MONETARY POLICY AND EXCESSIVE BANK RISK TAKING

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Monetary Policy and Excessive Bank Risk Taking*

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Abstract

If monetary policy is to aim at financial stability, how would it change? To analyze this question, this paper develops a general-form model with endogenous bank risk profiles. Policy rates affect both bank incentives to search for yield and the cost of wholesale funding. Financial stability objectives are then shown to make a monetary authority more conservative and more aggressive. Conservative as it sets higher rates on average. And aggressive because, in reaction to negative shocks, cuts are deeper but shorter-lived than otherwise. Keeping cuts short is crucial as bank risk responds primarily to stable low rates. Within the short span, cuts then must be deep to achieve standard objectives.

Keywords: Monetary policy, Financial stability
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1 Introduction

One of the prime suspects for the cause of the recent financial crisis is low monetary policy rates. Various authors have argued that the US Fed’s accommodative policies spurred risk taking incentives among the financial intermediaries that were at the heart of the crisis.\(^1\) Three recent papers investigate empirically the link between monetary policy to bank risk taking. Maddaloni et al. (2009) use data from the Euro Area Bank Lending Survey to show that lower overnight rates soften lending standards. This softening is beyond what can be explained by other factors affected by the rates, like the quality of the borrower’s collateral. Interestingly, moreover, they find evidence that keeping rates \emph{too low for too long} reduces credit standards even further. Jiménez et al. (2009) and Iaonnidou et al. (2009) use data from Spanish and Bolivian credit registers, respectively. In both countries monetary policy was largely determined abroad over the sample period. Controlling for various bank, borrower and market characteristics, these studies find robust evidence that lower short-term rates spur the granting of loans to worse quality borrowers.\(^2\)

Various authors have called for the formulation of a monetary policy that explicitly considers bank risk taking and financial stability.\(^3\) In this paper, we take the case for ‘leaning against the wind’ as given, and ask the following questions: if such an action is desirable, how would it affect optimal monetary policy? Would it involve a level shift in interest rates? Would it imply a different timing of monetary policy? We develop a general-form analytical framework to address these questions. It contains a representative bank and a monetary authority, whose choice variables, the risk profile and the path of policy rates respectively, interact to affect economic activity. The bank’s risky projects are long-term loans that are relatively illiquid: only a fraction terminate each period. Bank default probabilities rise in its maturity mismatch. The monetary authority puts a weight on preventing the event of default, the social costs of which are only partially internalized by the bank. The larger this weight, the more "financial stability" oriented we say the authority is. It controls the risk-free rate through which it can both influence inflation and economic activity, and the appetite of the bank to search for yield.

We solve the game between the bank and the authority analytically. The model’s main

\(^1\)These include Borio and Zhu (2008), Dell’Ariccia et al. (2008), Calomiris (2009), Brunnermaier (2009), Brunnermaier et al. (2009), Taylor (2009), Allen et al. (2009), Adrian and Shin (2009), and Diamond and Rajan (2009).

\(^2\)In the case of the recent crisis, worsening lending standards materialized primarily on the housing market. See Dell’Arricia et al. (2008), Mian and Sufi (2008), Keys et al. (forthcoming), Demyanyk and Van Hemert (forthcoming) and Taylor (2009).

\(^3\)Borio and White (2004), Borio and Zhu (2008) and Adrian and Shin (2008, 2009a). A related view is that monetary policy should focus on credit growth (Christiano et al. (2007, 2008)) or credit spreads (Taylor (2008), McCulley and Toloui (2008), Cúrdia and Woodford (2009)).
results are summarized by the figure above, which represents the response of the monetary authority to a negative economic shock. The dotted line graphs the policy of an authority that ‘leans against the wind’, while the solid line is that of an authority with standard-objectives. There are two main effects to the financial stability objective: a level effect and a dynamic effect shown in figure 1. The level effect means that an authority that ‘leans against the wind’ has a higher steady state interest rate. That is, on average it sets higher rates and is willing to put a degree of deflationary pressure on the economy to prevent the buildup of risks. The dynamic effect is what we term a v-shape. The financial stability objective makes the authority choose a short, deep rate cut in response to a negative shock. By making the cut short, it prevents the buildup of bank risk. The reason is that as risk is persistent, the bank cannot rebalance its portfolio towards less risk quickly. It only chooses to raise risk when it foresees that rates will remain low for long, while the economy starts picking up. This is closely related to the finding of Maddaloni et al. (2009) that keeping policy rates too low for too long is particularly damaging. But given that the financial stability objective favors a short period of low rates, it implies a deep cut, in order to approach its output and inflation objectives as well. Therefore, within the short window it has, it cuts rates deeply to boost economic activity over time. Overall, a policy concerned with financial stability is both conservative (high rates) as well as aggressive (deep but brief cuts).

Subsequently, we introduce a bank funding channel. As much discussed in the aftermath of the crisis, low policy rates strongly affect the cost of bank funding (refs). The reason is that banks have opted for more and more short-term funding on the wholesale market over time. By affecting the cost of bank funding, the monetary authority can directly affect bank leverage. This, in turn, feeds into default probabilities. We analyze the effects of the bank funding channel on optimal monetary policy. We find that the level effect of ‘leaning against the wind’ is strengthened.
Finally, we provide a numerical example. We define specific functional forms and simulate the model in order to further visualize the interactions. The simulations analyze how variables such as the illiquidity of assets, the lagged effects of monetary policy and the patience of the policy maker affect the dynamics of ‘leaning against the wind’.

Our work is related to the literature on the role of financial intermediation in the transmission of monetary policy.\textsuperscript{4} In this literature banks are mostly passive players, however, generating a credit friction. The analyses focus on how this friction affects monetary transmission. An exception is a recent contribution by De Walque et al. (2008) who develop a DSGE model with endogenous default probabilities for banks. They show that liquidity injections, which improve financial stability, have ambiguous effects on output fluctuations. Their aim and setup are quite different from ours as in our paper we consider how bank incentives affect monetary policy \textit{through} bank optimization. That is, how monetary policy affects the optimal buildup of bank risk. Our work also relates to the literature on monetary policy and bank regulation. This literature focusses primarily on the pros and cons of conducting these functions at the same institution.\textsuperscript{5} Instead, Cecchetti and Li (2008) ask what monetary policy should look like given the procyclicality of bank capital requirements. They conclude that rate cuts should be deeper during downturns in which banks are capital constrained.

We proceed as follows. Section 2 outlines the general framework, and derives the analytical results. Section 3 then discusses the numerical example. Finally, section 4 concludes.

\section{Model}

We examine how interest rates are affected - in levels and dynamically - when the monetary authority has explicit financial stability objectives, next to its standard concerns. The latter will be captured in our model by the output stabilization term (in output gap terms) and the former by a measure of excessive risk. We make three assumptions in our setup:

\textbf{Assumption 1} The bank takes more risk than is socially optimal.

\textbf{Assumption 2} Risk taking is procyclical.

\textbf{Assumption 3} Risk is persistent.

The first two assumptions will yield our level result. When the authority aims at reducing excessive risk then it will, on average, have higher interest rates because that reduces the bank’s risk motive. The third assumption underlies our dynamic result. When the authority

\textsuperscript{4}See Bernanke and Gertler (1989, 1995), Bernanke et al. (1996), Kiyo

\textsuperscript{5}See Goodhart and Schoenmaker (1995), Peek et al. (1999), and Iaonnidou (2005).
aims at reducing excessive risk then it will choose to confine rate cuts to the period that banks build down risk, and hike rates before the built-up of new risk begins.

We describe the economy by the general function:

\[ y_t(\alpha_t, \varepsilon_t, r^f_t, r^f_{t-1}, \ldots, r^f_0) , \]  

where \( y_t(\cdot) \) is the output gap; \( r^f_t, r^f_{t-1}, \ldots, r^f_0 \) are the current and all past interest rates. The standard arguments of the IS equation imply that:

\[ \frac{\partial y_t(\alpha_t, \varepsilon_t, r^f_t, r^f_{t-1}, \ldots, r^f_0)}{\partial r^f_{t-s}} < 0 \quad \forall s \leq t. \]  

Variable \( \varepsilon_t \) represents a persistent demand shock:

\[ \varepsilon_t = \theta \varepsilon_{t-1} + \nu_t, \]  

with \( \theta \in (0, 1) \) the persistence parameter, and \( \nu_t \) an iid shock. The impact on the business cycle is such that:

\[ \frac{\partial y_t(\cdot)}{\partial \varepsilon_t} > 0. \]

Finally, \( \alpha_t \) is the risk profile of the bank. The bank chooses a risk profile for its assets \( \alpha_t \in [0, 1] \), where a higher \( \alpha_t \) corresponds to a more risky profile. A riskier profile implies higher expected return, but also a higher volatility, and hence greater financial instability.

**Claim 1** There is a socially optimal level of bank risk taking, \( \alpha^w_t \), such that:

\[ \frac{\partial y_t(\cdot)}{\partial \alpha_t} > 0, \quad \forall \alpha_t \in [0, \alpha^w_t), \]  

\[ \frac{\partial y_t(\cdot)}{\partial \alpha_t} < 0, \quad \forall \alpha_t \in (\alpha^w_t, 1]. \]

That is: up to a certain point the social benefits of risk taking dominate the cost of greater financial instability. But beyond that point the opposite is true.

The monetary authority combines its two objectives in the following inter-temporal function:

\[ \min_{r^f_t, \quad t \geq 0} \mathbb{E} [L] = \min_{r^f_t, \quad t \geq 0} \left\{ \mathbb{E} \sum_{t=0}^{\infty} \delta^t \left[ (1 - \rho) [y_t(\cdot)]^2 + \rho (\alpha_t - \alpha^w_t)^2 \right] \right\} \]

s.t.: \( y_t(\cdot), \)
where \( \rho \in [0, 1] \) is the ‘leaning against the wind’ term. For a greater \( \rho \) the monetary authority places greater weight on preventing excessive risk taking and, thereby, the buildup of greater financial imbalances.

We further assume one bank, whose management is risk neutral. This bank can be seen as representing the banking sector’s aggregate balance sheet. The bank chooses a risk profile \( \alpha_t \) to maximize its profit, \( P_t \):

\[
\max_{\alpha_t} \left\{ \mathbb{E} \sum_{t=0}^{\infty} \delta^t P_t [\alpha_t, y_t (\cdot)] \right\}
\]

subject to:

\[
\alpha_t \geq \beta \alpha_{t-1}.
\]

A riskier profile raises expected revenues, but it also raises financial instability. The bank dislikes instability, but less so than society. It incurs smaller costs from a crisis or default than society, implying that there are externalities that it fails to internalize. The bank derives an optimal risk profile \( \alpha_t^b > \alpha_t^w \), or in other words it takes more risk than is socially optimal, (assumption 1).

As assets yield higher returns in good times, it follows that:

\[
\frac{\partial P_t (\cdot)}{\partial y_t (\cdot)} > 0.
\]

However, the business cycle affects asset returns differentially. Riskier assets are more sensitive to the state of the economy. In good times the yield curve is more upward sloping: the return differential between relatively safe short term assets and longer term investments is large. Therefore, both socially and bank optimal risk taking is higher in good times. This matches the procyclicality of risk taking (assumption 2):

\[
\frac{\partial \alpha_t^i}{\partial y_t (\cdot)} > 0 \text{ for } i = b, w.
\]

Finally, the constraint in (8), \( \alpha_t \geq \beta \alpha_{t-1} \) with \( \beta \in (0, 1) \), implies that the bank’s risk profile is persistent (assumption 3). Given that risky projects involve long maturities, they cannot all be instantaneously shed from the bank’s balance sheet. This illiquidity is a key facet of banking theory.\(^6\)

\(^6\)In fact, given that riskier projects generally involve longer maturities, we could write in more general notation: \( \beta (\alpha_t) \), with \( \beta' (\alpha_t) > 0 \). That is, the riskier a bank’s profile, the longer the maturities of its loans, the fewer loans terminate each period and, therefore, the more persistent its balance sheet becomes.
A Steady state

We first consider the implications of our set-up at the steady state, in terms of the level of interest rates and risk taking. We abstract therefore from the dynamic effects due to the presence of stochastic shocks, $\varepsilon_t$, and the illiquid nature of the assets, $\alpha_t \geq \beta \alpha_{t-1}$, and drop the time element $t$. We consider the equilibria of two games: a simultaneous moves Nash game and a Stackelberg game in which the monetary authority is the first mover. We will show why the latter is the more natural framework within which to analyze the issue of ‘leaning against the wind’. We are interested in how the comparative statics change with respect to the weight that the authority places on its financial stability objective, $\rho$.

We derive the two players’ reaction functions, in terms of each other’s instrument. Define $\alpha^b (r^f, \cdot)$ as the bank’s reaction function to the monetary authority’s interest rate. From (2) and (10) above, it follows that the bank reduces the risk it takes following an increase in the interest rate, i.e.:

$$\frac{\partial \alpha^b (\cdot)}{\partial r^f} < 0.$$  

(11)

Intuitively, high interest rates cool down the economy, which makes risk taking less attractive.

The monetary authority, in turn, reacts to the level of risk that the bank takes. Define $r^f (\alpha, \cdot)$ as the monetary authority’s reaction to bank’s risk taking. The effect of $\alpha$ on its objective function runs through $y (\cdot)$. An increase in $\alpha$ will, as discussed above, increase $y (\cdot)$ when $\alpha < \alpha^w$ and decrease $y (\cdot)$ when $\alpha > \alpha^w$. From (2), a higher $r^f$ is optimal for a higher $y (\cdot)$. Thus, we have that:

$$\frac{\partial r^f (\cdot)}{\partial \alpha} > 0 \quad \forall \quad \alpha < \alpha^w,$$

$$\frac{\partial r^f (\cdot)}{\partial \alpha} < 0 \quad \forall \quad \alpha > \alpha^w.$$

The reaction functions for the two players are depicted in figure 2 below.

The solid line is the reaction function of the bank and it is unambiguously downward sloping. The dotted line is the reaction function of the monetary authority.\textsuperscript{7} Up to $\alpha^w$ the optimal interest rate of the authority increases in $\alpha$, and decreases thereafter. The dot at the crossing of the two lines represents the Nash equilibrium of the simultaneous moves non-cooperative game. It therefore maps into the optimal action for each of the two players, taking as given the decision of the other. The corresponding interest rate and risk profile are $(r^f)^\ast$ for the authority, and $\bar{\alpha}^b$ for the bank.

\textsuperscript{7}Note that the slope of the two reaction functions depends on the sensitivity of the two players to each other’s instrument. Our choice here bears no consequence for the argument formulated.
We note that the Nash equilibrium of the simultaneous move non-cooperative game corresponds to the same interest rate level (and level of risk taking), irrespective of the weight that the monetary authority puts on the financial stability objective. In other words:

\[ r_f^N = \arg \min_{r_f} L \biggr|_{\rho=0} = \arg \min_{r_f} L \biggr|_{\rho>0}. \]

This ‘irrelevance’ of the financial objective in the monetary authority’s losses comes about from the fact that in such a set-up the risk taken by the banks is ‘given’. For \( \rho > 0 \) the authority puts weight on preventing financial instability, but it does so for a given \( \alpha \). By implication the level of excessive risk in the economy as given. When excessive risk is given, the authority cannot do much to prevent it. This, in turn, implies that the authority’s policy is unaffected by the choice of \( \rho \). However, when thinking about allowing for a financial stability objective in monetary policy, one would like to allow for interest rates to affect the buildup of risks in the financial sector. In the Stackelberg set-up the monetary authority chooses its optimal \( r_f \) given the reaction function of the bank, \( \alpha^b(r_f, \cdot) \).

It therefore chooses a point on the bank’s reaction function, which brings it closer to its own bliss point. In doing that, the authority considers how its interest rate decision affects the bank’s risk choice, effectively endogenizing the latter’s action. This is why we consider the Stackelberg setup as the natural one within which to consider ‘leaning against the wind’.

One final point is that for an authority with with \( \rho = 0 \) the Nash and Stackelberg equilibrium points are the same. The authority chooses the optimal point on the solid line. That is, it considers bank risk taking as a function of its interest rate \( \alpha^b(r_f, \cdot) \) and determines the

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8 See Appendix A for a discussion on the Nash versus Stackelberg game.
rate that minimizes its objective function. For \( \rho = 0 \), however, the simultaneous-moves Nash equilibrium must be at a point where \( y(\cdot) \) is zero. Since the authority has no other objective than minimizing deviations from \( y(\cdot) = 0 \), it will necessarily choose the \( r^f \) that makes \( y(\cdot) = 0 \) hold for any given \( \alpha \). But, then, the authority can do no better than this point in the Stackelberg game either: its objective is still the same. Thus, for \( \rho = 0 \) the dot represents the equilibria of both the simultaneous moves and Stackelberg games. However, for \( \rho > 0 \) the two set-ups diverge qualitatively in important respects, as explained above.

**Proposition 1** A monetary authority that ‘leans against the wind’ will, on average, impose a higher level of interest rates. Generally, \( \frac{dr^f}{d\rho} > 0 \).

**Proof.** Consider the Stackelberg set-up that leads to \( r^f_S \). Given that \( \alpha^b_t > \alpha^w_t \) and \( \frac{\partial \alpha_t}{\partial r^f} < 0 \) the authority gains on its financial stability objective by charging a higher interest rate. However, beyond the rate \( (r^f)^* = \arg \min y(\cdot)^2 \), that is, the optimal interest rate of the \( \rho = 0 \) authority, a higher interest rate comes at the cost of a loss in terms of the output gap. Formally, for \( r^f > (r^f)^* \big|_{\rho=0} \) we have that:

\[
\frac{\partial}{\partial r^f} [y_t(\cdot)]^2 > 0,
\]

while,

\[
\frac{\partial}{\partial r^f} (\alpha_t - \alpha^w_t)^2 < 0.
\]

It follows that the more an authority leans against the wind, the more it is willing to give up in terms of its output gap objective in order to obtain greater financial stability, and

\[
\frac{dr^f}{d\rho} > 0.
\]

This is depicted in figure 3, below. ■

**B Dynamic Effect**

We examine next the effects of a one period shock on the dynamic path of the interest rate \((r^f_t, \forall t)\) and bank risk taking \((\alpha_t, \forall t)\). At time \( t = 1 \) a random shock \( \nu_1 \) occurs, which determines the path of \( \epsilon_t \) through the persistence parameter \( \theta \). We assume that the central bank commits to the pre-announced interest-rate path that results from its optimization.\(^9\) The steady state analysis shown in the previous section explains how the instrument \( r^f \) and risk \( \alpha \) are related for various values of \( \rho \). It established a level effect in rates, such that an authority that ‘leans against the wind’ has a higher steady state interest rate. We now ask whether its policy also dynamically differs from that of an authority without a financial stability objective. Since we

\(^9\)In Appendix B we explain why (and how) this is a time-consistent policy.
consider a one period shock only, the dynamic aspect of our exercise relates to how an authority chooses to ‘spread’ a given policy across time When a negative shock hits, for instance, will it choose a short, deep cut or a longer, smoother response?

**Definition 1:** Define $\lambda$ as the profile of the monetary authority’s policy response, where a higher $\lambda$ means a deeper but shorter-lived policy. More specifically:

- assign $\lambda = 0$ to the optimal policy of the monetary authority with $\rho = 0$. This is the baseline case of an authority that does not lean against the wind;
- define a higher $\lambda$ as a policy that shifts forward part of the rate cut.

**Then:** Policy profile $i$ has a higher $\lambda$ than policy profile $j$ if:

$$\exists \hat{t}: \left( \left| r_{t,i}^f - \bar{r}^f \right| \geq \left| r_{t,j}^f - \bar{r}^f \right| \quad \forall t < \hat{t} \right) \wedge \left( \left| r_{t,i}^f - \bar{r}^f \right| \leq \left| r_{t,j}^f - \bar{r}^f \right| \quad \forall t > \hat{t} \right),$$

and for some $t < \hat{t}$ and some $t > \hat{t}$ the respective conditions are strictly binding. Here, $\bar{r}^f$ is the steady state interest rate and policy is thus defined in deviations from that steady state.

Monetary authorities with a $\lambda > 0$ profile apply deeper but shorter-lived policy rate cuts, as a result of a negative shock (figure 4).

We can now state this section’s main result:

**Proposition 2** Following a negative shock ($\nu_1 < 0$), a monetary authority that leans against the wind ($\rho > 0$) chooses a profile $\lambda > 0$ for its interest rates. It thus opts for a deeper but shorter response, compared to an authority, which only has standard objectives ($\rho = 0$). Generally, $\frac{d\lambda}{d\rho} > 0$. 

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Figure 4: Interest rate paths for alternative $\lambda$ profiles.

Figure 5: Risk paths for alternative policies

**Proof.** We outline our proof in figure 5 where we plot the level of risk taking for two alternative interest rate policies. We also plot the level of risk that is optimal for society. Last, the dashed (red) line represents how the constraint on risk ($\alpha_t \geq \beta \alpha_{t-1}$) prevents the reduction of risk from one period to the next. Consider first $\beta = 0$, i.e. no dynamic constraint on risk taking. First, by $\frac{\partial \alpha^b_t}{\partial \mu_t} \frac{\partial \mu_t}{\partial \varepsilon_t} = (+)(+) > 0$ a negative shock, $\nu_1 < 0$, implies that $\alpha^b_t$ decreases and then, as $\varepsilon_t \to 0$, gradually returns to $\alpha^b_t$, the bank’s steady state optimal risk taking. This is true for any policy irrespective of $\lambda$. Then, for $\beta > 0$, the constraint $\alpha_t \geq \beta \alpha_{t-1}$ will be binding from $t = 0$ up to a $t'$, at which point $\alpha^b_{t', \beta=0} = \beta \alpha^b_{t-1}$ (or $= \beta \alpha^b_t$). Set $\hat{t} = t'$. We observe that for $t < t'$ policy cuts $|r^f_t - \bar{r}^f|$ are less deep for $\lambda = 0$, generating risk taking that is closer to society’s optimal. For $t > t'$, policy cuts $|r^f_t - \bar{r}^f|$ implied by $\lambda > 0$ however, generate risk taking that is closer to society’s optimal. Then up to $t'$ the constrained paths of $\lambda = 0$ and $\lambda > 0$ are equivalent. But, subsequently, $\lambda > 0$ has lower risk taking. In terms
of financial stability, the $\lambda > 0$ thus offers an unambiguous gain on the financial stability objective, i.e.: 
\[
\frac{d}{d\lambda} \sum_{t=0}^{T} \delta^t [\rho (\alpha_t - \alpha_w^t)^2] < 0.
\]
However, it is also an unambiguous loss on 
\[
\sum_{t=0}^{T} \delta^t [(1 - \rho) [y_t (\cdot)]^2] \quad \text{by the definition that} \quad \lambda = 0 \quad \text{is the path of the} \quad \rho = 0 \quad \text{authority, which}
\]
minimizes $[y_t (\cdot)]^2$. It follows that the more weight the authority puts on preventing financial imbalances (higher $\rho$), the more it is willing to give up on minimizing $[y_t (\cdot)]^2$ to achieve a lower $(\alpha_t - \alpha_w^t)^2$, or in other words \[ \frac{d\lambda}{d\rho} > 0. \]

Intuitively, the bank builds up risk when the economy picks up again, while rates are still low. This is the pattern observed in the aftermath of the 2001-2003 recession, which some have argued contributed to the current crisis. An authority that leans against the wind wants to prevent this type of pattern but it is also willing to do so by allowing for greater output gap volatility. By raising rates quickly after an initial cut incentives to buildup risk later are mitigated.

In summary, the authority that leans against the wind has a higher steady state interest rate. But, compared to that rate, it makes a larger initial rate cut, following a shock However, it subsequently raises rates back more steeply than the authority with $\rho = 0$.

![Figure 6: Leaning against the wind](image)

Note that it need not be the case that the dotted line crosses the solid line, as in this example. The initial rate cut of the $\rho > 0$ authority is larger than that of the $\rho = 0$ authority compared to their respective steady states. But because the $\rho > 0$ authority has higher steady state levels, in absolute terms its rates may still always exceed those of the $\rho = 0$ authority.

**Corollary 1** *Proposition 1 does not extend to an upturn ($\nu_1 > 0$). No unambiguous statement can be made about the effect of a higher $\rho$ on the dynamics of monetary policy response to a positive shock.*
Proof. The proof of Proposition 1 is one of stochastic dominance: \( \int_0^t \alpha_b^h dt \) unambiguously smaller under a higher \( \lambda \), as \( \alpha_t^h \) is the same till \( \hat{t} \), and less afterwards. This does not extend to a positive shock, however. A higher \( \lambda \), which here implies steeper initial rate hike, does translate into a smaller \( \alpha_0^h \). But for \( t > \hat{t} \): \( \alpha_t^h|_{\lambda>0} > \alpha_t^h|_{\lambda=0} \). Thus, there is a parameter-dependent trade-off, instead of stochastic dominance, and no general proof can be derived.

The asymmetry between the negative and the positive shock emanates from the one-sided condition \( \alpha_t \geq \beta \alpha_{t-1} \). Intuitively, moving the asset portfolio from shorter to longer maturities is not very time consuming. But the converse is: building down risk takes time, as risky loans involve long-term commitments. The argument for the \( \nu \)-shaped response described above depends upon the persistence of risk.

C Bank funding

So far our analysis has only considered the effects of monetary policy on the bank’s asset side. However, policy rates have important transmission effects through bank funding too. This is especially true for wholesale financing, which largely occurs at short maturities. Low short term rates make bank financing cheaper. This directly affects bank profits - \( P_t(\alpha_t, y_t(\cdot), r^f) \) with \( \frac{\partial P_t(\cdot)}{\partial r^f} < 0 \). More importantly, however:

Assumption 4 Cheaper funding raises bank incentives to become leveraged.

When interest rates decrease debt funding becomes cheaper relative to equity funding. Though leverage can directly affect bank risk, within the confines of our model we identify the following channel. The less equity the bank holds, the less it internalizes the consequences of a potential bankruptcy. That is, leveraging increases the gap between the bank’s and society’s optimal risk taking, so that there is an additional effect:

\[
\frac{\partial (\alpha^b - \alpha^w)}{\partial r^f} < 0.
\]

By the last three sentences of the proof of Proposition 1, this strengthens the dynamic effect. That is, \( \lambda \) increases more strongly in \( \rho \). Likewise, \( \frac{d((r^f)^2)}{d\rho} \) becomes larger, which strengthens the steady-state result. This can be seen in figure 7:

where \( \rho_{BF} > 0 \) is an equivalent \( \rho \) but incorporating the bank funding channel. Overall, therefore, the introduction of bank funding adds to both the level and dynamic effects of leaning against financial imbalances.
3 Numerical example

To further visualize the interaction between monetary policy and bank risk taking, this section provides an example of the model for specific functional forms. The functional forms will also allow us to obtain assumptions 1 and 2 endogenously from the bank optimization problem. The persistence of risk, assumption 3, remain embodied in the parameter $\beta$, however. We simplify the structure by assuming that there are only two classes of assets: one risky and one risk-free asset. We now let $\alpha_t$ stand for the fraction of its portfolio that a bank invests in the risky asset. The return on the risky asset is termed $r^a_t$, while $r^f_t$ is the return on the risk-free asset. In our simplified economy the strategic interaction between the two players manifests itself in two ways: first, the fact that risky behavior on the part of the bank increases the potential for default, which is costly to society; second, a change in the monetary authority’s instrument, $r^f_t$, affects the bank’s appetite for risk. The incentive to search for yield is captured by the difference between the different return on the two asset classes: $r^a_t - r^f_t$. The economy is described by:

\begin{align}
    r^a_t &= \kappa_0 + \kappa_1 y_t \\
    y_t &= -\gamma^w \alpha^2_t + f \left( \sum_{s=0}^{t} r^f_s \right) + \varepsilon_t.
\end{align}

Equation (12) represents how the cycle affects the yield on the risky asset (parameters $\kappa_0$ and $\kappa_1$ are positive constants). Equation (13) represents aggregate demand, (IS), which is directly affected by the monetary authority through $r^f_t$, but is also subject to the state of the financial sector captured by the expected cost of default, $-\gamma^w \alpha^2_t$.\footnote{We interpret it as an opportunity cost to government expenditure, which enters the aggregate constraint.} Here $\gamma^w$ is the social cost
from the event of default, which occurs with probability $\alpha_t^2$. We thus assume, in reduced form, that the probability of bank failure is $\alpha_t^2$. With this formulation we obtain that the bank will optimally diversify its portfolio (i.e. $0 < \alpha_t < 1$) without having to assume risk aversion. There is also empirical evidence that bank default rates increase convexly in measures of risk taking, such as loan-to-asset ratios (Estrella et al. (2000), Kocagil et al. (2002), Halling and Hayden (2006)). In that sense, the quadratic form offers an approximation to this empirical feature.

Note that the choice for this ad hoc function relating bank risk to default rates is driven by the need to keep the model numerically soluble. Because of the asymmetric constraint $\alpha_t \geq \beta \alpha_{t-1}$ the model cannot be solved with Bellman equations. Rather, numerical simulation requires a grid search, which, in order to yield results, necessitates simplicity. We interpret $\gamma^w$ as the opportunity cost of government expenditure, since a government that commits funds to a bailout, can devote less funds to other projects. Finally, both variables are affected by the common demand shock, $\varepsilon_t$, which is as described in the general-form model, (3).

Constrained by (12) and (13), we summarize the bank’s optimization problem as follows:

$$
\max_{\alpha_t} P = \max_{\alpha_t} \left\{ \sum_{t=0}^{T} \delta^t \left[ \left( 1 - \alpha_t \right) r_f^t + \alpha_t r_a^t - \gamma_b^h \left( \alpha_t \right)^2 \right] \right\} ,
$$

where $\alpha_t \in [0, 1]$

$$
\alpha_t \geq \beta \alpha_{t-1} \forall t \in [1, T].
$$

When bank failure occurs, bank management experiences a loss worth $\gamma_b^h$. We allow this to differ from the loss experienced by society, $\gamma^w$ (in 13). In particular, we assume that the failing bank is always rescued and can continue to operate. Thus, $\gamma^w$ is the cost of the bailout, whereas $\gamma_b^h$ can be seen as a reputational or agent-based cost (part of bank management may be replaced, for instance). Furthermore, $\delta$ is the discount rate (assumed to be constant and independent of $r_f^t$). We thus focus only on the bank’s asset side, and leave its funding unmodelled.

The monetary authority’s objective is as given by (15). However, $\alpha_t^w = 0$ as we abstract from the social value of risk taking. In this example bank risk is only socially harmful as it raises the likelihood of default:

$$
\min_{r_f^t \geq 0, \forall t} L = \min_{r_f^t \geq 0, \forall t} \left\{ \sum_{t=0}^{T} \delta^t \left[ \left( 1 - \rho \right) y_t^2 + \rho \alpha_t^2 \right] \right\} .
$$

The monetary authority has direct control over the risk-free rate, $r_f^t$. It uses this to both target the output gap\textsuperscript{11} and to influence bank risk taking (i.e.: the bank’s incentive to search

\textsuperscript{11}Note that (13) is a reduced form representation of the standard two equation model of inflation and output.
for yield), subject to (12) and (13).

The functional form for \( f \left( \sum_{s=0}^{t} r_s^f \right) \) in the IS equation (13) is represented by:

\[
f \left( \sum_{s=0}^{t} r_s^f \right) = \sum_{s=0}^{t} \eta^{t-s} \left( \xi - r_s^f \right),
\]

where \( \eta \in (0, 1) \) is the rate of decay of monetary policy,\(^{12}\) and \( \xi > 1 \). Moreover, \( \xi < \kappa_0 \), such that the return on the risky asset is always higher than that on the risk-free asset, in equilibrium.

Replacing terms and rewriting in matrix notation the objective of the monetary authority becomes:

\[
\min_{r_f > 0} \left\{ \delta' \left[ (1 - \rho) \left( \varepsilon + \eta \left( \xi - r^f - \gamma^w \alpha^2 \right) \right)^2 + \rho \gamma^w \alpha^2 \right] \right\},
\]

where \( r^f \), \( \alpha \) and \( \varepsilon \) are \((T + 1) \times 1\) vectors with \( t = 0 \) values as first entry and \( t = T \) as last; \( \delta \) is a \((T + 1) \times 1\) vector with \( \delta^0 \) as first entry and \( \delta^T \) as last; \( \xi \) is a \((T + 1) \times 1\) vector with all entries \( \xi \); and \( \eta \) is the following \((T + 1) \times (T + 1)\) matrix:

\[
\eta = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
\eta & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\eta^T & \eta^{T-1} & \ldots & 1
\end{bmatrix}.
\]

We solve the optimization problems using numerical techniques. First, we write a procedure to solve for bank optimization at given interest rates. This procedure is then nested in the monetary authority’s optimization problem, which is solved through numerical gridpoint search. The GAUSS code of this program is available upon request.

We choose parameter values judgementally. The purpose of the exercise is to visually highlight some of the model’s comparative statics. We take the parameterization for figure 8 as a "baseline".\(^{13}\) It replicates both the level effect and the dynamic v-shape derived in the general form. The solid line represents the case of \( \rho = 0 \), while the broken line is the optimal policy of an authority that has a financial stability objective (\( \rho = 0.5 \) here).

This is a simplifying assumption that allows us to concentrate on the way financial stability concerns enter the model as well as the monetary authority’s objectives.

\(^{12}\)This decay factor formulation is chosen for analytical tractability. Empirically, the effect of monetary policy does not linearly decrease over time. Christiano et al. (2005) estimate that output, consumption and investment peak after about 1.5 years, and are back at initial levels after about 3 years, while inflation peaks at about 2 years.

\(^{13}\)Parameter values are \( \beta = 0.7, \gamma^w = 1, \gamma^b = 0.5, \eta = 0.6, \nu = -0.1, \theta = 0.7, \delta = 0.95, \lambda_0 = 1.15, \lambda_1 = 0.005, \xi = 0.05.\)
With a higher persistence of the shock ($\theta = 0.9$), the difference between the two policy paths becomes larger:

When the transmission of monetary policy to the real economy becomes faster (smaller $\eta$, here $\eta = 0.5$), the initial rate cut of the $\rho > 0$ authority becomes less pronounced. The reason is that there is less possibility of intertemporal substitution to satisfy both the targets of the central bank:
Figure 10: Quicker monetary policy transmission.

High patience increases the willingness of the policymaker to substitute intertemporally, which strengthens the dynamic effects described in the general form (here, $\delta = 0.99$):

Figure 11: Higher patience

When the risky assets become more liquid (here $\beta = 0.6$), the dynamic effect becomes less pronounced. It is the illiquidity of risky assets that drives the monetary authority’s decision to keep the rate cut short, after all:
When the bank internalizes more of the social costs of its own default (here, $\gamma^b = 0.9$), the policy paths of the $\rho = 0$ and $\rho > 0$ authorities become more similar:

A positive shock ($\nu = +0.1$) inverts the story. The $\rho = 0$ authority now chooses a shorter deeper rate hike than the $\rho > 0$ authority.
4 Conclusions

In an effort to account for the role of monetary policy in financial markets’ appetite for risk, we model the interaction between a monetary authority and a commercial bank. We thus allow for the bank’s action to be affected by the business cycle and, by consequence, the policy maker’s decisions. Similarly, the bank’s level of risk taking affects the ability of the monetary authority (and society) to achieve the desired level of output stabilization. We argue that a monetary authority that actively accounts for the level of risk that banks take will adjust their instrument in two important ways: first, the interest rate will be higher on average, and second, following a negative shock, the monetary authority will cut interest rates deeper but will revert faster to the steady state. The latter is necessary in order to discourage banks from taking risk, as risk taking increases when interest rates remain low for long periods.
APPENDIX

A  Nash vs. Stackelberg

Figure A.1 plots the two players reaction functions as well as their welfare (losses and profits for the monetary authority and bank respectively), in the instrument space $[\alpha, r^f]$. The Nash equilibrium, $N$, is the outcome when the two players take actions independently of each other, i.e.:

$$r^f_N \in \arg \text{ that sets } \alpha^b(r^f, \cdot) = r^{f(-1)}(\alpha, \cdot);$$
then: $$\alpha_N = \alpha^b(r^f_N, \cdot).$$

In the Stackelberg game, the monetary authority, which acts as the leader, moves along the bank’s reaction function in the direction that brings the outcome at a lower loss and closer to its bliss point. This is point $S$ on the graph, and it is the outcome of the following action:

$$r^f_S \in \arg \min L [\alpha^b(r^f, \cdot), \cdot];$$
then: $$\alpha_S = \alpha^b(r^f_S, \cdot).$$

The monetary authority moves in south-easterly direction in order to reduce its welfare losses.

The Stackelberg solution allows for the monetary authority to endogenize the bank risk
taking and therefore affect it in a way that is credible. This leads to a steady state outcome that corresponds to a higher interest rate and lower risk level, by comparison to Nash.

B Commitment

We have assumed that the monetary authority fully commits to the interest rate path.

**Proposition 3** If $\rho > 0$, such a commitment is time-consistent and therefore fully credible.

**Proof.** The result of relevance is the dynamic effect in proposition 1. It is here that, in response to a shock, the authority announces a path from which it could potentially deviate later. Allowing for deviations from the pre-announced path, we let the bank play a tit-for-tat strategy: if the monetary authority ever deviates from the path that it has announced, the bank resorts to risk-taking against the $\lambda = 0$ path. Note that the $\lambda = 0$ path is fully credible as it is the monetary authority’s optimal path that minimizes the stabilization objective, $[y_t(\cdot)]^2$. No monetary authority would wish to deviate to a $\lambda < 0$ as it would unambiguously lose out on both objectives in $L$. The potential benefit of deviating from an announced $\lambda > 0$ path is gaining on $[y_t(\cdot)]^2$. If, at the same time, risk behavior remains in accordance with the $\lambda > 0$ path, then the monetary authority sees a clear reduction (improvement) in its losses. We argue however, that this is not possible, as risk behavior will adjust immediately upon observing such deviation. Following the notation of the proof of Proposition 1, split the interest rate path into $r_t^f$ for $t < \hat{t}$ and $t > \hat{t}$. For $t > \hat{t}$ we have that $\alpha_t|_{\lambda>0} < \alpha_t|_{\lambda=0}$. But, the dynamic constraint on risk taking, $\alpha_t \geq \beta \alpha_{t-1}$ is only binding downwards. By the bank’s tit-for-tat strategy, then, if the monetary authority deviates from its path at any $t > \hat{t}$, it loses out unambiguously: the bank can directly adjust risk taking to the $\lambda = 0$ path. For $t < \hat{t}$ deviation would imply the exact same outcome for the path of $\alpha_t$ as just announcing $\lambda = 0$. The bank follows the same path of $\alpha_t$ for $t < \hat{t}$ under $\lambda = 0$ and $\lambda > 0$, after all (as depicted in figure 5). But in terms of its $[y_t(\cdot)]^2$ first announcing $\lambda > 0$ and later following $\lambda = 0$ cannot be an improvement either, by the fact that $\lambda = 0$ minimizes $[y_t(\cdot)]^2$. Hence, given this reaction from the part of the bank, the monetary authority gains nothing on either of its objectives by deviating from its pre-announced path. ■
References


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