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Competitive pressure: the effects on investments in product and process innovation

Jan Boone*

I analyze the effects of competitive pressure on a firm’s incentives to invest in product and process innovations. I present a framework incorporating the selection and adaptation effects of product market competition on efficiency and the Schumpeterian argument for monopoly power. The effects of competition on a firm’s innovations depend on whether a firm is complacent, eager, struggling, or faint, which is determined by the firm’s efficiency level relative to that of its opponents. Finally, the following tradeoff is pointed out: a rise in competitive pressure cannot raise both product and process innovations at the industry level.

1. Introduction

In this article I analyze the effects of competitive pressure on two types of innovations: product and process innovations. I show that an important determinant of the effect of competitive pressure on a firm’s incentives to undertake these innovations is the firm’s efficiency level relative to its industry’s efficiency distribution. This determines whether a firm is “faint,” “struggling,” “eager,” or “complacent.” Second, I derive conditions under which a rise in competitive pressure raises industrywide efficiency. Finally, I show that if a rise in pressure raises industrywide efficiency, then it reduces product variety.

The motivation for this analysis is the following. The idea of Porter (1990) and empirical articles by Baily and Gersbach (1995), Blundell, Griffith, and Van Reenen (1995), and Nickell (1996) is that competition is good for innovation. But the theoretical literature is less clear on this issue: both negative and positive effects of competition on innovation are found. Unfortunately, it is hard to compare the different results from the theoretical literature, as a common framework is missing. For instance, as an incentive to innovate, Aghion and Howitt (1992) identify a profit level, \( \pi \), while Martin (1993) uses the steepness of a firm’s profit function with respect to its own cost level, \( \frac{\partial \pi}{\partial c} \). Further, a rise in competition is seen by Aghion, Harris, and Vickers (1995) as

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a switch from Cournot to Bertrand competition, while Arrow (1962) views it as a rise in the number of firms in the industry. My article tries to bring these results together by considering a number of parametrizations of competition and the effects of competition on both the level and the steepness of the profit function.

To illustrate, consider the following informal story. An individual $j$ considers entering a chess tournament where he will play one opponent. I am interested in two of $j$’s decisions: whether or not he enters and how much effort he invests in preparation for the tournament. The first decision is determined by the utility level enjoyed by participating. Player $j$ participates if and only if this utility level exceeds the sum of some entry fee and the disutility of preparation (see below). I assume that $j$ only derives utility from winning the tournament (that is, not from participating per se). Further, assume that preparation (by reading some chess books, say) yields a disutility of effort and a small increase in $j$’s chess skills. Hence the preparation decision is determined by the increase in the probability that $j$ wins the tournament due to a small increase in $j$’s skills. There are two forms of the tournament. In tournament A, a player wins the tournament if he wins five out of nine matches. Tournament B consists of just one match, winning the match means winning the tournament. I ignore the problem of ties. Tournament A is called more competitive than B because the more matches played, the clearer the ability differences come out between the players. How do $j$’s utility level and preparation differ between tournaments A and B?

Consider the following four plausible possibilities.¹ First, if $j$ is far worse than his opponent, he will invest more effort in preparation in tournament B than in A. In the more competitive tournament A, $j$ has no chance of winning anyway. Hence, $j$’s investment in preparation has a very small, if any, effect on his probability of winning. In B, however, preparation can increase the probability of winning the tournament, for instance, if $j$’s opponent makes a silly mistake. Besides investing less in preparation in tournament A, $j$’s utility level is lower in tournament A because he has no chance of winning it. This I call the reaction of a faint player to competition. As competition is increased the faint player enjoys the tournament less and gives up on preparation. Second, if $j$ is only a slightly less able chess player than his opponent, his reaction to a rise in competition can be characterized as struggling. Playing tournament A is less enjoyable for $j$ than playing B, because of the lower probability of winning. However, the rise in competition challenges him to fight back and invest more in preparation. In other words, the preparation effort has a bigger effect on $j$’s probability of winning in tournament A than in B. So unlike the faint type, a struggler strives to improve his skills as competition is raised. Third, if $j$ is slightly better than his opponent, the rise in competition makes the tournament more attractive to him and spurs him to invest in preparation. This is the reaction of an eager player to a rise in competition. He enjoys the rise in competition and works harder. Finally, it is conceivable that player $j$ becomes complacent due to the rise in competition, if he is far better than his opponent. The rise in competition emphasizes his feelings of superiority. There is no chance that he will lose tournament A. In other words, preparing for the tournament has almost no effect on the probability of winning. Hence he prepares less and enjoys the more competitive tournament better.

More formally, in this article I consider the effects of competitive pressure on a firm’s incentives to undertake product innovation (associated with $\pi$, as explained below) and process innovation (associated with $[\partial \pi / \partial c]$). Letting $(+, -)$ denote the case

¹ Because I have not formally defined how skills and preparation translate into the probability of winning, the possibilities may seem arbitrary. Below, I formally define payoffs in terms of profit functions and show that these four cases indeed exist.
where a rise in pressure increases the firm’s incentive for product innovation but reduces the incentive for process innovation, it is clear that there are potentially four cases. I show in some simple examples that all four cases can occur and that the cases are ordered by a firm’s cost level relative to the industry cost distribution. Roughly speaking, as a firm’s cost level is raised from zero to a high level, it goes through the regimes as follows: (+, −), (+, +), (−, +), and (−, −). In the terminology introduced above, these cases are called complacent, eager, struggling, and faint respectively.

In fact, I define a parameter as measuring competitive pressure if it satisfies this ordering condition and if a rise in the parameter reduces the profits of the least efficient firm in the industry. With this definition of competitive pressure it is straightforward to identify the selection and adaption effects of competition and the Schumpeterian argument for monopoly power, which are interpreted here in the following way. The selection effect says that a rise in competition selects more-efficient firms from less-efficient ones. According to the adaption effect, a rise in competition spurs firms to raise their productivity. The Schumpeterian argument states that firms invest more to invent a new product if the product’s expected profits are higher, and profits are higher the more monopoly power a firm has.

Then, in order to analyze the claim made by Porter (1990) and others, that a rise in competitive pressure raises industrywide efficiency, I consider the effects of pressure while all firms simultaneously adjust their product and process innovation decisions. Conditions are derived under which a rise in competitive pressure indeed raises industry productivity. However, it is shown that if a rise in pressure raises industry productivity, it reduces the number of products introduced into the market. This is a drawback not often mentioned in the policy debate on competition. The intuition for this result is as follows. As pressure is increased, the least efficient firm in the market faces a direct rise in pressure and an indirect one via the reduction in its opponents’ costs. Both effects reduce the least efficient firm’s profits and induce its exit.

The relation between the theoretical literature and my work is discussed in Section 5. In terms of the empirical literature, this work is not in the vein of Cohen and Levin (1989), who survey studies on innovation and market structure. The reason is that with asymmetric firms there is no simple relation between product market competition and market structure. A firm may be a monopolist due to high barriers to entry and face no competition, or it may be a monopolist because it is the most efficient firm in the industry and competition is so intense that less-efficient firms cannot survive. In both cases the market structure is the same, yet competitive pressure differs starkly. Work by Baily and Gersbach (1995), Blundell, Griffith, and Van Reenen (1995), and Nickell (1996) is more in line with the analysis here, as they measure competitive pressure directly using variables like import penetration or exposure to competition from companies with best manufacturing processes.

The next section introduces competitive pressure, gives examples, and shows the framework used. Section 3 analyzes the effects of competitive pressure on a single firm’s incentives to innovate, and Section 4 on industrywide incentives to innovate. Section 5 shows how the recent theoretical literature fits into the framework here. Section 6 concludes.

2 The term “adaption effect” is borrowed from the literature on organizational change. For instance, Hannan and Freeman (1989, p. 12) view adaption as “organizational variability [that] reflects designed changes in strategy and structure of individual organizations in response to environmental changes, threats, and opportunities.” In my article, organizational variability has only one dimension: organizational efficiency in producing goods.

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2. Competitive pressure

The definition of competitive pressure used here is geared to my subject: innovation. Competitive pressure is defined in terms of its effect on a firm’s incentives to undertake product and process innovations. The result of product innovation is a new product to introduce into the market. Hence the incentive for product innovation is determined by the profit level associated with this new product. The result of process innovation is a reduction in a firm’s cost level. The steeper the slope of the firm’s profit function with respect to its own cost level, the larger the firm’s incentive to reduce costs.

The definition is shown to hold in six examples of parametrizations of pressure. The examples are chosen with the idea in mind that a rise in competitive pressure makes firms more exposed to each others’ actions. Since firms’ efficiency levels determine their actions (here, the choice of output or price level) a rise in pressure brings out cost differences more clearly. This view of competitive pressure differs from the idea that a rise in competition reduces profits, which is used by some of the articles discussed in Section 5. In that case, a rise in competitive pressure cannot be distinguished from an industrywide bad productivity shock or fall in demand.

I am aware that the definition of competitive pressure used here does no justice to the rich variety of interpretations held by economists. For instance, one may also be interested in the effects of competition on industry output and consumer surplus, which play no part here. Or one can probably think of parametrizations of pressure that do not satisfy the definition. Yet, I think for the purpose at hand, the definition covers enough parametrizations to warrant analysis.

**Framework.** Let \( \pi(c_i, c_{-i}; \theta_i) \) denote the profits of firm \( i \) as a function of its own (constant) marginal cost level \( c_i \), with \([\partial \pi(c_i, c_{-i}; \theta_i)/\partial c_i] < 0\), its opponents’ (constant) marginal cost levels \( c_{-i} \), and a parameter \( \theta_i \). Unless explicitly stated otherwise, I assume that firms differ only in their efficiency level \( c_i \). Hence the profit function \( \pi(\cdot) \) is not indexed by \( i \), because firms are completely symmetric except for their efficiency level \( c_i \). Now consider the effect of the parameter \( \theta_i \) on firm \( i \)’s profit level, \( \pi(c_i, c_{-i}; \theta_i) \), and the steepness of the profit function, \([\partial \pi(c_i, c_{-i}; \theta_i)/\partial c_i]\). Then restricting attention to the signs (that is, not considering the size) of the effects of \( \theta_i \) on \( \pi(c_i, c_{-i}; \theta_i) \) and \([\partial \pi(c_i, c_{-i}; \theta_i)/\partial c_i]\), there are the four cases shown in Table 1.

Consequently, a firm \( i \) is said to be complacent if a rise in parameter \( \theta_i \) raises \( i \)’s profit level but does not raise the steepness of \( i \)’s profit function. Eager, struggling, and faint firms are analogously defined.

In this article, I say that a parameter \( \theta_i \) measures competitive pressure on firm \( i \) if it satisfies the following definition.

**Definition 1.** For given profit function \( \pi(c_i, c_{-i}; \theta_i) \), a parameter \( \theta_i \) is said to measure competitive pressure on firm \( i \) if there exist the values \( c_i^{e_i}, c_i^{s_i}, \) and \( c_i^{f_i} \in \mathbb{R}_+ \cup \{+\infty\} \), where \( c_i^{e_i}, c_i^{s_i}, \) and \( c_i^{f_i} \) are functions of \( c_{-i} \) and \( \theta_i \), such that

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3 In Example 2 below, \( \theta_i \) is a discrete variable counting the number of opponents of firm \( i \). In that case firm \( i \) is said to be complacent if \( \pi(c_i, c_{-i}, \theta_i + 1) - \pi(c_i, c_{-i}, \theta_i) > 0 \) and

\[
[\partial \pi(c_i, c_{-i}, \theta_i + 1)/\partial c_i] - [\partial \pi(c_i, c_{-i}, \theta_i)/\partial c_i] < 0;
\]

firm \( i \) is called eager if \( \pi(c_i, c_{-i}, \theta_i + 1) - \pi(c_i, c_{-i}, \theta_i) > 0 \) and

\[
[\partial \pi(c_i, c_{-i}, \theta_i + 1)/\partial c_i] - [\partial \pi(c_i, c_{-i}, \theta_i)/\partial c_i] > 0, \text{ etc.}
\]
TABLE 1 Four Firm Types

<table>
<thead>
<tr>
<th></th>
<th>( \frac{\partial \pi(c_i, c_{-i}; \theta_i)}{\partial \theta_i} &gt; 0 )</th>
<th>( \frac{\partial \pi(c_i, c_{-i}; \theta_i)}{\partial c_i} \leq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial \pi(c_i, c_{-i}; \theta_i)}{\partial c_i} \leq 0 )</td>
<td>Complacent</td>
<td>Faint</td>
</tr>
<tr>
<td>( \frac{\partial \pi(c_i, c_{-i}; \theta_i)}{\partial c_i} &gt; 0 )</td>
<td>Eager</td>
<td>Struggling</td>
</tr>
</tbody>
</table>

(i) \( c_{ie} \leq c_{is} \leq c_{if} \) and
- \( c_i \in (0, c_{ie}) \) implies that firm \( i \) is complacent,
- \( c_i \in (c_{ie}, c_{es}) \) implies that firm \( i \) is eager,
- \( c_i \in (c_{es}, c_{if}) \) implies that firm \( i \) is struggling, and
- \( c_i > c_{if} \) implies that firm \( i \) is faint;

(ii) \( c_{is} < c_i \) if firm \( i \) is the least efficient firm in the market in the sense that \( c_i \geq c_j \) for each active firm \( j \).

In words, the definition restricts the sequencing of the complacent-faint cases as the cost level of firm \( i \) is increased from zero, and it excludes the possibility that a rise in competitive pressure raises the profits of the least efficient firm in the market.

The sequencing is important for the interpretation of \( \theta_i \) as a measure of competitive pressure. If the change in \( \theta_i \) leads to a reduction in profits for low-cost firms while it raises profits for high-cost firms, it seems counterintuitive to call this a rise in competition. And a rise in pressure increases a firm’s incentives to reduce costs unless it is far ahead (complacent) or far behind (faint) its opponents. The next section discusses the intuition for this sequencing property.

The second restriction implies the well-known idea that a rise in competitive pressure in an industry with symmetric firms (that is, \( c_i = c_j \) for all active firms \( i \) and \( j \)) reduces each firm’s profit level. But with asymmetric firms it allows for a situation where a rise in pressure raises the profits of the most efficient firms. Restriction (ii) will be used in the analysis of industrywide effects of competitive pressure.

Finally, note that the definition does not require all four cases to appear for a parameter \( \theta_i \) to measure competitive pressure. Consider, for instance, Example 1 below, where competitive pressure on firm \( i \) is measured by the number of \( i \)’s opponents (for a given average cost level of the opponents). In that case it turns out that \( c_{ie} = c_{if} = 0 \) for each firm \( i \), that is, such a rise in competitive pressure never raises a firm’s profits.

Now I turn to some examples to illustrate the definition. Then the definition is interpreted in terms of product and process innovations using a simple model in which firms choose whether or not to enter an industry with a new product and how much to invest to enhance the efficiency of the product’s production process.

Examples. The following examples present some explicit choices for competitive pressure \( \theta_i \) to illustrate the framework above. The first example is the only one where all four cases—complacent, eager, struggling, and faint—are present. In the other examples there are no complacent firms. I use the following notational convention. If, as for instance in Example 1, a parameter measures competitive pressure in the same way for all firms in the industry, it is denoted by \( \theta \). If, as in Example 3, the parameter measures pressure on a specific firm \( i \), it is denoted \( \theta_i \). In Appendix A it is proved that the parametrizations of pressure in Examples 2–6 satisfy Definition 1.
Example 1. Consider two firms, denoted 1 and 2, that face demand of the form
\[ p_i(x_i, x_j) = (1/x_i^{-\theta})(x_i^\theta + x_j^\theta) \] with \( i \neq j \) and \( 0 < \theta < 1 \). This demand function is derived from a CES utility function \( u(x_i, x_j) = (x_i^\theta + x_j^\theta)^{1/\theta} \), where \( \theta \) measures the degree of substitutability between the goods of firm 1 and 2. Firm \( i \) has marginal costs \( c_i \). I say that pressure is increased as goods become closer substitutes, that is, \( \theta \) rises. The Cournot Nash equilibrium profits of firm \( i \) can be written as
\[
\pi(c_i, c_j; \theta) = \frac{1 + (1 - \theta)\left(\frac{c_i}{c_j}\right)^\theta}{1 + \left(\frac{c_i}{c_j}\right)^\theta}, \quad \text{with } i, j = 1, 2 \text{ and } i \neq j.
\]

Without loss of generality, \( c_i \) can be normalized to one, so that profits for firm \( i \) can be written as \( \pi(c; \theta) = [1 + (1 - \theta)c^\theta]/(1 + c^\theta) \), where \( c \) measures the cost gap between firm 1 and 2. Appendix A shows the existence of \( c^\theta > 0 \). Analyzing the second derivative \( \partial^2 \pi / \partial c \partial \theta \) is very tedious; in fact, I am not able to characterize it completely. Instead, consider Figure 1, which shows \( \partial^2 \pi / \partial c \partial \theta \) as a function of \( c \) and \( \theta \).

The figure suggests two distinct cases: \( \theta > \overline{\theta} \) and \( \theta < \overline{\theta} \) (for some cutoff point \( \overline{\theta} \in (0, 1) \)), which I shall consider in turn. First, look at the case with \( \theta > \overline{\theta} \). Then for \( c \) close to zero one can see that \( \partial^2 \pi(c; \theta) / \partial c \partial \theta > 0 \). As proved in the Appendix, for low values of \( c \) (that is, \( c < c^\theta \)) it is also the case that \( \partial \pi(c; \theta) / \partial \theta > 0 \). Hence for \( \theta > \overline{\theta} \), a firm with \( c \) close to zero is complacent. For a slightly higher value of \( c \) one finds \( \partial^2 \pi(c; \theta) / \partial c \partial \theta < 0 \) and \( \partial \pi(c; \theta) / \partial \theta > 0 \), that is, an eager firm. Increasing \( c \) further makes both \( \partial \pi(c; \theta) / \partial \theta \) and \( \partial^2 \pi(c; \theta) / \partial c \partial \theta \) negative: a struggling firm. Finally, for \( \theta > \overline{\theta} \) and high values of \( c \), the term \( \partial \pi(c; \theta) / \partial \theta \) stays negative while \( \partial^2 \pi(c; \theta) / \partial c \partial \theta \) turns positive: a faint firm. Now turn to the second case, with \( \theta < \overline{\theta} \). Then for \( c \) close to zero one finds \( \partial^2 \pi(c; \theta) / \partial c \partial \theta < 0 \) and \( \partial \pi(c; \theta) / \partial \theta > 0 \), that is, an eager firm. Increasing \( c \) makes both \( \partial \pi(c; \theta) / \partial \theta \) and \( \partial^2 \pi(c; \theta) / \partial c \partial \theta \) negative: a struggling firm. In other words, for \( \theta < \overline{\theta} \) there are no complacent firms. This suggests that complacent firms only appear in very competitive environments (\( \theta \) close to one) where the cost gap between the leader and follower is very big.
Example 2. Consider the inverse demand function \( p(X) = 1 - X \), where \( X = \sum_{i=1}^{n+1} x_i \) equals the sum of output levels of firms 1 to \( n + 1 \). Firm \( i \) has constant marginal costs \( c_i \), and firm \( i \)'s opponents have average marginal costs \( m_{-i} = \frac{1}{n} \sum_{j \neq i} c_j \). Then I say that pressure on firm \( i \) is increased if \( n \) increases while keeping \( m_{-i} \), constant. With \( \theta_i = n \), firm \( i \)'s Cournot profits can be written as

\[
\pi(c_i, c_{-i}; \theta_i) = \{(1 - (\theta_i + 1)c_i + \theta_i m_{-i})/(\theta_i + 2)\}^2.
\]

Then \( c_{ie} = c_{ei} = 0 \) and

\[
c_{ij} = \max \left\{ 0, \left[ \frac{\theta_i + 2}{(\theta_i + 3)^2} - \frac{\theta_i + 1}{(\theta_i + 2)^2} \right] \left( 1 + \theta_i m_{-i} \right) + m_{-i} \frac{\theta_i + 2}{(\theta_i + 3)^2} \left( \frac{\theta_i + 2}{\theta_i + 3} - \frac{\theta_i + 1}{\theta_i + 2} \right) \right\}.
\]

In words, this example features neither complacent nor eager firms.

Example 3. Let us use the same setup as in Example 2, but now the number of opponents of firm \( i \) is fixed at \( n \). Here the pressure on firm \( i \) is increased if it faces opponents with lower costs. Hence \( \theta_i \) is defined here as \( \theta_i = 1/m_{-i} \). Then it follows that \( \pi(c_i, c_{-i}; \theta_i) = \{(1 - (n + 1)c_i + n(\theta_i))((n + 2))\}^2 \). Then \( c_{ie} = c_{ei} = c_{ij} = 0 \). In words, in this setup it is impossible to model the idea that a fall in opponents’ cost levels spurs a firm to improve its own efficiency. As there are only faint firms here, a rise in pressure reduces a firm’s profit level and its incentive to reduce costs.

Example 4. The setup is the same as in the previous two examples. Now firm \( i \)'s costs equal \( wc_i \), that is, labor is the only production factor and is paid a wage equal to \( w \). Following Porter (1990) a higher wage in the industry is interpreted as higher pressure on all firms to innovate. The idea is that a higher wage makes differences in efficiency more pronounced. Hence \( \theta = w \), and firm \( i \)'s profits can be written as

\[
\pi(c_i, c_{-i}; \theta) = \{(1 - (n + 1)\theta c_i + n\theta m_{-i})/(n + 2)\}^2.
\]

Then \( c_{ie} = 0, c_{ei} = [n/(n + 1)]m_{-i} \), and \( c_{ij} = c_{ei} + 1/[2\theta(n + 1)] \). Hence it is possible that higher wages raise firms’ incentives to reduce \( c_i \). This gives an interesting perspective on a result found by Van Reenen (1996) that “innovating firms are found to have higher wages” (p. 195).

Example 5. Consider a Hotelling beach of length 1 with consumers distributed uniformly over the beach with density 1. Firm 1 is located on the far left of the beach and firm 2 on the far right. A consumer at position \( x \in (0, 1) \) who buys a product from firm 1 incurs a linear travel cost \( tx \), and if she buys from firm 2 she incurs travel cost \( t(1 - x) \). Assume that each consumer buys one and only one product. Firm \( i \) has constant marginal costs \( c_i \) (\( i = 1, 2 \)). Then demand for the products of firm \( i \) equals \( q_i(p_i, p_j; t) = \frac{t}{2} + [(p_i - p_j)/2t] \). As travel costs decrease, consumers are more inclined to buy from the cheapest firm rather than the closest one. So as travel costs decrease, firms’ monopoly power is reduced and pressure is higher. Measuring pressure as \( \theta = 1/t \), the profits of firm \( i \) can be written as \( \pi(c_i, c_j; \theta) = [(3/\theta + c_j - c_i)^2]((18/\theta), \) with \( i, j = 1, 2 \) and \( i \neq j \). It follows that \( c_{ie} = c_{ei} = 0 \) and \( c_{ij} = c_j \).

Example 6. Consider two firms, denoted 1 and 2, where firm \( i \) (= 1, 2) faces demand
of the form \( p(x_i, x_j) = 1 - x_i - \theta x_j \) with \( i \neq j \) and \( 0 \leq \theta \leq 2 \) (where the upper bound on \( \theta \) is needed to allow for an equilibrium with two active firms). Firm \( i \) has constant marginal costs \( c_i \). I say that pressure on each firm is increased as goods become closer substitutes, that is, as \( \theta \) is increased. The Cournot Nash equilibrium profits of firm \( i \) can be written as \( \pi(c_i, c_j; \theta) = \frac{2(1 - c_i - \theta(1 - c_i))}{(4 - \theta^2)^2} \). Then \( c_{i'} = \max\{0, 1 - \frac{(4 + 3\theta)/8\theta(1 - c_j)}{1 - c_j}\} \)

Although the definition is shown to hold for these six examples, I do not claim that it holds for any parametrization of competitive pressure. To illustrate this, I finish this section with a counterexample.

There are two ways in which a parametrization of pressure may not satisfy the definition above. First, the sequencing may not hold. Second, a rise in pressure may raise profits for the least efficient firm in the industry. From a mathematical point of view, there are lots of ways in which the sequencing may not hold. For instance, as the cost level of firm \( i \) is raised from zero, it may pass through the regimes as complacent, eager, complacent, struggling, faint, and eager. The simplest way in which the sequencing does not hold is related to the second condition: a rise in pressure reduces profits for a low-cost firm while it increases profits for a high-cost firm. This happens in the counterexample because the high-cost firm has a first-mover advantage. Then a rise in competitive pressure makes the cost disadvantage more pronounced but also affects the first-mover advantage. In other words, if firms are different in more dimensions than costs, the cost level alone does not determine a firm’s competitive position relative to its opponents. This stresses the importance of the assumption that firms are completely symmetric except for their efficiency level.

Example 7 (counterexample). Consider the same demand and cost structure as in Example 6, now with \( 0 < \theta < \sqrt{2} \) to allow for an equilibrium with two active firms. Suppose firm 1 has a first-mover advantage: firm 2 chooses output level \( x_2 \) after observing 1’s choice \( x_1 \). As shown in Appendix A, it follows for \( c_1 = .4, c_2 = .35, \) and \( \theta = 1.1 \) that \( \frac{d\pi(c_1, c_2; \theta)}{d\theta} > 0 \) while \( \frac{d\pi(c_2, c_1; \theta)}{d\theta} < 0 \). In words, a rise in pressure decreases the profits of the low-cost firm while it increases the profits of the high-cost firm because the latter has a first-mover advantage.

3. Competitive pressure and product and process innovations

To interpret the taxonomy in Section 2, consider the following, which is essentially the Dasgupta and Stiglitz (1980) model with two important additional features. First, firms are asymmetric in their ability to reduce production costs. Second, I analyze the effects of a change in competitive pressure instead of looking at the correlation between concentration and research intensity.

Consider an agent \( i \in \{1, 2, 3, \ldots\} \) who has an idea for a new good (also denoted \( i \)) to introduce into the market and a process-innovation function \( d(c, i) \). This process-innovation function \( d(c, i) \) denotes the investment agent \( i \) must make to reduce the constant marginal production costs of good \( i \) to level \( c \).

I analyze the following deterministic two-stage game. In the first stage, each agent \( i \) decides whether or not to enter the market with a new product and, if he enters, how much to invest to improve the efficiency of this product’s production process. In the second stage, the number of firms in the market and their cost levels are common knowledge. The firms produce output and choose independently and simultaneously their strategic variable (output or price level).

The second-period Nash equilibrium payoffs to agent \( i \) with cost level \( c_i \) are written as \( \pi(c_i, c_{-i}, I; \theta) \), where \( I \) equals the total number of firms in the market. That is, the

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following three sources of competitive pressure are explicitly introduced as arguments in the profit function. First, the lower the cost levels \( c_{-i} \) of firm \( i \)'s opponents, the higher the competitive pressure on firm \( i \), as modelled in Example 3. Second, the higher the number of opponents, measured by \( I \), the higher the competitive pressure, as in Example 2. The third source of competitive pressure is \( \theta \), which measures how aggressive the interaction is between firms, as in Examples 1, 4, 5, and 6.

I make the following assumptions on the process-innovation function \( d(c, i) \) and the profit function \( \pi(c_r, c_{-r}, I, \theta) \).

**Assumption 1.** The functions \( d(c, i) \) and \( \pi(c_r, c_{-r}, I; \theta) \) satisfy the following properties:

(i) \( d(c, i) \) is twice differentiable in \( c \) and satisfies \([\partial d(c, i)]/\partial c < 0, [\partial^2 d(c, i)]/\partial c^2 > 0\), and \( \lim_{c \downarrow 0} [\partial d(c, i)]/\partial c = -\infty \) for each \( i = 1, 2, \ldots \);

(ii) \( d(c, i) \) is nondecreasing in \( i \) and \([\partial d(c, i)]/\partial c \) is nonincreasing in \( i \);

(iii) \( \pi(c_r, c_{-r}, I; \theta) \) is twice differentiable in \( c_r, c_{-r}, \) and \( \theta \);

(iv) \([\partial^2 \pi(c_r, c_{-r}, I; \theta)]/\partial c_j^2 < 0 \) and \([\partial^2 \pi(c_r, c_{-r}, I; \theta)]/\partial c_j^i \geq 0 \) for \( j \neq i \);

(v) \([\partial^2 d(c, i)]/\partial c^2 > [\partial^2 \pi(c_r, c_{-r}, I; \theta)]/\partial c^2 \) for each \( c_r, c_{-r}, I, \) and \( \theta \); and

(vi) there exists a value \( M \in \mathbb{R}_+ \) such that for each value of \( c_{-r}, I, \) and \( \theta \) it is the case that \( \lim_{c \downarrow 0} ||[\partial \pi(c_r, c_{-r}, I; \theta)]/\partial c|| < M \).

In words, Assumption 1(i) says that a reduction in marginal production costs \( c \) requires higher expenditure on process innovations. A small reduction in \( c \) is more expensive, the lower the initial value of \( c \). And reducing marginal costs to zero is infinitely expensive. Assumption 1(ii) implies that agents are arranged in such a way that low-\( i \) agents have (weakly) better ideas than high-\( i \) agents in the following sense. Higher-\( i \) agents spend (weakly) more to achieve a given marginal production cost level \( c \). And for higher-\( i \) agents, reducing costs slightly below \( c \) is (weakly) more expensive. Assumption 1(iii) ensures that the first derivatives of the function \( \pi \) with respect to \( c_r, c_{-r}, \) and \( \theta \) are continuous functions of \( c_r, c_{-r}, \) and \( \theta \) that the second derivative exists. This simplifies the analysis below. Further, Assumption 1(iv) states that a firm’s profits are higher if its own costs are lower and its opponents’ costs higher. The latter implies that with competitive pressure on firm \( i \) measured by the cost level of opponent \( j \), there are only struggling and faint firms. Or if one assumes that the output of firm \( j \) is decreasing in \( j \)'s marginal cost level, then this condition can be stated as firms’ products being substitutes, not complements. Assumption 1(v) guarantees the concavity of the objective function below. Finally, I assume that the incentive to reduce costs is finite at values of \( c \) close to zero. Together with Assumption 1(i), this implies that firms’ marginal costs are always strictly positive in equilibrium.

In this section attention is focused on how \( \theta \) affects the following Nash equilibrium outcome in the first stage, called connected equilibrium.

**Definition 2.** For a given profit function \( \pi(c_r, c_{-r}, I; \theta) \) and level of competitive pressure \( \theta \), the first-stage connected equilibrium \( (c_1, \ldots, c_r, I; \theta) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R} \) is defined by the following four properties:

(i) optimal cost levels: the cost level \( c_i \) of each entering agent \( i \) satisfies \( c_i \in \arg\max_{c_i} \{ \pi(c_r, c_{-r}, I; \theta) - d(c_r, i) \} \);

(ii) connectedness: if agent \( i \geq 2 \) enters the market, then agent \( i - 1 \) enters as well;

(iii) nonnegative payoffs: the last agent, denoted \( I \), to enter the market has a nonnegative payoff \( \pi(c_r, c_{-r}, I; \theta) - d(c_r, I) \geq 0 \); and

(iv) free entry: for each \( I' > I \) it is the case that \( \max_{c_r} \{ \pi(c_r, c_{-r}, I'; \theta) - d(c_r, I') \} < 0 \).

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In words, in a connected equilibrium each agent $i$ chooses cost level $c_i$ that maximizes profits minus process-innovation costs, taking the number of firms $I$ and its opponents’ cost levels $c_{-i}$ as given. The equilibrium is called connected because all agents in the set $\{1, 2, \ldots, I\}$ are active in the market. That is, I do not consider equilibria where the presence of firm $i$ keeps the more efficient firm $i-1$ out of the market. The last agent $I$ to enter with a new product earns nonnegative payoffs and agents $I' > I$ cannot profitably enter. Because the equilibrium is connected, $I$ also measures the total number of firms in the market. The definition implicitly assumes that payoffs are decreasing in $i$, and the next lemma shows that this is indeed the case.

**Lemma 1.** $V(i, c_{-i}, I; \theta) = \max_i \{\pi(c, c_{-i}, I; \theta) - d(c, i)\}$ is (weakly) decreasing in $i$.

**Proof.** For $j > i$ it is the case that

$$
\pi(c_i, c_{-i}, I; \theta) - d(c_i, i) \geq \pi(c_j, c_{-i}, I; \theta) - d(c_j, i) \geq \pi(c_j, c_{-j}, I; \theta) - d(c_j, j),
$$

where the first inequality follows from revealed preference and the second inequality from Assumption 1. In particular, Assumption 1 (ii) implies that $d(c, i)$ is weakly increasing in $i$ and Assumption 1 (iv) implies that $[\partial \pi(c, c_{-i}, I; \theta)]/\partial c_j \equiv 0$ for $j \neq i$.

Q.E.D.

**Partial effects.** In this section I consider the effects of competitive pressure $\theta$ on firm $i$’s incentives to invest in product and process innovations, for fixed product- and process-innovation decisions of the other firms. In other words, I consider

$$
[dV(i, c_{-i}, I; \theta)]/d\theta \text{ and } dc_i/d\theta
$$

for given $c_{-i}$ and $I$.

**Corollary 1.** Fix $c_{-i}$ and $I$, then for

(i) a complacent firm it is the case that $[dV(i, c_{-i}, I; \theta)]/d\theta > 0$ and $dc_i/d\theta \geq 0$,

(ii) an eager firm it is the case that $[dV(i, c_{-i}, I; \theta)]/d\theta > 0$ and $dc_i/d\theta < 0$,

(iii) a struggling firm it is the case that $[dV(i, c_{-i}, I; \theta)]/d\theta \leq 0$ and $dc_i/d\theta < 0$, and

(iv) a faint firm it is the case that $[dV(i, c_{-i}, I; \theta)]/d\theta \leq 0$ and $dc_i/d\theta \geq 0$.

**Proof.** This follows from the envelope theorem and the implicit function theorem.

Q.E.D.

This result says that for a struggling or faint firm, an increase in competitive pressure reduces the firm’s incentive to invest in product innovation as $dV(\cdot)/d\theta \leq 0$. For complacent and eager firms, such a rise in pressure improves the incentive to undertake product innovation. Using the sequencing property in Definition 1 (complacent and eager firms have lower costs than struggling and faint firms), this can be interpreted as the selection effect described by Vickers (1995, p. 13) as “when firms’ cost differ, competition can play an important role in selecting more efficient firms from less efficient ones.”

In terms of the endogenous growth literature, $dV(\cdot)/d\theta \leq 0$ can be interpreted as the Schumpeterian argument for monopoly power. As pressure on a struggling or faint firm is increased, monopoly power and profit levels are reduced. This reduces the incentive to invent a new product. For instance, Aghion and Howitt (1992) identify the Schumpeterian argument in this way. Since in their model there is only one active firm in the market (which by definition is then the least efficient firm in the market), Definition 1 (ii) implies that this firm is struggling or faint. In my model, inventing a
product does not necessarily yield a monopoly position. Hence for relatively efficient firms (complacent or eager), a rise in competitive pressure raises their incentive to undertake product innovations because this rise in pressure enables them to better exploit their cost advantage.

For eager and struggling firms, increasing pressure leads to higher investments in process innovation as \( dc_i/d\theta < 0 \). This can be interpreted as the argument put forward by, for instance, Porter (1990) and Nickell (1996) that firms adapt to increased competitive pressure by raising their productivity. This I call the adaption effect of competitive pressure.

So the adaption and selection effects suggest that increasing competitive pressure improves the average efficiency in an industry. The latter effect works through eliminating relatively inefficient firms from the market and the former through forcing existing firms to improve their efficiency. For complacent and faint firms, however, competitive pressure works in the opposite direction, reducing their incentives to improve efficiency.

The intuition for this is given in the chess story in the introduction. When a very talented (complacent) agent plays a weak (faint) agent, increasing pressure translates into making the difference between them more pronounced. This will reduce the faint agent’s effort to prepare for the contest, as he no longer has even the slightest chance of winning. That is, increasing his effort has hardly any effect on his probability of winning. Similarly, if the talented player is far better than her weak opponent, such a rise in pressure makes her complacent. She reduces her effort as well, since she has a sure win anyway. This argument for the talented player seems to be most persuasive if the number of contestants is small. Indeed, in the examples above, complacent firms only feature in Example 1, where there are two players. With more than two players, the gaps between players become smaller. Hence, it is more likely that a rise in pressure spurs talented players’ efforts to get ahead of one another.

From this follows the intuition for the sequencing of signs for \( dc_i/d\theta \). If a firm is not that far behind or that far ahead of its opponents (that is, it is either eager or struggling), a rise in pressure spurs it to improve its efficiency.

4. Industrywide effects

In this section I consider the effects of competitive pressure \( \theta \) in the case where all firms adjust their product- and process-innovation decisions. This generalization is motivated by the policy question: What happens to industrywide product and process innovation if competitive pressure is raised? From a theoretical point of view, the issue is that three different sources of competitive pressure on firm \( i \) interact: \( \theta \), \( I \), and \( c_{-i} \). Proposition 1 shows that a rise in competitive pressure cannot raise both product and process innovation at the industrywide level. Proposition 2 derives sufficient conditions under which a rise in competitive pressure raises industrywide efficiency for the case where all firms are symmetric.

**Proposition 1.** In a connected equilibrium, a rise in \( \theta \) either reduces at least one firm’s efficiency level or (weakly) decreases the number of products, \( I \), introduced into the market (or both).

**Proof.** See Appendix B.

Thus, raising competitive pressure \( \theta \), at best, implies a tradeoff between improving each firm’s efficiency or increasing the number of new products introduced into the market. The key to this result is condition (ii) in Definition 1. If competitive pressure
\( \theta \) reduces each firm’s cost level \( c_i \), then the least efficient firm \( I \) faces increased competitive pressure from two sources: from \( \theta \) directly and from \( c_j \) indirectly. By Definition 1 (ii) both the rise in \( \theta \) and the fall in \( c_j \) cause firm \( I \)’s profits to fall and hence product \( I \) may not be introduced. Conversely, if the rise in \( \theta \) is offset by a fall in competitive pressure on firm \( I + 1 \) due to a rise in some firms’ cost levels \( c_j \) (\( j \neq I + 1 \)), firm \( I + 1 \)’s profits may rise, thereby increasing the number of products introduced into the market. The result also implies that, at worst, a rise in \( \theta \) can reduce firms’ efficiency and the number of products introduced into the market. This is illustrated with an example below.

Now I turn to a formalization of the policy claim made by Baily and Gersbach (1995), Porter (1990), and others that raising competitive pressure raises industrywide productivity. This result is not trivial theoretically, as can be seen in Martin (1993), who finds the complete opposite result, as discussed below. To simplify the analysis, consider the case where all firms have the same process-innovation function denoted by \( d(c) \), that is, \( d(c, i) = d(c) \) for each \( i = 1, 2, \ldots \). Further, I focus on equilibria where all entering firms choose the same marginal cost level \( c \). The first-stage equilibrium can be characterized in the following way, which is similar to Definition 2 except that the connectedness requirement is irrelevant with symmetric firms.

**Definition 3.** An industry configuration \((c, I, \theta) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}\) is called a symmetric equilibrium if and only if

(i) \( c \in \text{argmax}_{c} \{ \pi(c, c, I, \theta) - d(c) \} \);
(ii) \( c_i = c \) for each \( i \in \{1, 2, \ldots, I\} \);
(iii) \( \pi(c, c, I, \theta) - d(c) \geq 0 \); and
(iv) for each \( I' > I \) it is the case that \( \max_{c_i} \{ \pi(c, c, I', \theta) - d(c_i) \} < 0 \).

Lemma B1 in Appendix B derives sufficient conditions for such a symmetric equilibrium to exist. One of these conditions is that firms are faint with respect to competitive pressure as measured by \( I \). This opens up the possibility for multiple equilibria, as can be seen in the following. For a given value of \( \theta \), start at an initial equilibrium \((c, I, \theta) \) and raise \( c \). Since firms’ profits are increasing in their opponents’ costs (Assumption 1 (iv)), this rise in \( c \) attracts more firms in the industry, thereby raising \( I \). Because firms are faint with respect to \( I \), this rise in \( I \) decreases their incentive to reduce costs, making the outcome with higher \( c \) and higher \( I \) potentially a new equilibrium.

In light of these multiple equilibria, it seems natural to ask whether a rise in competitive pressure \( \theta \) can improve upon the equilibrium with the lowest cost level for each firm. Thus I focus on the most efficient symmetric equilibrium, which is defined as follows.

**Definition 4.** For given \( \theta \), a symmetric equilibrium \((c, I, \theta) \) is called a most efficient symmetric equilibrium if and only if there does not exist a symmetric equilibrium \((c', I', \theta) \) with \( c' < c \).

The next result formalizes the policy claim that raising competitive pressure raises industrywide productivity. Note that due to the focus on symmetric equilibria, Definition 1 (ii) implies that firms cannot be complacent or eager.

**Proposition 2.** Let \((c', I', \theta')\) and \((c'', I'', \theta'')\) denote two most efficient symmetric equilibria with \( \theta'' > \theta' \). Assume that for each \( c \geq 0 \) and \( I \geq 1 \), the following two conditions, evaluated at \( c_i = c \) and \( c_{-i} = (c, \ldots, c) \), hold:

(i) Each firm \( i \) is struggling with respect to pressure as measured by \( \theta \);
(ii) each firm \( i \) is faint with respect to pressure as measured by \( I \).
Then it is the case that $c'' \leq c'$.

**Proof.** See Appendix B.

Thus, raising competitive pressure from $\theta'$ to $\theta''$ (weakly) raises the efficiency of each (surviving) firm if each firm is struggling with respect to pressure as measured by $\theta$ and faint with respect to pressure as measured by $I$. The intuition for these conditions is the following. First, the rise in $\theta$ should increase firms’ incentives to reduce their marginal costs. That is, firms should be struggling with respect to $\theta$. Second, Proposition 1 implies that in this case, where all firms become more efficient, firm $I'$ is likely not to enter. That is, $I'' \leq I'$. The condition that firms are faint with respect to pressure as measured by $I$ guarantees that this fall in $I$ cannot overturn firms’ raised incentives (due to $\theta$) to reduce costs. In other words, if firms were struggling with respect to pressure as measured by $I$, the fall in $I$ reduces their incentives to reduce costs, which could overturn the result. Summarizing, it is indeed possible that a rise in competitive pressure $\theta$ increases each (surviving) firm’s efficiency. However, there is no reason to suppose that this is always the case.

Returning to the case of asymmetric firms, the next example shows that a rise in competitive pressure $\theta$ can reduce both the number of firms in the market and the efficiency level of the remaining firm in the market, because this firm is neither faint nor complacent with respect to pressure as measured by $I$. The intuition for this result is that a rise in pressure $\theta$ that reduces the number of firms in the market through the selection effect can reduce overall pressure on the remaining firms. This reduction in overall pressure weakens the remaining firms’ incentives to enhance efficiency.

**Example 8.** Consider two firms facing the demand structure in Example 6. Firm 2 has given constant marginal costs equal to $c_2 = .75$. Firm 1 can invest to reduce its constant marginal costs according to the process-innovation function $d(.5, 1) = 0$, $d(.4, 1) = .0276$, and $d(.3, 1) = .064$. In words, firm 1 has marginal costs equal to .5 if it does not invest at all and can reduce marginal costs to .3 if it invests .064 in process innovations. Both firms have fixed costs equal to zero. It is straightforward to verify that the equilibrium outcomes for different values of pressure $\theta$ are as in Table 2.

Increasing pressure from $\theta = .5$ to .7 triggers more investment by firm 1 to reduce costs, by the adaption effect of competition. Increasing pressure further from $\theta = .7$ to 1.0 pushes the inefficient firm 2 out of the market through the selection effect. This reduction in pressure on firm 1 outweighs (trivially) the increase in $\theta$, and firm 1 invests less in process innovations.

The symmetric case in Proposition 2 has the advantage that aggregate efficiency is a scalar $c$. The case with asymmetric firms in Proposition 1 is more general, but it has no natural way to introduce aggregate efficiency. In particular, in Proposition 1 aggregate efficiency is said to rise only if all surviving firms reduce their costs, which is a rather strict condition. An intermediate case is the following. Assume each firm’s

<table>
<thead>
<tr>
<th>Level of Competition</th>
<th>Cost Level Firm 1</th>
<th>Entry Decision Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = .5$</td>
<td>$c_1 = .4$</td>
<td>Firm 2 enters</td>
</tr>
<tr>
<td>$\theta = .7$</td>
<td>$c_1 = .3$</td>
<td>Firm 2 enters</td>
</tr>
<tr>
<td>$\theta = 1.0$</td>
<td>$c_1 = .5$</td>
<td>Firm 2 does not enter</td>
</tr>
</tbody>
</table>
profit function can be written as \( \pi(c, C, l; \theta) \), where \( C \) is a scalar measuring both aggregate efficiency and competitive pressure on a firm via its opponents’ cost levels.

For instance, in Example 3 aggregate efficiency can be defined as an unweighted average of the firms’ cost levels, \( C = 1/I \sum_{i=1}^{I} c_i \). In addition, the profit functions in this example can be written such that pressure via a firm’s opponents’ costs enters in the form of this aggregate efficiency measure \( C \).

For cases where the variable measuring competitive pressure with respect to opponents’ cost levels coincides with the measure of aggregate efficiency \( C \), Table 3 summarizes the following four industrywide regimes. First, as Proposition 1 shows, a rise in competitive pressure \( \theta \) cannot raise both product and process innovation. Second, Proposition 2 considers a case where a rise in competitive pressure raises aggregate efficiency but reduces the number of products introduced into the market. Third, although this case has not been analyzed here, it is possible that a rise in competitive pressure increases the number of products introduced into the market while reducing aggregate efficiency. For instance, Martin (1993) stresses the case where all firms invest less in process innovation due to an increase in the number of firms in the market. Also, Aghion and Howitt (1996) present a model where a rise in competition increases investments in creating new product lines but reduces investments in the development of old product lines. Finally, Example 8 illustrates the possibility that a rise in competitive pressure \( \theta \) reduces both the number of firms in the market and the efficiency of the remaining firm.

5. Product market competition in the theoretical literature

The analysis above is related to two strands in the theoretical literature. First, I discuss the literature on competition and innovation, then I briefly mention the literature on competition and managerial incentives where the effect of competition on the profit function plays an important role.

The seminal theoretical work on the effect of competition on innovation incentives is Arrow (1962). Arrow compares the situation of a monopolist that can reduce its constant marginal costs from \( c \) to \( c' < c \) with the case in which an R&D laboratory can license a technology \( c' \) to a firm in a homogeneous-good market with Bertrand competition where at least two firms have access to current best technology \( c \). He shows that the incentive to invent \( c' \) is always bigger in the latter case. If \( c' \) is a drastic innovation, this is due to the (Arrow) replacement effect, which says that higher initial profits for the incumbent reduce its valuation of the innovation. This effect is absent in my analysis because all firms enter simultaneously, hence there is no incumbent with initial positive profits. If \( c' \) is nondrastic, it is due to (what I call) the adaption effect. That is, \( \left( \frac{\partial \pi}{\partial c} \right) / \partial n > 0 \) in Arrow’s model, where \( n \) is the number of firms in the industry. In contrast, Martin (1993, p. 446) finds that “the greater the number of firms in the market—the greater the degree of competition—the smaller the payoff associated with a marginal increase in firm efficiency.” Martin analyzes Example 2 in Section 2 with symmetric firms. In that case, one can show that \( c_i > c'_i \) for all firms.

### Table 3: Four Industrywide Regimes

<table>
<thead>
<tr>
<th>Due to Rise in Competition</th>
<th>Number of Products ( l ) Introduced into the Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Efficiency ( C )</td>
<td>Rises Impossible by Proposition 1</td>
</tr>
<tr>
<td>Falls</td>
<td>Martin (1993), Aghion and Howitt (1996)</td>
</tr>
</tbody>
</table>

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\(i = 1, \ldots, n\) and therefore \(\frac{\partial \pi(c_i, \ldots, \theta))}{\partial c_i} \frac{\partial \theta}{\partial \theta} \leq 0\). In other words, the symmetry assumption implies that all firms are faint in this example. Aghion, Harris, and Vickers (1995) have a duopoly model in which a firm is either one step ahead of its opponent, level, or one step behind, with profits \(\pi(+1)\), \(\pi(0)\), and \(\pi(-1)\) respectively. They assume that \(\{\partial[\pi(+1) - \pi(0)]/\partial \theta > 0\) and \(\{\partial[\pi(0) - \pi(-1)]/\partial \theta < 0\). In other words, a firm that is one step ahead is eager or struggling, while a firm that is one step behind is faint.

The industrywide tradeoff found here between product innovation and process innovation is reminiscent of a result found by Aghion and Howitt (1996). In their model, a rise in competition reallocates resources away from process innovation on old product lines to product-innovation research that creates new product lines. The intuition is that developing old product lines becomes less profitable as competition increases, that is, firms working on old product lines are faint. Although Propositions 1 and 2 derive conditions for the opposite (adaption) effect, these results are not inconsistent, as argued at the end of the previous section.

The literature on the effects of competition on managerial incentives starts with the observation that managers’ incentives are not directly linked to profits because they do not own the firms. However, profits affect managers’ compensation contracts specified by the firms’ owners. Hence the effect of competition on profit is important in this context as well. Hart (1983), Scharfstein (1988), and Aghion, Dewatripont, and Rey (1997) model the effect of a rise in competition as a reduction in profits. This disciplines management through the threat of liquidation. That is, they assume that firms are either struggling or faint. Besides this threat-of-liquidation effect, Schmidt (1997) distinguishes the value-of-a-cost-reduction effect, which is determined by the expression \(\{\partial[\pi(c; \theta))]/\partial c_i \} / \partial \theta\). Hermalin (1992) features the same effect and calls it the change-in-the-relative-value-of-actions effect. In Hermalin (1994) this effect appears again in the form of Example 2 (in Section 2 above) with \(\theta = n\). None of these articles is able to sign this effect consistently. Hermalin gives examples and parameter values where it is positive and other parameter values where it is negative. The framework presented here can play a clarifying role, because it suggests conditions to sign this effect in terms of a firm’s relative efficiency position in the industry. If the firm is eager or struggling, then this value-of-a-cost-reduction effect is positive. If the firm is complacent or faint, the effect is negative.

6. Conclusion

This article has analyzed the effects of competitive pressure on firms’ incentives to innovate, for a broad class of parametrizations of pressure. I have distinguished two types of innovations: product and process innovations. The effects of a rise in competitive pressure on a firm’s incentives to invest in these innovations depend on whether the firm is complacent, eager, struggling, or faint. A firm’s type is determined by its efficiency level relative to that of its opponents. This framework brings together the selection and adaption effects of competition on innovation and the Schumpeterian argument for monopoly power, found in the literature. Finally, conditions have been derived under which a rise in competitive pressure increases each firm’s investments in process innovations to improve efficiency. But if the rise in pressure induces more industrywide process innovation, it reduces industrywide product innovation.

\footnote{Note that \(\{\partial[\pi(c') - \pi(c, \ldots, \theta))]/\partial \theta\), with \(c' < c^*, \) can be written as \(\int_{c'}^{c^*} \{\partial[\pi(c)]/\partial c\} / \partial \theta dc\) in order to see the similarity between their assumptions and the framework used here.}
I conclude with three qualifications of the analysis above, which are left for future research. First, the results above hold only if firms compete on a level playing field. That is, if firms differ on more dimensions than just efficiency, the taxonomy will no longer hold. Above I have given the example of an “unlevel” playing field in the sense that one firm has a first-mover advantage.

Second, in the analysis above no explicit representation of the product space is offered and products are simply assumed to be different. Then a fall in profits due to a rise in pressure makes it less attractive to introduce a new product, and product-innovation research falls. However, if firms invest to explicitly position their products, this result can be overturned. Suppose product-innovation research is needed to move a product away horizontally from the industry standard, and more research is needed to move it further away. Then a rise in pressure may make it profitable to move further away from the standard to create your own niche in the market, and product-innovation research increases. This is one way to overturn the result on the industrywide tradeoff between product and process innovation.

Finally, starting from the idea that managers are not profit maximizers is another way to argue that a rise in competitive pressure can stimulate both product and process innovation at the industrywide level. In that case, it is reasonable to assume that a rise in pressure that makes a firm’s profit function more steep still encourages process innovation. However, a rise in pressure that reduces a firm’s profit level may in this case act as a wake-up call for managers. That is, to avoid bankruptcy, managers have to look for new products that can generate additional profits. Hence if all firms are struggling with respect to competitive pressure as measured by \( \theta \), the number of firms in the market, and opponents’ costs, the following is possible in this case. A rise in pressure \( \theta \) improves each firm’s productivity (by making profit functions steeper) and increases the number of products introduced into the market (by reducing each firm’s profit level).

Appendix A

In this Appendix I derive that there exists a unique value \( c_i^* \) for Example 1. Further, it is proved that the parameterizations of competitive pressure in Examples 2–6 satisfy Definition 1. This is done by (1) deriving the critical values \( c_i^*, c_j^* \), and \( c_i^* \), (2) showing that \( c_i^* \leq c_j^* \leq c_i^* \), and (3) verifying that for the least efficient firm in market \( i \) it is the case that \( c_i^* > c_i^* \). Finally, the expressions for \( \delta \pi_i / \delta \theta \) and \( \delta \pi_i / \delta \theta \) in the counterexample (Example 7) are derived.

Example 1. The profits of firm \( i \) can be written as \( \pi(c; \theta) = [1 + (1 - \theta)c^a](1 + c^b)^2 \), where \( c = c_i/c_j \) measures the cost gap between firms \( i \) and \( j \).

\[
\frac{\delta \pi}{\delta \theta} = \frac{-c^a}{(1 + c^b)^2} \cdot \{\ln c\} \cdot [1 + \theta + (1 - \theta)c^a] \cdot (1 + c^b). \]

Hence

\[
\frac{\delta \pi}{\delta \theta} > 0 \quad \text{if and only if} \quad (-\ln c) > \frac{1 + c^a}{1 + \theta + (1 - \theta)c^a}. \]

Since \( \lim_{c \to 0} (-\ln c) = +\infty \) and \( (-\ln c) \) is decreasing in \( c \) while \( (1 + c^a)(1 + \theta + (1 - \theta)c^a) \) is bounded and increasing in \( c \), for each given value of \( \theta \in (0, 1) \) there is a unique value \( c_i^* > 0 \) such that

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Example 2. \( \pi(c, c_1; \theta) = \frac{(1 - (\theta_1 + 1)c_1 + \theta m_{-1})((\theta_1 + 2) - c_1)}{(\theta_1 + 2)^3} \)

\[
\frac{\partial \pi}{\partial \theta_1} = \frac{2(1 - (\theta_1 + 1)c_1 + \theta m_{-1})(2m_{-1} - c_1)}{(\theta_1 + 2)^3}.
\]

For \( i \)'s profits and output to be positive, it must be the case that \( 1 - (\theta_1 + 1)c_1 + \theta m_{-1} > 0 \). Therefore \( \frac{\partial \pi}{\partial \theta_1} > 0 \) if and only if \( c_1 < 2m_{-1} - 1 \). However, one can show that there is no cost configuration \((c_1, c_2, \ldots, c_n)\) where all firms produce profitably and \( c_i < 2m_{-1} - 1 \) for at least one firm \( i \). Hence \( c_{1n} = 0 \).

Although differentiating \( \pi \) with respect to \( \theta_1 \) has no economic meaning (since \( \theta_1 \) is a discrete variable counting the number of \( i \)'s opponents), it is mathematically possible by viewing \( \theta_1 \) (temporarily) as a real variable. The observation that \( \frac{\partial \pi}{\partial \theta_1} < 0 \) for each \((c_1, c_2, \ldots, c_n)\) and \( \theta_1 \) implies that \( \pi(c, c_{-1}, \theta_1 + 1) - \pi(c, c_{-1}, \theta_1) < 0 \) for each \((c_1, c_{-1})\) and \( \theta_1 \). Hence there are neither complacent nor eager firms.

Instead of considering \( \delta \pi / \delta \theta_1 \mid \theta_1 > 0, \) I now analyze

\[
\left| \frac{\partial \pi(c, c_{-1}, \theta_1 + 1)}{\partial c_1} - \frac{\partial \pi(c, c_{-1}, \theta_1)}{\partial c_1} \right| > 0.
\]

This can be written as

\[
(\theta_1 + 2) - \frac{(1 - (\theta_1 + 1)c_1 + \theta m_{-1})((\theta_1 + 2) - c_1)}{(\theta_1 + 3)^2} > (\theta_1 + 1) - \frac{(1 - (\theta_1 + 1)c_1 + \theta m_{-1})((\theta_1 + 2) - c_1)}{(\theta_1 + 3)^2}
\]

or, equivalently,

\[
c_1 < \frac{\theta_1 + 2}{(\theta_1 + 3)^2} - \frac{\theta_1 + 1}{(\theta_1 + 2)^2} + \frac{\theta_1 + 2}{(\theta_1 + 3)^2} - \frac{\theta_1 + 1}{(\theta_1 + 2)^2}.
\]

Hence

\[
c_{1n} = \max \left\{ 0, \frac{\theta_1 + 2}{(\theta_1 + 3)^2} - \frac{\theta_1 + 1}{(\theta_1 + 2)^2} \right\}
\]

\[
c_{2n} = \max \left( 0, \frac{\theta_1 + 2}{(\theta_1 + 3)^2} - \frac{\theta_1 + 1}{(\theta_1 + 2)^2} \right)
\]

\[
(1/2) \quad c_{1n} = c_{2n} = 0 \quad \text{and} \quad c_{ij} = \max \left\{ 0, \frac{\theta_1 + 2}{(\theta_1 + 3)^2} - \frac{\theta_1 + 1}{(\theta_1 + 2)^2} \right\}.
\]

(3) \( c_i > c_{ij} \) holds trivially for the least efficient firm.

Example 3. \( \pi(c, c_{-1}; \theta) = \frac{(1 - ((n + 1)c_i + n/\theta_1)(n + 2))}{(n + 2)^2} \)

\[
\frac{\partial \pi}{\partial c_i} = -\frac{2}{(n + 2)^3} \frac{1 - ((n + 1)c_i + n/\theta_1)(n + 2)}{(n + 2)^2}, \quad \frac{\partial \pi}{\partial \theta_1} = \frac{2n}{(n + 2)^3}.
\]

For \( i \)'s profits and output to be positive, it must be the case that \( 1 - ((n + 1)c_i + n/\theta_1) > 0 \). Therefore \( \frac{\partial \pi}{\partial \theta_1} \mid \theta_1 = \frac{2n}{(n + 2)^3} \) for all relevant values of \( c_i \).

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\[ (1/2/3) c_{ir} = c_{ir}' = c_{ir}'' = 0. \]

**Example 4.** \( \pi(c_i, c_{-i}; \theta) = [(1 - (n + 1)\theta) c_i + n\theta m_y]/(n + 2) \] 

\[ \frac{\partial \pi}{\partial \theta} = 2 \cdot \frac{1 - (n + 1)\theta c_i + n\theta m_y}{(n + 2)^2} \] 

\[ \frac{\partial}{\partial \theta} \left\{ \frac{\partial \pi}{\partial c_i} \right\} = \frac{2(1 + n(1 + 1)\theta - 2(n + 1)\theta c_i + 1).}{(n + 2)^2} \]

For \( i \)'s profits and output to be positive, it must be the case that \( 1 - (n + 1)\theta c_i + n\theta m_y > 0 \). Hence, \( \partial \pi/\partial \theta > 0 \) if and only if \( c_i < [n/(n + 1)]m_y \). Further, \( (\partial \pi/\partial \theta)/\partial c_i > 0 \) if and only if \( c_i < (1 + 2\theta m_y)/(2(n + 1)\theta) = [n/(n + 1)]m_y + 1/[2(n + 1)\theta] \). Consequently, \( c_i' = 0 \) and \( c_i'' = c_i''' = 1/[2(n + 1)\theta] \).

(1) For the least efficient firm \( i \) it is the case that \( c_i > m_y \), hence \( c_i > [n/(n + 1)]m_y \) also holds.

**Example 5.** \( \pi(c_i, c_{-i}; \theta) = (3\theta + c_i - c_{-i})^2/[18(\theta) = (3 + \theta(c_i - c_{-i}))^2/18 \theta \]

\[ \frac{\partial \pi}{\partial \theta} = \frac{[3 + \theta(c_i - c_{-i})][3 + \theta(c_i - c_{-i})]}{18 \theta^2}, \quad \frac{\partial}{\partial \theta} \left\{ \frac{\partial \pi}{\partial c_i} \right\} = \frac{c_i - c_{-i}}{9}. \]

For \( i \)'s profits and output to be positive, it must be the case that \( 3 + \theta(c_i - c_{-i}) > 0 \). Similarly, for \( j \)'s profits and output to be positive, it must be the case that \( 3 + \theta(c_i - c_{-i}) > 0 \). Hence \( \partial \pi/\partial \theta < 0 \) and \( c_{i'} = 0 \). Further, \( (\partial \pi/\partial c_i)/\partial c_i > 0 \) if and only if \( c_i < c_{-i} \). Hence \( c_{i'} = 0 \) and \( c_{i'} = c_i \).

(1) For the least efficient firm \( i \).

**Example 6.** \( \pi(c_i, c_{-i}; \theta) = [(2(1 - c_i) - \theta(1 - c_{-i})]/(4 - \theta^2) \]

\[ \frac{\partial \pi}{\partial \theta} = \frac{2[2(1 - c_i) - \theta(1 - c_{-i})]}{(4 - \theta^2)} [4\theta(1 - c_i) - (4 + \theta^2)(1 - c_{-i})] \]

\[ \frac{\partial}{\partial \theta} \left\{ \frac{\partial \pi}{\partial c_i} \right\} = \frac{2}{(4 - \theta^2)^2} [16\theta(1 - c_i) - (8 + 6\theta^2)(1 - c_{-i})]. \]

For \( i \)'s profits and output to be positive, it must be the case that \( 2(1 - c_i) - \theta(1 - c_{-i}) > 0 \). Hence \( \partial \pi/\partial \theta > 0 \) if and only if \( c_i < 1 - [(4 + \theta^2)/4\theta] (1 - c_{-i}) \). That is, \( c_i' = \max(1 - [(4 + \theta^2)/4\theta] (1 - c_{-i}), 0) \). Further, \( (\partial \pi/\partial \theta)/\partial c_i > 0 \) if and only if \( c_i < 1 - [(4 + 3\theta^2)/8\theta] (1 - c_{-i}) \). That is, \( c_i'' = 0 \) and

\[ c_{i'} = \max(1 - [(4 + 3\theta^2)/8\theta] (1 - c_{-i}), 0). \]

(1) \( c_{i'} = 0 \), \( c_{i''} = \max(1 - [(4 + \theta^2)/4\theta] (1 - c_{-i}), 0) \) and \( c_{i'} = \max(1 - [(4 + 3\theta^2)/8\theta] (1 - c_{-i}), 0) \).

(2) \( c_i' \leq c_i'' \) if \( (4 + \theta^2)/4\theta \geq (4 + 3\theta^2)/8\theta \). It is routine to verify that this last inequality holds because the assumption that \( \theta \in (0, 2) \).

(3) I prove by contradiction that \( c_i \geq c_i \) implies \( c_i > c_{i'} \). That is, suppose that \( c_i \geq c_i \) and \( c_i < c_{i'} \). Then the last inequality implies \( 1 - c_i \geq [(4 + \theta^2)/4\theta] (1 - c_{-i}) \), which is impossible because \( 1 - c_i < 1 - c_{-i} \) by assumption and \( (4 + \theta^2)/4\theta > 1 \) for each \( \theta \in (0, 2) \).

**Example 7** (counterexample). Firm 2, taking \( x_1 \) as given, chooses \( x_2 \) to solve

\[ \max_{x_2} (1 - x_2 - \theta x_1 - c_2) x_2. \]

It follows that \( x_2 = \gamma(1 - \theta x_1 - c_2) \) and \( x_2 = \gamma(1 - \theta x_1 - c_2)^2 \). Firm 1, knowing 2’s reaction function, solves...
Further, Lemma B1. In addition to Assumption 1, suppose that for a given value of \( I \) is a rise in pressure on \( I \) de\textsuperscript{finition} but now applied to pressure as measured by the cost level of \( i \in \text{RAND 2000} \).

By part (ii) of Definition 1, it is the case that \( \pi_c = \frac{1}{2} \). By the envelope theorem, this can be written as

\[ \max_{x_i} \left[ 1 - x_i - \frac{\theta}{2} (1 - \theta x_i - c_i) - e_i \right] x_i. \]

Consequently, \( x_i = [1/(2 - \theta^2)](1 - \theta^2 + (\theta/2)c_i - c_i) \) and \( \pi_c = [1/(4 - 2\theta^2)](1 - \theta^2 + (\theta/2)c_i - c_i)^2. \). It follows that

\[ \frac{\partial \pi_c}{\partial \theta} = \frac{1}{2} \left( 1 - \theta x_i - c_i \right) \left( x_i + \theta \frac{\partial x_i}{\partial \theta} \right). \]

where

\[ x_i + \frac{\partial x_i}{\partial \theta} = \frac{1 - c_i}{2 - \theta^2} + \frac{(1 - c_i)(2 - \theta^2)}{2(2 - \theta^2)} + \frac{2\theta \left[ 1 - c_i - \frac{(1 - c_i)(2 - \theta^2)}{2} \right]}{(2 - \theta^2)^2} = \frac{(2 + \theta^2)(1 - c_i) - 2\theta(1 - c_i)}{(2 - \theta^2)^2}. \]

Hence

\[ \frac{\partial \pi_c}{\partial \theta} = -\frac{1}{2} \left( 1 - \theta x_i - c_i \right) \frac{(2 + \theta^2)(1 - c_i) - 2\theta(1 - c_i)}{(2 - \theta^2)^2}. \]

It follows at \( c_1 = 0.4, c_2 = .35, \) and \( \theta = 1.1 \) that \( \frac{\partial \pi_c}{\partial \theta} > 0 \) while \( \frac{\partial \pi_c}{\partial \theta} < 0. \)

**Appendix B**

This Appendix contains the proofs of Propositions 1 and 2. Further, Lemma B1 derives sufficient conditions for a symmetric equilibrium to exist.

**Proof of Proposition 1.** To find out whether a rise in \( \theta \) may induce some firms to exit, consider the effect of \( \theta \) on the value of entering for the least efficient firm in the market, \( V(c_i, c_{-i}, I, \theta) \). By the envelope theorem, this can be written as

\[ \frac{dV(c_i, c_{-i}, I, \theta)}{d\theta} = \sum_{j \neq i} \frac{\partial \pi(c_i, c_{-i}, I, \theta)}{\partial c_j} \frac{dc_j}{d\theta} + \frac{\partial \pi(c_i, c_{-i}, I, \theta)}{\partial \theta}. \]

By part (ii) of Definition 1, it is the case that \( \frac{dV(c_i, c_{-i}, I, \theta)}{d\theta} > 0 \). Further, again using part (ii) of this definition but now applied to pressure as measured by the cost level of \( I \)'s opponents (where a fall in \( c_i \) for \( i \neq I \) is a rise in pressure on \( I \)), one finds that \( \frac{dV(c_i, c_{-i}, I, \theta)}{dc_i} > 0 \) for each \( i \neq I \). Therefore, \( dc_i/d\theta \leq 0 \) for each \( i \in \{1, \ldots, I - 1\} \) implies \( dV(c_i, c_{-i}, I, \theta)/d\theta < 0 \). Hence, if \( V(c_i, c_{-i}, I, \theta) = 0 \), this rise in \( \theta \) induces firm \( I \) to leave the market.

Conversely, the only way in which a rise in \( \theta \) will solicit more product innovation, in the sense that product \( I + 1 \) enters the market, is that \( dc_{i+1}/d\theta > 0 \) for a number of firms \( i \). Q.E.D.

**Lemma B1.** In addition to Assumption 1, suppose that for a given value of \( \theta \) it is that case that

(i) there exists a value \( c_0 > 0 \) such that

(a) for each \( I \geq 1 \) it is the case that

\[ \left| \frac{\partial \pi(c_i, c_{-i}, I, \theta)}{dc_i} \right|_{(c_0, c_0, \ldots, \theta)} \leq |d(c_0)|, \]

where \( \mathbf{1} = (1, \ldots, 1) \) is a vector of ones of dimension \( (I - 1) \), and

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Proof of Lemma B1. Define the function \( I(c, \theta) \) as

\[
I(c, \theta) = \max_{I \in c, \theta} \{ I \pi(c; c, I; \theta) - d(c) \geq 0 \}.
\]  

(B1)

In words, \( I(c, \theta) \) equals the maximum number of opponents with cost level \( c \) that a firm can face while it is still profitable to enter. By Assumption 1 (iv), \( I(c, \theta) \) is nondecreasing in \( c \).

Define the function \( \phi(c, \theta) \) as

\[
\phi(c, \theta) = \left| \frac{\partial \pi(c; c, I; \theta)}{\partial c} \right|_{(c, I, \theta)}.
\]  

(B2)

Note that by Assumption 1 (i) and (vi) it is the case that

\[
\phi(0, \theta) > 0,
\]

while by assumption (i) in the lemma it is the case that

\[
\phi(c_0, \theta) \leq 0.
\]

Further, for intervals of \( c \) where the number of firms \( I(c, \theta) \) does not change, the function \( \phi(c, \theta) \) is continuous in \( c \) because Assumption 1 (i) and (iii) imply that \( \partial \pi(c; c, I; \theta) / \partial c \) is a continuous function of \( c \) and \( c_0 \). Finally, at points \( c \) where a small increase in \( c \) does change the integer value of \( I(c, \theta) \), the function \( \phi(c, \theta) \) jumps upward because of Assumption 2 (ii) in the lemma and the observation above that \( \partial \pi(c; c, I; \theta) / \partial c \) is nondecreasing in \( c \). In words, if a small increase in \( c \) raises the number of firms in the market, \( \partial \pi(c; c, I; \theta) / \partial c \) decreases because firms are wary of entering into each other's presence.

Summarizing, for given \( \theta \), the function \( \phi(c, \theta) \) is continuous, but for upward jumps, see Milgrom and Roberts (1994). Further, for values of \( c \) close enough to zero the function \( \phi(c, \theta) \) is strictly positive, while at \( c = c_0 \) the function is nonpositive. Milgrom and Roberts's theorem 1 implies that there exists a solution \( c \) for the equation \( \phi(c, \theta) = 0 \).

Now I will argue that a configuration \((c, I, \theta)\) where \( c \) solves \( \phi(c, \theta) = 0 \) and the number of firms equals \( I = I(c, \theta) \) is a symmetric equilibrium as defined in Definition 3. First, at such a configuration it is the case that

\[
\frac{\partial \pi(c; c, I; \theta)}{\partial c} \bigg|_{(c, I, \theta)} - d'(c) = 0,
\]

which together with Assumption 1 (v) implies that \( c \) satisfies property (i) of Definition 3. Since \( \phi \) is defined for symmetric configurations only, property (ii) of Definition 3 is satisfied as well. Second, the definition of \( I(c, \theta) \) implies that

\[
\pi(c, c, I; \theta) - d(c) \geq 0,
\]

while

\[
\max_{I, I'} \{ \pi(c, c, I + 1; \theta) - d(c, I') \} < 0,
\]

where, with a slight abuse of notation \( \iota \) denotes a vector with ones of dimension \((I - 1)\) in the first equation and of dimension \( I \) in the second equation. These two inequalities imply that conditions (iii) and (iv) of Definition 3 are satisfied as well. Finally, the second part of assumption (i) in the lemma ensures that at least one firm can profitably enter in equilibrium. \( \overline{Q.E.D.} \)

Proof of Proposition 2. Because of Assumption 1 (i) and (vi), \( \phi(0, \theta) > 0 \), where the function \( \phi(c, \theta) \) is defined in (B2) above. Also, as shown in the proof of Lemma B1, Assumption 1 together with condition (ii) in the proposition imply that \( \phi(c, \theta) \) is continuous in \( c \) but for upward jumps. In the proof of Lemma B1 it
is shown that \((c, I, \theta)\) is a symmetric equilibrium if \(c\) solves the equation \(\phi(c, \theta) = 0\) and \(I = I(c, \theta)\), where the function \(I(c, \theta)\) is defined in (B1) above. It follows from these observations that

\[
\begin{align*}
  c' &= \min\{c \mid \phi(c, \theta) = 0\} = \inf\{c \mid \phi(c, \theta) \leq 0\} \\
  c'' &= \min\{c \mid \phi(c, \theta') = 0\} = \inf\{c \mid \phi(c, \theta') \leq 0\}.
\end{align*}
\]

Then, using Lemma 1 in Milgrom and Roberts (1994), to prove that \(c'' \leq c'\) it is sufficient to show that\(\phi(c, \theta') \leq \phi(c, \theta')\)

for each \(c \succeq 0\). This inequality follows from conditions (i) and (ii) in the proposition, as the following two steps show. First, for given \(c\) and \(I\), \(\phi(c, \theta)\) is decreasing in \(\theta\) by condition (i). Second, this condition (i) also implies that the function \(I(c, \theta)\) is nonincreasing in \(\theta\). Together with condition (ii), which implies that for given \(c\) and \(\theta\) the function \(\phi(c, \theta)\) is nonincreasing in \(I\), one finds that a rise in \(\theta\) also decreases \(\phi(c, \theta)\) via a reduction in \(I\). Hence both effects together imply that \(\phi(c, \theta') \leq \phi(c, \theta')\) for each \(c \succeq 0\). Q.E.D.

References


