FINANCIAL DEVELOPMENT, LONG-TERM FINANCE AND THE MACROECONOMY:
THE ROLE OF SECONDARY MARKETS

By

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Financial Development, Long-term Finance and the Macroeconomy: The Role of Secondary Markets *

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Abstract

The paper develops a dynamic general equilibrium model of financial markets and macroeconomy. In the model, long-term debt is extended to firms in a primary market and then traded in a secondary market among financiers. Two financial frictions that are ex-ante and ex-post with respect to the secondary market trading date raise the cost of debt finance. In stationary equilibrium, while ex-ante frictions are always counterproductive, financing costs that are ex-post could promote macroeconomic growth. I show that a model consistent with the U.S. financial development experience of the last 30 years is likely to exhibit declining ex-post frictions.

Keywords: microfoundations of financial frictions, long-term investment, and secondary markets.

JEL Classification Numbers: E44, G2, O16, O47.

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1 Introduction

The absence of short-run cash-flows from an investment project could be related to poor as well as to slow output realization. While the former state is clearly a bad economic outcome, the same is not necessarily true for the latter. Financiers might lack the time and the capital to uncover which one is the true state of the world. Therefore, lowering lending imperfections and facilitating the trade of financial instruments in secondary markets could stimulate an economy’s ability to provide long-term finance. In this paper, I develop a general equilibrium model that is consistent with the stylized features of the U.S. financial development experience over the last 30 years, and show that the interactions between lending imperfections and the secondary market trade carry inconvenient implications for the aggregate supply of long-term debt and the macroeconomy.

The model has the essential features of a dynamic general equilibrium framework: (i) Entrepreneurs, depositors and financiers specialize in complementary activities by taking equilibrium prices as given; and, (ii) decisions of today affect the macroeconomic performance of tomorrow. In the model, long-term debt is vital to finance the productivity enhancing projects, where debt is extended to entrepreneurs in a primary investment market and then exchanged in a secondary market between financiers - with heterogeneous consumption propensities. Two financial frictions associated with long-term debt that prevail ex-ante and ex-post with respect to the secondary market trading date raise the cost of firms’ debt finance. Following Hennessy and Zechner (2011), I motivate these two frictions as a two-staged commitment problem: An entrepreneur’s commitment to not divert entrepreneurial capital before the secondary market trade, and after the trade entrepreneur’s commitment to repay are enforceable only if financiers show some effort.1

The general equilibrium analysis reveals that lowering the ex-post cost of long-term debt finance could reduce the corporate debt supplied and the macroeconomic output of the economy. The key mechanism underlying this highlighted result is driven by a general equilibrium feedback effect: As the ex-post cost of financing declines

1Chart 1 summarizes the investment-finance relationships in the model.
relative to the ex-ante cost, the demand in the secondary market rises, increasing the prices and the volume of secondary market trade, and crowding out the primary debt supply. When the ex-post financing cost is not too large, the secondary market effect could slow down the macroeconomy.

The model can account for the key stylized features of the U.S. financial development experience over the last few decades. Since 1980s, information technology has increased efficiency and lowered transaction costs throughout the economy. The sharp decline in transaction costs reduced financial imperfections and fostered financial development. The following three stylized observations are crucial for the analysis of the current paper:

1. Financial services industry grew disproportionately compared to the rest of the economy since 1980s. For instance, Philippon (2008) and Philippon and Reshef (2012) document that in 1990, the financial sector accounted for 5.65% of the U.S. GDP and 6.11% of the aggregate employment compensation. In 2005, these fractions became 7.69% and 7.65% respectively. Recent empirical evidence shows that as of February 2013, the total compensation of the financial sector (including profits, wages, salary and bonuses) as a fraction of GDP hit an all-time high, around 9% of the U.S. GDP.

2. Since 1980s, the U.S. secondary markets that allow the trade of long-term financial instruments grew exponentially. For instance, the volume of secondary debt markets have exhibited a tremendous development, rising from an annual volume of about 10 billion U.S. Dollars in 1990 to 400 billion U.S. Dollars in 2012.

3. Over the same time-period the aggregate stock of debt instruments available for the corporate sector was more or less stable - rising only slightly from 6.04% of the U.S. GDP in 1990 to 6.14% in 2005.

Did the growth of the U.S. financial industry over the recent decades contribute to the long-run macroeconomic growth of the U.S. economy? Or, has financial development simply gone too far as argued among others also by Rajan (2005). This
paper aims to shed light on this important question from a novel general equilibrium angle. The key argument of the paper is that the growth of the financial sector could facilitate as well as slow down the macroeconomic performance; and, the interactions between the financial frictions - that determine the size of the finance sector - and secondary financial markets are key to understand the conditions for counterproductive financial development.

The dynamic general equilibrium model that I study in this paper captures the first essential feature highlighted above. Specifically, in all stationary equilibria of the model the reductions in ex-ante and ex-post frictions stimulate the size of the financial sector and raise the compensation of financiers. The difference between two debt financing costs becomes prevalent when we compare the equilibrium implications of the model in matching the stylized empirical facts (ii) and (iii). Specifically, reducing the ex-ante cost of debt financing lowers the trading volume in the secondary market and stimulates the supply of corporate debt. On the contrary, reducing the ex-post cost of debt financing facilitates the trading volume in the secondary market, and have the potential for stagnating the debt finance supplied to the corporate sector. Therefore, a financial development experience that aims to account for the secondary debt market growth should incorporate declining ex-post frictions. As the key theoretical result of the paper shows ex-post financial frictions are not necessarily barriers to long-run economic growth.

Related Literature: The theoretical findings of the paper shed light on the dark-side of the secondary market trading. Several studies emphasized that the size of secondary financial markets could hinder the capital formation and macroeconomic performance. Bencivenga, Smith and Starr (1995, 1996 and 2000) are seminal studies in this literature. Different from this existing literature, in this paper I develop a model with micro-founded financial transaction costs - that are heterogeneous with respect to their timing relative to the secondary market trading date - and show that (a) secondary markets contract and macroeconomic performance expands unambiguously when ex-ante frictions decline and (b) reducing ex-post frictions stimulates the size of the secondary market while it might reduce the macroeconomic output.
The paper also contributes to the broad literature on Finance and Macroeconomy. Some of the important studies in this literature are Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Banerjee and Newman (1993), Antunes et al. (2008), and Aghion, Angeletos, Banerjee and Manova (2010). This paper is essentially related to the literature that identifies non-monotone real effects of financial sector development. Some studies in this area of research are Fatouh (2002), Castro, Clementi and MacDonald (2004), and Uras (2012). In these studies, the non-monotonicity between finance and macroeconomic performance arises if quantities of some particular macroeconomic variable, such as the steady-state capital stock (Castro, Clementi and MacDonald (2004)) or the population weight of firms with an access to a long-term production technology (Uras (2012)) do not reach a critical level. Different from the approach of these studies, in the current paper, the non-monotonicity between finance and macroeconomy depends not only on the stage of economic development but also on the structure of the financial sector and the underlying microfoundations that determine the cost of debt finance. The current study is the first in pointing out this specific issue about the potential counterproductivity of financial development.

There is a large number of empirical papers, such as La Porta, Lopez-de-Silanes, Shleifer, and Vishny, (1998), Beck, Levine and Loayza (2000) and Beck and Levine (2002), motivated to identify causal effects of legal and financial institutions which include creditor protection, debt enforcement, and investor rights on long-run macroeconomic performance. Most of the studies in this literature employ a wholistic approach and study how the quality of a collection of institutions affect the macroeconomic well-being. A more recent strand of the literature studies the quality of individual institutions in isolation and suggests empirical evidence to be skeptical about the macroeconomic implications of financial development. For example Castro et al. (2004) concentrate on institutions that foster investor protection and show that investor protection and macroeconomic output do not follow a positive correlation for high income countries. As another example, Hasselman et al. (2010) breakdown the debt enforcement institution into its components as collateral enforcement and

\footnote{Other important studies in this literature are Marcet and Marimon (1992), Acemoglu and Zilibotti (1997), Aghion and Bolton (1997), Azariadis and Kaas (2007), and Antunes et al. (2008).}
bankruptcy enforcement. The authors use cross-country panel regressions to study the effects of collateral and bankruptcy in isolation, and show that collateral enforcement has positive explanatory power on aggregate bank finance. The quality of bankruptcy institutions on the other hand turns out to be insignificant in explaining the aggregate size of outstanding bank loans. Motivated with the findings of this recent literature, my theoretical analysis in this paper decomposes the financing costs of long-term debt contracts into two microfounded frictions in a dynamic general equilibrium model and studies the impact of each friction on the behavior of corporate debt markets and the macroeconomy.

Closely related to this paper, there is also a large literature on financing frictions, entrepreneurship and macroeconomy. For example, Philippon (2010) derives optimality of the financial sector taxation when entrepreneurship requires the debt enforcement services of the financial industry. Similarly, Erosa (2001) shows that financial intermediation costs can explain the cross-country differences in per-capita income and average business size. Cagetti and De Nardi (2006) and Antunes et al. (2008) study the effects of enforceability of debt finance on entrepreneurship and macroeconomic development, Midrigan and Xu (2013) and Gilchrist et al.(2013) analyze enforcement constraints and misallocation. Amaral and Quintin (2006) and D’Erasmo and Boedo (2012) show that financial constraints generated by limited enforceability increases the size of the informal sector. This literature studies enforcement frictions with short-term (one period) debt contracts along-side a one-period enforcement problem. Different from the existing research my analysis incorporates two-period debt contracts in a dynamic general model and studies the macroeconomic implications of debt finance frictions.

The rest of the paper is organized as follows. Section 2 introduces the basic model environment. Sections 3-6 study the macroeconomic implications of several financial development experiments. Section 7 concludes the paper.

2 The Model

Consider an overlapping generations economy as in Diamond (1965), where time is indexed with $t$ and continues forever. There is a single good that can be consumed
or invested. Each period a continuum of risk-neutral agents enter the economy - indexed by \( i \in [0, M] \) - and live for two periods. I will call the periods in an agent’s life-cycle as \textit{young} and \textit{old} and denote the consumption of an individual \( i \) from generation \( t + 1 - j \) at time \( t \) with \( c_{jt}^{i} \). The lifetime utility of the agent \( i \) is

\[
U_{t}^{i} = c_{\text{young},t}^{i} + c_{\text{old},t}^{i}.
\]

At the beginning of the two-period life-cycle agents select into one of the following occupations: \textit{Worker, Financier}, or \textit{Entrepreneur}. There are three markets in the economy: the Deposit Market, the Primary Debt Market, and the Secondary Debt Market. Financiers are the intermediaries in the model; and therefore, they interact in all three markets, whereas workers and entrepreneurs can respectively access to the deposit and the primary debt markets only. The characteristics of the occupations and the markets are delineated as follows.

Young \textbf{workers} have access to a constant returns to scale technology that converts labor hours into the consumption good. In return of their labor efforts, workers receive a salary payment - worth of 1 unit of consumption - at the end of the youth period. Workers do not receive any wage income when old. In order to finance the old-period consumption, they have to deposit fractions of the youth salary with financiers at the competitive rate of deposit repayment, \( R_{d} \).

Each \textbf{financier} collects a sum of deposits \( \tilde{d} \) from young workers, convert them into units of finance \( \tilde{w} \) and provide investable funds in primary and secondary markets.

\textbf{Assumption 1.} \( \tilde{w}_{t}(\tilde{d}_{t}) = \min\{\tilde{d}_{t}, B\tilde{d}_{t}^{\beta}\} \), with \( B > 0 \) and \( 0 < \beta < 1 \).

Assumption 1 implies that the conversion of deposits-into-finance exhibits span of control, where each unit of deposit can be transformed into at most one unit of finance. The strict decreasing returns scale property becomes binding when the amount of deposits demanded by a financier is large enough, such that \( \tilde{d} > B^{1/\beta} \).

Financiers keep \( R_{f} \) units of consumption good from returns to finance as their own compensation. Similar to the deposit rate, \( R_{d} \), the financier’s compensation \( R_{f} \) will also be determined in the general equilibrium.
Entrepreneurs operate a production technology - a long-term project - that takes two periods to generate cash-flow and uses capital in addition to entrepreneur’s time as production inputs. Specifically, for each $\tilde{k}_t$ units of capital raised in the primary debt market with a two-period unit cost of finance $\tilde{R}_t$, a young entrepreneur’s production technology will deliver $\ell_{t+1}(\tilde{k}_t)$ units of the consumption good (cash) when the entrepreneur becomes old.

Assumption 2. $\ell_{t+1}(\tilde{k}_t) = A_t \tilde{k}_t^\alpha$, with $0 < \alpha < 1$.

Productivity. The productivity of the project, $A_t$, in any given period $t$ is a function of the aggregate output produced by entrepreneurial projects in period $t-1$. I denote the aggregate output produced by entrepreneurial projects with $W$, and formalize the productivity process in the economy with the following assumption.

Assumption 3. $A_t = \bar{A} [1 + g (W_{t-1})] \equiv \int_{M_{t-1}}^{M_t} \ell_{t-1} d_i$; and, $g'(.) > 0$ and $g''(.) \leq 0$.

This assumption states that there are inter-generational knowledge spillovers in the form of productivity enhancing investment: Entrepreneurial output of today determines the aggregate entrepreneurial productivity of tomorrow. Productivity enhancing investment of this form is a stylized feature of endogenous growth models with overlapping generations as for example in Bencivenga, Smith and Starr (1995, 1996 and 2000), and Aghion, Angeletos, Banerjee and Manova (2010). It is easy to see that with the entrepreneurial technology at assumption 2 and the productivity process specified at assumption 3, a unique and stable $A$ exists as long as $g'(W) > 0$ and $g''(W) \leq 0$.

Secondary Market Trading. Workers are the suppliers of investable funds and entrepreneurs are the users of funds. Financiers are the intermediaries: They convert deposits into finance and supply input for entrepreneurial production. Financiers perform a maturity transformation function in the economy. Positive measures of entrepreneurship could prevail in equilibria only with maturity transformation, because workers have access to earnings for only a short-while (one-period) whereas the entrepreneurial projects are long-term (two-period). The maturity transformation is sustained through secondary market trading. There are primary and sec-
ondary financiers of entrepreneurial investment projects. Primary financiers extend long-term debt to entrepreneurs at the beginning of a period \( t \) and receive debt claims against next period’s \((t + 1)\) cash-flows. Secondary financiers purchase these debt claims from primary financiers in the secondary market at the end of the period \( t \) and collect the capital returns from entrepreneurs upon the finalization of long-term projects in period \( t + 1 \). The timing of secondary market trading implies that primary financiers are old whereas secondary financiers are young when primary and secondary market investment takes place respectively.

I allow any convex combination of the two financing options - primary and secondary market finance - for a particular financier. That means, financiers can utilize deposits that they convert into units of finance to directly invest in new entrepreneurial projects as well as to purchase debt claims against returns from old entrepreneurial projects.

**Frictions.** In investigating the macroeconomic implications of financial development, I consider two frictions, that are ex-ante and ex-post with respect to the secondary market trading date. Both frictions are measured in terms of financier efforts, and are proportional to the per-unit capital repayment of the entrepreneur. I let \( \hat{R} \) denote the unit rate of capital repayment and assume that ex-ante and ex-post frictions cause disutility to the financier worth of

\[
\begin{align*}
    v_{\text{ex-ante}} &= \psi_1 \hat{R}_t \bar{k}_t, \quad \text{and} \\
    v_{\text{ex-post}} &= \psi_2 \hat{R}_t \bar{k}_t
\end{align*}
\]

units of consumption with \( 0 < \psi_1 < 1 \) and \( 0 < \psi_2 < 1 \). Throughout the analysis, I will assume that the effort of financiers are perfectly observable and fairly priced. Following Hennessy and Zechner (2011), we could motivate the sequence of ex-ante and ex-post financing frictions as a two-staged financial imperfection: Entrepreneur’s commitment to not divert entrepreneurial capital before the secondary market trading, and after the trade takes place entrepreneur’s commitment to repay are enforceable only if financiers show some effort. The parameters \( \psi_1 \) and \( \psi_2 \) are the key policy variables of the study, both of which jointly determine the stage of
financial development in a society.

**Timing.** In any given period $t$ there are two sub-periods. I name the first sub-period as the *day* and the second sub-period as the *night*. The events that occur in each respective sub-period are as follows:

- **Day:**
  1. Young agents select into occupations.
  2. Primary debt market opens, and old financiers provide two-period debt to young entrepreneurs against period $t + 1$ debt claims.
  3. Primary debt market closes.
  4. Old primary financiers incur the ex-ante cost of holding entrepreneurial debt.
  5. Entrepreneurs invest in long-term projects.

- **Night:**
  1. Young workers receive the wage income.
  2. Young financiers collect deposits from young workers.
  3. Secondary debt market opens. Young financiers use fractions of the deposits to purchase debt claims against period $t + 1$ long-term investment returns from old financiers. In other words, old-financiers, who have extended long-term debt to young entrepreneurs in the day sub-period, sell debt claims to young financiers.
  4. Secondary market closes.
  5. Old entrepreneurs collect investment returns from projects started in the day sub-period in $t − 1$.
  6. Old secondary financiers incur the ex-post cost of holding entrepreneurial debt and collect debt repayment from entrepreneurs.
  7. Old workers receive deposit repayment.
  8. Agents consume.
An interpretation of the model and the key question. The economic model I developed is closely related to Banerjee and Newman’s (1993) occupation choice model of financial development. In both models when financial frictions are infinitely large - which is the case here when $\psi_1$ and/or $\psi_2$ equals to 1 - the economy is in an autarkic (no-trade) state. For instance in the context of my model, the autarky is an equilibrium where every agent chooses to become a worker at the beginning of his life-time, earns a wage income worth of 1 unit of the consumption good and consumes it. When financial frictions are not so severe - $\psi_1$ and $\psi_2$ sufficiently smaller than 1 - the entrepreneurship sector emerges. Different from Banerjee and Newman (1993), in my theoretical framework financial intermediary sector exhibits growth following a reduction in financing frictions and channels funds from workers to entrepreneurs. The key question I am interested in addressing in sections 3 through 6 is whether the growth of the intermediary sector necessarily co-exist with the growth in the long-term debt supply and the macroeconomic output. The answer is not necessarily.

3 Financial Markets with Ex-ante Costs of Debt Finance

In this section, I ignore the ex-post costs of holding debt claims and consider an economy with ex-ante financing frictions only. This means only primary debt financiers - who directly contact entrepreneurs - incur the effort costs. Using the parameters introduced in the previous section, this implies that $\psi_1 > 0$ and $\psi_2 = 0$, as presented in Chart 2.

CHART 2 ABOUT HERE

The cost of holding debt ex-ante to the secondary market trading date can be motivated with a Hart and Moore (1986) type of friction: If an entrepreneur quits from a project right after the debt is extended, the project as well as the the debt claim becomes worthless. In order to enforce project implementability financiers might need to spend resources, where the implementation of the project becomes costlier.
with the size of the debt repayment.\(^3\)

### 3.1 Optimizing Behavior

**Workers.** Risk neutrality implies that workers deposit the unit wage compensation from the youth period as long as the competitive deposit repayment rate is large enough \(R_{d,t} \geq 1\), such that the equilibrium is non-autarkic. For \(R_{d,t} \geq 1\) a worker’s lifetime utility satisfies \(V_{w,t} = R_{d,t}\).

**Financiers.** The lifetime utility of a financier is denoted with \(V_{f,t}\) which equals to \(R_{f,t}\), financier’s end of life-time compensation. Risk-neutrality also implies that financiers consume their compensation at the end of the old period as well.

**Entrepreneurs.** An entrepreneur takes the two period cost of capital, \(\hat{R}_{t+1}\), as given and solves:

\[
\max_{k_t} A_t \hat{k}_t^\alpha - \hat{R}_{t+1} \hat{k}_t^\alpha.
\]

Entrepreneur’s optimal capital demand \(k_t\) and the life-time value of being an entrepreneur are given by

\[
k_t = \left( \frac{\alpha A_t}{\hat{R}_{t+1}} \right)^{\frac{1}{1-\alpha}},
\]

\[
V_{e,t} = (1-\alpha)A_t \left( \frac{\alpha A_t}{\hat{R}_{t+1}} \right)^{\frac{\alpha}{1-\alpha}}.
\]

### 3.2 The General Equilibrium Analysis

**Definition** The **dynamic general equilibrium** of the economy is characterized by streams of primary financier returns to capital - net of effort costs - \(\{R^1_t\}_{t=0}^{\infty}\) and secondary financier returns \(\{R^2_t\}_{t=0}^{\infty}\), return on deposits \(\{R_{d,t}\}_{t=0}^{\infty}\), cost of long-term debt \(\{\hat{R}_t\}_{t=0}^{\infty}\) and prices of debt claims in the secondary debt market.

\(^3\)As seminal examples of general equilibrium models with debt frictions, Kiyotaki and Moore (1997) and Matsuyama (2004) work with reduced form specifications of the Hart and Moore frictions as well.
\[(\{p_t\}_{t=0}^{\infty})\] at which

1. Agents optimize their life-time utility,

2. Life-time income from the three occupation choices are equalized, such that

\[V_{w,t} = V_{f,t} = V_{e,t},\] (2)

3. The life-time value from being a primary debt market financier \((V^1_{f,t})\) is equal to the life-time value of a secondary market financier \((V^2_{f,t})\), such that

\[V^1_{f,t} = V^2_{f,t},\] (3)

4. Deposit market, primary and secondary debt markets clear.

Defining \((1 - \psi_1) \equiv \frac{1}{1+\phi}\), in equilibrium primary financier’s unit return from entrepreneurial finance net of effort costs satisfies \(R^1_t = \frac{p_t}{1+\phi}\). Clearly, in a no-autarky equilibrium \(R^1_t > 1\), and positive measures of financier and entrepreneur populations prevail in the general equilibrium. The life-time value indifference between primary and secondary financiers then implies the results obtained in the following two lemma.

**Lemma 1** In a no-autarky equilibrium, (i) the primary and secondary financiers’ unit return from finance - net of effort costs - are equalized such that \(R^1_t = R^2_t = R_t\), and (ii) the unit cost of two-period finance for an entrepreneur satisfies \(\hat{R}_{t+1} = R_t R_{t+1} (1 + \phi)\).

**Proof** The unit investment return from secondary market finance is inversely related to the secondary market price of debt, such that \(R^2_{t+1} = \frac{\hat{R}_{t+1}}{p_{t+1}}\). The no-arbitrage condition

\[
\frac{\hat{R}_{t+1}}{p_t} = \frac{p_{t+1}}{1+\phi},
\]

\[
= R^2_{t+1} = R^1_{t+1}
\]

13
should hold; otherwise, all financiers strictly prefer to be either a financier in the
primary debt market or a financier in the secondary market, and under either case
finance cannot become available for entrepreneurs. Defining $R_t \equiv R_{t+1}^1 = R_{t+1}^2$,
$\hat{R}_{t+1} = R_t R_{t+1} (1 + \phi)$ holds in equilibrium. □

Lemma 2 The model exhibits a unique stationary equilibrium characterized by a
constant $R$ in all time periods and an invariant distribution of workers, financiers
and entrepreneurs across all cohorts.

Proof The stationarity of the productivity of entrepreneurial projects implies that
the rate of return from entrepreneurial finance eventually reaches a steady-state
level. Therefore, the return to finance in primary and secondary debt markets are
also constant implying a unique steady-state distribution between entrepreneurship
and the remaining occupations. Given worker’s exogenous wage compensation,
the assumption 1 implies a unique steady-state distribution between workers and
financiers. □

For the rest of the analysis, I will concentrate on the unique steady-state equilibrium
of the model while studying the effects of financial development on asset returns,
distribution of occupations and the macroeconomic performance.

Financier-Worker Ratio. The steady-state ratio between population measures of
financiers ($M_f$) and workers ($M_w$) is independent of the level of financing frictions.
To observe this, we need to first note that each financier solves the following pro-
gram to maximize the total quantity of finance produced by taking the return from
providing finance, $R$, and the unit deposit repayment, $R_d$ as given:

$$\max_d R \bar{w}(d) - R_d \bar{d}. \tag{4}$$

The first order conditions yield:

$$R \bar{w}'(\bar{d}) = R_d. \tag{5}$$
There are two special cases to consider: (i) \( \tilde{w}'(\tilde{d}) = 1 \), with \( R = R_d \), and (ii) \( \tilde{w}'(\tilde{d}) < 1 \), with \( R > R_d \). In case (i), the financier does not make any private returns from providing finance; and therefore, the equilibrium is an autarky. In case (ii), financial intermediation as well as entrepreneurial investment do prevail in the general equilibrium. I will concentrate on parameterizations of the model that support the case (ii) with \( d > w \) where \( d \) and \( w \) are optimum quantities of deposits collected and finance generated by each financier in a stationary equilibrium.\(^4\)

In order to derive the optimum quantity of deposits collected, \( d \), and the optimum quantity of finance produced, \( w \), I use the functional form assumption 1 in equation (5) to obtain

\[
\begin{align*}
    d &= \left( \frac{\beta BR}{R_d} \right)^{\frac{1}{1-\beta}}, \\
    w &= B \left( \frac{\beta BR}{R_d} \right)^{\frac{\beta}{1-\beta}}.
\end{align*}
\]

The indifference condition between being a worker and a financier solves for \( R_d \) and \( R_f \) as functions of \( R \) and the structural parameters \( B \) and \( \beta \):

\[
R_d = \frac{R_f}{\beta B^\beta (1 - \beta)^{1-\beta}}.
\]  
\[\text{(6)}\]

Using (6) in \( d \), at the optimum \( d > w \) holds when \( \left( \frac{\beta}{1-\beta} \right)^\beta < \frac{1}{B} \), where the optimum quantities of deposits demanded and finance produced by each financier have the closed-form expressions

\[
\begin{align*}
    d &= \frac{\beta}{1-\beta}, \text{ and} \\
    w &= B \left( \frac{\beta}{1-\beta} \right)^\beta.
\end{align*}
\]  
\[\text{(7)}\]

\(^4\)The case of \( d > w \) is also empirically plausible than the case with \( d = w \), since financial intermediaries convert only a fraction of their deposits into investable funds.
The ratio between population measures of financiers and workers is then independent of the level of the society’s financial market efficiency.

**Proposition 3.1** The deposit market clearance condition yields the population ratio between financiers and workers as

\[
\frac{M_f}{M_w} = \frac{1 - \beta}{\beta}. \tag{8}
\]

**Proof** Since each worker receives a unit salary payment at the end of the youth period, the aggregate stock of deposits equals to \(M_w\). The demand for deposits by each financier is \(\frac{\beta}{1-\beta}\). Therefore, the financier-worker distribution in the steady-state equals to \(\frac{1-\beta}{\beta}\). □.

**Financier-Entrepreneur Ratio.** In order to solve for the the population ratio between financiers and entrepreneurs, at first, we need to observe that the entrepreneur’s cost of finance in the steady-state equilibrium is

\[
\hat{R}(\phi) = R^2(1 + \phi).
\]

The unit price of a debt claim in the secondary market, \(p\), should then satisfy

\[
p(\phi) = R(1 + \phi).
\]

We can express the occupation indifference between being a worker, a financier - net of effort costs - and an entrepreneur as the following

\[
RB\beta^\beta(1 - \beta)^{1-\beta} = (1 - \alpha)A \left( \frac{\alpha A}{R^2(1 + \phi)} \right)^\frac{1}{1+\alpha}. \tag{9}
\]

Solving for \(R\) as a function of \(\phi\),

\[
R(\phi) = A\left( \frac{1 - \alpha}{B\beta^\beta(1 - \beta)^{1-\beta}} \right)^\frac{1}{1+\alpha} \left( \frac{\alpha}{1 + \phi} \right)^\frac{\alpha}{1+\alpha}. \tag{10}
\]
provides the closed-form solution for the key variable of interest of the general equilibrium analysis. The return from providing finance, $R$, is a decreasing function of the ex-ante financing friction $\phi$. Since $R_f$ (and also $R_d$) is also linear in $R$, the equilibrium life-time income of a financier is also decreasing in ex-ante financing frictions.\(^5\) This general equilibrium property implies that financial development makes the “intermediary-sector” attractive.

Given $R$, the two-period cost of capital $\hat{R}$ and the secondary market price of debt claims $p$ can both be expressed as functions of $\phi$ as well:

\[
\hat{R}(\phi) = R^2 (1 + \phi) \propto (1 + \phi)^{1-\alpha}, \tag{11}
\]

\[
p(\phi) = R (1 + \phi) \propto (1 + \phi)^{1+\alpha}. \tag{12}
\]

Ex-ante financing frictions make the entrepreneurial finance and the purchase of debt claims in the secondary market costlier.

In the stationary equilibrium, each financier invests a constant fraction of his financial endowment in the primary debt market. I denote the steady-state primary market investment of each financier by $x$. The aggregate finance invested in long-term entrepreneurial projects then equals to $M_f x$, and the aggregate compensation of primary financiers is $M_f x p$. Since in equilibrium each financier produces $w = B \left( \frac{\beta}{1-\beta} \right)^\beta$ units of finance, we can derive $x$ as

\[
x = \frac{w}{1+p} = \frac{w}{1+R(1+\phi)}. \tag{13}
\]

Since $p$ increases with $\phi$, the steady-state debt-finance provided by each financier rises with financial development. To understand whether the size of the financial sector expands with declining financial frictions, we need to derive an expression for the equilibrium measure of financiers in the economy ($M_f$) as a function of $\phi$. This, we can do by solving for the market clearance condition of the primary debt

\(^5\)The rise in the rate of return on capital as financial frictions contract is a standard feature of general equilibrium models of financial development, such as Aiyagari (1994) and Angeletos and Calvet (2006).
market:

\[
M_f \frac{w}{1 + R(1 + \phi)} = M_e \left( \frac{\alpha A}{R^2(1 + \phi)} \right)^{\frac{1}{1-\alpha}}
\]

\[
\Rightarrow M_f \frac{V_e (1 - \beta)}{R(1 + R(1 + \phi))} = M_e \left( \frac{\alpha A}{R^2(1 + \phi)} \right)^{\frac{1}{1-\alpha}}
\]

\[
\Rightarrow M_f (1 - \beta)(1 - \alpha)A \left( \frac{\alpha A}{R^2(1 + \phi)} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{1}{R(1 + R(1 + \phi))} \right) = M_e \left( \frac{\alpha A}{R^2(1 + \phi)} \right)^{\frac{1}{1-\alpha}}
\]

Deriving the equilibrium measure of financiers relative to the entrepreneurs \((M_f/M_e)\) yields:

\[
\frac{M_f}{M_e} = \frac{\alpha(1 - \beta)(1 + p)}{1 - \alpha} \frac{1 + R(1 + \phi)}{p} = \frac{\alpha(1 - \beta)}{1 - \alpha} \frac{1 + R(1 + \phi)}{R(1 + \phi)}. \tag{14}
\]

The population measures of workers, financiers and entrepreneurs then can be derived by solving equations (8) and (14) together:

\[
M_w = \frac{\alpha \beta (1 + p)}{\alpha + p} M, \tag{15}
\]

\[
M_f = \frac{\alpha (1 - \beta)(1 + p)}{\alpha + p} M, \tag{16}
\]

\[
M_e = \frac{(1 - \alpha)p}{\alpha + p} M. \tag{17}
\]

The equations (15), (16) and (17) imply that the population measure of workers and financiers are decreasing functions of \(\phi\) whereas the population measure of entrepreneurs rises with \(\phi\). Since the aggregate compensation of financiers equals to \(M_f R_f\), and \(R_f\) decreases with \(\phi\) as well, I obtain the following relationship between the size of the financial sector and a permanent reduction in ex-ante financing cost \(\phi\).

**Proposition 3.2**

*If financial development reduces the effort costs related to the ex-ante financing frictions, then in the stationary equilibrium*
i. The total quantity of financiers increases with financial development.

ii. The total compensation of financiers relative to the aggregate income rises with financial development.

The result in proposition 3.2 is a stylized feature of the U.S. financial development experience over the last few decades. Financial services industry grew disproportionately compared to the rest of the economy since 1990s. According to Philippon (2008) in 1990, the financial sector accounted for 5.65% of the U.S. GDP and 6.11% of the aggregate employment compensation. As of February 2013, the total compensation of the financial sector (including profits, wages, salary and bonuses) as a fraction of GDP hit an all-time high, around 9% of the U.S. GDP.

**Supply of Long-term Debt and the Secondary Market.** Equilibrium in the secondary debt market implies that the ratio between the primary financiers and the secondary financiers should satisfy

\[
\frac{M_{f1}}{M_{f2}} = \frac{1}{p} = \frac{1}{R(1 + \phi)}.
\]

(18)

The aggregate ratio between primary financiers and secondary financiers rises as the ex-ante cost of providing finance declines. Using equations (16) and (18) together, we can derive the aggregate quantity of long-term debt, \(K\), that finances society’s entrepreneurial investment as

\[
K = M_{f1} \cdot w = \frac{\alpha (1 - \beta) (1 + p)}{\alpha + p} \cdot \frac{1}{1 + p} \cdot Mw.
\]

(19)

Denoting the first partial derivative of \(p\) by \(p'\) with \(p' > 0\) as shown above, the first-partial derivative of \(K\) with respect to \(\phi\) can be derived as

\[
\frac{\partial K}{\partial \phi} = -\alpha (1 - \beta) \frac{p'}{(\alpha + p)^2} Mw < 0.
\]

(20)

There are two channels where financial development and the supply of long-term
debt interact: (i) Financial development expands the size of the financing sector, and (ii) the prices in the secondary market contract with financial development. Both channels stimulate the primary debt supply and raise the input that goes into entrepreneurial investment.

The size of the secondary market in the steady-state is measured as the aggregate fraction of secondary financiers in the economy multiplied by the the quantity of finance produced by each financier:

$$S = M_f^2 w$$

$$= \frac{\alpha(1-\beta)(1+p)}{\alpha + p} \cdot \frac{p}{1+p} \cdot M w.$$  \hspace{1cm} (21)

The first-partial derivative of \( S \) with respect to \( \phi \) is

$$\frac{\partial S}{\partial \phi} = \alpha(1-\beta) \frac{\alpha p'}{(\alpha + p)^2} M w > 0.$$  \hspace{1cm} (22)

Similar to the supply of long-term debt, financial development affects the size of the secondary market trading through two channels: (i) The increase in the size of the financial sector stimulates the secondary market trading whereas (ii) declining secondary market prices lower the size of the secondary market. The net effect is negative, financial development and secondary market size are negatively related when financial development reduces the financing frictions that are ex-ante with respect to the secondary market trading date.

**Proposition 3.3**

*If financial development reduces the effort costs related to the ex-ante financing frictions, then in the stationary equilibrium*

1. the quantity of long-term debt increases with financial development,
2. the size of the secondary market decreases with financial development.
Steady-state Consumption.

I measure the long-run performance of the economy by the steady-state per-capita consumption - net of effort costs. Denoting $c$ as the consumption per-capita in the steady-state of an old agent, from equation (9), we can derive:

$$c = w(1 - \beta)R,$$

$$= (1 + \phi) \frac{\alpha}{1 + \alpha} (w(1 - \beta)) \left( \frac{2\alpha}{1 + \alpha} \right) \left( 1^{-\alpha} \alpha \left( 1 - \alpha \right)^{1 - \alpha} \right) \frac{1}{1 + \alpha}. \quad (23)$$

Steady-state consumption is a function of $A$, the productivity of long-term projects, which is determined by the aggregate output produced by entrepreneurial projects ($W$):

$$W = MeA \left[ \frac{K}{Me} \right]^\alpha,

= MeA \left[ \frac{w(1 - \beta)}{p \frac{\alpha}{1 - \alpha}} \right]^\alpha.\quad (24)$$

Substituting $Me$ from (17) in $W$ we get

$$W = (1 - \alpha) \frac{1 - \alpha}{\alpha + p} A \left[ \frac{w(1 - \beta)}{\frac{\alpha}{1 - \alpha}} \right]^\alpha M,$$

where the steady-state long-term project productivity solves

$$A = \bar{A} \left[ 1 + g(W(A)) \right].$$

Ex-ante financing frictions impact per-capita consumption - derived at (23) - through two channels: (1) A direct effect - due to deadweight losses - and (2) an indirect effect through the impact of frictions on aggregate entrepreneurial output. The former effect is clearly negative. Therefore, in order to evaluate whether ex-ante financing frictions stimulate the per-capita consumption in steady-state we have to study comparative statics at the aggregate entrepreneurial output with respect to $\phi$.

Financial Development and Macroeconomic Performance. Although financial
development is expansionary for the aggregate capital invested in entrepreneurial projects, it also comes with a growing financial sector which crowds out the total number of entrepreneurs in the economy. Therefore, one needs to check whether the aggregate output produced by long-term projects does in fact rise with financial development. I define financial development as an event that occurs at the beginning of a period \( \tau \) (at stage-0 of the flow of events presented in section 2) and reduces the cost of primary market debt finance, \( \phi \), permanently.

Differentiating (24) with respect to \( \phi \), we can show that lowering the ex-ante cost of financing friction \( \phi \) improves the steady-state aggregate output produced by entrepreneurial projects when

\[
(1 - \alpha)^2 p^{-\alpha} (\alpha + p) < (1 - \alpha) p^{1-\alpha}.
\]  

(25)

Simplifying both sides at (25) yields

\[
(1 - \alpha) < p.
\]  

(26)

The rate of financial return, \( R \), must exceed 1 to have a no-autarky equilibrium with positive entrepreneurial investment. Since \( p > R \), as long as the equilibrium is non-autarkic, the condition at (26) is satisfied for all parameter values.

**Proposition 3.4** The inequality at (26) is satisfied for all parameter values that support an investment equilibrium. Lowering the primary financing friction - that is ex-ante with respect to the secondary market trading date - stimulates the macroeconomic output generated by long-term projects and as a result the per-capita consumption - net of effort costs - in the steady-state equilibrium.

**Transitory Dynamics.** A permanent reduction in \( \phi \) in an arbitrary period \( \tau \) stimulates the aggregate output from long-term projects in the consecutive period \( W_{\tau+1} \). Since the productivity of long-term projects is time-dependent as specified at assumption 3, a rise in current output from long-term projects translates into a higher entrepreneurial productivity in the future. Therefore, comparing the steady-state
values of productivity and entrepreneurial output \((A, W)\) against the productivity and output in period \(\tau + 1\) \((A_{\tau+1}, W_{\tau+1})\) reveals that \(A > A_{\tau+1}\) and \(W > W_{\tau+1}\).

As depicted in figure 1 these results imply that future generations benefit from financial development - in the form of an ex-ante financing friction alleviation - more than the current generation who invests in institutions that reduce ex-ante financing costs and promote the primary market trading. In order to observe this note that the current generation (cohort born in period \(\tau\)) clearly benefits from financial development since for constant \(A\), the per-capita consumption in period in \(\tau + 1\) rises as \(\phi\) declines. The steady-state per-capita consumption, \(c\), is strictly larger than the per-capita consumption in period \(\tau + 1\), \(c_{\tau+1}\), because the rise in \(W\) stimulates the growth in \(A\) until a steady-state level is reached.

FIGURE 1 ABOUT HERE

3.3 Discussion

When the ex-ante financing friction is large the economy exhibits an inefficiently undersized finance-sector because the rate of financial return is too small. Financial development stimulates the rates of return on providing finance, and causes an expansion in the size of the finance-sector, and a leads to a rise in the primary debt supply available for the entrepreneurial sector.

The key result that shows the steady-state consumption is a decreasing function of \(\phi\) has an intuitive interpretation. Permanent reductions in \(\phi\) generate three effects on the macroeconomy. (1) The population share of financiers \((M_f)\) rises as \(\phi\) decreases. (2) The contraction in \(\phi\) also mitigates the fraction of investable funds that needs to be paid to primary financiers in secondary market transactions: \(p\) is an increasing function of \(\phi\), and therefore the smaller \(\phi\) the larger is the fraction of the physical endowment stock that can be allocated to entrepreneurial production. (3) The output produced by entrepreneurial projects rises with the expansion of the debt stock which stimulates the future productivity of long-term projects in the economy.
The channels 1 and 2 foster a debt stock deepening whereas the channel 3 stimulates the total factor productivity by improving the allocative efficiency of capital and as a result stimulating inter-generational productivity spillovers. Lowering the effort cost of primary debt finance impacts the steady-state per-capita consumption - net of effort costs - through all three channels.

4 Financial Markets with Ex-post Costs of Debt Finance

In this section, I explore the behavior of an economy with debt financing costs that are ex-post with respect to the secondary market trading date. Using the parameters introduced in section 2, this implies that $\psi_1 = 0$ and $\psi_2 > 0$, as I illustrate in chart 3. Ex-post debt financing costs can be rationalized with a costly debt repayment friction.\textsuperscript{6} For instance, imperfect bankruptcy regulation would increase the efforts of a financier that he needs to incur in order to obtain the full repayment from an entrepreneurial project. Therefore, strict bankruptcy regulation could function as a mean of enforcing entrepreneurs’ debt repayment to financiers and stimulate the aggregate supply of entrepreneurial debt finance. My analysis in this section argues that the general equilibrium feedback effects - through secondary market adjustment - have the potential of undoing the aggregate benefits of strong bankruptcy regulation.

4.1 Optimizing Behavior

The optimization programs of workers, financiers, and entrepreneurs; the definition of the dynamic general equilibrium; and finally, the existence result for the

\textsuperscript{6}Seminal studies such as Townsend (1979) and Williamson (1986) provide costly state verification as a microfoundation for the debt repayment friction.
stationary equilibrium remains the same as in section 3. Since financier-worker distribution in the economy is independent of the financing frictions, the ratio between financiers and workers is still determined by

$$\frac{M_f}{M_w} = \frac{1 - \beta}{\beta}.$$ 

Applying the convenient definition $(1 - \psi_2) \equiv \frac{1}{1+\phi}$, the occupation indifference condition between being a financier and an entrepreneur solves the unit rate of return from providing finance, yielding the same steady-state expressions for the net financial returns $R(\phi)$

$$R(\phi) = A \frac{1}{1+\alpha} \left( \frac{1 - \alpha}{B \beta^\beta (1 - \beta)^{1-\beta}} \right) \left( \frac{\alpha}{1+\phi} \right)^\frac{\alpha}{1+\alpha},$$

(27)

and for the unit cost of capital finance $\hat{R}(\phi)$

$$\hat{R}(\phi) = R^2 (1 + \phi) \propto (1 + \phi)^\frac{1-\alpha}{1+\alpha},$$

as in section 3.

When financing frictions are ex-post with respect to the secondary market trading date, the primary financiers do not directly incur the cost of long-term debt finance. Therefore, $p = R$ in equilibrium, with

$$p(\phi) = A \frac{1}{1+\alpha} \left( \frac{1 - \alpha}{B \beta^\beta (1 - \beta)^{1-\beta}} \right) \left( \frac{\alpha}{1+\phi} \right)^\frac{\alpha}{1+\alpha},$$

which implies that the secondary market price of debt rises following a permanent reduction in ex-post financing frictions. Denoting the per-financier primary market debt invested in entrepreneurial projects again by $x$, the aggregate debt stock invested in long-term entrepreneurial projects equals to $M_f x$ which implies that the aggregate compensation of primary financiers must be $M_f x p$. Since in equilibrium each financier produces $w = B \left( \frac{\beta}{1-\beta} \right)^\beta$ units of finance, we can derive
and observe that $x$ increases with $\phi$ since $p$ is a decreasing function of the ex-post financing friction. That means the supply of long-term debt provided by each financier in the primary debt market contracts with a financial development that reduces the ex-post effort cost of providing finance.

We can express the market clearance in the primary debt market again as the following:

$$M_f \frac{w}{1+R} = M_e \left( \frac{\alpha A}{R^2(1+\phi)} \right)^{\frac{1}{1-\alpha}},$$

$$\Rightarrow M_f \frac{V_e(1-\beta)}{R(1+R)} = M_e \left( \frac{\alpha A}{R^2(1+\phi)} \right)^{\frac{1}{1-\alpha}},$$

$$\Rightarrow M_f(1-\beta)(1-\alpha)A \left( \frac{\alpha A}{R^2(1+\phi)} \right)^{\frac{1}{1-\alpha}} \left( \frac{1}{R(1+R)} \right) = M_e \left( \frac{\alpha A}{R^2(1+\phi)} \right)^{\frac{1}{1-\alpha}}.$$

Solving for $M_f/M_e$ ratio yields:

$$\frac{M_f}{M_e} = \left( \frac{\alpha(1-\beta)}{1-\alpha} \right) \left( \frac{1+R}{R(1+\phi)} \right) = \left( \frac{\alpha(1-\beta)}{1-\alpha} \right) \left( \frac{1+p}{p(1+\phi)} \right). \quad (29)$$

The equation (29) shows that the $M_f/M_e$ ratio - similar to the financial regime 1 with costly primary debt market finance - increases as $\phi$ declines. The population measures of workers, financiers and entrepreneurs have the following general equilibrium expressions:

$$M_w = \left( \frac{\alpha \beta}{1-\alpha} \right) \left( \frac{1+p}{p(1+\phi) + \frac{\alpha}{1-\alpha}(1+p)} \right) M, \quad (30)$$

$$M_f = \left( \frac{\alpha(1-\beta)}{1-\alpha} \right) \left( \frac{1+p}{p(1+\phi) + \frac{\alpha}{1-\alpha}(1+p)} \right) M, \quad (31)$$

$$M_e = \frac{p(1+\phi)}{p(1+\phi) + \frac{\alpha}{1-\alpha}(1+p)}. \quad (32)$$
As in section 3, I obtain the following relationship between the size of the financial sector and financial development.

**Proposition 4.1**

*If financial development reduces the effort costs related to the ex-post financing frictions, then in the general equilibrium*

1. The total quantity of financiers increases with financial development.
2. The total compensation of financiers relative to the aggregate income rises with financial development.

Results 3.2 and 4.1 show that reducing ex-ante and ex-post financing costs stimulate the financial sector size as well as the aggregate compensation captured by the financial sector.

**Supply of Long-term Debt and the Secondary Market.** Equilibrium in the secondary debt market as in section 3 continues to imply that

\[
\frac{M_{f1}}{M_{f2}} = \frac{1}{p} = \frac{1}{\hat{R}}. \tag{33}
\]

Since \( p \) is a decreasing function of \( \phi \), the fraction of secondary debt financiers rises as the ex-post financing costs decrease. Solving equations (31) and (33) together provides the aggregate supply long-term debt, \( K \), available for entrepreneurial investment as

\[
K = M_{f1}w, = \left( \frac{\alpha (1 - \beta)}{1 - \alpha} \right) \left( \frac{1 + p}{p(1 + \phi) + \frac{\alpha}{1 - \alpha} (1 + p)} \right) \cdot \frac{1}{1 + p} \cdot M_{w}. \tag{34}
\]

There are two counteracting channels where financial development and the supply of long-term debt interact: (i) Financial development expands the size of the intermediary sector; however, (ii) the prices in the secondary market increase with
financial development which suppresses the aggregate measure of primary market finance relative to the secondary market finance. Denoting the first partial derivative of \( p \) again with \( p' \) where \( p' < 0 \), the first-partial derivative of \( K \) with respect to \( \phi \) can be derived as

\[
\frac{\partial K}{\partial \phi} = \left( \frac{\alpha(1 - \beta)}{1 - \alpha} \right) \left[ -p' \left( 1 + \phi + \frac{\alpha}{1 - \alpha} \right) - p \right] \left( \frac{\alpha(1 - \beta)}{1 - \alpha} \right) \left[ (1 + p) + \frac{\alpha}{1 - \alpha} (1 + p) \right]^2,
\]

with \( \frac{\partial K}{\partial \phi} < 0 \) if \( \frac{\alpha^2}{1 - \alpha} - 1 \equiv \bar{\phi} < \phi \), and \( \frac{\partial K}{\partial \phi} > 0 \) if \( \bar{\phi} > \phi \). \( (35) \)

Therefore, reducing the ex-post financing frictions, stimulates the primary debt supply if and only if the initial size of the friction is large enough.

The size of the secondary market in the steady-state is again measured as

\[
S = M_{f2w} = \frac{\alpha(1 - \beta)(1 + p)}{\alpha + p + (1 - \alpha)\phi} \cdot Mw.
\]

Since both the total quantity of financiers in the economy as well as the secondary market financiers rise with financial development, we obtain

\[
\frac{\partial S}{\partial \phi} < 0.
\]

I summarize the two key results that I obtained regarding the relationship between the size of the ex-post financing frictions and the behavior of financial markets with the following proposition.

**Proposition 4.2**

*If financial development reduces the effort costs related to the ex-post financing frictions, then in the general equilibrium*
i. the supply of long-term debt increases with financial development if only if ex-post financing frictions are large enough, or in other words if \( \phi > \bar{\phi} \) with \( \bar{\phi} \) as defined at (35),

ii. the size of the secondary market decreases with financial development.

Since 1980s, the U.S. secondary markets that allow the trade of long-term financial instruments grew exponentially. For instance, the volume of secondary debt markets have exhibited a tremendous development, rising from an annual volume of about 10 billion U.S. Dollars in 1990 to 400 billion U.S. Dollars in 2012. Over the same time-period the aggregate stock of debt instruments available for the corporate sector was more less stable - rising only slightly from 6.04\% of the U.S. GDP in 1990 to 6.14\% in 2005. The theoretical financial development exercise presented in this section can provide a theoretical foundation for these stylized empirical facts.

Steady-state Consumption.

The steady-state expression for per-capita consumption net of effort costs has the same closed form expression as in section 3:

\[
\begin{align*}
c & = w(1 - \beta)R, \\
& = (1 + \phi)^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left(w(1 - \beta)\right)^{\frac{\alpha}{1-\alpha}} \left(\alpha^\alpha(1 - \alpha)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{38}
\end{align*}
\]

Ex-post financing frictions impact per-capita consumption through two channels as well: (1) The direct deadweight loss effect and (2) the indirect effect through the impact of frictions on aggregate entrepreneurial output. The former effect is negative - as it was also the case with ex-ante financing frictions. In order to evaluate whether ex-post frictions stimulate the per-capita consumption in steady-state we have to study comparative statics at the aggregate entrepreneurial output with respect to \( \phi \). The aggregate output from entrepreneurial projects is equal to:

\[
\begin{align*}
W & = M_e A \left[ \frac{K}{M_e} \right]^\alpha, \\
& = M_e A \left[ \frac{w(1 - \beta)}{p(1 + \phi)} \frac{\alpha}{1 - \alpha} \right]^\alpha.
\end{align*}
\]
When we define \( z \equiv \frac{1-\alpha}{\alpha} \), and use (32) in \( W \), \( W \) as a function of \( p \) is expressed as

\[
W = \frac{[p(1+\phi)]^{1-\alpha} A z^{1-\alpha} [w(1-\beta)]^\alpha M.}
\] (39)

An important implication of the model is revealed when we compare the steady-state aggregate output from entrepreneurial projects that we derived at (39) (with \( p = R \)), against the aggregate entrepreneurial output at (24) (with \( p = R(1 + \phi) \)): Holding everything else constant, shifting the effort cost of finance from secondary financiers to primary financiers such that \( p = R \) goes up to \( p = R(1 + \phi) \) lowers the entrepreneurial project output. In order to observe this property note that \( W_{\text{ex-\cdot post}} > W_{\text{ex-\cdot ante}} \) if and only if

\[
\frac{1}{\alpha + \alpha R + (1 - \alpha)R(1 + \phi)} > \frac{1}{\alpha + R(1 + \phi)},
\]

which is satisfied as long as \( \phi > 0 \). Since entrepreneurial investment is the engine of the productivity growth in the economy, the steady-state productivity of an economy governed by financial regime 2 (\( p = R \)) is higher than the steady-state productivity of an economy with financial regime 1 (\( p = R(1 + \phi) \)). I summarize this important result with the following proposition.

**Proposition 4.3** *Ceteris paribus, a financial regime that delegates the effort cost of debt finance to primary financiers has lower aggregate productivity compared to a regime where secondary financiers incur the cost of providing finance.*

**Financial Development and Macroeconomic Performance.** I consider again a financial development exercise at the beginning of a period \( \tau \) that reduces the cost ex-post financing friction \( \phi \) permanently. As in section 3, I will first study the effects of financial development on steady-state output produced by entrepreneurial projects and then draw conclusions for the per-capita consumption in the steady-state equilibrium.

In order to analyze the effects of a permanent reduction in \( \phi \) on steady-state output produced by entrepreneurs, I differentiate the expression at (39) with respect to \( \phi \).
and show that $\frac{\partial W}{\partial \phi} < 0$ if and only if:

$$[ (1 - \alpha) p^{-\alpha}(1 + \phi)^{1-\alpha} p' + p^{1-\alpha}(1 + \phi)^{-\alpha} ] [1 + p + zp(1 + \phi)] < [1 + z(1 + \phi)p' + zp][p^{1-\alpha}(1 + \phi)^{1-\alpha}],$$

which simplifies to:

$$\begin{align*}
(1 - \alpha)p^{-\alpha}(1 + \phi)^{1-\alpha} p' + p^{1-\alpha}(1 + \phi)^{-\alpha} (1 + p) & \equiv X \\
\alpha p^{1-\alpha}(1 + \phi)^{1-\alpha} p' + (1 - \alpha)p^{1-\alpha}(1 + \phi)^{2-\alpha} p' & \equiv Y.
\end{align*}$$

(40)

At inequality (40), all terms with $p'$ are negative. Since $|Y| > |X|$ (as defined at inequality (40)), it shows that the inequality (40) holds only if $p < \bar{p}$ where $\bar{p}$ is a threshold with $\bar{p} < 1$. The inequality (40) does not hold for any parameter values that satisfy a non-autarkic investment equilibrium.

**Proposition 4.4** A permanent reduction in ex-post financing frictions lowers the steady-state output produced by entrepreneurial projects.

Financial development through a reduction in ex-post financing frictions has two effects on steady-state per-capita consumption: (1) The direct (positive) impact that increases the consumption immediately following a reduction in financing frictions and (2) the indirect (negative) impact that contracts the output from long-term projects which suppresses $A$ in steady-state. The net effect of financial development on steady-state consumption depends on the relative weights between the two channels. Optimizing the steady-state consumption derived at (38) with respect to $\phi$ provides the level of ex-post secondary market frictions that maximizes the steady-state macroeconomic performance

$$\bar{A}g'(W) \left( \frac{\partial W}{\partial \phi} \right) = \alpha \frac{A}{1 + \phi},$$

(41)

where $\phi$ is the optimum level of ex-post debt financing frictions.
Transitory Dynamics. Following a permanent reduction of $\phi$ in period $\tau$, $A$ does not adjust until period $\tau+2$. Therefore, per-capita consumption in period $\tau+1$ rises. Since $A_t$ steadily adjusts downwards with perpetual contractions in $W_t$, the per-capita consumption in period $\tau+1$ exceeds the steady-state per-capita consumption. This is a result contrasting with the transitory dynamics derived in section 3. If $\phi < \hat{\phi}$, lowering enforcement costs further is counterproductive for steady-state consumption. Figure 2 summarizes the dynamic implications of ex-post financing frictions.

FIGURE 2 ABOUT HERE

4.2 Discussion

Comparing the results from sections 3 and 4 shows that understanding the microfoundations of debt finance frictions is important for uncovering the effects of financial development on steady-state macroeconomic performance.

Why is financial development counterproductive when providing debt finance is costly for secondary financiers? Because, excessive demand for secondary market financial claims resulting in from low secondary finance costs generates an allocation of resources in the decentralized equilibrium that is sub-optimal. Therefore, secondary finance friction $\phi$ could serve as a welfare-improving “tax-wedge” on financial gains when secondary financiers incur the effort cost of providing finance.

This analytical conclusion has important empirical implications and also relevance for financial market policies. Cross-country empirical evidence suggests that financial development and economic growth are not always positively associated. For example, Bandiera et al. (2000) suggest that the effects of the domestic financial liberalization on economic growth are mixed. The authors argue that for 1970-1994 time period, the relationship between banking deregulation and economic performance is negative and significant in Korea and Mexico\textsuperscript{7}, whereas it is positive and

\textsuperscript{7}Similar conclusions are reached for Korea by Demetriades and Luintel (2001) and Castro et al. (2004).
significant in Turkey and Ghana. Pagano and Jappelli (1994) show that borrowing limits and economic growth displayed a negative correlation during 1980s in OECD countries, and Bayoumi (1993) found similar results for the United Kingdom for the same time interval.

For developing countries, there are cases where well-intended financial development policies reduced economic growth. For example, the Latin American countries experienced secular banking deregulation experiences during 1990s, which according to Fostel and Geanakoplos (2008), generated an under-supply of market liquidity and undermined economic performance. The theoretical results presented in sections 3 and 4 offer an understanding for the conditions that policy makers should pay attention in order to avoid financial market policies with potential counterproductive consequences, especially when long-term investment and liquid secondary markets are essential for the real economic performance.

5 Financial Markets with Uniform Cost of Debt Finance

De facto costs of providing debt finance are related to the efficiency of courts in a society, and when court transaction costs decrease any type of financing friction might get alleviated. Therefore, in this section I study a numerical exercise with $\psi_1 = \psi_2 \equiv \psi$ as presented in chart 4, such that ex-ante and ex-post financing costs are both associated with the same level of financier effort. Again, I define $(1 - \psi) = \frac{1}{1+\phi}$ and study the aggregate implications of a contraction in $\phi$.

Following the same line of thinking applied in the previous two sections, we can show that financiers’ returns, entrepreneurial cost of finance and the secondary market price of debt in a stationary equilibrium are proportional to the financing friction
\( \phi \) as in the following:

\[
\begin{align*}
R & \propto (1 + \phi)^{\frac{2\alpha}{1+\alpha}}, \\
\dot{R} & \propto (1 + \phi)^{\frac{2-2\alpha}{1+\alpha}}, \\
p & \propto (1 + \phi)^{\frac{1-\alpha}{1+\alpha}},
\end{align*}
\]

with \( \frac{\partial R}{\partial \phi} < 0, \frac{\partial \dot{R}}{\partial \phi} > 0, \frac{\partial p}{\partial \phi} > 0. \)

Equilibrium in the primary debt market implies that

\[
\frac{M_f}{M_e} = \frac{\alpha(1 - \beta) R(1 + p)}{1 - \alpha p^2}.
\]

As I have also obtained in the previous two sections, \( M_f/M_e \) goes up; and therefore, the financial sector expands following a contraction in \( \phi \). Secondary market clearance condition

\[
\frac{M_{f1}}{M_{f2}} = \frac{1}{p},
\]

implies that \( M_{f1}/M_{f2} \) goes down as financing frictions decline. Therefore, the equilibrium quantity of aggregate long-term debt,

\[
K = M_{f1} \cdot \frac{1}{1 + \dot{R}} \cdot w,
\]

rises with financial development.

Finally, the steady-state consumption net of financing effort costs and the aggregate entrepreneurial production can be solved as

\[
\begin{align*}
c & = w(1 - \beta)R \\
& = (1 + \phi)^{\frac{2\alpha}{1+\alpha}} A^{\frac{1}{1+\alpha}} [w(1 - \beta)]^{\frac{2\alpha}{1+\alpha}} \left( \alpha^\alpha (1 - \alpha)^{1-\alpha} \right)^{\frac{1}{1+\alpha}}, \\
W & = \frac{(1 - \alpha)p^{2(1-\alpha)}R^\alpha}{\alpha R + \alpha Rp + (1 - \alpha)p^2} \left( \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} A[w(1 - \beta)]^\alpha M.
\end{align*}
\]
5.1 A Numerical Exercise

In order to understand the numerical implications of debt financing costs across three financial regimes, I simulate each regime using the same parameter values and conduct financial development experiments. Table 1 summarizes, the parameterization of $\alpha$, $\beta$, $B$, $\bar{A}$, $\theta$, and $\phi$, and the steady-state values of aggregate consumption, aggregate consumption net of enforcement costs, the relative size of financial sector ($M_f/M_e$), net financial return, the aggregate supply of long-term debt $K$, the aggregate entrepreneurial production, and finally the aggregate long-term productivity.

TABLE 1 ABOUT HERE

The model is parameterized such that the aggregate steady-state consumption for the model presented in this section with uniform financing frictions equals to 100 in the steady-state. Across all three financial regimes the parameter that captures the intermediary’s cost, $\phi$, equals to 0.1.

As also predicted by proposition 4.3, the model with ex-post cost of providing finance has the highest steady-state net consumption in per-capita. Steady-state consumption net of enforcement frictions in an economy where both types of intermediaries incur the financing costs exceeds that of the specification with only ex-ante financing frictions.

Finally, table 2 presents a hypothetical policy experiment where I increase the debt finance friction from 0.1 to 0.2 in all three financial regimes. Steady-state consumption net of effort costs decreases by 5.73 units in uniform finance costs model, and by 5.60 units in costly ex-ante financing model. In costly ex-post financing model the same hypothetical financial repression leads to a rise in net consumption by 0.50 unit.

TABLE 2 ABOUT HERE

35
6 Financial Markets with Heterogeneous Effort Costs of Debt Finance

In this section I study an extension to the model, where an exogenously determined \( \eta \) fraction of all entrepreneurial projects require ex-ante effort costs with respect to the secondary market trading date whereas \( 1 - \eta \) fraction of the entrepreneurial projects require ex-post effort costs, such that \( (1 - \psi_1) = \frac{\eta}{1 + \phi} \) and \( (1 - \psi_2) = \frac{1 - \eta}{1 + \phi} \) as presented in chart 5. I will derive a critical level of \( \eta^* \), and show that reducing financing frictions stimulate the steady-state aggregate output produced by entrepreneurial projects only if \( \eta \geq \eta^* \). I also analyze the behavior of \( \eta^* \) with respect to the equilibrium quantity of finance produced by each financier, and draw conclusions concerning the effects of the stage of economic development on finance-development nexus.

6.1 Equilibrium

I denote the steady-state per-financier debt provided in the primary debt market again with \( x \). The aggregate capital invested in entrepreneurial production equals to \( M_f x \) which implies that the aggregate compensation of primary savers must equal to \( M_f x R(1 + \eta \phi) \). We can derive \( x \) as a function of \( p \), or \( R \) and \( \phi \):

\[
x = \frac{w}{1 + p} = \frac{w}{1 + R(1 + \eta \phi)}.
\]

The primary debt market clearance yields

\[
M_e = \frac{z R(1 + \phi)}{1 + (1 + z) R(1 + \eta \phi)}, \quad (42)
\]

where \( z \equiv \frac{1 - \alpha}{\alpha} \). Using the expression for \( M_e \) in the aggregate production function, the steady-state aggregate output produced by entrepreneurial projects is stated as

\[
W = \frac{[R(1 + \phi)]^{1-\alpha}}{1 + (1 + z) R(1 + \eta \phi)} A \left( \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} [w(1 - \beta)]^{\alpha} M. \quad (43)
\]
Note that, $R$ again is determined uniquely by the endogenous occupation choice as in sections 2-5. Therefore, $\partial R / \partial \phi < 0$ continues to hold.

Define $\tilde{R}(\phi) = R(1 + \phi)$ where $\partial \tilde{R} / \partial \phi > 0$. Differentiating $W$ with respect to $\phi$ we can show that lowering $\phi$ would stimulate steady-state aggregate long-term project output only if:

$$
(1 - \eta)(1 + z)\left( R'(\phi)\phi + R(\phi) \right) < \tilde{R}'(\phi) \left( (1 + z)(\tilde{R}(\phi))^{1-\alpha} - (1 - \alpha)(\tilde{R}(\phi))^{-\alpha} [1 + (1 + z)\tilde{R}(\phi)] \right) \tag{44}
$$

**Proposition 6.1** There exists a critical $\eta^* < 1$ such that if and only if

$$
\eta > \eta^*,
$$

the inequality at (44) is satisfied, and financial development stimulates the steady-state aggregate long-term project output.

With financial regime 1, financial development promotes economic development whereas with financial financial regime 2 financial development could be counter-productive. Proposition 6.1 shows that when the financial regime is characterized as a combination of the two, the hybrid regime should resemble the financial regime 1 - with costly primary finance - as much as possible in order financial development and economic growth to be positively related.

Since the quantity of finance produced by each financier in the general equilibrium continues to be a function of $\beta$ with

$$
w = \frac{\beta}{1 - \beta},
$$

using (44) we can study the behavior of $\eta^*$ with respect to $w$. Note that, $R(\phi)$, $R'(\phi)$ and $\tilde{R}'(\phi)$ are proportional to $(\frac{1}{w})^{\frac{1-\alpha}{1+\alpha}}$. Therefore, it is useful to re-write (44) as:
\[ (1 - \eta)(1 + z) \left( \frac{R'(\phi)\phi + R(\phi)}{R'(\phi)} \right) \]
\[ \leq (1 + z) \left[ \ddot{R}(\phi) \right]^{1-\alpha} - (1 - \alpha) \left[ \ddot{R}(\phi) \right]^{-\alpha} [1 + (1 + z)\ddot{R}(\phi)] \quad (45) \]

**Proposition 6.2** The threshold \( \eta^* \) is an increasing function of \( w \).

**Proof** The left hand-side of (45) is constant in \( w \) whereas the right hand side is a decreasing function of \( w \). Hence, the threshold \( \eta^* \) that satisfies (45) with an equality increases as \( w \) rises. \( \square \)

### 6.2 Discussion

The ability of financiers in converting deposits into units of finance is expected to be higher in economically advanced societies. To this end, proposition 6.2 provides an important insight concerning the influence of economic development on how finance and macroeconomic performance might be related: In a high income economy, the fraction of primary financiers who incur the ex-ante costs of finance must be relatively large compared to a developing country such that financial development could promote the aggregate output produced by entrepreneurial projects. This implies that in high-income countries the chances of financial development policies to be counterproductive is relatively higher compared to low income countries. This result matches with empirical findings that point out a potential non-monotone relationship between finance and economic development: For instance, Castro et al. (2004) measure costly external finance by “limited investor protection” and present empirical evidence for the non-linear growth effects of financial development. An important empirical result from their work shows that investor protection and economic growth exhibit a negative relationship especially for high income countries. Similar conclusions are also reached by Reinhardt and Tokatlidis (2005). The authors show that following the implementation of domestic financial deregulation policies, the economic growth rates in low income countries rises relatively more compared to high income countries. The current theoretical analysis suggests a role for secondary market trading in explaining this empirical pattern.
7 Conclusion

I studied the interactions between debt financing costs, secondary market trading and macroeconomic performance using an overlapping generations model of occupation choice. The key analytical finding from the paper is the endogenous existence of a non-monotone effect of debt financing frictions on steady-state output. I characterized the behavior of asset prices and occupational allocations with respect to changes in financial development and showed that reducing the ex-ante financing costs are potentially much more important for macroeconomic performance compared to reducing the ex-post financing costs.

The results from sections 3 and 4 show that holding everything else constant, a financial regime that delegates the enforcement of debt repayment to primary financiers is less productive from a macroeconomic point of view relative to a regime where secondary financiers incur debt financing costs. Finally, as presented in section 5, countries with a high level of economic development are the most likely candidates to suffer from counterproductive financial development experiences.

The theoretical results presented in this paper provide empirically testable predictions and important policy conclusions. The regulatory framework as well as the complexity of financial instruments sold in secondary markets are expected to dictate whether primary or secondary financiers of long-term debt carry the burden of debt contracts. The conclusions from my analysis suggests that policy makers need to pay attention to the structure of financial markets and the distribution of effort costs of providing finance across financiers in order to avoid financial sector policies with potential counterproductive consequences.
References


Chart 1. Long-term Investment and Finance

Long-term Entrepreneur in Period $\tau$

$\tau$ \hspace{2cm} $\tau + 1$

Invest $k_\tau$ \hspace{2cm} Returns $\ell_{\tau+1}(k_\tau)$

Extend Debt $\uparrow$ \hspace{1cm} Trade $\leftrightarrow$ \hspace{1cm} Collect Returns $\downarrow$

Primary-Financier \hspace{1cm} Secondary-Financier

Financiers in Periods $\tau$ and $\tau + 1$

Chart 2. Ex-ante Financing Costs

Long-term Entrepreneur from Cohort $\tau$

$\tau$ \hspace{2cm} $\tau + 1$

$k_\tau$ \hspace{2cm} $\ell_{\tau+1}(k_\tau)$

$\psi_1 > 0$ \hspace{2cm} $\psi_2 = 0$

Primary-Financier \hspace{1cm} Secondary-Financier

Financiers from Cohorts $\tau$ and $\tau + 1$
Chart 3. Ex-post Financing Costs

Long-term Entrepreneur from Cohort $\tau$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\tau + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_\tau$</td>
<td>$\ell_{\tau+1}(k_\tau)$</td>
</tr>
</tbody>
</table>

$\psi_1 = 0$ $\quad$ $\psi_2 > 0$

Primary-Financier $\quad$ Secondary-Financier

Financiers from Cohorts $\tau$ and $\tau + 1$

---

Chart 4. Uniform Financing Costs

Long-term Entrepreneur from Cohort $\tau$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\tau + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_\tau$</td>
<td>$\ell_{\tau+1}(k_\tau)$</td>
</tr>
</tbody>
</table>

$\psi_1 = \psi$ $\quad$ $\psi_2 = \psi$

Primary-Financier $\quad$ Secondary-Financier

Financiers from Cohorts $\tau$ and $\tau + 1$
Table 1: A Numerical Comparison Across Models

<table>
<thead>
<tr>
<th></th>
<th>Uniform Costs</th>
<th>Ex-ante Costs</th>
<th>Ex-post Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agg. Cons.</td>
<td>100</td>
<td>96</td>
<td>102</td>
</tr>
<tr>
<td>Net Cons.</td>
<td>93.38</td>
<td>93.00</td>
<td>100.09</td>
</tr>
<tr>
<td>$M_f/M_e$</td>
<td>2.48</td>
<td>2.73</td>
<td>2.51</td>
</tr>
<tr>
<td>$R$</td>
<td>1.22</td>
<td>1.21</td>
<td>1.31</td>
</tr>
<tr>
<td>$K$</td>
<td>23.27</td>
<td>23.96</td>
<td>23.68</td>
</tr>
<tr>
<td>$W$</td>
<td>68.78</td>
<td>63.90</td>
<td>73.25</td>
</tr>
<tr>
<td>$A$</td>
<td>7.12</td>
<td>6.68</td>
<td>7.53</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>$B$</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$w = B \left( \frac{\beta}{1-\bar{\beta}} \right)^{\bar{\beta}}$</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Productivity process: $A_{L,t} = \bar{A}_L \left[ 1 + \theta W_L \left( A_{L,t-1} \right) \right]$

**Chart 5. Heterogeneous Financing Costs**

**Long-term Entrepreneur from Cohort $\tau$**

<table>
<thead>
<tr>
<th></th>
<th>$\tau$</th>
<th>$\tau + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_\tau$</td>
<td>$\ell_{\tau+1}(k_\tau)$</td>
<td></td>
</tr>
</tbody>
</table>

$\psi_1 > 0$ $\quad$ $\psi_2 = \psi_1 \frac{1-\eta}{\eta}$

Primary-Financier $\quad$ Secondary-Financier

Financiers from Cohorts $\tau$ and $\tau + 1$
Table 2: $\phi$ increases from 0.1 to 0.2.

<table>
<thead>
<tr>
<th>Δ in Net Cons.</th>
<th>Uniform Costs</th>
<th>Ex-ante Costs</th>
<th>Ex-post Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.73</td>
<td>-5.60</td>
<td>0.50</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 1](image-url)

Figure 1. Permanent Reduction in $\phi$
Figure 2.

A: Permanent Reduction in $\phi$ if $\phi < \hat{\phi}$

B: Permanent Reduction in $\phi$ if $\phi > \hat{\phi}$