[Review of the book Differential Games in Economics and Management Science, E.J. Dockner & S. Jorgensen, 2000]

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Bookreview

Dockner, Engelbert, Steffen Jρrgensen, Ngo Van Long and Gerhard Sorger (2000), *Differential Games in Economics and Management Science*, Cambridge University Press

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Differential games is an important tool for analyses in economics and management that involve more than one actor and conditions that change over time. Since the two aspects often occur, differential games is a natural tool for many analyses. Examples are strategic interactions between firms and governments under conditions of accumulation of capital, knowledge or pollution. However, the tool is not generally used because the material is rather complicated. The standard reference is Ba∏ar and Olsder (1995), and rightly so, but this book is not easily accessible for a larger academic community and, as the authors of the book under review write, examples are not drawn from economics and management science. The material was introduced into the literature in a clear way by Starr and Ho (1969a, 1969b) (a reference that is missing in the book under review) but since then more theory has developed and many applications have appeared so that a book like this was needed.
The first question is whether the book serves the main purpose of being better accessible and providing a wide range of applications. The problem is that an intermediate level of mathematics is required to understand the intricacies of control theory and differential games. This book chooses a middle course between mathematical precision and a toolbox for practitioners and can be criticized by people who prefer one of the two, but the book succeeded, in my view, quite well in its purpose. By presenting heuristic arguments for the theorems, the book gives insight in the theory and by presenting a large number of examples and applications, the book teaches how to work with the theorems. The choice

of applications is largely driven by previous work of the authors, which is understandable and fine, but it would have been nice if the reference list had also covered other major application areas such as fiscal policy games. May be the organization of this part could also have been improved. The distinction between examples and applications is a bit odd (e.g. section 7.1.4), some repetition occurs, and sometimes a concept is introduced that is explained later (like the Hotelling rule on page 126).

The core chapter of the book is chapter 4. Since differential games combines game theory and optimal control theory, it makes sense to start with introductory chapters 2 and 3 into the two fields. Although the authors write on page 3 that differential games is a subclass of game theory now and not an extension of optimal control theory anymore, I think that this book shows that it still is predominantly optimal control theory. The book chooses the Markovian Nash equilibrium as the central concept. Given Markovian strategies for the other players, each player has to solve an optimal control problem and core theorems 4.1 and 4.2 are essentially control theorems. I consider this, by the way, as a strong point of differential games and not a weak point. The fit with game theory, however, is more problematic and had deserved more attention. Three issues are important here. First, the book chooses to work in continuous time but game theory traditionally works in discrete time, because one feels uncomfortable with the concept of instantaneous reaction. This shows, for example, in chapter 6 on trigger strategies where the authors have to deviate from this general framework and introduce a reaction time. Later in the chapter this is repaired but the final result becomes complicated and it is not so clear whether new insights in trigger strategy equilibria are acquired. Moreover, game theory got further in this topic by introducing renegotiation proof equilibria. Second, some discussion on the concept of information structure would have been useful. Traditionally differential games focuses on information on the current state, initial state or past states and distinguishes Nash and Stackelberg equilibria concepts (see chapter 5). However, game theory does not use the concept of Stackelberg equilibria anymore but describes hierarchical play as a Nash equilibrium with a special information structure. In fact, figure 2.2 in chapter 2 is a Stackelberg game. It would have been a simultaneous Nash game if the two nodes in the middle had been one information set. The book misses a chance to prevent possible confusion here. Last but not least, the issue of subgame perfectness is not overall clear but this requires some introduction and will be discussed below. The contribution of differential games to the main body of game theory is, in my view, especially the introduction of a (deterministic) state equation that represents accumulation of capital, knowledge, pollution and so on. Traditional dynamic game theory is mainly concerned with repeated games and Bayesian updates of uncertain factors.

Most applications in the book (and outside) have a standard pattern of comparing open-loop and Markov perfect Nash equilibria for the problem under consideration. This can be seen in sections 9.5, 10.1, 11.1, 12.1.2 and 12.3.1. Also the treatment of differential games with special structures (chapter 7) shows this pattern: see for linear state games, examples 7.1, 7.2 and 7.3 and for linear quadratic games, theorem 7.2. The approach in the book is mostly the usual approach that is found in the literature. The open-loop Nash equilibrium is derived with Pontryagin's maximum principle with degenerate Markovian strategies, and the Markov perfect Nash equilibrium is derived with the Hamilton-Jacobi-Bellman equations. However, the core chapter 4 is set up a bit differently. The idea here is again that given Markovian strategies for the other players, each player has to solve an optimal control problem and can either use the maximum principle (theorem 4.2) or the HJB equations (theorem 4.1). Then it is shown that a Markovian Nash equilibrium is always time-consistent (theorem 4.3) but not always subgame perfect. Theorem 4.4 states that if the conditions of theorem 4.1 hold not only on but also off the equilibrium path, the equilibrium is subgame perfect. Finally, on page 105 it is written that any stationary Markovian Nash equilibrium (for infinite time problems) is subgame perfect, provided it is independent of the initial state. This story is correct but it would have helped the reader to explain whether or not the use of the HJB equations in finite time problems (such as in section 11.1) automatically leads to a Markov perfect Nash equilibrium. In the chapter on control theory (chapter 3) it is shown that the HJB equations result from dynamic programming, and in the chapter on game theory (chapter 2) it is shown that subgame perfectness is a natural extension of dynamic programming (page 24). Given the main thrust of the book, I think these issues had deserved even more clarity than the book already provides. For example, it would have been interesting to present an application with a time-consistent Markovian Nash equilibrium that is not subgame perfect. Ba∏ar and Olsder (1995) used the terms global and stagewise to make distinctions (pages 141-

142). Later it became common to make distinctions between closed-loop and feedback. The authors consider this to be unfortunate on page 59 but it may in fact be useful. It is also important to relate this topic to the widely used concept of (pre-)commitment (pages 107 and 243). In discrete time, these issues are easier to understand but the authors prefer to work in continuous time and confusion may arise.

The authors chose to write one chapter on stochastic differential games (chapter 8) with some applications on pages 275 and 282. The topic deserves a whole book, but I think it is worth to give the reader of this book some idea of this important topic.

In general, I expect the book to be useful for researchers in economics and management who want to use the tool of differential games and that it will be widely cited in the near future. I would like to congratulate the authors with a nice contribution to the literature.

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