# The Politics of Pension Reform Under Population Ageing

Leers, T.; Meijdam, A.C.; Verbon, H.A.A.

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# The Politics of Pension Reform under Aging

Theo Leers Lex Meijdam Harrie A. A. Verbon¤

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Abstract

In this paper we address the question whether in case of population aging a transition from an unfunded to a more funded pension scheme is politically feasible in a representative democracy. We consider two parties: a right-wing party which is willing to trade o¤ intragenerational equity against e¢ciency gains in intergenerational redistribution, and a left-wing party which does not want to adjust the level of intragenerational distribution. We show that, in an economy with an exogenously given interest rate, only a thus de…ned right-wing government will propose a social-security reform. Moreover, we demonstrate that such a policy proposal may lead to electoral success if it entails an appropriate mix of distributional e¢ciency and equity.

JEL classi…cation: J14, H55

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¤All authors are a¢liated with the Department of Economics and CentER, Tilburg University. Lex Meijdam is also a¢liated with the Vrije Universiteit Amsterdam. Department of Economics and CentER, Tilburg University, PO Box 90153, 5000LE Tilburg, The Netherlands. E-mail: H.A.A.Verbon@kub.nl

# 1 Introduction

The aging of the population is one of the most important phenomena of the beginning of this new millennium[1], and is considered to going to be a threat for future Pay-As-You-Go (PAYG) …nanced social-security arrangements in many countries. Such schemes are known to have an internal rate of return lower than the rate of return of funded schemes (i.e., the interest rate on savings) and aging deteriorates their rate of return even further. Some economists therefore advocate a complete (and fast) transition from unfunded to funded schemes by 'privatizing' social security (see e.g., Feldstein [1996]). In the 2000 US presidential campaign, the debate about 'privatization' of the social-security system boiled down to the issue of the implementation of private retirement accounts which, linked to tax cuts, was a key issue in the program of the Republican candidate George W. Bush. The Democratic candidate, Al Gore, basically ran the campaign on the promise to keep the current social-security arrangements intact and to simultaneously try to minimize the crowding e¤ects to other policy …elds by devoting a part of the budget surplus to a running down of the national debt.

The Republican privatization proposal does not explicitly address adjustments of the social-security bene…ts. However, it is well known that PAYG-pension schemes are Pareto e¢cient (see Verbon [1989], or Breyer [1989]). That is, although the transformation of a PAYG-scheme in a capital reserve system increases utility in the long run, it harms at least one generation.[2] Naturally, the group that will be harmed after the introduction of private retirement accounts will be the currently retired who will see their bene…t decrease due to the retrieval of tax money for the implementation of private accounts. Thus, this implementation will not be unanimously supported. But, of course, in democracies decisions do not have to be made under unanimity. Even though unanimity is not a prerequisite, in a direct democracy the prospect for pension reform remains rather bleak. In this respect, the classic result of Browning (1975), stating that a 'too high' bene…t level in a PAYG-scheme will be adopted, and that one might, moreover, expect an upward pressure on the bene…t level if the population is aging, is quite unequivocal.[3] However, in a representative democracy,

[1] In the year 2030 more than 30 percent of the total population in the OECD countries will be over 60 years of age whereas in 1990 this was only 18 percent. See e.g. The Economist, 'All our tomorrows: a survey of the economics of ageing', January 27th 1996, or more recently The United Nations World Population Prospects: The 1998 Revision. In particular, the elderly dependency ratio in the US is expected to rise from 19 % in the year 2000 to 35 % in the year 2040.

[2] If reducing or abolishing the PAYG-scheme goes along with the reduction or elimination of a negative external e¤ect, as for example the distortion of the labor-leisure decision caused by a PAYG-payroll tax as in Homburg (1990), then a Pareto-improving transition to a funded system is possible. Likewise, this can be achieved if the increase in savings that results from a reduction of the PAYG-scheme has positive external e¤ects; for example under an endogenous-growth scenario as in Belan et al. (1998). Finally, assuming endogenous fertility, Groezen et al. (2000) show that a combined policy of decreasing the PAYG-tax and introducing child allowances can also result in a Pareto-improvement.

[3] The intuition of Browning's result is that under majority voting the median voter is relatively old, who, in considering the marginal costs of any bene…t level, considers her past contributions as sunk costs which she does not include in trading o¤ the marginal costs (her future contributions) and the marginal bene…ts (her future bene…ts). Aging implies that the median voter will be older,

direct voting on issues by the population does, in general, not occur. Instead, elected politicians, representing political parties, trade o¤ gains for some groups against losses for other groups. At one point in time, exogenous shocks like population aging can make it politically advantageous for political decision makers to decrease one group's utility in favor of an increase in utility of another group. Pension reform then might become feasible. The prospect of an aging population could therefore have been one of the reasons why in the 2000-presidential campaign social-security reform turned out to be one of the key issues and was apparently no longer considered to be the third rail of American politics.[4]

This paper analyses the politics of transitions from unfunded to funded pensions in reaction to population aging in a representative democracy. The support for political parties is assumed to depend on the utility that groups derive from political decisions on relevant issues. A political party maximizes its (re-)election prospect by maximizing some (politically) weighted sum of utilities. We will assume that parties (and therefore also politicians) are only liable to current generations, but, in spite of that, we exclude the possibility to exploit government debt to shift the burden of the transition to future generations. There are several reasons for this exclusion. Firstly, in many countries, the possibility for the government to use debt as an instrument is constrained by exogenous institutional restrictions. In the US, for example, each year the Board of Trustees of the Social Security trust fund has to design, by law, a schedule of current and future tax rates to …nance the program over a 75-years horizon, under the (apparently implicit) assumption that the Trust Fund is not allowed to have a negative wealth position. Secondly, even without such restrictions, a government cannot unlimitedly increase public debt. A representative government, that is only liable to current generations, will already have raised debt to its maximum level, leaving no room for additional government de…cits.[5] Finally, as the purpose of the transition to funded social security is primarily to relieve the burden for the current young and unborn generations, it seems not very useful to saddle these generations with additional public debt. In fact, the transition then just replaces implicit debt by explicit debt and does not improve their welfare.

A basic insight that results from our analysis is that, when the interest rate is exogenously determined at the world capital market, a transition from a PAYG-scheme to a capital reserve system in reaction to aging can only come about if politicians have an instrument at their disposal to introduce substitution e¤ects on the savings decision. The point is that, if taxes (and bene…ts) only generate income e¤ects, politicians will not be able to e¤ectively stimulate savings by the young. An increase in savings is needed, however, for a conversion from a PAYG-scheme to a funded system. By decreasing PAYG-transfers, and stimulating savings, politicians will realize utility gains for the younger generations. If the resulting increase in political support by

so that for her the marginal costs of increasing the pension bene…t will be even lower.

[4] See for example the Economist survey on the economics of aging of January, the 27th, 1996: "Even in America, where the quest for a balanced budget has caused many assumptions to be reexamined, conversations about social security always seem to lead up to the phrase: It's like the third rail: touch it, and you're dead."

[5] For an analysis of decision making on public debt in a representative democracy see Meijdam, Van de Ven and Verbon (1996).

the young makes up for the decrease in political backing by the elderly, whose utility decreases as a result of the transition, a pension reform will be politically feasible. To enable politicians to a¤ect the savings decision, and thus to realize a social security reform, we introduce a capital tax, i.e., a tax on savings. The proceeds of this tax are assumed to be rebated in a lump-sum fashion, so that it can be viewed as an instrument for intragenerational redistribution among the elderly only.

We consider two political parties di¤ering in ideology. The way we formulate ideology was inspired by the positions in the debate on pension reform taken by the candidates in the 2000 US presidential campaign, alluded to above. One party, which we dub right-wing, is willing to trade o¤ intragenerational equity against e¢ciency gains in intergenerational redistribution. The other, labeled left-wing, is opposed to any adjustment of the level of intragenerational redistribution. That is, the left-wing party will never use the capital tax to stimulate savings and thus to realize a social-security reform. We show that, as a consequence of this, in case of aging, a left-wing policy implies that the economy is trapped at the pre-existing level of savings. The right-wing party, however, is inclined to sacri…ce intragenerational equity by relaxing the disincentives the capital tax entails for savings in order to reform the pension system and thus to increase the e¢ciency of intergenerational redistribution. This indeed allows for a partial transition from PAYG to funded pensions in case of an aging population.

Given that additional debt creation is excluded, the decrease in the PAYG-bene…t that is necessary to realize a transition decreases the oldest living generations' utility, while raising the youngest living generations' utility. We show, however, that in principle a transition may be politically feasible if aging occurs. More precisely, we demonstrate that a right-wing reform plan may beat the left-wing policy proposal that keeps the PAYG-scheme intact in the elections if at the outset a (strictly positive) distortionary tax on savings is in place. A striking consequence of this result is that subsidies on savings, which in the real world frequently occur, are not a suitable point of departure for starting a conversion policy. Moreover, in practice, it may be very hard to …nd a politically feasible reform plan as it is crucial that such a plan is optimally tuned. In particular, a proposal for a fast transition process with high cuts in pension bene…ts will not carry the election. The intuition for this result is that a fast transition will strongly harm the current retirees while it does not su¢ciently bene…t the young to make up for this, as for them the positive e¤ect of a low PAYG-tax when young is largely o¤set by the negative e¤ect of an even lower tax - and thus a lower PAYG-bene…t - when old. Only plans with very gradually declining taxes and thus very slowly rising private savings may survive in the political arena.

The paper is organized as follows. Section 2 presents the well-known two-period overlapping-generations (OLG) model for the private sector that forms the basis of our analysis. In section 3 we describe the target function and the instruments of the representative government. The e¤ects of a right-wing and a left-wing policy, when applied to an aging economy, are analyzed in Section 4. In this section, we also discuss the political feasibility of the right-wing proposal for a transition to funded social-security. Section 5 concludes.

# 2 The private sector

Consider an economy with an exogenously given interest rate r, determined on the world capital market and assumed to be constant over time. Production is described by a standard constant-returns-to-scale production function, so that the wage rate is also …xed. The economy is populated by a large number of non-altruistic agents who live for two periods, so that in each period both a young and an old generation are alive. When young, the agent inelastically supplies one unit of labor.[6] We assume lifetime utility of a representative agent born on date t to be additively separable and the felicity functions to be logarithmic[7]:

$$U_t \acute{} U(c_t^y; c_{t+1}^o) = \log(c_t^y) + \frac{1}{1+½}\log(c_{t+1}^o); \tag{1}$$

where $c_t^y$ denotes consumption during youth and $c_{t+1}^o$ denotes the consumption of the same individual while old. The parameter ½ indicates the pure rate of time preference.

A part ¿ of the labor income, that is normalized to 1, is taxed away by the government in a lump-sum fashion to be transferred to the currently living old. The remaining wage income is used for savings[8] for old age ($s_t$) and for consumption while young:

$$c_t^y = 1 \text{ ¡ } ¿_t \text{ ¡ } s_t: \tag{2}$$

When old, savings may be taxed by a (possibly negative) capital tax » explained in more detail in the next section.[9] The old agent consumes his net savings, including their returns, and the transfer payment ¢ from the government which the (atomistic) agent takes as given. The consumption at time t + 1 of an old agent born on date t is given by

$$c_{t+1}^o = (1 \text{ ¡ } »_{t+1} + r)s_t + ¢_{t+1}: \tag{3}$$

Maximizing lifetime-utility (1) subject to the single-period budget constraints (2) and (3) yields

$$c_{t+1}^o = \frac{1 \text{ ¡ } »_{t+1} + r}{1+½} c_t^y: \tag{4}$$

[6] Allowing for endogenous labor supply considerably complicates the analysis, but does not a¤ect the main results. The reason for this is that the supply of labor can be eliminated from the model using the …rst-order conditions. The dynamical system that results in that case is equivalent to the one in case of exogenous labor supply.

[7] Felicity is assumed to be logarithmic for simplicity. For most of our results it su¢ces to assume that $U(c_t^y; c_{t+1}^o)$ is homothetic in …rst-period and second-period consumption so that the optimal ratio $\frac{c_t^y}{c_{t+1}^o}$ depends on the interest rate and the discount factor only.

[8] In addition to public pensions and savings, the elderly's consumption is often …nanced by private occupational pension schemes that are typically fully funded. In this paper we assume that private occupational pension schemes can be identi…ed with individual savings for old age.

[9] Alternatively, we could assume a tax on capital income instead of a tax on capital. This would, however, not a¤ect the results.

# 3 Political decision making

As noted in the Introduction, we assume that decisions on social security are made in a representative democracy.[10] In this representative democracy, there is not an in…nitely long lived government that optimizes social welfare, but a sequence of governments exists where, as in a two-party democracy, each period a di¤erent political party can be voted into o¢ce. As a result, decisions made by any government are unrelated to past or future incumbent parties. The political parties take account only of the interests of both the current young and the current old. So, at time t; they maximize a politically weighted average of the utilities of generations currently alive (that is, of all current voters):

$$W_t = \psi U_{t-1} + (1 + n_t)U_t; \tag{5}$$

where $n_t$ stands for the rate of population growth at time t. Throughout the paper it is assumed that the economy is dynamically e¢cient ($r > n_t$; 8t). The political weight $\psi > 0$ can be interpreted as a measure for the political in‡uence of an old agent relative to a young one, which is determined exogenously, for example through an implicit process of lobbying and/or rent seeking of the di¤erent generations currently alive.[11] Foundations for this approach can be found in Hettich and Winer (1988) and Grossman and Helpman (1994).[12] Maximizing $W_t$ can also be interpreted as maximizing the (re-)election probability, allowing for the impact of (in our case two) interest groups (see Coughlin et al. [1988]). In a two-party system, this implies that if a political party, at time t; proposes a platform with less political support relative to the support for the platform of the competing party, it will lose the next election.

If we abstract from administrative costs, the (balanced) budget constraint for the government at date t is given by

$$(1 + n_t)\tau_t + \theta_t s_{t-1} = \phi_t. \tag{6}$$

Two remarks are in order. Firstly, notice that we assume that the revenues of the capital tax are used for …nancing social-security bene…ts. Consequently, capital taxation is purely intragenerational redistribution. Of course, in our representative-agent

[10] For an overview of decision making in a representative democracy we refer to Gärtner (1994) and Verbon (1993).

[11] The political weight $\psi$ is the equilibrium result of the activities of the pressure groups representing the young and old, respectively. Therefore, as Becker (1983) points out, even if the marginal absolute pressure of, for example the old, were in…nitely large, this does not hold for the marginal relative pressure - that is captured by the parameter $\psi$ - , because the young will react with increased counteractivities. It is beyond the framework of this paper and our simple characterization of the political process to take explicitly into account the relation between group size and the activities of pressure groups that try to in‡uence the representative government. Therefore we ignore the fact that organizing larger groups aggravates the free-rider-problem and therefore lowers the relative political in‡uence (Olson [1965], Becker [1983]), and take $\psi$ to be constant.

[12] Notice that the target function of the representative government is a truncated version of a Benthamite social welfare function. Bernheim (1989) calls this "the most natural class of welfare functions for a representative government" (p.124). Although his approach inhibits the presence of some explicit sort of altruism, it is comparable to our framework where the representative government also has to display some sort of 'political altruism' towards both groups of current voters in order to survive politically.

model this intragenerational redistribution has no e¤ect (as it is just shifting of funds), except for a disincentive to save. We do allow for it, however, because, as noted earlier and to be shown rigorously later on, the existence of a capital tax is of importance for the political feasibility of a transition to a funded pension scheme.

Secondly, the capital tax in place at time $t$ is imposed upon savings formed at time $t - 1$. Given our perfect-foresight assumption, savers at time $t - 1$ will be able to tell the size of the capital tax at time $t$. The government at time $t$ can no longer change the impact of this capital tax on savings held by the current elderly, born at time $t - 1$, as these are given at time $t$. So, the only remaining motive for altering the capital tax will be a modi…cation of the distribution of income over the elderly people.

Obviously, we cannot explicitly model political decision-making on intragenerational redistribution in our representative agent model. However, for the possibility of pension reform we only need to identify two political parties that di¤er in their ideology with respect to the trade-o¤ they allow to exist between intergenerational and intragenerational redistribution. As stated in the Introduction, this way of modelling ideology was inspired by the positions taken by the candidates in the 2000 US presidential campaign. One party, which we dub left-wing, is not willing to sacri…ce intragenerational equity for e¢ciency in intergenerational redistribution. This party thus sets a level for the PAYG-tax $\tau_t$ and, independently from this PAYG-tax, effectuates a constant rate of capital taxation $\theta_t = \theta > 0; \ \forall t$ (which we assume to be exogenously determined as we do not explicitly model intragenerational redistribution). The other party, labeled right-wing, is inclined to give up some intragenerational equity for a more e¢cient pension system. That is, the right-wing party relates the amount of intragenerational redistribution (through the capital tax $\theta_t$) to the amount of intergenerational redistribution (via $\tau_t$). In particular, we assume that the capital tax chosen by the right-wing party depends negatively on the desired level of the PAYG-tax:

$$\theta_t = \theta(\tau_t) \quad \text{with } \theta'_t \equiv \frac{d\theta_t}{d\tau_t} < 0, \tag{7}$$

i.e., more intergenerational redistribution goes along with less intragenerational redistribution.[13] Notice that, if relation (7) holds, the net return on savings depends on the capital tax and thus on the level of the social-security tax when old. So, the future PAYG-tax causes a substitution e¤ect in addition to the normal income e¤ect. As a consequence of this, the intertemporal allocation of consumption varies over time with $\tau$ (see equation 4). As we will show below, this may allow for a partial transition from PAYG to a funded system in case of aging, by stimulating individuals to save more if the PAYG-system loses in importance.

In each period $t$; the incumbent government determines the level of the social-security tax $\tau_t$ in that period (and if $\theta_t$ is a function of $\tau_t$ they thus implicitly determine the level of the capital tax). We assume that in doing so it takes private savings in

[13] It might be emphasized here that our analysis pertains to an aging population, so that the PAYG-tax is likely to increase anyway. Equation (7) thus implies that, in this case, the right-wing party will be inclined to decrease the capital tax.

period t as given.[14] Because the current and future taxes are only linked through current savings, this assumption implies that future taxes are also taken as given. The optimal PAYG-tax in period t follows from maximizing (5) subject to the relevant budget constraints. This gives the following …rst-order condition:

$$c_t^o = \frac{¸}{1 + ½} c_t^y : \tag{8}$$

It follows directly from (4) and (8) that in the steady state »(¿) ´ » = ¸ ¡ 1 ¡ r has to hold.

# 4 Aging and pension reform

This section considers the pension policies of left–wing and right-wing governments who have to deal with an increasing elderly dependency ratio. Population aging is interpreted as a once-and-for-all decrease in the rate of population growth (n). First, we investigate whether, in the presence of an aging population, these governments will pursue a (partial) transition from the PAYG-system to a funded pension scheme. We show that a right-wing government will reform the pension system by stimulating private savings while a left-wing government will only increase transfers. Subsequently, we analyze the political feasibility of a pension reform. That is, we investigate the possibilities for the right-wing party which favors a transition to a capital reserve system to win the election and to pursue this policy.

Compared to the case where the PAYG-system remains unaltered, a pension reform necessitates a fall in the utility of the elderly living at the time the reform is initiated, but enables an increase in utility for current (and future) young generations. A reform is only politically feasible, that is, politicians will only be able to enact a transition to funded pensions, if the decrease in political backing by the elderly (due to their experienced loss in utility) is (more than) compensated by the additional political support of the young. Regarding the latter, however, lifetime utility of currently young voters is also determined by decisions of future politicians (who decide on their bene…t level and, maybe, on the net return on their savings). In turn, future politicians' decisions depend on measures of politicians succeeding them. This implies that politicians have to be forward looking by taking account of the total sequence of future decisions in response to their own decision making. Analytically, this implies that our model has to be solved as a (saddlepoint-stable) dynamic system with two state variables, the PAYG-tax and the private savings. The solution of this dynamic system is presented in Section 4.2. The economic intuition of our result, however, can also be grasped by considering a conventional graphical analysis which will be

[14] That is, we analyze the Nash-solution of the game between politicians and the private sector. Alternatively, one could assume that the government acts as a Stackelberg-leader towards the private sector. However, the government is not a natural leader in this game. Within one period (one generation), private savings may react to the PAYG-tax (the current capital tax does not a¤ect current savings) but the tax may also react to the level of savings. Therefore, the Nash-solution seems most appropriate.

presented in the next subsection. In Section 4.3 we will discuss whether a conversion policy can be a winning platform in an election.

## 4.1 A sketch of the options for pension reform

In this subsection, we examine the policy options resulting from the two alternative cases under consideration, i.e., one where $\theta_t = \theta$ (left-wing policy) and one where $\theta_t^0 < 0$ (right-wing policy). To get a clear picture of the alternative cases, we consider what will happen if the policy under consideration will be upheld indefinitely. Thus, we first consider the case where $\theta_t = \theta$ holds indefinitely, after which we turn to the case where $\theta_t^0 < 0$ permanently holds. Notice that the steady-state condition $\theta(\tau) = \theta = \gamma - 1 - r$ has to hold in both cases. This implies that, if the capital tax $\theta_t$ is a function of the social-security tax $\tau_t$, i.e., $\theta(\tau_t)$, the PAYG-tax will be uniquely determined in the steady state. If a demographic shock occurs, the PAYG-tax may change as a result, but will (and even has to) return to its previous value in the new steady state, as a change in the demographic composition of the population is assumed to have no direct effect on political affairs (captured by the parameter $\gamma$) and on the gross return on savings (1 + r). When the capital tax is not related to the PAYG-tax, however, the steady state social-security tax cannot be determined uniquely. A continuum of steady states exists. Each steady state represents a level of intergenerational redistribution (determined by the social-security tax $\tau$) in combination with a unique level of intertemporal consumption smoothing (denoted by the level of private savings s). Which steady state will eventually come about is path dependent (we thus have an example of hysteresis at hand). However, given an initial steady state with some level of the PAYG-tax, the ensuing changes following a demographic shock can unambiguously be determined, as will become clear below. In order to make the two cases comparable, we start from the assumption that the social-security tax in the initial steady state is the same for both cases.

### Left-wing policy

In Figure 1, budget constraints of a representative individual are sketched. It is assumed that the line $AA^0$ represents the consumption possibilities of an individual for some given level of the PAYG-tax before a demographic shock.

[Insert Figure 1 about here.]

The political first-order condition (8) is in the figure represented by the straight line OP starting from the origin. As this condition has to hold, the allocation of consumption to the current generations $(c_t^y; c_t^o)$ has to be on this line. Point E denotes the initial steady-state equilibrium. Now, consider what will happen if at time t = 0 a once-and-for-all demographic shock occurs. As in this case the capital tax is not related to the PAYG-tax (i.e., $\theta^0 = 0$), the demographic shock and the ensuing shifts in the PAYG-tax only cause income effects. Keeping the social-security tax at its initial level, the lifetime budget constraint for the current young shifts inward to $BB^0$, where young individuals will select $E^0$ as their preferred consumption allocation, with young-age (old-age) consumption equal to $c_0^{y0}$ ($c_1^{o0}$). The elderly, however, at the time of the

shift are left with consumption possibilities in point Q, giving a level of consumption equal to $c_0^{o0}$. As the resulting allocation of consumption to the current generations at time 0 (i.e., $c_0^{y0}$ and $c_0^{o0}$) is not on the line OP, the government will alter the PAYG-tax ¿ along the budget restriction of the PAYG-system (which is represented in Figure 1 by the dotted line Q**b**) leading to an allocation **b**. So, an incumbent left-wing government increases the social-security tax and the young have to bear an extra part of the costs of aging, additional to a situation in which the PAYG-scheme would not have been expanded. Naturally, lifetime utility of the young will be below the utility level that would have been obtained in case of an unchanged PAYG-tax, i.e., the utility level corresponding to $E^0$. If no further shocks occur, **b** will be the new steady state: future politicians can only change the intergenerational redistribution along the budget constraint Q**b** so that they have no other option than to stick to **b**, with $(c_t^y)_{»^0=0} = \mathbf{b}^y$ and $(c_t^o)_{»^0=0} = \mathbf{b}^o$.

The upshot is that, in the absence of an explicit instrument to stimulate savings, left-wing politicians cannot aim for the higher old-age consumption levels $c_1^{o0}$ for the current young because that would imply that current young-age consumption has to be too large for being compatible with the political optimum.[15] As a result of the increased redistribution from the young to the elderly, savings will be una¤ected in the new steady state **b** compared to the steady state before the shock, E. That is, the economy is trapped at the pre-existing level of savings and there is no transition towards a more funded pension scheme. This will be shown more explicitly in the next subsection.

### Right-wing policy

Right-wing politicians face di¤erent constraints than left-wing politicians, as the level of the capital tax now is negatively related to the level of the PAYG-tax in place. Notice, …rstly, that the new steady state will have to be at $E^0$ as the steady-state condition $»(¿) = ¸ ¡ 1 ¡ r$ …xes the long-run social-security tax ¿. Secondly, a potential substitution e¤ect allows for di¤erent redistribution policies in the short run. As before, with the initial value of the PAYG-tax, ¿, after a demographic shock has occurred, the system is in point Q: Now the right-wing politicians will, just as their left-wing colleagues, react by adjusting the PAYG-tax such that $c_0^y$ and $c_0^o$ are on the line OP. However, the substitution e¤ect that is operative now makes it possible for the current young to realize a lifetime consumption allocation $(c_0^y; c_1^o)$ o¤ the line OP. As $»^0 < 0$; a current increase in the social-security tax along the line Q**b** (and thereby future increases in this tax) will entail a positive substitution e¤ect for the current young (due to the lower capital tax they are facing when old) stimulating old-age consumption versus young-age consumption. Consequently, the PAYG-tax does not have to increase all the way to **b** in order to satisfy the political …rst-order condition. As individual savings generate a higher return than 'public savings' through the PAYG-system, this shift provides the potential for utility gains

[15] The point is that given homotheticity of the consumption function the proportion of young-age versus. old-consumption will remain constant for an individual, see equation (4), as long as substitution e¤ects are absent.

for the young compared to the steady state Ê.

[Insert Figure 2 about here.]

In Figure 2, consumption for the current old is increased in point $Q^0$ compared to the consumption in point Q. With this increase in ¿; the lifetime budget constraint of the young rotates to $DD^0$; and their lifetime allocation of consumption $E^{00}$ is such that at time t = 0 the distribution of consumption among the young ($c_0^y$) and the elderly ($c_0^o$) will obey the political …rst-order condition, that is, this distribution is on the line OP. At $E^{00}$ the young experience a higher utility than at Ê, the status-quo lifetime allocation. The cost of this shift towards more old-age consumption is obviously a decrease in consumption for the current old, compared to Ê. The next generation of politicians will set the PAYG-tax (and thereby implicitly the capital tax) such that the consumption of the new-born young generation and the elderly obey the political …rst-order condition again.[16] In Figure 2, it has been assumed that the new steady state, $E^0$, immediately emerges at time t = 1. As a result, from period t = 2 onwards, the social-security tax and the capital tax will remain constant (and equal to their initial steady-state values). In general, however, the PAYG-tax will asymptotically converge to the steady state. During the transition process, the capital tax will remain lower than in the initial steady state, and the PAYG-tax will be higher, leading to a higher old-age consumption than generated by the allocation Ê for future generations. Eventually, the new steady state $E^0$ will be reached. As is clear from Figure 2, this new steady state corresponds to a higher lifetime utility relative to the level of lifetime utility under Ê.

We thus …nd that, by having an instrument to boost savings, a right-wing government can enact a partial transition to a more funded system. It can be expected, and will be shown in the next subsection, that during and after the transition savings will be higher than in the steady state before the demographic shock.

## 4.2 The comparative dynamics of aging

We now present a more formal derivation of the above results. The analysis is based on a linearized version of the model presented in the previous sections. The short-run e¤ects are traced by comparative dynamics.[17]

Let $n_t = n + ¼h_t$, where $h_t$ describes the time pattern of a perturbation of the steady-state value of the rate of population growth and ¼ is a measure for the magnitude. The model is linearized around the initial steady state by di¤erentiation of the equations (2)-(4), (6) and (8) with respect to ¼. This yields the following system describing the changes in the level of private savings ($s_t$) and in the social security tax ($¿_t$):

[16] More precisely, the capital tax at time t + 1, i.e., $»_{t+1}$, determines the savings decision at time t, while the choice of $»_t$, through the choice of $¿_t$, has within-generation redistributive e¤ects only.

[17] The method of comparative dynamics was …rst introduced by Judd (1982) for a continuous-time model. It can easily be transformed to discrete time, however. The method is explained in part B of the Appendix.

$$\begin{bmatrix} \frac{@s_t}{@¼} \\ \frac{@¿_{t+1}}{@¼} \end{bmatrix} = J \begin{bmatrix} \frac{@s_{t¡1}}{@¼} \\ \frac{@¿_t}{@¼} \end{bmatrix} + M \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}; \qquad t = 0;1;:::; \tag{9}$$

where J is the Jacobian matrix and M is a matrix describing the e¤ects of the current and next-period change in the rate of population growth.[18] This system can be used to describe the evolution of savings and the PAYG-tax in reaction to aging, given the predetermined level of savings of the old in period t = 0: It is assumed that aging occurs unexpectedly, i.e., $h_0 = h_1 = ¢¢¢ = h < 0$.[19]

For expositional reasons, we …rst analyze the right-wing policy, i.e., the policy with a negative relation between intragenerational and intergenerational redistribution ($»^0_t < 0$), so that a larger PAYG-tax is accompanied by a lower tax on capital. Then we derive what happens in case of a left-wing policy where no relation between both forms of redistribution exists, i.e., $»^0_t = 0$: For the algebra underlying the following graphical analysis, we refer to parts A and B of the Appendix.

## Right-wing policy

The e¤ects of an unexpected decrease in the rate of population growth when a larger PAYG-tax on labor goes along with a lower capital tax (i.e., $»^0 < 0$) are displayed in Figure 3. This …gure displays the phase diagram of the linear system (9) for $h_0 = h_1 = ¢¢¢ = h < 0$: The phaselines of this system $¢s = 0$ and $¢¿ = 0$ denote combinations of $\frac{@s_{t¡1}}{@¼}$ and $\frac{@¿_t}{@¼}$ for which $\frac{@s_t}{@¼} = \frac{@s_{t¡1}}{@¼}$ and $\frac{@¿_{t+1}}{@¼} = \frac{@¿_t}{@¼}$, respectively. The intersection of these lines gives the new steady state of the model. The derivation of these phaselines can be found in part B of the Appendix and yields the following equations:[20]

$$¢s = 0: \qquad \frac{@¿_t}{@¼} = ¡\frac{(2+½)(1+r)}{(1+r)+(1+n)(1+½)}\frac{@s_{t¡1}}{@¼} ¡ \frac{(1+½)}{(1+r)+(1+n)(1+½)}¿h; \tag{10.a}$$

$$¢¿ = 0: \qquad \frac{@¿_t}{@¼} = ¡\frac{(2+½)(1+r)}{(1+r)+(1+n)(1+½)¡Á}\frac{@s_{t¡1}}{@¼} ¡ \frac{(1+½)}{(1+r)+(1+n)(1+½)¡Á}¿h; \tag{10.b}$$

with $Á = \frac{»^0 c^y}{1+½} < 0$:

The phaselines before the shock (which can be found by inserting h = 0 into equations (10)) intersect in the origin. aging leads to an outward shift of the phaselines and, consequently, to a new steady state indicated by $E^0$, which corresponds to $E^0$ in Figures 1 and 2. As is immediately clear from the graph, taxes will not be a¤ected but savings are larger in $E^0$. If there were no change in the PAYG-tax after the demographic change, the system would get stuck at Q. However, as discussed earlier,

[18] Both matrices are described in further detail in the Appendix.

[19] This implies that $\frac{@s_{¡1}}{@¼} = 0$.

[20] As the initial steady-state level of the capital tax is only of importance for the political feasibility of the pension reform and does not qualitatively a¤ect the dynamics underlying the transition from PAYG to capital reserve, for expositional simplicity we assume that » = 0 in the initial steady state, that is ¸ = 1 + r. The analysis can easily be generalized to positive and negative initial values for », where » < 0 refers to an initial subsidy on savings.

Q is not a feasible point as the political …rst-order condition does not hold. So, a transition process will start, bringing the system to its new steady state, $E'$. The arrows indicate the dynamics in the points between the phaselines. As the model is saddlepoint stable, there is a unique stable trajectory (drawn as a dotted line) converging to the new steady state.

[Insert Figure 3 about here.]

Starting at the time of the demographic shock (t = 0), the following process develops. At t = 0; savings by the old are given and only the social-security tax can react. This initial e¤ect on the PAYG-tax can be found by applying the method developed by Blanchard and Kahn (1980) to system (9). This exercise is executed in part B of the Appendix. In Figure 3, the initial change in the social-security tax is indicated by point $Q'$ which is on the intersection of the stable manifold of the system and the $\frac{@¿}{@¼}$-axis. That is, the right-wing politicians initially react to the unexpected decrease in the population growth rate n by an enlargement of the PAYG-scheme to compensate the currently living old. The increase in the tax is not large enough, however, to make the consumption for the currently living old as high as the level of old-age consumption for subsequent generations, which would require a jump to E. As a result of the jump to $Q'$; savings increase, which is mainly due to the lower capital tax in the following period (along with the lower PAYG-bene…t) which accompanies the increase in the future PAYG-tax. In the following periods, the PAYG-tax and private savings evolve along the stable manifold until the new steady state is reached. Notice that the adjustment path implies that the initially enlarged PAYG-scheme is gradually and partly replaced by a capital reserve system, where old-age consumption is to a larger degree …nanced out of private savings. The long-run e¤ect on the level of savings follows from inserting $\frac{@¿}{@¼} = 0$ into equations (10), producing $\left[\frac{@s}{@¼}\right]_{»'<0} = ¡\frac{1+½}{(1+r)(2+½)}¿h > 0$. As the burden of aging is allocated to the periods of life according to equation (4), the long-run e¤ects on young-age and old-age consumption are given by $\frac{1+½}{1+r}\frac{@c^o}{@¼} = \frac{@c^y}{@¼} = ¡\left[\frac{@s}{@¼}\right]_{»'<0} < 0$:

## Left-wing policy

With a constant capital tax (not related to the level of the social-security tax), the economy appears to be trapped at its attained level of savings before aging, and the only possible consumption paths are those which do not alter the intergenerational link between currently living young and the elderly, or, alternatively, paths with a constant ratio $\frac{c_t^y}{c_t^o}$; t = 0; 1; : : :. The intuition for this result can be seen most clearly by once more inspecting Figure 1. In this case, the intergenerational and the intertemporal link between young-age and old-age consumption is …xed in time, and politicians have no desire to break the intergenerational link (as the political …rst-order condition tells them to maintain it) and no instrument to break the intertemporal link between old-age consumption and young-age consumption within the same generation.

[Insert Figure 4 about here.]

Figure 4 depicts the phase diagram for this case. This …gure is a squeezed version of the diagram in Figure 3. In Figure 3, the stable manifold and the two phaselines are all three distinguishable and only intersect in the (new) steady state. For the phaselines the di¤erence between the left-wing and the right-wing policy is given by the parameter Á in equation (10). With a …xed tax on capital, Á = 0 holds and the two phaselines ( ¢s = 0 and ¢¿ = 0, respectively) coincide. Consequently, there is an in…nite number of steady states. Moreover, no saddlepoint dynamics exist, as the system contains one unit root and one unstable root (given dynamic e¢ciency, see part B of the Appendix for details). Hence, any change of the PAYG-tax that does not immediately reach a new steady state will lead to implosive or explosive paths as indicated by the dotted arrows in Figure 4. In other words, at the time of the shock, the tax will have to jump immediately to its new steady-state value. As, at the time of the shock, the level of savings of the then living old is predetermined, the social-security tax has to jump to the phaseline ¢s = ¢¿ = 0 at the point where $\frac{@s_0}{@¼} = 0$ (see the solid curved arrow in Figure 4), which point is indicated by **b** in the …gure. So, the change in the level of private savings due to an unexpected decrease in the rate of population growth n is equal to zero in the short run as well as in the long run. The left-wing politicians in this system are thus caught by a savings trap: aging leads to a rise of the PAYG-tax with the long-run e¤ect being equal to the short-run e¤ect, and there is no way in which politicians can incite individuals to increase their savings. Therefore, a partial transition from PAYG to a more funded system is not feasible. The e¤ect on the level of the PAYG-tax follows from inserting $\frac{@s_0}{@¼} = \frac{@s_1}{@¼} = ¢¢¢ = \frac{@s}{@¼} = 0$ into equations (10), producing $\frac{@¿_0}{@¼} = \frac{@¿_1}{@¼} = ¢¢¢ = \left[\frac{@¿}{@¼}\right]_{»^0=0} = ¡\frac{(1+½)}{(1+r)+(1+n)(1+½)}¿h > 0$. Again, the burden of aging is spread between young-age and old-age consumption according to equation (4), i.e., $\frac{1+½}{1+r}\frac{@c^o}{@¼} = \frac{@c^y}{@¼} = ¡\left[\frac{@¿}{@¼}\right]_{»^0=0} < 0$: Note that in the long run this fall in both young-age and old-age consumption is larger than in case of a (partial) social security reform as $\left[\frac{@¿}{@¼}\right]_{»^0=0} > \left[\frac{@s}{@¼}\right]_{»^0<0}$ due to dynamic e¢ciency (r > n). Consequently, with a negative relation between intragenerational redistribution and intergenerational redistribution (i.e., $»^0 < 0$), steady-state lifetime utility is higher than without such a relation.

## 4.3 Political feasibility of transition

In the Introduction, we noted that in the US the debate about 'privatization' of the social-security system has been interpreted as meaning that workers should be given individual retirement accounts, and tax cuts should make it more attractive to try to generate returns from these assets. This is precisely what the policy with $»^0_t < 0$ is achieving: in this policy, part of the budget initially available for the pension bene…ts is channeled to the young (and, thus, away from the elderly), and at the same time their savings are encouraged by promising them a lower tax on their returns. The policy with $»^0_t = 0$ is basically a promise to evenly spread the burden of aging among the young and the elderly. Thus, the two analyzed policies (that is, with $»^0_t < 0$ and $»^0_t = 0$, respectively) can indeed be linked to real-world policies a right-wing (Republican) and a left-wing (Democratic) political party, respectively, would advocate. If no further shocks are to be expected after a demographic shock, an

elected left-wing government will leave the PAYG-system in place. Private savings will remain at its initial level as well. A right-wing government (i.e., one with $\theta_t^0 < 0$) will, if elected, partially replace the intergenerational transfer system by an intertemporal transfer system (private savings) which promises a higher rate of return. But which of the two parties will be elected? Is a transition to a more funded pension scheme as proposed by the right-wing government politically feasible? That is the question we address in this subsection.

In order to analyze this question, we have to compare the political support for both parties as measured by their target function (5), as the party with the largest political support will win the election. It will be clear from the previous subsections that a left-wing government will, in reaction to aging, initially increase the PAYG-tax more than a right-wing government. Consequently, the current elderly are better off under a left-wing government and thus will support the left-wing party more than the right-wing party. In contrast to a left-wing government, a right-wing government will initiate a pension reform by stimulating private savings, however. For a transition policy to be politically feasible, it is necessary (but not sufficient) that the increase in savings initiated by a right-wing government raises the utility of the young such that their political back up for the right-wing policy is larger than for the left-wing policy. The analysis in part C of the Appendix learns that this is only possible if the level of the capital tax in the initial steady state is strictly positive. The intuition for this result is obvious: the utility of the young can only be raised by increasing savings if the savings decision is distorted so that the initial level of savings is inefficiently low. A striking consequence of this result is that a transition to funded pensions is not possible if there is a subsidy on savings, as is frequently the case in the real world.

The condition that the capital tax is positive in the initial steady-state is not sufficient for a transition to be politically feasible, however. Numerical calculations show that there exist combinations of $0 < \theta < 1 + r$ and $\theta^0 < 0$ such that, at the time of the demographic shock, the right-wing reform plan outperforms the left-wing policy in the sense of having a higher value of the target function $W_0$: But in general this will not be the case. In particular, a transition from PAYG-pensions to a capital reserve system is only politically feasible if $\theta^0$ is small in absolute size. A small value of $\theta^0$ implies that the initial increase in the PAYG-tax is sufficiently large, relative to the increase implemented by a left-wing party, to curtail the loss in political support by the initial generation of elderly as much as possible. Moreover, a small value of $\theta^0$ implies that the speed of the transition process is rather modest. A slow speed of adjustment means, in turn, that the initial increase in the PAYG-tax is only very gradually annulled. As a consequence, the initial young generation is not confronted with a 'too low' benefit level when old relative to their own PAYG-contribution when young. This is necessary for their political support for the right-wing party to be sufficiently large to get/remain in office, even though the elderly favor a left-wing government.

Finally, we have to check whether the timepath of taxes promised by the right-wing party is time consistent. That is, we have to make sure that the right-wing party, if it wins the initial elections, will not be defeated in future elections. If this were the case, current savers would in the future be confronted with higher capital

taxes than promised by the right-wing party. Hence, they would not increase savings and the transition would fail. It can be shown, however (see part C of the Appendix), that this will not happen: if the utility gain for the young outweighs the utility loss for the old (in terms of political support) at the start of the transition process, it will also do so at any point in time during this process. Consequently, if the right-wing party manages to win the elections immediately after the demographic shock, it will, in the absence of further shocks, be able to remain in office.

# 5 Concluding remarks

In this paper we analyzed whether aging admits the (partial) transition from a PAYG-financed to a funded social-security system in a representative democracy. The motivation is that aging deteriorates the internal rate of return of PAYG-systems, making it worthwhile to consider the viability of such a transition. As emphasized by Feldstein (1996), the long-run gains of a transition are sizeable. Such a conversion, however, is also known to be non-Pareto improving: some generation must bear the cost. Obviously, redistributional issues are at the heart of the debate on social-security reform. In a direct democracy where the median voter is relatively old (as in Browning's [1975]-model), the old voters can block transition as such a transition is most likely not to their advantage. On the other hand, assuming that the government operates in a representative democracy, it will trade off the costs for the elderly against the gains for the young. The question we try to answer is whether in that situation the decrease in political support by the elderly can be dominated by the increased political back up by the young.

Our main conclusion, based on a positive general-equilibrium model with a given rate of interest, is that a (partial) transition from PAYG to a funded social-security system can be politically feasible if three conditions are met. Firstly, there should initially be a distortion in the individual savings decision that causes an inefficiently low level of savings (that is, an initially positive capital tax). Secondly, a substitution effect towards more old-age consumption has to be introduced at the start of the conversion policy. In particular, the PAYG-tax and the tax on savings should be negatively related, i.e., an increase in the PAYG-tax should be accompanied by a decline in the tax on capital. Issuing individual accounts coupled to a decrease in the capital tax, as advocated in the 2000-election program of George W. Bush, corresponds to such a conversion policy. Finally, the conversion policy should be very gradual. Otherwise, the young will be confronted with too meagre benefits when old and, therefore, their political support is not sufficiently large to get the right-wing party into office. This demonstrates that a fast transition as advocated by Feldstein (1996) is not likely to be politically feasible.

What our model also highlights is that a conversion policy demands specific rearrangements of existing redistribution policies. As individual pension accounts can only be implemented if part of the funds available for financing the pension benefits are channelled to the young, a conversion policy typically implies redistribution from the elderly to the young. In our model the capital tax is used as a financing device for social security, apart from the PAYG-tax. Although we have not explicitly modeled

intragenerational inequality, the capital tax might thus be interpreted as leading to intragenerational redistribution from rich (the wealth holders) to poor old individuals. One interpretation of our results, therefore, is that a conversion can be realized if the capital tax gets less progressive. Stated differently, perverse redistribution within generations from poor to rich (elderly) might be unavoidable to start a conversion policy. That is why a conversion policy demands the incumbent government to be of a right-wing signature.

In the absence of a conversion policy, however, the costs of aging are generally evenly spread among the young and the elderly, and no (regressive) redistributions within generations are required. Placed in this perspective, more efficiency (i.e., more funding elements in social security) goes along with less equity (i.e., a larger wealth inequality among the old). Therefore, not only the loss in support from the old should be small enough in a representative democracy, but within the older generation the interests of the poor old individuals should moreover count less. This stresses once again that the conversion issue is strongly mixed up with both intergenerational and intragenerational issues. If trading off intragenerational equity for intergenerational efficiency induces additional losses in political support (as we did not assume in our paper), social-security reform might be even harder to achieve than our results suggest.

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# Appendix

The Appendix consists of three parts. The …rst part describes the linearized version of the model and introduces the method of comparative dynamics. The second part shows the pension-reform options given di¤erent relationships between the PAYG-tax and the capital tax. In the last part, it is pointed out that a (partial) transition from a PAYG-scheme to a more funded pension in an aging economy is politically feasible whenever in the initial steady state savings are discouraged, that is, a strictly positive capital tax is in place.

## A Comparative dynamics

This part of the Appendix describes the method of comparative dynamics for the model presented in Section 2 and Section 3. Let $h_t$ describe the time pattern of the perturbation of the steady-state value of the population growth rate n and ¼ the magnitude of the shock. As mentioned in section 4.1 we can write $n_t = n + ¼h_t;\ t = 0; 1; : : :$. The e¤ects of a marginal decrease or increase in n can be traced by di¤erentiation of the …rst-order conditions (4) and (8) with respect to ¼ around the initial steady state, i.e.,

$$\frac{@c^o_{t+1}}{@¼} = \frac{1+r}{1+½}\frac{@c^y_t}{@¼} ¡ Á\frac{@¿_{t+1}}{@¼}; \tag{A.1}$$

$$\frac{@c^o_t}{@¼} = \frac{¸}{1+½}\frac{@c^y_t}{@¼}; \tag{A.2}$$

with $Á = \frac{»^0 c^y}{1+½}$ and

$$\begin{aligned}
\frac{@c^y_t}{@¼} &= ¡\frac{@¿_t}{@¼} ¡ \frac{@s_t}{@¼};\\
\frac{@c^o_t}{@¼} &= (1+r)\frac{@s_{t-1}}{@¼} + \frac{@´_t}{@¼};\\
\frac{@´_t}{@¼} &= (1+n)\frac{@¿_t}{@¼} + ¿h_t;
\end{aligned}$$

where $´_t$ is de…ned as $´_t ´ ¢_t ¡ »_t s_{t-1}$. Furthermore, the linearized version of the objective function of the political parties at time t, $W_t = ¸U(c^y_{t-1}; c^o_t) + (1+n_t)U(c^y_t; c^o_{t+1})$, is given by

$$\frac{@W_t}{@¼} = \frac{¸}{c^y}\frac{@c^y_{t-1}}{@¼} + \frac{¸}{1+½}\frac{1}{c^o}\frac{@c^o_t}{@¼} + \frac{1+n}{c^y}\frac{@c^y_t}{@¼} + \frac{1+n}{1+½}\frac{1}{c^o}\frac{@c^o_{t+1}}{@¼} + U(c^y; c^o)h_t: \tag{A.3}$$

Equations (A.1) and (A.2) can be condensed to the following linear system in the state variables s (savings per capita) and ¿ (the social security tax)

$$\begin{bmatrix} \frac{@s_t}{@¼} \\ \frac{@¿_{t+1}}{@¼} \end{bmatrix} = J\begin{bmatrix} \frac{@s_{t-1}}{@¼} \\ \frac{@¿_t}{@¼} \end{bmatrix} + M\begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}; \qquad t = 0; 1; : : : \tag{A.4}$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} -\frac{(1+r)(1+½)}{¸} & -\frac{(1+n)(1+½)+¸}{¸} \\ \frac{(1+r)(¸+(1+r)(1+½))}{(1+n+Á)¸} & \frac{¸(2+r+n)+(1+n)(1+r)(1+½)}{(1+n+Á)¸} \end{bmatrix}; \tag{A.5}$$

and M is

$$M = \begin{bmatrix} -\frac{1+½}{¸} & 0 \\ \frac{¸+(1+r)(1+½)}{(1+n+Á)¸}¿ & -\frac{1}{1+n+Á}¿ \end{bmatrix}: \tag{A.6}$$

As already noted in the main text, the initial steady-state level of the capital tax is only of importance for the political feasibility of the pension reform and does not qualitatively a¤ect the dynamics underlying the transition from PAYG to capital reserve. Therefore, we may, for expositional simplicity, assume that » = 0 in the initial steady state, that is ¸ = 1 + r. The subsequent analysis can easily be generalized to positive and negative initial values for ». Under the assumption that ¸ = 1 + r, the matrices J and M, given by (A.5) and (A.6), respectively, boil down to

$$J = \begin{bmatrix} -(1+½) & -\frac{(1+n)(1+½)+(1+r)}{1+r} \\ \frac{(1+r)(2+½)}{1+n+Á} & \frac{1+r+(1+n)(2+½)}{1+n+Á} \end{bmatrix}; \tag{A.7}$$

and

$$M = \begin{bmatrix} -\frac{1+½}{1+r} & 0 \\ \frac{2+½}{1+n+Á}¿ & -\frac{1}{1+n+Á}¿ \end{bmatrix}: \tag{A.8}$$

This system comprises one predetermined or backward-looking variable (s) and one jump or forward-looking variable (¿). The eigenvalues of the Jacobian matrix J are given by,

$$\begin{aligned} "_1 &= \frac{2+r+n-(1+½)Á-\sqrt{©}}{2(1+n+Á)}; \\ "_2 &= "_1 + \frac{\sqrt{©}}{1+n+Á}; \end{aligned} \tag{A.9}$$

with

$$© = (2+r+n-(1+½)Á)^2 - 4(1+r)(1+n+Á):$$

## B The di¤erent pension-reform options

This part of the Appendix describes the dynamics of the system (A.4), with the matrices J and M given by (A.7) and (A.8), respectively, for di¤erent relationships between the PAYG-tax and the capital tax, i.e., di¤erent values of »$^0$:

For $\lambda^0 < 0$, it holds for the roots of the system (A.4) that $\varepsilon_1 \in (0;1)$ and $\varepsilon_2 > 1$. So the system is saddlepoint stable and can be solved to find the initial effect of changes in n on the social security tax (see Blanchard and Kahn, 1980):

$$\frac{\partial \tau_0}{\partial n} = \frac{\varepsilon_1 - \gamma_{11}}{\gamma_{12}} \frac{\partial s_{t-1}}{\partial n} + m_{11} \sum_{i=0} \varepsilon_2^{-i-1} h_i + m_{12} \sum_{i=0} \varepsilon_2^{-i-1} h_{i+1} \qquad \text{(A.10)}$$
$$- m_{21} \sum_{i=0} \varepsilon_2^{-i-1} h_i - m_{22} \sum_{i=0} \varepsilon_2^{-i-1} h_{i+1};$$

where $\gamma_{ij}$ and $m_{ij}$ $(i;j = 1;2)$ denote elements of $\Gamma$ and $M$. To distinguish between current and future effects, the terms of (A.10) can be rearranged as follows

$$\frac{\partial \tau_0}{\partial n} = \frac{\varepsilon_1 - \gamma_{11}}{\gamma_{12}} \frac{\partial s_{t-1}}{\partial n} + \left[ \frac{\varepsilon_1 - \gamma_{11}}{\gamma_{12}} m_{11} - m_{21} \right] \frac{h_0}{\varepsilon_2} \qquad \text{(A.11)}$$
$$+ \left[ \frac{\varepsilon_1 - \gamma_{11}}{\gamma_{12}} \left( \frac{m_{11}}{\varepsilon_2} + m_{12} \right) - \frac{m_{21}}{\varepsilon_2} - m_{22} \right] \sum_{j=1} \varepsilon_2^{-j} h_j :$$

Using the facts that $\frac{\partial s_{t-1}}{\partial n} = m_{12} = 0$, $\sum_{j=1}^{1} \varepsilon_2^{-j} = \frac{1}{\varepsilon_2 - 1}$; and, assuming that the once-and-for-all decrease in n is unexpected, i.e., $h_0 = h_1 = \ldots = h < 0$, (A.11) boils down to

$$\frac{\partial \tau_0}{\partial n} = \frac{1}{1 - \varepsilon_2} \left[ m_{21} + m_{22} - \frac{\varepsilon_1 - \gamma_{11}}{\gamma_{12}} m_{11} \right] h: \qquad \text{(A.12)}$$

The evolution of both s and $\tau$ can then be derived from the system equations (A.4). Given the evolution of these state variables, we can calculate the time paths of the variables such as young-age and old-age consumption, lifetime utility, and the net PAYG-benefit.

The phaselines of the dynamical system (A.4), that is, the sets of points where it holds that $\Delta s = \frac{\partial s_t}{\partial n} - \frac{\partial s_{t-1}}{\partial n} = 0$ and $\Delta \tau = \frac{\partial \tau_{t+1}}{\partial n} - \frac{\partial \tau_t}{\partial n} = 0$, $\forall t$, respectively can be derived as follows,

$$(I - \Gamma) \begin{bmatrix} \frac{\partial s}{\partial n} \\ \frac{\partial \tau}{\partial n} \end{bmatrix} = M \begin{bmatrix} h \\ h \end{bmatrix}; \quad \text{with} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}:$$

More explicitly, these lines, which are drawn in Figure 3, are given by

$$\Delta s = 0: \qquad \frac{\partial \tau_t}{\partial n} = -\frac{(2+\nu)(1+r)}{(1+r)+(1+n)(1+\nu)} \frac{\partial s_{t-1}}{\partial n} - \frac{(1+\nu)}{(1+r)+(1+n)(1+\nu)} \tau h; \qquad \text{(A.13)}$$

$$\Delta \tau = 0: \qquad \frac{\partial \tau_t}{\partial n} = -\frac{(2+\nu)(1+r)}{(1+r)+(1+n)(1+\nu)-\lambda} \frac{\partial s_{t-1}}{\partial n} - \frac{(1+\nu)}{(1+r)+(1+n)(1+\nu)-\lambda} \tau h: \qquad \text{(A.14)}$$

From the dynamics, it immediately follows that the social-security tax initially jumps to a value below the $\Delta \tau = 0$-phaseline (A.14) (see Figure 3),

$$0 < \frac{\partial \tau_0}{\partial n} < -\frac{(1+\nu)}{(1+r)+(1+n)(1+\nu)-\lambda} \tau h: \qquad \text{(A.15)}$$

The new steady state is given by the intersection of both phaselines, i.e., $\frac{@s}{@¼} = ¡\frac{(1+½)}{(2+½)(1+r)}¿h > 0$ and $\frac{@¿}{@¼} = 0$.

Without a relation between the capital tax and the PAYG-tax ($»^0 = Á = 0$); the Jacobian matrix J and the matrix M are given by

$$J = \begin{bmatrix} 1 & ¡(1+½) \\ \frac{(1+r)(2+½)}{1+n} & \frac{1+r+(1+n)(2+½)}{1+n} \end{bmatrix}; \qquad (A.16)$$

and M is

$$M = \begin{bmatrix} 1 & ¡\frac{1+½}{1+r}¿ \\ \frac{2+½}{1+n}¿ & ¡\frac{1}{1+n}¿ \end{bmatrix}: \qquad (A.17)$$

System (A.4) still comprises one predetermined or backward-looking variable (s) and one jump or forward-looking variable (¿). The eigenvalues of the Jacobian matrix J are now

$$"_1 = 1; \qquad "_2 = \frac{1+r}{1+n} > 1;$$

where the last inequality follows from the assumption that the economy is dynamically e¢cient. Because of the unit root, there are no saddlepoint dynamics and the new steady state immediately occurs. The economy …nds itself in a savings trap. As can be seen by inserting $Á = 0$ into (A14), the phaselines (A.13) and (A14) now coincide (see Figure 4). The initial increase in the PAYG-tax is given by these phaselines with $\frac{@s_{¡1}}{@¼} = 0$, i.e.,

$$\frac{@¿_0}{@¼} = ¡\frac{(1+½)}{(1+r)+(1+n)(1+½)}¿h. \qquad (A.18)$$

Notice that (A.18) can also be calculated through (A.12). Comparing (A.18) with (A.15) shows that the initial increase of the PAYG-tax in case of no relation between » and ¿ ($»^0 = 0$) is larger than the initial increase in case of a negative relation between both ($»^0 < 0$). Because of the unit root of the Jacobian, the steady-state increase in the PAYG-tax has to be equal to its initial increase and savings remain at their initial level, i.e., $\frac{@¿}{@¼} = ¡\frac{(1+½)}{(1+r)+(1+n)(1+½)}¿h$ and $\frac{@s}{@¼} = 0$.

## C The political feasibility and time consistency of a pension reform

In this part of the Appendix, we …rst show that a (partial) transition from a PAYG-scheme to a more funded pension in an aging economy can be politically feasible for a right-wing party (i.e., $»^0 < 0$) if the initial steady state is characterized by a strictly positive capital tax, that is, if $» > 0$. A (partial) transition is politically feasible if, at any instant, the value of the political target function given an extension of the existing PAYG-scheme (executed by a left-wing incumbent party with $»^0 = 0$) is smaller than

the value of the target function given a social security reform implemented by a right-wing incumbent party. The initial value of the political target function (5) is given by

$$W_0 = \gamma U(c^y_{-1}; c^o_0) + (1 + n_0)U(c^y_0; c^o_1): \tag{A.19}$$

If one, without loss of generality, abstracts from discounting ($\rho = 0$), the initial marginal change in this target function is given by

$$\begin{aligned}\frac{\partial W_0}{\partial \tau} &= -\frac{1}{c^o}\frac{\partial c^o_0}{\partial \tau} + (1+n)\frac{1}{c^y}\frac{\partial c^y_0}{\partial \tau} + (1+n)\frac{1}{c^o}\frac{\partial c^o_1}{\partial \tau} + U(c^y; c^o)h_0 \\ &= -\frac{1}{\gamma c^y}\frac{\partial c^o_0}{\partial \tau} + (1+n)\frac{1}{c^y}\frac{\partial c^y_0}{\partial \tau} + (1+n)\frac{1}{\gamma c^y}\frac{\partial c^o_1}{\partial \tau} + U(c^y; c^o)h_0;\end{aligned} \tag{A.20}$$

where the second equality follows from the fact that in the steady state it holds that $c^o = \gamma c^y$. Using the fact that $\frac{\partial s_{-1}}{\partial \tau} = 0$ and assuming that the once-and-for-all decrease in $n$ is unexpected i.e. $h_0 = h_1 = \cdots = h < 0$, (A.20) boils down to

$$c^y\frac{\partial W_0}{\partial \tau} = \frac{1+n}{\gamma}\left(\gamma + (1+n)\frac{\partial \zeta_1}{\partial \tau} + \theta\frac{\partial s_0}{\partial \tau}\right) + \phi h; \tag{A.21}$$

with $\phi = \frac{\zeta(\gamma+1+n)+\gamma U(c^y;c^o)}{\gamma}$.

If the capital tax is not related to the size of the PAYG-scheme ($\theta' = 0$), we have that $\left.\frac{\partial s_0}{\partial \tau}\right|_{\theta'=0} = 0$, and $\left.\frac{\partial \zeta_1}{\partial \tau}\right|_{\theta'=0} = \left.\frac{\partial \zeta_0}{\partial \tau}\right|_{\theta'=0}$. That is, the steady-state increase in the tax is equal to its initial increase under the pension policy implemented by a left-wing government. If the capital tax is negatively related to the size of the PAYG-scheme ($\theta' < 0$), we know that $\left.\frac{\partial s_0}{\partial \tau}\right|_{\theta'<0} > 0$. Moreover, from comparing (A.18) with (A.15) it follows that $\left.\frac{\partial \zeta_0}{\partial \tau}\right|_{\theta'<0} > \left.\frac{\partial \zeta_1}{\partial \tau}\right|_{\theta'<0}$. From (A.21) we then get that $\theta > 0$ is a necessary condition for $\left.\frac{\partial W_0}{\partial \tau}\right|_{\theta'<0} > \left.\frac{\partial W_0}{\partial \tau}\right|_{\theta'=0}$. So, only if initially a strictly positive capital tax is in place, the social-security reform proposed by the right-wing party may be politically feasible. Numerical calculations show, however, that $\theta > 0$ is not sufficient for the right-wing party to carry the election. Apart from that, the parameter $\theta'$ has to be sufficiently close to zero, making the policy look like a left-wing policy apart from a small decrease in both the PAYG-tax and the capital tax, compared to the taxes adopted by the left-wing party.

Finally, we show that a right-wing reform policy is time consistent. That is, if the right-wing party is able to win the election immediately after a demographic shock (i.e., $\left.\frac{\partial W_0}{\partial \tau}\right|_{\theta'<0} - \left.\frac{\partial W_0}{\partial \tau}\right|_{\theta'=0} > 0$) and starts a transition, it will remain in office (i.e., $\left.\frac{\partial W_t}{\partial \tau}\right|_{\theta'<0} - \left.\frac{\partial W_t}{\partial \tau}\right|_{\theta'=0} \geq 0; t = 1; 2; \ldots$). Firstly, consider the dynamics under a right-wing reform policy ($\theta' < 0$): Let $\left.\frac{\partial s}{\partial \tau}\right|$ denote the steady-state change in the level of private savings, and recall that $\varepsilon_1$, with $0 < \varepsilon_1 < 1$, denotes the stable root of the dynamical system (A.4), then it is convenient to write the dynamical system in this case as follows:

$$\begin{bmatrix} \left.\frac{\partial s_{t-1}}{\partial \tau}\right|_{\theta'<0} \\ \left.\frac{\partial \zeta_t}{\partial \tau}\right|_{\theta'<0} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial s}{\partial \tau}\right|_{\theta'<0} & 0 - \left.\frac{\partial s}{\partial \tau}\right|_{\theta'<0} \\ 0 & \left.\frac{\partial \zeta_0}{\partial \tau}\right|_{\theta'<0} \end{bmatrix} + \begin{bmatrix} 0 \\ \left.\frac{\partial \zeta_0}{\partial \tau}\right|_{\theta'<0} \end{bmatrix}\varepsilon_1^t: \tag{A.22}$$

Rearranging the rows of (A.22) gives

$$\left(\frac{\partial \epsilon_t}{\partial \mu}\right)_{\theta^0<0} = \varepsilon_1^t \left(\frac{\partial \epsilon_0}{\partial \mu}\right)_{\theta^0<0}; \tag{A.23}$$

$$\left(\frac{\partial s_{t-1}}{\partial \mu}\right)_{\theta^0<0} = (1-\varepsilon_1^t)\left(\frac{\partial s}{\partial \mu}\right)_{\theta^0<0}. \tag{A.24}$$

Furthermore, equivalent to (A.13), the $\dot{s}=0$-phaseline (which does not depend on [the sign of] $\theta^0$) can be written as $\frac{\partial \epsilon_t}{\partial \mu} = -\lambda_1 \frac{\partial s_{t-1}}{\partial \mu} - \lambda_2 \epsilon(h)$, with $\lambda_1, \lambda_2 > 0$. As, in case of $\theta^0 < 0$, the new steady state lies on the intersection of the $\dot{s}=0$-phaseline and the horizontal axis (see Figure 3), it immediately follows that $\left(\frac{\partial s}{\partial \mu}\right)_{\theta^0<0} = -\left(\frac{\lambda_2}{\lambda_1}\epsilon(h)\right) > 0$. Consequently, we can write (A.24) as

$$\left(\frac{\partial s_{t-1}}{\partial \mu}\right)_{\theta^0<0} = -(1-\varepsilon_1^t)\frac{\lambda_2}{\lambda_1}\epsilon(h). \tag{A.25}$$

Now, assume through an arbitrary period $t-1$, a continuous sequence of right-wing governments has been in office, starting form period 0. Then, if a left-wing party (with $\theta^0 = 0$) would be voted into office at time $t$, it would implement an increase in the social-security tax which would be on the $\dot{s}=0$-phaseline given the level of saving in the previous period (i.e., $\left(\frac{\partial s_{t-1}}{\partial \mu}\right)_{\theta^0<0}$) (recall the analysis of a left-wing policy in the main text), that is,

$$\left(\frac{\partial \epsilon_t}{\partial \mu}\right)_{\theta^0=0} = -\lambda_1 \left(\frac{\partial s_{t-1}}{\partial \mu}\right)_{\theta^0<0} - \lambda_2 \epsilon(h). \tag{A.26}$$

Substituting (A.25) into (A.26) then gives

$$\left(\frac{\partial \epsilon_t}{\partial \mu}\right)_{\theta^0=0} = \lambda_1(1-\varepsilon_1^t)\frac{\lambda_2}{\lambda_1}\epsilon(h) - \lambda_2\epsilon(h) = -\varepsilon_1^t\lambda_2\epsilon(h) = \varepsilon_1^t \left(\frac{\partial \epsilon_0}{\partial \mu}\right)_{\theta^0=0}; \tag{A.27}$$

where the last equality follows from the fact that at time $t = 0$, a left-wing party proposes an increase in the PAYG-tax such that $\left(\frac{\partial s_0}{\partial \mu}\right)_{\theta^0=0}$ (that is, an increase to the point where the $\dot{s}=0$-phaseline cuts the vertical axis [see Figure 4]).

With respect to the initial change in the target function of the political parties, it follows from (A.21) that $\left(\frac{\partial W_0}{\partial \mu}\right)_{\theta^0<0} = W\left(\left(\frac{\partial \epsilon_1}{\partial \mu}\right)_{\theta^0<0}, \left(\frac{\partial s_0}{\partial \mu}\right)_{\theta^0<0}\right)$ and $\left(\frac{\partial W_0}{\partial \mu}\right)_{\theta^0=0} = W\left(\left(\frac{\partial \epsilon_1}{\partial \mu}\right)_{\theta^0=0}, \left(\frac{\partial s_0}{\partial \mu}\right)_{\theta^0=0}\right)$; with $W(\cdot)$ a linear function. For $\theta^0 < 0$, it is straightforward to write $\left(\frac{\partial \epsilon_1}{\partial \mu}\right)_{\theta^0<0}$ and $\left(\frac{\partial s_0}{\partial \mu}\right)_{\theta^0<0}$ as functions of $\left(\frac{\partial \epsilon_1}{\partial \mu}\right)_{\theta^0<0}$ by applying the accompanying dynamical system. For $\theta^0 = 0$; the same applies, as it holds that $\left(\frac{\partial \epsilon_1}{\partial \mu}\right)_{\theta^0=0} = \left(\frac{\partial \epsilon_0}{\partial \mu}\right)_{\theta^0=0}$. Ergo, at time $t = 0$, we have, in case of political feasibility of a right-wing reform policy, that

$$W\left(\frac{\partial \epsilon_0}{\partial \mu}\right)_{\theta^0<0} - W\left(\frac{\partial \epsilon_0}{\partial \mu}\right)_{\theta^0=0} > 0. \tag{A.28}$$

So given that (A.28) is true, it holds in the same manner that, for $t = 1; 2; : : : ;$

$$
\begin{aligned}
&\left(\frac{\partial W}{\partial \mu}\right)_{\theta<0} \geq \left(\frac{\partial W}{\partial \mu}\right)_{\theta=0} \\
&= \left(\frac{\partial W}{\partial \mu}\right)_{\theta<0} \int_1^t \varepsilon \, d\tau - \left(\frac{\partial W}{\partial \mu}\right)_{\theta=0} \qquad \text{(A.29)} \\
&= \int_1^t \left[\left(\frac{\partial W}{\partial \mu}\right)_{\theta<0} - \left(\frac{\partial W}{\partial \mu}\right)_{\theta=0}\right] d\tau \geq 0;
\end{aligned}
$$

where the first equality follows from equations (A23) and (A27). The second equality follows from the fact that $W(\cdot)$ is a linear function, and the last (in)equality from the fact that $\int_1^t \varepsilon > 0$ for any finite time t and $\lim_{t \to \infty} \int_1^t \varepsilon = 0$: So, (A.29) states that, at any instant during transition phase, the value of the political target function given a continuation of the right-wing reform policy, will be larger than the value of the same function in case of a reversal to a left-wing policy. Consequently, an initially politically feasible right-wing policy will not be abolished in the future and the partial transition is time consistent.

Figure 1

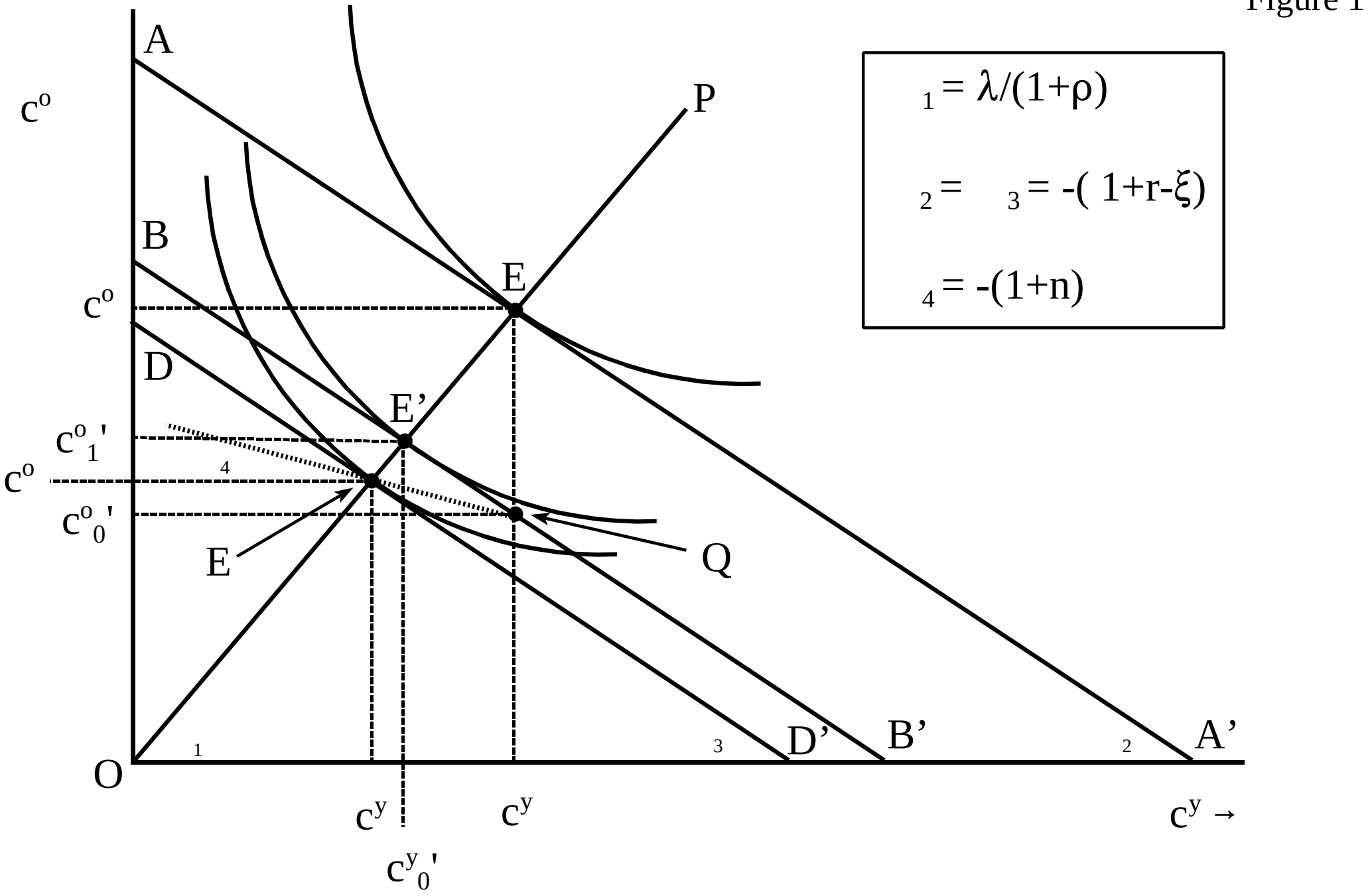
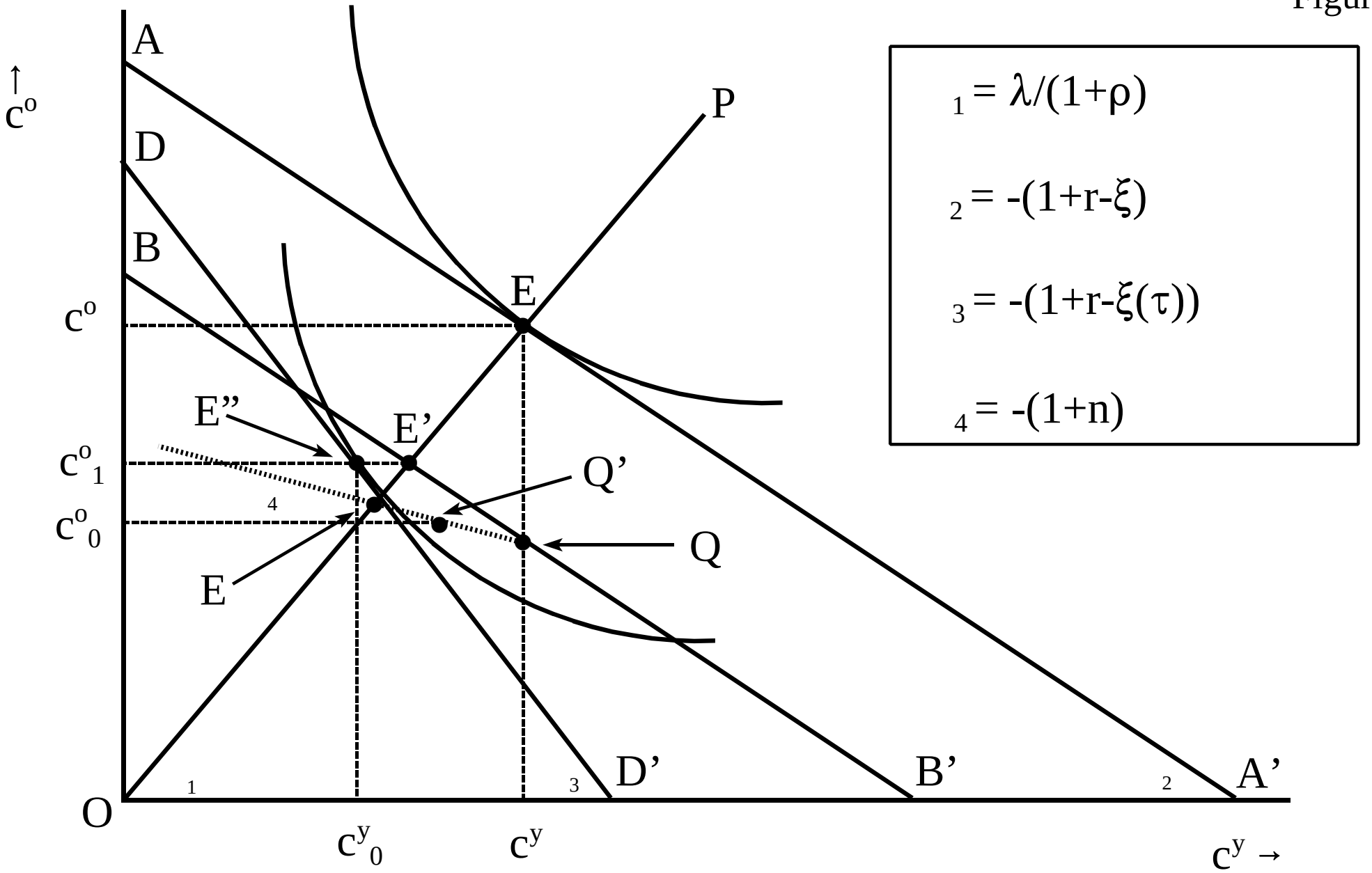

A
$c^o$
P
B
E
$c^o$
D
E'
$c^o_1$'
$c^o$
4
$c^o_0$'
E
Q
D'
B'
A'
1
3
2
O
$c^y$
$c^y$
$c^y$ →
$c^y_0$'
$_1 = \lambda/(1+\rho)$
$_2 =$ $_3 = -(1+r-\xi)$
$_4 = -(1+n)$

Figure 2




$_1 = \lambda/(1+\rho)$

$_2 = -(1+r-\xi)$

$_3 = -(1+r-\xi(\tau))$

$_4 = -(1+n)$

Figure 3

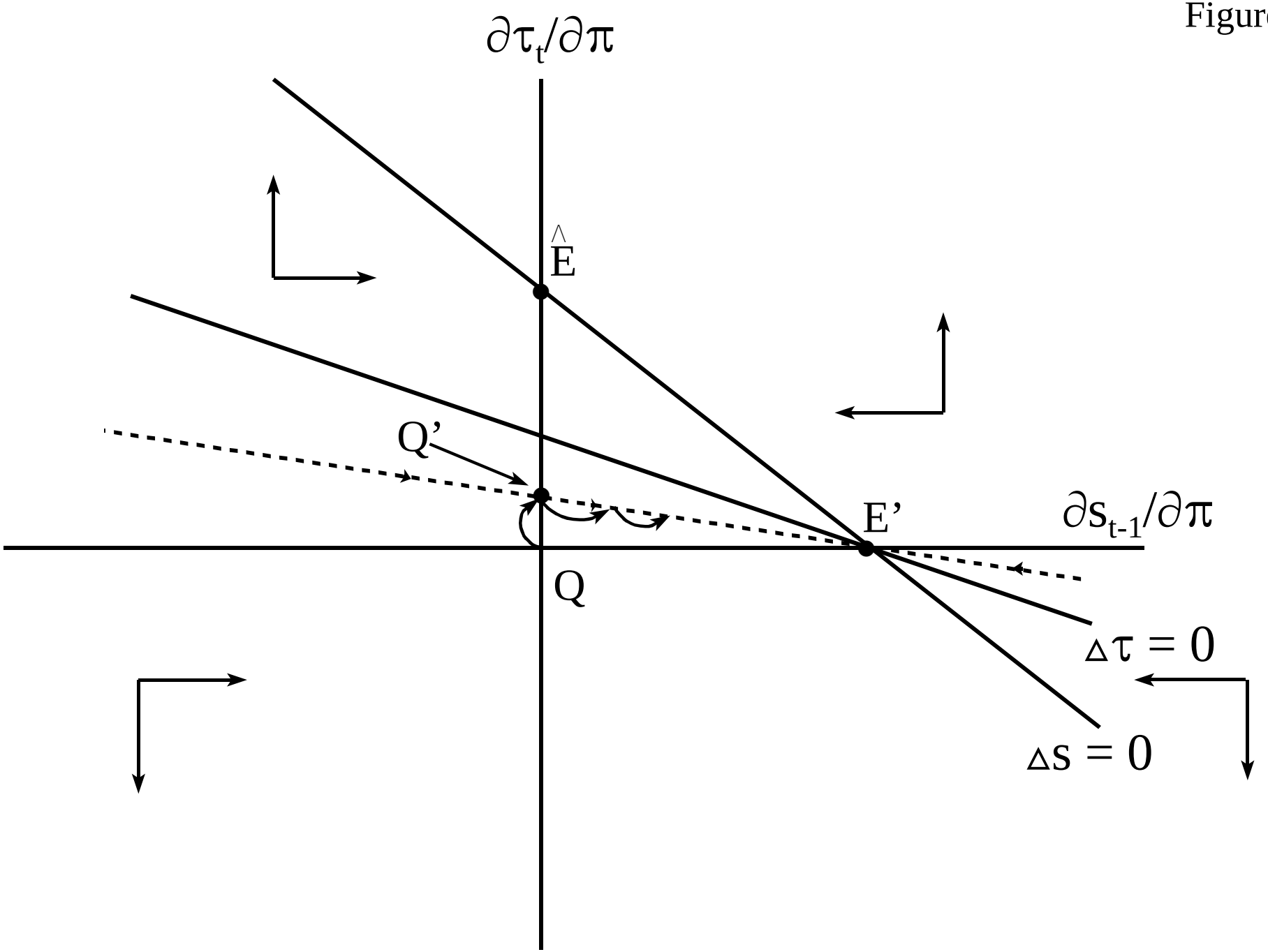
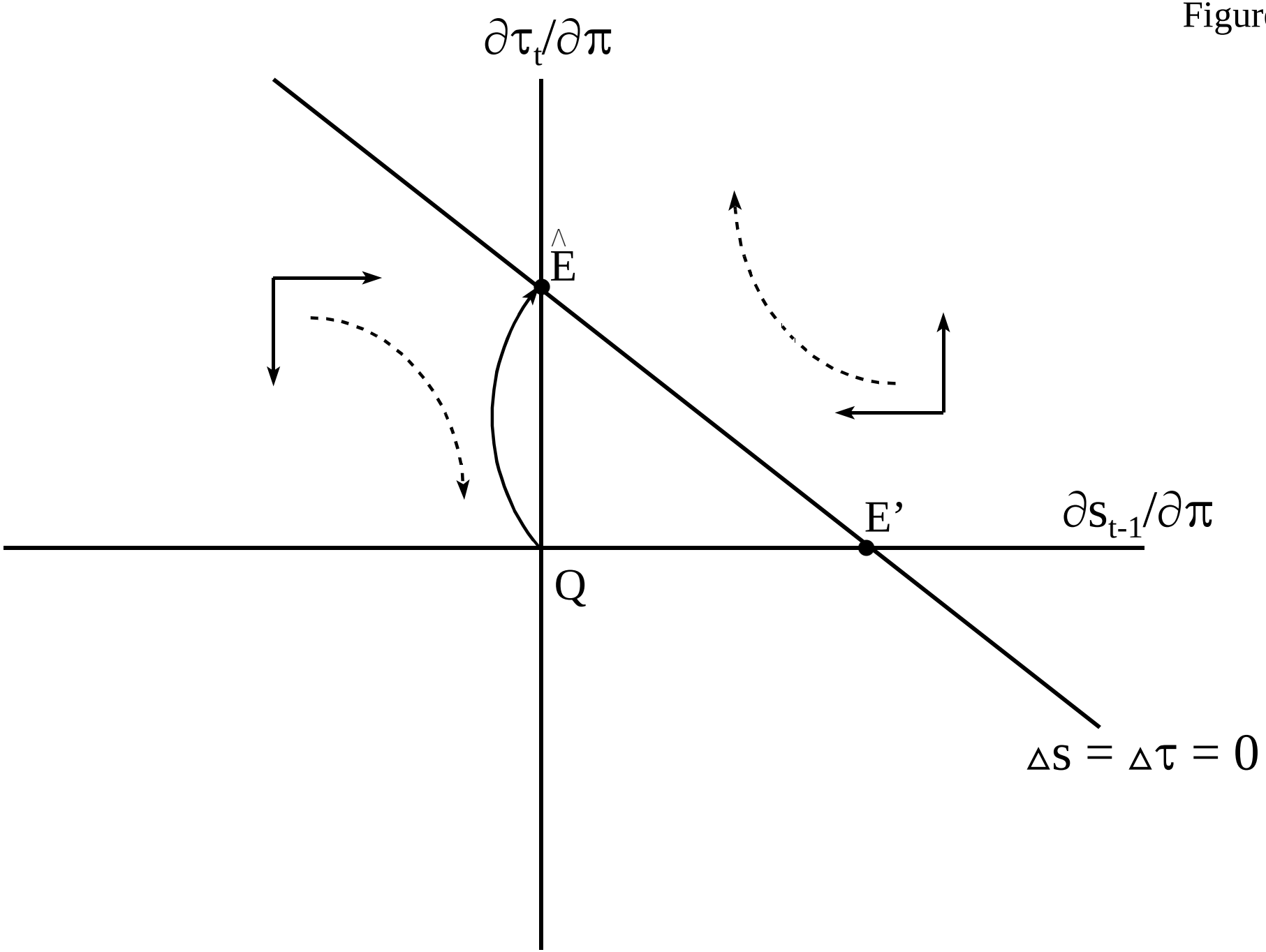

$\partial\tau_t/\partial\pi$
$\hat{E}$
Q'
E'
$\partial s_{t-1}/\partial\pi$
Q
$\Delta\tau = 0$
$\Delta s = 0$

Figure 4


$\partial\tau_t/\partial\pi$
$\hat{E}$
E'
$\partial s_{t-1}/\partial\pi$
Q
$\Delta s = \Delta\tau = 0$