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Theory and Methodology

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Abstract

This paper considers a cost allocation problem that arises from a delivery problem associated with the Chinese postman problem (CPP). A delivery problem is described by a connected undirected graph in which each edge belongs to a different player, a cost function on the edges of this graph and a fixed vertex which is referred to as the post office. Assume that the post office is providing some service to the players. The nature of this service, which can be thought of as mail delivery, requires that a server will travel along the edges of the graph and returns to the post office. The cost allocation problem is concerned with the cost of providing the service to all players. A specific cost allocation rule is introduced and characterized. Further, the class of delivery problems gives rise to a new class of cooperative combinatorial optimization games called delivery games. It is shown that the outcome of the allocation rule with respect to a bridge-connected Euler graph is a core element of the corresponding delivery game.

Keywords: Chinese postman problem; Cost allocation; Delivery game

1. Introduction

Since the early days of operations research and non-cooperative game theory there has been interaction between the two fields. We mention:

(i) Duality result in mathematical programming and minimax results in zero sum game theory (Dantzig, 1963).
(ii) Linear complementarity problems and the theory of bimatrix games (Lemke and Howson, 1964).
(iii) Markov decision theory and the theory of stochastic games (Shapley, 1953).
(iv) Optimal control theory and the theory of differential games (Isaacs, 1965).

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In the last two decades one also sees many interesting interrelations between operations research and cooperative game theory, where often two problems come together: on the one hand an optimization problem and on the other hand a cost allocation problem. We mention

(i) Minimum spanning tree games and spanning network games (Claus and Kleitman, 1973; Bird, 1976; Megiddo, 1978; Granot and Huberman, 1981).

(ii) Linear production games and mathematical programming games (Owen, 1975; Granot, 1986).

(iii) Flow games (Kalai and Zemel, 1982; Derks and Tijs, 1986).

(iv) Traveling salesman games (Potters et al., 1992; Tamir, 1989).

(v) Sequencing games (Curiel et al. (1989)).

This paper will focus on a new interrelation between operations research and cooperative game theory. We consider delivery problems corresponding to situations where from a central depot (post office) deliveries are to be made to customers and where on the one hand the central depot wants to deliver as cheaply as possible and on the other hand there is the problem of allocating a fair share of the overall costs to each customer. This problem is related to the Chinese postman problem (CPP), introduced by Mei-Ko Kwan (1962). Here an undirected connected graph and a cost function on the edges are involved. The problem is to find a route of minimal costs that visits each edge (street) at least once. Edmonds and Johnson (1973) provide a polynomial algorithm that solves this problem.

In fact, this paper considers a cost allocation problem that arises from the CPP. Let there be an undirected connected graph (street plan). In the model we identify each edge (street) with a player of the player set N and there is a fixed vertex called the post office, where all routes of the postman will start and finish. The postman has to make an S-tour to deliver a coalition $S \subseteq N$. An S-tour is a tour of the postman in which the mail can be delivered in each street of coalition S. So, an S-tour visits each edge of S at least once, but may use streets outside S to reach the streets of S. For an S-tour we have that the postman charges for each time a specific street is visited the corresponding travel costs. We assume that in any S-tour each street of S pays once his own specific travel costs itself. Hence, each player of S has individually fixed costs. The sum over the members of S of these fixed costs in an S-tour is called the separable costs of an S-tour. Note that these separable costs are independent of the chosen S-tour. The remaining travel costs corresponding to an S-tour we will call the non-separable costs. These have to be paid in some way by the members of S. Obviously, a coalition S wants to be served by an S-tour which minimizes the non-separable costs. Such an S-tour is called a minimal S-tour. Note that the grand coalition N, which consists of all edges, faces a CPP to obtain a minimal N-tour. The general problem is whether one can find an allocation of the non-separable costs of a minimal N-tour that is stable in the sense that no (sub)coalition has an incentive to split off and to have the mail delivered separately. For this purpose we investigate delivery games, a class of cooperative combinatorial optimization games that arise from delivery problems. The costs of a coalition S in a delivery game is given by the non-separable costs of a minimal S-tour. The core of this game consists of all allocations which are stable in the way described above.

Section 2 provides the formal description of the delivery problem. In Section 3 we consider delivery problems corresponding to bridge-connected Euler graphs, i.e. graphs in which all components that arise by removing the bridges are Euler graphs. For this class of delivery models we introduce and characterize a specific rule γ that divides the non-separable costs of a minimal N-tour among all players. Cooperative delivery games related to delivery models are introduced in Section 4. It is shown that, in general, delivery games may have an empty core, i.e. no stable allocation exists. For bridge-connected Euler graphs, however, it can be shown that the outcome of γ is always a core element. Finally, in Section 5, an extension of the rule γ is provided to a more general class of graphs. It turns out that this extension can be characterized along the same lines as γ but need not be in the core of the corresponding delivery game.

2. The delivery problem

For the formal description of a delivery problem we need some preliminary definitions. Let $G = (V, E, i)$ be an undirected connected graph where V is the set of vertices, E the set of edges and i the incidence mapping that assigns to each edge of E an unordered pair of not necessarily different vertices of V. A *walk* in G is a finite sequence of edges and vertices of the form $v_1, e_1, v_2, \ldots, e_k, v_{k+1}$ with $k \geqslant 0, v_1, \ldots, v_{k+1} \in V, e_1, \ldots, e_k \in E$ such that $i(e_j) = \{v_j, v_{j+1}\}$ for all $j \in \{1, \ldots, k\}$. Such a walk is a *closed walk* if $v_1 = v_{k+1}$. A closed walk in which all edges are distinct is called a *closed trail.* A *path* in G is a walk in which all vertices (except, possibly $v_1 = v_{k+1}$) and edges are distinct. A closed path, i.e. $v_1 = v_{k+1}$, containing at least one edge is called a *circuit.*

We assume that each edge corresponds to one player. Formally, there is a one-to-one function $g : E \to N$, where $N = \{1, \ldots, |E|\}$ is the set of players. Let $v_0 \in V$ and $S \subseteq N$. An *S-tour* w.r.t. v_0 is a closed walk $v_0, e_1, v_1, \ldots, v_{k-1}, e_k, v_0$ in G such that $S \subset \{g(e_j) \mid j \in \{1, \ldots, k\}\}$.

An S-tour visits each edge that corresponds to a player of S and starts and finishes in the same vertex v_0. Note that an edge can be visited more than once by an S-tour. Note that an S-tour may also use edges outside the coalition S. We will assume that any S-tour for all $S \subseteq N$ will start and finish in the same vertex. This vertex is denoted by v_0 and we will refer to it as the post office. Further, we denote the set of all S-tours by $D(S)$.

To the edges of the graph G we assign travel costs $t : E \to [0, \infty)$. Suppose a coalition S is served according to the S-tour $v_0, e_1, v_1, \ldots, v_{k-1}, e_k, v_0 \in D(S)$. The total costs of this tour are $\sum_{j=1}^{k} t(e_j)$. We will assume that each player $g(e) \in S$ (once) pays the travel costs $t(e)$ by himself. In this way we already allocated the *separable costs* $\sum_{e:g(e) \in S} t(e)$ of an S-tour. Note that these separable costs are independent of the chosen S-tour. The remaining *non-separable costs* C_S for coalition S,

$$C_S(v_0, e_1, \ldots, e_k, v_0) := \sum_{j=1}^{k} t(e_j) - \sum_{e:g(e) \in S} t(e), \tag{1}$$

have to be allocated in some way to the members of S. This implies that each time the postman revisits an edge of S or visits an edge that corresponds to a player outside S, the coalition S has to pay the travel costs of this edge. Since each coalition S wants to be delivered as cheaply as possible, it will choose an S-tour in $D(S)$ that minimizes the non-separable costs as given in Eq. (1). Such an S-tour is called *minimal.*

As an illustration we consider the following example.

Example 1. Consider the delivery problem with $N = \{1, 2, 3, 4, 5\}$ as described in Fig. 1. Here $g(e_j) = j$ for all $j \in N$ and all edges have travel costs 1.

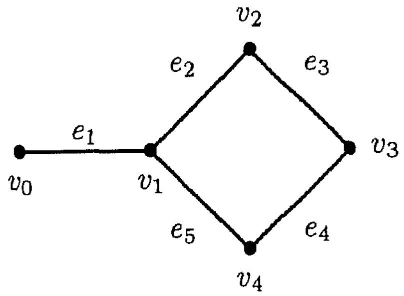

Fig. 1.

Each N-tour will contain e_1 at least twice. The N-tour $v_0, e_1, v_1, e_2, v_2, e_3, v_3, e_4, v_4, e_5, v_1, e_1, v_0$ visits the edges e_2, e_3, e_4, e_5 once and e_1 is visited twice. Hence, this is a minimal N-tour and the non-separable costs for coalition N are equal to $t(e_1) = 1$. Note that this minimal N-tour is also a $\{2\}$-tour for coalition $\{2\}$. Then the non-separable costs for coalition $\{2\}$ of this $\{2\}$-tour are equal to $t(e_1) + t(e_3) + t(e_4) + t(e_5) + t(e_1) = 5$. A minimal $\{2\}$-tour is $v_0, e_1, v_1, e_2, v_2, e_2, v_1, e_1, v_0$ where the non-separable costs are equal to $t(e_1) + t(e_2) + t(e_1) = 3$.

In the following a delivery problem is denoted by $\Gamma = (N, (V, E, i, v_0), t, g)$. Here, N is the set of players, (V, E, i, v_0) is a connected graph in which v_0 is the post office, $t : E \to [0, \infty)$ assigns the travel costs to the edges and g gives the one–one correspondence between players and edges. The class of delivery problems corresponding to the player set N is denoted by DP(N). We will study the cost allocation problem of the non-separable costs of a minimal N-tour.

3. A cost allocation rule for bridge-connected Euler graphs

This section considers delivery problems $\Gamma = (N, (V, E, i, v_0), t, g)$ where $G = (V, E, i)$ is a bridge-connected Euler graph. For this class of delivery problems we introduce and characterize a specific cost allocation rule γ that divides the non-separable costs of a minimal N-tour.

First we recall the definition of an Euler graph. A connected graph G is called an *Euler graph* if there exists a closed trail that includes every edge of G. This implies that for a delivery problem in which the underlying graph is an Euler graph there exists an N-tour that visits each edge of E exactly once. Obviously, this tour is a minimal N-tour since the non-separable costs equal zero. Hence, an (obvious) division of the non-separable costs in this case is the vector $(0, 0, \ldots, 0) \in \mathbb{R}^N$.

Let $G = (V, E, i)$ be an arbitrary connected graph. Then the set of *bridges* $B(G)$ in G is defined by

$$B(G) := \{b \in E \mid (V, E - \{b\}, i_{|E-\{b\}}) \text{ is not connected}\}.$$

Hence, in case $i(b) = \{v, w\}$ with $v \neq w$ we have that $b \in B(G)$ if and only if there is no path between v and w in the graph $(V, E - \{b\}, i_{|E-\{b\}})$. Now, we can introduce the class of bridge-connected Euler graphs. A connected graph $G = (V, E, i)$ is called a *bridge-connected Euler graph* if all the components of the graph $\hat{G} = (V, E - B(G), i_{|E-B(G)})$ are Euler graphs or singletons.

Note that the graph $\hat{G}$ is derived from G by removing all bridges in G. Since all components of $\hat{G}$ are Euler graphs there exists in each component a closed trail. The components are connected by the elements of $B(G)$. Hence, we can make a closed walk in G that visits each bridge twice and all other edges in G once. Obviously, this closed walk is an N-tour in the delivery problem $(N, (G, v_0), t, g)$. Since each N-tour starts and finishes in v_0 it follows from the definition of a bridge that each bridge is visited at least twice in any N-tour. This implies that this specific closed walk is a minimal N-tour for the delivery problem $(N, (G, v_0), t, g)$. Moreover, it readily follows that the non-separable costs of this minimal N-tour equal $\sum_{b \in B(G)} t(b)$ (cf.(1)).

The following example illustrates a bridge-connected Euler graph.

Example 2. Let G be the graph as shown in Fig. 2. The bridges of G are the edges b_1, b_2 and b_3. The graph $\hat{G}$ is derived from G by removing all bridges of G. Since the components E_0, E_1, E_2 and E_3 of $\hat{G}$ are Euler graphs or singletons, we have that G is a bridge-connected Euler graph.

The class of delivery problems corresponding to bridge-connected Euler graphs and player set N is denoted by DE(N). For introducing a rather intuitive rule on DE(N) that divides for each $\Gamma \in$ DE(N) the non-separable costs of a minimal N-tour among the members of N we need the notion of followers of a bridge with respect to v_0 in a bridge-connected Euler graph $G = (V, E, i)$. Let $v_0 \in V$ be the post office and

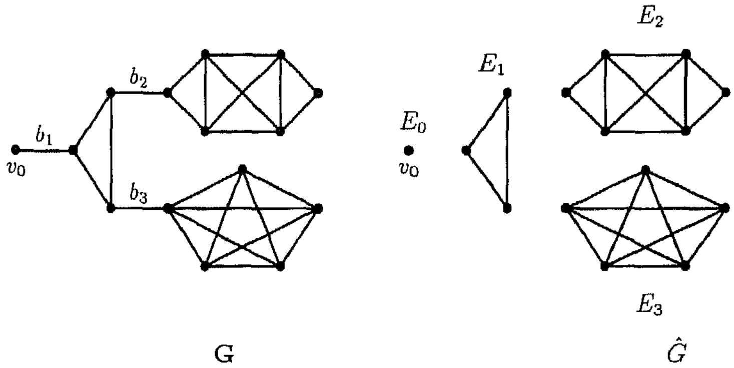

Fig. 2.

let $b \in B(G)$ be a bridge. Then $e \in E$ is a *follower* of b with respect to v_0 if and only if each path $v_0, e_1, \ldots, e_k, v_{k+1}$ with $e_k = e$ also contains b. The set of followers of b will be denoted by $F_b(G, v_0)$. Note that $b \in F_b(G, v_0)$ and that the set of followers depends on the location of v_0 in the graph. Let $b \in B(G)$. Clearly the postman needs this bridge to visit each edge in $F_b(G, v_0)$. We propose that each player in $g(F_b(G, v_0))$ will pay an equal share of the travel costs of b. So, if a player needs several bridges to be visited, he has to contribute a fair share of the costs of all these bridges. Formally, the division rule $\gamma : \mathrm{DE}(N) \to \mathbb{R}^N$ is defined for all $\Gamma = (N, (V, E, i, v_0), t, g) \in \mathrm{DE}(N)$ with $G = (V, E, i)$ by

$$\gamma_{g(e)}(\Gamma) = \sum_{b \in B(G), e \in F_b(G, v_0)} \frac{t(b)}{| F_b(G, v_0) |} \quad \text{for all } e \in E. \tag{2}$$

Note that $\gamma_{g(e)}(\Gamma)$ represents the costs of player $g(e)$.

Now we will provide a characterization of the division rule γ. For this we need the notions of bridge-clusters of a bridge-connected Euler graph and the condensation of a graph with respect to an extreme bridge. Let $\Gamma = (N, (V, E, i, v_0), t, g) \in \mathrm{DE}(N)$. Let $G = (V, E, i)$. A *bridge-cluster* is a set of edges that needs the same set of bridges to be connected to the post office. So, formally, we have at most $|B(G) + 1|$ bridge clusters $C_0(G, v_0)$ and $\{C_b(G, v_0)\}_{b \in B(G)}$. Here,

$$C_0(G, v_0) = E - \bigcup_{b \in B(G)} F_b(G, v_0), \tag{3}$$

is the set of edges that do not need any bridge to be connected to v_0. Note that this set may be empty. For $b \in B(G)$,

$$C_b(G, v_0) = F_b(G, v_0) - \bigcup_{\{b' \in B(G) \cap F_b(G, v_0), b' \neq b\}} F_{b'}(G, v_0), \tag{4}$$

is the cluster of edges that needs the set $\{b^* \in B(G) \mid b \in F_{b^*}(G, v_0)\}$ to be connected to v_0. A bridge $b \in B(G)$ is called an *extreme bridge* if it has no other bridge as a follower, or equivalently if $C_b(G, v_0) = F_b(G, v_0)$. The following example gives the bridge-clusters and extreme bridges of a bridge-connected Euler graph.

Example 3. Take the graph G from Example 2. Then $C_0(G, v_0) = \emptyset$ and for each $j \in \{1, 2, 3\}$ the bridge-cluster $C_j(G, v_0)$ consists of b_j and the edges of component E_j. The extreme bridges are b_2 and b_3.

In the following we describe the procedure to construct a so-called *condensed graph* of a bridge-connected Euler graph with respect to an extreme bridge. Let $\Gamma = (N, (G, v_0), t, g)$ and $G = (V, E, i)$. Let $b \in B(G)$ be an extreme bridge of G. Let $v_1^* \in i(b)$ be such that there exists a path in the graph $(V, E - \{b\}, i_{|E-\{b\}})$ between v_0 and v_1^*. Let $F_b(G, v_0)$ be the set of followers of b and let $V(F_b(G, v_0))$ be the vertices incident with the edges of $F_b(G, v_0)$. The graph $\underline{G}$ is derived from G by removing all edges in $F_b(G, v_0)$ and vertices $V(F_b(G, v_0)) \setminus \{v_1^*\}$. With $|F_b(G, v_0)| = m$, the graph G^* arises from the graph $\underline{G}$ by constructing a circuit defined by $v_1^*, e_1^*, v_2^*, \ldots, v_m^*, e_m^*, v_1^*$ where $\{v_1^*, v_2^*, \ldots, v_m^*\} \cap V = v_1^*$ and $\{e_1^*, \ldots, e_m^*\} \cap E = \emptyset$ to the vertex v_1^*. The graph G^* is called the *condensed graph* of G with respect to the extreme bridge b. Note that G^* is also a bridge-connected Euler graph. Moreover, the number of edges in G and in G^* coincide and $|B(G^*)| = |B(G)| - 1$.

Example 4. Take the graph G of example 2. Fig. 3 shows the graph G^* that arises from G by condensation with respect to the extreme bridge b_2. In an obvious way we can now define *the condensed delivery problem* $\Gamma_b = (N, (V^*, E^*, i^*, v_0), t^*, g^*)$ with respect to an extreme bridge $b \in B(G)$. Here, $G^* = (V^*, E^*, i^*)$ is the condensed graph of G with respect to b. Further, the travel costs $t^* : E^* \to [0, \infty)$ are defined by

$$t^*(e) = \begin{cases} t(e) & \text{if } e \in E - F_b(G), \\ 0 & \text{otherwise,} \end{cases}$$

and for the one–one map $g^* : E^* \to N$ it holds that $g^*(e) = g(e)$ if $e \in E - F_b(G)$ and $g^*(\{e_1^*, \ldots, e_m^*\}) = g(F_b(G))$. So, in fact we have defined a whole class of condensed delivery problems.

Now consider the following three properties for a division rule $f : \mathrm{DE}(N) \to \mathbb{R}^N$.

Efficiency: Let $G = (V, E, i)$ and let $\Gamma = (N, (G, v_0), t, g) \in \mathrm{DE}(N)$. Then it holds that $\sum_{i \in N} f_i(\Gamma) = \sum_{b \in B(G)} t(b)$.

Bridge-cluster symmetry: Let $G = (V, E, i)$ and let $\Gamma = (N, (G, v_0), t, g) \in \mathrm{DE}(N)$. Then $f_{g(e_1)}(\Gamma) = f_{g(e_2)}(\Gamma)$ for all $e_1, e_2 \in C_0(G, v_0)$ and for any $b \in B(G)$ it holds that $f_{g(e_1)}(\Gamma) = f_{g(e_2)}(\Gamma)$ for all $e_1, e_2 \in C_b(G, v_0)$.

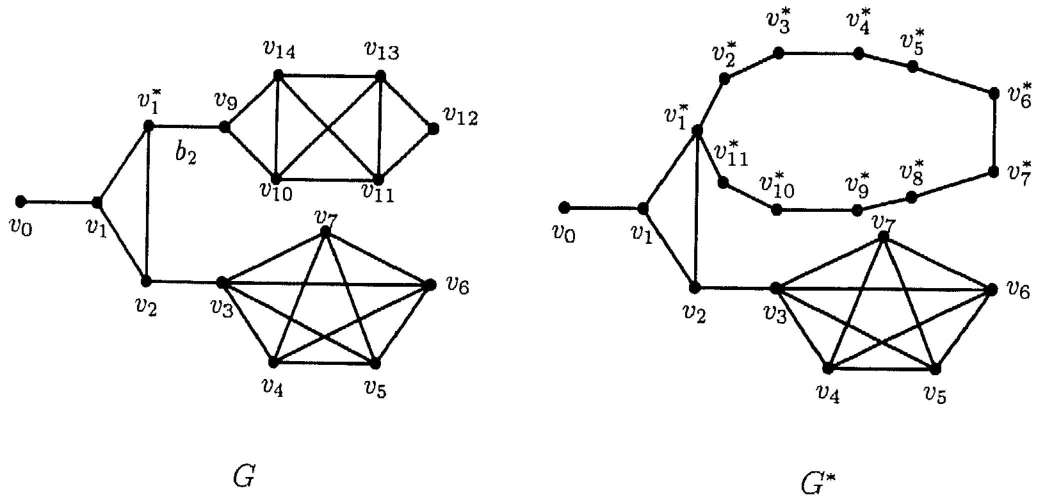

Fig. 3.

Condensation independence: Let $G=(V,E,i)$ and let $\Gamma=(N,(V,E,i,v_0),t,g)\in \mathrm{DE}(N)$. If b is an extreme bridge of G and $\Gamma_b=(N,(V^*,E^*,i^*,v_0),t^*,g^*)$ is a condensed problem of Γ with respect to b, then $f_{g(e)}(\Gamma)=f_{g(e)}(\Gamma_b)$ for all $e\in E-F_b(G,v_0)$.

Efficiency states that the non-separable costs of a minimal N-tour are fully accounted for. Bridge-cluster symmetry states that each group of players that need the same set of bridges to be connected with the post office will contribute the same part to the non-separable costs. Condensation independence compares cost shares in a delivery problem and a corresponding condensed delivery problem. All players who are not in the bridge-cluster corresponding to the removed extreme bridge face in this reduced graph the same problem to reach the post office as in the original graph. The condensation independence claims that in both situations this rule assigns to each member of this group of players the same cost share.

Theorem 1. *The allocation rule* $\gamma:\mathrm{DE}(N)\to\mathbb{R}^N$ *is the unique rule that satisfies efficiency, bridge-cluster symmetry and the condensation property.*

Proof. It is easy to check that γ satisfies the three properties. Let $f:\mathrm{DE}(N)\to\mathbb{R}^N$ be a rule that satisfies the three properties. We show that $f=\gamma$ by induction to the number of bridges of $G=(V,E,i)$. First, let $|B(G)|=0$. Then efficiency and bridge-cluster symmetry implies that $f(\Gamma)=(0,\ldots,0)=\gamma(\Gamma)$. Assume that if $|B(G)|=q$ then $f(\Gamma)=\gamma(\Gamma)$. Let $|B(G)|=q+1$. Let b be an extreme bridge of G. Let $|F_b(G,v_0)|=m$ and let $\Gamma_b=(N,(V^*,E^*,i^*,v_0),t^*,g^*)$ be a condensed problem of Γ with respect to b. Obviously, the underlying graph of Γ_b has q bridges. Then, by induction we have that

$$\gamma_{g(e)}(\Gamma_b)=f_{g(e)}(\Gamma_b)\quad\text{for all } e\in E^*. \tag{5}$$

The condensation property yields that

$$\gamma_{g(e)}(\Gamma)=\gamma_{g(e)}(\Gamma_b)\quad\text{for all } e\in E-F_b(G,v_0), \tag{6}$$

$$f_{g(e)}(\Gamma)=f_{g(e)}(\Gamma_b)\quad\text{for all } e\in E-F_b(G,v_0). \tag{7}$$

From Eqs. (5)–(7) it follows immediately that

$$\gamma_{g(e)}(\Gamma)=f_{g(e)}(\Gamma)\quad\text{for all } e\in E-F_b(G,v_0). \tag{8}$$

Efficiency and Eq. (8) gives that

$$\sum_{e\in F_b(G,v_0)}\gamma_{g(e)}(\Gamma)=\sum_{e\in F_b(G,v_0)}f_{g(e)}(\Gamma). \tag{9}$$

Since b is an extreme bridge we have that $C_b(G,v_0)=F_b(G,v_0)$. Bridge-cluster symmetry implies that

$$\gamma_{g(e)}(\Gamma)=\gamma_{g(b)}(\Gamma)\quad\text{for all } e\in F_b(G,v_0), \tag{10}$$

$$f_{g(e)}(\Gamma)=f_{g(b)}(\Gamma)\ \text{for all } e\in F_b(G,v_0). \tag{11}$$

Substitution of Eqs. (10) and (11) in Eq. (9) and the fact that $|F_b(G,v_0)|=m$ gives

$$m\gamma_{g(b)}(\Gamma)=mf_{g(b)}(\Gamma)$$

and, consequently, $\gamma_{g(b)}(\Gamma)=f_{g(b)}(\Gamma)$. From Eqs. (10) and (11) it then follows that

$$\gamma_{g(e)}(\Gamma)=f_{g(e)}(\Gamma)\quad\text{for all } e\in F_b(G,v_0). \tag{12}$$

This finishes the proof. □

4. Delivery games

In this section we introduce to each delivery problem a corresponding cooperative delivery game. First, we recall some well known facts concerning cooperative games. A *cooperative cost game* is a pair (N,c) where N is a finite set of players and c is a mapping $c: 2^N \to \mathbb{R}$ with $c(\emptyset) = 0$. Here, 2^N is the collection of all subsets of N (coalitions).

Cooperative game theory focuses on 'fair' and/or 'stable' division rules from the value $c(N)$ of the grand coalition. A core element $x = (x_i)_{i\in N} \in \mathbb{R}^N$ is such that no coalition $S \subset N$ has an incentive to split off, i.e.

$$\sum_{i\in N} x_i = c(N) \text{ and } \sum_{i\in S} x_i \leqslant c(S) \text{ for all } S \in 2^N.$$

The core Core(c) consists of all core elements. A game is called balanced if its core is non-empty.

In Section 2 we defined a minimal S-tour of a coalition S. By defining the cost value of a coalition S as the non-separable costs of a minimal S-tour (cf. (1)), we obtain the delivery game corresponding to the delivery problem.

Definition 1. The cooperative delivery game (N,c) corresponding to $\Gamma = (N,(V,E,i,v_0),t,g) \in \mathrm{DP}(N)$ is defined for all $S \subseteq N$ by

$$c(S) := \min_{v_0,e_1,\ldots,e_k,v_0 \in D(S)} \left(\sum_{j=1}^{k} t(e_j) - \sum_{e:g(e)\in S} t(e) \right).$$

In the following example a delivery game is considered.

Example 5. Take the delivery problem of Example 1. In Example 1 it is shown that the non-separable costs of a minimal N-tour for coalition N equal 1 and that the non-separable costs of the minimal $\{2\}$-tour for coalition $\{2\}$ equal 3. Hence, $c(N) = 1$ and $c(\{2\}) = 3$. Since for each coalition S the edge e_1 is contained two times in all S-tours, we have that $c(S) \geqslant t(e_1) = 1$. Since $c(N) = 1$ we have that e.g. $(\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5}) \in \mathrm{Core}(c)$.

The following example shows that a delivery game is not necessarily balanced.

Example 6. Consider the delivery problem with $N = \{1,2,3,4,5\}$ as described in Fig. 4 in which $g(e_j) = j$ and $t(e_j) = 1$ for all $j \in \{1,\ldots,5\}$.

Then $c(N) = 1$ and $c(S) = 0$ if $S \in \mathscr{A} := \{\{1,2,5\},\{3,4,5\},\{1,2,3,4\}\}$. Suppose $x \in \mathrm{Core}(c)$. Then

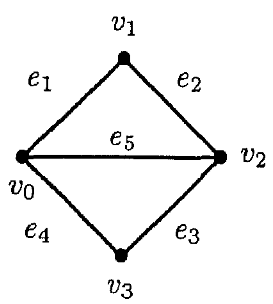

Fig. 4.

$$0 < 2c(N) = \sum_{S\in\mathscr{A}}\sum_{i\in S} x_i \leqslant \sum_{S\in\mathscr{A}} c(S) = 0.$$

Contradiction, so no core elements exists.

For $\Gamma \in \mathrm{DE}(N)$, i.e. a delivery problem corresponding to bridge-connected Euler graph, it will be shown that $\gamma(\Gamma)$ is an element of the core of the corresponding delivery game. Consequently, each delivery game that corresponds to a delivery problem of $\mathrm{DE}(N)$ is balanced.

Theorem 2. *Let* $\Gamma = (N, (G, v_0), t, g) \in \mathrm{DE}(N)$ *and let* (N, c) *be the corresponding delivery game. Then* $\gamma(\Gamma) \in \mathrm{Core}(c)$.

Proof. Obviously, $\sum_{e\in E} \gamma_{g(e)}(\Gamma) = c(N)$. Let $S \subset N$ and $g^{-1}(S) = A \subset E$. Then A can be partitioned into at most $|B(G)| + 1$ sets $A_0, \{A_b\}_{b\in B(G)}$ with $A_0 = A \cap C_0(G, v_0)$ and $A_b = A \cap C_b(G, v_0)$. Then

$$\begin{aligned}\sum_{i\in S}\gamma_i(\Gamma) &= \sum_{e\in A}\sum_{b\in B(G):e\in F_b(G,v_0)} \frac{t(b)}{|F_b(G,v_0)|}\\ &= \sum_{b'\in B(G)}\sum_{e\in A_{b'}}\sum_{b\in B(G):e\in F_b(G,v_0)} \frac{t(b)}{|F_b(G,v_0)|} = \sum_{b'\in B(G)}\sum_{b\in B(G):F_b(G,v_0)\cap A_{b'}\neq\emptyset} \frac{|A_{b'}|\,t(b)}{|F_b(G,v_0)|}\\ &= \sum_{b\in B(G):F_b(G,v_0)\cap A\neq\emptyset}\ \sum_{b'\in B(G):F_b(G,v_0)\cap A_{b'}\neq\emptyset} \frac{|A_{b'}|\,t(b)}{|F_b(G,v_0)|} \leqslant \sum_{b\in B(G):F_b(G,v_0)\cap A\neq\emptyset} t(b)\\ &\leqslant c(S). \qquad \square\end{aligned}$$

Hamers (1997) shows that delivery games corresponding to bridge-connected Euler graphs need not be concave, but that delivery games corresponding to bridge-connected cyclic graphs are concave. Here, a connected graph $G = (V, E, i)$ is called a bridge-connected cyclic graph if the components of the graph $\hat{G} = (V, E - B(G), i_{|E-B(G)})$ are cycles or single vertices. A cycle is the union of edge-disjoint circuits. Note that the class of bridge-connected cyclic graphs is contained in the class of bridge-connected Euler graphs. Granot et al. (1996) imposed necessary and sufficient conditions on a graph such that the associated delivery game is balanced, totally balanced and concave, respectively.

5. Concluding remarks

This section discusses an extension of the division rule γ to a more general class of delivery problems. For this, we need the observation that any connected graph $G = (V, E, i)$ can be considered as a generalization of a bridge-connected Euler graph. This follows from the fact that the set of edges E can be partitioned into two subsets: the edges that are contained in a circuit and the edges that are a bridge. Hence, after the removal of all bridges in an arbitrary connected graph we get components which either consists of a single vertex or consists of a connected (sub)graph in which each edge is contained in at least one circuit. Let $v_0 \in V$ be the post office in G, then $B(G)$ is the set of bridges of G and the set of followers $F_b(G, v_0)$ of $b \in B(G)$ is defined in the same way as in Section 3. Moreover, a bridge-cluster is again a set of edges that need the same set of bridges to be connected to the post office (cf. (3) and (4)). We restrict our attention to graphs in which v_0 is connected only to bridges, which implies that $C_0(G, v_0) = \emptyset$, or v_0 is not connected to any bridge of G. For this class of graphs we have that, whenever $C_0(G, v_0) \neq \emptyset$, each edge needs at least one edge of $C_0(G, v_0)$ to be connected to v_0.

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