leaders is much shorter than the horizon over which pension promises have to be met by adequate funding. Hence, union leaders and politicians tend to downplay the long-term costs of pension promises in favor of higher short-term wages and benefits (Mitchel and Smith (1994), Schieber (2011)), as well as higher state spending in the short run. Shnitser (2013) stresses the role of the institutional design of pension plans in explaining the large variation in funding discipline. Institutional rules facilitating transparency and pre-commitment to expert-based financial and actuarial advice, in particular, appear to be conducive to mitigating underfunding and limiting the shift of costs to future tax payers. Kelley (2014), and Elder and Wagner (2014) employ a public choice perspective to explain underfunding issues.

Essentially, the literature has explored three broad directions of solutions to the underfunding problem: higher contributions, lower benefits and higher investment returns. Novy-Marx and Rauh (2014b) explore by how much contributions need to be raised to reach full funding in 30 years time. They compute a necessary increase of the order of 2.5 times the prevailing contribution level. The option of reducing benefits has long been seen as virtually impossible, as in many states public pensions are interpreted as hard contractual obligations, protected by civil law and state constitutions (Monahan, 2012). However, recently several states have enacted benefit cuts and other measures to scale back the generosity of pensions. Novy-Marx and Rauh (2014a) explore performance-linked indexation rules as an alternative to automatic cost-of-living adjustments, like the one in the Wisconsin Retirement System or the conditional indexation rule in the Netherlands (Ponds and van Riel (2009) and Beetsma and Buccioli (2010, 2011a and 2011b)). Shnitser (2013) stresses that simply scaling back generosity or imposing higher contribution rates will not be enough. She claims that foremost institutional design has to be reframed to practices and rules that have proven to be successful in promoting to funding discipline. A third route proposed to solve funding deficits is to raise expected investment returns. This option is already being exploited by U.S. state pension funds, as many of them have high portfolio shares in equity, substantially higher than of pension funds outside the U.S. (Andonov et al. (2013) and OECD (2011)). However, finance-based papers (Black (1989), Peskin (2001), Bader and Gold (2003), Lucas and Zeldes (2009) and Pennacchi and Rastad (2011)) recommend that the strategic asset allocation should be set in line with the perspective of the tax payers as the party bearing the residual risk, which suggests to limit mismatch risk between the funds' assets and the market value of their liabilities. This may imply no equity investment at all when one aims at full protection of accrued benefits in the short run. Lucas and Zeldes (2009), employing a long-term perspective, suggest that the optimal portfolio should include some equity holdings as future pension liabilities are related to future wage growth and this growth is to some extent correlated with equity returns.

We simulate a representative pension fund over a period of three-quarters of a century. The standard or "classic" ALM results reveal that raising the share of the amortization cost that is contributed, speeding up the amortization payment or halving indexation to consumer-price inflation leads to a substantial long-term improvement of the funding ratio. A policy that is of particular interest is a policy of inflation indexation that is conditional on the funding ratio. A funding ratio below one leads to less-than-full or even zero indexation, while a funding ratio above one leads to more than full indexation. Such type of indexation policy has become popular among Dutch pension funds. This way of using indexation has a tendency of improving the financial position of the fund, but at the same time compressing the spread in the distribution of possible funding ratios and, thereby, also compressing the spread in the distribution of contribution rates, as its amortization component is linked to the funding ratio. Hence, a well-designed policy of conditional indexation may be of particular interest for policymakers concerned with a redesign of the U.S. system of state pension plans.

Our value-based ALM results show that the current pension contract yields a substantial net benefit to all cohorts of fund participants, which in turn implies a substantial financial burden on all cohorts of tax payers. In present value terms, during the 75-years horizon under the base plan the fund participants receive almost 28 trillion dollars in net benefit, while the tax payers contribute approximately 17 trillion dollars. Increasing the amortization contribution rate and speeding up amortization without addressing the benefit level improves the residual value of the fund at the end of our projection horizon, but it does not affect the contract value to the participants over our simulation horizon. A doubling of the contribution rate by the participants does affect their net benefit from the pension contract. The cohorts that have not yet entered the labor force lose about a quarter of the value of their pension contract, while older participants lose about 10%. In money terms, the tax payers benefit by an amount of almost 4.5 trillion dollars, or between a quarter and one-third of the contract value for young and older tax payers, respectively. Reducing benefits by cutting indexation can be quite effective too. Halving the indexation to CPI inflation reduces the contract value by around 20% for both young and old participants, while making indexation conditional on the funding ratio is even more effective, leading to a reduction in the participants' contract value on the order of 25-30%. In dollar terms the alleviation of the burden on the tax payers is about 3-4 trillion dollars. Older tax payers benefit by 10-15% of their contract value, young tax payers by about double those figures. Changes in the pension fund's investment portfolio do not affect the participants values, though they do lead to minor shifts in value between tax payers and the fund's residual value.

The remainder of this paper is structured as follows. Section 2 presents the model, including a description of the demography, the pension fund and the calculation of the fund's benefits and liabilities. Sections 3 - 5, respectively, describe the data, our economic scenario generator and the valuation of the cash flows following from the pension contract. Section 6 summarizes the baseline settings, while Section 7 presents the various reforms we will consider. Section 8 discusses the simulation results for the classic ALM and the value-based ALM. Finally, Section 9 concludes the main text of this paper. The appendices are not intended for publication, but will be posted on the authors' websites.

2 The model

This section describes the demography, wage developments and the pension fund. The survival probabilities are deterministic. Hence, there is no longevity risk.

2.1 The population

The participant population of the pension fund consists of individuals of ages 25 to 99 years. Individuals enter the fund at the age of 25 and remain with the fund for the rest of their life. Further, we assume that they retire at age a_R . The number of male and female participants of age a at time t is denoted as M_t^a and F_t^a respectively, where $a \in [25, 99]$. The population size for the rest of the simulation for each age group is determined using the projections of survival probabilities. That is, $M_{t+n}^{a+n} = q_{a,t}^{m,n} M_t^a$ and $F_{t+n}^{a+n} = q_{a,t}^{f,n} F_t^a$, where $q_{a,t}^{m,n}$ and $q_{a,t}^{f,n}$ are the probabilities that a male, respectively female, person aged a in period t will survive another n years.

2.2 Wages

Crucial for the calculation of the pension liabilities are the wage developments. The wage level of the cohort of age a at time t is W_t^a and it is updated each time period. We do not take into account idiosyncratic wage risk, hence we assume a uniform wage level within each cohort. The wage levels across cohorts will be set to follow a certain career profile. We set the wage levels of males and females equal, although one could in principle allow for different wage levels or career profiles depending on gender. The nominal wage level evolves as follows:

$$W_t^a = W_{t-1}^{a-1} w_{t-1}^{a-1},$$

where w_{t-1}^{a-1} is the gross wage growth rate from period $t-1$ to t for a cohort aged $a-1$ in period $t-1$. This factor can be disaggregated into the economy-wide gross wage growth rate w_{t-1} and a component \tilde{w}_{t-1}^{a-1} attributable to the progression of the individual career:

$$w_{t-1}^{a-1} = w_{t-1} \tilde{w}_{t-1}^{a-1}. \quad (1)$$

We refer to \tilde{w}_{t-1}^{a-1} as the promotion rate from period $t-1$ to t for someone aged $a-1$ in period $t-1$. Note that \tilde{w}_{t-1}^{a-1} is always positive if the career profile has an upward sloping shape. Promotion rates differ across ages. However, we assume that the career profile remains constant over time, which implies that $\tilde{w}_{t-1}^{a-1} = \tilde{w}^{a-1}$ in all periods. Hence, the ratio of the wage levels of two workers of different ages is constant over time. Economy-wide wage growth rate w_{t-1} is stochastic and will be modeled as explained in Section 4 below.

Given the current wage rate, based on (1), we calculate an age- a individual's expected wage i periods into the future as:

$$\mathbb{E}_t [W_{t+i}^{a+i}] = W_t^a \prod_{j=1}^i \mathbb{E}_t [w_{t+j}^{a+j}], \forall i \in \{1, a_R - 1 - a\}, \quad (2)$$

where, using (1), $E_t [w_{t+j}^{a+j}] = \tilde{w}^{a+j} \cdot E_t [w_{t+j}]$, for $j \geq 1$.

2.3 The pension fund

State pension funds can differ in many respects. Some of the differences are parametric, such as the value of the discount rate to be applied when calculating the liabilities, while other differences are more fundamental. We model a pension fund based on the most common features across the entire population of pension funds available to us from the Public Plans Database of the Center for Retirement Research at Boston College (2014).

2.3.1 Assets

Market value of assets The market value of the fund's assets at the beginning of the next year, A_{t+1} , is equal to the asset value A_t at the beginning of this year, multiplied by its gross rate of return R_t , plus the net money inflow times the gross return over the half year over which it is on average invested:

$$A_{t+1} = A_t R_t + (C_t - B_t) R_t^{1/2}, \quad (3)$$

where C_t is the total amount of contributions received (calculated below) and B_t the total amount of benefits paid out. Since the benefits and contributions are (usually) paid on a monthly basis, while our model runs on a yearly basis, we assume that the payment of the benefits and contributions takes place in the middle of the calendar year and, hence, the net money inflow is invested on average for half a year.

Actuarial assets Pension fund assets in the U.S. are not measured at their market value when they are used as an input for pension policy. Rather, pension funds in the U.S. apply a certain smoothing procedure to come up with an actuarial value A_t^{act} of their assets in period t . Its detailed calculation is laid out in Appendix A. In short, the actuarial value of the assets at the end of the year is equal to the actuarial value at the end of previous year, plus the net cash flows into the fund, plus the projected return on the assets, and a recognition of the smoothed excess of actual above expected investment income. In the special case that initial actuarial assets equal the initial market value of the assets, $A_0^{act} = A_0$, and the smoothing period is shrunk to a single period, one has that $A_t^{act} = A_t$, for all $t \geq 0$. Hence, in this specific case the process for actuarial assets coincides with that of the market value of the assets.

2.3.2 Calculation of the liabilities

In this section we will formalize the calculation of the liabilities, closely following Munnell et al. (2008b) and Novy-Marx and Rauh (2011). Total liabilities L_t are calculated as the present

value of the projected future benefit payments to all current pension fund participants, taking into account the survival probabilities. The group of participants comprises the employees and the retired. The total liabilities are the sum of the liabilities L_t^m and L_t^f to the male and female participants. The liabilities to each gender, in turn, are calculated by multiplying the individual liability $L_t^{a,\zeta}$ to an age- a and gender- ζ individual by the number of gender- ζ individuals in this cohort and then summing over the cohorts. Hence, the total liabilities are calculated as:

$$L_t = L_t^m + L_t^f = \sum_{a=25}^{99} \left(M_t^a L_t^{a,m} + F_t^a L_t^{a,f} \right), \quad (4)$$

$$L_t^{a,\zeta} = \sum_{i=\max(a_R-a,0)}^{99-a} \left(\tilde{R}_t^{(i)} \right)^{-i} q_{a,t}^{\zeta,i} K_{t,t+i}^{a+i},$$

where $K_{t,t+i}^{a+i}$ is the projection at time t of the pension pay-out i years ahead for a participant of age a and $\tilde{R}_t^{(i)}$ is the interest rate used to discount to period t the cash flows materializing i periods into the future. We notice that $\tilde{R}_t^{(0)} = 1$ and $p_{a,t}^{\zeta,0} = 1$. U.S. public pension plans usually use a discount rate that is flat over the entire projection horizon and equal to the expected asset portfolio return \bar{R}_t . Summarizing, the liabilities to an individual of a particular cohort depend on the years they will (still) receive benefits, the level of the benefits, the discount rate and the survival probabilities.

Under plausible parameter settings the value of the liabilities peaks at around retirement age. The reason is that the workers close to retirement have reached the maximum time span over which pension rights can be accumulated, while most of their benefits occur in the relatively near future, so that the effect of discounting is relatively limited. Younger individuals have fewer years of accruals, while any given benefits occur further into the future. Older individuals expect to receive fewer remaining benefit payments. Moreover, the average of the highest wages during a career, which forms the basis for the pension benefits (see below), falls with age in a cross-section of retirees. The reason is that nominal wage growth tends to exceed the cost-of-living adjustment (COLA) they have received while in retirement.

Calculation of pensioners' benefits To protect the living standards of the retired as much as possible, pensioners receive a COLA, which we denote as π_t . The COLA usually covers a certain share of inflation and it is often capped at a predetermined level (Center for Retirement Research at Boston College, 2014):

$$\pi_t = 1 + \min\{\alpha \cdot cpi, cap\},$$

where cpi stands for consumer-price inflation, α is the fraction of inflation compensated and cap is the maximum indexation rate. (Both cpi and cap are expressed as a fraction of unity.)

The current pay-out to a retiree equals the current pension rights, B_t^a , while the projected pay-out i periods from now equals the current pension rights adjusted for expected future

indexation:¹

$$K_{t,t}^a = B_t^a, \forall a \in [a_R, 99],$$

$$K_{t,t+i}^{a+i} = B_t^a \prod_{j=1}^i E_t [\pi_{t+j}], \forall i \in [1, 99 - a], \forall a \in [a_R, 98],$$

where B_t^a equals the pension rights at the moment of retirement increased by the past indexation since then:

$$B_t^a = B_{t-(a-a_R)}^{a_R} \prod_{j=0}^{a-(a_R+1)} \pi_{t-j}, \forall a \in [a_R + 1, 99].$$

For example, the pension rights of a 70-year old person, who retired at the age $a_R = 65$, are equal to the benefit calculated when that person reached the age of 65, plus the indexation that has been awarded over the 5 years since then.

A common feature of the state pension plans in the U.S. is that they are based on the average wage preceding the moment of exit from the workforce. Concretely, the benefit of somebody retiring in year $t - (a - a_R)$ can be calculated as the product of the accrual rate ε , the number of years in the workforce (here, 40) and the average wage level over the past z years:

$$B_{t-(a-a_R)}^{a_R} = \frac{40\varepsilon}{z} \sum_{l=1}^z W_{t-(a-a_R)-l}^{a_R-l}, \forall a \in [a_R, 99].$$

The averaging period z varies from one to five years, with the majority of public plans applying a three-year average (Munnell et al., 2012).

Because the pension rights for the retired are already determined and, hence, cannot be affected by future service or salary increases, they are independent of the method we use to recognize the liabilities – see below.

Calculation of workers' projected benefits The calculation of the projected pay-outs to current employees is similar to that of the pensioners, i.e. as the pension rights B_t^a at time t of someone of age a , adjusted for future indexation during retirement:

¹In practice, COLAs are only included in the calculation of the liabilities to the pensioners if there exists an explicit policy rule on how the indexation is awarded. COLAs are not taken into account if indexation is determined on an ad-hoc basis.

$$K_{t,t+i}^{a+i} = B_t^a, i = a_R - a, \forall a \in [25, a_R - 1],$$

$$K_{t,t+i}^{a+i} = B_t^a \prod_{j=a_R+1-a}^i E_t[\pi_{t+j}], \forall i \in [a_R + 1 - a, 99 - a], \forall a \in [25, a_R - 1],$$

where the first line gives the projected benefit during the first year of retirement, while the second line gives the projected benefit during the ensuing years in retirement. Determining the pension rights B_t^a is not as straightforward for the working population as it is for the pensioners. Unlike pensioners, the active participants continue working and, hence, they are expected to accrue more years of service and (in most cases) to have a salary at retirement date that is higher than their current salary. Hence, depending on the method used, their pension rights B_t^a can be recognized in different ways. We will now discuss the various ways in which B_t^a can be determined.

Methods for recognizing liabilities There are different methods to recognize liabilities. Here, we will explain the so-called *Accrued Benefit Obligation* (ABO) method, the *Projected Benefit Obligation* (PBO) method and the *Projected Value of Benefits* (PVB) method.² The latter is also known as the Present Value of Future Benefits method.

Under the *ABO method*, only the pension rights accrued up until time t are taken into account. Hence, B_t^a is based on the number of years since entry into the workforce and the average of at most the past z wage levels:

$$B_t^a = \frac{(a - 25)\varepsilon}{\min(a - 25, z)} \sum_{l=1}^{\min(a-25, z)} W_{t-l}^{a-l}, \forall a \in [26, a_R],$$

$$B_t^{25} = 0.$$

The youngest cohort of age 25 has just entered the fund and has no rights accrued yet. The rights of the other young cohorts who do not yet have z years in service are based on the average of the available wage history. For the cohorts that have at least z years of service, the pension rights are the product of the years in the workforce, the accrual rate and the average pay over the past z years. Hence, for an individual worker the ABO pension rights increase with each additional year of service.

Under the *PBO method*, we also take into account the effect of expected future salary increases on the rights accrued up to now. Hence, under this method B_t^a is the projected benefit level at

²There is also the Vested Benefit Obligation (VBO), which considers only employees who have served at least the fund's vesting period, if their is one. However, this variant is not meaningful in our context, because we assume that everybody works for 40 years.

retirement when expected future salary advances are taken into account:

$$B_t^a = \frac{(a - 25)\varepsilon}{z} \sum_{l=1}^z \mathbf{E}_t \left[W_{t+(a_R-a)-l}^{a_R-l} \right], \forall a \in [26, a_R - 1],$$

$$B_t^{25} = 0.$$

Again, the youngest cohort, the 25 years old, has no accrual yet and, hence, it has no pension rights in terms of the PBO. However, for a given age and assuming that the overall wage level is expected to increase, the pension rights for the other working cohorts are higher under the PBO method than under the ABO method. When the worker nears its retirement age, the difference of accrued rights in PBO and ABO terms shrinks. The two measures merge for a given individual at the moment of their retirement.

Finally, state jobs are relatively secure, so that the pension fund might in addition consider the rights that the employees will acquire in the future if they continue working in their job until retirement. The *PVB method* takes this into account. Therefore, it defines the pension rights B_t^a including future accrual due to new service:

$$B_t^a = \frac{40\varepsilon}{z} \sum_{l=1}^z \mathbf{E}_t \left[W_{t+(a_R-a)-l}^{a_R-l} \right], \forall a \in [25, a_R - 1]. \quad (5)$$

Note that in this case the generation that has just entered the labor force already has a stake in the pension liabilities, as opposed to ABO and PBO. At any given age before retirement, rights computed according to the PVB method exceed rights computed according to ABO or PBO, because PVB takes into account both expected future increases in the wage level and expected future accrual. All three measures merge for a given individual at the moment of their retirement.

2.4 Contributions and benefits

2.4.1 The entry-age normal costing method and the accrued liability

The value of the additional pension rights earned in a given year is called the *normal cost*. The present value of the future normal cost will be used to calculate what is called the fund's *accrued liability*, which will be compared with the fund's actuarial assets to establish the required amortization payments.

The fund's actuaries determine the normal cost based on the liability recognition method used by the pension fund. The two most common methods for calculating the normal cost are the so-called *projected unit credit* method, which is mostly used in the private sector, and the so-

called *entry-age normal costing* (EAN) method, which is dominant in public plans and which is related to the PVB. Here, we explain only the EAN method, as it is the method we will use in our calculations. Under the EAN method, the employer's annual normal cost associated with an individual participant is calculated as a payment throughout the projected years of service needed to finance the PVB obligation. Since the accrued benefits due to salary growth increase more than linearly, the method implies a component of front-loading, because the employer is pre-paying some of the benefits to be earned in the future (Munnell et al., 2008b).

The payment of the normal cost can be calculated in dollar terms or as a percentage of projected salary levels. Here, we will explain the latter, as it is the more commonly used method. The *normal cost rate* (NCR) of an active participant is calculated at the entry age as the ratio of the present value of future benefits and the present value of career salary levels:

$$NCR_t^{25,\zeta} = \frac{L_t^{25,\zeta}}{PVW_t^{25,\zeta}}, \quad \zeta \in \{f; m\},$$

where $L_t^{25,\zeta}$ is the liability to someone of gender ζ who enters the labor force in period t as calculated in (4) on the basis of the PVB method (as in (5), i.e. taking into account future accrual associated with future service), and $PVW_t^{25,\zeta}$ is the present value of all future wages throughout the participant's career as projected at entry. Hence, the normal cost rate is the percentage payment of a worker's projected career salary needed to cover the cost of the projected benefits for that worker.

The $PVW_t^{a,\zeta}$ associated with a worker of age a and gender ζ is calculated as follows:

$$PVW_t^{a,\zeta} = \sum_{i=0}^{a_R-1-a} \left(\tilde{R}_t^{(i)} \right)^{-i} q_{a,t}^{\zeta,i} E_t [W_{t+i}^{a+i}],$$

where the expected wage level $E_t [W_{t+i}^{a+i}]$ is computed according to (2). The present value of a worker's future normal cost ($PVFNC$) are the normal costs that will be recognized throughout its remaining years of service:

$$PVFNC_t^{a,\zeta} = PVW_t^{a,\zeta} \times NCR_{t-(a-25)}^{25,\zeta}.$$

Hence, this is the product of the present value of future wages and the normal cost rate determined at the time when the worker entered the fund. The so-called *accrued liability* (L_{accr}) for this participant will then be the difference between the liabilities to this participant, calculated as the present value of the projected benefits, and the present value of the future normal cost:

$$L_{accr,t}^{a,\zeta} = L_t^{a,\zeta} - PVFNC_t^{a,\zeta}.$$

If we follow the individual worker over time, we will see that the present value of the future normal cost will decrease, as there will be fewer remaining years to pay the normal cost.

Therefore, accrued liability associated with an individual increases over time. The accrued liability of pensioners is simply equal to the present value of their future benefits, since they should have paid the whole normal cost before reaching the retirement age. Therefore,

$$PVFNC_t^{a,\zeta} = 0 \Rightarrow L_{accr,t}^{a,\zeta} = L_t^{a,\zeta}, \forall a \in [a_R, 99].$$

Finally, the fund's accrued liability is the sum of the individual accrued liabilities over genders and cohorts:

$$L_{accr,t} = L_{accr,t}^m + L_{accr,t}^f = \sum_{a=25}^{99} \left(M_t^a L_{accr,t}^{a,m} + F_t^a L_{accr,t}^{a,f} \right).$$

2.4.2 Contributions

The annual contribution to the pension fund is paid by both the employer and the workers. Combined the two payments form the total contribution, which we determine as a sum of two components: the normal cost and the amortization payment of the so-called *unfunded actuarial accrued liability (UAAL)*, defined as the difference between the fund's accrued liability and the actuarial value of the assets:

$$UAAL_t = L_{accr,t} - A_t^{act}.$$

The amortization payment, like the normal cost, can be determined as a dollar amount or a percentage of projected salaries. We will use the former. The amortization payment cannot be negative. Hence, if there is a funding surplus, the normal cost covering contribution cannot be reduced. We assume a smoothing period of u years for the amortization of the *UAAL*, implying that the required amortization payment in period t is:

$$AMORT_t = \begin{cases} \frac{1}{u} UAAL_t & \text{if } UAAL_t \geq 0, \\ 0 & \text{if } UAAL_t < 0. \end{cases}$$

Munnell et al. (2008a) show that only about half of all the plans pay the annually required contribution. Hence, in our simulations we will include a parameter λ of the fraction of the required amortization payment that is actually paid.

The aggregate volume of contributions that should be paid to the pension fund in year t by all the participants and the employer together is:

$$C_t = \sum_{a=25}^{a_R-1} \left(NCR_{t-(a-25)}^{25,m} M_t^a W_t^a + NCR_{t-(a-25)}^{25,f} F_t^a W_t^a \right) + AMORT_t.$$

The total contribution rate c_t is expressed as a percentage of the total wage sum in year t :

$$c_t = \frac{C_t}{\sum_{a=25}^{a_R-1} (M_t^a + F_t^a) W_t^a}. \quad (6)$$

The total contribution rate can be split into a contribution rate c_t^W paid by the worker and a contribution rate c_t^T paid by the employer (i.e., the tax payer):

$$c_t = c_t^W + c_t^T.$$

Typically, the worker pays a fixed contribution rate, in which case c_t^W is constant, while the employer pays the remaining part. It is also possible that the employee contribution is a fixed proportion of actuarially required contribution, in which case c_t^W may change over time. However, in our simulations we will focus on the case of a constant worker contribution rate.

2.4.3 Total benefits

The total amount of benefits to be paid out in year t is:

$$B_t = \sum_{a=a_R}^{99} (M_t^a + F_t^a) B_t^a.$$

2.5 Sponsor support

Forecasting the development of a pension fund over a long horizon implies that there exist scenarios in which the fund's assets get depleted. Therefore, we assume that, whenever the pension fund's assets become insufficient to finance the net cash outflow, the employer/tax payer, provides *sponsor support* by replenishing the fund's assets. The sponsor support, SS_t , comes on top of the employer/tax payer's regular contribution:

$$SS_t = \begin{cases} 0 & \text{if } A_t \geq B_t - C_t, \\ (B_t - C_t) - A_t & \text{if } A_t < B_t - C_t, \end{cases}$$

so that the sponsor support covers the difference between the assets and the net cash outflow. Once the sponsor support becomes positive, the fund's assets are depleted and the fund effectively continues to be run on a pay-as-you-go basis. The sponsor support is part of the total contribution included in the calculation of the actuarial assets (see Appendix A).

2.6 The funding ratio

The evaluation of the pension fund's financial position is usually based on its funding ratio (FR), which we calculate here as the ratio of its actuarial assets over its liabilities:

$$FR_t = \frac{A_t^{act}}{L_t}.$$

The funding ratio measures the financial health of the pension fund and is an important quantity to guide the policies of the fund.

3 The data

All our data are for the U.S. Our dataset comprises macroeconomic data, data from financial markets, data on state pension funds and demographic data. The economic scenario generator described in the following section is based on historical data spanning the third quarter of 1971 up to and including the last quarter of 2012. We use historical time series of the short interest rate, stock returns, price inflation and wage inflation, which are the state variables of the VAR model presented below. The short interest rate is 3-month Treasury rate series and it is obtained from two sources. The data until 1982 come from the Handbook of Monetary Economics (McCulloch, 1990). The data from 1982 onwards is available through the Economic Data from the Federal Reserve Bank of St. Louis (2014a). For the stock returns we use the value-weighted NYSE/Amex/Nasdaq return including dividends, which is available from the Center for Research in Security Prices (2014b). For price inflation we use the CPI index provided in Treasury and Inflation Indices dataset by the Center for Research in Security Prices (2014a). Finally, wage inflation is the compensation of employees, wages and salary accruals, retrieved from the Economic Database of Federal Reserve Bank of St. Louis (2014b). These data are all quarterly or of higher frequency and transformed into quarterly data. To value the cash flows we use a term structure of interest rates based on U.S. Treasury zero-coupon rates for maturities from 1 to 10 years from the Federal Reserve Board (2014). Pension plan data are obtained from the Public Plans Database of the Center for Retirement Research at Boston College (2014). As far as the demography is concerned, we use the National Population Estimates for 2010 by single year of age and sex provided by the United States Census Bureau (2014). Further, we use the survival rates derived from the Cohort Life Tables for the Social Security Area by Year of Birth and Sex provided by the U.S. Social Security Administration (2014). These life tables are available for birth years with 10 years intervals. Therefore, we linearly interpolate the survival rates for the birth years in between the ones directly available.

4 The economic scenario generator

An economic scenario is one possible realization of economic variables over a certain period of time in the future. It is generally not known which scenario will occur in reality, hence our simulations make use of a number of possible scenarios to evaluate the performance of an object of interest under all these realizations. Here, we use a vector auto-regressive (VAR) model to generate scenarios. We estimate the VAR on historical data for the U.S. Then, using the estimated VAR, we generate a set of scenarios and use this same set for all alternative policies that we study.

We use a first-order VAR model:

$$X_{t+1} = \begin{bmatrix} y_{t+1} \\ xs_{t+1} \\ cpi_{t+1} \\ w_{t+1} \end{bmatrix} = \alpha + \Gamma X_t + \varepsilon_{t+1}, \quad (7)$$

where y_{t+1} is the short-term interest rate, xs_{t+1} is the excess return on stocks, cpi_{t+1} is consumer price inflation and w_{t+1} is nominal wage growth, all in natural logarithms. The noise term ε_{t+1} follows a multivariate normal distribution:

$$\varepsilon_{t+1} \sim \mathbb{N}(0, \Sigma). \quad (8)$$

Further,

$$\alpha = (I - \Gamma)\mu, \quad (9)$$

where I denotes the identity matrix and μ is the unconditional mean of the vector X_t for all t . Equation (8) implies that:

$$E_t [X_{t+1}] = (I - \Gamma)\mu + \Gamma X_t. \quad (10)$$

The estimates $\hat{\alpha}$ and $\hat{\Gamma}$ for α and Γ , respectively, are obtained by estimating equation (7) using OLS. We obtain $\hat{\Sigma}$ as the variance-covariance estimate of the vector of error terms. Given $\hat{\Gamma}$ and $\hat{\alpha}$, the estimate μ is:

$$\hat{\mu} = (I - \hat{\Gamma})^{-1} \hat{\alpha}. \quad (11)$$

The estimates $\hat{\alpha}$, $\hat{\Gamma}$, $\hat{\Sigma}$ and $\hat{\mu}$ are used for pricing the cash flows – see below.

5 Valuation

A pension plan is a financial contract whose payoff depends on market contingencies. For example, the contract might specify that benefits are higher and/or contributions are lower

when market outcomes are good, and vice versa. In that case, the implicit return on the pension deal depends on the plan specifics as well as the market conditions. If we perceive the pension contract as a combination of contingent claims, we can value the pension deal using the derivative pricing techniques of risk-neutral valuation introduced by Black and Scholes (1973). Specific valuation issues of contingent claims in pension contracts are discussed in Hoevenaars and Ponds (2008), Lekniute (2011) and Ponds and Lekniute (2011).

To value a contract we use the scenarios for the underlying variables produced by our scenario generator. One can calculate the market value of a contingent claim by using deflators and real-world scenarios, or using risk-neutral scenarios. Both alternatives should result in the same outcome. Here, we will use risk-neutral valuation. We want to value the pension contract to the different stakeholders of the fund. Therefore, we will calculate the value of the net benefits of each cohort of pension fund participants and of each cohort of tax payers and of the residual assets in the fund. We will value the cash flows over a horizon of T years.

The value $V_0^{P,a}$ of the pension contract to participants of the cohort aged a at the starting moment $t = 0$ is:

$$V_0^{P,a} = E_0^Q \left[(R_1^f)^{-0.5} NB_1^a + \sum_{t=2}^T \left((R_t^f)^{-0.5} NB_t^a \prod_{j=1}^{t-1} (R_j^f)^{-1} \right) \right],$$

where $R_j^f = 1 + r_j^f$ is the gross return on a short-term risk-free bond and r_j^f the corresponding (net) rate of return in time period j . E_0^Q is the risk-neutral expectation under the \mathbb{Q} measure of the cash flows discounted against the risk-free rate. Further, NB_t^a is the net benefit, which is the contribution for workers and the benefit payment for pensioners, hence NB_t^a is negative for workers and positive for retirees. Therefore, for a cohort that is already retired at $t = 0$, $V_0^{P,a} > 0$. NB_t^a occurs in the middle of time period t . Hence, it is discounted only for half of period t . Similarly, the value of the tax payers' contributions is calculated as

$$V_0^T = -E_0^Q \left[(R_1^f)^{-0.5} C_1^T + \sum_{t=2}^T \left((R_t^f)^{-0.5} C_t^T \prod_{j=1}^{t-1} (R_j^f)^{-1} \right) \right].$$

where C_t^T is the tax-payers' contribution in dollars. Finally, at the end of evaluation horizon the pension fund generally has some assets left, of which the value is analogously calculated as:

$$V_0^A = E_0^Q \left[A_T \prod_{j=1}^T (R_j^f)^{-1} \right].$$

In line with the literature (e.g., see Cochrane, 2005), we assume that the stochastic discount rate $-m_{t+1}$ for the real-world scenario's is given by the following function of the state vector

generated by our VAR model and the shocks to this state vector:

$$-m_{t+1} = e'X_t + \frac{1}{2}(\beta_0 + \beta_1 X_t)' \Sigma (\beta_0 + \beta_1 X_t) + (\beta_0 + \beta_1 X_t)' \varepsilon_{t+1}, \quad (12)$$

where β_0 and β_1 are a vector and a matrix of parameters respectively and e indicates the position of the short rate in the state vector:

$$e = (1, 0, 0, 0, 0)'$$

Appendix B shows the details for the pricing of the cash flows and the transformation to the risk-neutral scenarios needed to calculate the value of the pension contract.

6 Baseline settings for our U.S. state plan

The annual averages in the scenario set generated as explained in Section 4 and used for further calculations, are 11.7% for the stock returns, 6.3% for 10-year Treasury bonds, 5.7% for nominal wage growth and 3.7% for consumer price inflation.

We set the simulation horizon to 75 years. At the start $t = 0$ of the simulation horizon we set the population of our pension fund equal to the aggregate number of participants in 2010 of all the state plans covered by the data from the Center for Retirement Research at Boston College (2014). As a result, our pension fund initially has approximately 24 million participants. While these data do not include all the U.S. public pension plans, it does include a very substantial share of the U.S. public sector workers at the sub-national level. Hence, the magnitude of the net burden on the tax payers that we will expose below provides a good indication of the seriousness of the public sector pension underfunding problem for the U.S., although it is likely only a lower bound to the problem. The initial demography of the fund is assumed to be identical to that of the U.S. population in 2010. Hence, the relative sizes of the cohorts are equal to that for the U.S. in 2010.

The fund's demography evolves as follows over the simulation horizon. Individuals enter the labor market at the age of 25. Hence, for the first 25 years of the simulation horizon we determine the sizes of the 25-year old cohort by taking the sizes of the cohorts younger than 25 in the U.S. population in 2010 and projecting the size of each such cohort when it reaches the age of 25 by using the survival probabilities. The number of new entrants for the remaining 50 years of a simulation run is calculated by extending over time the trend in the size of the 25-year old cohort, i.e. by calculating the average growth rate of the 25-year old cohort over the first 25 years and taking this as the annual growth rate for the next 50 years. At the start of the simulation there are about 260 thousand 25-year old males and about 250 thousand 25-year old females entering the fund. During the first 25 years the number of new 25-year

olds decreases on average by 0.34% per year. Extrapolating this trend over the next 50 years yields roughly 200 thousand and 190 thousand new male and female entrants, respectively, in the final simulation year.

Our baseline simulation is based on the most common characteristics of the U.S. state pension plans. Its liabilities are computed by using the entry age normal actuarial method for liability recognition and are based on the following actuarial assumptions. Individuals retire at the age of 65. Hence, a full career means that they work for 40 years. We distinguish between men and women in terms of the survival rates, but we assume no gender differences in wage levels. The career (age-wage) profile is obtained from Bucciol (2012). We assume that the shape of this profile does not change over time, hence the wage earned at a given age increases each year with the common wage growth rate in the economy. The $t = 0$ average wage rate is set such that the fund's initial liabilities L_0 are equal to 3,400 billion dollars, which is the aggregate amount of liabilities in 2010 over all pension funds in the Public Plans Database of the Center for Retirement Research at Boston College (2014).³ The benefit factor, or accrual rate, is 2% for each additional year of service, while the earnings base for the retirement benefit is the average of the final three years of pay during the career. During retirement there is always full indexation for consumer price inflation, also when inflation is negative.

Projected benefits are heightened up with an assumed salary increase of 5.4% for each working year and with an assumed rate of annual price inflation of 3.5% for years in retirement. Note that the salary increase used in the projections does not necessarily have to be equal to the realization or even the average of the wage growth in the economy. It is based on the average for the funds in our sample. Future retirement benefits are discounted at a fixed rate of 8% a year, which is the current median for the U.S. state pension funds. Hence, the implicit assumption is that the fund's assets earn an average annual return of 8%. We assume the initial accrued pension rights, B_0^a , to have been fully indexed up to time $t = 0$.

Contributions to the fund are calculated as follows. The annually required contribution is set to the normal cost plus the required amortization payment. The latter is fixed at zero in the case of a fund surplus (one-sided policy). The normal cost is calculated as a percentage of the projected career salary based on the EAN actuarial method. The amortization payment is determined by spreading the unfunded actuarial accrued liability $UAAL$ in equal annual payments over the next 30 years, with a moving 30-years window. Hence, we use the so-called open amortization period. The employees pay a fixed 6% contribution of their salary, while the employer pays the remainder of the required contribution. The actual contribution is set to 100% of the normal cost, plus 50% of the required amortization payment. Our sample data show that, assuming that the normal cost is paid in full, the actual amortization payment is on average 10% of the required amortization payment. However, these data are for the year 2010,

³Effectively, this implies that we have to scale down the wage relative to the actual average U.S. wage in 2010. Implicitly, this rescaling corrects for the potential presence of vesting periods, which are not explicitly included in our model of the pension fund, and the difference between average public and economy-wide wages.

the year following the year with the worst economic performance since the Great Depression and a year with unusually large declines in the value of stocks. This suggests that the share of the required amortization payment paid in that year may have been unusually low. Because of the lack of potentially more representative data we thus assume that 50% of the required amortization payment is paid in a year. However, below we will also examine variations on the baseline in which less or more of the required amortization payment is paid in a year.

The funding position of the pension fund is calculated as the ratio of the actuarial assets computed by averaging the excess returns of the last 5 years and the amount of liabilities. We set the initial funding ratio FR_0 equal to 75%, which is close to the average 2010 funding ratio of 73% of the pension funds in our database. Based on FR_0 and the initial liabilities, which are thus both matched to our data, we obtain the initial actuarial assets of the fund:

$$A_0^{act} = FR_0 \cdot L_0.$$

Although more asset classes exist, we simplify the asset mix somewhat and assume that the fund invests 50% of its assets in fixed income and the other 50% in risky assets. This implies that an average annual return on the fund's portfolio is 9%.

7 The reforms

We consider a number of variations on the baseline plan, which we refer to as Plan 0.0, to explore what various policy changes imply for the contract values of the various stakeholders. This can give us leads about the effectiveness of different measures in increasing the financial sustainability of the pension fund and, hence, in reducing reliance on the support from the tax payers.

We consider three groups of measures, which we summarize in Table 1. The first set of measures, Plans 1.1-1.4, addresses the contribution rate. Plans 1.1 and 1.2 vary the fraction of the required amortization payment actually paid, Plan 1.3 shortens the period over which the amortization payment is spread, while Plan 1.4 doubles the contribution payment by the participants from 6% to 12%, but leaves the total contribution rate unchanged. In other words, in Plan 1.4 the financial health of the pension fund itself is unaffected, while the burden on the tax payers is alleviated.

The second set of alternatives addresses the degree of indexation. As under the baseline plan, under each alternative plan, when CPI inflation is negative, then indexation is always full (thus negative). Plan 2.1 halves indexation when CPI inflation is positive. Under Plan 2.2, if CPI inflation is positive, then indexation to CPI inflation is conditional on the level of the funding ratio FR . Specifically, if $cpi \geq 0$, indexation is 0, if $FR < 0.5$, and $(2 * FR - 1) * cpi$, if $FR \geq 0.5$. In other words, if CPI inflation is positive, indexation is zero for funding ratios

Table 1: Summary of alternative plans

Plan	Description
0.0	Baseline
Contribution	
1.1	0% amortization paid
1.2	100% amortization paid
1.3	Amortization spread over 10 years
1.4	Participants' contribution rate doubled to 12%
Indexation	
2.1	Indexation at $0.5 * CPI$
2.2	Conditional indexation
Portfolio composition	
3.1	100% stocks
3.2	0% stocks

below one half, while it increases linearly with the funding ratio for funding ratios of at least one half. Hence, a funding ratio above unity implies more than full indexation. This way of providing conditional indexation is closely related to the way most Dutch pension funds index their pension rights, (see Ponds and van Riel (2009) and Beetsma and Buccioli (2011b)). We continue to set contributions on the basis of a normal cost that is still calculated under the assumption of full indexation. We do this in order to see the isolated effect of the reduction in indexation. Otherwise, the comparison with the baseline would be contaminated by a fall of the normal cost in response to reduced indexation, which through lower contributions would in turn dampen the beneficial effect of less indexation on the fund's financial health.

The third group of measures concerns the composition of the fund's asset portfolio, which we vary from zero to 100% stocks.

8 Results

8.1 The "classic" ALM results

This subsection discusses the "classic" (i.e., not applying market-based valuation) ALM simulation results under the baseline and the alternative policies. Here, and in the sequel, we simulate a set of 5000 economic scenarios each over a horizon of 75 years. This horizon allows us to take into account a number of cohorts that have not entered the fund at the start of the evaluation horizon. Moreover, it is a commonly-used horizon for pension policy evaluation in the U.S.

Table 2a reports for the various plans the 5%, 50% and 95% percentiles after 25 years for

the funding ratio FR , the pension result PR , the total contribution rate (c), the normal cost payment (cNC), the amortization payment ($cAmort$) and the sponsor support payment (cSS). The pension result is defined as the ratio of cumulative granted indexation to cumulative price inflation, i.e. $PR_t = \prod_{\tau=0}^t (1 + \pi_\tau) / \prod_{\tau=0}^t (1 + cpi_\tau)$. Hence, the lower it is, the larger the deterioration in the purchasing power of the benefits. All contribution measures are expressed in percentages of wage sum. Table 2b reports all the corresponding numbers after 75 years.

Consider first the results for the baseline. After 75 years the median value of the funding ratio is slightly above its initial value of 75%. However, we observe a wide spread in the development of the funding ratio. Its fifth percentile after 75 years is 0, implying that the fund is effectively run on a pay-as-you-go basis, with benefits on a period-by-period basis paid from participants' and tax payers contributions. The substantial probability of deterioration of the funding ratio is not surprising in view of the fact that only 50% of the required amortization costs is paid when the fund's financial health is poor. The pension result is by construction equal to unity because full indexation is always granted, including a negative adjustment in case of deflation. Table 3 reports the probability that all assets are fully depleted within the 75-year simulation horizon. This probability is 9% for the baseline, while, conditional on a full depletion taking place, the median year in which this happens is 65 years from the start of the simulation.

We now discuss the variations on the baseline setting. Under all the alternatives the component of the contribution rate based on the normal cost increases from approximately 16% after 25 years to somewhat less than 18% after 75 years, under the full indexation assumption. When no amortization payments are made (Plan 1.1), the total contribution rate is equal to the normal cost. However, this leads to a dramatic expected deterioration of the funding ratio. Even the median funding ratio after 75 years is 0 and the chances that the funds assets are fully depleted within the simulation horizon rises to 59%. By contrast, a shift to 100% amortization under Plan 1.2 tends to produce a gradual improvement of the funding ratio; the median funding ratio after 75 years is 1.27. The spread in the distribution of the funding ratio is still quite high, but extremely low funding ratios are not very likely at the end of the horizon. Indeed, the probability of a full depletion of the funds assets within the simulation horizon is virtually zero. The flip side of the improvement in the financial position of the fund is, of course, that the total contribution rate c rises. The median contribution rate after 25 years is slightly more than 21%, hence more than 1 percentage point above the median under the baseline and 5 percentage points above the median under Plan 1.1. In the long run, however, the improved financial position of the fund allows contribution rates to fall again. After 75 years, no amortization payment is needed anymore under Plan 1.2 on average, whereas the base plan with only half of required amortization paid still requires 3% amortization on average. Reducing the amortization period to 10 years has a similar and, in fact, even stronger positive effect on the fund's financial position. Compared to Plan 1.2 the distribution of funding ratios after 75 years shifts even further upwards. Of course, the contribution rates in the meantime tend to be even higher, with a ninety-fifth percentile of 34% after 25 years, as compared to 30% under

Table 2: Classic ALM results

(a) after 25 years

	0.0	1.1	1.2	1.3	1.4	2.1	2.2	3.1	3.2
FR 5%	32%	15%	44%	52%	32%	40%	45%	24%	21%
FR 50%	69%	56%	79%	87%	69%	81%	81%	109%	32%
FR 95%	155%	144%	162%	170%	155%	174%	148%	493%	49%
PR 5%	100%	100%	100%	100%	100%	54%	41%	100%	100%
PR 50%	100%	100%	100%	100%	100%	67%	69%	100%	100%
PR 95%	100%	100%	100%	100%	100%	80%	121%	100%	100%
c 5%	16%	16%	16%	16%	16%	16%	16%	16%	23%
c 50%	20%	16%	21%	21%	20%	18%	18%	16%	25%
c 95%	25%	16%	30%	34%	25%	23%	22%	26%	26%
cAmort 5%	0%	0%	0%	0%	0%	0%	0%	0%	6%
cAmort 50%	4%	0%	5%	5%	4%	2%	2%	0%	8%
cAmort 95%	8%	0%	14%	18%	8%	7%	6%	9%	10%
cNC	16%	16%	16%	16%	16%	16%	16%	16%	16%
cSS 5%	0%	0%	0%	0%	0%	0%	0%	0%	0%
cSS 50%	0%	0%	0%	0%	0%	0%	0%	0%	0%
cSS 95%	0%	0%	0%	0%	0%	0%	0%	0%	0%

(b) after 75 years

	0.0	1.1	1.2	1.3	1.4	2.1	2.2	3.1	3.2
FR 5%	0%	0%	40%	58%	0%	33%	48%	0%	0%
FR 50%	79%	0%	127%	156%	79%	171%	109%	356%	0%
FR 95%	636%	522%	732%	799%	636%	917%	226%	7856%	1%
PR 5%	100%	100%	100%	100%	100%	17%	11%	100%	100%
PR 50%	100%	100%	100%	100%	100%	25%	65%	100%	100%
PR 95%	100%	100%	100%	100%	100%	37%	1580%	100%	100%
c 5%	18%	18%	18%	18%	18%	18%	18%	18%	31%
c 50%	21%	18%	18%	18%	21%	18%	18%	18%	32%
c 95%	32%	18%	35%	36%	32%	26%	24%	32%	33%
cAmort 5%	0%	0%	0%	0%	0%	0%	0%	0%	13%
cAmort 50%	3%	0%	0%	0%	3%	0%	0%	0%	14%
cAmort 95%	14%	0%	17%	18%	14%	9%	7%	14%	15%
cNC	18%	18%	18%	18%	18%	18%	18%	18%	18%
cSS 5%	0%	0%	0%	0%	0%	0%	0%	0%	1%
cSS 50%	0%	18%	0%	0%	0%	0%	0%	0%	7%
cSS 95%	7%	26%	0%	0%	7%	0%	0%	8%	11%

Note: classic ALM results for the 5%, 50% and 95% percentiles of the funding ratio (FR), the pension result (PR), the contribution rate (c), which is split into amortization ($cAmort$) and normal cost (cNC), and sponsor support (SS) for the base Plan 0.0 and alternative contribution (Plans 1.1-1.4), indexation (Plans 2.1-2.2) and investment (Plans 3.1-3.2) policies.

Table 3: Likelihood of full depletion of assets

	0.0	1.1	1.2	1.3	1.4	2.1	2.2	3.1	3.2
Probability	9.0%	59.0%	0.0%	0.0%	9.0%	0.1%	0.0%	11.8%	97.2%
Year, 5%	47.5	29	-	-	47.5	54.75	-	34	51
Year, 50%	65	47	-	-	65	66.5	-	57	61
Year, 95%	75	70	-	-	75	74	-	74	72

Note: the first line reports the probability that the fund's assets are fully depleted within the 75-year simulation horizon. The following lines show the percentiles for the distribution of the years of depletion, conditional on scenarios in which depletion takes place within the simulation horizon. As an example, under the baseline Plan 0.0, given that depletion takes place, in less than 5% of the cases this happens within the first 28 years of the simulation period. If there is no single year that stands exactly in the position of the quantile, then averages of the surrounding years are taken. Hence, the moment a quantile is reached can be a fraction of a full year.

Plan 1.2 and 25% under the base plan. However, at the end of the full horizon, the median contribution rate for the amortization has fallen to zero and, hence, the total contribution rate is 3 percentage points below the level under the baseline Plan 0.0. The reported numbers under Plan 1.4 are identical to those under the baseline. Funding ratios, total contributions and amortization contributions are identical, though of the total contributions now twice as much as before is paid by the participants.

The second set of alternative measures introduces changes in the indexation rate. Plan 2.1 always yields lower indexation, implying an expected improvement in the financial position of the funding over the long horizon. Although a long-term halving of the indexation rate has a substantial effect on the purchasing power of the pension benefits, the improvement in the funding ratio is not as spectacular as one might a priori expect, because the total contribution in the meantime falls below that under the baseline. While the normal cost rate remains the same, the amortization contribution rate becomes lower than under the baseline. After 25 years, the median total contribution rate is 2 percentage points lower than under the baseline. At the end of the full horizon, the median amortization contribution rate has fallen to zero and the difference in the median total contribution with the baseline has risen to 3 percentage points.

A shift from the baseline to conditional indexation (Plan 2.2) also tends to improve the financial position of the fund. Compared to Plan 2.1, where indexation is halved, the spread in the distribution of the funding ratios becomes smaller, with the fifth percentile under Plan 2.2 at the end of the 75-year horizon located at 48% instead of 33% under Plan 2.1. The compression in the distribution of funding ratios is not surprising, because the policy is specifically aimed at stabilizing the funding ratio with smaller indexation given when the fund's financial situation is unhealthy and higher, possibly more than full, indexation given when the fund runs a surplus. As a result, the distribution of the total contribution rates is also more compressed. At the end

of the full horizon, the median amortization contribution rate is zero.

Our third set of measures looks at changes in the pension fund's asset portfolio. Not surprisingly, a policy of investing 100% of the portfolio in stocks, Plan 3.1, raises the median funding ratio compared to the baseline, but it also raises the spread in funding ratios. After 75 years there is a non-negligible probability that the assets are down to zero. A complete shift out of stocks, Plan 3.2, leads over time to a downward shift of the entire distribution of the funding ratios, such that even the ninety-fifth percentile of the distribution after 75 years is almost zero under this plan. This is not surprising, as the normal cost is based on liabilities calculated with a discount rate higher than the expected portfolio return under Plan 3.2, implying a systematic shortage of resources in the fund.

Overall, it can be concluded that under many of the settings studied here the pension plan is likely to be unsustainable. The factors contributing to this conclusion are the fund's initial underfunding and the long amortization period in combination with the amortization contribution rate being lower than the required rate. A policy of reducing the indexation rate helps in keeping the plan financially sustainable, although such a policy may not be as effective as is a priori expected. Also a policy of conditional indexation helps in keeping the fund financially healthy. The advantage of such a policy is that it compresses the distribution of funding ratios, because the indexation rate is explicitly linked to the funding ratio.

8.2 The value-based ALM results

The classic ALM results discussed above have provided us with a number of interesting results regarding the effectiveness of alternative policies to strengthen the financial sustainability of our pension fund. Value-based ALM is useful to assess changes in the market value of the pension contract to its various stakeholders when shifting from the baseline plan to another plan. To this end it is instructive to show the balance sheet in Table 4. The left-hand side reports the value of the initial assets of the pension fund (A_0) and the present value of the contributions by all cohorts of tax payers (V_0^T) over the 75 years evaluation period. On the right-hand side we find the present value of the net benefits aggregated over all cohorts of participants (V_0^P), which consists of net benefits received over the evaluation horizon, plus the present value of the stock of pension rights remaining at the end of the evaluation horizon. To evaluate these remaining pension rights we use the EAN method, which means that we take the PVB liabilities, i.e. both future accrual and wage advances are taken into account, and subtract the present value of the future normal cost. What remains is the present value of the fund residue (V_0^R), which is the difference between the present value of the final assets at the end of the evaluation horizon and the present value of final pension rights of the participants.⁴

⁴Alternatively, the residual value could have been attributed to the tax payers, since it is net of the value of the pension rights of the participants alive at the end of the evaluation horizon. However, in the sequel we keep it separate from the tax payers' value based on their contributions and sponsor support SS_t .

Table 4: The balance sheet

A_0	V_0^P
V_0^T	
	V_0^R

We evaluate at $t = 0$ the baseline contract (denoted by a tilde) and changes relative to the baseline. Reforms are zero-sum games in value terms across the funds' stakeholders:

$$\Delta V_0^P + \Delta V_0^T + \Delta V_0^R = 0.$$

where V_0 is the value of the new plan and $\Delta V_0 \equiv V_0 - \tilde{V}_0$ the change in value from the baseline. For example, if the contributions of the tax payers to the pension fund increase, this implies that the value of the fund residue, the value of the net benefits to the participants, or both, must increase. The initial assets from the balance sheet do not appear in this expression, as we start with the same value of initial assets under all alternative plans. Hence, the change in the value of the initial assets is always 0. We are also interested in the relative change ΔRV_0 in the values of the various stakeholders, computed as:

$$\Delta RV_0 = \frac{V_0 - \tilde{V}_0}{|\tilde{V}_0|} * 100\%,$$

We report the results in a number of ways. In particular, we report values for the entire group of participants, the entire group of tax payers, individual cohorts of participants, individual cohorts of tax payers and for the residue of the fund. The value of the participant cohort aged a in period 0 is denoted by $\tilde{V}_0^{P,a}$ or $V_0^{P,a}$ and follows directly from the cash flows associated with the pension contract. The aggregate cash flows in a year from all the tax payers together are also immediately available. However, calculating the values $\tilde{V}_0^{T,a}$ or $V_0^{T,a}$ for the individual cohorts of tax payers aged a in period 0 requires an assumption about the allocation of the aggregate cash flows across these cohorts. We assume that the demographic structure of the tax payer population is identical to that of the participant population and that the cash flows assigned to the individual tax payer cohorts are proportional to the shares of the individual cohorts in aggregate income. Hence, for each cohort of tax payers, the contribution is based on the size of the cohort and the income level of an individual cohort member. For tax payers of working age, the relative contribution of each cohort is fixed through the age-wage profile, which is constant over time. The relative contribution of each cohort of retired tax payers is based on the average (across all moments under all scenarios) income under the baseline plan.

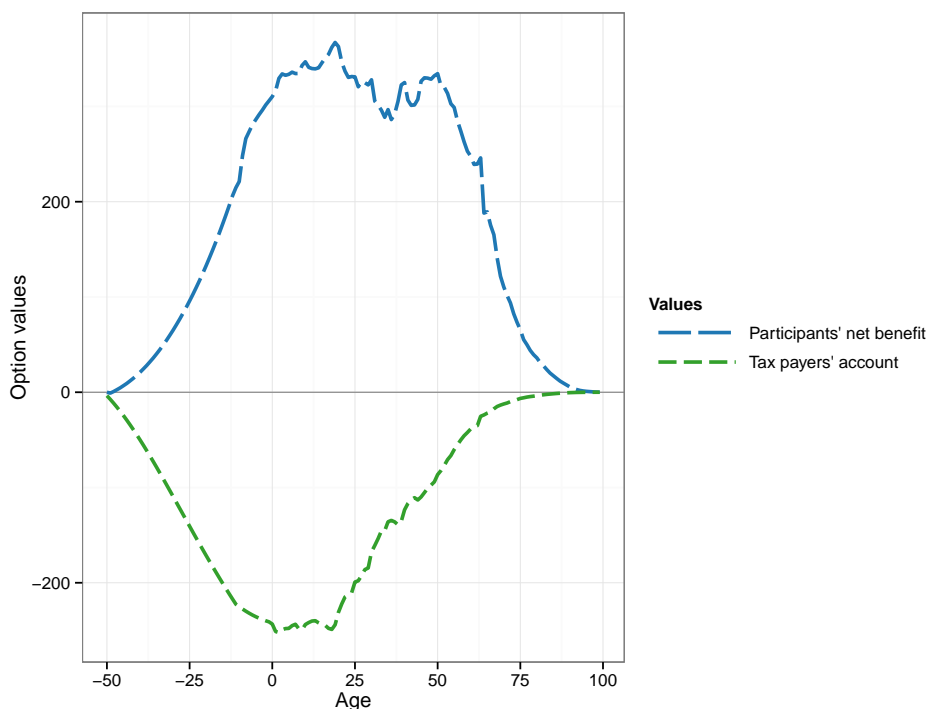
8.2.1 Baseline

Figure 1 shows for the baseline, Plan 0.0, the value of the contract $\tilde{V}_0^{P,a}$ and $\tilde{V}_0^{T,a}$ in billions of dollars for the various cohorts of fund participants and tax payers, respectively. The youngest generation is the one to be born in 50 years (age -50 at $t = 0$), whereas the oldest is 99 years old at the start of the simulation period. The figure gives an impression of the order of magnitude of the amounts at stake for the various cohorts of stakeholders in the aggregate state-level public sector in the U.S. In present value terms, under the baseline plan over the 75-year simulation horizon, the aggregate net benefit over all participant cohorts is approximately 28 trillion dollars, while the aggregate contribution over all tax payer cohorts is approximately 17 trillion dollars.⁵ These are sizable numbers and, as indicated earlier, they likely form a lower bound on the actual amount of redistribution from tax payers to fund participants. Importantly, also, the figure shows the relative distribution of the net benefits across the various participant cohorts and of the financing burden across the various cohorts of tax payers. It is useful to notice that the positive and negative areas in the figure do not sum to zero, because the value of the residue of the fund after 75 years generally differs from the value of the fund's initial assets. We see that the value of the contract is positive for all participant cohorts. This is not surprising for cohorts that have already been working for a while, i.e. the cohorts older than 25 at $t = 0$, because their contributions are "sunk", while the benefits are still ahead of them. As of the age of fifty, the contract value starts falling steeply with age. For those that are already retired, i.e. of age 65 and over at $t = 0$, the amount of future benefits to be received is shrinking with age. In addition, older cohorts have been accumulating lower rights, *ceteris paribus*, because the real wage levels were lower during their work career than for younger cohorts.

For the cohorts of age 25 and younger at $t = 0$, the positive contract value is more remarkable, because in a completely actuarially-neutral system, those who have not yet been contributing would experience zero value from their pension contract: the value of their future benefits must be offset by the value of their future contributions. However, this value is only zero for the cohort born at $t = 50$, because over the evaluation horizon this cohort will pay zero contributions and thus build up zero rights. For future generations that are born before $t = 50$, the value of the benefits they will receive over the evaluation horizon plus the value of the accrued rights at the end of the evaluation period exceeds the value of the contributions to be made during the evaluation period. This net benefit can only be financed out of two sources: a reduction in the residual value \tilde{V}_0^R of the fund or contributions from the tax payers. From the figure we observe that the value of the plan $\tilde{V}_0^{T,a}$ is negative for all cohorts of tax payers.

⁵In our discussion of these numbers we ignore the fact that part of the tax payer population are also participants in the fund. Thus, we focus on the gross redistribution between the groups and not on the net redistribution, which would require more information to construct reliable numbers.

Figure 1: Stakeholders' option values under the base plan



Participants' net benefit and tax payers' option values per age cohort under Plan 0.0, in billion dollars

8.2.2 Outcomes of the reforms

We report changes in dollar values and in relative values for groups of cohorts of participants and tax payers. The results are reported in Table 5a (changes in dollar values) and 5b (relative changes in values) for two groups of participants and two groups of tax payers. Negative relative values denote a deterioration for the particular stakeholders under consideration. The groups of the young (superscript Y) comprise all the cohorts that are younger than 25 or yet unborn at $t = 0$, while the groups of the old (superscript O) comprise all the cohorts of age 25 or older at $t = 0$. The table also report changes in the value of the residue at the end of the evaluation horizon. As explained above, theoretically, for each alternative plan, the sum of value changes, $\Delta V_0^{P,Y} + \Delta V_0^{P,O} + \Delta V_0^{T,Y} + \Delta V_0^{T,O} + \Delta V_0^R$ must be zero. However, due to numerical inaccuracies small deviations from zero may be possible.

Under Plans 1.1-1.3 neither the contributions made by the plan participants nor the indexation rules are changed, so $\Delta V_0^{P,Y} = \Delta V_0^{P,O} = 0$. A reduction in the amortization payment (Plan 1.1) benefits current tax payers, because they see their amortization contribution rate fall. The value of the funds residue falls, in line with the expected deterioration of the funding ratio over time. The policy change also has an effect on the future cohorts of tax payers, as they have to cover the deficit with sponsor support payments if assets get depleted completely. An increase in the amortization payment, Plan 1.2, implies higher contribution payments by the tax payers, so both young and old tax payers experience a loss of value. A similar conclusion holds for a

Table 5: Effects of plan changes on stakeholders

(a) Level effects

Plan	Description	$\Delta V_0^{P,Y}$	$\Delta V_0^{P,O}$	$\Delta V_0^{T,Y}$	$\Delta V_0^{T,O}$	ΔV_0^R
Contribution						
1.1	0% amortization paid	0	0	-465	709	-217
1.2	100% amortization paid	0	0	-1562	-1148	2676
1.3	amortization 10 years	0	0	-2914	-1992	4828
1.4	part. contr. rate doubled	-3335	-1105	2963	1477	0
Indexation						
2.1	indexation is 0.5*CPI	-2453	-3065	2578	383	2570
2.2	conditional indexation	-3386	-4605	3573	574	3871
Portfolio composition						
3.1	100% stocks	0	0	-277	-105	359
3.2	0% stocks	0	0	132	51	-164

(b) Relative effects

Plan	Description	$\Delta RV_0^{P,Y}$	$\Delta RV_0^{P,O}$	$\Delta RV_0^{T,Y}$	$\Delta RV_0^{T,O}$	ΔRV_0^R
Contribution						
1.1	0% amortization paid	0%	0%	-4%	16%	-3%
1.2	100% amortization paid	0%	0%	-12%	-26%	32%
1.3	amortization 10 years	0%	0%	-23%	-44%	57%
1.4	part. contr. rate doubled	-23%	-8%	23%	33%	0%
Indexation						
2.1	indexation is 0.5*CPI	-17%	-22%	20%	9%	31%
2.2	conditional indexation	-24%	-34%	28%	13%	46%
Portfolio composition						
3.1	100% stocks	0%	0%	-2%	-2%	4%
3.2	0% stocks	0%	0%	1%	1%	-2%

Note: the effects of switching from baseline Plan 0.0 to Plans 1.1-3.2 on future plan participants ($\Delta V_0^{P,Y}$, $\Delta RV_0^{P,Y}$), current plan participants ($\Delta V_0^{P,O}$, $\Delta RV_0^{P,O}$), future tax payers ($\Delta V_0^{T,Y}$, $\Delta RV_0^{T,Y}$), current tax payers ($\Delta V_0^{T,O}$, $\Delta RV_0^{T,O}$) and the residue of the pension fund (ΔV_0^R , ΔRV_0^R). Panel (a) reports changes in billions of dollars. Panel (b) reports relative changes as a percentage of baseline value. Negative numbers imply a deterioration of the value for that stakeholder.

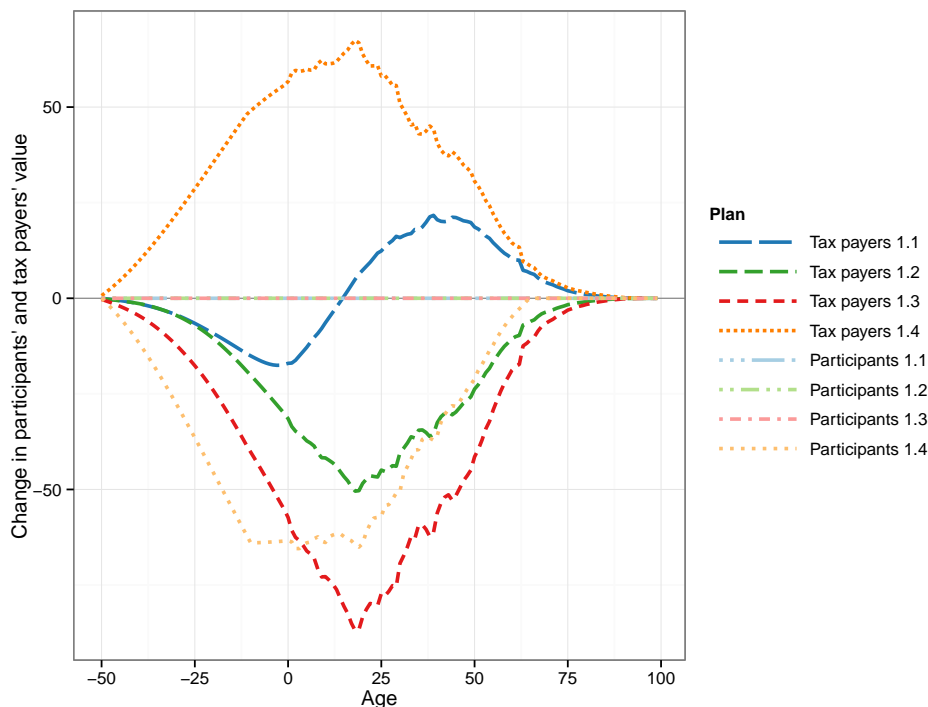
shortening of the period over which the amortization takes place, Plan 1.3. The residue value improves, but at the cost of higher contributions by the current and future tax payers, who see the value of their stake in the pension arrangement fall by a total of roughly 2.9 and 2.0 trillion dollars, respectively. A doubling of the contribution rate by the participants, Plan 1.4, reduces the total contribution paid by the tax payers and, thus, shifts value from both groups of participants to the tax payers. The total value shift from the participants to the tax payers is more than 4.4 trillion dollars. In percentage terms the older participants lose 8% of the contract value, while the younger participants lose 23%. The financial position of the fund is unchanged throughout and, hence, the change in the residue value is zero.

Changes in indexation policy shift value between the participants and the tax payers. Halving indexation, Plan 2.1, lowers the plan value to the participants and raises the plan value to the tax payers, who have to pay smaller amortization and sponsor support contributions. The total value loss to the participants is about 5.5 trillion dollars, or 17% of the contract value of the younger participants and 22% of the contract value of the older participants, of which about 3 trillion is shifted to the tax payers and the remainder to an increased residue value of the fund, because the funding ratio is expected to improve in the long-run (relative to the baseline), while the accrued pension rights of the participants at the end of the evaluation horizon are lower. Conditional indexation, Plan 2.2, produces qualitatively similar, but quantitatively even larger, value shifts across the stakeholders. This is because under Plan 2.2 no indexation is given a times when it is most valuable, and more is given when it is less valuable. Young participants lose about a quarter of their contract value, while older participants lose even a third of their contract value.

Our third set of changes concerns changes in the composition of the asset portfolio of the pension fund. In all of the previous plans the asset mix was kept constant at 50% fixed income and 50% risky assets. When the portfolio allocation is the same across the plans, the total amount of risk remains unchanged, but it is shifted among stakeholders due to policy changes, like changes in the contribution or indexation rules. However, changing the asset mix changes the risk of the pension fund. Under a symmetrical contract this should not lead to value transfers, as under such a contract higher volatility is rewarded with higher expected returns. However, the pension plan policy in question is not symmetrical from the tax payers' perspective. When bad returns materialize, tax payers have to cover the deficit by increased amortization and sponsor support payments. However, good returns do not necessarily lead to lower contribution payments as the contribution can never fall below the normal cost level. To sum up, the contribution payments are limited on the downside, but unlimited on the upside. When riskier investments are made (Plan 3.1) the upwards potential of the returns goes to the residue of the pension fund, but the downwards risk is allocated to the tax payers in the form of higher amortization and sponsor support payments. Therefore we see negative value changes for the tax payers and an increase in residue value. In the case of a derisking of the portfolio (Plan 3.2), the opposite effects can be seen. A much lower downside risk of portfolio returns

results in a lower probability of high amortization and sponsor support payments, hence the tax payers are better off. However, the lower expected portfolio returns implied by the less risky asset mix affect the residue value negatively. Since neither the indexation policy nor the participant contribution rules have changed, the fund participants are unaffected by changes in the composition of the fund's asset portfolio.

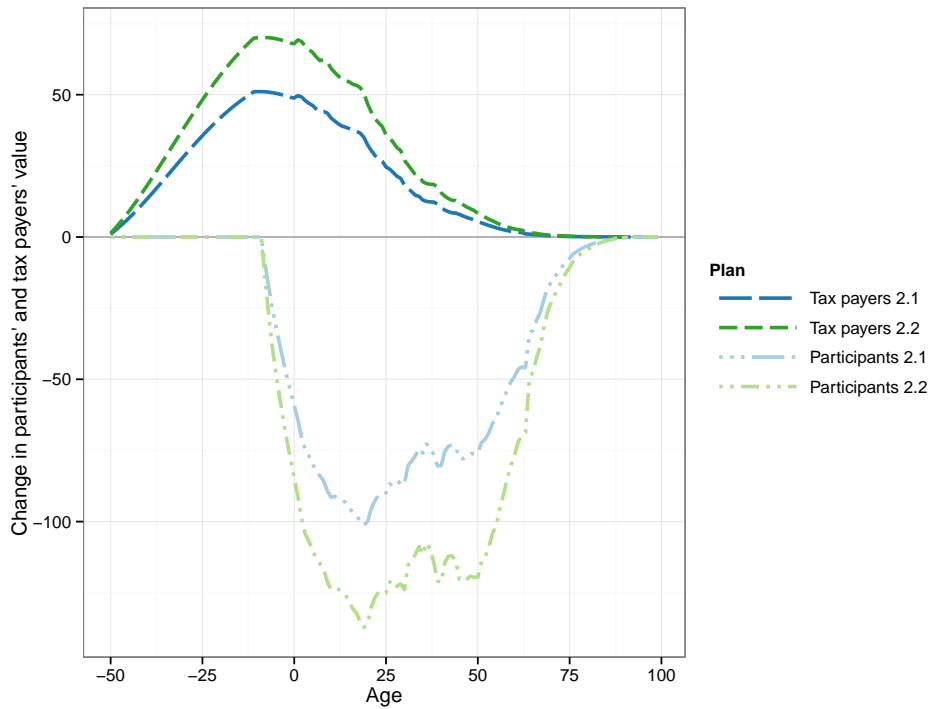
Figure 2: Stakeholders' value changes due to contribution reforms



Change in stakeholder values (in billion dollars) when policy is changed from the base Plan 0.0 to plans with no amortization payment (Plan 1.1), full amortization payment (Plan 1.2), 10 years amortization smoothing (Plan 1.3) and double contribution rate by employees (Plan 1.4).

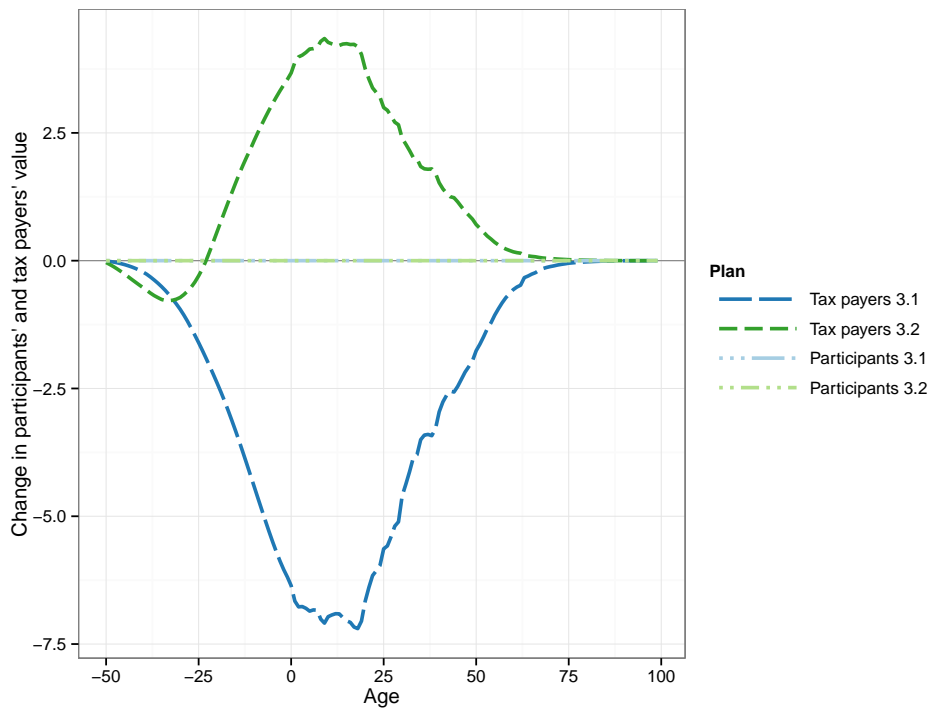
Figures 2 - 4 depict the value consequences of contract changes for all individual cohorts of participants and tax payers. In line with the figures reported in the above tables, we see indeed that moving from the baseline plan to Plan 1.1 with zero amortization the older tax payers are better off, as part of the contribution burden is shifted to the future, while younger cohorts of tax payers have to make up for the losses of the fund. Moving to 100% amortization, Plan 1.2, reduces the value for all tax payers, but ensures that the residual value of the fund is raised. Speeding up amortization, Plan 1.3, also affects all tax payer cohorts negatively over the simulation horizon, while doubling the contribution by the participants affects all participants' values negatively and all tax payers' values positively. Not surprisingly, a shift from full indexation to compensation of only half of the CPI, i.e. a shift from Plan 0.0 to Plan 2.1, lowers the net benefit value for all current participants and the first 35 cohorts of future participants. All cohorts that enter the fund later see no change in value, because they will not receive any pension benefit over the simulation horizon. Hence, over the simulation horizon they are unaffected by the change in indexation. However, since the change in indexation does

Figure 3: Stakeholders' value changes due to indexation reforms



Change in stakeholder values (in billion dollars) when policy is changed from the base Plan 0.0 to plans with half *CPI* indexation (Plan 2.1) and conditional indexation (Plan 2.2).

Figure 4: Stakeholders' value changes due to investment strategy reforms



Change in stakeholder values (in billion dollars) when policy is changed from the base Plan 0.0 to plans with 100% risky assets (Plan 3.1) and 100% fixed income assets (Plan 3.2).

affect benefit payments, it affects the final value of the assets and thus the residual value of the fund. The final liabilities are affected only through the part of the liabilities that belongs to the final pensioners, as this part incorporates all actual indexation received since retirement. The part of the liabilities belonging to the cohorts that have not yet retired at the end of the evaluation horizon does not change due to the assumption of full indexation that, independent of actual indexation, is retained throughout the different plans. Given that inflation is mostly positive, getting only half of the inflation indexed leads to lower pension benefit payments and consequently a lower value of the contract. Similarly, since indexation tends on average to be lower under conditional indexation, also Plan 2.2 affects the value for the participants in a negative way. The flip side is that all tax payer cohorts benefit from the two plan changes. Shifts in the portfolio composition of the pension fund are completely neutral in value terms for all the participants, because their benefits and contribution payments are unaffected, while we see small changes in the value for the tax payers. All tax payer cohorts suffer from a move to a 100% equity portfolio, while most tax payer cohorts benefit from a complete move out of equity. However, in the latter case, the cohorts born in periods around $t = 50$ and later lose from this shift, because the expected depletion of the fund's assets requires these cohorts to make additional amortization and sponsor support contributions.

9 Concluding remarks

This paper has explored the financial sustainability and redistributive aspects of a typical U.S. state DB pension fund under unchanged policies. The results confirm what is generally feared, namely that current pension policies are unlikely to be financially sustainable. Therefore, we explored a number of alternative policies, involving changes in the composition of the investment portfolio of the pension fund, the structure of pension contributions and policies aimed at reducing benefits, in particular policies that reduce indexation. Some of these measures can indeed be quite effective at improving the fund's financial position. Indexation conditional on the funding ratio is promising, because it can improve the fund's financial position, while keeping the potential spread in the funding ratio limited. The reason is that indexation rises with the size of the funding ratio and may even become more than full for large funding ratios. We also applied a market-based valuation approach to the cash flows generated by the fund and we found that under the baseline plan all cohorts of participants experience substantial net benefits at the expense of all cohorts of tax payers. Policies that raise the contributions by the participants or that reduce the indexation of benefits are quite effective at shifting part of the participants' net benefits back to the tax payers. We estimate the effect of halving the indexation to CPI inflation at a contract value reduction of around 20% for the participants, while conditional indexation lowers the participants' contract value by around 25-30%. In dollar terms the alleviation of the burden on the tax payers is on the order of 3 – 4 trillion dollars for the changes in indexation policy we consider, while a doubling of the participants' contribution

rate produces an alleviation of the tax payers' burden by around 4.5 trillion dollars.

The sustainability and redistribution issues exposed by this paper may well be a lower bound to the true extent of the future problems facing the U.S. civil servants' pension arrangements. First, our dataset did not include all civil servants' pension funds. A priori, there is little reason to assume that the funds that have not been included are financially healthier than the funds that were part of our dataset. Second, since we did not want to take a specific stand on the future equity premium, we have worked under the assumption that the future stock returns are in line with those in the past. However, if future equity premia turn out to be lower than those in the past, state pension funds' financial problems may become much more severe than suggested in the present paper.

A rather general conclusion from our analysis is that the substantial net benefit of the promises to fund participants at the cost of tax payers could be an argument for fund boards and participants to initiate steps towards improving their funding by raising contributions or by cutting back on benefit levels. The looming financial burden on tax payers and the prospective deterioration of public services may chase away individuals of working age from specific states. An outcome with falling spending on public goods and rising taxes merely paid to finance civil servants' retirement schemes may become politically unsustainable and result in states defaulting. This could well result in larger value losses to scheme participants than those associated with orderly reforms backed by a majority of the population.

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Appendices

Not for publication – for referees' use

A Calculation of the actuarial assets

Define the Investment Income Amount Of Immediate Recognition (*IIAOIR*) as a target investment income based on the *expected* return \bar{R}_t :

$$IIAOIR_t = A_t(\bar{R}_t - 1) + (C_t - B_t)(\bar{R}_t^{1/2} - 1). \quad (13)$$

The so-called Investment Income Market Total (*IIMT*) denotes the actual realization of investment income, i.e. the difference between the market value of assets at the end of the year and the market value of assets at the beginning of the year, less the net cash inflow associated with contributions and benefits. From equation (3) we get:

$$IIMT_t = A_{t+1} - A_t - (C_t - B_t) = A_t(R_t - 1) + (C_t - B_t)(R_t^{1/2} - 1). \quad (14)$$

Finally, the Investment Income Amount For Phased In Recognition (*IIAFPPIR*) is realized investment income in excess of expected investment income:

$$IIAFPIR_t = IIMT_t - IIAOIR_t.$$

Hence, *IIAFPIR*_{*t*} is positive when the actual investment return exceeds its expected value, and vice versa. The usual smoothing procedure to calculate actuarial assets involves taking the average of the excess investment incomes over the past. We define the Total Recognized Investment Gain (*TRIG*) as the average of *IIAFPIR* over a smoothing horizon of *v* years:

$$TRIG_t = \frac{1}{v} \sum_{i=0}^{v-1} IIAFPPIR_{t-i}.$$

Then, the actuarial value of the assets at the beginning of year *t+1* will be determined by raising the actuarial value of assets at the beginning of year *t* with the net money inflows (contribution payments minus the benefit pay-outs), investment income of immediate recognition and the smoothed value of the excess investment income over the past *v* years:

$$A_{t+1}^{act} = A_t^{act} + (C_t - B_t) + IIAOIR_t + TRIG_t. \quad (15)$$

B Pricing according to risk-neutral scenarios

This appendix provides the details of our valuation of the cash flows associated with the pension contract.

B.1 General pricing framework

Consider some derivative (e.g., the pension contract) with a payoff of Z_τ at time τ , which is a function of the path X_1, X_2, \dots, X_τ of the state vector. The price of the derivative at time t is then given by

$$P_t = \sum_{\tau=t+1}^{\infty} E_t \left[Z_\tau \exp \sum_{s=t+1}^{\tau} m_s \right].$$

In a complete market it is possible to sell the derivative at time $t+1$ for its price P_{t+1} . Hence the following must hold:

$$P_t = E_t [P_{t+1} \exp m_{t+1}], \quad (16)$$

where P_{t+1} is the total price based on the total return index where any payoff is reinvested in the same index. If we use lower-case letters to denote log-values so that

$$p_t = \log P_t,$$

knowing that X_{t+1} has a Gaussian distribution, and using the properties of the log-normal distribution, we can derive from equation (16):

$$\begin{aligned} \exp p_t &= E_t [\exp p_{t+1} \exp m_{t+1}] = E_t [\exp(p_{t+1} + m_{t+1})] \\ &= \exp (E_t [p_{t+1} + m_{t+1}] + 1/2 \text{Var}_t [p_{t+1} + m_{t+1}]), \end{aligned}$$

Hence,

$$p_t = E_t [p_{t+1} + m_{t+1}] + 1/2 \text{Var}_t [p_{t+1} + m_{t+1}]. \quad (17)$$

Note that

$$E_t [m_{t+1}] = -e' X_t - \frac{1}{2} (\beta_0 + \beta_1 X_t)' \Sigma (\beta_0 + \beta_1 X_t), \quad (18)$$

$$\text{Var}_t [m_{t+1}] = (\beta_0 + \beta_1 X_t)' \Sigma (\beta_0 + \beta_1 X_t). \quad (19)$$

We will now apply our pricing framework to the various assets that are relevant for us.

B.1.1 Nominal bonds

Denote by $p_t^{(n)}$ the period- t price of a zero coupon bond that matures at time $t + n$ and pays one unit of currency at maturity date. We assume that it is an affine function of the state variables:

$$p_t^{(n)} = -D_n - H'_n X_t. \quad (20)$$

Its nominal yield $Y_t^{(n)}$ satisfies the following relationship:

$$\left(1 + Y_t^{(n)}\right)^{-n} = P_t^n. \quad (21)$$

If we denote $\ln(1 + Y_t^{(n)})$ as $y_t^{(n)}$, it follows from equations (20) and (21) that

$$\begin{aligned} \ln\left(1 + Y_t^{(n)}\right) &= -\frac{1}{n} \ln P_t^n, \\ y_t^{(n)} &= -\frac{1}{n} p_t^{(n)} = \frac{1}{n} D_n + \frac{1}{n} H'_n X_t. \end{aligned} \quad (22)$$

Applying the pricing equation (17) to zero-coupon bonds and using equation (20) we get:

$$\begin{aligned} p_t^{(n)} &= \mathbb{E}_t \left[p_{t+1}^{(n-1)} + m_{t+1} \right] + 1/2 \text{Var}_t \left[p_{t+1}^{(n-1)} + m_{t+1} \right] \\ &= -D_{n-1} - H'_{n-1} \mathbb{E}_t [X_{t+1}] \\ &\quad + \mathbb{E}_t [m_{t+1}] \\ &\quad + \frac{1}{2} \text{Var}_t [-H'_{n-1} X_{t+1}] \\ &\quad + \frac{1}{2} \text{Var}_t [m_{t+1}] \\ &\quad + \text{Cov}_t [-H'_{n-1} X_{t+1}, m_{t+1}]. \end{aligned} \quad (23)$$

Using (18), (19) and (10) we can rewrite (23) as

$$\begin{aligned} p_t^{(n)} &= -D_{n-1} - H'_{n-1} ((I - \Gamma)\mu + \Gamma X_t) \\ &\quad - e' X_t - \frac{1}{2} (\beta_0 + \beta_1 X_t)' \Sigma (\beta_0 + \beta_1 X_t) \\ &\quad + \frac{1}{2} (H'_{n-1} \Sigma H_{n-1}) \\ &\quad + \frac{1}{2} (\beta_0 + \beta_1 X_t)' \Sigma (\beta_0 + \beta_1 X_t) \\ &\quad + H'_{n-1} \Sigma (\beta_0 + \beta_1 X_t), \end{aligned}$$

where the last part follows from

$$\begin{aligned}
\text{Cov}_t [-H'_{n-1}X_{t+1}, m_{t+1}] &= \text{Cov}_t[-H'_{n-1}((I - \Gamma)\mu + \Gamma X_t + \varepsilon_{t+1}), \\
&\quad - e'X_t - \frac{1}{2}(\beta_0 + \beta_1 X_t)' \Sigma (\beta_0 + \beta_1 X_t) - (\beta_0 + \beta_1 X_t)' \varepsilon_{t+1}] \\
&= \text{E}_t [(-H'_{n-1}\varepsilon_{t+1})(-(\beta_0 + \beta_1 X_t)' \varepsilon_{t+1})] \\
&= \text{E}_t [(H'_{n-1}\varepsilon_{t+1})(\varepsilon'_{t+1}(\beta_0 + \beta_1 X_t))] \\
&= H'_{n-1} \text{E}_t [\varepsilon_{t+1} \varepsilon'_{t+1}] (\beta_0 + \beta_1 X_t) \\
&= H'_{n-1} \Sigma (\beta_0 + \beta_1 X_t).
\end{aligned}$$

We can further rewrite $p_t^{(n)}$ as

$$\begin{aligned}
p_t^{(n)} &= -D_{n-1} - H'_{n-1}(I - \Gamma)\mu - H'_{n-1}\Gamma X_t \\
&\quad - e'X_t \\
&\quad + \frac{1}{2}H'_{n-1}\Sigma H_{n-1} \\
&\quad + H'_{n-1}\Sigma\beta_0 + H'_{n-1}\Sigma\beta_1 X_t \\
&= -D_{n-1} - H'_{n-1}(I - \Gamma)\mu + \frac{1}{2}H'_{n-1}\Sigma H_{n-1} + H'_{n-1}\Sigma\beta_0 \\
&\quad - (e' + H'_{n-1}\Gamma - H'_{n-1}\Sigma\beta_1)X_t.
\end{aligned}$$

The last equation is already of the affine structure as in equation (20) with parameters

$$\begin{aligned}
D_n &= D_{n-1} + H'_{n-1}(I - \Gamma)\mu - \frac{1}{2}H'_{n-1}\Sigma H_{n-1} - H'_{n-1}\Sigma\beta_0, \\
H_n &= e + (\Gamma - \Sigma\beta_1)'H_{n-1}.
\end{aligned} \tag{24}$$

For $n = 0$ we have that:

$$p_t^{(0)} = \ln P_t^{(0)} = \ln 1 = 0,$$

which is given by equation (20) with parameters

$$\begin{aligned}
D_0 &= 0, \\
H_0 &= 0.
\end{aligned} \tag{25}$$

Using (25) in (24) we get

$$\begin{aligned}
D_1 &= 0, \\
H_1 &= e.
\end{aligned} \tag{26}$$

This implies

$$p_t^{(1)} = -e'X_t = -y_t^{(1)}, \tag{27}$$

because e indicates the position of the short rate in the state vector. The deflator is calibrated on the short rate so that this constraint is satisfied.

B.1.2 Investable assets

The excess return on investable assets other than the nominal bond is defined as follows and can be rearranged using (27):

$$r_{t+1} - y_t^{(1)} = \ln\left(\frac{P_{t+1}}{P_t}\right) - y_t^{(1)} = p_{t+1} - p_t + p_t^{(1)}, \quad (28)$$

$$p_{t+1} = r_{t+1} - y_t^{(1)} + p_t - p_t^{(1)}. \quad (29)$$

Using (25) in (23) we get

$$p_t^{(1)} = \mathbb{E}_t[m_{t+1}] + \frac{1}{2} \text{Var}_t[m_{t+1}]. \quad (30)$$

From (17), (28) and (30) it follows that

$$\begin{aligned} \mathbb{E}_t[r_{t+1} - y_t^{(1)}] &= \mathbb{E}_t[p_{t+1}] - (\mathbb{E}_t[p_{t+1} + m_{t+1}] + 1/2 \text{Var}_t[p_{t+1} + m_{t+1}]) \\ &\quad + \mathbb{E}_t[m_{t+1}] + \frac{1}{2} \text{Var}_t[m_{t+1}] \\ &= \mathbb{E}_t[p_{t+1}] - \left(\mathbb{E}_t[p_{t+1}] + \frac{1}{2} \text{Var}_t[p_{t+1}] + \text{Cov}_t[p_{t+1}, m_{t+1}] \right) \\ &= -\frac{1}{2} \text{Var}_t[p_{t+1}] - \text{Cov}_t[p_{t+1}, m_{t+1}]. \end{aligned}$$

Using (29) and the fact that p_t and $p_t^{(1)}$ are known at time t , so that

$$\text{Var}_t[p_{t+1}] = \text{Var}_t[r_{t+1} - y_t^{(1)} + p_t - p_t^{(1)}] = \text{Var}_t[r_{t+1} - y_t^{(1)}], \quad (31)$$

we get

$$\mathbb{E}_t[r_{t+1} - y_t^{(1)}] = -\frac{1}{2} \text{Var}_t[r_{t+1} - y_t^{(1)}] - \text{Cov}_t[r_{t+1} - y_t^{(1)}, m_{t+1}]. \quad (32)$$

The excess return on assets other than the nominal bond can also be written as

$$r_{t+1} - y_t^{(1)} = e'_i X_{t+1} = e'_i ((\mathbf{I} - \Gamma)\mu + \Gamma X_t + \varepsilon_{t+1}),$$

where e_i is a unit vector representing the location of investable asset i in the state vector. It follows from equation (32) and (12) that

$$\begin{aligned} e'_i ((\mathbf{I} - \Gamma)\mu + \Gamma x_t) &= -\frac{1}{2} \text{Var}_t[e'_i \varepsilon_{t+1}] - \text{Cov}_t[e'_i \varepsilon_{t+1}, -(\beta_0 + \beta_1 x_t)' \varepsilon_{t+1}] \\ &= -\left(\frac{1}{2} e'_i \Sigma e_i\right) - \mathbb{E}_t[(e'_i \varepsilon_{t+1} - 0)(-(\beta_0 + \beta_1 x_t)' \varepsilon_{t+1} - 0)] \\ &= -\left(\frac{1}{2} e'_i \Sigma e_i\right) - \mathbb{E}_t[-e'_i \varepsilon_{t+1} \varepsilon'_{t+1} (\beta_0 + \beta_1 x_t)] \\ &= -\left(\frac{1}{2} e'_i \Sigma e_i\right) + e'_i \Sigma (\beta_0 + \beta_1 x_t). \end{aligned}$$

Hence,

$$\begin{aligned} e_i' \left((I - \Gamma)\mu + \Gamma X_t + \frac{1}{2}\Sigma e_i - \Sigma\beta_0 - \Sigma\beta_1 X_t \right) &= 0 \\ \Leftrightarrow e_i' \left(\left((I - \Gamma)\mu - \Sigma\beta_0 + \frac{1}{2}\Sigma e_i \right) + ((\Gamma - \Sigma\beta_1)X_t) \right) &= 0. \end{aligned}$$

This is satisfied for all values of X_t if:

$$\begin{aligned} ((I - \Gamma)\mu - \Sigma\beta_0)_i + \frac{1}{2}\Sigma_{ii} &= 0, \\ (\Gamma - \Sigma\beta_1)_{i*} &= 0, \end{aligned} \tag{33}$$

where i represents the i -th element of a vector, ii represents the element in i -th row and i -th column, while $i*$ stands for i -th row of a matrix. Hence, the first equation yield one condition, whereas the second equation yields as many conditions as there are state variables. It follows that the conditions in (33) must be satisfied for investable assets. These conditions determine the parameters β_0 and β_1 of the discount factor.

B.2 Parameter optimization

The empirical values $\hat{\alpha}$ and $\hat{\Gamma}$ for α and Γ in equation (7) are determined by regressing the state variables on their lagged values, using multivariate OLS. Then, $\hat{\Sigma}$ is the variance of the residuals, i.e. the difference between the observed and the fitted values. We use these values of $\hat{\alpha}$, $\hat{\Gamma}$ and $\hat{\Sigma}$ in the sequel.

Given $\hat{\Gamma}$ and $\hat{\alpha}$, the estimate $\hat{\mu}$ of the average μ can be obtained from (9):

$$\hat{\mu} = (I - \hat{\Gamma})' \hat{\alpha}. \tag{34}$$

The empirical values \hat{D}_n and \hat{H}_n for D_n and H_n in equation (22) are obtained by regressing the historical time series of zero-coupon yields on the time series of the state variables using multivariate OLS.

The optimal $\tilde{\beta}_0$ and $\tilde{\beta}_1$ are obtained through an optimization procedure with several restrictions imposed. The model parameters \tilde{D}_n and \tilde{H}_n are implied by these values of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ through the recursion in equation (24).

B.2.1 Optimization for β_1

We obtain $\tilde{\beta}_1$ by minimizing the criterion function given by the euclidean norm derived from the following restriction:

Constraint 1: we impose that the exposures of the nominal yields towards the state variables for maturities $n = [15, 30]$ quarters as predicted by the model coincide with those obtained from the historical data ($\tilde{H}_n = \hat{H}_n$).

We want to satisfy the constraint by finding an optimal $\Sigma\beta_1$ value that minimizes the criterion function of the following form:

$$\sum_{n \in \tau} \sum_{sv} \left(e + (\hat{\Gamma} - \Sigma\beta_1)' H_{n-1}^{sv} - \hat{H}_n^{sv} \right)^2, \quad (35)$$

where $\tau = \{15, 30\}$ and H_n^{sv} indicates the exposure to the state variable sv of the interest rate for maturity n .

The optimization is done by varying the rows of matrix $\Sigma\beta_1$ that do not correspond to investable assets. The rows of matrix $\Sigma\beta_1$ corresponding to investables are set to be equal to the respective rows of $\hat{\Gamma}$ (from (33)).

The matrix $\Sigma\beta_1$ with some rows variable is then used in (24) to calculate the exposures of nominal yields to state variables (H_n), and these are compared to values of regression parameters (\hat{H}_n), the differences being minimized.

When the optimal values ($\tilde{\Sigma\beta}_1$) of matrix $\Sigma\beta_1$ are found, $\hat{\Sigma}$ is used to extract $\tilde{\beta}_1$:

$$\tilde{\beta}_1 = \hat{\Sigma}^{-1}(\tilde{\Sigma\beta}_1). \quad (36)$$

The optimal values $\tilde{\beta}_1$, and \tilde{H}_n implied by it via (24), are used in the remaining optimization procedure and further in the model.

B.2.2 Optimization for β_0

We obtain $\tilde{\beta}_0$ by minimizing the criterion function given by the euclidean norm derived from the following restrictions:

Constraint 1. \tilde{D}_n must be equal to \check{D}_n implied by the current term structure and \tilde{H}_n for maturities $\{15, 30\}$. Using \tilde{H}_n and the current observation of the state variables, we calculate the current yield curve, as predicted by the model without the intercept. Then the intercept \tilde{D}_n is calculated to match the current yield curve by setting it equal to the difference between the observed current yield curve and the one predicted by the model without a constant. This is done for maturities $n = \{15, 30\}$, following the recursion in (24).

Constraint 2. For each investable asset, $\hat{\alpha}$ obtained from the regression must satisfy the first restriction of (33). From (33) and (9) we get that $\left(\alpha - \hat{\Sigma}\beta_0 \right)_i + \frac{1}{2}\hat{\Sigma}_{ii} = 0$.

We want to satisfy these constraints by finding an optimal β_0 value that minimizes the criterion function of the following form:

$$\sum_{n \in \tau} \left(\left(D_{n-1} + \tilde{H}'_{n-1}(I - \hat{\Gamma})\hat{\mu} - \frac{1}{2}\tilde{H}'_{n-1}\hat{\Sigma}\tilde{H}_{n-1} - \tilde{H}'_{n-1}\hat{\Sigma}\beta_0 \right) - \check{D}_n \right)^2 + \sum_i \left(\left(\hat{\Sigma}\beta_0 \right)_i - \frac{1}{2}\hat{\Sigma}_{ii} - \hat{\alpha}_i \right)^2, \quad (37)$$

where $\tau = \{15, 30\}$.

The optimal value $\tilde{\beta}_0$, and \tilde{D}_n implied by it, are used further in the model.

B.3 Fixed-income returns

The model generates scenarios for the term structure of interest rates. The fixed-income portfolio returns have to be extracted from this information. A unit coupon bearing bond priced at par satisfies the equation

$$1 = cp \sum_{t=1}^{\tau} \left(R_0^{(4t)} \right)^{-4s} + \left(R_0^{(4\tau)} \right)^{-4\tau},$$

where cp is the annual coupon payment and τ is the bond maturity in years. The coupon of a par bond with τ years maturity can then be determined in the following way:

$$cp = \frac{1 - \left(R_0^{(4\tau)} \right)^{-4\tau}}{\sum_{t=1}^{\tau} \left(R_0^{(4t)} \right)^{-4t}}.$$

When the coupon is determined, the cash flows of the bond are known. Hence, by discounting them with the interest rate term structure at the beginning of the period and the term structure at the end of the period the return can be calculated by subtracting the former value from the latter value. The interest rate term structure follows from the affine structure model. The bond return is thus obtained as follows:

$$\left(cp \sum_{t=1}^{\tau} \left(R_1^{(4t-1)} \right)^{-(4t-1)} + \left(R_1^{(4\tau-1)} \right)^{-(4\tau-1)} \right) - \left(cp \sum_{t=1}^{\tau} \left(R_0^{(4t)} \right)^{-4t} + \left(R_0^{(4\tau)} \right)^{-4\tau} \right).$$

For the fixed-income portfolio we assume a portfolio consisting of 10-year maturity bonds that are priced at par. The portfolio is rebalanced at the beginning of every time period so that it again consists of 10-year maturity bonds.

C Risk-neutral sampling

The price of a derivative paying a cash flow Z (which is a function of the path X_1, X_2, \dots, X_τ of the state vector) at time τ is

$$P_0 = E_0 \left[Z_\tau \exp \sum_{t=1}^{\tau} m_t \right].$$

Using (12) this equation can be rewritten as

$$\begin{aligned} P_0 &= \int Z(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau) \exp \left[- \sum_{t=1}^{\tau} e' X_{t-1} \right] \\ &\quad \exp \left[- \sum_{t=1}^{\tau} \left(\frac{1}{2} (\beta_0 + \beta_1 X_{t-1})' \Sigma (\beta_0 + \beta_1 X_{t-1}) + (\beta_0 + \beta_1 X_{t-1})' \varepsilon_t \right) \right] \\ &\quad \frac{1}{(2\pi)^{K\tau/2} |\Sigma|^{\tau/2}} \exp \left[- \sum_{t=1}^{\tau} \frac{1}{2} \varepsilon_t' \Sigma^{-1} \varepsilon_t \right] d\varepsilon_1 d\varepsilon_2 \dots d\varepsilon_\tau. \end{aligned}$$

Hence,

$$\begin{aligned} P_0 &= \int Z(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau) \exp \left[- \sum_{t=1}^{\tau} y_{t-1}^{(1)} \right] \frac{1}{(2\pi)^{K\tau/2} |\Sigma|^{\tau/2}} \\ &\quad \exp \left[- \sum_{t=1}^{\tau} \left(\frac{1}{2} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1}))' \Sigma^{-1} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})) \right) \right] d\varepsilon_1 d\varepsilon_2 \dots d\varepsilon_\tau. \end{aligned}$$

This integral can be evaluated numerically in a Monte Carlo simulation by drawing a number of time series $\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau\}$ from the multivariate normal probability distribution

$$f = \frac{1}{(2\pi)^{K\tau/2} |\Sigma|^{\tau/2}} \exp \left[- \sum_{t=1}^{\tau} \left(\frac{1}{2} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1}))' \Sigma^{-1} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})) \right) \right], \quad (38)$$

calculating for each time series the derivative payoff discounted at a risk-free rate

$$\exp \left[- \sum_{t=1}^{\tau} y_{t-1}^{(1)} \right] \cdot Z(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau),$$

and taking a simple average over the drawings. Hence, we have transformed the original problem in which we draw from the multivariate normal density function of ε_i and discount cash flows at the stochastic discount factor into a problem in which we draw from the multivariate normal density function with mean $-\Sigma(\beta_0 + \beta_1 X_{t-1})$ and the same original variance-covariance matrix Σ , but discount cash flows at the risk-free rate.

The marginal distribution of $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau$ for $t \leq \tau$ can be obtained by integrating over the remaining variables. Now, consider $t = \tau - 1$. For fixed values of $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{\tau-1}$, variable ε_τ has

a normal distribution, so it can be integrated over analytically:

$$f(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{\tau-1}) = \frac{1}{(2\pi)^{(\tau-1)K/2} |\Sigma|^{(\tau-1)/2}} \exp \left[- \sum_{t=1}^{\tau-1} \left(\frac{1}{2} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1}))' \Sigma^{-1} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})) \right) \right]. \quad (39)$$

By repeating this procedure for $\varepsilon_{\tau-1}, \varepsilon_{\tau-2}, \dots, \varepsilon_1$ we find that

$$f(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_t) = \frac{1}{(2\pi)^{Kt/2} |\Sigma|^{t/2}} \exp \left[- \sum_{s=1}^t \left(\frac{1}{2} (\varepsilon_s + \Sigma(\beta_0 + \beta_1 X_{s-1}))' \Sigma^{-1} (\varepsilon_s + \Sigma(\beta_0 + \beta_1 X_{s-1})) \right) \right]. \quad (40)$$

The conditional density $f(\varepsilon_t | \varepsilon_{t-1}, \dots, \varepsilon_1)$ is obtained using the Bayes' formula:

$$f(\varepsilon_t | \varepsilon_{t-1}, \dots, \varepsilon_1) = \frac{f(\varepsilon_t, \dots, \varepsilon_1)}{f(\varepsilon_{t-1}, \dots, \varepsilon_1)}.$$

Using (39) and (40) we get

$$f(\varepsilon_t | \varepsilon_{t-1}, \dots, \varepsilon_1) = \frac{1}{(2\pi)^{K/2} |\Sigma|^{1/2}} \exp \left[- \frac{1}{2} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1}))' \Sigma^{-1} (\varepsilon_t + \Sigma(\beta_0 + \beta_1 X_{t-1})) \right],$$

which is normally distributed as:

$$\varepsilon_t | \varepsilon_{t-1}, \dots, \varepsilon_1 \sim \mathbb{N}(-\Sigma(\beta_0 + \beta_1 x_{t-1}), \Sigma).$$

We can draw $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau$ from the distribution $f(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_\tau)$ by first drawing ε_1 from its marginal distribution, then drawing ε_2 from the conditional distribution $f(\varepsilon_2 | \varepsilon_1)$, and so on. Scenarios are produced in pairs using antithetic variables so as to increase the efficiency of the simulation.

D Generating scenarios

The economic scenarios are generated using the parameters $\hat{\alpha}, \hat{\Gamma}, \hat{\Sigma}, \tilde{\beta}_0, \tilde{\beta}_1, \tilde{D}_n$ and \tilde{H}_n .

First, the development of the state variables is modeled for a chosen number of years in the future, by using (7). The initial value of the state vector is set at the average μ . After the state vectors have been simulated, the development of the term structure of the interest rate can be simulated by using (22).

The same procedure is followed for both the real-world and risk-neutral scenarios. The only difference lies in the dynamics of shock ε . For the real-world simulation the error terms are

drawn from a normal distribution $\varepsilon_{t+1} \sim \mathbb{N}(0, \Sigma)$ (from (8)). Under the risk-neutral scenarios, the noise term is normally distributed with $\varepsilon_{t+1} \sim \mathbb{N}(-\Sigma(\beta_0 + \beta_1 X_t), \Sigma)$. Hence, the transformation from a real-world scenario to the risk-neutral one is done by merely changing the mean of the distribution.