Some properties of a generalized two-error components matrix (problem 01.5.1)

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Published in:
Econometric Theory

Publication date:
2001

Econometric Theory, **17**, 2001, 1025–1031. Printed in the United States of America.

PROBLEMS AND SOLUTIONS

PROBLEMS

01.5.1. *Some Properties of a Generalized Two-Error Components Matrix*, proposed by Franc J.G.M. Klaassen and Jan R. Magnus. The standard two-error components model has a variance matrix where all variances (diagonal elements) are equal to each other and all covariances (off-diagonal elements) are equal. The most convenient way to write the matrix is as $\alpha J + \beta(I_T - J)$, where $J = (1/T)\iota\iota'$, ι denotes a $T \times 1$ vector of ones, and I_T denotes the identity matrix of order T. Consider a generalization of this matrix, namely the $T \times T$ matrix

$$\Omega = \begin{pmatrix} \alpha_1 J_1 + \beta_1(I_{T_1} - J_1) & \gamma\iota_1\iota_2' \\ \gamma\iota_2\iota_1' & \alpha_2 J_2 + \beta_2(I_{T_2} - J_2) \end{pmatrix},$$

where ι_1 and ι_2 denote vectors of ones of dimension T_1 and T_2, respectively, $J_1 = (1/T_1)\iota_1\iota_1'$, $J_2 = (1/T_2)\iota_2\iota_2'$, and $T = T_1 + T_2$. Let $\Delta = \alpha_1\alpha_2 - \gamma^2 T_1 T_2$. Show that:

(a) The determinant of Ω is given by $|\Omega| = \beta_1^{T_1-1}\beta_2^{T_2-1}\Delta$.
(b) Ω is positive definite if and only if α_1, α_2, β_1, β_2, and Δ are all positive.
(c) Assume that Ω is positive definite. Then Ω^p has the same structure as Ω for any real p. In particular,

$$\Omega^p = \begin{pmatrix} \phi_1 J_1 + \beta_1^p(I_{T_1} - J_1) & \delta\iota_1\iota_2' \\ \delta\iota_2\iota_1' & \phi_2 J_2 + \beta_2^p(I_{T_2} - J_2) \end{pmatrix}.$$

(d) Obtain ϕ_1, ϕ_2, and δ for the special cases $p = -1$ and $p = -\frac{1}{2}$.
(e) Consider the transformation $y = \Omega^p x$. Let $x' = (x_1', x_2')$ and $y' = (y_1', y_2')$, where x_1 and y_1 are of dimension T_1 and x_2 and y_2 are of dimension T_2. Then,

$$y_{1t} = y_1^* + \beta_1^p x_{1t} (t = 1,\ldots,T_1), \qquad y_1^* = (\phi_1 - \beta_1^p)\bar{x}_1 + \delta T_2 \bar{x}_2,$$

and

$$y_{2t} = y_2^* + \beta_2^p x_{2t} (t = 1,\ldots,T_2), \qquad y_2^* = (\phi_2 - \beta_2^p)\bar{x}_2 + \delta T_1 \bar{x}_1,$$

where $\bar{x}_1 = (1/T_1)\iota_1' x_1$ and $\bar{x}_2 = (1/T_2)\iota_2' x_2$. This property (for $p = -\frac{1}{2}$) is especially useful in obtaining (feasible) GLS estimates, as in Klaassen and Magnus (2001).

REFERENCES

Klaassen, F.J.G.M. & J.R. Magnus (2001) Are points in tennis independent and identically distributed? Evidence from a dynamic binary panel data model. *Journal of the American Statistical Association* 96, forthcoming.

 0266-4666/01 $9.50