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THE ADVERSE AND BENEFICIAL EFFECTS OF FRONT-LOADED PENSION CONTRIBUTIONS

By

Ed Westerhout

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Ed Westerhout*

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Abstract

In funded defined benefit pension schemes, contribution and accrual rates are typically age-independent. This implies that pension contributions are front-loaded. As contributions and accruals usually relate to earned labour income, this front-loading may affect labour market efficiency. For it implies that the labour supply of younger workers is implicitly taxed and that of older workers implicitly subsidized. This paper shows that front-loading of pension contributions may be welfare-reducing. It also shows that the welfare loss is weakened if one accounts for government spending that is financed with a labour income tax. In this case, front-loading may even be welfare-increasing. In particular, the more elastic is the labour supply of older workers relative to that of younger workers, the more likely it is that front-loading produces a welfare gain.

*Ed Westerhout is affiliated with the department Tax Economics at Tilburg University and with CPB Netherlands Bureau for Economic Policy Analysis. This paper was presented at the 2019 IIPF Conference, Glasgow, and at the 2020 Netspar International Pension Workshop, Leiden. I thank my discussants, Louise Sheiner and Daniel van Vuuren respectively, for very helpful comments. I further thank Reyer Gerlagh, Jurre de Haan, Marcel Lever, Sander Muns, Siert Vos and Sweder van Wijnbergen for discussion and comments on an earlier version of this paper. All errors are my own. Affiliations: Department Tax Economics, Tilburg University; CPB Netherlands Bureau for Economic Policy Analysis; Netspar. E-mail: E.W.M.T.Westerhout@uvt.nl. Tel. +31 6 499 66 435.
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Pensions, Front-loaded pension contributions, Labour supply, Retirement, Distortionary taxes

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1 Introduction

Linda, 25 years old, has an annual income of EUR 20,000. She considers increasing her number of working hours per week from 20 to 24. If she does, she will have to contribute EUR 1,866.40 extra to her pension scheme. Her pension rights will increase less, namely EUR 1,226.40. Sara, 60 years old, happens to be in the same position as Linda, except for her age. If Sara adds 4 hours to her working week, she will pay only EUR 933.20 more contributions to the pension scheme, half as much as Linda, whereas her pension rights will increase as much as Linda’s, EUR 1,226.40.\footnote{This calculation assumes that both persons work from the age of 25 to 65, that they receive a pension from the age of 65 to 85, that labour income is constant, pension benefits are 75 percent of labour income, pensions are fully indexated against price inflation and the annual capital market rate of return is 2 percent. Furthermore, all amounts in the example are expressed in terms of euros at the age of retirement.}

Worldwide, pension reform is at the top of the policy agenda. Against the background of ageing populations and changing labour market patterns, many countries have reformed their pension schemes or are considering pension reform. In particular, some countries have made the move to individual defined contribution (DC) schemes (Chile) or have switched more gradually away from defined benefit (DB) towards defined contribution schemes (Australia, US, UK). Other countries have transformed their traditional pay-as-you-go schemes into notional defined contribution schemes (Italy, Latvia, Poland, Sweden). Thirdly, some countries introduced collective defined contribution schemes to replace traditional collective defined benefit schemes (Canada, Denmark, the Netherlands). Furthermore, many countries have adopted measures to stepwise increase the pension retirement age (see Bonenkamp et al. (2017) for a more detailed overview).

The current government in the Netherlands is considering reform of the second-pillar pension scheme (Regering, 2017; Ministerie van SZW, 2019). One of the elements of the reform package is the replacement of the current scheme of linear, age-independent accrual rates with a degressive scheme in which accrual rates decline with age. The aim is to make the second pillar of the Dutch pension scheme more actuarially fair. This would help to end the redistribution between workers of different age that is implied by a linear accrual scheme. It would also reduce the redistribution from the relatively poor to the relatively rich (Chen and Van Wijnbergen, 2019). And, as underlined in Ministerie van SZW (2019), it would improve the connection with the current dynamic labour market, ease the introduction
of more flexibility in the accumulation and pay-out phase and make it more attractive for the self-employed to opt in into the pension scheme that is currently obligatory only for salary workers.

This paper argues that this type of pension reform may have important labour market effects. The argument rests on two observations. The first is that a linear accrual scheme implies redistribution from young to old workers. As pension contributions increase in value over time because of the interest earned, pension contributions are front-loaded relative to accruals. Hence, young workers pay more than they earn in terms of pension accruals, whereas for old workers the reverse holds true. The second observation is that both pension contributions and accruals typically relate to earned labour income. That means that the income transfers between young and old workers may be perceived as implicit taxes (in case of young workers) and implicit subsidies (in case of old workers). If the pension reform studied here implies that these taxes and subsidies disappear, this may affect the labour market behaviour of both young and old workers.

One may expect that the reform will improve labour market efficiency as it removes two distortions on labour markets. As we will show, this expectation is correct in a stylized world in which labour income goes untaxed. In a world with a labour income tax, the effect upon labour market efficiency is ambiguous, however. In particular, if labour supply behaviour of older workers is sufficiently more elastic than that of younger workers, the reform is welfare-reducing. In light of policies aimed at increasing labour market participation on account of population ageing, this may be considered an important result, not only for the Netherlands, but also for a number of other countries. For DB pension schemes in many OECD countries feature linear accrual and contribution rates (Whitehouse, 2006; Van Vuuren, 2014; Chen and Van Wijnbergen, 2019).23

The combination of linear accrual rates and linear contribution rates is typical for DB schemes and for collective defined contribution schemes like those in the Netherlands. Individual defined contribution schemes are opposite, however: in these schemes contributions and accruals match each other by definition. Over time, DB schemes are losing market share worldwide, whereas DC schemes become gradually more important. This is illustrated in Willis Towers Watson (2018) who show that for the seven largest pension markets that together hold 91 percent of pension assets in the world, the share of DC has increased about 50 percent in only 20 years time. However, the DB scheme is still a major type of pension scheme, figuring in 17 out of 30 OECD countries (Whitehouse, 2006).

Our paper does not focus explicitly on pay-as-you-go (PAYG) schemes in which typically life-time contributions and accruals are different (we do analyze a collective DB scheme, however, which by its nature contains a PAYG element). Indeed, Fenge et al. (2006) show that a PAYG scheme may imply implicit taxation over the whole working
This paper builds upon earlier literature on the labour market implications of pension schemes. In particular, Krueger and Kubler (2006), Nishiyama and Smetters (2007) and Fehr and Habermann (2008) explore the connection between the labour market and pay-as-you-go pension schemes, whereas Disney (2004), Beetsma et al. (2013) and Bonenkamp and Westerhout (2014) focus on funded pension schemes. Further, Bonenkamp (2009) and Bovenberg and Gradus (2015) also discuss front-loaded pension contributions, but do not explore the potential labour implications as we do here.\footnote{There is an older literature on back-loading of pension benefits (Ippolito, 1985; Bodie et al., 1988; Kotlikoff and Wise, 1988). This literature focuses on the then more popular final-wage schemes. The present paper discusses average-wage schemes.}

The paper also relates to the literature on labour income taxation in general and age-dependent labour income taxation in particular (Atkinson and Sandmo, 1980; Erosa and Gervais, 2002; Fenge et al., 2006; Conesa et al., 2009; Weinzierl, 2011; Bastani et al., 2013; Farhi and Werning, 2013).\footnote{See Stantcheva (2017) for an analysis of age-dependent human capital policies.}

Our paper distinguishes from this literature in focusing on the taxes and subsidies that are implied by pension schemes with front-loaded contributions. Further, the paper is connected to the literature on the role of financial incentives in labour supply and retirement (Gruber and Wise, 1999; Chan and Stevens, 2004; Asch et al., 2005; Mastrobuoni, 2009; Hanel, 2010; Blundell et al., 2016) and to the literature on salience and taxation (Chetty et al., 2009; Brinch et al., 2017; Dolls et al., 2018).

This paper is structured as follows. Section 2 demonstrates how the combination of linear contribution and accrual rates makes pension contributions front-loaded relative to accruals. Sections 3 and 4 use the insights obtained from section 2 to explore the labour market and welfare effects of front-loading. Both sections adopt a stylized two-period model of the labour market, without (section 3) or with (section 4) government spending. Up till then, the paper describes an individual pension scheme. Next, section 5 explores how our results would change if we replaced the assumption used till then of an individual scheme with that of a collective scheme. Section 6 discusses some qualifications and section 7 offers concluding remarks.

phase of the life cycle. The two types of schemes do have something in common, however. Like our funded scheme, the PAYG scheme studied in Fenge et al. (2006) features declining implicit tax rates over the life cycle.
2 The implicit taxes and subsidies implied by front-loaded pension contributions

Generally, DB pension schemes apply linear contribution and accrual rates, i.e. pension contribution rates and accrual rates do not differentiate with respect to participant characteristics like age (Van Vuuren, 2014; Chen and Van Wijnbergen, 2019), although there are exceptions (Whitehouse, 2006). One could argue that this type of uniformity is intrinsically linked with the collective nature of the pension scheme. Sometimes uniformity is also legally required (Bovenberg and Gradus, 2015).

This uniformity creates an imbalance between contributions and accruals. A euro of pension contributions made by a young worker has a greater economic value than the same euro contributed by an old worker as the former euro has a longer time to accumulate and earn interest on it. Absent indexation, the economic value of a euro pension accrual for a young worker equals that for an old worker, however. Hence, uniformity implies that pension contributions are front-loaded relative to pension accruals. If pensions are indexed, to price or to wage inflation, then pension accruals also grow in value over time, but even then contributions are front-loaded, as long as the rate of return on pension wealth exceeds the rate of indexation.

Let us establish these claims more formally. The life of a household consists of a working phase and a retirement phase. The latter can for our purposes be confined to one period. We use $T$ to denote the length of life and $T_R = T - 1$ to denote the length of the working phase. Upon retirement, the household aims to have accumulated a given amount of pension wealth. This stock of wealth can, given there is no life uncertainty, be translated into a stream of pension benefits. The benefits can be indexed to the price level or wage level or not indexed at all (flat). But the stock of wealth can also represent an amount of money that is handed over to the household upon retirement. All interpretations are compatible with our framework.

As to the pension scheme, we make an assumption that may seem odd at first sight. We take the pension scheme to be an individual scheme in which lifetime contributions and accruals are equal to one another. This approach greatly simplifies the analysis. But this individual scheme differs from the collective schemes that we want to explore and in which lifetime contributions are in general not equal to lifetime accruals. In section 5

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6Bovenberg and Gradus (2015) talk about back-loading of pension benefits which refers to the same thing.

7This paper focuses on average-wage schemes, but could easily be generalized to cover also final-wage schemes.
we will show that the difference between the individual and the collective scheme is not relevant, however, for the purposes of this paper.

We use \( r \) to denote the real rate of return on the capital market, \( g \) to denote the rate of labour income growth and \( \mu \) to denote the real rate of indexation of pension accruals. For the sake of exposition, we take these variables constant through time. Below, we will discuss under what conditions a more general framework yields the same results. Furthermore, we assume that the rate of return exceeds the rate of indexation: \( r > \mu \). It is useful to consider this assumption more in detail, as this assumption is crucial for our analysis.

We argue that the assumption that \( r > \mu \) is very mild. In order to have the opposite condition, i.e. \( r \leq \mu \), we would need to assume that pension benefits are fully indexated against wage inflation and that the economy is dynamically inefficient - during all the working phase of the life cycle or at least a large part of it. To elaborate on the latter, dynamic inefficiency is more the exception than the rule (Jordà et al., 2019). As regards indexation, pension benefits are often indexated not against wage inflation, but against the lower rate of price inflation (Whitehouse, 2006). Moreover, indexation is not always full, but less than 100 percent.

Let us now start to describe the pension wealth that arises on account of the contributions to the pension fund. Equation (1) describes the accumulation of pension wealth, \( \Pi_t \),

\[
\Pi_t = (1 + r)(\Pi_{t-1} + \pi y_t)
\]

where \( \pi \) denotes the (age-independent) contribution rate and \( y \) denotes labour income. Pension wealth accumulates over time, due to the returns earned on it and due to the injections of new pension contributions.

Given that \( \Pi_0 = 0 \), it is straightforward to derive the total of pension contributions at the time of retirement:

\[
\Pi_{T_R} = \sum_{i=1}^{T_R} \pi y_i (1 + r)^{T_R-i+1} = \sum_{i=1}^{T_R} \pi y_T (1 + r) \left( \frac{1 + r}{1 + g} \right)^{T_R-i}
\]

The most RHS expression of equation (2) reflects the assumption that labour income grows at rate \( g \).

Let us denote the amount of pension wealth that the worker wants to have accumulated at the date of retirement as \( b \). Absent any intergenerational transfers, \( \Pi_{T_R} \) must then be equal to \( b \). This principle implies the
following expression for the contribution rate:

$$\pi = \frac{b}{\sum_{i=1}^{T_R} \left( \frac{1 + r}{1 + g} \right)^{T_R-i} (1 + r) y_{T_R}}$$  \hspace{1cm} (3)$$

A worker obtains pension rights by working. This can be seen in the following accumulation equation,

$$A_t = (1 + \mu)(A_{t-1} + \alpha y_t) \quad 1 \leq t \leq T_R \quad (4)$$

where $\alpha$ denotes the (age-independent) accrual rate. Every year of work adds $\alpha y_t$ to the stock of pension rights. On top of that, the stock of pensions accrued increases on account of indexation at rate $\mu$.

Given that $A_0 = 0$, we derive the following expression for the total of pension accruals at the time of retirement:

$$A_{T_R} = \sum_{i=1}^{T_R} \alpha y_i (1 + \mu)^{T_R-i+1} = \sum_{i=1}^{T_R} \alpha y_{T_R}(1 + \mu) \left( \frac{1 + \mu}{1 + g} \right) T_R-i \quad (5)$$

As in the case of $\Pi_{T_R}$, the most RHS expression of equation (5) shows that $A_{T_R}$ is proportional with $y_{T_R}$.

Equating $A_{T_R}$ to $b$ gives the following expression for the accrual rate:

$$\alpha = \frac{b}{\sum_{i=1}^{T_R} \left( \frac{1 + \mu}{1 + g} \right)^{T_R-i} (1 + \mu) y_{T_R}} \quad (6)$$

If we now combine equation (6) with equation (3), we can derive an expression for the contribution rate in terms of the accrual rate:

$$\pi = \alpha \left[ \frac{\sum_{i=1}^{T_R} \left( \frac{1 + \mu}{1 + g} \right)^{T_R-i} (1 + \mu)}{\sum_{i=1}^{T_R} \left( \frac{1 + r}{1 + g} \right)^{T_R-i} (1 + r)} \right] \quad (7)$$

This expression shows that the pension contribution rate is lower than the accrual rate (recall $r > \mu$). This reflects the front-loaded nature of pension contributions: given that pension contributions are tilted towards the beginning of the life cycle and accruals more to the end of the life cycle, the contribution rate must be lower than the accrual rate.

Another way to show the front-loaded nature of pension contributions is by elaborating how pension contributions and accruals evolve with age. We
measure both contributions and accruals in age-$T_R$ euros. Pension contributions at age $j$ ($j = 1, \ldots, T_R$) read as $\pi y_j(1 + r)^{T_R - j + 1}$. During the working phase, pension contributions thus grow with the factor $(1 + g)/(1 + r)$. Pension accruals at age $j$ read as $\alpha y_j(1 + \mu)^{T_R - j + 1}$. The annual growth rate of accruals is thus $(1 + g)/(1 + \mu)$. Recalling that $r > \mu$, we thus derive that pension contributions decline over the working phase relative to pension accruals. Since the totals of contributions and accruals coincide, this implies front-loading of contributions.

Two points require further discussion. The first is whether front-loading will also hold true if $r$, $g$ and $\mu$ vary through time? The answer is yes, if the following condition holds true: $\sum_{i=1}^{T_R} (1 + \mu_i) \prod_{j=i+1}^{T_R} ((1 + \mu_j)/(1 + g_j)) < \sum_{i=1}^{T_R} (1 + r_i) \prod_{j=i+1}^{T_R} ((1 + r_j)/(1 + g_j))$. Hence, $r_j > \mu_j$ in all years $1 \leq j \leq T_R$ is a sufficient condition for front-loaded pension contributions. It is not a necessary condition, however. Indeed, pension contributions will be front-loaded also if $r_j$ is equal to or smaller than $\mu_j$ in some years $j$, as long as $r$ exceeds $\mu$ in some average sense (with average sense defined precisely by the above inequality condition).

The second point is that in defining pension accruals one might adhere to what Ippolito (1985) has called the legal approach: workers account only for the nominal, not the indexed value of their pension rights. Ippolito (1985) explains this can be rationalized if the pension contract can be terminated at any time. Ippolito (1985) has argued against the legal approach on empirical grounds, but let us assume here that workers adopt the legal approach in assessing their accruals. Are pension contributions then still front-loaded?

The answer is that under the legal approach contributions are even more front-loaded. Under the rational approach adopted in this paper, workers account for the future indexation of their pension rights at the moment they obtain these rights. Under the legal approach however, they only account for the future indexation of their rights when the future arrives. See appendix A for a formal derivation.

The policy reform that this paper discusses involves replacing the scheme of linear contribution and accrual rates with a scheme that is more actuarially fair. There are two types of reform that achieve actuarial fairness. One is the move to a scheme of progressive contributions in which the value of the contribution in a year matches the value of the accrual in the same year. Formally, this boils down to changing the constant contribution rate $\pi$ into

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8One could also motivate this approach by arguing that workers are imperfectly informed about their pension wealth or over-pessimistic about the future.
an age-dependent contribution rate \( \pi_t \), defined as \( \alpha((1+\mu)/(1+r))^{TR-t} \). The alternative is the move to a scheme of degressive accruals in which the value of the accrual in every year is brought in line with the value of the contribution in that year. Formally, the constant accrual rate \( \alpha \) should be changed into an age-dependent accrual rate \( \alpha_t \), defined as \( \pi((1+r)/(1+\mu))^{TR-t} \). The Dutch government has announced to go for the latter option. Note that the two types of reform have in common that they remove the implicit taxes and subsidies that are due to linear contribution and accrual rates. Still, they are different in an important respect. A move to progressive contributions changes directly the life-cycle profile of disposable income; this effect is absent when a scheme of degressive accruals is implemented. We will return to the implications of this difference below.

Having illustrated the front-loaded nature of pension contributions, we now move to a more compact version of the model. In this version, the working life consists of two periods (\( TR = 2 \)). This simplifies the model, but does not change its basic features. We assume the pension benefit during retirement equals labour income in the current period times a replacement rate \( \beta \): \( b = \beta y_3 \).

Imposing \( TR = 2 \), equations (3) and (6) can then be written as follows:

\[
\pi = \frac{\beta}{(1 + r) \left( 1 + \frac{1+r}{1+g} \right)} \quad (8)
\]

\[
\alpha = \frac{\beta}{(1 + \mu) \left( 1 + \frac{1+\mu}{1+g} \right)} \quad (9)
\]

3 Front-loaded contributions in a first-best world

The previous section has demonstrated that the combination of linear contribution and accrual rates implies implicit taxes and subsidies. To explore the labour market and welfare effects of these what we will call front-loading policies, we set up a life-cycle model. We take the angle of an optimal tax problem in a world with identical households who can change their hours of work but whose productivity is taken as given. We thus abstract from positive or negative effects of front-loading on productivity.\(^9\)

\(^9\) We calculate \( y_3 \) as \((1+g)y_2\).

\(^{10}\) Kotlikoff and Wise (1988) discuss the contract view according to which back-loaded pension benefits may act as an incentive for the worker not only to remain in the firm, but also to work harder. Hence, front-loading of contributions may increase labour productivity. On the other hand, front-loading makes it unattractive for salary workers to become self-employed later in their career. To the extent that such a change of job market status
More specifically, we adopt a household life-cycle model with three periods in which the household is young and working, middle-aged and working, and retired. The household chooses its labour supply in the first two periods and consumption in the third period such as to maximize its lifetime utility. The front-loading of pension contributions enters this decision problem by changing the price of leisure in the first and second period. As we will see, front-loading does not affect lifetime wealth, since implicit tax revenues and implicit subsidies are equal in present-value terms. Furthermore, in this section, there is no government spending and no labour income taxation. We will leave these two features for the next section.

We describe the preferences of the household by the following intertemporal utility function,

$$U = u_1(v_1) + \frac{u_2(v_2)}{1 + \delta} + \frac{c_3}{(1 + \delta)^2}$$  \( (10) \)

where \( v_i \) denotes leisure in period \( i = 1, 2 \), \( c_3 \) denotes consumption when retired and \( \delta \) denotes the individual discount rate. The utility function features positive and decreasing marginal utility with respect to leisure in both periods.

Our utility function in equation (10) deviates from the more standard function that attaches positive and decreasing marginal utility to consumption in all three periods. We choose for the form in equation (10) as a more common utility function would complicate the derivations enormously, whereas the results would remain essentially unchanged. To illustrate, should we replace \( c_3 \) in equation (10) with \( c_3^2 \), \( \omega < 1 \), we would obtain very similar results. The same holds true if we replaced \( c_3 \) with \( c_3^2 \), \( \omega < 1 \) and also added terms \( c_1^2 \) and \( (c_2^2)/(1 + \delta) \).

The household budget constraint equals consumption during retirement to the sum of labour income earned in the two working periods net of pension contributions and the pension benefit in the retirement period,

$$c_3 = (1 + r)^2 q_1(1 - \pi)l_1 + (1 + r)q_2(1 - \pi)l_2 + b$$  \( (11) \)

where \( l_i = 1 - v_i \) denotes labour supply in period \( i = 1, 2 \) and \( q_i \) denotes productivity in period \( i = 1, 2 \). As before, \( \pi \) denotes the pension contribution rate and \( b \) the pension benefit. We allow productivity to grow over the periods.

11Basically, both alternative utility functions would introduce income effects into the analysis. This would leave unchanged our qualitative results on a marginal reform and a discrete reform. It would reinforce our results on the discrete reform in a quantitative sense if in both periods 1 and 2 leisure is a normal good.
life-cycle. We abstract from economy-wide productivity growth however, so the one generation we study can be considered representative for the whole working population.

As indicated in the previous section, the worker builds up his pension on the basis of a constant accrual rate $\alpha$ and indexation at rate $\mu$. Hence,

$$b = \alpha(1 + \mu)^2 q_1 l_1 + \alpha(1 + \mu)q_2 l_2$$  \hfill (12)

Substituting this result in equation (11) gives an integrated household budget constraint:

$$c_3 = ((1 + r)^2(1 - \pi) + (1 + \mu)^2 \alpha)q_1 l_1 + ((1 + r)(1 - \pi) + (1 + \mu)\alpha)q_2 l_2$$  \hfill (13)

We can formulate this constraint more concisely by using the definitions of the implicit pension tax and implicit pension subsidy:

$$c_3 = (1 + r)^2(1 - t_1)q_1 l_1 + (1 + r)(1 + s_2)q_2 l_2$$  \hfill (14)

Here, $t_1 \equiv \pi - \alpha((1 + \mu)/(1 + r))^2$ stands for the implicit tax rate in period 1. Similarly, $s_2 \equiv -\pi + \alpha((1 + \mu)/(1 + r))$ refers to the implicit subsidy rate in period 2. Before elaborating the household maximization problem, it is useful to dwell a little further on the expressions for the pension tax and subsidy, however.

Therefore, we recall the expressions we have derived for $\pi$ and $\alpha$ (equations (8) and (9) respectively). Using these expressions, we can derive expressions for $t_1$ and $s_2$ in terms of the replacement rate $\beta$:

$$t_1 = \frac{\beta(1 + \mu)}{(1 + r)^2 \left(1 + \left(\frac{1 + \mu}{1 + g}\right)\right) \left(1 + \left(\frac{1 + r}{1 + g}\right)\right)}$$  \hfill (15)

$$s_2 = \frac{\beta(1 + \mu)}{(1 + r)(1 + g) \left(1 + \left(\frac{1 + \mu}{1 + g}\right)\right) \left(1 + \left(\frac{1 + r}{1 + g}\right)\right)}$$  \hfill (16)

These two expressions show clearly that both the implicit tax and subsidy are strictly positive as long as $r > \mu$. One might think that these expressions also show that the pension tax and subsidy rate are proportional with the replacement rate, but, generally, they are not. The reason is that the variable $g$ enters into the two expressions. $g$, which measures the growth of labour income from the first to the second period of the working phase, is determined by the labour supply decisions of the household, which in turn may be affected by the pension scheme itself.
The expressions in equations (15) and (16) make also clear that \( t_1 \) and \( s_2 \) relate endogenously to the policy variable in our model, the replacement rate \( \beta \). It turns out to be too difficult to solve the model analytically in terms of \( \beta \), however. Instead, it is more convenient to treat \( s_2 \) as the policy variable and to relate \( t_1 \) to \( s_2 \) through the pension fund budget constraint (we will derive this condition below). When deriving analytical expressions, this approach is warranted as long as the endogeneity of \( g \) does not undermine the positive relationship between \( \beta \) and \( s_2 \) in equation (16). Numerical simulations showed that this is not the case.

Let us return to the household maximization problem. Maximizing household intertemporal utility with respect to leisure in periods 1 and 2 gives rise to two leisure demand functions,

\[
v_i = u_i^{-1}(\hat{w}_i) \equiv x_i(\hat{w}_i) \quad i = 1, 2
\]

where \( \hat{w}_1 \) is a shortcut for \( q_1(1-t_1)((1+r)/(1+\delta))^2 \) and \( \hat{w}_2 \) is a shortcut for \( q_2(1+s_2)((1+r)/(1+\delta)) \). Throughout the analysis, we will assume that leisure in periods 1 and 2 is strictly between zero and one, thereby avoiding the trivial cases in which labour supply or leisure are strictly zero.

Combining things, we can express \( U \) as a function of \( t_1 \) and \( s_2 \),

\[
U = u_1(x_1(q_1(1-t_1))) + u_2(x_2(q_2(1+s_2)))
\]

\[+ \hat{q}_1(1-t_1)(1-x_1(\hat{q}_1(1-t_1))) + \hat{q}_2(1+s_2)(1-x_2(\hat{q}_2(1+s_2)))
\]

where \( \hat{q}_1 \) and \( \hat{q}_2 \) are shortcuts for \( q_1((1+r)/(1+\delta))^2 \) and \( q_2((1+r)/(1+\delta)) \) respectively.

The pension fund budget constraint is already implied by the expressions for the implicit tax and subsidy rate in equations (15) and (16). We follow an easier route, however. We recall our assumption that the pension scheme is an individual scheme. This implies that the present value of contributions equals that of accruals and, thus, that the present value of implicit taxes

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equals that of implicit subsidies. If we denote the present value of implicit taxes as \( T_1 \) and that of implicit subsidies as \( S_2 \) and use the definitions \( T_1 = (1 + r)^2 q_1 t_1 l_1 \) and \( S_2 = (1 + r)q_2 s_2 l_2 \), we derive the following constraint:

\[
(1 + r) q_1 t_1 (1 - x_1(q_1(1 - t_1)))) = q_2 s_2 (1 - x_2(q_2(1 + s_2))) \tag{19}
\]

Total differentiation of this pension fund constraint gives an expression for \( dt_1/ds_2 \),

\[
\left( \frac{dt_1}{ds_2} \right)_F = \frac{q_2 l_2 (1 + s_2 \eta_2)}{(1 + r) q_1 t_1 (1 - t_1 \eta_1)} \tag{20}
\]

where we add subscript \( F \) to refer to the casus of a first-best world. Furthermore, \( \tilde{s}_2 \) and \( \tilde{t}_1 \) are shorthand notations for \( s_2/(1 + s_2) \) and \( t_1/(1 - t_1) \) respectively. As will be clear by now, \( l_1 \) and \( l_2 \) relate to \( t_1 \) and \( s_2 \), so we suppress this in equation (20) and henceforth.

In equation (20), we use \( \eta_i \), \( i = 1, 2 \) to denote the elasticity of labour supply in period \( i \) with respect to that period’s wage rate: \( \eta_i \equiv -x_i(\hat{w}_i)/((1 - x_i(\hat{w}_i))/\hat{w}_i) \), \( i = 1, 2 \). These two labour supply elasticities are positive on account of our assumed utility function.

Note that the numerator of the fraction at the RHS of equation (20) corresponds to the derivative of subsidy outlays to the subsidy rate. Similarly, the denominator of this fraction corresponds to the derivative of tax revenues to the tax rate. The former is always positive. The latter is only positive if the economy is on the left side of its Laffer curve. We will assume that this is the case. Hence, we have that \( (dt_1/ds_2)_F > 0 \).

Now we are ready to assess the welfare properties of front-loading policies. Note that we must elaborate \( dU/ds_2 \) as \( \partial U/\partial s_2 + \partial U/\partial t_1(dt_1/ds_2) \). Upon using equations (18) and (20), we then derive the following expression for \( dU/ds_2 \):

\[
\left( \frac{dU}{ds_2} \right)_F = \frac{(1 + r)}{(1 + \delta)^2 q_2 l_2 \left( 1 - \left( \frac{1 + \tilde{s}_2 \eta_2}{1 - t_1 \eta_1} \right) \right)} \tag{21}
\]

Equation (21) allows us to derive two propositions that distinguish between a marginal reform and a discrete (more than marginal) reform. A marginal reform pertains to a marginal increase in \( s_2 \) (and \( t_1 \)) from the initial point \( s_2 = t_1 = 0 \). Given that real-world pension policies imply values for \( s_2 \) and \( t_1 \) that are very different from zero, this marginal reform has no real-world counterpart. But it is a useful device to understand the effects of a discrete reform.

\[\text{It is easy to verify that combining equations (15) and (16) gives the same expression.}\]
Proposition 1:
In a first-best world, the introduction of a marginal degree of front-loading in pension contribution policies exerts no effect upon welfare.

Proof
The welfare effect of a marginal reform is zero, as can be seen by plugging in $t_1 = \bar{s}_2 = 0$ into equation (21).

Proposition 2:
In a first-best world, the introduction of a discrete degree of front-loading in pension contributions is strictly welfare-decreasing.

Proof
We view a discrete reform as a succession of marginal changes. The welfare effect of the first marginal change, that is the increase in $s_2$ (and $t_1$) from the initial point $s_2 = t_1 = 0$, is zero (see proposition 1). Let us denote the subsidy rate and tax rate after this first marginal change as $s_2^1$ and $t_1^1$. The second marginal change concerns again a marginal increase in $s_2$ (and $t_1$), but now from the point $s_2 = s_2^1 > 0$, $t_1 = t_1^1 > 0$. Inspection of equation (21) shows that $dU/ds_2 < 0$ if $s_2, t_1 > 0$. The welfare effect of the second change is thus negative. The argument for the second marginal change can be repeated for subsequent marginal changes. This proves proposition 2.

Equation (21) shows that the welfare loss due to front-loading is higher, the higher the implicit subsidy rate and implicit tax rate that are associated with front-loading, i.e. the more generous is the pension scheme. Furthermore, higher labour supply elasticities for young and old workers contribute to a higher welfare loss.

A question that cannot be answered by equation (21) is how the implicit tax and the implicit subsidy contribute separately to the welfare loss from front-loading. To answer this question, we split front-loading policies into two hypothetical policies: i) a distortionary tax in period 1 of which the revenues are rebated to households in a lump-sum fashion - to be denoted as H1; and ii) a distortionary subsidy in period 2 that is financed by imposing lump-sum taxes upon households - to be denoted as H2. We express the welfare losses of these two policies in equations (22) and (23):

$$\left( \frac{dU}{dt_1} \right)_{H1} = -\eta_1 \hat{t}_1 \hat{y}_1$$  

(22)
These two equations reflect the famous Harberger formula (Harberger, 1964). They indicate that the marginal welfare cost of taxation (subsidization) is increasing in the tax (subsidy) rate $t_1(s_2)$ and proportional with the associated elasticity $\eta_1(\eta_2)$.

We now combine these two types of policies, using the factor $(dt_1/ds_2)F$ as calculated in equation (20) to ensure that the lump-sum transfers associated with H1 cancel exactly against the lump-sum taxes associated with H2. This produces a welfare loss equal to $(dU/ds_2)_{H1H2}$:

$$
\left( \frac{dU}{ds_2} \right)_{H1H2} = \left( \frac{dU}{dt_1} \right)_{H1} \left( \frac{dt_1}{ds_2} \right)_{F} + \left( \frac{dU}{ds_2} \right)_{H2} - \eta_1 t_1 \eta_1 q_1 l_1 1 \left( q_2 l_2 \left( 1 + s_2 \eta_2 \right) \frac{1 + r}{1 + r} \right)\frac{1 + t_1 \eta_1 \eta_1}{1 + s_2} \frac{1 + s_2}{1 + \delta} \tag{24}
$$

Appendix B shows that this effect is exactly the welfare effect of front-loading as we derived above (equation (21)).

As indicated above, the next section will introduce government spending. But before moving to that section, we take a look at the effect of front-loading upon aggregate labour income. For this effect, measured as $d(q_1 l_1 + q_2 l_2)/ds_2$, we derive the following form:

$$
\left( \frac{d(q_1 l_1 + q_2 l_2)}{ds_2} \right)_{F} = -q_2 l_2 \left( \frac{\eta_1}{1 - t_1} \left( \frac{1 + s_2 \eta_2}{1 - t_1 \eta_1} \right) - \frac{\eta_2}{1 + s_2} \right) \tag{25}
$$

This expression tells us three things. First, the effect upon aggregate labour income can generally not be signed as it combines the opposite effects upon the labour income of young workers (negative) and upon that of old workers (positive). Second, in case the reform is marginal, the effect in equation (25) is proportional with $(\eta_1/(1 + r) - \eta_2)$. Without restricting the values of $\eta_1$, $\eta_2$ and $r$, this effect can also not be signed. Thirdly, the higher the initial values of $t_1$ and $s_2$, the less positive or the more negative will be the effect of front-loading upon aggregate labour income.

4 Front-loaded contributions in a second-best world

The previous section analysed the effects of front-loading in a first-best world: apart from front-loading, no other distortions prevailed. This section adds a distortion to the model. In particular, we introduce a government
that levies labour income taxes in order to finance a certain amount of go-
vernment spending. Aggregate labour income serves as the tax base. As
we will see, addition of labour income taxation which is common in many
countries may turn the conclusions of the previous section on its head.

Let us use \( p \) to denote the rate of general taxation and \( P \) to denote the
financing requirement of the government. The expressions for the wage rate
in period 1 and 2 are natural generalizations of the corresponding equations
in the previous section:

\[
\begin{align*}
  w_1 &= q_1(1 - p - t_1) \quad (26) \\
  w_2 &= q_2(1 - p + s_2) \quad (27)
\end{align*}
\]

Different from the previous section, the pension fund constraint in equation
(19) is now a function of three policy variables, namely \( t_1, s_2 \) and \( p \).
Total differentiation yields the following,

\[
(1 + r)q_1 l_1 (1 - \eta_1 \tilde{t}_1) dt_1 - q_2 l_2 (1 + \eta_2 \tilde{s}_2) ds_2
- \eta_2 \tilde{s}_2 l_2 q_2 + \eta_1 \tilde{t}_1 l_1 (1 + r) q_1 dp = 0
\]

where \( \tilde{t}_1 \) is a shortcut for \( t_1/(1 - p - t_1) \) and \( \tilde{s}_2 \) is a shortcut for \( s_2/(1 - p + s_2) \).

Next to this pension fund constraint, we now have a government bud-
get constraint, which relates the labour income tax rate to the financing
requirement \( P \):

\[
P = p(q_1 l_1 + q_2 l_2) \quad (29)
\]

Total differentiation of this government budget constraint gives a second
equation in the three policy variables,

\[
- q_1 l_1 \tilde{p}_1 \eta_1 dt_1 + q_2 l_2 \tilde{p}_2 \eta_2 ds_2
+ ((1 - \tilde{p}_1 \eta_1) q_1 l_1 + (1 - \tilde{p}_2 \eta_2) q_2 l_2) dp = 0
\]

where \( \tilde{p}_1 \) is a shortcut for \( p/(1 - p - t_1) \) and \( \tilde{p}_2 \) is a shortcut for \( p/(1 - p + s_2) \).

Combining equations (28) and (30) gives expressions for \( dt_1/ds_2 \) and
\( dp/ds_2 \),

\[
\begin{align*}
  \left( \frac{dt_1}{ds_2} \right)_S &= \left( \frac{q_2 l_2}{(1 + r) q_1 l_1} \right) \Omega_B \quad (31) \\
  \left( \frac{dp}{ds_2} \right)_S &= \frac{q_2 l_2 \left( \tilde{p}_1 \eta_1 - \Omega_B - \tilde{p}_2 \eta_2 \right)}{(1 - \tilde{p}_1 \eta_1) q_1 l_1 + (1 - \tilde{p}_2 \eta_2) q_2 l_2} \quad (32)
\end{align*}
\]
where

$$\Omega_B \equiv \left( \frac{(1 + \tilde{s}_2 \eta_2) - \Omega_A \tilde{p}_2 \eta_2}{(1 - \tilde{t}_1 \eta_1) - \Omega_A \tilde{p}_1 \eta_1} \right)$$

(33)

and

$$\Omega_A \equiv \left( \frac{(1 + r)q_1 l_1 \tilde{t}_1 \eta_1 + q_2 l_2 \tilde{s}_2 \eta_2}{(1 - \tilde{p}_1 \eta_1)q_1 l_1 + (1 - \tilde{p}_2 \eta_2)q_2 l_2} \right)$$

(34)

and where the subscript $S$ refers to the casus of a second-best world.

To evaluate the welfare effect of front-loading in this second-best model, we derive an expression for $\frac{dU}{ds_2}$, which must now be elaborated as

$$\frac{dU}{ds_2} = \frac{(1 + r)(1 + \delta)}{q_2 l_2} (1 - \Omega_B)$$

(35)

where $\Omega_B$ is defined in equation (33).

A benchmark case

We first evaluate the welfare effect in equation (35) for a particular case, in which the two labour supply elasticities obey the condition $\eta_1/(1 + r) - \eta_2 = 0$. The case is not very realistic as we will explain below, but is useful as a benchmark case. As before, we look at a marginal reform and a discrete reform and state our results in the form of two propositions.

Proposition 3:

In the benchmark case of positive public spending ($P > 0$, $\eta_1/(1 + r) - \eta_2 = 0$), the introduction of a marginal degree of front-loading in pension contribution policies exerts no effect upon welfare.

Proof

Note that $s_2 = t_1 = 0$ implies that $\tilde{s}_2 = \tilde{t}_1 = 0$. Hence, $\Omega_A = 0$ and $\Omega_B = 1$. Given that $\eta_1/(1 + r) = \eta_2$, we find that $dU/ds_2$ equals zero.

This result is identical to that in proposition 1: the addition of public spending exerts no effect upon welfare in the benchmark case.

18
Proposition 4:
In the benchmark case of positive public spending \( P > 0, \eta_1/(1+r) - \eta_2 = 0 \), a discrete degree of front-loading in pension contribution policies is strictly welfare-reducing.

Proof
As in the proof of proposition 2, we view a discrete policy reform as a succession of marginal changes. The welfare effect of the first marginal change, that is the increase in \( s_2 \) (and \( t_1 \)) from the initial point \( s_2 = t_1 = 0 \), is zero (see proposition 3). The second and subsequent marginal changes have \( s_2, t_1 > 0 \) as a starting point. \( s_2, t_1 > 0 \) implies \( \tilde{s}_2, \tilde{t}_1 > 0 \). Equation (34) shows that then \( \Omega_A > 0 \).\(^{14}\) Equation (33) can be used to derive that \( \Omega_B > 1 \). To see this, note that \( \tilde{p}_1 \) is increasing in \( t_1 \) and \( \tilde{p}_2 \) is decreasing in \( s_2 \). Note also that \( \eta_1/(1+r) = \eta_2 \) in the benchmark case. Then one can verify that if \( s_2 \) and \( t_1 \) turn positive, the denominator of the expression for \( \Omega_B \) will fall relative to the numerator.\(^{15}\) Combining the result \( \Omega_B > 1 \) with \( \tilde{p}_1 > \tilde{p}_2 \) if \( \tilde{s}_2, \tilde{t}_1 > 0 \) yields that \( dU/ds_2 < 0 \). This proves proposition 4.

An additional result emerges if we compare the benchmark case with positive public spending with the previous case without any public spending. We phrase this in a corollary to proposition 4.

Corollary to proposition 4:
A discrete degree of front-loading in pension contribution policies is more welfare-reducing in the benchmark case of positive public spending \( P > 0, \eta_1/(1+r) - \eta_2 = 0 \) than in the case without any public spending.

Proof
In case \( s_2, t_1 > 0 \), \( \Omega_B \) in equation (33) is strictly larger than \( (1 + \tilde{s}_2 \eta_2)/(1 - \tilde{t}_1 \eta_1) \) in equation (21). Hence, compared to the case of zero public spending, the first term at the RHS of equation (35) is now more negative. In addition, the second term at the RHS of equation (35) is now negative, whereas it is zero in case of zero public spending.

\(^{14}\) The denominator of the expression for \( \Omega_A \) in equation (34) equals the derivative of labour income tax revenues to the labour income tax rate. We assume that it is positive, which means that the economy is on the left side of the corresponding Laffer curve.

\(^{15}\) We assume that the numerator and the denominator of the expression for \( \Omega_B \) are positive. This is the case in all the numerical simulations presented below.
The explanation for the result stated in the corollary can be found in the
effect upon aggregate labour income,
\[
\left( \frac{dq_1 l_1 + q_2 l_2}{ds_2} \right)_S = -q_2 l_2 \left( \frac{\eta_1}{(1 + r)(1 - p - t_1)} \Omega_B - \frac{\eta_2}{1 - p + s_2} \right) - \left( \frac{\eta_1 q_1 l_1}{1 - p - t_1} + \frac{\eta_2 q_2 l_2}{1 - p + s_2} \right) \left( \frac{dp}{ds_2} \right)_S
\]
(36)
where \( (dp/ds_2)_S \) is given in equation (32).

Equation (36) shows that in the benchmark case of positive public spend-
ing a marginal degree of front-loading has a zero effect on aggregate labour
income (use equation (32) to show that \( dp/ds_2 = 0 \) in case \( s_2 = t_1 = 0 \)).
A discrete degree of front-loading produces an unambiguous decline of ag-
gregate labour income however. Indeed, both terms at the RHS of equation
(36) turn negative if we move from a marginal to a discrete degree of front-
loading (use equation (32) to show that \( dp/ds_2 > 0 \) in case \( s_2, t_1 > 0 \)).
Through the government budget constraint, a decline of aggregate labour
income necessitates an increase in the labour income tax rate. This adds
to the welfare loss due to front-loading and explains why the welfare loss is
now bigger than in the case of zero public spending.

A more realistic case

A rather different story emerges once we depart from the rather specific
assumptions of the benchmark case. In case of a marginal reform, the wel-
fare effect reads as follows,
\[
\left( \frac{dU}{ds_2} \right)_{S,M} = -\frac{q_2 l_2}{(1 + \tilde{\rho})^2} \left( \frac{q_1 l_1 (1 + r)^2 + q_2 l_2 (1 + r)}{(1 - \tilde{p} \eta_1) q_1 l_1 + (1 - \tilde{p} \eta_2) q_2 l_2} \right) \tilde{p} \left( \frac{\eta_1}{1 + r} - \eta_2 \right)
\]
(37)
where we use index \( M \) to refer to a marginal reform. Further, we use \( \tilde{p} \) as s
shortcut for \( p/(1 - p) \).

The welfare effect in equation (37) is proportional with \( \eta_1/(1 + r) - \eta_2 \).
Without any information on labour supply elasticities and the capital market
rate of return, it can be positive or negative. Empirical evidence is helpful
to pin down the sign of \( \eta_1/(1 + r) - \eta_2 \), however.

As regards the capital market rate of return, this can be interpreted as
the rate of return on a portfolio that consists of bonds and equity. Using
data from 1870 to 2015, Jordà et al. (2019) show that the safe interest rate has varied between 1 and 3 percent for most countries and peacetime
periods, that the average real rate of return on real estate and equity was
7 percent and that both rates of return are very volatile over the long run. To stay on the conservative side, we will take a portfolio rate of return of 3 percent annually.

The empirical literature on the relationship between labour supply elasticities and age is relatively scarce. The estimates in this literature point in the same direction, however: the labour supply elasticities for older workers are higher than those for middle-aged workers. For example, French (2005) estimates a life-cycle model of labour supply, retirement and savings. He calculates that the labour supply elasticity is between 0.3 for a 40-year old man, but 1.1 for a male worker of 60 years old. Fenge et al. (2006), using micro data for Germany, finds that the labour supply elasticity rises with age. This is especially true for males: compensated elasticities are estimated at 0.010 for males aged 20-39 and 0.215 for males aged 40-59. For females, the results are less outspoken: for this group, the figures are 0.527 for the 20-39 group and 0.565 for the 40-59 group. The figures reported in French and Jones (2012) are in line with those in French (2005): for permanent wage changes, the estimated labour supply elasticity is 0.17 for 40-year old workers and 1.17 for workers with age 60.16 Erosa et al. (2016) report that labour supply elasticities for males are a U-function of age and highest for the oldest age group considered. For high school individuals, they report elasticities of 2.01, 1.62, 1.90 and 2.74 for the 25-34, the 35-44, the 45-54 and the 55-61 age group respectively. For college individuals, a similar pattern holds true. In addition, they find that the age differences are almost entirely due to variations along the extensive margin. Karabarbounis (2016) presents a similar finding. The elasticities for older workers are higher than those of middle age due to higher extensive-margin elasticities. The finding in Bargain et al. (2014) that the extensive margin dominates the intensive margin in 17 European countries and the US also suggests higher labour supply elasticities for older workers as the participation decision is particularly relevant for this age group. In a similar vein, Blundell et al. (2016) argue that as the age of the worker rises, wages fall, health deteriorates and wealth increases, moving the worker closer to the participation margin where labour supply elasticities are higher.

Hence, we define as a more realistic case the case $\eta_1 / (1 + r) - \eta_2 < 0$. Equation (37) shows that in this case a marginal degree of front-loading is strictly welfare-increasing. We summarize this in proposition 5.

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16For temporary wage changes, the picture is similar. In this case, the labour supply elasticities for the two groups are 0.36 and 1.28 respectively.
Proposition 5:
In the more realistic case of positive public spending \((P > 0, \eta_1/(1+r) - \eta_2 < 0)\), a marginal degree of front-loading in pension contribution policies is strictly welfare-increasing.

Proof
This follows directly from equation (37).

This effect reverses the effects above for the case of zero spending and the benchmark case of positive public spending. The reason for the first-order welfare loss of front-loading lies in the effect upon aggregate labour income. To see this, we write down the effect upon aggregate labour income in case of a marginal degree of front-loading:

\[
\left( \frac{d(q_1 l_1 + q_2 l_2)}{d\theta} \right)_{S,M} = \left( \frac{q_2 l_2}{1-p} \left( 1 + \frac{(\eta_2 q_2 l_2 + \eta_1 q_1 l_1)\tilde{p}}{(1 - \tilde{p}\eta_1)q_1 l_1 + (1 - \tilde{p}\eta_2)q_2 l_2} \right) \right) \left( \frac{\eta_1}{(1+r) - \eta_2} \right)
\]

As this equation shows, front-loading produces a first-order increase in aggregate labour income. Through the government budget constraint, this implies a fall of the labour income tax rate, which fully explains the first-order effect of front-loading upon welfare, for, as we have seen in the previous section, the direct welfare effect of a marginal degree of front-loading is zero.

An empirically very interesting case is the discrete equivalent of the case considered in proposition 5, i.e. with positive public spending and \(\eta_1/(1+r) - \eta_2 < 0\). We may expect that increasing the degree of front-loading from zero to increasingly higher levels will reduce the welfare gain and may eventually turn the welfare gain into a welfare loss. For in the case of zero public spending and the benchmark case of positive public spending, moving from a marginal to a discrete reform turns a zero welfare effect into a welfare loss. Unfortunately, the expression in equation (35) is too complex to help us to verify this expectation. Therefore, we resort to numerical simulations for an illustration of the effects in this case.

The numerical calculations assume that \(r = 1.094, \mu = 0.0\) and \(g = 0.451\). These figures are based on an annual capital market rate of return of 3\%, 100\% indexation of pensions against price inflation, an annual rate of productivity growth of 1.5\% and a unit period of 25 years. We further assume that the discount rate equals the capital market rate of return: \(\delta = r\). For \(u_i \; i=1,2\) we adopt the specification \(-l_i^{1+1/\eta_i}/(1 + 1/\eta_i)\), which features
constant labour supply elasticities. For \( \eta_1 \) we use a value of 0.3. This is close to the value of 0.31 reported in Keane (2011) for a simple average of Hicks elasticities across male and female labour supply literatures.

The simulations highlight the role of \( \eta_2 \) and \( P \). For both of them, we adopt four different values. \( \eta_2 \) is taken to be equal to \( \eta_1/(1+r) \) (the benchmark case), \( \eta_1 \), \( 1.5\eta_1 \) or \( 2\eta_1 \). The evidence referred to above suggests that the implied ratios \( \eta_2/\eta_1 \) are somewhat conservative: in reality \( \eta_2/\eta_1 \) might be even higher. Note however that \( \eta_2 \) in our model refers to a much broader group than workers close to retirement. This suggests a somewhat lower value for the \( \eta_2/\eta_1 \) ratio than that implied by the empirical estimates. For \( P \), we take a wide range: the values are 0, 0.281, 0.563 and 0.844. Finally, in all simulations, the replacement rate \( \beta \) runs from 0 to 100\% in 100 steps. This range is wide enough to cover all real-world cases.

Figures 1-1 to 4-4 show the results for household utility \( U \). The first index refers to the value of \( \eta_2 \), the second index to the value of \( P \). Figures 1-1 to 1-4 display the results where \( \eta_2 = \eta_1/(1+r) \) for the four different values of \( P \). The four figures reflect propositions 1 to 4: welfare does not change upon a marginal reform and decreases in case of a discrete reform.

Figures 2-1 to 2-4 display the results where \( \eta_2 \) equals \( \eta_1 \), again for the four different values of \( P \) listed above. The case of figure 2-1, with \( P = 0 \), is like that of figure 1-1. With zero public spending, there is no existing labour market distortion and front-loading policies reduce welfare as before. Figure 2-2 displays a different message, however. Now, as \( P = 0.281 \), the labour market is distorted due to labour income taxation. Starting with a zero replacement rate, a marginal increase raises welfare. Increasing the replacement rate further initially increases welfare more, but later starts to reduce welfare. The latter effect is so large that an economy with a 100\% replacement rate features lower welfare than an economy without a pension scheme. But now take a look at figures 2-3 and 2-4, with public spending equal to 0.563 and 0.844 respectively. Compared with the case of figure

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17In the simulation in which \( \beta = P = 0; \eta_1 = 0.3; \eta_2 = 0.6 \), GDP equals 2.814. In other simulations, GDP takes a different value, but with the same order of magnitude. Hence, the values for \( P \) we adopt in our simulations are in the order of magnitude of 0, 10, 20 and 30 \% of GDP.

18The OECD average of the net replacement rate for the average worker is 58.6 \%. In South Africa, the lowest number applies, namely 18.5 \%; in India, the highest number applies, 94.8 \% (OECD, 2019). Note that the role of front-loaded contributions is smaller than these figures might suggest: the replacement rate in our model, \( \beta \), corresponds only to pension benefits that are funded (and to which linear contribution and accrual rates apply), whereas in the real world generally a large part of pensions is PAYG-financed.
the existing distortion is now larger. This changes the balance between the new distortion and the existing distortion such that an economy with a 100% replacement rate features higher welfare than an economy without a pension scheme.

The cases where $\eta_2$ equals $1.5\eta_1$ or $2\eta_1$ (in figures 3-1 to 3-4 and 4-1 to 4-4 respectively) are even more pronounced. If $\eta_2$ equal $1.5\eta_1$, an economy with a pension scheme features lower welfare than an economy without a pension scheme only in case $P = 0.281$ and a replacement rate higher than 93%. If $\eta_2$ equals $2\eta_1$, an economy with a pension scheme features higher welfare than an economy without a pension scheme, even for a replacement rate of 100% and a value for $P$ as low as 0.281.

The simulations suggest that the case for front-loading is pretty strong. For the simulations with realistic values for public spending point to a welfare gain, even when the labour supply elasticities for the young and the old are assumed equally large. Higher values for the elasticity of old workers increase the welfare gain. However, we should keep in mind that the evidence on labour supply elasticities is scarce.

There is a close connection between our results and those of the literature on optimal taxation. This literature tells us that optimal policies apply relatively low tax rates on goods for which demand is relatively elastic. According to the empirical estimates referred to above, the demand for leisure by older workers is more elastic than that by younger workers. This suggests that younger workers should be taxed more heavily than older workers. The labour income tax in our model does not distinguish between young and old workers, however. A pension scheme with linear contribution and accrual rates can thus fill the gap that is left by tax policies. By implicitly differentiating tax rates with respect to age, these pension schemes can bring the economy closer to the first best and achieve a welfare gain. Be aware of the word ‘can’, however. Front-loading can also produce too much of a difference between tax rates on young and old workers and produce a welfare loss, as is nicely illustrated in figures 1-1 to 4-4.

Our result that we should have lower taxation for older than middle-aged workers because of a difference in labour supply elasticities is similar to that in Karbarbounis (2016). The result is also reminiscent of that in Erosa and Gervais (2002) and Conesa et al. (2009). These papers find that it may be optimal to tax capital income for the same reason that we find that front-loading may imply a welfare gain: the inability to condition tax rates on age. This result is not general, however. Weinzierl (2011) concludes the opposite, because low taxes for young workers may alleviate liquidity constraints. Liquidity constraints are not relevant in our case. We
will return to this issue below.\footnote{Fahri and Werning (2013) also find that the optimal labour income tax is increasing with age. Golosov et al. (2016) qualify their result, however.}

5 The case of a collective pension scheme

Throughout the paper, we have assumed that the pension scheme is an individual scheme in which life-cycle contributions and accruals are equal, whereas the front-loading policies that we study are characteristic of collective schemes in which life-cycle contributions and accruals are generally different. In this section, we will demonstrate that this difference between an individual and a collective scheme does not have any affect upon our results. In particular, we will use the model without government spending to demonstrate that the welfare result derived for the case of an individual scheme carries over to that of a collective scheme.

To move to the collective equivalent of the two-period model of section 3 requires two changes. The first is clarified in appendix C which analyzes the collective pension scheme in the multi-period setting that we also adopted in section 2. It shows that the transformation from an individual scheme to a collective one changes the relation between implicit tax revenues and implicit subsidies. Whereas $T_1/S_2$ equals 1 in an individual scheme, it equals $1 + r > 1$ in a collective scheme (appendix C shows that we need to impose dynamic efficiency, i.e. $r > 0$, in order to let the sum of utilities of future generations converge). In a collective scheme, implicit tax revenues are thus larger than implicit subsidies, a reflection of, as appendix C clarifies, an implicit debt inherited from the time the scheme was started.

Hence, the pension fund constraint, which governs the relation between changes in the implicit tax rate $t_1$ and the implicit subsidy rate $s_2$, reads as follows in the case of a collective scheme,

$$\left(\frac{dt_1}{ds_2}\right)_F^C = (1 + r) \left( \frac{q_2 l_2 (1 + \tilde{s}_2 \eta_2)}{(1 + r) q_1 l_1 (1 - t_1 \eta_1)} \right)$$

where we use superscript C to refer to the collective scheme.

The corresponding expression for the welfare effect differs from the one in section 3 only in the factor $(1 + r)$:

$$\left(\frac{dU_1}{ds_2}\right)_F^C = \frac{(1 + r)}{(1 + \delta)^2 q_2 l_2} \left( 1 - (1 + r) \left( \frac{1 + \tilde{s}_2 \eta_2}{1 - t_1 \eta_1} \right) \right)$$

\footnote{Fahri and Werning (2013) also find that the optimal labour income tax is increasing with age. Golosov et al. (2016) qualify their result, however.}
Note that we now index $U$ to refer to the period in which the generation was born.

From the expression in equation (40) one might conclude that there is a difference between the collective scheme and the individual scheme. According to equation (40), a marginal reform produces a nonzero effect upon $dU_1/ds_2$. But note that this expression refers to the generation who is young when the scheme is started. In the case of a collective scheme this generation is not representative of the whole population.

This brings us to the second change. To do justice to the interests of all generations, we need to define a social welfare function that includes the interests of all generations involved, i.e. old, young and future generations:

$$W = (1 + r)U_0 + U_1 + \frac{1}{1 + r}U_2 + ...$$

(41)

This social welfare function uses the capital market rate of return as discount rate. This is necessary if one wants to avoid that redistribution between generations in itself will affect social welfare.

The welfare effect of a marginal increase in the degree of front-loading can be derived by elaborating $dW/ds_2$:

$$\left(\frac{dW}{ds_2}\right)_F^C = (1 + r)\left(\frac{dU_0}{ds_2}\right)_F^C + \left(\frac{dU_1}{ds_2}\right)_F^C + \frac{1}{1 + r}\left(\frac{dU_2}{ds_2}\right)_F^C + ...$$

$$= (1 + r)\left(\frac{\partial U_0}{\partial s_2}\right)_F + \left(\frac{1 + r}{r}\right)\left(\frac{dU_1}{d s_2}\right)_F^C$$

(42)

The expression on the second line of equation (42) deserves further clarification. It consists of two terms. The first of these represents the effect upon the generation who is old when the scheme is started. This effect relates only to the change in $s_2$, not the change in $t_1$, which explains why we use the partial derivative operator. The second term follows by noting that the welfare effects for future generations are identical to that for the young generation.

Elaborating the expression by substituting the expressions for $\partial U_0/\partial s_2$ and $(dU_1/ds_2)_F^C$ yields the following:

$$\left(\frac{dW}{ds_2}\right)_F^C = \frac{(1 + r)}{(1 + \delta)^2} \frac{(1 + r)^2}{r} \left(1 - \left\{ \frac{1 + \bar{s}_2 \eta_2}{1 - t_1 \eta_1} \right\}\right)$$

(43)

Comparing this expression with the one derived for the case of an individual scheme in section 3, we see that the two are identical, except for a proportionality factor $(1 + r)^2/r$, which reflects that the collective scheme accounts for the interests of all generations alive and those yet to be born.
This shows that front-loading policies work out the same for the case of a collective scheme and that of an individual scheme. The reason is that the only addition of a collective scheme to an individual scheme is a windfall gain enjoyed by the transition generation and paid for by the steady-state generations. In other words, the collective scheme adds redistribution between the transition generation and the steady-state generations. If redistribution itself does not affect social welfare, this cannot change the welfare effects of front-loading policies.

6 Qualifications

As always, our analysis rests on assumptions of which some can be debated. One debatable assumption is that workers are rational and forward-looking decision-makers who not only have access to all relevant information but also are capable to interpret it. Evidence suggests that people are myopic (Van Rooij et al., 2012), unable to interpret all relevant information (Lusardi and Mitchell, 2014) and sensitive to the way they are informed about taxes (Chetty et al., 2009; Dolls et al., 2018). If we take this to the limit, we could assume that labour-supply decisions are unaffected by the implicit pension taxes and subsidies that we have explored. In that case, our results would disappear.

We argue that this alternative assumption is too extreme, however. It conflicts with a bunch of empirical evidence that taxes do affect labour supply behaviour (Keane, 2011). It also conflicts with the evidence in Disney (2004) that the ‘tax’ and ‘savings’ components of pension contributions exert very different effects upon labour supply. Furthermore, assuming that pension taxes and subsidies exert no effects upon labour supply conflicts with evidence on retirement behaviour. For example, Kotlikoff and Wise (1988) and Samwick (1998) find that the accrual rate of retirement wealth affects the probability of retirement. Similarly, a large literature, recently summarized in Blundell et al. (2016), finds that financial incentives determine the retirement behaviour of older workers.

The evidence on myopia and information frictions does suggest two things, however. First, the implicit taxes and subsidies studied here may have smaller effects that tax changes of similar magnitude that result from deliberate policy reform. Second, this argument may apply in particular to labour supply decisions that people take early in their careers. This is in line with the empirical evidence on labour supply elasticities of younger and older workers referred to above. It also accords with the finding that people
tend to adopt high discount rates (Frederick et al., 2002). Further, it is consistent with the finding of Brinch et al. (2017) that agents do not account for benefit increases into their decision-making if these benefits occur in the future.

A second assumption that is crucial for our analysis is that wage profiles are exogenous. Theoretically, it would be possible to adjust wage profiles such as to neutralize the implications of front-loading. Then, the policy reform discussed here would have no labour market implications. One can doubt whether this argument applies, however. Saez et al. (2019), analyzing an employer-borne payroll tax rate cut for young workers in Sweden, provides strong evidence that there is no such an effect on wage profiles. In our case, one would expect the effect to be even smaller as the taxes and subsidies we discuss are only implicit and not the result of a deliberate attempt to change labour market outcomes. Furthermore, an implicit subsidy on labour later in the career may be beneficial for employers as it may induce workers to invest more in the firm, raising their productivity (Lazear, 1979). One would therefore not expect employers to agree with changes in wage profiles that would neutralize these effects.

A related issue is whether wage and productivity profiles coincide or, stated alternatively, whether older workers are less productive than what they are paid for. If this is true, employers would experience a windfall gain if older workers decided to earlier leave their firms. However, it is still unclear whether older workers are indeed overpaid. The recent studies of Van Ours and Stoeldraijer (2011) and Mahlberg et al. (2013), for the Netherlands and Austria respectively, find no evidence for it. Moreover, if a windfall gain will occur, it will only be temporary on account of competition on output and input markets.

Related is our concept of labour market equilibrium: the labour market accommodates any change in labour supply. This abstracts from demand factors that, according to, e.g., Frimmel et al. (2018) and Rabaté (2019) do play a role. It goes well beyond the scope of this paper to include demand factors, however.

Weinzierl (2011) stresses the role of liquidity constraints. These may prevent young workers from consuming more than their income in order to exploit their rising life-cycle wage profile. Lowering labour income taxes on young workers may relieve these liquidity constraints and thus raise welfare. In our model, these constraints are lacking. Does that mean that our results on the move towards more actuarially fair policies in the Netherlands are biased? The answer depends on the way one wants to achieve actuarial fairness. If a flat profile of contribution rates is replaced with an increasing
profile, the answer is yes. However, if actuarial neutrality is achieved through an adjustment of the accrual profile (moving from a flat to a decreasing profile), the answer must be no: the policy reform does not relieve liquidity constraints. Appendix D illustrates these claims formally adopting a more general model with liquidity constraints. As mentioned above, the latter is the type of policy reform that the Dutch government has announced.

7 Concluding remarks

The policy reform, announced by the Dutch government, to move from a scheme of linear accruals to one of degressive accruals, may have profound effects. It will end or at least reduce the redistribution between younger and older workers, make the pension scheme more transparent and help to increase labour mobility. It will also remove tax and subsidy elements that are implied by the combination of linear contribution and accrual rates. As this paper has shown, this element of the policy reform may reduce labour market efficiency.

Essentially, the reason for the potential welfare loss is that the government loses a policy instrument. The taxes and subsidies that are implied by the combination of linear contribution and accrual rates make the labour income tax effectively age-dependent. The policy reform thus changes the labour income tax into a uniform tax that does not differentiate between generations. This thus also suggests a way out: the labour market effects of the policy reform can be mitigated by turning the labour income tax into one that is conditional on the age of the taxpayer.

References


Fahri, Emmanuel and Iván Werning, 2013, Insurance and Taxation over the Life Cycle, Review of Economic Studies 80, 596-635.


Hanel, Barbara, 2010, Financial incentives to postpone retirement and further effects on employment – Evidence from a natural experiment, Labour Economics 17, 474-486.


Ours, Jan C. van, and Lenny Stoeldraijer, 2011, Age, Wage and Productivity in Dutch Manufacturing, *De Economist* 159, 113-137.


Saez, Emmanuel, Benjamin Schoefer and David Seim, 2019, Payroll


Figure 1: Cases 1-1 to 1-4

![Replacement rates and welfare for Cases 1-1 to 1-4](image-url)
Figure 2: Cases 2-1 to 2-4

- Welfare, Case 2-1
- Welfare, Case 2-2
- Welfare, Case 2-3
- Welfare, Case 2-4
Figure 3: Cases 3-1 to 3-4
Figure 4: Cases 4-1 to 4-4
Appendix A

This appendix shows that pension accruals are more back-loaded under the legal approach than under the rational approach if $\mu > 0$; in the absence of indexation, $\mu = 0$, the two approaches coincide.

Let us use superscripts $L$ and $R$ for the legal and the rational approach respectively. We make four observations.

First, by definition $A^L_0 = A^R_0 = 0$.

Second, $A^L_{TR} = A^R_{TR}$. To prove this, define the flow of accruals under the rational approach, $D^R_i$, as $\alpha y_i (1 + \mu)^{TR-i+1}$. Hence, $A^R_{TR}$ follows from adding all flows:

$$A^R_{TR} = \sum_{i=1}^{TR} \alpha y_i (1 + \mu)^{TR-i+1}$$

Next, recall the accumulation equation (4) which defines the stock of accruals under the legal approach:

$$A^L_i = (1 + \mu)(A^L_{i-1} + \alpha y_i)$$

Repeated forward substitution of this equation implies an expression for $A^L_{TR}$ which is identical to the one we obtained for $A^R_{TR}$:

$$A^L_{TR} = \sum_{i=1}^{TR} \alpha y_i (1 + \mu)^{TR-i+1}$$

Thirdly, the flow of accruals under the legal approach, $D^L_i$, is defined as $D^L_i = \alpha y_i (1 + \mu) + \mu A^L_{i-1}$. Hence, we derive that for $\mu > 0$ the flow of accruals in the first year of the life cycle is larger under the rational approach:

$$D^R_1 = \alpha y_1 (1 + \mu)^{TR} > D^L_1 = D^L_1 = \alpha y_1 (1 + \mu)$$

In subsequent years, $D^R_i$ grows with a factor $(1 + g)/(1 + \mu)$: $D^R_i = ((1 + g)/(1 + \mu))D^R_{i-1}$. The growth rate of $D^L_i$ is $(1 + g)$: $D^L_i = (1 + g)D^L_{i-1}$. Hence, in the last year of the life-cycle, the two accrual flows are equal. Furthermore, $D^2_i$, zero for $i = 1$, becomes more positive over the life-cycle if $\mu > 0$: $D^2_i > D^2_{i-1}$. Hence, we make our fourth observation: for $\mu > 0$ the flow of accruals in the last year of the life cycle is smaller under the rational approach.

$$D^R_{TR} = \alpha y_{TR} (1 + \mu) < D^L_{TR} = D^L_{TR} + D^R_{TR} = \alpha y_{TR} (1 + \mu) + \mu A^R_{TR-1}$$

Taken together, these four observations imply that pension accruals are more back-loaded under the legal than under the rational approach in case of positive indexation.

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Appendix B

In order to derive that equation (24) and equation (21) are identical, repeat equation (24):

$$\frac{dU}{ds_2}_{H1H2} = -\eta_1 \tilde{t}_1 \hat{q}_1 \left( \frac{q_2 l_2}{(1 + r)q_1 l_1} \right) \left( \frac{1 + \eta_2 \hat{s}_2}{1 - \eta_1 \tilde{t}_1} \right) - \eta_2 \hat{s}_2 l_2 \hat{q}_2$$

Given that we defined $\hat{q}_1$ as $q_1((1 + r)/(1 + \delta))^2$ and $\hat{q}_2$ as $q_2((1 + r)/(1 + \delta))$, we can rewrite this as follows:

$$\frac{dU}{ds_2}_{H1H2} = \frac{(1 + r)}{(1 + \delta)^2} q_2 l_2 \left( -\eta_1 \tilde{t}_1 \left( \frac{1 + \eta_2 \hat{s}_2}{1 - \eta_1 \tilde{t}_1} \right) - \eta_2 \hat{s}_2 \right)$$

$$= \frac{(1 + r)}{(1 + \delta)^2} q_2 l_2 \left( 1 - (1 + \eta_2 \hat{s}_2) - \frac{\eta_1 \tilde{t}_1}{1 - \eta_1 \tilde{t}_1} (1 + \eta_2 \hat{s}_2) \right)$$

$$= \frac{(1 + r)}{(1 + \delta)^2} q_2 l_2 \left( 1 - \left( \frac{1 + \eta_2 \hat{s}_2}{1 - \eta_1 \tilde{t}_1} \right) \right)$$

The last line is the expression for the welfare effect of front-loading as derived in the main text (equation (21)).

Appendix C

We model the collective scheme identical to the individual scheme in section 2, except on one point: the collective scheme accounts for transition generations, the generations that were born before the collective scheme started to operate. These generations enjoy a windfall gain when this scheme is started as their life-cycle pension contributions will be lower than the pension rights they accrue. The windfall gain for a generation of age $i$ equals the cumulative value of implicit taxes at age $i$, as entry at age $i$ means that these implicit taxes will be foregone. These windfall gains have to be paid for by those generations that will be living all their life under the collective scheme. We will call these steady-state generations, as they live in an economy that grows at a constant rate. The redistribution between transition generations and steady-state generations is identical with the redistribution that the start of a pay-as-you-go scheme implies. Such a scheme also carries an implicit debt that is to be paid for by workers till infinity (Sinn, 2000).

It is easy to see that the collective scheme thus differs from the individual scheme in the pension fund constraint. We derive this pension fund constraint as follows. Aggregate pension contributions must equal aggregate accruals (and aggregate benefits) if we measure everything in euros at the
same point in time. Hence, aggregate excess contributions of steady-state
generations cancel against aggregate excess accruals of transition gener-
ations.

To make this concrete, let us start the scheme start at 1. Given that
people spend $T_R$ years in the working phase, the transition phase will take
$T_R - 1$ years. Year $T_R$ then marks the beginning of the steady-state phase
of the scheme. The pension fund budget constraint now equates the excess
accruals of the generations born in years 1 to $T_R - 1$ to the excess contribu-
tions of the generations born in year $T_R$ and thereafter, where all items are
discounted to $T_R$:

$$
\sum_{j=1}^{T_R-1} \sum_{i=1}^{j} \left( (\alpha(1+\mu)^i - \pi(1+r)^i)T^{T_R+i} \right) (1+r)^{T_R-j}
$$

$$
= \sum_{i=T_R}^{\infty} (\Pi_{T_R} - A_{T_R})(1+r)^{T_R-i} 
$$

(44)

and where $\Pi_{T_R}$ and $A_{T_R}$ are defined in equations (2) and (5). In order for
the infinite sum of utilities of steady-state generations to converge, we need
to assume that $r > 0$, which implies dynamic efficiency (see section 2).

For comparison, we take the accrual rate in the collective scheme equal
to that in the individual scheme. Hence, equation (44) can be elaborated to
calculate the pension contribution rate in the collective scheme. For brevity
however, we will move directly to the $T_R = 2$ case.

Equation (44) simplifies considerably in the $T_R = 2$ case:

$$
(\alpha(1+\mu) - \pi(1+r))y_2(1+r)
$$

$$
= \left( (\alpha y_1(1+\mu)^2 + \alpha y_2(1+\mu)) - (\pi y_1(1+r)^2 + \pi y_2(1+r)) \right) \left( \frac{1+r}{r} \right) 
$$

(45)

Elaboration of equation (45) gives the following expression for the pension
contribution rate,

$$
\pi^C = \alpha \left( \frac{1+\mu}{1+r} \right) \left( 1 - \frac{(r-\mu)}{(1+r)(2+g)} \right) 
$$

(46)

where we have used index $C$ to refer to the collective scheme.

Equation (46) shows that the pension contribution rate is below the
accrual rate, like in the individual scheme (recall that $r > \mu$). The front-
loading of pension contributions is thus preserved when we move to the case
of a collective scheme. But the contribution rate in the collective scheme
is higher than the one in the individual scheme. Obviously, this reflects reflecting the implicit debt that features the collective scheme. That this is so can be seen by a simple comparison. Rewrite the pension contribution rate in the individual scheme as specified in equation (7) for the case $T_R = 2$:

$$\pi = \alpha \left( \frac{1 + \mu}{1 + r} \right) \left( 1 - \frac{(r - \mu)}{(2 + r + g)} \right)$$  \hspace{1cm} (47)$$

Comparing the expression for $\pi$ in equation (47) with that for $\pi^C$ in equation (46) shows that $\pi^C > \pi$ (recall that $r > 0$).

For implicit taxes and subsidies, we derive the following expressions:

$$T^C_1 = -\alpha y_1 \left( \frac{(1 + \mu)(\mu - r)(1 + g)}{(2 + g)} \right)$$ \hspace{1cm} (48)$$

$$S^C_2 = \alpha y_1 \left( \frac{(1 + \mu)(r - \mu)(1 + g)}{(1 + r)(2 + g)} \right)$$ \hspace{1cm} (49)$$

The expression for the ratio of the two cash flows is remarkably simple:

$$\frac{T^C_1}{S^C_2} = 1 + r$$ \hspace{1cm} (50)$$

Recalling $r > 0$, this ratio is larger than one. Again, this reflects the implicit debt that features the collective scheme (the ratio equals one in the individual scheme).

**Appendix D**

This appendix shows formally why adding liquidity constraints does not modify the results of our analysis.

In order to explore liquidity constraints, we take a slightly modified version of our utility function in which workers consume in the first and second period of their lives. Also, without implications, we omit the third period and take zero values for the capital market rate of return and the rate of time preference, $r = \delta = 0$:

$$U = u_1(v_1) + u_2(v_2) + u_3(c_1) + u_4(c_2)$$  \hspace{1cm} (51)$$

The lifetime budget constraint changes accordingly:

$$q_1(1 - \pi)(1 - v_1) + q_2(1 - \pi)(1 - v_2) = c_1 + c_2$$  \hspace{1cm} (52)$$
The liquidity constraint expresses that workers are not allowed to borrow in the first period against second-period income:

\[ q_1(1 - \pi)(1 - v_1) - c_1 \geq 0 \]  

(53)

The optimization problem of the government is now best illustrated by setting up a Lagrangean function:

\[
\mathcal{L} = \{ u_1(v_1) + u_2(v_2) + u_3(c_1) + u_4(c_2) \}
+ \lambda\{ q_1(1 - \pi)(1 - v_1) + q_2(1 - \pi)(1 - v_2) - c_1 - c_2 \}
+ \nu\{ q_1(1 - \pi)(1 - v_1) - c_1 \}
\]  

(54)

In general, all types of consumption, i.e. \( c_1, c_2, v_1 \) and \( v_2 \), are functions of the implicit pension tax \( t_1 \) and the implicit pension subsidy \( s_2 \). In turn, these two policy instruments can be written in terms of the pension contribution rate \( \pi \) and the pension accrual rate \( \alpha \). Recalling that we assume a zero capital market rate of return, the definitions of \( t_1 \) and \( s_2 \) read as follows: \( t_1 = \pi - \alpha(1 + \mu)^2 \) and \( s_2 = -\pi + \alpha(1 + \mu) \). Further, the generalization implies we need to define two more elasticities that express how a change in the price of leisure in one period affects leisure demand in the other period:

\[ \eta_{12} = -(\partial x_1/\partial \hat{w}_2)/(1 - x_1(\hat{w}_2))/\hat{w}_2) \]  
\[ \eta_{21} = -(\partial x_2/\partial \hat{w}_1)/(1 - x_2(\hat{w}_1))/\hat{w}_1) \]

As clarified in the main text, we can have two types of policy reform. One is to achieve actuarial neutrality by adjusting the life cycle profile of the contribution rate. This means that the contribution rate in both periods 1 and 2 is changed in order to match the accrual period in that period. Using equation (54) and noting that any change in \( \pi \) changes \( t_1 \) and \( s_2 \), we can derive the following expression for \( \partial \mathcal{L}/\partial \pi \),

\[
\frac{\partial \mathcal{L}}{\partial \pi} = q_2l_2\left( \lambda - (\lambda + \nu)\{ 1 + \hat{s}_2(\eta_2 - \eta_{12}/\omega) \} / (1 - t_1(\eta_1 - \eta_{21}/\omega) ) \right)
\]  

(55)

where \( \omega \) is a shortcut for \((q_2l_2s_2)/(q_1l_1t_1)\).

As equation (55) makes clear, provided that the cross-elasticities \( \eta_{12} \) and \( \eta_{21} \) are sufficiently small, the expression for the welfare effect of an increase in the contribution rate is more negative if \( \nu > 0 \), i.e. if the liquidity constraint is binding. Even proposition 1 in the main text, which says that the welfare effect of a marginal reform is zero if \( t_1 = s_2 = 0 \), does not apply here. Indeed, \( \partial \mathcal{L}/\partial \pi < 0 \) if \( \nu > 0 \) and \( t_1, s_2 \geq 0 \).

The alternative type of policy reform is to achieve actuarial neutrality by adjusting the life cycle profile of the accrual rate. This means that the
accrual rate in both periods is changed in order to match the contribution period in that period. Using equation (54), we can derive the following expression for $\frac{\partial L}{\partial \alpha}$:

$$
\frac{\partial L}{\partial \alpha} = q_2 t_2 \left( \lambda - \lambda \left\{ \frac{1 + \tilde{s}_2 (\eta_2 - \eta_1 \omega)}{1 - t_1 (\eta_1 - \eta_2 \omega)} \right\} \right) \tag{56}
$$

As equation (56) makes clear, the welfare effect of an increase in the accrual rate is close to the one derived in the main text. If we assume that cross-elasticities are zero, the results are identical ($\lambda = 1$ in the model in the main text). Hence, the analysis in the main text continues to apply if workers are liquidity constrained and if the reform takes the form of adjusting accrual rates.