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Published in:
Chapters in Game Theory

Publication date:
2002

Document Version
Peer reviewed version

Link to publication in Tilburg University Research Portal

Citation for published version (APA):
van Damme, E. E. C. (2002). Game theory and the market. In P. E. M. Borm, & H. J. M. Peters (Eds.), *Chapters in Game Theory* (pp. 51-81). (Theory and Decision Library C; No. 31). Kluwer Academic Publishers.

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Download date: 11. Oct. 2024

Game Theory and the Market

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December 23, 2003

Abstract

We show that both cooperative and non-cooperative game models can substantially increase our understanding of the functioning of actual markets. In the first part of the paper, we provide a brief historical sketch of the differences and complementary between the two types of models, by going back to the work of the founding fathers, Von Neumann, Morgenstern and Nash. In the second part, we illustrate our main point by means of examples of bargaining, oligopolistic interaction and auctions.

1 Introduction

Based on the assumption that players behave rationally, game theory tries to predict the outcome in interactive decision situations, i.e. situations in which the outcome is determined by the actions of all players and no player has full control. The theory distinguishes between two types of models, cooperative and non-cooperative. In models of the latter type, emphasis is on individual players and their strategy choices, and the main solution concept is that of Nash equilibrium (Nash, 1951). Since the concept as originally proposed by Nash is not completely satisfactory - it does not adequately take into account that certain threats are not credible, many variations have been proposed, see Van Damme (2002), but in their main idea these all remain faithful to Nash's original insight. The cooperative game theory models, instead, focus on coalitions and outcomes,

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and for cooperative games a wide variety of solution concepts have been developed, in which few or no unifying principles can be distinguished. (See other papers in this volume for an overview). The terminology that is used sometimes gives rise to confusion; it is not the case that in non-cooperative games do not wish to cooperate and that in cooperative games players automatically do so. The difference instead is in the level of detail of the model; non-cooperative models assume that all possibilities for cooperation have been included as formal moves in the game, while cooperative models are "incomplete" and allow players to act outside of the detailed rules that have been specified.

One of us had the privilege and the luck to follow undergraduate courses in game theory with Stef Tijs. There were courses in non-cooperative theory as well as in cooperative theory and both were fun. When that author had passed his final (oral) exam, he was still puzzled about the relationships between the models and the solution concepts that had been covered and he asked Stef a practical question: when to use a cooperative model and when to use a non-cooperative one? The answer is not recalled, but clearly the question is a nonsensical one: it all depends on what one wants to achieve and what is feasible to do. Frequently, it will not be possible to write down an explicit non-cooperative game, and even if this is possible, one should be aware that players may attempt to violate the rules that the analyst believes to apply. On the other hand, a cooperative model may be pitched at a too high level of abstraction and may contain too little detail to allow the theorist to come up with a precise prediction about the outcome. In a certain sense, the large variety of solution concepts that one finds in cooperative game theory is a natural consequence of the model that is used being very abstract It also follows from these considerations that cooperative and non-cooperative models are complements to each other, rather than competitors.

Our aim in this paper is to demonstrate the complementarity between the two types of game theory models and to illustrate their usefulness for the analysis of actual markets. Section 2 provides a historical perspective and briefly discusses the views expressed in Von Neumann and Morgenstern (1953) and Nash (1953). Section 3 focuses on bargaining games, while Section 4 discusses oligopoly games and markets. Auctions are the topic of Section 5. Section 6 concludes.

2 Von Neumann, Morgenstern and Nash

As Von Neumann and Morgenstern (1953) argue, there is not much point in forming a coalition in 2-person zero-sum games. In this case, both the cooperative and the non-cooperative theory predict the same outcome. Furthermore, in 2-person non-zero-sum games, there is only one coalition that can possibly form and it will form when it is attractive to form it and when the rules of the game do not stand in the way. The remaining question then is how the players will divide the surplus, a question that we will return to in Section 3. The really interesting problems start to appear when there are at least three players. Von Neumann and Morgenstern (1953, Chapter V) argue that in this case the game cannot sensibly be analyzed without coalitions and side-payments, for, even if these are not explicitly allowed by the rules of the game, the players will try to form coalitions and make side payments outside of these formal rules.

To illustrate their claim, the founding fathers of game theory, start from a simple non-cooperative game. Assume there are three players and each player can point to one of the others if he wants to form a coalition with him. In this case, the coalition $\{i, j\}$ forms if and only if i points to j and j points to i. The rules also stipulate that if $\{i, j\}$ forms, the third player, k, has to pay 1 money unit to each of i and j. Formally therefore this game of coalition formation can be represented by the following normal form (non-cooperative) game:

	1	3
2	1, 1, -2	0, 0, 0
3	1, -2, 1	1, -2, 1

1

	1	3
2	1, 1, -2	-2, 1, 1
3	0, 0, 0	-2, 1, 1

2

Figure 1: A non-cooperative game of coalition formation.
(Player 1 chooses a row, player 2 a column, and players 3 a matrix.)

This game has several pure Nash equilibria, it also has a mixed Nash equilibrium in which each player chooses each of the others with equal probability. Von Neumann and Morgenstern start their analysis from a non-cooperative point of view, i.e. as if the above matrix tells the whole story:

> ”Since each player makes his personal move in ignorance of those of the others, no collaboration of the players can be established during the course of play” (p. 223).

Nevertheless, Von Neumann and Morgenstern argue that the whole point of the game is to form a coalition, and they conclude that, if players are prevented to do so within the game, they will attempt to do so outside. They realize that this raises the question of why such outside agreements will be kept, and they pose the crucial question "what, if anything, *enforces* the "sanctity" of such agreements? They answer this question in the following way

> "There may be games which themselves - by virtue of the rules of the (...) provide the mechanism for agreements and their enforcement. But we cannot base our considerations on this possibility since a game *need* not provide this mechanism; (...) Thus there seems no escape from the necessity of considering agreements concluded outside the game. If we do not allow for them, then it is hard to see what, if anything, will govern the conduct of a player in a simple majority game" (p. 223).

The reader may judge for himself whether, and in which circumstances, he considers this argument to be convincing. In any case, if one accepts the argument that a convincing theory cannot be formulated without auxiliary concepts such as "agreements" and "coalitions", then one also has to accept that side-payments will form an integral part of the theory. This latter argument is easily seen by considering a minor modification of the game of Figure 1. Suppose that if the coalition $\{1,2\}$ would form the payoffs would be $(1+\varepsilon, 1-\varepsilon, -2)$ and that if $\{1,3\}$ would form, the payoffs would be $(1+\varepsilon, -2, 1-\varepsilon)$: what outcome of the game would result in this case? Von Neumann and Morgenstern argue that the advantage of player 1 is quite illusory: if player 1 would insist on getting $1+\varepsilon$ in the coalition $\{1,2\}$, then 2 would prefer to form the coalition with 3,and similarly with the roles of the weaker players reversed. Consequently, in order to prevent the coalition of the two "weaker" players from forming, player 1 will offer a side payment of ε to each of them. Consequently, Von Neumann and Morgenstern conclude

> "It seems that what a player can get in a definite coalition depends not only on what the rules of the game provide for that eventuality, but also on the other (competing) possibilities of coalitions for himself and for his partner. Since the rules of the game are absolute and inviolable, this means that under certain conditions *compensations* must be paid among coalition partners; i.e. that a player must have to pay a well-defined price to a prospective

> coalition partner. The amount of the compensations will depend on what other alternatives are open to each of the players" (p. 227).

Obviously, if one concludes that coalitions and side payments have to be considered in the solution, then the natural next step is to see whether the solution can be determined by these aspects alone, and it is that problem that Von Neumann and Morgenstern then the one he is negotiating a coalition with set out to solve in the remaining 400 pages of the their book.

John Nash refused to accept that it was necessary to include elements outside the formal structure of the game to develop a convincing theory of games. His thesis (Nash, 1950a), of which the mathematical core was published a bit later as Nash (1951) opens with

> "Von Neumann and Morgenstern have developed a very fruitful theory of two-person zero-sum games in their book *Theory of Games and Economic Behavior*. This book also contains a theory of n-person games of a type which we would call cooperative. This theory is based on an analysis of the interrelationships of the various coalitions which can be formed by the players of the game. Our theory, in contradistinction, is based on the *absence* of coalitions in that it is assumed that each participant acts independently, without collaboration or communication with any of the others. The notion of an *equilibrium point* is the basic ingredient in our theory."

Hence, Nash was the first to introduce the formal distinction between the two classes of games. After having given the formal definition of a non-cooperative game, Nash then defines the equilibrium notion, proves that any finite game has at least one equilibrium, derive properties of equilibria, discusses issues of robustness and equilibrium selection and finally discussed interpretational issues. In the remainder of this Section, we give a brief sketch.

A non-cooperative game is a tuple $< S_i, u_i >_{i \in I}$ where I is a nonempty set of players, S_i is the strategy set of player i and $u_i : S \to \mathbb{R}$ (where $S = X_{i \in I} S_i$) is the payoff function of player i. This formal structure had already been introduced by Von Neumann and Morgenstern, who had also argued that, for finite S_i, it was natural to introduce mixed strategies. A mixed strategy σ_i of player i is a probability distribution on S_i.

In what follows we write k to denote a generic pure strategy and we write σ_i^k for the probability that σ_i assigns to k. If $\sigma = (\sigma_1, \ldots, \sigma_I)$ is a combination of mixed strategies, we may write $u_i(\sigma)$ for player i's expected payoff when σ is played. Von Neumann and Morgenstern had proved the important result that for rational players it was sufficient to look at expected payoffs. In other words, it is assumed that payoffs are Von Neumann Morgenstern utilities. Nash now defines an equilibrium point as a mixed strategy combination σ^* such that each player's mixed strategy σ_i^* maximizes his payoff if the strategies of the others (denoted by σ_{-i}^*) are held fixed, hence

$$u_i(\sigma^*) = \max_{\sigma_i} u_i(\sigma_i, \sigma_{-i}^*) \text{ for all } i \in I$$

Nash's main result is that in finite games (i.e. I and all S_i are finite sets) at least one equilibrium exists. The proof is so elegant that it is worthwhile to give it here. For $i \in I$ and $k \in S_i$, write

$$U_i^k(\sigma) = \max(0, u_i(k, \sigma_{-i}) - u_i(\sigma))$$

and consider the map $f : \Sigma \to \Sigma$ defined (componentwise) by

$$f_i^k(\sigma) = (\sigma_i^k + U_i^k(\sigma))/(1 + \sum_{\ell \in S_i} U_i^\ell(\sigma)),$$

then f is a continuous map, that maps the convex set Σ (of all mixed strategy profiles) into itself, so that, by Brouwer's fixed point theorem, a fixed point σ^* exists. It is then easily seen that such a σ^* is an equilibrium point of the game.

The section "Motivation and Interpretation" from Nash's thesis was not included in the published version (Nash 1951). In retrospect, this is to be regretted as it led to misunderstandings and delayed progress in game theory for some time. Nash provided two interpretations, one, "the rationalistic interpretation" arguing why equilibrium is relevant when the game is played by fully rational players, the other "the mass action representation" arguing that equilibrium might be obtained as a result of ignorant players learning to play the game over time when the game is repeated. We refer the reader to Van Damme (1995) for further discussion on these interpretations, here we confine ourselves to the remark that the rationalistic interpretation, the view of a solution as a convincing theory of rationality, had already been proposed in Von Neumann and

Morgenstern, see Section 17.3 of their book. However, the founding fathers had not followed up their own suggestion. In addition, they had come to the conclusion that it was necessary to consider set-valued solution concepts. Again, Nash was not convinced by their arguments and he found it a weak spot in their theory.

3 Bargaining

In this Section we illustrate the complementarity between game theory's two approaches for the special case of bargaining problems.

As referred to already at the end of the previous Section, the theory that Von Neumann and Morgenstern developed generally allows multiple outcomes. Consider the special case of a simple bargaining problem. Assume there is one seller who has one object for sale, who does not value this object himself, and that there is one buyer that attaches value 1 to it, with both players being risk neutral. For what price will the object be sold? Von Neumann and Morgenstern discuss this problem in Section 61 of their book where they come to the conclusion that "a satisfactory theory of this highly simplified model should leave the entire interval (i.e. in this case [0,1]) availabel for p". (p. 557)

The above is unsatisfactory to Nash. In Nash (1950b), he writes

> "In *Theory of Games and Economic Behavior* a theory of n-person games is developed which includes as a special case the two-person bargaining problem. But the theory developed there makes no attempt to find a value for a given n-person game, that is, to determine what it is worth to each player to have the opportunity to engage in the game (...) It is our opinion that these n-person games should have values."

Nash then postulates that a value exists and he sets out to identify it. To do so, he uses the axiomatic method, that is

> "One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely" (Nash, 1953, p. 129)

In his 1950b paper, Nash adopts the cooperative approach, hence, he assumes that the solution can be identified by using only information about what outcomes and coalitions are possible. Without loss of generality, let us normalize payoffs such that each player has payoff 0 if players do not cooperate and that cooperation pays, i.e. there is at least one payoff vector u with $u_1, u_2 > 0$ that is feasible. In this case, the solution then should just depend on the set of payoffs that are possible when players do cooperate. Let us write $f(S)$ for the solution when this set is S. This set will be convex, as players can randomize. Obviously, such trivialities as $f(S) \in S$ and $f_i(S) \geqslant 0$ for $i = 1, 2$ should be satisfied. In addition, the solution should be independent of which utility function is used to represent the given players preferences and should be symmetric ($u_1 = u_2$) when the game is symmetric. All these things are undebatable. It is quite remarkable that only one additional axiom is needed to uniquely determine the solution for each bargaining problem. This is the Axiom of Independence of Irrelevant Alternatives:

If $S \subset T$ and $f(T) \in S$, then $f(S) = f(T)$

Again the proof of this major result is so elegant, that we cannot resist to give it. Define $g(S)$ as that point in S that maximizes $u_1 u_2$ in $S \cap \mathbb{R}^2_+$. Then by rescaling utilities we may assume $g(S) = (1, 1)$, and it follows that the line $u_2 + u_2 = 2$ is a supporting hyperplane for S at $(1, 1)$. (It separates the convex set S from the convex set $\{(u_1, u_2); u_1 u_2 \geqslant 1\}$.) Now let T be the set $\{(u_1, u_2) \in \mathbb{R}^2_+; u_1 + u_2 \leqslant 2\}$. Then, by symmetry, $f(T) = (1, 1)$, hence IIA implies $f(S) = (1, 1)$. We have, therefore, established that there is only one solution satisfying the IIA axiom: it is the point where the product of the players' utilities is maximized. As a corollary we obtain that, in the simple seller-buyer example that we started out with, the solution is a price of $\frac{1}{2}$.

One interpretation of the above solution is that it will result when players can bargain freely. Obviously, when the players would be severely restricted in their bargaining possibilities then a different outcome may result. For example, in the above buyer-seller game, if the seller can make a take it or leave it offer, the buyer will be forced to pay a price of (almost) one. The advantage of non-cooperative modelling is that it allows to analyze each specific bargaining procedure and to predict the outcome on the basis of detailed modelling of the rules; the drawback (or realism?) of that model is that the outcome may crucially depend on these details. Indeed, if, in our simple example, the buyer would have all the bargaining power, the price would be (close to) zero. The symmetry assumption in Nash's axiomatic model represents something like players having equal bargaining power and this is obviously violated in these take it or leave it games. It is not clear how such asymmetric games could be relevant for players that are otherwise completely symmetric. Nash (1953) contains important modelling advise for non-cooperative game theorists. He writes that in the non-cooperative approach

> "the cooperative game is reduced to an non-cooperative game. To do this, one makes the players' steps of negotiation in the cooperative game become moves in the non-cooperative model. Of course, one cannot represent all possible bargaining devices as moves in the non-cooperative game. The negotiation process must be formalized and restricted, but in such a way that each participant is still able to utilize all the essential strength of his position" (Nash (1953, p. 129).

Nash also writes that the two approaches are complementary and that each helps to justify and clarify the other. To complement his cooperative analysis, Nash studies the following simultaneous demand game: each player i demands a certain utility level u_i that he should get; if the demands are compatible, that is $(u_1, u_2) \in S$, then each player gets what he demanded, otherwise disagreement (with payoff 0) results. At first it seems that this non-cooperative game does not fulfill our aims, after all any Pareto optimal outcome of S corresponds to a Nash equilibrium of the game, and so does disagreement. Nash, however, argues that one of these equilibria is distinguished in the sense that it is the only one that is robust against small perturbations in the data. Of course, this unique robust equilibrium is then seen to correspond to the cooperative solution of the game. Specifically, Nash assumes that players are somewhat uncertain about what outcomes are feasible. Let $p(u)$ be the probability that u is feasible with $p(u) = 1$ if $u \in S$ and p a continuous function that falls rapidly to zero outside of S. With uncertainty given by p, player i's payoff function is now given by $u_i p(u)$ and it is easily verified that any maximum of the map $u_1 u_2 p(u)$ is an equilibrium of this slightly perturbed game. Note that all these equilibria converge to the Nash solution (the maximum of $u_1 u_2$ on S) when p tends to the characteristic function of S and that, for nicely behaved p, the perturbed game will only have equilibria close to the Nash solution. Consequently, only the Nash solution constitutes a robust equilibrium of the original demand game.

The above coincidence certainly is not an isolated result, the Nash solution also arises in other natural non-cooperative bargaining models. As an example, we discuss Rubinstein's (1981) alternating offer bargaining game. Consider the simple seller buyer game that we started this Section with and assume bargaining proceeds as follows, until agreement is reached or the game has come to an end. In odd numbered period ($t = 1, 3, \ldots$), the seller proposes a price to the buyer and the buyer responds by accepting or rejecting the offer; in even numbered period ($t = 2, 4, \ldots$), the roles of the players are reversed and the buyer has the initiative; after each rejection, the game stops with positive but small probability α. Rubinstein shows that this game has a unique (subgame perfect) equilibrium, and that, in equilibrium, agreement is reached immediately. Let p_s (resp. p_b) be the price proposed by the seller (resp. the buyer). The seller realizes that if the buyer rejects his first offer, the buyer's expected utility will be $(1-\alpha)(1-p_b)$, hence, the seller will not offer a higher utility, nor a lower. Consequently, in equilibrium we must have

$$1 - p_s = (1-\alpha)(1-p_b)$$

and, by a similar argument

$$p_b = (1-\alpha)p_s$$

It follows that the equilibrium prices are given by

$$p_s = 1/(2-\alpha) \qquad p_b = (1-\alpha)/(2-\alpha)$$

and as α tends to zero (when the first mover advantage vanishes and the game becomes symmetric), we obtain the Nash bargaining solution.

We conclude this Section with the observation that also in Von Neumann and Morgenstern (1953) both cooperative and non-cooperative approaches are mixed. In Section 2, we discussed the 3-player zero-sum game and the need to consider coalitions and side-payments. In Section 22.2 of the *Theory of Games and Economic Behavior*, the general such game is considered: if coalition $\{j, k\}$ forms, then player i has to pay a_i to this coalition ($i \neq j \neq k$). What coalition will form and how will it split the surplus? To answer this question, Von Neumann and Morgenstern consider a demand game. They assume that each player i specifies a price p_i for his participation in each coalition. Obviously, if p_i is too large, j and k will prefer to cooperate together rather than to form a coalition with i. Given p_i, j cannot expect more than $a_k - p_i$ in $\{i, j\}$ while k cannot expect more than $a_j - p_i$ in $\{i, k\}$, hence i will price himself out of the market if

$$(a_k - p_i) + (a_j - p_i) < a_i$$

Consequently, each player i cannot expect more than

$$p_i = (-a_i + a_j + a_k)/2$$

If the game is essential and it pays to form a coalition, i.e. $a_1 + a_2 + a_3 > 0$, then the above system of three questions with three unknown ($i = 1, 2, 3$) has a unique solution. Each player i can reasonably demand p_i: we can predict how the coalition that will form will split the surplus, but all three possible coalitions are equally likely.

4 Markets

In this Section, we briefly discuss the application of game theory to oligopolistic markets. In line with the literature, most of the discussion will be based on non-cooperative models, but we will see that also here cooperative analysis plays its role.

In a non-cooperative *oligopoly game,* the players are firms, the strategy sets are, in general, compact and connected subsets of an Euclidean space, and the payoffs are the profits of the firms. As Nash's existence theorem only applies to finite games, a first question is whether equilibrium exists. Here we will confine ourselves to the specific case where the strategy set of player i, denoted X_i, is a closed and connected interval in $\mathbb{R}$. Hence, in essence we assume that each firm sells just one product, of which it either sets the price or the quantity. We speak of a *Cournot game* when the strategies are quantities, of a *Bertrand game* when the strategies are prices. Write X for the Cartesian product of all X_i. For player $i \in N$ his *best response correspondence* is the map $\mathcal{B}_i$ that assigns to each $x \in X$ the set of all $y_i \in X_i$ that maximize this player's payoff against x. Note that in the two-player case, $\mathcal{B}_i$ (viewed as a function of x_{-i}) will typically be decreasing in the case of a Cournot game and be increasing in the case of Bertrand. In the former case, we speak of strategic substitutes, in the latter of strategic complements. We write $\mathcal{B}$ for the vector of all $\mathcal{B}_i$. When for each player $i \in N$ the profit function is continuous on X and is quasi–concave in $x_i \in X_i$ for fixed $x_{-i} \in X_{-i}$, then the conditions of the Kakutani fixed point theorem are satisfied ($\mathcal{B}$ is an upper-hemi continuous map, for which all image sets are non-empty compact and convex), hence, the oligopoly game has a Nash equilibrium. When products are differentiated, these conditions will typically be satisfied, but with homogeneous products, they may be violated. For example, in the Bertrand case, without capacity constraints and with no possibility to ration demand, the firm with the lowest price will typically attract all demand, hence, demand functions and profit functions are discontinuous. Dasgupta and Maskin (1986) contains useful existence theorems for cases like these. (Also see Furth (1986).) Of course, the equilibrium is not necessarily unique.

The first formal analysis of an oligopolistic market was performed by Cournot, who analyzed a duopoly in which two firms sell a homogeneous (consumption) good to the consumers, see Cournot(1838). He writes

> "Let us now imagine two proprietors and two springs of which the qualities are identical, and which, on account of their similar positions, supply the same market in competition. In this case the price is necessarily the same for each proprietor. [...]; and *each of them independently* will seek to make this income as large as possible."

Cournot(1838), cited from Daughety(1988, p. 63)

In Cournot's model, a strategy of a firm is the quantity supplied to the market. Cournot argued that if firm i supplies q_i firm j will have an incentive to supply the quantity q_j that is the best response to q_i and he defined an equilibrium as a situation in which each of the duopolists is at a best response. Hence, the solution that Cournot proposed, the Cournot equilibrium, can be viewed as a Nash equilibrium. Nevertheless, Cournot's interpretation of the equilibrium seems to have been very different from the modern "rationalistic" interpretation of equilibrium, it seems to be more in line with the "mass action interpretation" of Nash. The following citations are revealing of this:

> "After one hundred and fifty years the Cournot model remains the benchmark of price formation under oligopoly. Nash equilibrium has emerged as the central tool to analyze strategic interactions and this is a fundamental methodological contribution which goes back to Cournot's analysis." Vives(1989, p.511)

> "After the appearance of the Nash equilibrium, what we witness is the gradual injection of a certain ambiguity into Cournot's account in order to make

> it interpretable in terms of Nash. Following Nash, Cournot is reread and reinterpreted. This may have several different motivations, of which we here present concrete evidence of two. In one case, it is a way of anchoring, or stabilizing, the new and still floating idea of the Nash equilibrium. By showing that somebody in the past – and all the better if it is an eminent figure – seems to have had 'the same idea' in mind, the Nash equilibrium is given a history, it is legitimised, and the case for game theory is strengthened. In the other case, the motivation is to detract from the originality of Nash's idea, maintaining that 'it was always there', i.e. Nash has said nothing new."
> Leonard(1994, p.505)

Bertrand(1883) criticized Cournot for taking quantities as strategic variables and he suggested to take prices instead. It differs a lot for the outcome what the strategic variables are. In a Cournot game, a player assumes that the opponent's quantity remains unchanged, hence, this corresponds to assuming that the opponent raises his price if I raise mine. Clearly such a situation is less competitive than one of Bertrand competition in which a firm assumes that the opponent maintains his price when it raises its own price. Consequently, prices are frequently lower in the Bertrand situation. In fact, when the firms produce identical products, marginal cost are constant and there are no capacity constraints, already with two firms, Bertrand price competition results in the competitive price, that is the price is equal to the marginal costs.

This result, that in a Bertrand game with homogeneous products and constant marginal cost, the competitive price is already obtained with two firms is sometimes called the Bertrand paradox and it seems to have bothered many economists in the past. Edgeworth(1897) suggested that firms have capacity constraints and that such constraints might resolve the paradox; after all, with capacity constraints, the reaction of the opponent will be less aggressive, hence, the market less competitive. However, capacity constraints raise another puzzle. Suppose one firm sets the competitive price, but is not able to supply total demand at that price. After this firm has sold its full capacity, a 'residual' market remains and the other firm makes most profits when it charges the 'residual monopoly price' in this market. As Edgeworth observed, given the high price of the second firm, the first firm has an incentive to raise its price to just below this price. Obviously, at these higher prices, there is then a game of each firm trying to undercut the other, which is driving prices down again. As a consequence, a pure strategy equilibrium need not exist. We are led to *Edgeworth cycles,*.see also Levitan and Shubik(1972).

However, we note here that there always exists an equilibrium in mixed strategies: firms set prices randomly, according to some distribution function. It may be shown, see Levitan and Shubik(1972), Kreps and Scheinkman(1983), Osborne and Pitchik(1986) and Deneckere and Kovenock(1992), that for small capacities a Cournot type outcome results, i.e. supplies are sold against a market clearing price, while for sufficiently large capacities, the Bertrand outcome is the equilibrium, i.e. firms set the competitive price. For the remaining intermediate capacity levels, there is no equilibrium in pure strategies.

Kreps and Scheinkman(1983) also analyze the situation where firms can choose their capacity levels.They assume that firms play the following two period game:
- In the first period firms choose their capacity levels k_1 and k_2,
- Knowing these capacities, in the second period firms play the Bertrand Edgeworth price game.

In this situation, high capacity levels are attractive as they allow to sell a lot, but they are likewise unattractive as they imply a very competitive market; in contrast, low levels imply high prices but low quantities. Kreps and Scheinkman(1983) show that with efficient rationing in the second period, firms will choose the Cournot quantities in the first period and the corresponding market clearing prices in the second. Hence, the Cournot model can be viewed as a shortcut of the two-stage Bertrand-Edgeworth model. However, it turns out that the solution of the game depends on the rationing scheme, as Davidson and Deneckere(1986) have shown.

All the oligopoly games discussed thus far are games with imperfect information, as players take their decisions simultaneously. Hence, when a player takes his decision, he does not know about the decisions of the other players. Oligopoly games with perfect information, in which players take their decisions sequentially with they being informed about all the previous moves, are nowadays called Stackelberg games, after Stackelberg(1934). Moving sequentially is a way in which too intense competition might be avoided, for example, if players succeed in avoiding simultaneous price setting, prices will typically be higher. Von Stackelberg assumed that one of the players is the 'first mover', the *leader,* and the other is the *follower.* In Stackelberg's model, first 'the leader' decides and next, knowing what the leader has done, 'the follower' makes his decision, hence, we have a game with *perfect information.*We believe that Stackelberg meant 'leader' and 'follower' more as a behavior rule, rather than an exogenously imposed ordering of the moves, hence, in our view, he assumed asymmetries between different player types. In any case, this behavior does not lead to a Nash equilibrium of the simultaneous move game. The best a follower can do, is to play a best response against the action of the leader

$$x_F = B_F(x_L).$$

The leader knowing this, will therefore play

$$x_L = \arg\max_{x \in X_L} \pi_L(x; B_F(x)).$$

In a Cournot setting, this typically applies that the leader will produce more, and the follower will produce less than his Cournot quantity, hence, the follower is in a weaker position, and it pays to lead: there is a first-mover advantage. (Bagwell (1995), however, has argued that this first-mover advantage is eliminated if the leader's quantity can only be observed with some noise. Specifically, he considers the situation where, if the leader choose x_L the follower observes x_L with probability $1 - \varepsilon$, while the follower sees a randomly drawn $\tilde{x}$ with the remaining positive probability ε, where $\tilde{x}$ has full support. As now the signal that the follower receives is completely uniformative, the follower will not condition on it, hence, it follows that in the unique pure equilibrium, the Cournot quantities are played. Hence, there is no longer a first mover advantage. Van Damme and Hurkens (1996) however show that there is always a mixed equilibrium, that there are good arguments for viewing this equilibrium as the solution of the game, and that this equilibrium converges to the Stackelberg equilibrium when the noise vanishes.

We note that, in this approach to the Stackelberg game with perfect information, leader and follower are determined exogenously. Now it is easy to see that, in Cournot type games, it is most advantageous to be the leader, while in Bertrand type games, the follower position is most advantageous. Hence, the question arises which player will take up which player role. There is a recent literature that addresses this question of endogenous leadership. In this literature, there are two-stage models in which players choose the role they want to play in a timing game. The trade-off is between moving early and enjoy the advantage of commitment, or moving late and having the possibility to best respond to the opponent. Obviously, when firms are 'identical' there will be no way to determine an endogenous leader, hence, these models assume some type of asymmetry: endogenous leaders may emerge from different capacities, different efficiency levels, different information, or product differentiation. In cases like these, one could argue that player i will become the leader when he profits more from it than player j does, hence, that player i will lead if

$$\pi_i^L - \pi_i^F > \pi_j^L - \pi_j^F,$$

or equivalently when

$$\pi_i^L + \pi_j^F > \pi_i^F + \pi_j^L,$$

in other words, that the leadership will be determined as if players had joint profits in mind. Based on such considerations, many papers come to the conclusion that the dominant or most efficient firm will become the leader, see Ono (1982), Deneckere and Kovenock (1992), Furth and Kovenock (1993), and Van Cayseele and Furth (1996). To get some intuition for this result, let consider a simple asymmetric version of the 2-firm Bertrand game. Assume that the product is perfectly divisible, that the demand curve is given by $D(p) = 1$ for $p < 1$ and $D(p) = 0$ for $p > 1$, and that firm 2 has a capacity constraint of k. If firm 2 acts as a leader, firm 1 will undercut and firm 2's profit is zero. Firm 2's profit is also zero if price setting is simultaneous and in this case firm 1's profit is zero as well. If firm 1 commits to be leader, he will be undercut by firm 2, but given that firm 2 has a capacity constraint, firm 1 is not hurt that much by it. Firm 1 will simply commits to the monopoly price and profits will be $1 - k$ for firm 1 and k for firm 2. Hence, only in the case where firm 1 takes up the leadership position will profits be positive for each firm, and we may expect firm 1 to take up the leadership position.

Hurkens and Van Damme (1996, 1999) argue that the above profit calculation is not convincing and that the leadership position should result from individual risk considerations. Be that as it may, the interesting result that they derive is that these risk considerations do lead to exactly the above inequalities, hence, Van Damme and Hurkens obtain that both in the price and in the quantity game, the efficient firm will lead. Note then, that the efficient firm obtains the most preferred position in the case of Cournot competition, but not in the case of Bertrand competition.

Above, we already briefly referred to the work of Edgeworth on Bertrand competition with capacity constraints. Edgeworth was also the one who introduced the Core as the concept that models unbridled competition. Shubik (1959) rediscovered this concept in the context of cooperative games, and the close relation between the Core of the cooperative exchange game and the competitive outcome was soon discovered. Hence, also here we see the close relation between cooperative and non-cooperative theory. In the remainder of this Section, we illustrate this relationship for the most simple 3-person exchange game, a game that, incidentally, also was analyzed in Von Neumann and Morgenstern(1953). The founding fathers indeed already mention the possibility of applying their theory in the context of an oligopoly. Specifically, in the Sections

62.1 and 62.4 of their book, they calculated their solutions, the Stable Set, of a three-person non-constant sum game that arises in a situation with one buyer and two sellers. Shapley(1958) generalized their analysis to a game with $n(\geq 1)$ buyers and $m(\geq 1)$ sellers, see also Shapley and Shubik(1969). We will confine ourselves to the case with $m = 2$ and $n = 1$. Furthermore, for simplicity, we will assume that the sellers are identical, that they each have one single indivisible object for sale, that they do not value this object, and that the buyer is willing to pay 1 for it. Denoting the consumer by player 3, the situation can be represented by the (cooperative) 3-person characteristic function game given by $v(S) = 1$ if $3 \in S$ and $| S | \geqslant 2$; and $v(S) = 0$ otherwise. In this game, the Core consists of a single allocation (0,0,1), corresponding to the consumer buying from either producer for a price of 0, hence, the Core coincides with the competitive outcome, illustrating the well-known Core equivalence theorem.

When, in the mid 1970s, one of us took his first courses in game theory with Stef Tijs, he considered the solution prescribed by the Core in the above game to be very natural. As a consequence, he was bothered very much by the fact that the Shapley value of this game was not an element of the Core and that it predicted a positive expected utility for each of the sellers. (As is well-known, the Shapley value of this game is (1,1,4)/6). Why could the sellers expect a positive utility in this game? The answer is in fact quite simple: the sellers can form a cartel! Obviously, once the sellers realize that their profits will be competed away if they do not form a cartel, they will try to form one. Hence, in this price competition game, coalitions arise quite naturally and, as a consequence, the Core actually provides a misleading picture. . If the sellers succeed in forming a stable coalition, they transform the situation into a bilateral monopoly in which case the negotiated price will be $\frac{1}{2}$. By symmetry, each of the sellers will get $\frac{1}{4}$ in this case. But, anticipating this, the consumer will try to form a coalition with any of the sellers, if only to prevent these sellers from entering into a cartel agreement. As Von Neumann and Morgenstern (1953) already realized, and as we discussed in Section 2, the game is really one in which players will rush to form a coalition and the price that the buyer will pay will depend on the ease with which various coalitions can form. But then the outcome will be determined by the coalition formation process, hence, following Nash's advise, non-cooperative modelling should focus on that process.

Let us here study one such process. Let us assume that the players bump into each other at random and that, if negotiations between two players are not successful (which, of course, will not happen in equilibrium), the match is dissolved and the process starts afresh. The remaining question is what price, p, the consumer will pay to the seller if a buyer-seller coalition is formed. (By symmetry, this price does not depend on which seller the buyer is matched with.) The outcome is determined by the players' outside options, i.e. by what players can expect if the negotiations break down. The next table provides the utilities players can expect depending on the first coalition that is formed

	Utility		
First Coalition	1	2	3
$\{1,3\}$	p	0	$1-p$
$\{2,3\}$	0	p	$1-p$
$\{1,2\}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

For the coalition $\{1,3\}$, the outside option of the seller is $\frac{1}{3}(p+\frac{1}{4})$, while the buyer's outside option is $\frac{2}{3}(1-p)+\frac{1}{6}$. (This follows since all three 2-person coalitions are equally likely to form in the next round.) The coalition loses $\frac{1}{3}(p+\frac{1}{4})$ if it does not come to an agreement, hence, it will split this surplus evenly. It follows that the price p must satisfy

$$p = \frac{1}{3}(p+\frac{1}{4}) + \frac{1}{6}(p+\frac{1}{4}),$$

Hence, $p=\frac{1}{4}$. Since all coalitions are equally likely, the expected payoff of a seller equals $\frac{1}{6}$, while the buyer's expected payoff equals $\frac{2}{3}$. The conclusion is that expected payoffs are equal to the Shapley value of the game. Furthermore, the outcome, naturally, lies outside of the Core. We refer that reader who thinks that we have skipped over too many details in the above derivation to Montero (2000), where all such details are filled in.

Of course, the exact price will depend on the details of the matching process and different processes may give rise to different prices, hence, different cooperative solution concepts. Viewed in this way, also Von Neumann and Morgenstern's solution of this game appears quite natural. As they write (Von Neumann and Morgenstern (1953, pp. 572, 573), the solution consists of two branches, either the sellers compete (and then the buyer gets the surplus), a situation they call the classical solution, or the sellers form a coalition, and in this case, they will have to agree on a definite rule for how to split the surplus obtained; as different rules may be envisaged, multiple outcome may be a possibility.

5 AUCTIONS

In this Section, we illustrate the usefulness of game theory in the understanding of real life auctions. The Section consists of three parts. First, we briefly discuss some auction theory. Next,we discuss an actual auction and provide a non-coopertive analysis to throw light on a policy issue. In the third part, we demonstrate that also in this non-cooperative domain, insights from cooperative game theory are very relevant.

Four basic auction forms that are typically distinguished. The first type is the Dutch auction. If there is one object for sale, the auction proceeds by the seller starting the auction clock and continuously lowering the price until one of the bidders pushes the button, or shouts "mine"; that bidder then receives the item for the price at which he stopped the clock. The other three basic auction forms are the English (ascending) auction in which the auctioneer continuously increases the price until one bidder is left; this bidder then receives the item at the price where his final competitor dropped out. Both the Dutch and the English auction involve a dynamic element. The two basic static auction forms are the sealed bid first price auction and the Vickrey auction. In the first price auction, bidders simultaneously and independently enter their bids, typically in sealed envelopes, and the object is awarded to the highest bidder who is required to pay his bid. In the Vickrey auction, players enter their bids in the same way, and the winner is again the one with the highest bid, however, the winner "only" pays the second highest bid

As auctions are conducted by following explicit rules they can be represented as (non-cooperative) games. Milgrom and Weber (1982) have formulated a fairly general auction model. In this model, there are n bidders, that occupy symmetric positions. The game is one with incomplete information, each bidder i has a certain type θ_i that is known only to this bidder himself. In addition, there may be residual uncertainty, represented by θ_0, where 0 denotes the chance player. If $\theta = (\theta_0, \theta_1, \ldots, \theta_n)$ is the vector of types (including that of nature), then θ is called the state of the world, and θ is assumed to be drawn from a commonly known distribution F on a set Θ that is symmetric with respect to the last n arguments (Symmetry thus means that F is invariant with respect to permutations of the bidders.) In addition to his type, each player i has a value function, $v_i(\theta)$, where again the assumption of symmetry is maintained, i.e. if θ_i and θ_j are interchanged, then v_i and v_j are interchanged as well. Under the additional assumption of affiliation (which roughly states that a higher value of θ_i makes a higher value of θ_j more likely), Milgrom and Weber derive a symmetric equilibrium for this model. For the Vickrey auction, the optimal bid is characterized by

$$b_i(\theta_i) = E(v_i | v_i = v^1 = v^2)$$

where v^k denotes the k-th largest component of the vector $v(\theta) = (v_1(\theta), \ldots, v_n(\theta))$. In words, in the Vickrey auction, the player bids the expected value of the object to him, conditional on his value being the highest, and this value also being equal to the second highest value. For the Dutch (first price) auction, the optimal bid is lower, and the formula will not be given here. (See Wilson, 1992). We also note that, in addition to giving insights into actual auctions, game theory has also contributed to characterizing optimal auctions, where optimality either is defined with respect to seller revenue or with respect to some efficiency criterion (Myerson, 1981; Wilson, 1992).

In many cases, the seller will have more than one item for sale. In case the objects are identical (such as in the case of shares, or treasury bills), the generalizations of the model and the theory are relatively straightforward: only one price is relevant; players can indicate how much they demand at each possible price and the seller can adjust price (either upward, or downward, or in a static sealed bid format) to equate supply and demand. The issue is more complicated in case the objects are heterogenous. With m objects, the relevant price region would be $\mathbb{R}^m_+$ and, of course, one could imagine bidders expressing their demand ($d_i : \mathbb{R}^m_+ \to 2^{\{1,\ldots,m\}}$) for all possible price vectors, but this may get very complicated. Alternatively, each bidder expresses bids for collections of items, hence, if $S \subset \{1, \ldots, m\}$, then $B_i(S)$ is the maximum that i is willing to pay if he is awarded set S, where the auction rule would be completed by a winner determination rule. At present, there is active research on such combinatorial auctions. In connection with spectrum auctions in the US, game theorists designed the simultaneous multi round ascending auction, a generalization of the English auction. In this format, the objects are sold simultaneously in a sequence of rounds with at least one price increasing from one round to the next. In its most elementary form, each bidder can bid on all items and the auction continues to raise prices as long as at least one new bid is made; when the auction ends, the current highest bidders are awarded the objects at these respective prices. To speed up the auction, activity rules may be introduced that force the bidders to bid seriously already early on. We refer to Milgrom (2000) for more detailed description and analysis.

Having briefly gone over the theory, our aim in the remainder of this Section is to show how game theory can contribute to better insight and to more rational discussion in several policy areas. Our examples are drawn from the Dutch policy context, and our frst example relates to electricity. Electricity prices in the Netherlands are high, at least they are higher than in the neighboring Germany. As a result of the price difference, market parties are interested in exporting electricity from Germany into the Netherlands. Such imports into the Netherlands are limited by the limited capacity of the interconnectors at the border, which in turn implies that the price difference can persist. In 2000, it was decided to allocate this scarce capacity by means of an auction; on the website www.tso-auction.org, the interested reader can find the details about the auction rules and the auction outcomes. We discuss here a simplified (Cournot) model that focuses on some of the aspects involved.

As always in auction design, decisions have to be made about what is to be auctioned, whether the parties are to be treated symmetrically, and what the payment mechanism is going to be. Of course, these decisions have to be made to contribute optimally to the ultimate goal. In this specific case, the goal may be taken as to have an as low price for electricity in the Netherlands as possible. The simple point now is that adopting this goal implies that players cannot be treated symmetrically. The reason is that they are not in symmetric positions: some of them have electricity generating capacity in the Netherlands, while others do not, and members of the first group may have an incentive to block the interconnector in order to guarantee a higher price for the electricity that is produced domestically. To illustrate this possibility, we consider a simple example.

Suppose there is one domestic producer of electricity, who can produce at constant marginal cost c. Furthermore, assume that demand is linear, $D(p) = 1 - p$. If the domestic producer is shielded from competition, and is not regulated, he will produce the monopoly quantity q_m, found by solving:

$$\max_{q_m} q_m(1 - q_m) - cq_m,$$

Hence the quantity q_m, the price p_m and the profit π_m will be given by:

$$\begin{aligned} q_m &= (1-c)/2 \\ p_m &= (1+c)/2 \\ \pi_m &= (1-c)^2/4 \end{aligned}$$

Assume that the interconnector has capacity $k > 0$, and that in the neighboring country electricity is also produced at marginal cost c. In contrast to the home country, the foreign country is assumed to have a competitive market, so that the price in the foreign country $p_f = c$. As a result $p_f < p_m$ and there is interest in transporting electricity from the foreign to the home country. If all interconnector capacity would be available for competitors of the monopolist, the monopolist would instead solve the following problem:

$$\max_{q_m} q_m(1 - q - k) - cq,$$

hence, if he produces q, the total production is $q + k$, and the price $1 - q - k$. The quantity q_c that the monopolist produces in this competitive situation is:

$$q_c = (1 - k - c)/2,$$

while the resulting price p_c and the profit for the monopolist π_c are given by:

$$\begin{aligned} p_c &= (1 + c - k)/2 \\ \pi_c &= (1 - c - k)^2/4 \end{aligned}$$

The above calculations allow us to compute how much the capacity is worth for the competing (foreign) generators. If they acquire the capacity, they can produce electricity at price c and sell it at price p_c, thus making a margin $p_c - c = (1 - c - k)/2$ on k units, resulting in a profit of

$$\pi_f = k(1 - c - k)/2$$

At the same time, the loss in profit for the monopolist is given by

$$\triangle\pi = \pi_m - \pi_c = k(1 - c - k/2)/2$$

We see that

$$\pi_f < \triangle\pi,$$

so that the capacity is worth more to the monopolist. The intuition for this result is simple, and is already given in Gilbert and Newbery (1982): competition results in a lower price; this price is relevant for all units that one produces, hence, the more units that a player produces, the more he is hurt. It follows that, if the interconnector capacity would be sold in an ordinary auction, with all players being treated equally, then all the capacity would be bought by the home producer, who would then not use it. Consequently, a simple standard auction would not contribute to the goal of realizing a lower price in the home electricity market.

The above argument was taken somewhat into account by the designers of the interconnector auction, however it was not taken to its logical limit. In the actual auction rules, no distinction is being made between those players that do have generating capacity at home and those that don't: a uniform cap of 400 Mw of capacity is imposed on all players, (hence, the rule is that no player can have more than 400 Mw of interconnector capacity at its disposal, which corresponds with some 25 percent of all available capacity). This rule has obvious drawbacks. Most importantly, the price difference results because of the limited interconnector capacity that is available, hence, one would want to increase that capacity. As long as the price difference is positive, and sufficiently large, market parties will have an incentive to build extra interconnector capacity: the price margin will be larger than the investment cost. However, in such a situation, imposing a cap on the amount of capacity that one may hold, may actually deter the incentive to invest. Consequently, it would be better to have the cap only on players that do have generating capacity in the home country, and that profit from interconnector capacity being limited.

To prevent players with home generating capacity from buying, but not using interconnector capacity, the auction rules include "use it or lose it" clause. Clearly, such clauses are effective in ensuring that the capacity is used, however, they need not be effective in guaranteeing a lower price in the home electricity market. This can be easily seen in the explicit example that was calculated above. Suppose that a "use it or lose it" clause would be imposed on the monopolist, how would it change the value of the interconnector capacity for this monopolist? Note that the value is not changed for the foreign competitors, this is still π_f , as they will use the capacity in any way. The important insight now is that the clause also does not change the value for the monopolist: if the monopolist is forced to use k units at the interconnector, he will simply adjust by using k units less of his domestic production capacity. By behaving in this way, he will still produce q_m in total and obtain monopoly profits of π_m. Hence a "use it or lose it" clause has no effect, neither on the value of the interconnector for the incumbent, nor on the value for the entrants. Therefore, the value is larger for the incumbent, the incumbent will acquire the capacity and the price will remain unchanged, hence, the benefits of competition will not be realized.

This simple example has shown that the design that has been adopted can be improved: it would be better to impose capacity caps asymmetrically, and it should not be expected that "use it or lose it" clauses are very effective in lowering the price. Of course, the actual situation is much richer in detail than our model. However, the actual situation is also very complicated and one has to pick cherries to come to better grips with the overall situation. We hope it is clear that a simple model like the one that we have discussed in this section provides an appropriate starting point for coming to grips with a rather complicated situation.

Our second example relates to the high stakes telecommunications auctions that took place in Europe at the beginning of te third Millenium. During 2000, various European countries auctioned licenses for third generation mobile telephony (UMTS) services. Already a couple of years earlier, some of these countries had auctioned licenses for second generation (DCS-1800) services. In this subsection, we briefly review some aspects of the Dutch auctions. For further detail, see Van Damme (1999, 2001, 2002).

Van Damme (1999) describes the Dutch DCS-1800 auction and argues that, as a consequence of time pressure imposed on Dutch officials by the European Commission, that auction was badly designed. The main drawback was that the available spectrum was divided into very unequal lots: 2 large ones of 15 MHz each and 16 small ones of on average 2.5 MHz, which were sold simultaneously by using a variant of the multiround ascending auction that had been pioneered in the US. The rules stipulated that newcomers could bid on all lots, but that incumbents (at the time, KPN and Libertel) could bid only on the small lots. In this situation, new entrants had the choice between bidding on large lots, or trying to assemble a sufficient number of small lots so that enough spectrum would be obtained in total to create a viable national network. The latter strategy was risky. First of all, by bidding on the small lots one was competing with the incumbents. Secondly, one faced the risk of not obtaining enough spectrum. This is what is called in the literature "the exposure problem": if say 6 small lots were needed for a viable network, one had the risk of finding out that one could not obtain all six because of the intensity of competition, one might be left with three lots which would be essentially worthless. (At the time of auction, it was not clear whether such blocks could be resold, the auction rules stating that this was up to the Minister to decide.)

The structure of supply that was chosen had an interesting consequence. Most newcomers found it too risky to bid on the small lots, hence, bidding concentrated on the large lots and the price was driven up there. In the end, the winners of the large lots, Dutchtone and Telfort paid Dfl. 600 mln and Dfl. 545 mln, respectively for their licenses. Compared to the prices paid on the small lots, these prices are very high: Van Damme (1999) calculates that, on the basis of prices paid for the small lots, these large lots were worth only Dfl. 246 mln, hence, less than half of what was paid. There was only one newcomer, Ben, who dared to take the risk of trying to assemble a national license from small lots and it was successful in doing so; it was rewarded by having to pay only a relatively small price for its license. It seems clear that if the available spectrum had been packaged in a different way, say 3 large lots of 15 MHz each and 10 small lots of an average 2.5 MHz each, the price difference would have been smaller, and the situation less attractive for the incumbents. Perhaps one might even argue that the design that was adopted in the Dutch DCS-1800 auction was very favorable for the incumbents.

In any case, the 1998 DCS-1800 auction led to a five player market, at least one player more than in most other European markets. This provides relevant background for the third generation (UMTS) auction that took place in the summer of 2000, and which was really favorable for the incumbents. At that time, the two "old" incumbents (KPN and Libertel) still had large market shares, with the market shares of the newer incumbents (Ben, Dutchtone and Libertel) being between 5 and 10 percent each. In this situation, it was decided to auction five 3G-licenses, two large ones (of 15 MHz each) and three smaller ones (of 10 MHz each). It is also relevant to know that the value of a license is larger for an incumbent than for a newcomer to the market, and this because of two reasons. First, an incumbent can use its existing network, hence, it will have lower cost in constructing the necessary infrastructure. Secondly, if an incumbent does not win a 3G-license, it will also risk to lose its 2G-customers. Finally, it is relevant to know that it was decided to use a simultaneous ascending auction.

The background provided in the previous paragraph makes clear why the Dutch 3G-auction was unfavorable to newcomers. First, the supply of licenses (2 large, 3 small) exactly matches the existing market structure (5 incumbents, of which 2 large ones). Secondly, an ascending auction was used, a format that allows incumbents to react to bids and thus to outbid new entrants. Thirdly, the value of a license being larger for an incumbent than for an entrant implies that an incumbent will also have an incentive to outbid a newcomer. In a situation like this, an entrant cannot expect to win a license, so why should it bother to participate in this auction? On the basis of these arguments, one should expect only the incumbents to participate and, hence, the revenues to remain small, see Maasland (2000).

The above arguments seem to have been well understood by the players in the market. Even though many potential entrants had expressed an interest to participate in the Dutch 3G-auction at first, all but one subsequently decided not to participate. In the end, only one newcomer, Versatel, participated in the auction. This participant had equally well understood that it could not win; in fact, it had started court cases (both at the European and the Dutch level) to argue that the auction rules were "unfair" and that it was impossible for a newcomer to win. If Versatel knew that it could not win a license in this auction, why did it then participate? A press release that Versatel posted on its website the day before the auction givens the answer to this question.

> "We would however not like that we end up with nothing whilst other players get their licenses for free. Versatel invites the incumbent mobile operators to immediately start negotiations for access to their existing 2G networks as well as entry to the 3G market either as a part owner of a license or as a mobile virtual network operator."

The press release that Versatel realizes, and want the competitors to realize, that it has power over the incumbents. By participating in the auction, Versatel drives up the price that the winners (the incumbents) will have to pay. (Viewed in this light, the court cases that Versatel had started signals to the incumbents that Versatel know that it cannot win, hence, that it must participate in the auction with another objective in mind.) On the other hand, by dropping out, Versatel does the incumbents a favour, since the auction will end as soon as Versatel does drop out. The press release signals that Versatel is willing to drop out, provided that the incumbents are willing to let Versatel share in the benefits that they obtain in this way. All in all then, Versatel appears to be following a smarter strategy than the newcomers that did not participate in the auction.

For the reader who has studied Von Neumann and Morgenstern (1953), the above may all appear very familiar. Recall the basic three-player non-zero sum game from that book, with one seller, two buyers, one indivisible object, and one buyer attacking a higher value to this object than the other. Why would the weaker participate in the game, if he knows right from the start that he will not get the object anyway. The answer that the founding fathers give is that he has power over both other players, by being in the game, he forces the other seller to pay a higher price and he benefits the seller; by stepping out he benefits the buyer, and by forming a coalition with one of these other players, he can exploit his power. This argument is also contained, and popularized, in Brandenburger and Nalebuff (1996), a book that also clearly demonstrates the value of combining cooperative and competitive analysis. If one knows that Nalebuff was an advisor to Versatel, then it is no longer that surprising that Versatel has used this strategy.

One would like to combinue this story with a happy end for game theory, but unfortunately that is not possible in this situation. Even though Versatel's strategy was clever, it was not successful. Versatel stayed in the auction, but it did not succeed in reaching a sharing agreement with one of the incumbents, even though negotiations seem to have been conducted with one of them, the BT-subsidiary Telfort. Perhaps, the other parties had not fully realized the cleverness of Versatel and, as Edgar Allen Poe already remarked, it pays to be one level smarter than your opponents, but not more. When it became clear that negotiations would not be successful, Versatel dropped out. In the end only the Dutch government was the beneficiary of Versatel's strategy.

6 Conclusion

In this paper, we have attempted to show that the cooperative and non-cooperative approaches to games are complementary, not only for bargaining games, as Nash had already argued and demonstrated, but also for market games. Specifically, we have demonstrated this for oligopoly games and for auctions. We have shown that these approaches are not only complementary, but also that each approach may give essential insights into the situation and that, by combining insights from both vantage points, a deeper understanding of the situation may be achieved.

The strength of the non-cooperative approach is that allows detailed modelling of actual institutions. Hence, many different institutional arrangements may be modelled and analysed, thus allowing an informed, rational debate about institutional reform. Indeed, the non-cooperative models show that outcomes can depend strongly on the rules of the game. The strength of this approach is at the same time its weakness: why would players play by the rules of the game? Von Neumann and Morgenstern argued that, whenever it is advantageous to do so, players will always seek for possibilities to evade constraints, in particular, they will be motivated to form coalitions and make side-payments outside the formal. This insight is relevant for actual markets and even though competition lows attempt to avoid cartels and bribes, one should expect these to be not fully successful.

The cooperative approach aims to predict the outcome of the game on the basis of much less detailed information, it only takes account of the coalitions that can form and the payoffs that can be achieved. One lesson that the theory has taught us is that frequently this information is not enough to pin down the outcome. The multiplicity of cooperative solution concepts testifies to this. Hence, in many situations we may need a non-cooperative model to make progress. Such a non-cooperative model may also alert to the fact that the efficiency assumption that frequently is routinely made in cooperative models may not be appropriate. On the other hand, when the cooperative approach is really successful, such as in the 2-person bargaining context, it is really powerful and beautiful.

We expect that that the tension between the two models will continu to be a powerful engine of innovation in the future.

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