Techniques Choice, Misallocation and Total Factor Productivity

Uras, R.B.; Wang, P.

CentER Discussion Paper

No. 2014-074

TECHNIQUES CHOICE, MISALLOCATION AND TOTAL FACTOR PRODUCTIVITY

By

Burak R. Uras, Ping Wang

2 December, 2014

This is also an EBC discussion paper no. 2014-012

ISSN 0924-7815
ISSN 2213-9532

Tilburg University

Techniques Choice, Misallocation and Total Factor Productivity*

Burak R. Uras †
Tilburg University
European Banking Center

Ping Wang ‡
Washington University in St. Louis
Federal Reserve Bank of St. Louis/NBER

November 2014

Abstract: We develop a generalized production framework with endogenous "production techniques" that serve to organize raw factor inputs in an efficient manner. We establish a positive relationship between production flexibility and cost efficiency. By allowing firms to differ in technology scales, capital constraints and technique limitations, we illustrate an amplification of the detrimental effects of technique limitations by production flexibility. We apply the structure to studying, both theoretically and quantitatively, the consequences of capital and technique misallocation across firms for the TFP and the interplay of their TFP effects with production flexibility. Using firm-level data from U.S. manufacturing industries, we find that, due to the amplification effect of production flexibility, technique misallocation generates more TFP losses than capital misallocation. Our quantitative results suggest that the relative importance of technique misallocation on TFP is substantial for a broad range of manufacturing industries – with larger TFP gains from removing technique misallocation in industries using more flexible production technologies.

JEL Classification: D24, E23, O11, O33.

Keywords: Capital and Technique Misallocation, Production Flexibility, Aggregate Productivity.

*We would like to thank Costas Arkolakis, Chang-Tai Hsieh, Sam Kortum, Ed Prescott, Robert Porter, and Chong Yip for their helpful comments.

†Assistant Professor of Economics; E-mail: r.b.uras@uvt.nl

‡Seigle Family Professor in Arts and Sciences; E-mail: pingwang@wustl.edu

"Why Are Total Factor Productivities Different? My candidate for the factor is the strength of the resistance to the adoption of new technologies and to the efficient use of currently operating technologies, and this resistance depends upon the policy arrangement a society employs. What is needed is a theory of how arrangement affects total factor productivity." [Prescott (1998, p. 549)]

1 Introduction

Thanking to the increased availability of reliable micro datasets, there is a growing literature over the past decade uncovering the sources of factor misallocation and the consequences for aggregate total factor productivity.[1] This literature not only helps explaining productivity gaps across countries or sectors, but also provides valuable policy prescriptions for means to advance a macroeconomy. In this paper, we revisit this important issue by developing a generalized production framework in which we augment raw measures of factor inputs with production techniques that serve to organize factor inputs to govern their efficient usage under currently operating technologies. This enables us to examine the separate roles played by misallocation of factor inputs and misallocation of production techniques in aggregate productivity. Our approach in turn delivers new insights toward isolating the causes and the consequences of misallocation in techniques from that of misallocation in capital.

Almost half a century after the pivotal work by Houthakker (1955-1956), there is a renewed interest in the microfoundation of the aggregate production function.[2] A key ingredient in the literature is the introduction of factor-specific production techniques. The primary focus of this line of research has been on qualifying the shape of the aggregate production function based on the distribution of production techniques or on the adoption or assimilation of a global frontier technology by local firms. It is, however, in our belief that the incorporation of production techniques is a natural channel for assessing the macroeconomic consequences of micro-level misallocation as well. Why? To answer this, let us refer the reader to two strands of management science literature largely ignored by economists. One strand is on the design for manufacturability (DFM) that specifies the ability to reorganize under the given input capacity in order to cope with market uncertainty.[3] This line of research emphasizes that DFM is a crucial way for a firm to operate effectively. We argue that DFM can be captured by production techniques which are subject to a manufacturer's knowledge scale as well as technological availabilities. Another strand of the literature is on production flexibility, in

[1] See Restuccia and Rogerson (2013) for a comprehensive survey of the literature.

[2] See two pioneer studies by Kortum (1997) and Jones (2005).

[3] See Youssef (1994) for a literature review.

particular, studies highlighting operation flexibility, process flexibility and machine flexibility.[4] This line of research elaborates that high manufacturing flexibility can improve upon the competitiveness and create future options for a firm. We notice the importance of the interplay between DFM and flexibility in affecting the performance of a firm. Such matters can be conveniently captured in a framework with generalized Constant Elasticity of Substitution (CES) technologies where production flexibility can be measured by the elasticity of substitution between technique-augmented factor inputs.

Thus, we proceed by going one-step further, sharpening the microfoundation by modeling techniques choice in addition to the neoclassical firm optimization, while allowing firms to differ in their technology scales as well as in their capital input constraints and techniques limitations. We then develop a structure to aggregate these heterogeneous firms to produce an industry-level total factor productivity (TFP) measure, which enables us to isolate, both theoretically and quantitatively, the TFP effects of capital misallocation from that of technique misallocation.

Specifically, we consider a manufacturing firm producing with the use of two factor inputs, capital and labor. Each input is augmented by a factor-specific production technique. The two production techniques are chosen from a technology menu, depending on the technology scale measuring the capacity of DFM. The two technique-augmented factor inputs are then combined in a CES form to produce the output. For better illustration, we conveniently divide the production decisions of the firm into two steps. In the first step, the manufacturer decides on its capital and labor demands, as in the standard neoclassical framework. In the second step, the manufacturer chooses a suitable combination of production techniques from a convex technology menu. As a key property of this benchmark framework, we derive a positive association between production flexibility and the cost efficiency of the firm: Flexible substitutability between technique-augmented factor inputs reduces the unit cost of production. This result echoes the micro-level evidence documented in the management science literature.

Building upon this benchmark production framework, we incorporate distortionary wedges into firm's optimal factor and techniques choice. For better comparison with previous studies, the factor input wedge is modeled as a capital financing distortion due to financial market imperfections. The technique choice wedge – new to the literature – is a distortion resulting from a technique capability limitation. While both distortions, as expected, reduce the cost efficiency of the firm, we establish another key property: Flexibility of production amplifies the cost distortions arising from the technique capability constraint. With a distribution of heterogeneous firms differing in

[4] See, for example, Gerwin (1993) and Roller and Tombak (1993).

technology scales and capital constraints and technique capabilities, we show that both capital and technique distortions suppress firm entry, increase exit and also generate intensive margin resource misallocation among producers. By aggregating over heterogeneous firms, we construct a new deep-structural measure of industry-level TFP to reexamine the consequences of factor and technique misallocation. This industry-level TFP not only depends on firms' physical (quality) productivities (TFPQ) and the dispersion of their revenue productivities (TFPR), but is, more importantly, also influenced by the covariance between firms' physical productivities and the distortions. When this covariance is negative (which is true for a wide range of parameter conditions allowed by technique capabilities), those firms hit by adverse production frictions get to also become "inherently" less productive.

Upon learning the theoretical interplay between techniques choice and production flexibility and its consequences for misallocation and aggregate productivity, an immediate question arises: Is this new mechanism quantitatively important to explain TFP of manufacturing industries? To address this, we utilize firm-level balance sheet data from Compustat North America, in conjunction with industry-level factor elasticity, to separately back out manufacturing firms' capital and technique distortions. We then compute firms' physical and revenue productivities as well as industry-level TFPs. As expected, we find that, firms' TFPR rise with both type of distortions, so the dispersion in TFPR leads to lower industry TFP. However, since capital and technique distortions reduce firms' TFPQ, we also estimate a negative covariance between TFPQ and TFPR at the firm level. This latter channel counteracts the adverse consequences of capital and technique distortions on the industry TFP.

An important contribution of our paper is the investigation of quantitative consequences of the misallocation of factor inputs and production techniques for firms' performance and industry TFP. Our quantitative results are threefold: First, technique misallocation measured by the variation in firms' technique capability distortions accounts for more than half of the intra-industry dispersion of revenue productivities for all manufacturing industries. Second, the relative importance of technique misallocation is largely driven by production flexibility with larger TFP gains from removing technique misallocation in industries using more flexible production technologies. Third, the presence of technique adjustability – and the resulting negative covariance between TFPR and TFPQ – reduces the adverse consequence of capital misallocation. To this end, we conduct a counterfactual exercise with the standard neoclassical framework to show that capital misallocation measured by the variation in firms' capital distortions becomes quantitatively important for the aggregate industry TFP when techniques choice is absent. We thus echo the Lawrence Klein Lecture delivered by

Prescott (1998): TFP differences depend crucially on *the strength of the resistance to the efficient use of currently operating technologies*. The underlying policy arrangement causing such resistance is captured in our framework by the technique capability distortions and the resulting variation in such distortions due to intra-industry technique misallocation – these are shown to be essential for the aggregate industry productivity.

<u>Related Literature</u>

The paper is related to two strands of literature. One strand is on understanding the sources of factor misallocation and the consequences for aggregate productivity. Another strand is on the introduction of factor-specific production techniques to qualify the shape of the aggregate production function and to investigate the adoption or assimilation of a global frontier technology by local firms.

The first strand of literature goes back to Banerjee and Duflo (2005) who identify the large dispersion in the marginal product of capital among firms in India as an important source for underperformance in overall output. More closely related to our paper are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Restuccia and Rogerson show that when production distortions hit physically productive firms, this has quantitatively important consequences for the total factor productivity of the macroeconomy. Hsieh and Klenow find that when the dispersion in production distortions are alleviated in India and China to the extent of the U.S., the TFP gap between the U.S. economy and these two countries could shrink up to 40%. Jones (2013) further elaborates that misallocation at the micro level leads to lower TFP at the macro level, thereby helping explain cross-country TFP gaps. To understand the sources of misallocation, Banerjee and Moll (2010), Midrigan and Xu (2013), Buera and Shin (2013) and Moll (2014) construct dynamic general equilibrium models of misallocation with capital market imperfections, whereas Jovanovic (2014) studies misallocation using an assignment framework with heterogeneous firms and workers.

We also work in an environment of factor misallocation, where firms face factor input distortions a la Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Yet, we go beyond by studying technique misallocation, where firms are subject to different technique capabilities. Moreover, with our emphasis on production flexibility, we generalize the Cobb-Douglas production function commonly used in the literature by allowing different non-unity elasticities of substitution for firms in different industries. Since we consider techniques choice, dispersion in capital and techniques distortions not only generates an heterogeneity in revenue productivities but also an endogenous covariance between distortionary wedges and physical productivities. Both channels have quantitatively important consequences for the TFP effects of factor and technique misallocation.

The second strand of literature owes to the seminal work by Houthakker (1955-1956), where firms produce with different Leontief technologies (local production) with production techniques following a Pareto distribution. The aggregate (global) production across production units then exhibits the Cobb-Douglas form. Kortum (1997) shows that if researchers sample production techniques from Pareto distributions, then productivity growth is proportional to the growth of the research stock and accounts for the empirical regularities concerning productivity growth and researcher employment observed over the past 50 years. Jones (2005) generalizes Houthakker's result and shows that as long as the techniques arrive to firms following a Pareto distribution, firms' global production would be Cobb-Douglas. Wang, Wong and Yip (2013) develop a technology assimilation framework using the global technology approach and show that the lack of assimilation of the frontier technology can be instrumental for differentiating between trapped and growth miracle economies. Also related to our paper is the framework of Caselli (1999), Acemoglu (2003), and Caselli and Coleman (2006). In Caselli (1999), firms decide both on production factors and techniques, whereas in Acemoglu (2003) firms undertake both labor- and capital-augmenting technological improvements. Caselli and Coleman (2006) investigate the implications of endogenous techniques choice for the cross-country technology frontier.

As in this second strand of literature, we also explore an alternative production framework incorporating the concept of production techniques at the firm level. Yet, we differ by modeling techniques choice under a generalized CES framework with a distribution of firms heterogeneous in their technology scales as well as in capital and technique distortions. Thus, we are able to highlight the role of production flexibility as well as to differentiate technique misallocation from factor misallocation. By developing a structure to aggregate these heterogeneous firms to produce industry-level TFP, we can then study the TFP effects of capital and technique misallocation in isolation, both theoretically and quantitatively.

2 A Benchmark Model with Production Techniques Choice

The benchmark economy features a representative firm, manufacturing a product with two factor inputs, capital and labor. Different from the neoclassical production framework, we augment raw measures of factor inputs with production techniques that serve to organize factor inputs in an effective manner to enhance the performance of the production process. The ability to choose potentially more profitable production techniques is subject to a technology menu available to the firm. For the time being, we assume that there are no frictions on the availability of production techniques.

These benchmarking assumption will be relaxed in Sections 3 to 5 below.

2.1 The Basic Environment

Denote capital as K and labor as L. The *production techniques* are captured by a pair (a_K, a_L) which augment the two factor inputs (K, L) to govern their usage and coordinate their match. The concept of production techniques is in line with the literature on the property of firm and aggregate production developed by Houthakker (1955-1956), Kortum (1997) and Jones (2005). It also captures factor-augmenting technology improvement modeled by Caselli (1999), Acemoglu (2003), and Caselli and Coleman (2006). That is, one may rename a_K and a_L, respectively, as capital-augmenting and labor-augmenting techniques. Our framework follows more closely the former literature, assuming that the availability of techniques is subject to the following technology constraint:

$$H(a_K, a_L) = z \tag{1}$$

Throughout the paper, we will assume that $H(a_K, a_L) = a_K^{\alpha} a_L^{1-\alpha}$. This technology constraint specifies the *full menu* of production techniques entailing different combinations of (a_K, a_L) under a given efficiency measure z. The trade-off between the two techniques is qualitatively similar to concept of the iso-quant, which can be referred to as the *iso-tech*. The firm is called more *knowledgeable* in engineering the process of production if it has a higher level of z. The level of knowledge is important for productivity measures.

We then depart from Houthakker-Kortum-Jones where a Cobb-Douglas "global" production function can be derived as an envelope of the Leontief "local" production function with techniques drawn from an independent Pareto distribution. We instead propose that the representative firm's production function takes the Constant Elasticity Substitution (CES) form:

$$Y = [\lambda (a_K K)^{\rho} + (1 - \lambda)(a_L L)^{\rho}]^{\frac{1}{\rho}}.$$

The parameter $\rho \in (-\infty, 1]$ captures the flexibility of the production technology in allowing the firm to substitute between the technique-augmented factor inputs, $a_K K$ and $a_L L$, with $1/(1 - \rho)$ measuring the elasticity of substitution. The parameter λ is thus the effective capital share of production. With (a_K, a_L) and the associated knowledge level z, there is no need to add another scaling parameter to the production function.

2.2 Firm's Optimization

The representative firm optimizes by jointly choosing factor inputs and production techniques. Although there is no particular sequencing, it is convenient to divide the production decisions into two steps: (i) In the first step, the firm decides on capital and labor demand to achieve a given level of output; and (ii) in the second step the firm chooses a suitable combination of production techniques from the convex technology menu to ensure the efficiency of the production process. While Step 1 is the standard optimization under the neoclassical production framework, Step 2 is referred to as the *techniques choice* problem.

Specifically, facing the market wage and capital rental (w, r), the firm in Step 1 solves the following cost minimization problem:

$$\begin{aligned} \min_{K,L} \quad & rK + wL \\ s.t. \quad & [\lambda(a_K K)^\rho + (1-\lambda)(a_L L)^\rho]^{\frac{1}{\rho}} = Y \end{aligned} \tag{2}$$

The solution yields a unit cost function conditional on a particular pair of production techniques, $\tilde{c}(a_K, a_L; r, w)$.

In Step 2, the firm pins down techniques choice to achieve the lowest unit cost of production under a given techniques menu:

$$\begin{aligned} \min_{a_K, a_L} \quad & \tilde{c}(a_K, a_L; r, w) \\ s.t. \quad & H(a_K, a_L) = z. \end{aligned} \tag{3}$$

As we illustrate in Figure 1, in this model we are primarily interested in an interior solution for the techniques choice.[5] Thus, in our model economy, one may capture the concept of *process innovation* by an expansion in z, which reduces the unit cost of production via more efficient organization of factor inputs.

In the benchmark model, the firm is allowed to choose any combination of techniques from the menu, i.e., from a full menu. In Section 3, we will introduce technique accessibility constraints into the benchmark and investigate the consequences of limited menu of techniques on firm-level outcomes.

[5] Note that the axes in figure 1 are $1/a_K$ and $1/a_L$. We adopt this convenience, so that the higher ranked isocosts are associated with higher levels of unit cost of production.

Neoclassical Cost Minimization

We turn now to solving the neoclassical cost-minimization problem. Throughout the paper, we shall relegate all detailed mathematical derivations and proofs to the Appendix. Manipulating the two first-order conditions yields:

$$\frac{K}{L} = \left(\frac{w}{r}\right)^{\frac{1}{1-\rho}} \left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{1-\rho}} \left(\frac{a_K}{a_L}\right)^{\frac{\rho}{1-\rho}}. \tag{4}$$

Thus, the capital-labor ratio is inversely related to the factor price ratio, which is standard. How the capital-labor ratio responds to the production technique ratio depends crucially on production flexibility. When the two technique-augmented factor inputs are *Pareto complements* ($\rho < 0$), the capital-labor ratio is negatively related to the production technique ratio. This is quite intuitive: under Pareto complementarity, it is profitable to balance between the two technique-augmented factor inputs. In this case, if the organization of factor inputs is biased towards one particular factor, then it is expected that the firm would employ more of another factor to ensure balanced factor usage. When the two technique-augmented factor inputs are *Pareto substitutes* ($\rho < 0$), the opposite is true: the firm employs more of the input associated with a better technique.

The unit cost function conditional on a given pair of production techniques (a_K, a_L) can be derived as:

$$\tilde{c}(a_K, a_L; r, w) = \left[\left(\frac{r}{a_K}\right)^{\frac{\rho}{\rho-1}} \lambda^{\frac{1}{1-\rho}} + \left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}} (1-\lambda)^{\frac{1}{1-\rho}}\right]^{\frac{\rho-1}{\rho}}. \tag{5}$$

This equation indicates that the unit cost of production is a CES aggregator of the technique-deflated factor costs. The endogenous adjustments in production techniques are the key to differentiate this unit cost function from the standard neoclassical one, to which we shall turn.

Techniques Choice

With a full menu, solution to techniques choice problem gives the optimized technique ratio:

$$\frac{a_K}{a_L} = \frac{r}{w} \left(\frac{1-\lambda}{\lambda}\right)^{\frac{1}{\rho}} \left(\frac{1-\alpha}{\alpha}\right)^{\frac{\rho-1}{\rho}}. \tag{6}$$

It turns out that the optimized technique ratio depends positively on the factor price ratio. Intuitively, when a factor input becomes pricier, it is profitable to devote more effort toward enhancing the technique associated with that factor in order to minimize the neoclassical unit cost.

Plugging (6) in (4) solves for the optimized capital-labor ratio:

$$\frac{K}{L} = \frac{w}{r} \frac{\alpha}{1-\alpha} \frac{\lambda}{1-\lambda}. \tag{7}$$

While the optimized capital-labor ratio continues to be inversely related to the factor price ratio, the factor cost share $\frac{rK}{wL}$ maintains constant. The K/L ratio also depends on both the neoclassical factor income share $\frac{\lambda}{1-\lambda}$ and additionally the techniques usage share $\frac{\alpha}{1-\alpha}$.

In order to determine the levels of production techniques, we combine (6) with (1) to derive:

$$
\begin{aligned}
a_K &= z\left(\frac{w}{r}\right)^{-(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)\left(\frac{1-\rho}{\rho}\right)}\left(\frac{1-\lambda}{\lambda}\right)^{(1-\alpha)\left(\frac{1}{\rho}\right)}, && (8)\\
a_L &= z\left(\frac{w}{r}\right)^{\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha\left(\frac{1-\rho}{\rho}\right)}\left(\frac{1-\lambda}{\lambda}\right)^{-\alpha\left(\frac{1}{\rho}\right)}. && (9)
\end{aligned}
$$

That is, both techniques are increasing in the level of knowledge. Moreover, they depend on the factor price ratio rather than the individual factor prices. While a higher wage-rental ratio induces a technique choice to be more biased toward labor, production flexibility affects the choice of techniques via factor income and technique usage shares.

2.3 Unit Cost of Production

By combining the techniques choice problem with the neoclassical cost-minimization problem, we can solve the unit cost of production as a function of factor prices:

$$
c(w,r) = \frac{1}{z}\left(\left(\frac{\alpha}{\lambda}\right)^{\frac{1}{\rho}}\frac{r}{\alpha}\right)^{\alpha}\left(\left(\frac{1-\alpha}{1-\lambda}\right)^{\frac{1}{\rho}}\frac{w}{1-\alpha}\right)^{1-\alpha}. \tag{10}
$$

As we delineated before, better knowledge in techniques (higher z) engineers a process innovation, thereby reducing the unit cost of production. Interestingly, while the conditional unit cost function is a CES aggregator of factor prices, the final solution of the unit cost function, after taking into account the techniques choice problem, becomes a Cobb-Douglas aggregator of factor prices weighted by technique usage rather than factor income shares. Moreover, this Cobb-Douglas aggregator depends on the ratios of technique usage to factor income shares, $\frac{\alpha}{\lambda}$ and $\frac{1-\alpha}{1-\lambda}$, and production flexibility ρ. Before elaborating the impact of production flexibility on unit cost of production, let us summarize the firm optimization results that we obtained so far in the following proposition.

Proposition 2.1 *(Firm Optimization) Firm's optimizing decision possesses the following properties:*

(i) *While the capital-labor ratio is inversely related to the factor price ratio, a higher factor price induces a technique choice more biased toward that factor.*

(ii) *The unit cost function is a Cobb-Douglas aggregator of factor prices weighted by technique usage shares, scaled by both technique usage and factor income shares, where better knowledge in techniques enhances the levels of techniques chosen and reduces the unit cost of production.*

Proof All proofs are relegated to the Appendix.

Now, we turn to studying the implications of production flexibility for the cost of production. To begin, using (10) we establish the following limit properties.

Lemma 2.2 *In the limit cases with extreme flexibility measures, the unit cost of production converges to:*

(i) *($\rho \to -\infty$)*

$$c(w,r) = \frac{1}{z}\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha};$$

(ii) *($\rho \to 1$)*

$$c(w,r) = \frac{1}{z}\left(\frac{r}{\lambda}\right)^{\alpha}\left(\frac{w}{1-\lambda}\right)^{1-\alpha}.$$

Thus, while the factor prices are always weighted by technique usage shares, how much they affect the unit cost depend crucially on production flexibility. When flexibility is shut down ($\rho \to -\infty$), the production technology (the CES aggregator) precludes technique-augmented factor inputs from substituting by each other. As a result, factor prices are deflated only by their technique usage shares. With a greater technique usage share, a factor price would not raise the unit cost of production as much. When flexibility is perfect, on the contrary, factor prices are deflated only by their income shares. In this case, an increase in the price of a factor with a greater income share would become less damaging to the unit cost of production.

With the extreme cases addressed, we now inquire what happens with intermediate levels of flexibility.

Proposition 2.3 *(Production Flexibility and Unit Cost) Production flexibility (ρ) monotonically reduces the unit cost of production for any given pair of factor prices.*

This theoretical result indicates a positive impact of production flexibility on firm performance. This result echoes an extensive list of findings highlighted in the management science literature.[6] In the next section, we introduce frictions into our benchmark model and investigate the interactions between production flexibility and factor-and-technique-driven distortions.

[6] See Roller and Tombak (1993), Gerwin (1993) and Adler, Goldoftas and Levine (1999)

3 Production Frictions and Distortions

In this section, we incorporate production frictions into the benchmark framework and investigate the distortionary consequences of such frictions on firm-level performance. We focus our study on two types of frictions: (i) capital finance imperfections that result in constrained capital input, and (ii) technique capability limitations that lead to constrained technique usage.

3.1 Capital Input Constraint

Suppose that capital input is constrained by

$$K \leq \bar{K},$$

where $\bar{K}$ is an upper limit due for example to a financial imperfection.[7] When the capital constraint is binding, the firm's capital finance cost increases. This distortion can be conveniently expressed as $r(1+\eta)$ with the wedge $1+\eta$ capturing the additional capital distortion resulting from capital finance imperfection.

The constrained neoclassical optimization problem can be solved to yield the capital-labor ratio:

$$\frac{K}{L} = \left(\frac{w}{r(1+\eta)}\right)^{\frac{1}{1-\rho}} \left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{1-\rho}} \left(\frac{a_K}{a_L}\right)^{\frac{\rho}{1-\rho}}, \tag{11}$$

and the conditional unit cost of production:

$$\tilde{c}(a_K, a_L; r, w) = \left[\left(\frac{r(1+\eta)}{a_K}\right)^{\frac{\rho}{\rho-1}} \lambda^{\frac{1}{1-\rho}} + \left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}} (1-\lambda)^{\frac{1}{1-\rho}}\right]^{\frac{\rho-1}{\rho}}. \tag{12}$$

Applying comparative statics at 12 with respect to η yields the following intuitive result.

Proposition 3.1 *(Input Wedge and Unit Cost) For a given pair of techniques (a_K, a_L), the unit cost of production rises as the capital input wedge (η) increases.*

Not surprisingly, capital distortions result in a higher capital user cost, which in turn increases the unit cost of production.

3.2 Technique Capability Constraint

We next incorporate that techniques choice is subject to a *technique capability* constraint as in the following:

$$\frac{a_K}{a_L} \leq \nu,$$

[7] Using our framework, one can also investigate the consequences of labor market frictions. For brevity, we omit the discussion on labor input constraints

where ν measures the limitation on technique choice.[8] Such technique biased constraints may prevail because of limited availability of one technique relative to another or limited ability in reorganizing one factor relative to another. As we illustrate in Figure 2, a binding constraint implies that the firm operates under an inefficiently low a_K/a_L ratio compared to the unconstrained optimum.

When the technique capability constraint is binding, it generates a wedge $1+\phi$ between techniques substitution from the technology side and that of from the cost side:

$$-\left.\frac{da_K}{da_L}\right|_{\text{iso-tech}} = \frac{\partial H/\partial a_L}{\partial H/\partial a_K} = (1+\phi)\frac{\partial \tilde{c}/\partial a_L}{\partial \tilde{c}/\partial a_K} = -(1+\phi)\cdot \left.\frac{da_K}{da_L}\right|_{\text{iso-cost}}. \tag{13}$$

As shown in Figure 3, a wedge $1+\phi > 1$ arising from the binding accessibility constraint makes the firm operate along a higher iso-cost curve.[9] Then, with both capital and technique distortions, the techniques choice problem leads to a constrained technique ratio – expressed as the following:

$$\frac{a_K}{a_L} = \frac{r(1+\eta)}{w}\left(\frac{1-\lambda}{\lambda}\right)^{\frac{1}{\rho}}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{\rho-1}{\rho}}(1+\phi)^{\frac{1-\rho}{\rho}}. \tag{14}$$

It is interesting to observe that while the constrained technique ratio is always increasing in the capital input wedge to accommodate rising cost of capital, the effect of the technique wedge appears to depend crucially on the flexibility of production, which requires further investigation. First, we would like to point out the following result concerning the relationship between ρ and ϕ:

Lemma 3.2 *(Production Flexibility and Technique Distortions)*

(i) *(Pareto Complement) When $\rho < 0$, the technique wedge exhibits $\phi > 0$.*

(ii) *(Pareto Substitute) When $0 < \rho < 1$, the technique wedge exhibits $-1 < \phi < 0$.*

Consider the Pareto complement case. We can see from (13) that the technique wedge creates a distortionary margin raising the shadow cost of capital-augmenting technique relative to the labor-augmenting technique. As this wedge rises, techniques choice is more biased toward labor-augmenting. Now, let's examine the Pareto substitute case. As the wedge is widened ($1+\phi$ approaching zero), techniques choice becomes again more biased toward labor-augmenting. Thus, we obtain the following:

Proposition 3.3 *(Distortion Wedges and Techniques Choice) Either a reduced capital input wedge (higher η) or a widened technique accessibility wedge (higher $|\phi|$) induces technique choice to be biased toward labor-augmenting regardless of the elasticity of substitution between technique-augmented factor inputs.*

[8] For brevity, we omit the discussion of the alternative, $\frac{a_K}{a_L} \geq \nu$

[9] We would like to note that this wedge may be, more generally, as a result of various frictions and distortions associated with techniques choice.

For the rest of the analysis, we will focus on the Pareto complement case with $\rho < 0$ and $\phi > 0$. In Section 5, we will provide empirical evidence from various manufacturing industries for the estimates of ρ and argue that for the manufacturing sector $\rho < 0$ is the empirically plausible portion of the parameter space.

3.3 Distortions, Production Flexibility, and Unit Costs

We again combine the techniques choice problem with the neoclassical optimization problem to solve the factor input ratio:

$$\frac{K}{L} = \frac{w}{r(1+\eta)} \frac{\alpha}{1-\alpha} \frac{\lambda}{1-\lambda} \frac{1}{1+\phi}, \tag{15}$$

Thus, the capital-labor ratio always decreases with either distortionary wedge, whether it raises the user cost of capital or the shadow cost of capital-augmenting technique. Moreover, the levels of production techniques can be derived as:

$$a_K = z\left(\frac{r(1+\eta)}{w}\right)^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)\left(\frac{1-\rho}{\rho}\right)}\left(\frac{1-\lambda}{\lambda}\right)^{(1-\alpha)\left(\frac{1}{\rho}\right)}(1+\phi)^{(1-\alpha)\left(\frac{1-\rho}{\rho}\right)}, \tag{16}$$

$$a_L = z\left(\frac{r(1+\eta)}{w}\right)^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha\left(\frac{1-\rho}{\rho}\right)}\left(\frac{1-\lambda}{\lambda}\right)^{-\alpha\left(\frac{1}{\rho}\right)}(1+\phi)^{-\alpha\left(\frac{1-\rho}{\rho}\right)}. \tag{17}$$

We can see that while an increase in the capital input wedge induces techniques choice to be biased toward capital-augmenting, widening the technique capability wedge generates an opposite effect. Intuitively, widening the technique capability wedge implies more distortions on capital-augmenting technique relative to the labor-augmenting technique, thereby inducing a technique choice biased toward labor-augmenting. This result is very interesting from the theoretical and – as we delineate below – from the quantitative point of view. In order to undo the distortionary effects of the capital wedge (η), the firm raises its "capital productivity". However, by how much it can do this is limited by the capital-technique wedge (ϕ). We summarize the impact of distortionary wedges on firms' $K - L$ and $a_K - a_L$ decisions as the following.

Proposition 3.4 *(Distortion Wedges and Firm Decisions)*

(i) *The capital-labor ratio decreases with either wedge.*

(ii) *A higher capital input wedge induces techniques choice to be biased toward capital-augmenting, but widening the technique capability wedge generates an opposite effect.*

The unit cost of production can now be derived as:

$$c(r,w) = \frac{1}{z}\left(\left(\frac{r(1+\eta)}{\alpha}\right)\left(\frac{\lambda}{\alpha}\right)^{-\frac{1}{\rho}}\right)^{\alpha}\left(\left(\frac{w}{1-\alpha}\right)\left(\frac{1-\lambda}{1-\alpha}\right)^{-\frac{1}{\rho}}\right)^{1-\alpha}\left(\frac{(1+\phi)^{\alpha}}{1+\alpha\phi}\right)^{\frac{1-\rho}{\rho}}. \tag{18}$$

Capital wedge η clearly raises the unit cost of production. The technique wedge affects the unit cost of production via the term $\left(\frac{(1+\phi)^{\alpha}}{1+\alpha\phi}\right)^{\frac{1-\rho}{\rho}}$, which depends crucially on production flexibility. Observe that $\frac{(1+\phi)^{\alpha}}{1+\alpha\phi}$ is hump-shaped, reaching the minimum value of zero at $\phi = -1$ and the maximum value of one at $\phi = 0$; it never exceeds one for all $-1 < \phi < 1$. Thus, in the Pareto complement case with $\rho < 0$ and $\phi > 0$, $\left(\frac{(1+\phi)^{\alpha}}{1+\alpha\phi}\right)^{\frac{1-\rho}{\rho}} > 1$, implying that the unit cost of production is higher in the presence of technique distortions. Therefore, regardless of the elasticity of substitution between technique-augmented factor inputs, increasing either the capital input wedge or the technique wedge lead to a higher unit cost of production.

The next question to be addressed is whether the distortionary effects of the two wedges on the unit cost of production are influenced by the flexibility of production. We are basically interested in signing $\frac{\partial^2 c}{\partial\phi\partial\eta}$, $\frac{\partial^2 c}{\partial\phi\partial\rho}$ and $\frac{\partial^2 c}{\partial\eta\partial\rho}$, under the benchmark of Pareto complementarity with $\rho < 0$ and $\phi > 0$. In this case, we depict in Figure 4 the combinations of the pairs of wedges (η, ϕ) that keep the unit cost of production constant. Since both wedges raise the unit cost, this locus is downward sloping. While the capital input wedge raises the unit cost of production, one can see from (18) that this wedge does not directly interact with production flexibility. Therefore, a more flexible production technology would not help mitigate the detrimental effects of capital distortions on the unit cost of production, i.e., $\frac{\partial^2 c}{\partial\eta\partial\rho} = 0$. For the distortionary technique wedge, we can observe that $\frac{\partial^2 c}{\partial\phi\partial\rho} > 0$, implying that flexibility elevates the detrimental effects of technique distortions. Thus, when techniques choice is subject to frictions, a high production flexibility might not be as desirable from firms' cost efficiency point of view. We summarize the results concerning the interactions between distortions, production flexibility and the unit cost as the following.

Proposition 3.5 *(Production Flexibility, Distortion Wedges and Unit Cost of Production)*

(i) *Increasing the capital input wedge or widening the technique capability wedge raises the unit cost of production.*

(ii) *Production flexibility does not mitigate the detrimental effects of capital distortions on the unit cost of production.*

(iii) *Under Pareto complementarity, greater flexibility of production amplifies the detrimental effects of technique distortions.*

Finally, we also examine the interaction between capital and technique distortions. In the Pareto complement case with $\rho < 0$ and $\phi > 0$, it is clear that $\frac{\partial c^2}{\partial \eta \partial \phi} > 0$, implying reinforcing cost deficiency of η and ϕ wedges.

Proposition 3.6 *(Interaction of Input and Technique Distortions) The detrimental effects of capital and technique distortions on the unit cost of production are reinforcing each other.*

This result is very important for the quantitative implications of distortions on economic performance. In particular, it suggests that factor input distortions become more harmful for the performance of the firm when the access to production techniques is limited.

4 Firm Dynamics, Production and Misallocation

The theoretical analysis so far taught us that both capital financing and technique accessibility frictions would raise the unit cost of production. In order to investigate the aggregate implications of each type of friction, we consider a heterogeneous firms set-up, where we allow firms to differ in their productivities as well as in the distortions they face due to the two types of frictions. We model the dynamic firm entry-exit following the seminal work by Melitz (2003). In so doing, we are able to isolate the effects of input and technique distortions on extensive and intensive margin misallocation. Building upon our theoretical aggregation results, we will develop industry-level aggregate TFP measures and decompose the TFP effects of capital-and-technique-generated misallocation.

4.1 Entry

There is a distribution of firms, which can produce a particular manufacturing industry's output. We assume that in order to enter the industry, each firm must pay f_e, as a fixed entry cost - measured in units of labor. Upon entry, in order to produce a firm needs to incur f units of fixed production costs. In addition to this, each firm will pay the operation cost cy, where c is the unit cost of production. The unit cost of production will endogenously vary across firms depending on the firm-specific variations in technology scale, capital and technique distortions, namely in $\{z, \eta, \phi\}$.

4.2 Demand

Firms are monopolistic competitors a la Dixit-Stiglitz, which allows us to derive firm profits as

$$\pi = \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} (\sigma - 1)^{-1} c(r, w)^{1-\sigma} I,$$

where $\sigma/(\sigma-1)$ – with $\sigma > 1$ – is the mark-up implied by Dixit-Stiglitz preferences and I is the aggregate income in the society which we treat as exogenous. Then, the larger ϕ and η the smaller are firm profits due to higher unit cost of production. Recall that, via the unit cost function, production flexibility ρ does not influence the effect of η on firm profits, but it amplifies the distortionary effects of ϕ on firm profits.

4.3 Heterogeneity

We analyze the effects of $\{z, \eta, \phi\}$ on firms' entry-exit (extensive margin) and their production scale (intensive margin). In particular, we need to characterize the cutoffs that govern firms' entry-exit decisions and study the impact of production flexibility on firm dynamics. For illustrative purposes, we shall loosely refer to z, η and ϕ as the productivity shock, the capital financing shock and the technique capability shock respectively. The productivity shock (z) is standard, which is assumed to follow a Pareto distribution. The capital financing shock (η) is similar to Hsieh and Klenow's capital wedge τ_k, which is assumed to follow a log normal distribution. The technique capability shock (ϕ) is a novel contribution of our framework, which is also assumed to follow a log normal distribution.

The timing of events that we consider is given as follows:

1. Firms pay f_e to enter, where entry decision depends on expected profits.
2. All shocks are realized.
3. Firms' production and exit decisions depend on realized profits.
   (a) If realized profits are too low, firms exit immediately without engaging in production.
   (b) If realized profits are high enough, firms pay fixed cost f and produce at unit cost $c(r, w)$.

Similar to Melitz (2003), entry is pinned down when a firm's expected profit is equal to the entry cost f_e. Since all shocks are realized upon entry, firms are *ex ante* identical when it comes to expected profits and the decision to enter the industry. Exit and production scale decisions, however, exhibit heterogeneity across firms – to generate misallocation at extensive and intensive margins.

In order to establish some theoretical properties, let us consider one idiosyncratic shock at a time. That is, when we consider a particular shock, the other two are known *ex ante* and assumed to be homogeneous among all firms.

Consider, say, the productivity shock. It is straightforward to show that there is a cutoff productivity level z^* determining the threshold level of z at which a firm is indifferent between *to-produce* and *not-to-produce*: $f = \pi(z^*)$. Similar cutoffs apply to the capital financing shock (η^*) and the technique capability shock (ϕ^*). We can establish:

Proposition 4.1 *(Firm Heterogeneity and Entry)*

(i) *Each shock can affect both the allocation of production factors (intensive margin) and the production/exit decisions (extensive margin).*

(ii) *While production flexibility only matters for the extensive margin in the presence of productivity or capital financing shock, it matters for both intensive and extensive margin in the presence of the technique capability shock.*

This important theoretical result illustrates that the production flexibility kicks in and has an heterogeneous impact through the *intensive* margin misallocation driven by input-and-technique oriented frictions. Therefore, in our quantitative analysis we focus on intensive margin misallocation and the quantitative relevance of flexibility in mitigating technique distortions. Before turning to the quantitative exercises, in the next section we develop methods to measure firm-level *revenue* and *physical* productivity, which we will utilize in isolating the impacts of input-and-technique-driven distortions on intensive margin misallocation and in turn on manufacturing TFPs.

4.4 Firm-level Revenue Productivity: TFPR

In this section, we study firms' allocations, marginal revenue products and derive proxies to measure the intensive margin misallocation of resources across firms implied by the two distortionary wedges. As we delineated above, we continue to focus on the Pareto complement case with $\rho < 0$ and $\phi > 0$.

4.4.1 Misallocation of Capital

Let $(1+\eta_i)$ denote firm i's capital financing wedge, and suppose that $\phi_i = 0$ for all i. This means the capital wedges faced by different firms in an industry are allowed to vary, capturing the *misallocation of capital*. Applying (15), we can see that the capital-labor ratio for firm i possesses the following property:

$$\frac{K_i}{L_i} \propto \frac{1}{1+\eta_i},$$

which is identical to one in Hsieh and Klenow (2009).[10] Therefore, the misallocation resulting from capital distortions can be can be directly compared to Hsieh and Klenow.

[10] Hsieh and Klenow utilize this K/L ratio equation to back firm specific η's (τ_K's in their model)

With Dixit-Stiglitz preferences, the revenue function is given by $Y_i^{\frac{\sigma-1}{\sigma}}$ where Y_i is the output produced by firm i and σ is the elasticity of substitution across producers. We can derive the marginal revenue product of capital (MRPK) as the following:

$$MRPK_i \propto (1+\eta_i)^{1-\rho(1-\alpha)},$$

which differs from the formulation in Hsieh and Klenow given by $MRPK_i^{HK} \propto 1+\eta_i$. It is clear that with $\rho = 0$ our MRPK measure collapses to $MRPK^{HK}$. In general, under the Pareto complement benchmark, the exponent $1-\rho(1-\alpha)$ exceeds one, implying that our MRPK measure is more responsive to capital distortions. Moreover, the higher production flexibility is, the less responsive a firm's marginal revenue product of capital is to such distortions. Intuitively, a firm with a distortionary capital wedge can potentially raise its capital technique a_K to mitigate the detrimental effect of capital distortions. By how much a firm can exercise this depends on production flexibility. With greater flexibility (ρ closer to zero), the detrimental effect is more easily mitigated, thereby requiring lower revenue productivity to achieve the given level of output. As a result, the MRPK dispersion among firms facing heterogeneous capital financing shocks is lower. This result is crucial for measuring the TFP losses due to misallocation of capital and labor, because the dispersion in MRPKs across firms can generate aggregate TFP losses as pointed out by Hsieh and Klenow.

Similarly, we can derive:

$$MRPL_i \propto (1+\eta_i)^{\rho\alpha}.$$

In contrast to Hsieh and Klenow where the marginal revenue product of labor is independent of capital distortions, our model suggests that, in the Pareto complement benchmark, capital distortions can result in lower MRPL due to endogenous techniques choice. While greater production flexibility requires higher labor revenue productivity to achieve the given level of output, the MRPL dispersion across firms is lower.

Aggregating MRPK and MRPL at the firm-level, we can apply (18) to obtain the firm-specific total factor revenue productivity:

$$TFPR_i \propto (1+\eta_i)^{\alpha},$$

which is increasing in the severity of capital distortion but such a detrimental effect is independent of ρ.

Proposition 4.2 *(Capital Distortions and Misallocation and TFPR) The effects of capital distortions on the firm-level TFPR and the dispersion of TFPRs are independent of production flexibility.*

4.4.2 Misallocation of Techniques

Denote firm i's technique capability wedge by $(1+\phi_i)$ and suppose that $\eta_i = 0$ for all i. The wedges faced by different firms in an industry are also allowed to differ - capturing the *misallocation of techniques*. We can again build on (15) to obtain:

$$\frac{K_i}{L_i} \propto \frac{1}{1+\phi_i},$$

and use (18) to get:

$$TFPR_i \propto \left(\frac{(1+\phi)^{\alpha}}{1+\alpha\phi}\right)^{\frac{1-\rho}{\rho}}.$$

These together with the property of $\frac{\partial^2 c}{\partial\phi\partial\rho} > 0$ enable us to establish:

Proposition 4.3 *(Technique Distortions and Misallocation, Production Flexibility and TFPR) The effects of technique distortions on the firm-level TFPR and the dispersion of TFPRs are elevated by production flexibility.*

Dispersions in TFPR will enable us to evaluate the intensive margin misallocation generated by η and ϕ and its impact on aggregate TFP - as it's standard in the literature. However, in our framework η and ϕ distortions influence the aggregate TFP through firms' physical productivity (TFPQ) as well, which we shall turn next.

4.5 Firm-level Physical Productivity: TFPQ

Different from the standard neoclassical analysis, in our framework the capital and technique oriented distortions affect the physical productivity of the firm via techniques choice. Following Foster, Haltiwanger and Syverson (2008) we call physical productivity as the Total Factor Productivity in Quality (TFPQ) and investigate the impact of the interactions between techniques choice and production distortions on TFPQ.

4.5.1 Intensive Margin TFPQ Gap

In our framework, idiosyncratic capital distortions are alleviated by endogenous technique adjustments. Endogenous techniques selection, in turn, implies TFPQ differences across firms. In order to illustrate this TFP differential, let $(\hat{K}, \hat{L})$ be the factor demands by a firm A that chooses techniques $(\hat{a}_K, \hat{a}_L)$ in the absence of capital distortions ($\eta = 0$). Let (K^*, L^*) and (a_K^*, a_L^*) be the solution to the firm B that faces a capital distortion of $\eta > 0$. Firms A and B are identical otherwise. Since capital distortions induce a higher unit cost of production, it is clear that $Y_A > Y_B$, where

$$\begin{aligned} Y_A &= \left[\lambda(\hat{a}_K\hat{K})^\rho + (1-\lambda)(\hat{a}_K\hat{K})^\rho\right]^{\frac{1}{\rho}}, \\ Y_B &= \left[\lambda(a_K^*K^*)^\rho + (1-\lambda)(a_L^*L^*)^\rho\right]^{\frac{1}{\rho}}. \end{aligned}$$

The question we need to answer is: Given (a_K^*, a_L^*), what would be the output of the initially capital constrained firm B if it were to produce by employing factor inputs $(\hat{K}, \hat{L})$ instead of (K^*, L^*)? If there were no techniques choice as in the standard neoclassical production framework, then we would have had $Y_A(\hat{K}, \hat{L}) = Y_B(\hat{K}, \hat{L})$. With techniques choice, we clearly have $Y_A(\hat{K}, \hat{L}) > Y_B(\hat{K}, \hat{L})$. We shall say such *intensive margin physical productivity gap* is present because the ratio $TFPQ_{int}(\hat{K}, \hat{L}) \equiv \frac{Y_B(\hat{K},\hat{L})}{Y_A(\hat{K},\hat{L})}$ falls below one. We denote such TFPQ differential as the intensive margin TFPQ-gap, because they are driven by the intensive margin capital dispersion - and not by some intrinsic productivity differences, such as the heterogeneity in firms' knowledge scale.z.

Lemma 4.4 *(Capital Distortions and $TFPQ_{int}$) The intensive margin dispersion in capital distortions causes an intensive margin spread in TFPQs across manufacturers (defined as $TFPQ_{int}$). The higher the capital dispersion between any two firms the lower is the $TFPQ_{int}$ differential between them.*

We would like to note that this intensive margin productivity gap is largely ignored in the standard neoclassical production framework. Therefore, standard capital misallocation exercises tend to over-state the TFP gains from reallocating capital. We will address this issue in Section 5 when we conduct quantitative policy exercises.

The next question we would like to address is: What is the implication of production flexibility for the intensive margin TFPQ differences - arising from capital misallocation? Now, using (16) and (17) with $\phi = 0$ and defining $\chi_K \equiv \lambda((1+\eta)^{-(1-\alpha)}a_K^*\hat{K})^\rho$ and $\chi_L \equiv (1-\lambda)((1+\eta)^\alpha a_L^*\hat{L})^\rho$, we obtain:

$$TFPQ_{int}(\hat{K}, \hat{L}) = \frac{Y_B(\hat{K}, \hat{L})}{Y_A(\hat{K}, \hat{L})} = (1+\eta)^{-\alpha}\left[\frac{(1+\eta)^\rho\chi_K + \chi_L}{\chi_K + \chi_L}\right]^{\frac{1}{\rho}} \tag{19}$$

Define $\bar{\rho}$ as one solving:

$$\frac{1}{\bar{\rho}^2}\left\{\log\left[(1+\eta)^{\bar{\rho}}\chi_K + \chi_L\right] - \log\left[\chi_K + \chi_L\right]\right\} = \frac{(1+\eta)^{\bar{\rho}-1}\chi_K + \chi_L}{(1+\eta)^{\bar{\rho}}\chi_K + \chi_L}.$$

It is easy to show that $\bar{\rho} > 0$. We then establish the following property regarding the impact of production flexibility on intensive margin TFPQ gap.

Lemma 4.5 *For all $\rho < \bar{\rho}$, $\frac{dTFPQ_{int}(\hat{K},\hat{L})}{d\rho} > 0$.*

Then, the following proposition follows immediately.

Proposition 4.6 *(Production Flexibility and the Intensive Margin TFPQ Effect of Capital Distortions) For the Pareto complement case we are interested in (with $\rho < 0$), production flexibility mitigates the intensive margin TFPQ losses resulting from capital distortions.*

4.5.2 Extensive Margin TFPQ Gap

As an alternative, next we study a TFPQ gap between a firm which can adjust the techniques and another firm which cannot (the standard neoclassical cost minimizer) to understand the extensive margin TFPQ gap induced by fully constrained technique choice. In the absence of techniques choice, we simply set, for comparison purposes, $a_K = a_L = z$. Thus, we can write the firm with a neoclassical production as:

$$\begin{aligned} Y_{neo} &= [\lambda(zK)^\rho + (1-\lambda)(zL)^\rho]^{\frac{1}{\rho}} \\ &= z\left[\lambda K^\rho + (1-\lambda)L^\rho\right]^{\frac{1}{\rho}}. \end{aligned}$$

For given levels factor demands, we can define the *extensive margin physical productivity ratio* by:

$$TFPQ_{ext} \equiv \frac{Y(K,L)}{Y_{neo}(K,L)}.$$

Define $\kappa \equiv \frac{w}{r(1+\eta)}\frac{\alpha}{1-\alpha}\frac{\lambda}{1-\lambda}\frac{1}{1+\phi}$. The following lemmas and propositions as well as Figure 5 characterize $TFPQ_{ext}$ over $\frac{K}{L} \in [0,\infty)$.

Lemma 4.7 *(Extensive Margin TFPQ)*

(i) *When $\kappa > 1$, $TFPQ_{ext}$ increases in K/L for all $K/L \in [1,\kappa]$, with $\lim_{\frac{K}{L}\to\infty} TFPQ_{ext} > 1 > \lim_{\frac{K}{L}\to 0} TFPQ_{ext}$.*

(ii) *When $\kappa < 1$, $TFPQ_{ext}$ decreases in K/L for all $K/L \in [\kappa,1]$, with $\lim_{\frac{K}{L}\to\infty} TFPQ_{ext} < 1 < \lim_{\frac{K}{L}\to 0} TFPQ_{ext}$.*

Lemma 4.8 *(Extensive Margin TFPQ and Distortionary Wedges)*

(i) *When $\kappa > 1$, $\max\ TFPQ_{ext} - \min\ TFPQ_{ext}$ is decreasing in η but increasing in ϕ for all $K/L \in [0,\infty)$.*

(ii) *When $\kappa < 1$,* $\max\ TFPQ_{ext} - \min\ TFPQ_{ext}$ *is increasing in η but decreasing in ϕ for all $K/L \in [0, \infty)$.*

We next examine the effect of capital financing distortions on the extensive margin TFPQ gap locally around $K/L = \kappa$, that is, evaluating the sign of $\frac{\partial TFPQ_{ext}}{\partial \eta}|_{\frac{K}{L}=\kappa}$.

Proposition 4.9 *(Capital Distortions and Misallocation and Extensive Margin TFPQ) For $\kappa > 1$, techniques choice amplifies the detrimental effect of capital distortions on physical productivity relative to the neoclassical benchmark (i.e., $\frac{\partial TFPQ_{ext}}{\partial \eta}|_{\frac{K}{L}=\kappa} < 0$). For $\kappa < 1$, it dampens such a detrimental effect ($\frac{\partial TFPQ_{ext}}{\partial \eta}|_{\frac{K}{L}=\kappa} > 0$).*

We are also interested how production flexibility influences the $TFPQ_{ext}$ - again around $K/L = \kappa$. Define $\theta_K \equiv \lambda K^\rho$ and $\theta_L \equiv (1-\lambda)L^\rho$, $\varphi_K \equiv \frac{a_K}{z}(1+\eta)^{\alpha-1}$ and $\varphi_L \equiv \frac{a_L}{z}(1+\eta)^{\alpha}$. Then, it can be derived,

$$TFPQ_{ext} \equiv \frac{Y}{Y_{neo}} = (1+\eta)^{-\alpha}\left[\frac{(1+\eta)^\rho \varphi_K \theta_K + \varphi_L \theta_L}{\theta_K + \theta_L}\right]^{\frac{1}{\rho}}. \tag{20}$$

Further define $\tilde{\rho} > 0$ as one solving:

$$\frac{1}{\tilde{\rho}^2}\left\{\log\left[(1+\eta)^{\tilde{\rho}}\varphi_K\theta_K + \varphi_L\theta_L\right] - \log\left[\theta_K + \theta_L\right]\right\} = \frac{(1+\eta)^{\tilde{\rho}-1}\varphi_K\theta_K + \varphi_L\theta_L}{(1+\eta)^{\tilde{\rho}}\varphi_K\theta_K + \varphi_L\theta_L}.$$

We then obtain:

Lemma 4.10 *(Extensive Margin TFPQ) For all $\rho < \tilde{\rho}$, $\frac{dTFPQ_{ext}(\hat{K},\hat{L})}{d\rho} < 0$.*

Again, since $\tilde{\rho} > 0$, the following result is immediate.

Proposition 4.11 *(Technique Distortions and Misallocation, Production Flexibility and Extensive Margin TFPQ) For the Pareto complement case we are interested in (with $\rho < 0$), production flexibility reduces extensive margin effect of techniques choice on TFPQ relative to the neoclassical benchmark.*

5 Quantitative Analysis

We have established several important properties of a generalized production framework with endogenous techniques choice in an economy with frictions. We are now prepared to utilize firm-level data from Compustat North America database to conduct quantitative exercises to illustrate the effects of misallocation of capital and misallocation of techniques on aggregate industry TFP. Specifically, we express the variance in aggregate-industry TFPR – a measure of intensive margin misallocation, compute firm-level TFPQ and calibrate the aggregate industry TFP.

5.1 Data

We use firm-level balance sheet data from Compustat's Industrial database in 2005. Our industry-level exercises focus on 4-digit SIC manufacturing industry clusters that we list in Table 1.

The key firm-level variables that we need for the quantitative analysis are annual figures for capital and labor costs, employment, capital stock and sales, which we proxy by firm's Capital Expenditures, Compensation of Employees, Number of Employees, Stock of Plant-Property-and-Equipment, and Value of Sales.

The quantitative analysis also requires values for industry-level structural parameters, namely, the capital share of income in production (λ_s), the capital's technique share in the technology menu (α_s), and finally the flexibility of the production technology (ρ_s), where s denotes a manufacturing industry from the groups of industries we list in Table 1. We utilize U.S. industry aggregate data to calibrate the industry level structural parameters λ_s and α_s and adopt the existing U.S. industry-level measures for ρ_s.

In order to calibrate α_s and λ_s, we rewrite (15) for firm i in industry s as:

$$\frac{K_{si}}{L_{si}} = \frac{w_{si}}{r(1+\eta_{si})} \frac{\alpha_s}{1-\alpha_s} \frac{\lambda_s}{1-\lambda_s} \frac{1}{1+\phi_{si}}. \tag{21}$$

We assume that, at the aggregate industry level, the capital distortions and technique adjustment frictions wash out, that is, the industry averages of η_{si} and ϕ_{si} across all i in any s satisfy:

$$\overline{\eta_{si}} = \overline{\phi_{si}} = 0. \tag{22}$$

Then, using (21) and (22) and industry averages of capital expenditures, $(\overline{K_{si}r(1+\eta_{si})(1+\phi_{si})})$, and labor expenditures $(\overline{L_{si}w_{si}})$ we obtain:

$$\frac{\alpha_s \lambda_s}{(1-\alpha_s)(1-\lambda_s)} = \frac{\overline{K_{si}r(1+\eta_{si})(1+\phi_{si})}}{\overline{L_{si}w_{si}}}. \tag{23}$$

We next compute the industry labor share in production, $1-\lambda_s$, utilizing the labor share of U.S. manufacturing plants obtained from NBER-CES Manufacturing Database. This database contains all manufacturing industries for the years in between 1958-2005 at 4-digit SIC level and provides the labor compensation and value added for each industry, which allow us to compute industry level $1-\lambda_s$. With this, we then apply (23) to back out industry level capital share in the technology menu, α_s. Finally, we use industry-level capital-labor elasticity for the U.S. economy from Oberfield and Raval (2012) to proxy our production flexibility parameter ρ_s, in which $\rho_s < 0$ holds for all manufacturing industries, implying Pareto complementarity. Table 2 presents the benchmark structural parameters for U.S. manufacturing industries.

We now evaluate the equation (21) at the firm-level in order to back-out firm-specific ϕ_{si}. Specifically, by taking the fraction $\frac{\alpha_s\lambda_s}{(1-\alpha_s)(1-\lambda_s)}$, firm specific capital expenditures $K_{si}r(1+\eta_{si})$, and compensation of employees $L_{si}w_{si}$ as given, we back out firm specific technique wedge as:

$$1+\phi_{si}=\frac{\alpha_s\lambda_s}{(1-\alpha_s)(1-\lambda_s)}\frac{w_{si}L_{si}}{K_{si}r(1+\eta_{si})}.$$

In order to extract the firms' factor distortions, η_{si}, we compute the average price of capital in each industry, $\bar{r}_s$ as the ratio between firms' average capital expenditures and the average property, plant and equipment stock. We then derive the firm-specific capital wedge:

$$1+\eta_{si}=\frac{r_{si}}{\bar{r}_s},$$

where r_{si} is the ratio between a particular firm's capital expenditures and the stock of property, plant and equipment. Table 3 also presents the values of $\bar{r}_s$ across industries in our sample.

5.2 Capital and Technique Misallocation and TFPR Dispersion

Recall that TFPR is an indicator of the intensive margin distortion at the firm-level, which in aggregation exercises measures the misallocation of capital, as shown by Hsieh and Klenow (2009). While quantifying this measure of TFP losses, we will provide a variance decomposition exercise toward understanding the sources of *intra-industry* TFPR dispersion. In order to fully understand the quantitative consequences of technique capability, we derive the TFPR dispersion in relation to the misallocation of capital as well as to the misallocation of techniques. To highlight our results, we also consider a standard neoclassical framework with Cobb-Douglas production technology – a special case with $\rho_s=1$ for every industry s – as assumed in Hsieh and Klenow.

In an environment with capital and technique distortions, we can rewrite the TFPR derived above for firm i in industry s as:

$$TFPR_{si}\propto(1+\eta_{si})^{\alpha_s}\left(\frac{(1+\phi_{si})^{\alpha_s}}{1+\alpha\phi_{si}}\right)^{\frac{1-\rho_s}{\rho_s}}.$$

Taking logs yields,

$$\ln TFPR_{si}\propto\alpha_s\ln(1+\eta_{si})+\left(\frac{1-\rho_s}{\rho_s}\right)\ln\left(\frac{(1+\phi_{si})^{\alpha_s}}{1+\alpha\phi_{si}}\right).$$

Table 3a presents the intra-industry variances of $\ln(1+\eta_{si})$ and $\ln\left(\frac{(1+\phi_{si})^{\alpha}}{1+\alpha\phi_{si}}\right)$.

If η and ϕ are independently and identically distributed across firms in a given industry, then with adjustable techniques the intra-industry variance of TFPR can be derived as:

$$var(\ln TFPR_{si})=\alpha_s^2 var(\ln(1+\eta_{si}))+\left(\frac{1-\rho_s}{\rho_s}\right)^2 var\left(\ln\left(\frac{(1+\phi_{si})^{\alpha_s}}{1+\alpha_s\phi_{si}}\right)\right). \quad (24)$$

This expression reveals that a rise in the production flexibility ρ widens the intra-industry TFPR dispersion if and only if:

$$\frac{\rho-1}{\rho^3} > 0,$$

which holds under $\rho < 0$. As we noted in Table 2, this is the empirically plausible case for the broad range of manufacturing industries that we consider. Table 3b documents the TFPR dispersion in each industry by decomposing the TFPR dispersion into (i) the variances in capital distortions (η) and (ii) the variances in technique capability distortions (ϕ). While the former measures capital misallocation, the latter reflects technique misallocation.

There are two important findings to highlight. First, with techniques choice, production flexibility is found to amplify the contribution of technique misallocation to the dispersion in TFPR. Second, as a result of the aforementioned amplification effect, technique misallocation turns out to explain a larger fraction of the TFPR dispersion than capital misallocation, across all industries in our sample. Specifically, as can be observed in Table 3, capital misallocation captured by the variance of $\ln(1+\eta_{si})$ is larger than technique misallocation measured by the variance of $\ln\left(\frac{(1+\phi_{si})^\alpha}{1+\alpha\phi_{si}}\right)$ in most industries, but the latter is *amplified* by the production flexibility parameter ρ_s, thereby leading to a greater contribution to the TFPR variance. That is, with the amplification effect of production flexibility, technique misallocation becomes more important for the TFPR dispersion compared to the capital misallocation in manufacturing industry clusters.

For comparison purposes, we also quantify the TFPR dispersion under the standard Cobb-Douglas production specification. In this case, the capital-labor ratio becomes:

$$\frac{K_{si}}{L_{si}} = \frac{w_{si}}{r(1+\eta_{si})}\frac{\lambda_s}{1-\lambda_s}\frac{1}{1+\tau_{si}},$$

and the TFPR can be expressed as:

$$\ln TFPR_{si} \propto \lambda_s[\ln(1+\eta_{si}) + \ln(1+\tau_{si})],$$

where τ_{si} is the residual firm specific distortions, which could be classified as output distortions or labor input distortions. We thus have:

$$var(\ln TFPR_{si}) = \lambda_s^2\left[var(\ln(1+\eta_{si})) + var(\ln(1+\tau_{si}))\right]. \tag{25}$$

In contrast to our formula (24), the coefficient associated with the capital input distortion is λ_s – the capital share in industry s's output. Table 4a presents the variances of $\ln(1+\eta_{si})$ and $\ln(1+\tau_{si})$, whereas Table 4b decomposes the the TFPR dispersion into variances in (i) capital input distortions (η) and (ii) residual distortions (τ), where the former captures capital misallocation.

It is clear that, with the Cobb-Douglas specification in the absence of techniques choice, the misallocation of capital has a quantitatively large contribution to the TFPR dispersion – as emphasized by Hsieh and Klenow (2009) and many other studies.

5.3 Industry TFP

We next turn to computing the physical productivity of the firm, the TFPQ. Again, we rewrite it for a particular firm i in industry s:

$$TFPQ_{si} = \xi_s^{tech} \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{\left[\lambda_s(\frac{a_{Ksi}}{z_{si}}K_{si})^{\rho} + (1-\lambda_s)(\frac{a_{Lsi}}{z_{si}}L_{si})^{\rho}\right]^{\frac{1}{\rho}}}, \tag{26}$$

where ξ_s^{tech} is an industry scaling factor. Since a_{Ki}/z_{si} and a_{Li}/z_{si} are functions of η_{si} and ϕ_{si}, TFPR and TFPQ might exhibit a systematic covariance as we delineated in Section 4.5 because of intensive margin TFPQ adjustment resulting from endogenous techniques selection. The endogenous component of the covariance between TFPQ and TFPR is quite crucial for assessing the impact of factor misallocation on aggregate industry total factor productivity. Specifically, if firms with low levels of physical productivity are hit with larger capital distortions, the aggregate TFP effects of misallocation would be smaller.[11] Under the Cobb-Douglas production technology in the absence of techniques choice, the TFPQ reduces to the following:

$$TFPQ_{si} = \xi_s^{neo} \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\lambda_s} L_{si}^{1-\lambda_s}},$$

which is independent of the capital input distortion, as in Hsieh and Klenow (2009).

As shown in Foster, Haltiwanger and Syverson (2008) as well as in Hsieh and Klenow, one may compute industry-level aggregate TFP using the information of firms' TFPRs and TFPQs. We follow this line of methodology, assuming that TFPQ and TFPR are jointly drawn from a log-normal distribution. Then, there is a closed form expression for the industry-wide TFP aggregator:

$$\begin{aligned} \ln TFP_s &= \frac{1}{M_s}\left(\sum_{i=1}^{M_s} \ln TFPQ_{si}\right) \\ &\quad + \frac{\sigma-1}{2}\left[var(\ln TFPQ_{si}) - var(\ln TFPR_{si}) - 2cov(\ln TFPQ_{si}, \ln TFPR_{si})\right], \end{aligned} \tag{27}$$

where we set σ equal to 3 as in Hsieh and Klenow. We would like to note that this TFP expression is a generalized TFP formula stemming from the Hsieh and Klenow framework. The generalization is that it allows for a covariance between TFPQ and TFPR, which is of particular importance for

[11] Restuccia and Rogerson (2008) and Buera et al. (2013)

our analysis. Under the Cobb-Douglas production technology in the absence of techniques choice, the TFP expression reduces to the Hsieh and Klenow aggregator:

$$\ln TFP_s = \frac{1}{M_s}\left(\sum_{i=1}^{M_s} \ln TFPQ_{si}\right) + \frac{\sigma-1}{2}\left[var(\ln TFPQ_{si}) - var(\ln TFPR_{si})\right].$$

For our general production framework with techniques choice, the contribution of capital input and technique distortions to industry TFP depend crucially on the endogenous component of the covariance between TFPQ and TFPR.

We conduct counterfactual exercises by collapsing either capital or technique distortions (η_{si} or ϕ_{si}) to zero for every firm in an industry s, thus removing the misallocation of capital or techniques, respectively. We find that either would decrease the TFPR dispersion and reduce the negative covariance between TFPQ and TFPR. While the former effect is desirable for raising the industry TFP, the latter is not as we delineated in Section 4.5. Our counterfactual exercises also indicate that elimination of either type of misallocation would have an impact on the average TFPQ as well. The TFPQ effect might result with an increase as well as a decrease in average lnTFPQ – depending on whether the average firm in the industry was being "taxed" or "subsidized" with its idiosyncratic distortion. For the case of Cobb-Douglas production without techniques choice, counterfactual analysis shows that, as expected with the formula above, setting η_{si} to zero for all firm will only affect the TFPR variance. Tables 5-7 present the results from our industry TFP exercises.

Table 5a presents the baseline TFP calibration under our generalized production framework. There are two features that we would like to highlight. With the exception of the Paper & Printing industry in all manufacturing industries, the TFPR dispersion is substantially large ranging from 0.23 to 1.72 and averaging around 0.8. Furthermore, except for the Food & Tobacco industry, the covariance between TFPQ and TFPR is negative (averaging around -0.14) – a key quantitative result that is expected to be produced under our generalized production framework. Table 5b provides the comparable TFP measures for the case of Cobb-Douglas production without techniques choice.

Table 6 presents the counterfactual results by setting capital input distortions to zero for all firms in each industry s. With endogenous techniques choice, its affect on the TFP turns out to be quantitatively small. This is because that the benefits from reducing TFPR dispersion by removing capital misallocation is counteracted by the reductions in the negative covariance between TFPQ and TFPR. We also note that the value of average lnTFPQ does not change by a significant margin. On the contrary, with the standard Cobb-Douglas production without techniques choice, eliminating capital misallocation has a quantitatively large aggregate TFP effect in every industry.

Table 7 summarizes the counterfactual results for removing technique misallocation by setting technique distortions to zero for all firms in each industry s. As we already noted in the previous subsection, technique misallocation contributes to a large fraction of the TFPR variance across firms. Therefore, eliminating this type of misallocation lowers the TFPR dispersion sharply and increases the TFP substantively while the reductions in the negative covariance between TFPQ and TFPR does not counteract this with the exception of the Paper & Printing and Metal Manufacturing industries.

Another important result to note is that reducing the TFPR dispersion caused by technique misallocation has the highest impact in industries with highest degrees of production flexibility. For instance, Food & Tobacco and Transportation Equipment industries, which have the highest level of production flexibility in our sample of industries, reducing TFPR dispersion due to technique misallocation lead to quantitatively the highest amounts of TFP gains. On the contrary, Primary & Fabricated Metal Manufacturing is one with the least production flexibility; technique misallocation also turn out to be the least important for the aggregate TFP of this industry. These results are aligned with our theoretical predictions concerning the interactions between production flexibility and technique distortions.

5.4 Summary of Quantitative Results

Combining the quantitative results in the two previous subsections, we can conclude the following:

1. More than 50% of the intra-industry TFPR dispersion is accounted by misallocation of techniques among firms. This observation consistently prevails in all manufacturing industries in the sample.

2. The relative importance of technique misallocation in explaining the dispersion of the TFPR, compared to capital misallocation, is largely driven by the magnitude of industry production flexibility. Reducing the TFPR dispersion caused by technique misallocation has greater impact in industries with higher degrees of production flexibility.

3. The effects of capital misallocation on the TFPQ and as a result on the covariance between TFPQ and TFPR are quite essential in understanding the consequences of capital misallocation for the aggregate industry TFP.

4. In the standard neoclassical framework without techniques choice, the misallocation of capital remains to have a quantitatively large impact for the aggregate industry TFP.

6 Concluding Remarks

We have developed a generalized production framework, incorporating the concept of design for manufacturability where firms decide on their factor inputs as well as production techniques that govern the efficient use of factors under currently operating technologies. We have characterized factor demands, techniques choice and the unit cost of production and examined the role of production flexibility. By allowing firms to differ in technology scales and capital financing and technique capability constraints, we have modeled firm entry-exit and aggregated heterogeneous firms in each industry to produce an industry TFP measure. We have then examined the consequences of capital and technique misallocation for aggregate productivity as well as the interplay of the TFP effects of misallocation with production flexibility. While both capital financing and technique capability distortions raise the unit cost of production, production flexibility reduces it. The detrimental effects of capital financing distortions is independent of the extent of production flexibility, but the detrimental effects of technique capability distortions is amplified by production flexibility.

We have undertaken a number of quantitative exercises by calibrating our model to fit the observations from firm-level data in U.S. manufacturing industries. Our quantitative results suggest the following. Due to an amplification effect of production flexibility, technique misallocation yields larger TFP losses than capital misallocation. This means, TFP gains from removing technique misallocation are larger in industries with more flexible production. Once techniques choice is absent and production takes the Cobb-Douglas form, the importance of capital misallocation for aggregate productivity stands out.

An important policy implication arising from our analysis is that to achieve greater production efficiency, mitigating technique misallocation should be granted with higher priority than reducing capital misallocation. This is of particular importance in industries with higher degrees of production flexibility, such as Food & Tobacco and Transportation Equipment industries – which are not typical candidates for industry policy considerations in the development literature. Such policy arrangement may include, but not limited to, tax incentives for adjusting production techniques and subsidies to design for manufacturability, both on equal ground for all firms in the same industry.

References

[1] Acemoglu, D., 2003. Labor-and-capital-augmenting technical change. *Journal of the European Economic Association*, 1, 1-37.

[2] Adler, P., Goldoftas, B., and Levine, D.I., 1999. Flexibility Versus Efficiency? A Case Study of Model Changeovers in the Toyota Production System. *Organization Science*, 10, 43-68.

[3] Banerjee, A.V., Duflo, E., 2005. Growth theory through the lens of development economics. *Handbook of Economic Growth*, 1, 473-552.

[4] Banerjee, A., Moll, B., 2010. Why Does Misallocation Persist? American Economic Journal: Macroeconomics, 2, 189-206.

[5] Buera, F.J., Shin, Y., 2013. Financial Frictions and the Persistence of History: A Quantitative Exploration. *Journal of Political Economy*, 121, 221-272.

[6] Caselli, F., 1999. Technological Revolutions. *American Economic Review*, 89, 78-102.

[7] Caselli, F., Coleman, J., 2006. The World Technology Frontier. *American Economic Review*, 499-522.

[8] Foster, L., Haltiwanger, J., and Syverson, C., 2008. Reallocation, Firm Turnover and Efficiency: Selection on Productivity or Profitability. *American Economic Review.*

[9] Gerwin, D., 1993. Manufacturing Flexibility: A Strategic Perspective. *Management Science*, 39, 395-410.

[10] Houthakker, H.S., 1955-1956. The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis. *Review of Economic Studies*, 23, 27-31.

[11] Hsieh, C., Klenow, P, 2009. Misallocation and Manufacturing TFP in China and India. *Quarterly Journal of Economics*, 124, 1403-1448.

[12] Jones, C., 2005. The shape of production functions and the direction of technical change. *Quarterly Journal of Economics*, 120, 517-549.

[13] Jones C.I., 2013. Misallocation, input-output economics, and economic growth. In: D. Acemoglu, M. Arellano and E.Dekel, Eds. *Advances in Economics and Econometrics.* Tenth World Congress. Volume II. Cambridge, Cambridge University Press.

[14] Jovanovic, B., 2014. Misallocation and Growth. *American Economic Review*, 104, 1149-1171.

[15] Kortum, S. S., 1997. Research, patenting, and technological change. *Econometrica*, 1389-1419.

[16] Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71, 1695-1725.

[17] Midrigan, V., Xu, D., 2013. Finance and Misallocation: Evidence from Plant-Level Data. American Economic Review, 104, 422-458.

[18] Moll, B., 2014. Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation? *American Economic Review* (forthcoming).

[19] Restuccia, D., Rogerson, R., 2008. Policy Distortions and Aggregate Productivity with Heterogeneous Establishments. *Review of Economic Dynamics*, 11, 707-720.

[20] Restuccia, D., Rogerson, R., 2013. Misallocation and Productivity. *Review of Economic Dynamics*, 16, 1-10.

[21] Roller, L., Tombak, M.M., 1993. Competition and Investment in Flexible Technologies. *Management Science*, 39, 107-114.

[22] Youssef, M. A., 1994. Design for manufacturability and time-to-market. *International Journal of Operations & Production Management*, 14, 6-21.

[23] Wang, P., Wong, T., and Yip, C.K., 2013. Technology Assimilation and Development Accounting. Working Paper, Washington University in St. Louis.

Table 1. Classification of Manufacturing Industries

Industry Cluster	4-digit SIC Classification
Food-Tobacco	2000-2199
Paper-Printing	2600-2799
Chemichal-Petrol-Rubber/Plastic	2800-3099
Primary/Fabricated Metal	3300-3499
Machinery (Industrial+Commercial+Computer)	3500-3599
Electrical Equipment	3600-3699
Transportation Equipment	3700-3799

Table 2. Structural Parameters

Industry Cluster	α	λ	ρ	Effective Capital Share with Adjustable Techniques
Food-Tobacco	0.13	0.72	-0.15	0.38
Paper-Printing	0.14	0.62	-1.85	0.27
Chemichal-Petrol-Rubber/Plastic	0.14	0.72	-1.05	0.42
Primary/Fabricated Metal	0.25	0.55	-2.85	0.41
Machinery	0.17	0.54	-0.92	0.24
Electrical Equipment	0.22	0.57	-0.96	0.37
Transportation Equipment	0.17	0.55	-0.90	0.25

Elasticity of capital in technology menu, α, is calibrated. The income share of capital in CES production function, λ, is calibrated using labor share of U.S. manufacturing plants from NBER-CES Manufacturing Database. The production flexibility, ρ, is from Oberfield and Raval (2012). The effective capital share is computed using $\frac{K}{L} = \frac{w}{r}\frac{\alpha}{1-\alpha}\frac{\lambda}{1-\lambda}$.

Table 3a. Manufacturing Distortions (Production with Technique Choice)

Industry Cluster	St. Dev($\ln(1+\eta_{si})$)	St. Dev($\ln\left(\frac{(1+\phi_{si})^{\alpha}}{1+\alpha\phi_{si}}\right)$)	$\bar{r}_s$	# of Observations
Food-Tobacco	0.874	0.125	0.13	172
Paper-Printing	0.776	0.037	0.12	142
Chemichal-Petrol-Rubber/Plastic	1.134	0.463	0.17	882
Primary/Fabricated Metal	0.827	0.165	0.11	177
Machinery	1.017	0.097	0.18	352
Electrical Equipment	1.010	0.286	0.17	601
Transportation Equipment	0.798	0.443	0.12	162

η_{si} and ϕ_{si} are the firm-level factor and technique adjustment distortions respectively. $\bar{r}_s$ is the industry-level cost of borrowing averaged over all firms in an industry s.

Table 3b. TFPR Dispersion (Production with Technique Choice)

Industry Cluster	Var(lnTFPR)	Contr. η-Var (%)	Contr. ϕ-Var (%)	ρ
Food-Tobacco	1.449	1.3	98.7	-0.15
Paper-Printing	0.061	79	21	-1.85
Chemichal-Petrol-Rubber/Plastic	0.832	3	97	-1.05
Primary/Fabricated Metal	0.361	46	54	-2.85
Machinery	0.070	42	58	-0.92
Electrical Equipment	0.968	13	87	-0.96
Transportation Equipment	1.718	0.5	99.5	-0.90

$\ln TFPR_{si}$ is proportional to $\alpha \ln(1+\eta_{si}) + \left(\frac{1-\rho}{\rho}\right) \ln\left(\frac{(1+\phi_{si})^{\alpha}}{1+\alpha\phi_{si}}\right)$. Contributions are the percentages of the $\ln TFPR$ variance explained by η and ϕ heterogeneity across firms.

Table 4a. Manufacturing Distortions (Neoclassical Production)

Industry Cluster	St. Dev($\ln(1+\eta_{si})$)	St. Dev($\ln(1+\tau_{si})$)	$\bar{r}_s$	# of Observations
Food-Tobacco	0.874	1.149	0.13	172
Paper-Printing	0.776	1.024	0.12	142
Chemichal-Petrol-Rubber/Plastic	1.134	1.668	0.17	882
Primary/Fabricated Metal	0.827	1.277	0.11	177
Machinery	1.017	1.515	0.18	352
Electrical Equipment	1.010	1.640	0.17	601
Transportation Equipment	0.798	1.242	0.12	162

η_{si} is the firm-level factor distortion and τ_{si} is the residual distortion. $\bar{r}_s$ is the industry-level cost of borrowing averaged over all firms in an industry s.

Table 4b. TFPR Dispersion (Neoclassical Production)

Industry Cluster	Var(lnTFPR)	Contr. η-Var (%)	Contr. τ-Var (%)
Food-Tobacco	0.013	42.4	57.7
Paper-Printing	0.010	50.1	49.9
Chemichal-Petrol-Rubber/Plastic	0.037	39.1	60.9
Primary/Fabricated Metal	0.077	42.4	57.6
Machinery	0.127	43.6	56.4
Electrical Equipment	0.414	35.8	64.2
Transportation Equipment	0.044	41.7	58.3

$\ln TFPR_{si}$ is proportional to $\lambda \ln(1+\eta_{si}) + \lambda \ln(1+\tau_{si})$. Contributions are the percentages of the $\ln TFPR$ variance explained by η and τ heterogeneity across firms.

Table 5a. Baseline TFP (Production with Technique Choice)

Industry Cluster	lnTFP	Mean(lnTFPQ)	Var(lnTFPQ)	Var(lnTFPR)	2×Cov(.,.)	ρ
		+	+	-	-	
Food-Tobacco	1.24	2.53	0.17	1.45	0.02	-0.15
Paper-Printing	1.93	1.87	0.07	0.06	-0.05	-1.85
Chem.-Pet.-Plastic	1.66	1.91	0.38	0.83	-0.19	-1.05
Metal	1.94	1.91	0.12	0.23	-0.14	-2.85
Machinery	1.72	1.76	0.19	0.36	-0.13	-0.92
Electrical Equ.	1.32	1.79	0.25	0.97	-0.25	-0.96
Transport. Equ.	1.05	2.36	0.21	1.72	-0.19	-0.90

$TFPQ_{si} = \xi_s^{tech} \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{\left[(\frac{a_{Ksi}}{z_{si}}K_{si})^{\rho}+(\frac{a_{Lsi}}{z_{si}}L_{si})^{\rho}\right]^{\frac{1}{\rho}}}$, and
$\ln TFP_s = \frac{1}{M_s}\left(\sum_{i=1}^{M_s} \ln TFPQ_{si}\right) + \frac{\sigma-1}{2}\left[var(\ln TFPQ_{si}) - var(\ln TFPR_{si}) - 2cov(\ln TFPQ_{si}, \ln TFPR_{si})\right]$,
$Cov(.,.) \equiv Cov(\ln TFPQ, \ln TFPR)$.

Table 5b. Baseline TFP (Neoclassical Production)

Industry Cluster	lnTFP	Mean(lnTFPQ)	Var(lnTFPQ)	Var(lnTFPR)
		+	+	-
Food-Tobacco	2.42	2.02	0.42	0.01
Paper-Printing	2.27	2.05	0.23	0.009
Chem.-Pet.-Plastic	2.34	1.33	1.05	0.04
Metal	2.33	2.15	0.25	0.08
Machinery	2.36	2.06	0.43	0.13
Electrical Equ.	1.89	1.92	0.38	0.41
Transport. Equ.	2.62	2.29	0.39	0.04

$TFPQ_{si} = \xi_s^{neo} \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\lambda_s} L_{si}^{1-\lambda_s}}$, and
$\ln TFP_s = \frac{1}{M_s}\left(\sum_{i=1}^{M_s} \ln TFPQ_{si}\right) + \frac{\sigma-1}{2}\left[var(\ln TFPQ_{si}) - var(\ln TFPR_{si})\right]$

Table 6a. The Effects of Collapsing to $\eta_{si} = 0$ for all firm i in sector s (Prod. with Tech. Choice)

Industry Cluster	% Δ in lnTFP	% pts. cont. of Δ in Mean(lnTFPQ)	% pts. cont. of Δ in Var(lnTFPQ)	% pts. cont. of Δ in Var(lnTFPR)	% pts. cont. of Δ in 2×Cov(.,.)	ρ
Food-Tobacco	+ 7.8 %	- 0.2	+ 0.3	+ 9.3	-1.4	-0.15
Paper-Printing	- 3.4 %	- 1.7	-0.05	+ 1.3	-2.5	-1.85
Chem.-Pet.-Plastic	- 7.6 %	-2.3	+ 0.2	+ 5.8	- 11.3	-1.05
Metal	- 1.6 %	- 3.2	- 0.5	+ 7.1	- 5.0	-2.85
Machinery	- 6.6 %	- 1.7	- 0.2	+ 2.6	- 7.3	-0.92
Electrical Equ.	+ 3.2 %	- 2.1	- 1.0	+ 14.5	- 8.1	-0.96
Transport. Equ.	- 0.6 %	- 1.9	- 0.02	+ 16.3	- 14.8	-0.90

% Δ in lnTFP is the percentage change in lnTFP following a hypothetical exercise that collapses the factor friction, η_{si}, to zero for every firm i in the industry s. % pts. cont. is the percentage points contribution of a particular TFP component to the change in lnTFP. $Cov(.,.) \equiv Cov(\ln TFPQ, \ln TFPR)$.

Table 6b. The Effects of Collapsing to $\eta_{si} = 0$ for all firm i in sector s (Neoclassical Production)

Industry Cluster	% Δ in lnTFP	% pts. cont. of Δ in Mean(lnTFPQ)	% pts. cont. of Δ in Var(lnTFPQ)	% pts. cont. of Δ in Var(lnTFPR)
Food-Tobacco	+ 9.7 %	0	0	+9.7
Paper-Printing	+ 1.8 %	0	0	+ 1.8
Chem.-Pet.-Plastic	+ 6.5 %	0	0	+ 6.5
Metal	+ 8.9 %	0	0	+ 8.9
Machinery	+3.1 %	0	0	+ 3.1
Electrical Equ.	+ 15.3 %	0	0	+ 15.3
Transport. Equ.	+ 17.1 %	0	0	+17.1

% Δ in lnTFP is the percentage change in lnTFP following a hypothetical exercise that collapses the factor friction, η_{si}, to zero for every firm i in the industry s. % pts. cont. is the percentage points contribution of a particular TFP component to the change in lnTFP.

Table 7. The Effects of Collapsing to $\phi_{si} = 0$ for all firm i in sector s (Prod. with Tech. Choice)

Industry Cluster	% Δ in lnTFP	% pts. cont. of Δ in Mean(lnTFPQ)	% pts. cont. of Δ in Var(lnTFPQ)	% pts. cont. of Δ in Var(lnTFPR)	% pts. cont. of Δ in 2×Cov(.,.)	ρ
Food-Tobacco	+ 112.3 %	- 5.1	+ 1.0	+ 114.6	+ 1.8	-0.15
Paper-Printing	- 3.4 %	- 3.3	- 0.3	+ 2.4	- 2.2	-1.85
Chem.-Pet.-Plastic	+ 32.3 %	- 6.4	+ 2.3	+ 48.8	- 12.3	-1.05
Metal	- 7.6 %	- 7.0	- 1.1	+ 6.7	- 6.2	-2.85
Machinery	+ 9.6 %	- 6.3	+ 1.1	+ 18.6	- 3.8	-0.92
Electrical Equ.	+ 31.7 %	- 12.3	- 0.9	+ 62.9	- 18.0	-0.96
Transport. Equ.	+ 135.4 %	- 11.5	+ 6.6	+ 161.3	- 21.0	-0.90

% Δ in lnTFP is the percentage change in lnTFP following a hypothetical exercise that collapses the technique adjustment friction, ϕ_{si}, to zero for every firm i in the industry s. % pts. cont. is the percentage points contribution of a particular TFP component to the change in lnTFP. $Cov(.,.) \equiv Cov(\ln TFPQ, \ln TFPR)$.

Figure 1: Technique Optimizing Problem

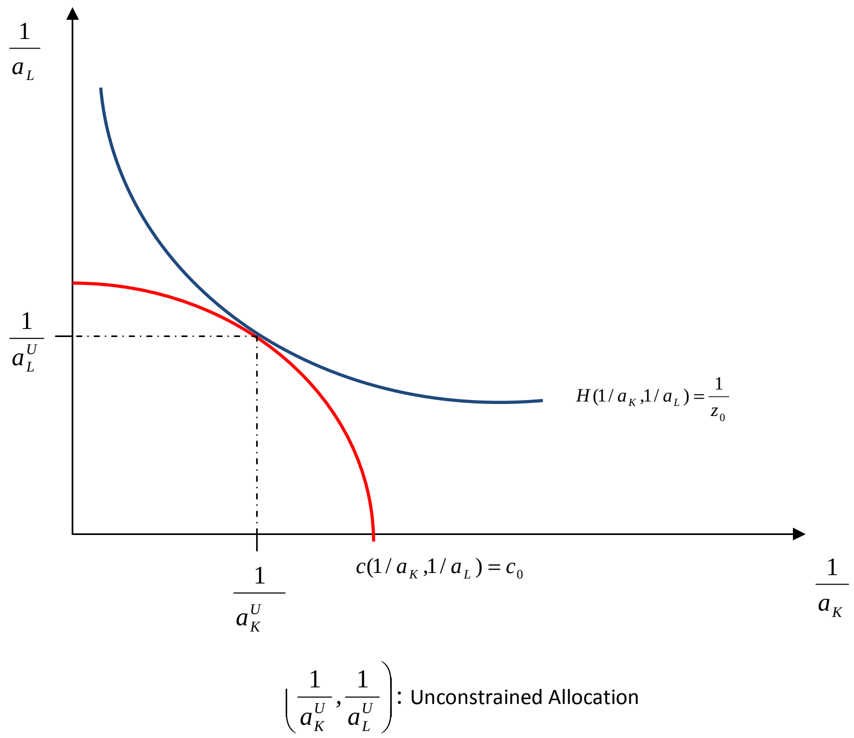


Figure 2: Technique Accesibility Constraint

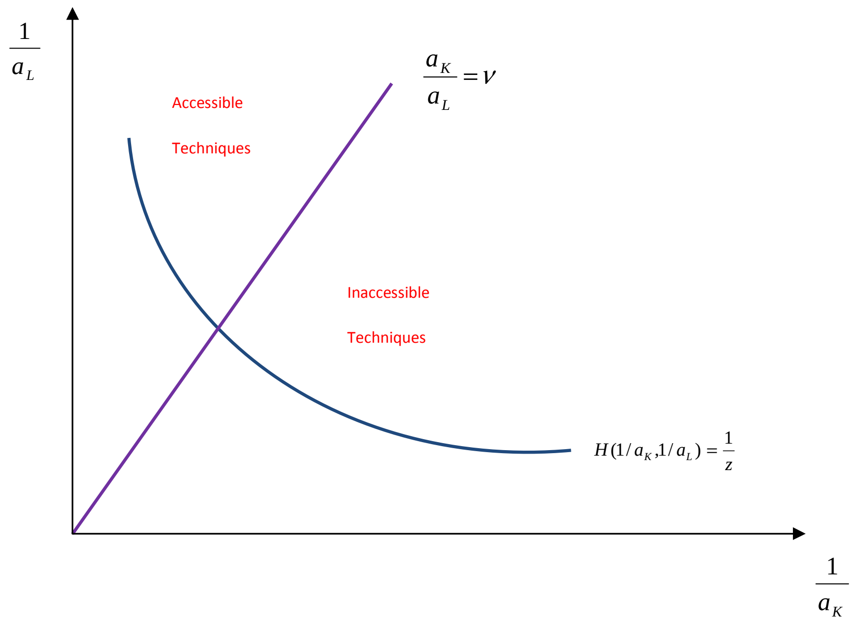


Figure 3: Technique Optimizing Problem: Constrained vs. Unconstrained Solution

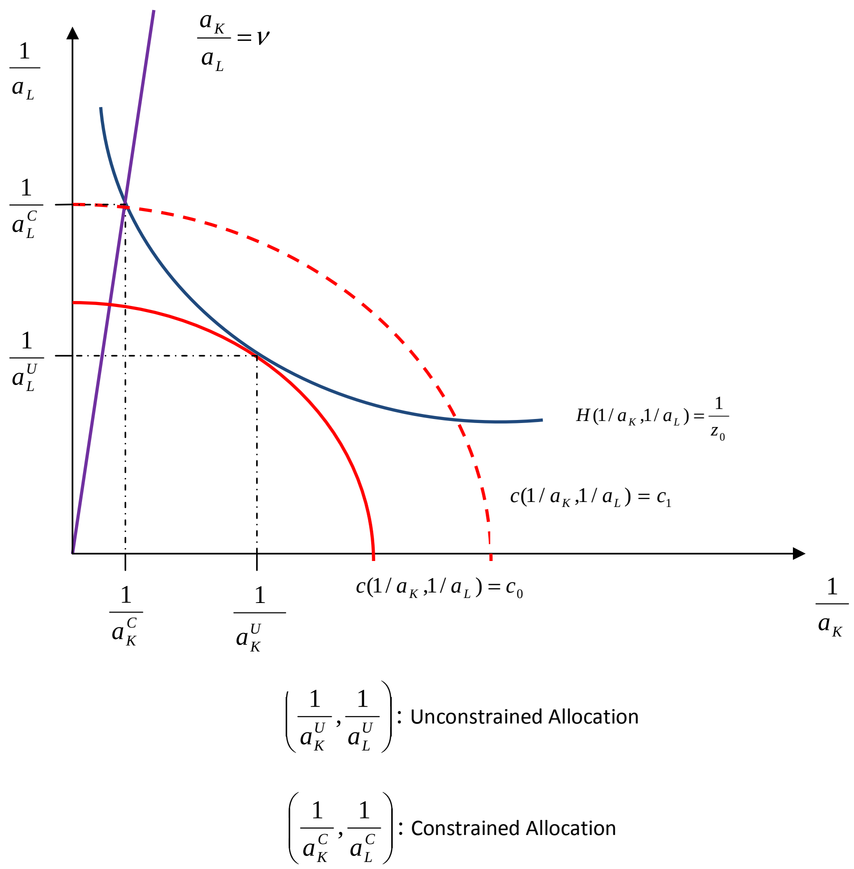
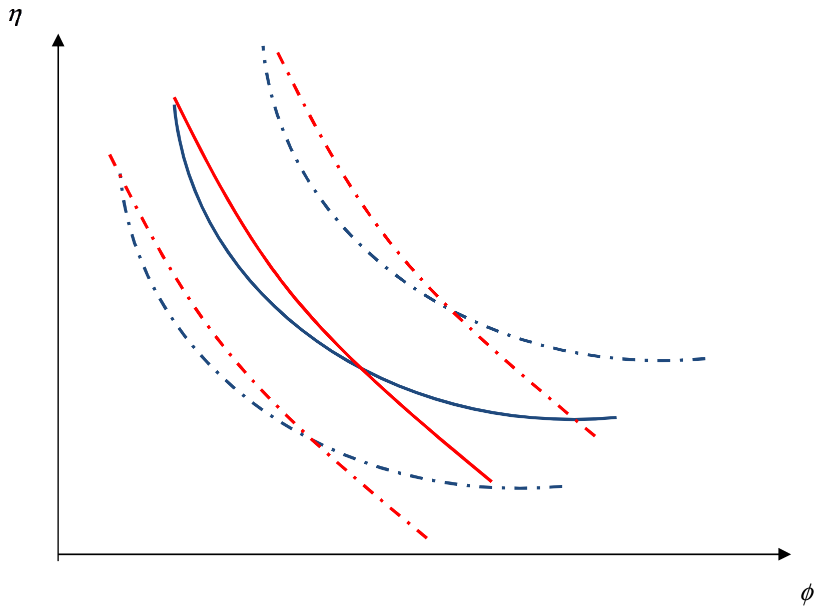
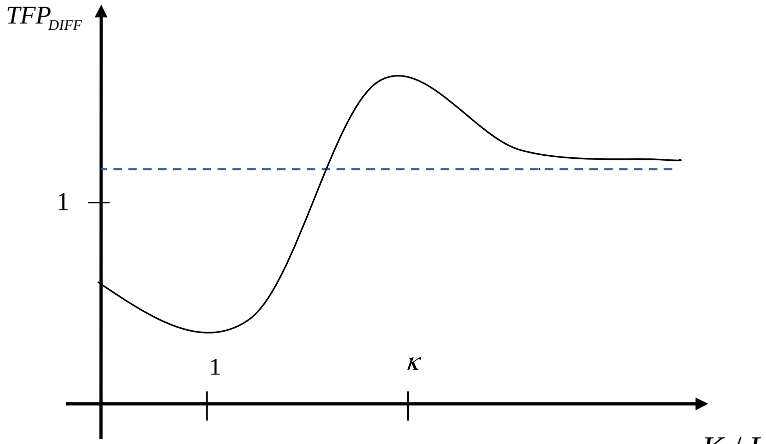


Figure 4: Isocosts with η and ϕ

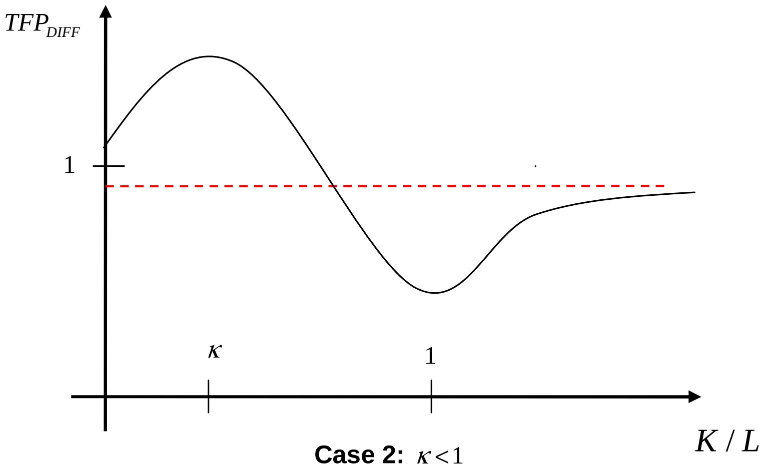


Blue: A firm with a low ρ Red: A firm with a high ρ

Figure 5: TFP Differential



Case 1: $\kappa > 1$



Case 2: $\kappa < 1$

Appendix

(Not Intended for Publication)

In this Appendix, we provide detailed mathematical derivations and proofs of various Lemmas and Propositions.

Benchmark Step-1 Solution: The Cost Minimizing $K-L$

The first-step is the standard neoclassical cost minimization problem presented at (2). Denoting the Lagrange multiplier associated with constraint by μ_1, solving for μ_1 will provide the marginal cost of producing one extra unit of output. First order conditions with respect to K and L are derived as the following:

$$\begin{aligned} K \quad &: \quad r = \mu_1 \lambda a_K^{\rho} K^{\rho-1} [(a_K K)^{\rho} + (a_L L)^{\rho}]^{\frac{1-\rho}{\rho}} \\ \Rightarrow \quad & \quad K = \mu_1^{\frac{1}{1-\rho}} r^{\frac{-1}{1-\rho}} \lambda^{\frac{1}{1-\rho}} a_K^{\frac{\rho}{1-\rho}} Y \qquad (28) \\ L \quad &: \quad w = \mu_1 (1-\lambda) a_L^{\rho} L^{\rho-1} [(a_K K)^{\rho} + (a_L L)^{\rho}]^{\frac{1-\rho}{\rho}} \\ \Rightarrow \quad & \quad L = \mu_1^{\frac{1}{1-\rho}} w^{\frac{-1}{1-\rho}} (1-\lambda)^{\frac{1}{1-\rho}} a_L^{\frac{\rho}{1-\rho}} Y \qquad (29) \end{aligned}$$

Putting (28) and (29) together yields to the K/L ratio at (4). Plugging K and L from (28) and (29) into the constraint at (2) and solving for μ_1 provides the unit cost of production that we presented at (5).

Benchmark Step-2 Solution: The optimized a_K-a_L

The second-step cost minimization problem is stated at (3). Denoting the lagrange multiplier associated with the constraint by μ_2, we can derive the first order conditions with respect to a_K and a_L:

$$a_K \quad : \quad r^{\frac{\rho}{\rho-1}} \lambda^{\frac{1}{1-\rho}} a_K^{-\frac{\rho}{\rho-1}-1} \left[\left(\frac{r}{a_K}\right)^{\frac{\rho}{\rho-1}} \lambda^{\frac{1}{1-\rho}} + \left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}} (1-\lambda)^{\frac{1}{1-\rho}} \right]^{\frac{\rho-1}{\rho}-1} = \mu_2 \alpha a_K^{\alpha-1} a_L^{1-\alpha}, \qquad (30)$$

$$a_L \quad : \quad w^{\frac{\rho}{\rho-1}} (1-\lambda)^{\frac{1}{1-\rho}} a_L^{-\frac{\rho}{\rho-1}-1} \left[\left(\frac{r}{a_K}\right)^{\frac{\rho}{\rho-1}} \lambda^{\frac{1}{1-\rho}} + \left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}} (1-\lambda)^{\frac{1}{1-\rho}} \right]^{\frac{\rho-1}{\rho}-1} = \mu_2 (1-\alpha) a_K^{\alpha} a_L^{-\alpha}. \qquad (31)$$

Solving (30) and (31) together provide us with the optimized a_K/a_L ratio at (6). Plugging (6) in (4) solves for the optimized K/L ratio as a function of model parameters α and λ; and factor prices r and w as we expressed at (7). In order to determine the levels of a_K and a_L as functions of z, w/r, α and ρ, we plug (6) in $H(a_K, a_L)$ and obtain the expressions at (8) and (9). Plugging a_K and a_L in $\tilde{c}(.)$ provides:

$$c(w,r) = \frac{1}{z} r^{\alpha} w^{1-\alpha} \left[\lambda^{\alpha}(1-\lambda)^{1-\alpha}\right]^{-\frac{1}{\rho}} \left[\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \right]^{\frac{\rho-1}{\rho}}$$

Finally, putting the related terms together yields the unit cost expression at (10).

Proof of Proposition 2.3

We re-write the unit cost function as:

$$c(w,r) = \frac{1}{z} r^{\alpha} w^{1-\alpha} \left[\underbrace{\left(\lambda^{\alpha}(1-\lambda)^{1-\alpha}\right)^{-1}}_{\Omega_1} \right]^{\frac{1}{\rho}} \left[\underbrace{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}_{\Omega_2} \right]^{\frac{1-\rho}{\rho}} \tag{32}$$

The first term inside the square brackets (Ω_1) is greater than 1 whereas the second term (Ω_2) is smaller than 1. Therefore, the impact of more flexibility on cost of production depends on λ and α. Specifically, if

$$\lambda^{\alpha}(1-\lambda)^{1-\alpha} < \alpha^{\alpha}(1-\alpha)^{1-\alpha} \tag{33}$$

then more flexibility is desirable. In order to see the parameter conditions for which the (33) holds; note that

$$\arg\max_{\lambda} \ \lambda^{\alpha}(1-\lambda)^{1-\alpha} = \alpha,$$

which implies that the highest value the LHS of (33) could attain equals to $\alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Therefore, $\lambda^{\alpha}(1-\lambda)^{1-\alpha} \leq \alpha^{\alpha}(1-\alpha)^{1-\alpha}$ for all $\lambda, \alpha \in [0,1]$.

Solution with frictions in the Factor Choice Stage

The constrained first-step problem is the following:

$$\begin{aligned} \min_{K,L} \quad & wL + rK && (34)\\ s.t. \quad & Y = [\lambda(a_K K)^{\rho} + (1-\lambda)(a_L L)^{\rho}]^{\frac{1}{\rho}} && (35)\\ & K \leq \bar{K} && (36) \end{aligned}$$

Let's denote the lagrange multiplier with respect to the constraint (35) with μ_1 and the lagrange multiplier with respect to 36 with η. Assuming a binding capital constraint ($\eta > 0$) and solving for μ_1 will again provide us with the unit cost of producing one extra unit of output. First order conditions with respect to K and L are as the following:

$$\begin{aligned} K \quad : \quad & r(1+\eta) = \mu_1 \lambda a_K^{\rho} K^{\rho-1} [(a_K K)^{\rho} + (a_L L)^{\rho}]^{\frac{1-\rho}{\rho}} \\ \Rightarrow \quad & K = \mu_1^{\frac{1}{1-\rho}} (r(1+\eta))^{\frac{-1}{1-\rho}} \lambda^{\frac{1}{1-\rho}} a_K^{\frac{\rho}{1-\rho}} Y && (37)\\ L \quad : \quad & w = \mu_1 (1-\lambda) a_L^{\rho} L^{\rho-1} [(a_K K)^{\rho} + (a_L L)^{\rho}]^{\frac{1-\rho}{\rho}} \\ \Rightarrow \quad & L = \mu_1^{\frac{1}{1-\rho}} w^{\frac{-1}{1-\rho}} (1-\lambda)^{\frac{1}{1-\rho}} a_L^{\frac{\rho}{1-\rho}} Y && (38) \end{aligned}$$

Putting (37) and (38) together solves for the optimized K/L ratio expressed at (11). Plugging K and L from (37) and (38) into (35),

$$Y = \left[\left(\frac{r(1+\eta)}{a_K} \right)^{\frac{\rho}{\rho-1}} \lambda^{\frac{1}{1-\rho}} \mu_1^{\frac{\rho}{1-\rho}} + \left(\frac{w}{a_L} \right)^{\frac{\rho}{\rho-1}} (1-\lambda)^{\frac{1}{1-\rho}} \mu_1^{\frac{\rho}{1-\rho}} \right]^{\frac{1}{\rho}} Y,$$

and solving for μ_1 gives the unit cost of production at (12).

Solution with frictions in the Technique Choice Stage

The constrained technique choice problem yields the first order conditions with respect to a_K and a_L expressed at (13). Using the respective partial derivatives of $c(.)$ and $H(.)$ implies:

$$a_K: \quad (r(1+\eta))^{\frac{\rho}{\rho-1}}\lambda^{\frac{1}{1-\rho}}a_K^{-\frac{\rho}{\rho-1}-1}\left[\left(\frac{r(1+\eta)}{a_K}\right)^{\frac{\rho}{\rho-1}}\lambda^{\frac{1}{1-\rho}}+\left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}}(1-\lambda)^{\frac{1}{1-\rho}}\right]^{\frac{\rho-1}{\rho}-1} = (1+\phi)\mu_2\alpha a_K^{\alpha-1}a_L^{1-\alpha}$$

$$a_L: \quad w^{\frac{\rho}{\rho-1}}(1-\lambda)^{\frac{1}{1-\rho}}a_L^{-\frac{\rho}{\rho-1}-1}\left[\left(\frac{r(1+\eta)}{a_K}\right)^{\frac{\rho}{\rho-1}}\lambda^{\frac{1}{1-\rho}}+\left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}}(1-\lambda)^{\frac{1}{1-\rho}}\right]^{\frac{\rho-1}{\rho}-1} = \mu_2(1-\alpha)a_K^{\alpha}a_L^{-\alpha}$$

which together lead to the a_K/a_L ratio at (14). This can then be substituted into (1) to obtain the levels of a_K and a_L expressed at (16) and (17). The unit cost of production as a function of the two distortions, η and ϕ, can be derived as:

$$\begin{aligned} c(r,w) &= \left[\left(\frac{(1+\eta)\,r}{a_K}\right)^{\frac{\rho}{\rho-1}}\lambda^{\frac{1}{1-\rho}}+\left(\frac{w}{a_L}\right)^{\frac{\rho}{\rho-1}}(1-\lambda)^{\frac{1}{1-\rho}}\right]^{\frac{\rho-1}{\rho}} \\ &= \frac{1}{z}(r(1+\eta))^{\alpha}w^{1-\alpha}\left[\lambda^{\alpha}(1-\lambda)^{1-\alpha}\right]^{-\frac{1}{\rho}}\left[\left(\frac{\alpha(1+\phi)}{1-\alpha}\right)^{1-\alpha}+\left(\frac{1-\alpha}{\alpha(1+\phi)}\right)^{\alpha}\right]^{\frac{\rho-1}{\rho}} \end{aligned}$$

Putting related terms together and manipulating, we obtain the expression at (18).

Proof of Proposition 3.2

Distortionary wedge against a particular production technique (for instance a_K in the particular case that we analyze) implies that the relative use of that technique is lower compared to an unconstrained benchmark. Using (6), (13) and (14) a sub-optimally low a_K usage implies that: when $\rho < 0$ then $\phi > 0$ and when $0 < \rho < 1$, then $-1 < \phi < 0$.

Proof of Proposition 4.1

With z-shocks, firms' entry decision is then determined via

$$f_e = \int_{z^*}^{\bar{z}} \pi(z_i)f(z_i)di. \tag{39}$$

We can re-write

$$\pi(z_i) = h_1(z_i)h_2(\rho), \tag{40}$$

where the $h_2(\rho)$ component includes all the terms that include production flexibility parameter ρ. Then:

$$f = h_1(z^*)h_2(\rho) \tag{41}$$

$$f_e = h_2(\rho)\int_{z^*}^{\bar{z}} h_1(z_i)f(z_i)di. \tag{42}$$

With η-shocks, we can isolate the component that includes the ρ terms as in the following:

$$\pi(\eta_i) = j_1(\eta_i)j_2(\rho), \tag{43}$$

$$f = j_1(\eta^*)j_2(\rho), \tag{44}$$

$$f_e = j_2(\rho)\int_{\eta^*}^{\bar{\tau}_K} j_1(\eta_i)f(\eta_i)di, \tag{45}$$

where the cutoff η^* determines the threshold level of η at which a firm is indifferent between to-produce and not-to-produce.

With ϕ-shocks, this time we write:

$$\pi(\phi_i) = v_1(\phi_i;\rho)v_2(\rho) \tag{46}$$

Therefore:

$$\begin{aligned} f &= v_1(\phi^*;\rho)v_2(\rho), && (47)\\ f_e &= v_2(\rho)\int_{\phi^*}^{\bar{\phi}} v_1(\phi_i;\rho)f(\phi_i)di, && (48)\end{aligned}$$

where the cutoff ϕ^* determines the threshold level of η at which a firm is indifferent between to-produce and not-to-produce.

Proof of Proposition 4.3

We can derive the firm-specific component of MRPK and MRPL as the following:

$$\begin{aligned} MRPK_i &\propto (1+\phi_i)^{-(1-\alpha)\left(\frac{1-\rho}{\rho}\right)}, && (49)\\ MRPL_i &\propto (1+\phi_i)^{\alpha\left(\frac{1-\rho}{\rho}\right)}, && (50)\end{aligned}$$

and the TFPR as

$$TFPR_i \propto \left(\frac{(1+\phi)^{\alpha}}{1+\alpha\phi}\right)^{\frac{1-\rho}{\rho}}. \tag{51}$$

Proof of Lemma 4.5

Note that:

$$\frac{\partial \log\left(TFPQ_{int}(\hat{K},\hat{L})\right)}{\partial\rho} = \underbrace{-\frac{1}{\rho^2}\left\{\log\left[(1+\eta)^{\rho}\chi_K+\chi_L\right]-\log\left[\chi_K+\chi_L\right]\right\}}_{I} + \underbrace{\frac{(1+\eta)^{\rho-1}\chi_K+\chi_L}{(1+\eta)^{\rho}\chi_K+\chi_L}}_{II} \tag{52}$$

The RHS of the expression at (52) increases in ρ until some threshold $\bar{\rho} > 0$ is reached, and then it decreases in ρ. To see this, note that for $\rho < 0$, the first term of the RHS is always positive, and so is the second term. Therefore for $\rho < 0$, increases in ρ are always associated with decreases in the local TFPQ loss between B and A. For $\rho > 0$ there is a threshold $\bar{\rho}$ defined implicitly using the following equation

$$\frac{1}{\bar{\rho}^2}\left\{\log\left[(1+\eta)^{\bar{\rho}}\chi_K+\chi_L\right]-\log\left[\chi_K+\chi_L\right]\right\} = \frac{(1+\eta)^{\bar{\rho}-1}\chi_K+\chi_L}{(1+\eta)^{\bar{\rho}}\chi_K+\chi_L},$$

such that whenever $\rho < \bar{\rho}$, the TFPQ losses are still declining in ρ and for $\rho > \bar{\rho}$ the TFPQ losses start to increase in ρ. Note that the existence of a $\bar{\rho} < 1$ depends on the parameter conditions of the economy. If $\bar{\rho} > 1$, then for all values of ρ, increases in ρ reduces the TFPQ losses resulting from techniques adjustability.

Proof of Lemma 4.7

Let us observe:

$$
\begin{aligned}
Y &= z\left[\lambda\left(\frac{a_K}{z}K\right)^{\rho} + (1-\lambda)\left(\frac{a_L}{z}L\right)^{\rho}\right]^{\frac{1}{\rho}}, \\
\text{with } \frac{a_K}{z} &= \left(\frac{r(1+\eta)}{w}\right)^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)\left(\frac{1-\rho}{\rho}\right)}\left(\frac{1-\lambda}{\lambda}\right)^{(1-\alpha)\left(\frac{1}{\rho}\right)}(1+\phi)^{(1-\alpha)\left(\frac{1-\rho}{\rho}\right)}, \\
\frac{a_L}{z} &= \left(\frac{r(1+\eta)}{w}\right)^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha\left(\frac{1-\rho}{\rho}\right)}\left(\frac{1-\lambda}{\lambda}\right)^{-\alpha\left(\frac{1}{\rho}\right)}(1+\phi)^{-\alpha\left(\frac{1-\rho}{\rho}\right)}.
\end{aligned} \tag{53}
$$

Note that for $\frac{K}{L} = 1$, the optimized a_K and a_L combination equals to $\frac{a_K}{z} = 1$ and $\frac{a_L}{z} = 1$; which is the exogenously specified technique level in the neoclassical production function above. Similarly, when $\frac{K}{L} = \kappa$, then the optimized $\frac{a_K}{z}$ and $\frac{a_L}{z}$ are as derived at equation (53). Hence, for all K/L within the $[\kappa, 1]$ boundaries, when K/L ratio is closer to κ then the neoclassical TFPQ is dominated by the TFPQ with adjustable techniques, and vice versa when K/L ratio is closer to 1.

Proof of Lemma 4.8

$\frac{\partial\kappa}{\partial\eta} > 0$, and $TFPQ_{neo}$ at κ is constant with respect to the changes in η. Furthermore, the TFPQ with adjustable techniques ($TFPQ$) satisfies $TFPQ \to TFPQ_{neo}$ as η decreases for $\kappa < 1$, and $TFPQ \to TFPQ_{neo}$ as η increases for $\kappa > 1$.

Proof of Proposition 4.9

We can derive

$$
\frac{\partial \log\left(TFPQ_{ext}\right)}{\partial\rho} = \underbrace{-\frac{1}{\rho^2}\left\{\log\left[(1+\eta)^{\rho}\varphi_K\theta_K + \varphi_L\theta_L\right] - \log\left[\theta_K + \theta_L\right]\right\}}_{I} + \underbrace{\frac{(1+\eta)^{\rho-1}\varphi_K\theta_K + \varphi_L\theta_L}{(1+\eta)^{\rho}\varphi_K\theta_K + \varphi_L\theta_L}}_{II} \tag{54}
$$

The RHS of the expression at (54) increases in ρ until some threshold $\bar{\rho} > 0$ is reached, and then it decreases in ρ. To see this, note that for $\rho < 0$, the first term of the RHS is always positive, and so is the second term. Therefore for $\rho < 0$, increases in ρ are always associated with decreases in the TFPQ differential at $\frac{K}{L} = \kappa$. For $\rho > 0$ there is a threshold $\bar{\rho}$ defined implicitly using the following equation

$$
\frac{1}{\tilde{\rho}^2}\left\{\log\left[(1+\eta)^{\tilde{\rho}}\varphi_K\theta_K + \varphi_L\theta_L\right] - \log\left[\theta_K + \theta_L\right]\right\} = \frac{(1+\eta)^{\tilde{\rho}-1}\varphi_K\theta_K + \varphi_L\theta_L}{(1+\eta)^{\tilde{\rho}}\varphi_K\theta_K + \varphi_L\theta_L},
$$

such that whenever $\rho < \bar{\rho}$, the TFPQ differential is declining in ρ and for $\rho > \bar{\rho}$ the TFPQ differential starts to increase in ρ. Note that the existence of a $\bar{\rho} < 1$ depends on the parameter conditions of the economy. If $\bar{\rho} > 1$, then for all values of ρ, increases in ρ reduces the TFPQ differential. We summarize this theoretical result as the following.

Proof of Lemma 4.10

The proof follows from the same lines as in the proof of Lemma 4.5.