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Monotonicity and Egalitarianism

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Abstract

This paper identifies the maximal domain of transferable utility games on which aggregate monotonicity (no player is worse off when the worth of the grand coalition increases) and egalitarian core selection (no other core allocation can be obtained by a transfer from a richer to a poorer player) are compatible. On this domain, which includes the class of large core games, we show that these two axioms characterize a unique solution which even satisfies coalitional monotonicity (no member is worse off when the worth of one coalition increases) and strong egalitarian core selection (no other core allocation can be obtained by transfers from richer to poorer players).

Keywords: TU-games, aggregate monotonicity, coalitional monotonicity, egalitarian core, strong egalitarian core, egalitarian stability
JEL classification: C71

1 Introduction

A *transferable utility game* models cooperative situations where players generate joint revenues while being able to act in coalitions. One of the main issues is to allocate the joint revenues, the worth of the grand coalition, among the cooperating players. The most famous solution, the Shapley (1953) value, which assigns an average of all marginal contributions, generally violates a central stability principle: it does not necessarily select from the *core* when this is possible. This means that it may conflict coalitional rationality, a strengthening of individual rationality which says that no coalition can be better off by itself. However, for the specific class of *convex games* (cf. Shapley (1971)), the Shapley value is a core selector.

A solution that satisfies core selection on the class of all games with nonempty core is the nucleolus (cf. Schmeidler (1969)), which asssigns the imputation that lexicographically minimizes the excesses of all coalitions. However, Megiddo (1974) showed that the nucleolus violates an elementary solidarity principle ensuring that no player is worse off when the worth of the grand coalition increases, i.e. it violates *aggregate monotonicity.* Hokari (2000a) showed that the nucleolus even violates aggregate monotonicity on the class of convex games. However, a modification of the nucleolus, the so-called per-capita nucleolus, satisfies aggregate monotonicity and core selection on the class of all games with nonempty core. Calleja et al. (2009) studied the aggregate monotonic core, the set of all allocations that can be assigned by solutions satisfying aggregate monotonicity and core selection.

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The Shapley (1953) value even satisfies the stronger property *coalitional monotonicity*, which requires that no member is worse off when the worth of one coalition increases. Young (1985) and Housman and Clark (1998) showed that coalitional monotonicity and core selection are incompatible on the class of all games with nonempty core. Needless to say, these properties are compatible on the class of convex games.

The principle of egalitarianism stems from the belief that all humans are fundamentally equal and should be treated as equally as possible. This is often justified on the basis of a thought experiment where members of a society decide upon moral issues behind the veil of ignorance, i.e. without being aware of their identity and characteristics a priori. A standard measure for egalitarianism in economic distributions is the Lorenz criterion. An allocation *Lorenz dominates* another allocation if the former cumulatively assigns to each subgroup of ex post poorest agents more than the latter does.

In the context of coalitional games, the Lorenz dominating allocation of the worth of the grand coalition is equal division. This is not really satisfactory since it does not take the economic opportunities of subcoalitions into account. Instead, motivated by a coherent use of egalitarian norms, Dutta and Ray (1989) studied the Lorenz undominated elements of the so-called Lorenz core. Remarkably, and particularly because the Lorenz core is not closed, such element is unique whenever it exists, despite the partial ordering induced by the Lorenz criterion. Unfortunately, existence of the Dutta and Ray (1989) solution is not guaranteed. However, for convex games, it exists, belongs to the core, and Lorenz dominates any other core allocation. Moreover, Hokari (2000b) showed that it satisfies coalitional monotonicity. Recently, Calleja et al. (2019) axiomatically characterized the Dutta and Ray (1989) solution for convex games using aggregate monotonicity.

In search for a larger domain of existence to extend the potential range of applications, Dutta and Ray (1991) studied the Lorenz undominated elements of the equal division core. Although existence is guaranteed under mild conditions, such element is not necessarily unique, even for convex games. Uniqueness on the class of convex games is guaranteed for the Lorenz undominated elements of the core (cf. Hougaard et al. (2001)), to which we refer as the *strong egalitarian core*. This set is nonempty for all games with nonempty core and boils down to the Dutta and Ray (1989) solution on the class of convex games. Alternatively, the strong egalitarian core can be described as the set of core allocations for which no other core allocation can be obtained by transfers from richer to poorer players. The larger *egalitarian core* (cf. Arin and Iñarra (2001)) consists of all core allocations for which no other core allocation can be obtained by a single transfer from a richer to a poorer player.

Several selectors of the (strong) egalitarian core for the class of all games with nonempty core have been proposed in the literature. The *Lmin solution* (cf. Arin and Iñarra (2001)) assigns the core allocation which lexicographically maximizes the minimal payoffs. The *Lmax solution* (cf. Arin et al. (2003)) assigns the core allocation which lexicographically minimizes the maximal payoffs. The *least squares solution* (cf. Arin et al. (2008)) assigns the core allocation which minimizes the sum of squared payoffs. The *coalitional Nash solution* (cf. Compte and Jehiel (2010)) assigns the core allocation which maximizes the product of payoffs. However, these solutions do not inherit the monotonicity properties of the Dutta and Ray (1989) solution for convex games.

This paper studies the compatibility of aggregate monotonicity and egalitarian core selection. On the class of convex games, egalitarian core selection characterizes the Dutta and Ray (1989) solution, which satisfies aggregate monotonicity. On the class of equal division stable games, i.e. games where the core contains equal division of the worth of the grand coalition, aggregate monotonicity and egalitarian core selection characterize the equal division solution. However, on the class of all games with nonempty core, aggregate monotonicity and egalitarian core selection are incompatible. This means that the aggregate monotonic egalitarian core, the set of all allocations that can be assigned by solutions satisfying aggregate monotonicity and egalitarian core selection, is empty.

We identify the maximal domain of games on which aggregate monotonicity and egalitarian core selection are compatible. In other words, we identify the maximal domain of games for which the aggregate monotonic egalitarian core is nonempty. This turns out to be the class of *egalitarian stable games*, i.e. games for which the *procedural egalitarian solution* (cf. Dietzenbacher et al. (2017)) selects from the core. This class not only contains all convex games and all equal division stable games, but also all games with a large core (cf. Sharkey (1982)). Interestingly, on this class, aggregate monotonicity and egalitarian core selection characterize the *Lmax* solution, which even satisfies coalitional monotonicity and strong egalitarian core selection. This means that the aggregate monotonic egalitarian core for egalitarian stable games is single-valued.

Our results at least imply that the *Lmax* solution is from a monotonicity perspective more appealing than the *Lmin* solution, the least squares solution, and the coalitional Nash solution. This complements the conclusions of Llerena and Mauri (2016), where an approach based on consistency leads to the *Lmax* solution.

This paper is organized as follows. Section 2 provides some preliminary notions and notations for monotonicity and egalitarianism in transferable utility games. Section 3 studies the compatibility of aggregate monotonicity and egalitarian core selection. Section 4 formulates some concluding remarks and suggestions for future research.

2 Preliminaries

Let N be a nonempty and finite set. Denote $2^N = \{S \mid S \subseteq N\}$. For any $x \in \mathbb{R}^N$, let $\overline{x} \in \mathbb{R}^{|N|}$ be obtained from x by permuting its coordinates in such a way that $\overline{x}_1 \leq \ldots \leq \overline{x}_{|N|}$. For any $x, y \in \mathbb{R}^N$, *x Lorenz dominates y*, denoted by $x \succ_{Lor} y$, if $\overline{x} \neq \overline{y}$ and $\sum_{k=1}^{\ell} \overline{x}_k \geq \sum_{k=1}^{\ell} \overline{y}_k$ for all $\ell \in \{1, \ldots, |N|\}$. For any $x, y \in \mathbb{R}^N$, *x Lmin dominates y*, denoted by $x \succ_{Lmin} y$, if there is $k \in \{1, \ldots, |N|\}$ such that $\overline{x}_k > \overline{y}_k$ and $\overline{x}_\ell = \overline{y}_\ell$ for all $\ell \in \{1, \ldots, k-1\}$. For any $x, y \in \mathbb{R}^N$, *x Lmax dominates y*, denoted by $x \succ_{Lmax} y$, if there is $k \in \{1, \ldots, |N|\}$ such that $\overline{(-x)}_k > \overline{(-y)}_k$ and $\overline{(-x)}_\ell = \overline{(-y)}_\ell$ for all $\ell \in \{1, \ldots, k-1\}$.

A *transferable utility game*, or simply *game*, is a pair (N, v), where N is a nonempty and finite set of *players* and $v : 2^N \to \mathbb{R}$ assigns to each *coalition* $S \in 2^N$ its *worth* such that $v(\emptyset) = 0$. Let TU^N_{all} denote the class of all games with player set N. For convenience, a game is denoted by $v \in \mathrm{TU}^N_{all}$. Throughout this paper, TU^N denotes a generic class of games.

Let $v \in \mathrm{TU}^N$. The *core* $C(v) \subseteq \mathbb{R}^N$ consists of all allocations which distribute the worth of the grand coalition in such a way that no coalition can be better off by itself, i.e.

$$C(v) = \left\{ x \in \mathbb{R}^N \;\middle|\; \sum_{i \in N} x_i = v(N), \forall_{S \in 2^N} : \sum_{i \in S} x_i \geq v(S) \right\}.$$

The *egalitarian core* $EC(v) \subseteq \mathbb{R}^N$ (cf. Arin and Iñarra (2001)) consists of all core allocations from which no other core allocation can be obtained by a transfer from a richer to a poorer player, i.e.

$$EC(v) = \left\{ x \in C(v) \;\middle|\; \forall_{i,j \in N: x_i > x_j} \exists_{S \in 2^N : i \in S, j \notin S} : \sum_{k \in S} x_k = v(S) \right\}.$$

The *strong egalitarian core* $SEC(v) \subseteq \mathbb{R}^N$ (cf. Hougaard et al. (2001)) consists of all core allocations from which no other core allocation can be obtained by transfers from richer to poorer players, i.e.

$$SEC(v) = \left\{ x \in C(v) \;\middle|\; \forall_{y \in C(v)} : y \not\succ_{Lor} x \right\}.$$

Note that $SEC(v) \subseteq EC(v) \subseteq C(v)$. Moreover, $SEC(v) \neq \emptyset$ if $C(v) \neq \emptyset$. The game v is *balanced* if $C(v) \neq \emptyset$, and *convex* (cf. Shapley (1971)) if $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$ for all $S, T \in 2^N$. Let TU^N_{bal} denote the class of all balanced games and let TU^N_{conv} denote the class of all convex games. Then $\mathrm{TU}^N_{conv} \subseteq \mathrm{TU}^N_{bal} \subseteq \mathrm{TU}^N_{all}$.

A *solution* $f : \mathrm{TU}^N \to \mathbb{R}^N$ assigns to any game $v \in \mathrm{TU}^N$ a payoff allocation $f(v) \in \mathbb{R}^N$. Throughout this paper, f denotes a generic solution.

A solution satisfies *aggregate monotonicity* if no player is worse off when the worth of the grand coalition increases, and *coalitional monotonicity* if no member is worse off when the worth of one coalition increases. Note that coalitional monotonicity implies aggregate monotonicity.

Aggregate monotonicity
$f(v) \leq f(v')$ for all $v, v' \in \mathrm{TU}^N$ with $v(N) \leq v'(N)$ and $v(S) = v'(S)$ for all $S \subset N$.

Coalitional monotonicity
$f_S(v) \leq f_S(v')$ for all $v, v' \in \mathrm{TU}^N$ with $v(S) \leq v'(S)$ and $v(T) = v'(T)$ for all $T \in 2^N \setminus \{S\}$.

A solution satisfies *egalitarian core selection* if it is an element of the egalitarian core, and *strong egalitarian core selection* if it is an element of the strong egalitarian core. Note that strong egalitarian core selection implies egalitarian core selection.

Egalitarian core selection
$f(v) \in EC(v)$ for all $v \in \mathrm{TU}^N$.

Strong egalitarian core selection
$f(v) \in SEC(v)$ for all $v \in \mathrm{TU}^N$.

The following solutions for balanced games satisfy strong egalitarian core selection. The *Lmin solution* $Lmin : \mathrm{TU}^N_{bal} \to \mathbb{R}^N$ (cf. Arin and Iñarra (2001)) assigns to any balanced game $v \in \mathrm{TU}^N_{bal}$ the core allocation which lexicographically maximizes the minimal payoffs, i.e.

$$\{Lmin(v)\} = \left\{ x \in C(v) \;\middle|\; \forall_{y \in C(v)} : y \not\succ_{Lmin} x \right\}.$$

The *Lmax solution* $Lmax : \mathrm{TU}^N_{bal} \to \mathbb{R}^N$ (cf. Arin et al. (2003)) assigns to any balanced game $v \in \mathrm{TU}^N_{bal}$ the core allocation which lexicographically minimizes the maximal payoffs, i.e.

$$\{Lmax(v)\} = \left\{ x \in C(v) \;\middle|\; \forall_{y \in C(v)} : y \not\succ_{Lmax} x \right\}.$$

The *least squares solution* $LS : \mathrm{TU}^N_{bal} \to \mathbb{R}^N$ (cf. Arin et al. (2008)) assigns to any balanced game $v \in \mathrm{TU}^N_{bal}$ the core allocation which minimizes the sum of squared payoffs, i.e.

$$LS(v) = \underset{x \in C(v)}{\operatorname{argmin}} \sum_{i \in N} x_i^2.$$

The *coalitional Nash solution* $CN : \mathrm{TU}^N_{bal} \to \mathbb{R}^N$ (cf. Compte and Jehiel (2010)) assigns to any balanced game[1] $v \in \mathrm{TU}^N_{bal}$ the core allocation which maximizes the product of payoffs, i.e.

$$CN(v) = \underset{x \in C(v)}{\operatorname{argmax}} \prod_{i \in N} x_i.$$

Example 1
Let $N = \{1, 2, 3, 4\}$ and let $v \in \mathrm{TU}^N_{bal}$ be given by

$$v(S) = \begin{cases} 24 & \text{if } S = \{1, 2, 3, 4\}; \\ 14 & \text{if } S \in \{\{2, 4\}, \{3, 4\}\}; \\ 0 & \text{otherwise.} \end{cases}$$

The $Lmin$ solution is given by $Lmin(v) = (5, 5, 5, 9)$. The $Lmax$ solution is given by $Lmax(v) = (3, 7, 7, 7)$. The least squares solution is given by $LS(v) = (4, 6, 6, 8)$. The coalitional Nash solution is given by $CN(v) = (1 + \sqrt{11}, 9 - \sqrt{11}, 9 - \sqrt{11}, 5 + \sqrt{11})$. $\triangle$

On the class of convex games, all these solutions coincide with the Dutta and Ray (1989) solution. A solution which extends the Dutta and Ray (1989) solution for convex games to all games is the *procedural egalitarian solution* (cf. Dietzenbacher et al. (2017)). This solution is defined on the basis of an iterative procedure in which intercoalitional egalitarian considerations are central.

The procedural egalitarian solution
Let $v \in \mathrm{TU}^N_{all}$. Define $P^{v,0} = \emptyset$. Let $k \in \mathbb{N}$. The function $\chi^{v,k}$ assigns to each $S \in 2^N \setminus \{\emptyset\}$ the payoff allocation $\chi^{v,k}(S) \in \mathbb{R}^S$ defined by

$$\chi_i^{v,k}(S) = \begin{cases} \gamma_i^{v,k-1} & \text{if } i \in S \cap P^{v,k-1}; \\ \frac{v(S) - \sum_{j \in S \cap P^{v,k-1}} \gamma_j^{v,k-1}}{|S \setminus P^{v,k-1}|} & \text{if } i \in S \setminus P^{v,k-1}. \end{cases}$$

The collection $\mathcal{A}^{v,k} \subseteq 2^N \setminus \{\emptyset\}$ is defined by

$$\mathcal{A}^{v,k} = \left\{ S \in 2^N \setminus \{\emptyset\} \;\middle|\; \sum_{i \in S} \chi_i^{v,k}(S) = v(S), \forall_{i \in S} \forall_{T \in 2^N : i \in T} : \chi_i^{v,k}(T) \leq \chi_i^{v,k}(S) \right\}.$$

The set $P^{v,k} \in 2^N \setminus \{\emptyset\}$ is defined by $P^{v,k} = \bigcup_{S \in \mathcal{A}^{v,k}} S$. The vector $\gamma^{v,k} \in \mathbb{R}^{P^{v,k}}$ is defined by $\gamma_i^{v,k} = \chi_i^{v,k}(S)$ for all $i \in P^{v,k}$, where $S \in \mathcal{A}^{v,k}$ and $i \in S$.

[1]Formally, the coalitional Nash solution is not well-defined for all balanced games, but this definition suffices for our purposes.

The iteration $n^v \in \{1, \ldots, |N|\}$ is defined by $n^v = \min\{k \in \mathbb{N} \mid P^{v,k} = N\}$. The vector of *egalitarian claims* $\widehat{\gamma}^v \in \mathbb{R}^N$ is defined by $\widehat{\gamma}^v = \gamma^{v,n^v}$. The collection of *egalitarian admissible coalitions* $\widehat{\mathcal{A}}^v \subseteq 2^N \setminus \{\emptyset\}$ is defined by $\widehat{\mathcal{A}}^v = \mathcal{A}^{v,n^v}$. The set of *strong egalitarian claimants* $D^v \in 2^N$ is defined by $D^v = \bigcap\{S \in \widehat{\mathcal{A}}^v \mid \forall_{T \in \widehat{\mathcal{A}}^v} : S \not\subset T\}$. The *procedural egalitarian solution* $PES(v) \in \mathbb{R}^N$ is defined by

$$PES(v) = \left((\widehat{\gamma}_i^v)_{i \in D^v}, (\min\{\widehat{\gamma}_i^v, \lambda\})_{i \in N \setminus D^v}\right),$$

where $\lambda \in \mathbb{R}$ is such that $\sum_{i \in N} PES_i(v) = v(N)$.

The egalitarian procedure underlying the procedural egalitarian solution starts dividing the worth of each coalition equally among its members. The payoff of a player in a coalition is fixed if none of its members is allocated a higher payoff in any other coalition. In the next iteration, each such player is allocated this fixed payoff in each coalition and the remaining worth is equally divided among the other members. Again, the payoff of a player in a coalition is fixed if none of its members is allocated a higher payoff in any other coalition. This process continues until all players have a fixed payoff. The fixed payoffs are called the *egalitarian claims* and the coalitions in which they are attainable are called *egalitarian admissible*. Members of all inclusion-wise maximal egalitarian admissible coalitions are called *strong egalitarian claimants*. The procedural egalitarian solution assigns to the strong claimants their claims and divides the remaining worth of the grand coalition as equally as possible among the other players provided that they are not allocated more than their claims.

Example 2
Let $N = \{1, 2, 3, 4\}$ and let $v \in \mathrm{TU}^N_{bal}$ be the game from Example 1. The following table presents the egalitarian procedure underlying the procedural egalitarian solution.

S	$\{2,4\}$	$\{3,4\}$	$\{1,2,3,4\}$
$v(S)$	14	14	24
$\chi^{v,1}(S)$	$(\cdot, \underline{7}, \cdot, \underline{7})$	$(\cdot, \cdot, \underline{7}, \underline{7})$	$(6, 6, 6, 6)$
$\chi^{v,2}(S)$	$(\cdot, \mathbf{7}, \cdot, \mathbf{7})$	$(\cdot, \cdot, \mathbf{7}, \mathbf{7})$	$(\underline{3}, \mathbf{7}, \mathbf{7}, \mathbf{7})$
$\chi^{v,3}(S)$	$(\cdot, \mathbf{7}, \cdot, \mathbf{7})$	$(\cdot, \cdot, \mathbf{7}, \mathbf{7})$	$(\mathbf{3}, \mathbf{7}, \mathbf{7}, \mathbf{7})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then $n^v = 2$, $\widehat{\gamma}^v = (3, 7, 7, 7)$, $\widehat{\mathcal{A}}^v = \{\{2,4\}, \{3,4\}, \{1,2,3,4\}\}$, $D^v = \{1, 2, 3, 4\}$, and the procedural egalitarian solution is given by $PES(v) = (3, 7, 7, 7)$. $\triangle$

In Example 2, the procedural egalitarian solution coincides with the *Lmax* solution. This is not the case in general.

Example 3
Let $N = \{1, 2, 3\}$ and let $v \in \mathrm{TU}^N_{bal}$ be given by

$$v(S) = \begin{cases} 4 & \text{if } S \in \{\{1,3\}, \{2,3\}, \{1,2,3\}\}; \\ 0 & \text{otherwise.} \end{cases}$$

Then $n^v = 1$, $\widehat{\gamma}^v = (2, 2, 2)$, $\widehat{\mathcal{A}}^v = \{\{1,3\}, \{2,3\}\}$, $D^v = \{3\}$, and $PES(v) = (1, 1, 2)$. On the other hand, $Lmin(v) = Lmax(v) = LS(v) = CN(v) = (0, 0, 4)$. $\triangle$

Let $v \in \mathrm{TU}^N_{all}$. The vector of egalitarian claims is an *aspiration* (cf. Bennett (1983)), i.e. $\sum_{j \in S} \widehat{\gamma}^v_j \geq v(S)$ for all $S \in 2^N$ and for each $i \in N$ there is an $S \in 2^N$ with $i \in S$ such that $\sum_{j \in S} \widehat{\gamma}^v_j = v(S)$. In fact, since a player's payoff is the lowest among all members of the corresponding coalitions when it is fixed for the first time, for each $i \in N$ there is an $S \in 2^N$ with $i \in S$ such that $\sum_{j \in S} \widehat{\gamma}^v_j = v(S)$ and $\widehat{\gamma}^v_i \leq \widehat{\gamma}^v_j$ for all $j \in S$. All such coalitions are contained in the collection of egalitarian admissible coalitions

$$\widehat{\mathcal{A}}^v = \left\{ S \in 2^N \setminus \{\emptyset\} \;\middle|\; \sum_{i \in S} \widehat{\gamma}^v_i = v(S) \right\}.$$

In games where the grand coalition is egalitarian admissible, all players are strong claimants, and the procedural egalitarian solution assigns to all players their claims. Such games are called *egalitarian stable.*

Egalitarian stability
$v \in \mathrm{TU}^N$ is *egalitarian stable* if $N \in \widehat{\mathcal{A}}^v$.

A game is egalitarian stable if and only if the procedural egalitarian solution selects from the core. Let TU^N_{es} denote the class of all egalitarian stable games. Then $\mathrm{TU}^N_{conv} \subseteq \mathrm{TU}^N_{es} \subseteq \mathrm{TU}^N_{bal} \subseteq \mathrm{TU}^N_{all}$. For games with one or two players, egalitarian stability is equivalent to balancedness and convexity. For games with more than two players, the three notions differ from each other, e.g. the game in Example 2 is egalitarian stable but not convex, and the game in Example 3 is balanced but not egalitarian stable. Like balanced games and convex games, the class of egalitarian stable games is closed under increment of the worth of the grand coalition. In fact, like balancedness, egalitarian stability is a *prosperity property* (cf. Van Gellekom et al. (1999)), i.e. any game becomes egalitarian stable when the worth of the grand coalition is sufficiently increased.

3 Monotonicity and Egalitarianism

This section studies the compatibility of aggregate monotonicity and egalitarian core selection for solutions for transferable utility games. For all games with one player, egalitarian core selection simply implies an efficient allocation of the worth of the grand coalition, which is aggregate monotonic. For all balanced games with two players, egalitarian core selection characterizes *constrained egalitarianism* (cf. Dutta (1990)), the solution which divides the worth of the grand coalition as equally as possible subject to individual rationality, which is aggregate monotonic. On the class of balanced games with three or more players, aggregate monotonicity and egalitarian core selection are incompatible. This is shown by the following example.

Example 4
Let $N = \{1, 2, 3\}$ and let $v \in \mathrm{TU}^N_{bal}$ be the game from Example 3. Then $EC(v) = \{(0, 0, 4)\}$. Let $v' \in \mathrm{TU}^N_{bal}$ be given by

$$v'(S) = \begin{cases} v(S) + 2 & \text{if } S = \{1, 2, 3\}; \\ v(S) & \text{otherwise.} \end{cases}$$

Then $EC(v') = \{(2, 2, 2)\}$. This means that aggregate monotonicity and egalitarian core selection are incompatible on the class of balanced games. △

However, on several subclasses of balanced games, aggregate monotonicity and egalitarian core selection are compatible. For instance, on the class of convex games, the Dutta and Ray (1989) solution satisfies coalitional monotonicity and strong egalitarian core selection. Moreover, it is characterized by egalitarian core selection.

Another subclass of balanced games on which aggregate monotonicity and egalitarian core selection are compatible is the class of *equal division stable games*, i.e. games where the core contains equal division of the worth of the grand coalition. The *equal division solution* $ED : \mathrm{TU}^N_{all} \to \mathbb{R}^N$ assigns to any game $v \in \mathrm{TU}^N_{all}$ the allocation which equally divides the worth of the grand coalition, i.e.

$$ED(v) = \left(\frac{v(N)}{|N|}\right)_{i \in N}.$$

Equal division stability
$v \in \mathrm{TU}^N$ is *equal division stable* if $ED(v) \in C(v)$.

Let TU^N_{eds} denote the class of all equal division stable games. Then $\mathrm{TU}^N_{eds} \subseteq \mathrm{TU}^N_{es} \subseteq \mathrm{TU}^N_{bal} \subseteq \mathrm{TU}^N_{all}$. Clearly, on the class of equal division stable games, the equal division solution satisfies coalitional monotonicity and strong egalitarian core selection. In fact, it is characterized by strong egalitarian core selection. However, as the following example shows, it is not characterized by egalitarian core selection.

Example 5
Let $N = \{1, 2, 3, 4\}$ and let $v \in \mathrm{TU}^N_{eds}$ be given by

$$v(S) = \begin{cases} 4 & \text{if } S = \{1,2,3,4\}; \\ 2 & \text{if } S \in \{\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}; \\ 0 & \text{otherwise.} \end{cases}$$

Then $EC(v) = \mathrm{conv}(\{(1,1,1,1),(0,0,2,2)\})$. This means that egalitarian core selection does not characterize a unique solution on the class of equal division stable games. △

If we impose aggregate monotonicity in addition to egalitarian core selection, we do obtain a full axiomatic characterization of the equal division solution on the class of equal division stable games.

Theorem 3.1
The equal division solution is the unique solution for equal division stable games satisfying aggregate monotonicity and egalitarian core selection.

Proof. Clearly, the equal division solution satisfies coalitional monotonicity and strong egalitarian core selection on the class of equal division stable games. Let $f : \mathrm{TU}^N_{eds} \to \mathbb{R}^N$ be a solution for equal division stable games satisfying aggregate monotonicity and egalitarian core selection. Let $v \in \mathrm{TU}^N_{eds}$. Suppose that $f(v) \neq ED(v)$. Then $f_i(v) > ED_i(v)$ for some $i \in N$. Define $R \in 2^N \setminus \{\emptyset, N\}$ by $R = \mathrm{argmax}_{i \in N} f_i(v)$. Let $\xi \in \mathbb{R}$ be such that

$$\max\left\{\frac{v(N)}{|N|}, \max_{i \in N \setminus R} f_i(v)\right\} < \xi < \max_{i \in N} f_i(v).$$

Define $v' \in \mathrm{TU}^N_{eds}$ by

$$v'(S) = \begin{cases} \sum_{i \in R} f_i(v) + |N \setminus R|\xi & \text{if } S = N; \\ v(S) & \text{otherwise.} \end{cases}$$

Then $v'(N) > v(N)$. By aggregate monotonicity, $f(v') \geq f(v)$. Moreover, $f_i(v') > f_i(v)$ for some $i \in N$. For all $i \in N$ with $f_i(v') > f_i(v)$ and all $S \subset N$ with $i \in S$,

$$\sum_{j\in S} f_j(v') > \sum_{j\in S} f_j(v) \geq v(S) = v'(S).$$

By egalitarian core selection, this means that $f_i(v') \leq f_j(v')$ for all $i, j \in N$ with $f_i(v') > f_i(v)$. This implies that $f_i(v') = f_i(v) > \xi$ for all $i \in R$, and $f_i(v') = \xi$ for all $i \in N \setminus R$. For all $S \in 2^N \setminus \{\emptyset, N\}$,

$$\sum_{i\in S} f_i(v') > \sum_{i\in S} \frac{v(N)}{|N|} \geq v(S) = v'(S).$$

In particular, $\sum_{i\in S} f_i(v') > v'(S)$ for all $S \subset N$ with $S \cap R \neq \emptyset$. This contradicts that f satisfies egalitarian core selection. Hence, $f(v) = ED(v)$. □

Example 6
Let $N = \{1, 2, 3, 4\}$ and let $v \in \mathrm{TU}^N_{eds}$ be the game from Example 5. Let $\varepsilon > 0$ and let $v' \in \mathrm{TU}^N_{eds}$ be given by

$$v'(S) = \begin{cases} v(S) + 4\varepsilon & \text{if } S = \{1, 2, 3, 4\}; \\ v(S) & \text{otherwise.} \end{cases}$$

Then $EC(v') = \{(1 + \varepsilon, 1 + \varepsilon, 1 + \varepsilon, 1 + \varepsilon)\}$. △

On the class of convex games, the Dutta and Ray (1989) solution coincides with the procedural egalitarian solution. On the class of equal division stable games, the equal division solution coincides with the procedural egalitarian solution. In fact, on any domain of games which is closed under increment of the worth of the grand coalition, all solutions satisfying aggregate monotonicity and egalitarian core selection assign the egalitarian claims to the players. This is described by the following key lemma.

Lemma 3.1
Let TU^N be a domain of games which is closed under increment of the worth of the grand coalition. If a solution $f : \mathrm{TU}^N \to \mathbb{R}^N$ satisfies aggregate monotonicity and egalitarian core selection, then $f(v) = \widehat{\gamma}^v$ for all $v \in \mathrm{TU}^N$.

Proof. Let $f : \mathrm{TU}^N \to \mathbb{R}^N$ be a solution satisfying aggregate monotonicity and egalitarian core selection. Let $v \in \mathrm{TU}^N$. Define $Q_0 = \emptyset$ and $Q_k = \{i \in N \mid \forall_{j\in N\setminus Q_{k-1}} : \widehat{\gamma}^v_j \leq \widehat{\gamma}^v_i\}$ for all $k \in \mathbb{N}$. Then $Q_{k-1} \subseteq Q_k$ for all $k \in \mathbb{N}$ and $Q_k = N$ for all $k \geq |N|$. We show by induction that $\max_{i\in N\setminus Q_{k-1}} f_i(v) \leq \max_{i\in N\setminus Q_{k-1}} \widehat{\gamma}^v_i$ and $f_{Q_k}(v) = \widehat{\gamma}^v_{Q_k}$ for all $k \in \mathbb{N}$. Suppose that $\max_{i\in N} f_i(v) > \max_{i\in N} \widehat{\gamma}^v_i$. Define $R_1 \in 2^N \setminus \{\emptyset, N\}$ by $R_1 = \operatorname{argmax}_{i\in N} f_i(v)$. Let $\xi_1 \in \mathbb{R}$ be such that

$$\max\left\{\max_{i\in N} \widehat{\gamma}^v_i, \max_{i\in N\setminus R_1} f_i(v)\right\} < \xi_1 < \max_{i\in N} f_i(v).$$

Define $v_1 \in \mathrm{TU}^N$ by

$$v_1(S) = \begin{cases} \sum_{i\in R_1} f_i(v) + |N \setminus R_1|\xi_1 & \text{if } S = N; \\ v(S) & \text{otherwise.} \end{cases}$$

Then $v_1(N) > v(N)$. By aggregate monotonicity, $f(v_1) \geq f(v)$. Moreover, $f_i(v_1) > f_i(v)$ for some $i \in N$. For all $i \in N$ with $f_i(v_1) > f_i(v)$ and all $S \subset N$ with $i \in S$,

$$\sum_{j \in S} f_j(v_1) > \sum_{j \in S} f_j(v) \geq v(S) = v_1(S).$$

By egalitarian core selection, this means that $f_i(v_1) \leq f_j(v_1)$ for all $i, j \in N$ with $f_i(v_1) > f_i(v)$. This implies that $f_i(v_1) = f_i(v) > \xi_1$ for all $i \in R_1$, and $f_i(v_1) = \xi_1$ for all $i \in N \setminus R_1$. For all $S \in 2^N \setminus \{\emptyset, N\}$,

$$\sum_{i \in S} f_i(v_1) > \sum_{i \in S} \widehat{\gamma}_i^v \geq v(S) = v_1(S).$$

In particular, $\sum_{i \in S} f_i(v_1) > v_1(S)$ for all $S \subset N$ with $S \cap R_1 \neq \emptyset$. This contradicts that f satisfies egalitarian core selection. Hence, $\max_{i \in N} f_i(v) \leq \max_{i \in N} \widehat{\gamma}_i^v$. For all $i \in Q_1$ and all $S \in \widehat{\mathcal{A}}^v$ with $i \in S$ and $\widehat{\gamma}_i^v \leq \widehat{\gamma}_j^v$ for all $j \in S$,

$$\sum_{j \in S} f_j(v) \leq \sum_{j \in S} \widehat{\gamma}_j^v = v(S) \leq \sum_{j \in S} f_j(v).$$

Hence, $f_{Q_1}(v) = \widehat{\gamma}_{Q_1}^v$.

Let $k \in \mathbb{N}$ with $Q_k \neq N$ and assume that $\max_{i \in N \setminus Q_{k-1}} f_i(v) \leq \max_{i \in N \setminus Q_{k-1}} \widehat{\gamma}_i^v$ and $f_{Q_k}(v) = \widehat{\gamma}_{Q_k}^v$. Suppose that $\max_{i \in N \setminus Q_k} f_i(v) > \max_{i \in N \setminus Q_k} \widehat{\gamma}_i^v$. Define $R_{k+1} \in 2^N \setminus \{\emptyset, N\}$ by $R_{k+1} = Q_k \cup \operatorname{argmax}_{i \in N \setminus Q_k} f_i(v)$. Let $\xi_{k+1} \in \mathbb{R}$ be such that

$$\max\left\{\max_{i \in N \setminus Q_k} \widehat{\gamma}_i^v, \max_{i \in N \setminus R_{k+1}} f_i(v)\right\} < \xi_{k+1} < \max_{i \in N \setminus Q_k} f_i(v).$$

Define $v_{k+1} \in \mathrm{TU}^N$ by

$$v_{k+1}(S) = \begin{cases} \sum_{i \in R_{k+1}} f_i(v) + |N \setminus R_{k+1}|\xi_{k+1} & \text{if } S = N; \\ v(S) & \text{otherwise.} \end{cases}$$

Then $v_{k+1}(N) > v(N)$. By aggregate monotonicity, $f(v_{k+1}) \geq f(v)$. Moreover, $f_i(v_{k+1}) > f_i(v)$ for some $i \in N$. For all $i \in N$ with $f_i(v_{k+1}) > f_i(v)$ and all $S \subset N$ with $i \in S$,

$$\sum_{j \in S} f_j(v_{k+1}) > \sum_{j \in S} f_j(v) \geq v(S) = v_{k+1}(S).$$

By egalitarian core selection, this means that $f_i(v_{k+1}) \leq f_j(v_{k+1})$ for all $i, j \in N$ with $f_i(v_{k+1}) > f_i(v)$. This implies that $f_i(v_{k+1}) = f_i(v) > \xi_{k+1}$ for all $i \in R_{k+1}$, and $f_i(v_{k+1}) = \xi_{k+1}$ for all $i \in N \setminus R_{k+1}$. For all $S \subset N$ with $S \not\subseteq Q_k$,

$$\sum_{i \in S} f_i(v_{k+1}) > \sum_{i \in S} \widehat{\gamma}_i^v \geq v(S) = v_{k+1}(S).$$

In particular, $\sum_{i \in S} f_i(v_{k+1}) > v_{k+1}(S)$ for all $S \subset N$ with $S \cap (R_{k+1} \setminus Q_k) \neq \emptyset$. This contradicts that f satisfies egalitarian core selection. Hence, $\max_{i \in N \setminus Q_k} f_i(v) \leq \max_{i \in N \setminus Q_k} \widehat{\gamma}_i^v$. For all $i \in Q_{k+1}$ and all $S \in \widehat{\mathcal{A}}^v$ with $i \in S$ and $\widehat{\gamma}_i^v \leq \widehat{\gamma}_j^v$ for all $j \in S$,

$$\sum_{j \in S} f_j(v) \leq \sum_{j \in S} \widehat{\gamma}_j^v = v(S) \leq \sum_{j \in S} f_j(v).$$

Hence, $f_{Q_{k+1}}(v) = \widehat{\gamma}_{Q_{k+1}}^v$. $\square$

We are interested in the maximal subclass of balanced games on which aggregate monotonicity and egalitarian core selection are compatible. Note that aggregate monotonicity and egalitarian core selection are compatible on any domain consisting of one single balanced game, and any such domain is included in an inclusion-wise maximal one. Formally, we are interested in a domain which is closed under increment of the worth of the grand coalition and inclusion-wise maximal in terms of compatibility of aggregate monotonicity and egalitarian core selection. This is the class of egalitarian stable games, where the procedural egalitarian solution satisfies coalitional monotonicity and strong egalitarian core selection.

Lemma 3.2
The procedural egalitarian solution satisfies coalitional monotonicity on the class of egalitarian stable games.

Proof. Let $v, v' \in \mathrm{TU}^N_{\mathrm{es}}$ and let $S \in 2^N$ be such that $v(S) \leq v'(S)$ and $v(T) = v'(T)$ for all $T \in 2^N \setminus \{S\}$. First, we show by induction that for all $k \in \mathbb{N}$, $\gamma_S^{v',k} \geq \widehat{\gamma}_S^v$ if $S \in \mathcal{A}^{v',k}$, and $\gamma^{v',k}_{P^{v',k}} = \widehat{\gamma}^v_{P^{v',k}}$ if $S \notin \mathcal{A}^{v',k}$. For all $i \in P^{v',1}$ and all $T \in \widehat{\mathcal{A}}^v$ with $i \in T$ such that $\widehat{\gamma}_i^v \leq \widehat{\gamma}_j^v$ for all $j \in T$,

$$\gamma_i^{v',1} \geq \chi_i^{v',1}(T) = \frac{v'(T)}{|T|} \geq \frac{v(T)}{|T|} \geq \widehat{\gamma}_i^v.$$

This means that $\gamma_S^{v',1} \geq \widehat{\gamma}_S^v$ if $S \in \mathcal{A}^{v',1}$. Suppose that $S \notin \mathcal{A}^{v',1}$. For all $T \in \mathcal{A}^{v',1}$,

$$\sum_{i \in T} \gamma_i^{v',1} \geq \sum_{i \in T} \widehat{\gamma}_i^v \geq v(T) = v'(T) = \sum_{i \in T} \gamma_i^{v',1}.$$

This means that $\gamma^{v',1}_{P^{v',1}} = \widehat{\gamma}^v_{P^{v',1}}$.

Let $k \in \mathbb{N}$ and assume that $\gamma_S^{v',k} \geq \widehat{\gamma}_S^v$ if $S \in \mathcal{A}^{v',k}$, and $\gamma^{v',k}_{P^{v',k}} = \widehat{\gamma}^v_{P^{v',k}}$ if $S \notin \mathcal{A}^{v',k}$. If $S \in \mathcal{A}^{v',k}$, then $S \in \mathcal{A}^{v',k+1}$ and $\gamma_S^{v',k+1} = \gamma_S^{v',k} \geq \widehat{\gamma}_S^v$. Suppose that $S \notin \mathcal{A}^{v',k}$. Then $\gamma^{v',k}_{P^{v',k}} = \widehat{\gamma}^v_{P^{v',k}}$. For all $i \in P^{v',k+1} \setminus P^{v',k}$ and all $T \in \widehat{\mathcal{A}}^v$ with $i \in T$ such that $\widehat{\gamma}_i^v \leq \widehat{\gamma}_j^v$ for all $j \in T$,

$$\gamma_i^{v',k+1} \geq \chi_i^{v',k+1}(T) = \frac{v'(T) - \sum_{j \in T \cap P^{v',k}} \gamma_j^{v',k}}{|T \setminus P^{v',k}|} \geq \frac{v(T) - \sum_{j \in T \cap P^{v',k}} \widehat{\gamma}_j^v}{|T \setminus P^{v',k}|} \geq \widehat{\gamma}_i^v.$$

This means that $\gamma_S^{v',k+1} \geq \widehat{\gamma}_S^v$ if $S \in \mathcal{A}^{v',k+1}$. Suppose that $S \notin \mathcal{A}^{v',k+1}$. For all $T \in \mathcal{A}^{v',k+1}$,

$$\sum_{i \in T} \gamma_i^{v',k+1} \geq \sum_{i \in T} \widehat{\gamma}_i^v \geq v(T) = v'(T) = \sum_{i \in T} \gamma_i^{v',k+1}.$$

This means that $\gamma^{v',k+1}_{P^{v',k+1}} = \widehat{\gamma}^v_{P^{v',k+1}}$. Hence, $\widehat{\gamma}_S^{v'} \geq \widehat{\gamma}_S^v$ if $S \in \widehat{\mathcal{A}}^v$, and $\widehat{\gamma}_S^{v'} = \widehat{\gamma}_S^v$ if $S \notin \widehat{\mathcal{A}}^v$. This implies that

$$PES_S(v') = \widehat{\gamma}_S^{v'} \geq \widehat{\gamma}_S^v = PES_S(v).$$

Hence, the procedural egalitarian solution satisfies coalitional monotonicity on the class of egalitarian stable games. $\square$

Lemma 3.3
The procedural egalitarian solution coincides with the Lmax solution on the class of egalitarian stable games.

Proof. Let $v \in \mathrm{TU}^N_{\mathrm{es}}$. Denote $x = PES(v)$. Define $R_0 = \emptyset$ and $R_k = \{i \in N \mid \forall_{j \in N \setminus R_{k-1}} : x_j \leq x_i\}$ for all $k \in \mathbb{N}$. Then $R_{k-1} \subseteq R_k$ for all $k \in \mathbb{N}$ and $R_k = N$ for all $k \geq |N|$. Let $y \in C(v)$. Let $k \in \mathbb{N}$ and assume that $y_i = x_i$ for all $i \in R_{k-1}$. Let $i \in R_k \setminus R_{k-1}$ and let $S \in \widehat{\mathcal{A}}^v$ with $i \in S$ be such that $x_i \leq x_j$ for all $j \in S$. Then $S \subseteq R_k$ and

$$\sum_{j \in S \setminus R_{k-1}} y_j = \sum_{j \in S} y_j - \sum_{j \in S \cap R_{k-1}} y_j \geq v(S) - \sum_{j \in S \cap R_{k-1}} x_j = \sum_{j \in S} x_j - \sum_{j \in S \cap R_{k-1}} x_j = \sum_{j \in S \setminus R_{k-1}} x_j.$$

This means that $y_j > x_j$ for some $j \in S$ or $y_j = x_j$ for all $j \in S$. In general, $y_i > x_i$ for some $i \in R_k \setminus R_{k-1}$ or $y_i = x_i$ for all $i \in R_k \setminus R_{k-1}$. This means that there does not exist a $k \in \{1, \ldots, |N|\}$ such that $\overline{(-y)}_k > \overline{(-x)}_k$ and $\overline{(-y)}_\ell = \overline{(-x)}_\ell$ for all $\ell \in \{1, \ldots, k-1\}$. Hence, the procedural egalitarian solution coincides with the *Lmax* solution on the class of egalitarian stable games. □

Theorem 3.2
The maximal domain[2] *on which aggregate monotonicity and egalitarian core selection are compatible is the class of egalitarian stable games.*

Proof. By Lemma 3.2 and Lemma 3.3, the procedural egalitarian solution satisfies aggregate monotonicity and egalitarian core selection on the class of egalitarian stable games.

Let TU^N be a domain of games which is closed under increment of the worth of the grand coalition and on which aggregate monotonicity and egalitarian core selection are compatible. Let $f : \mathrm{TU}^N \to \mathbb{R}^N$ be a solution satisfying aggregate monotonicity and egalitarian core selection. Let $v \in \mathrm{TU}^N$. By Lemma 3.1, $f(v) = \widehat{\gamma}^v$. Then

$$\sum_{i \in N} \widehat{\gamma}^v_i = \sum_{i \in N} f_i(v) = v(N).$$

This means that $N \in \widehat{\mathcal{A}}^v$, so v is egalitarian stable. Hence, $\mathrm{TU}^N \subseteq \mathrm{TU}^N_{es}$. □

On the class of convex games, the *Lmin* solution, the *Lmax* solution, the least squares solution, and the coalitional Nash solution all coincide with the coalitional monotonic Dutta and Ray (1989) solution. On the class of equal division stable games, the *Lmin* solution, the *Lmax* solution, the least squares solution, and the coalitional Nash solution all coincide with the coalitional monotonic equal division solution. On the full class of egalitarian stable games, only the *Lmax* solution coincides with the coalitional monotonic procedural egalitarian solution.

Example 7
Let $N = \{1, 2, 3, 4\}$ and let $v \in \mathrm{TU}^N_{es}$ be the game from Example 1. Let $v' \in \mathrm{TU}^N_{es}$ be given by

$$v'(S) = \begin{cases} v(S) + 4 & \text{if } S = \{1, 2, 3, 4\}; \\ v(S) & \text{otherwise.} \end{cases}$$

Then $Lmin(v) = Lmax(v) = LS(v) = CN(v) = (7, 7, 7, 7)$. This means that the *Lmin* solution, the least squares solution, and the coalitional Nash solution do not satisfy aggregate monotonicity on the class of egalitarian stable games. △

[2] i.e. the inclusion-wise maximal domain of games which is closed under increment of the worth of the grand coalition

In fact, aggregate monotonicity and egalitarian core selection characterize the *Lmax* solution on the class of egalitarian stable games.

Theorem 3.3
The Lmax solution is the unique solution for egalitarian stable games satisfying aggregate monotonicity and egalitarian core selection.

Proof. The *Lmax* solution satisfies strong egalitarian core selection on the class of egalitarian stable games. By Lemma 3.3, the *Lmax* solution coincides with the procedural egalitarian solution on the class of egalitarian stable games. By Lemma 3.2, this means that the *Lmax* solution satisfies coalitional monotonicity on the class of egalitarian stable games.

Let $f : \mathrm{TU}^N_{es} \to \mathbb{R}^N$ be a solution for egalitarian stable games satisfying aggregate monotonicity and egalitarian core selection. Let $v \in \mathrm{TU}^N_{es}$. By Lemma 3.1, $f(v) = \widehat{\gamma}^v$. Then Lemma 3.3 implies that

$$f(v) = \widehat{\gamma}^v = PES(v) = Lmax(v).$$

Hence, $f = Lmax$. □

Corollary 3.4
The Lmax solution is the unique solution for egalitarian stable games satisfying coalitional monotonicity and strong egalitarian core selection.

The equal division solution satisfies coalitional monotonicity on the class of egalitarian stable games, but does not satisfy egalitarian core selection. The *Lmin* solution, the least squares solution, and the coalitional Nash solution satisfy strong egalitarian core selection on the class of egalitarian stable games, but do not satisfy aggregate monotonicity. This means that the properties in Theorem 3.3 are independent and remain independent when aggregate monotonicity is strengthened to coalitional monotonicity, and egalitarian core selection is strengthened to strong egalitarian core selection, as in Corollary 3.4.

The characterization of the *Lmax* solution is essentially valid on any subdomain of egalitarian stable games which is closed under increment of the worth of the grand coalition, including the class of convex games and the class of equal division stable games. Another such subdomain of egalitarian stable games is the class of large core games.

Large core
$v \in \mathrm{TU}^N$ is a *large core game* if for all $x \in \mathbb{R}^N$ with $\sum_{i \in S} x_i \geq v(S)$ for all $S \in 2^N$, there exists a $y \in C(v)$ such that $y \leq x$.

Theorem 3.5
All large core games are egalitarian stable.[3]

Proof. Let $v \in \mathrm{TU}^N_{bal}$ be a large core game. Since $\sum_{i \in S} \widehat{\gamma}^v_i \geq v(S)$ for all $S \in 2^N$, there exists a $y \in C(v)$ such that $y \leq \widehat{\gamma}^v$. For all $S \in \widehat{\mathcal{A}}^v$,

$$\sum_{i \in S} y_i \leq \sum_{i \in S} \widehat{\gamma}^v_i = v(S) \leq \sum_{i \in S} y_i.$$

This means that $y = \widehat{\gamma}^v$, so $N \in \widehat{\mathcal{A}}^v$. Hence, v is egalitarian stable. □

The egalitarian stable game in Example 2 is not a large core game, so the class of egalitarian stable games strictly includes the class of large core games.

[3] In fact, in large core games, all aspirations are core elements.

4 Concluding Remarks

This paper studies the compatibility of aggregate monotonicity and egalitarian core selection for solutions for transferable utility games. The maximal domain on which these axioms are compatible is the class of egalitarian stable games, i.e. games for which the procedural egalitarian solution selects from the core. On this class, these axioms characterize the *Lmax* solution, which even satisfies coalitional monotonicity and strong egalitarian core selection. The *Lmax* solution and the procedural egalitarian solution coincide if and only if the underlying game is egalitarian stable. For games which are not egalitarian stable, aggregate monotonicity and egalitarian core selection are incompatible and there is a trade-off. The *Lmax* solution satisfies egalitarian core selection on the full class of balanced games, but satisfies aggregate monotonicity only on the class of egalitarian stable games. The procedural egalitarian solution satisfies aggregate monotonicity on the class of all games, but satisfies egalitarian core selection only on the class of egalitarian stable games. This consideration is up to the decision maker.

The class of egalitarian stable games not only contains the class of convex games and the class of equal division stable games, but also the class of large core games. Biswas et al. (1999) showed that all *exact games* (cf. Schmeidler (1972)) with at most four players are large core games. Estévez-Fernández (2012) showed that all *stable core games* with at most five players are large core games. This means that all exact games with at most four players and all stable core games with at most five players are egalitarian stable. On the other hand, the egalitarian stable game with four players in Example 2 is neither an exact game, nor a stable core game. Whether all exact games and all stable core games with an arbitrary number of players are egalitarian stable is an interesting open question for future research. This would contribute to a better understanding of the compatibility of aggregate monotonicity and egalitarian core selection.

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