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Some properties of a generalized two-error components matrix (problem 01.5.1)

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PROBLEMS AND SOLUTIONS

PROBLEMS

01.5.1. *Some Properties of a Generalized Two-Error Components Matrix*, proposed by Franc J.G.M. Klaassen and Jan R. Magnus. The standard two-error components model has a variance matrix where all variances (diagonal elements) are equal to each other and all covariances (off-diagonal elements) are equal. The most convenient way to write the matrix is as $\alpha J + \beta(I_T - J)$, where $J = (1/T)u'u'$, u denotes a $T \times 1$ vector of ones, and I_T denotes the identity matrix of order T . Consider a generalization of this matrix, namely the $T \times T$ matrix

$$\Omega = \begin{pmatrix} \alpha_1 J_1 + \beta_1(I_{T_1} - J_1) & \gamma t_1 t_2' \\ \gamma t_2 t_1' & \alpha_2 J_2 + \beta_2(I_{T_2} - J_2) \end{pmatrix},$$

where t_1 and t_2 denote vectors of ones of dimension T_1 and T_2 , respectively, $J_1 = (1/T_1)t_1 t_1'$, $J_2 = (1/T_2)t_2 t_2'$, and $T = T_1 + T_2$. Let $\Delta = \alpha_1 \alpha_2 - \gamma^2 T_1 T_2$. Show that:

- (a) The determinant of Ω is given by $|\Omega| = \beta_1^{T_1-1} \beta_2^{T_2-1} \Delta$.
- (b) Ω is positive definite if and only if $\alpha_1, \alpha_2, \beta_1, \beta_2$, and Δ are all positive.
- (c) Assume that Ω is positive definite. Then Ω^p has the same structure as Ω for any real p . In particular,

$$\Omega^p = \begin{pmatrix} \phi_1 J_1 + \beta_1^p(I_{T_1} - J_1) & \delta t_1 t_2' \\ \delta t_2 t_1' & \phi_2 J_2 + \beta_2^p(I_{T_2} - J_2) \end{pmatrix}.$$

- (d) Obtain ϕ_1, ϕ_2 , and δ for the special cases $p = -1$ and $p = -\frac{1}{2}$.
- (e) Consider the transformation $y = \Omega^p x$. Let $x' = (x_1', x_2')$ and $y' = (y_1', y_2')$, where x_1 and y_1 are of dimension T_1 and x_2 and y_2 are of dimension T_2 . Then,

$$y_{1t} = y_1^* + \beta_1^p x_{1t} \quad (t = 1, \dots, T_1), \quad y_1^* = (\phi_1 - \beta_1^p) \bar{x}_1 + \delta T_2 \bar{x}_2,$$

and

$$y_{2t} = y_2^* + \beta_2^p x_{2t} \quad (t = 1, \dots, T_2), \quad y_2^* = (\phi_2 - \beta_2^p) \bar{x}_2 + \delta T_1 \bar{x}_1,$$

where $\bar{x}_1 = (1/T_1)t_1' x_1$ and $\bar{x}_2 = (1/T_2)t_2' x_2$. This property (for $p = -\frac{1}{2}$) is especially useful in obtaining (feasible) GLS estimates, as in Klaassen and Magnus (2001).

REFERENCES

- Klaassen, F.J.G.M. & J.R. Magnus (2001) Are points in tennis independent and identically distributed? Evidence from a dynamic binary panel data model. *Journal of the American Statistical Association* 96, forthcoming.