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On the Option Effects of Short-Time Work Arrangements*

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Abstract

We analyse the short term work (STW) regulations that several OECD countries introduced after the 2007 financial crisis. We view these measures as a collection of real options and study the dynamic effect of STW on the endogenous liquidation decision of the firm. While STW delays a firm's liquidation, it is not necessarily welfare enhancing. Moreover, it turns out that firms use STW too long. We show (numerically) that providers of capital benefit more than employees from STW. Benefits for employees can even be negative. A typical Nordic policy performs better than a typical Anglo-Saxon policy for all stakeholders.

Keywords: Temporary unemployment, Real options, Dynamic cost-benefit analysis

JEL classification: G33, G38, H53

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1 Introduction

Many commonly-used economic policies have welfare effects that depend crucially on economic agents' timing decisions. As an example take short-time work (STW) arrangements, which are used to reduce the number of lay-offs in economically challenging times by temporarily allowing employers to reduce the hours worked by their employees. Even though there are many differences in the precise rules governing STW in different countries (such as eligibility criteria, duration, etc.) the basic idea is similar: rather than laying off workers, firms are allowed to put employees on reduced hours. Affected workers are compensated for the resulting loss in wage income, partly by employers and partly by the government.[1]

In order to make a full welfare analysis of an STW policy one needs to know by how much both the costs to the government and the benefits to the workers should be discounted. Both these discount factors depend on the timing decisions of the firm: when (if ever) it enters STW, when (if ever) it exists, when (if ever) it liquidates. These decisions, in turn, will depend on the underlying, uncertain, state of the economy as relevant to the firm. The discounted costs and benefits of STW will, thus, depend on firms' timing decisions under future uncertainty. To facilitate the analysis of STW it helps to view it as providing the firm with a collection of *real options*. First it has an option to enter STW. Once this option is exercised it has an option to either liquidate or leave STW. If the firm exists STW it again has an option to liquidate.

In order to value a real option of any type a firm must weigh the marginal costs and benefits of waiting versus those of exercise at any time. Therefore, real options are not easily dealt with in a discrete time model, because such a framework is typically not rich enough to determine *exactly* when and where the marginal net benefits of immediate action are equal to the marginal net benefits of waiting.[2]

This paper analyses three issues related to STW that have been addressed to some extent in the literature, but will be studied here in a dynamic stochastic framework that focusses on the

[1] In 2011, STW measures were in place in 25 of 33 OECD countries with take-up rates being as high as 7.4% of employees in some countries in 2009 (Cahuc and Carcillo, 2011). It has been estimated, for example, that STW has saved 5,000–6,000 jobs in the Netherlands alone (Hijzen and Venn, 2011).

[2] This is essentially the same reason why economists model utility or profit maximization on the real line rather than the rationals: an abstraction that provides analytical tractability.

incentives it creates for employers. First, it has been argued (Abraham and Houseman, 1995) that, while from the employee's point of view STW is preferable to lay-offs, they are close substitutes for employers. However, since laying off staff is an (at least partly) irreversible decision, while STW is not, the option of STW has economic value to the firm.

A second, related, issue is the concern that STW can lead to inefficient reductions in working hours (cf. Rosen, 1985) for which some empirical evidence has been found (Cahuc and Carcillo, 2011). We find a theoretical reason for this inefficiency, linked to the option value that STW presents. Most countries have provisions that preclude firms from signing up to STW if and when they like. This adds a degree of irreversibility to the decision to leave STW once a firm is using it. This irreversibility, in turn, creates an option value of waiting, which gives the firm a disincentive to return employees to full hours. This effect is driven purely by irreversibility and uncertainty over the firm's prospects and affects even risk-neutral firms.

A third question is related to who benefits most from STW. The measure is usually introduced by appealing to the advantages that it has for employees. Indeed, STW delays or prevents employees from being laid-off and, hence, reduces the present value of the sunk-costs of being made redundant, estimated to be some 11% of life-time earnings (Davis and von Wachter, 2011). However, STW is also beneficial to the providers of capital and shareholders. Providers of capital benefit, because they get fully reimbursed throughout the STW period. Shareholders benefit, because STW gives them additional options in managing the firm, which have a positive value.

Our findings are that, firstly, from the firm's perspective STW and liquidation are not close substitutes. This is because liquidation is irreversible, while using STW is not. So, STW gives the firm additional flexibility to deal with unfavourable economic circumstances, which has economic value for the firm. Secondly, from a social welfare perspective, firms use STW too long. This again is related to the option value of STW: leaving STW is an irreversible decision. This irreversibility gives the firm an option to use STW longer than it otherwise would. Thirdly, in numerical simulations we find that, on the whole, the providers of capital benefit more from STW than providers of labour. In addition, all interested parties (providers of capital and labour, and shareholders) are better off in a typical Nordic programme as opposed to a typical Anglo-Saxon programme. Finally, the benefit-to-cost ratio is significantly below unity, indicating the importance of a stochastic dynamic approach to analysing measures like STW. In general we find that the benefits and costs of

STW measures are very sensitive to the policy's parameters and that no clear policy prescriptions emerge. This point is also made in some recent empirical work on STW measures by Boeri and Bruecker (2011). Our numerical results are based on STW programmes as they are run in several OECD countries.

The paper uses a real options approach to the analysis of unemployment insurance by focussing on the effects of irreversibility and uncertainty on the value of STW measures to the firm. The use of real options analysis is well established in the analysis of economic decision making. Since the seminal contributions of McDonald and Siegel (1986) and Dixit and Pindyck (1994) there has been a burgeoning literature on applications of the real options approach. For example, Abel and Eberly (1994) use the framework to analyse optimal investment in the production factors in a firm, Bar–Ilan and Strange (1996) use it to investigate the consequences of construction lags on optimal investment decision. In a recent contribution, Kellogg (2014) gives evidence that the decision rules that theoretic real options models prescribe are consistent with actual firm behaviour.

Not much theoretical work has been conducted in the use of STW to dampen the effects of the recent recession. Some contributions, like Bentolila et al. (2012) focus on labour market flexibility, in particular the use of temporary workers, as an explanation for the different effects of the recession in different labour. markets. To our knowledge, our paper is the first that applies a real options framework to STW. Much of the literature on STW (see, fore example Blanchard and Tirole, 2007 for a recent contribution) focus on the effect of unemployment insurance and protection on optimal wage contracts between employer and employee. We choose not to model the workers' side of the labour market at all. There are concerns that STW unduly advantages insiders although there is no conclusive evidence that this really takes place (Cahuc and Carcillo, 2011). In addition, it can be argued that during times of economic hardship, which STW measures are designed to alleviate, the labour market will not be very "liquid", making it different for employees to switch jobs. Finally, a focus on the firm allows for analytical solutions and a clear understanding how irreversibility and uncertainty drive the value of STW.

The paper is organized as follows. In Section 2 we set up the model and derive the firm's optimal policy by valuing three subsequent (real) options implied by a (stylized) model of STW. The effects of STW on liquidation probabilities and welfare are analysed in Sections 3 and 4, respectively. Some concluding remarks are given in Section 5.

2 Optimal Use of Short Term Work by Firms

As mentioned in Section 1, the details of STW measures vary substantially across countries, although the main idea is the same: employers can put staff on reduced hours, while the government partially compensates for lost wage earnings. In this section we will study the effect of STW on the firm's liquidation decision. The main idea is that STW provides the firm with an option to postpone an irreversible liquidation decision. As far as the details of the STW measure, we make the following additional simplifying assumptions. First the firm can use STW only once, second the option to use STW is infinitely lived, third the use of STW does not involve sunk costs and fourth the firm can decide itself when to stop STW.

The second and third assumptions are, arguably, the most unrealistic ones. Both are made for technical convenience and can be relaxed. However, when options are not infinitely lived no analytical results can be obtained, although it has been shown that even for moderate finite life times the (numerically obtained) solutions are very similar to those obtained analytically under an infinite time horizon (Gryglewicz et al., 2008). The assumption that the use of STW does not involve sunk costs speeds up the decision to avail of STW. This is a standard result from the literature[3] and the same intuition applies here as well.

Our assumptions imply that the firm has three subsequent options. First the option option to enter STW. Second, once entered, the firm has the options to (i) exit STW and return to normal production, or (ii) to liquidate. Third, if the firm decided to exit STW and return to normal production, then it now has an option to liquidate. These three options will be valued successively in this section, starting with the final one.

2.1 The value of an active firm that has already used STW

In this section we analyse the value of a firm that is currently active in a market and has already used STW. Such a firm no longer has an option to enter STW and we will assume that the only option that is left to the firm is to liquidate at a time of its choosing. Liquidation implies laying off all workers and retiring all capital stock. We make the following two simplifying assumptions. First, upon liquidation the firm does not receive a scrap value for its capital stock and, second,

[3] See, for example, Dixit and Pindyck (1994).

liquidation does not involve sunk costs. These assumptions can easily be relaxed, at the cost of more notation, without qualitatively changing the conclusions of the model.

The crucial ingredient in our model is that the evolution of the firm's revenues is subject to uncertainty. Uncertainty is modeled on a measurable space $(\Omega, \mathscr{F})$. We consider a family of probability measures P_y, $y \in \mathbb{R}_+$, on $(\Omega, \mathscr{F})$. A particular firm is assumed to have a cash inflow that is given by $Q_N Y$, where Q_N is the production level of the firm and Y is the stochastically evolving price level, which under P_y, evolves according to the geometric Brownian motion (GBM),

$$dY_t = \mu Y_t dt + \sigma Y_t dz_t, \quad Y_0 = y, \ \mathsf{P}_y\text{-a.s.},$$

where $(z_t)_{t\geq 0}$ is a Wiener process. Information is modeled by the filtration generated by this GBM, augmented with the P_y-null sets, and is denoted by $(\mathscr{F}_t)_{t\geq 0}$.

It is assumed that under normal conditions the firm produces a quantity Q_N at a cost c_N, and that it discounts profits at a constant rate $r > \mu$.[4]

The present value (under P_y) of an operational firm without the liquidation option is

$$F_N(y) = \mathsf{E}_y \left[\int_0^\infty e^{-rt} \left(Q_N Y_t - c_N \right) dt \right] = \frac{Q_N y}{r-\mu} - \frac{c_N}{r}.$$

The firm's value with the liquidation option is the solution to the optimal stopping problem

$$\begin{aligned} F_N^*(y) =& \sup_{\tau \in \mathscr{M}} \mathsf{E}_y \left[\int_0^\tau e^{-rt} \left(Q_N Y_t - c_N \right) dt \right] \\ =& F_N(y) + \sup_{\tau \in \mathscr{M}} \mathsf{E}_y \left[e^{-r\tau} (-F_N(Y_\tau)) \right], \end{aligned} \tag{1}$$

where $\mathscr{M}$ is the set of stopping times relative to the filtration $(\mathscr{F}_t)_{t\geq 0}$. Because the planning horizon is infinite and $(Y_t)_{t\geq 0}$ is strongly Markovian with continuous sample paths (a.s.) the optimal policy will be to liquidate at the first hitting time of an endogenously determined trigger Y_N^*, i.e. at the stopping time $\check{\tau}(Y_N^*) := \inf\{t \geq 0 | Y_t \leq Y_N^*\}$.[5]. The optimal stopping problem (1) can, therefore, be formulated as a maximization problem over the threshold:

$$\begin{aligned} F_N^*(y) &= F_N(y) + \sup_{Y^*} \mathsf{E}_y \left[e^{-r\tau(Y^*)} (-F_N(Y_{\tau(Y^*)})) \right] \\ &= F_N(y) + \sup_{Y^*} \mathsf{E}_y \left[e^{-r\check{\tau}(Y^*)} \right] (-F_N(Y^*)). \end{aligned}$$

[4]This assumption ensures that the present value of profits is finite in our infinite horizon model.

[5]See, for example, Stokey (2009)

The Laplace transform of GBM can easily be computed via Dynkin's formula (see, for example, Øksendal, 2000) as

$$\mathsf{E}_y\left[e^{-r\check{\tau}(Y^*)}\right] = \left(\frac{y}{Y^*}\right)^{\beta_2},$$

where $\beta_2 < 0$ is the negative root of the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0. \tag{2}$$

The positive root of this equation is denoted by $\beta_1 > 1$. Therefore, the optimal stopping problem then reduces to

$$F_N^*(y) = F_N(y) + \sup_{Y^*}\left(\frac{y}{Y^*}\right)^{\beta_2}(-F_N(Y^*)).$$

The objective function is continuous and concave so that a global maximum is attained on $[0,\infty]$, which we denote by Y_N^*.

The following proposition can easily be established using standard techniques. Since the problem is standard, the proof will be omitted.

Proposition 1 *A firm that has already used STW should liquidate at the first hitting time (from above) of the trigger*

$$Y_N^* = \frac{\beta_2}{\beta_2-1}\frac{r-\mu}{Q_N}\frac{c_N}{r}.$$

The value of this firm, when the current state is $y > 0$, equals

$$F_N^*(y) = \begin{cases} \frac{Q_N y}{r-\mu} - \frac{c_N}{r} + \left(\frac{y}{Y_N^*}\right)^{\beta_2}\left[\frac{c_N}{r} - \frac{Q_N Y_N^*}{r-\mu}\right] & \textit{if } y > Y_N^*, \\ 0 & \textit{if } y \le Y_N^*. \end{cases} \tag{3}$$

The value function in (3) has a straightforward interpretation. For $y \le Y_N^*$, the firm liquidates immediately and its value is, thus, zero. For $y > Y_N^*$ the firm's value consists of the expected present value of always producing Q_N at cost c_N, corrected for the fact that at some point in the future the threshold Y_N^* may be reached. The expected discount factor of this event is $(y/Y_N^*)^{\beta_2}$.

2.2 The value of a firm currently using STW

Once the firm has decided to enter STW it has two, inter-related options:

1. leave STW and return to normal production;

2. leave STW and liquidate.

This problem has two aspects: (i) the optimal decision time has to be determined and (ii) the optimal decision at that time has to be determined.

The value of an active firm using STW with the two exit options described above, can now be written as the optimal stopping problem

$$F_P^*(y) = \sup_{\tau \in \mathscr{M}} \mathsf{E}_y \Big[\int_0^\tau e^{-rt}(Q_P Y_t - c_P)dt + e^{-r\tau} \max \big\{ F_N^*(Y_\tau), 0 \big\} \Big]. \tag{4}$$

Of course, STW only makes sense if it allows firms to reduce costs by lowering the wage bill through reduced hours for its workers. Therefore, the per-period costs during STW are assumed to be constant and equal to $c_P \in (0, c_N)$. The *quid pro quo* is that the firm will produce less than before, say $Q_P \in (0, Q_N)$. We will assume that, on average, normal production is more profitable than production in STW.

Assumption 1 *The production and cost levels in STW, Q_P and c_P, are such that*

$$\frac{Q_N}{c_N} > \frac{Q_P}{c_P}. \tag{5}$$

Essentially this assumption says that Q_N is a more efficient production level than Q_P. If it is violated then the firm may never wish to leave STW once it has entered.

The following assumption ensures that a firm that uses STW liquidates later than a firm that does not. If this assumption is violated, then the STW policy does not have its intended result and would better be scrapped.

Assumption 2 *For the unique solution $\hat{Y}_H > Y_N^*$ to the equation*

$$\begin{aligned}
&\frac{\beta_1\beta_2}{\beta_1-\beta_2}\left(\frac{Q_P Y_N^*}{r-\mu} - \frac{c_P}{r}\right)\left[\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_1} - \left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_2}\right] - \frac{Q_P Y_N^*/(r-\mu)}{\beta_1-\beta_2}\left[\beta_1\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_1} - \beta_2\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_2}\right] \\
&= \frac{Q_N - Q_P}{r-\mu}\hat{Y}_H + \beta_2\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_2}\left[\frac{c_N}{r} - \frac{Q_N Y_N^*}{r-\mu}\right],
\end{aligned} \tag{6}$$

it holds that

$$\begin{aligned}&\frac{(\beta_2-1)Q_pY_N^*/(r-\mu)-\beta_2c_P/r}{\beta_1-\beta_2}\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_1}+\frac{(1-\beta_1)Q_pY_N^*/(r-\mu)+\beta_1c_P/r}{\beta_1-\beta_2}\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_2}\\&<\frac{Q_N-Q_P}{r-\mu}\hat{Y}_H+\left(\frac{\hat{Y}_H}{Y_N^*}\right)^{\beta_2}\left[\frac{c_N}{r}-\frac{Q_NY_N^*}{r-\mu}\right].\end{aligned}\tag{7}$$

It is intuitively clear that the firm decides to stop STW and revert to the normal production level once the process $(Y_t)_{t\geq 0}$ hits an endogenously determined trigger Y_H^* from below, i.e. at the stopping time $\hat{\tau}(Y_H^*):=\inf\{t\geq 0|Y_t\geq Y_H^*\}$, or to liquidate once $(Y_t)_{t\geq 0}$ hits an endogenously determined trigger $Y_L^*<Y_H^*$ from above, i.e. at the stopping time time $\check{\tau}(Y_L^*)$. The proposition below shows that this intuition is correct and that the triggers are, in fact, uniquely determined. In order to formulate the proposition we denote the expected value (under P_y) of operating in STW forever by $F_P(y)$, i.e.,

$$F_P(y)=\mathsf{E}_y\left[\int_0^\infty e^{-rt}(Y_tQ_P-cP)dt\right]=\frac{Q_Py}{r-\mu}-\frac{c_P}{r}.$$

Proposition 2 *If Assumptions 1 and 2 are satisfied, then there is a unique trigger $Y_H^*>Y_N^*$, such that returning to normal production is optimal as soon as Y_H^* is hit from below for the first time. There is also a unique trigger $Y_L^*<Y_N^*$, such that liquidation is optimal as soon as Y_L^* is hit from above for the first time. The triggers Y_L^* and Y_H^* are uniquely determined, together with two constants $\hat{A}$ and $\check{A}$ by the equations*

$$\begin{aligned}&\hat{A}(Y_H^*)^{\beta_1}+\check{A}(Y_H^*)^{\beta_2}=F_N^*(Y_H^*)-F_P(Y_H^*)\\&\beta_1\hat{A}(Y_H^*)^{\beta_1-1}+\beta_2\check{A}(Y_H^*)^{\beta_2-1}=\frac{\partial F_N^*(Y_H^*)}{\partial y}-\frac{\partial F_P(Y_H^*)}{\partial y}\\&\hat{A}(Y_L^*)^{\beta_1}+\check{A}(Y_L^*)^{\beta_2}=F_N^*(Y_L)-F_P(Y_L),\quad \textit{and}\\&\beta_1\hat{A}(Y_L^*)^{\beta_1-1}+\beta_2\check{A}(Y_L^*)^{\beta_2-1}=-\frac{\partial F_P(Y_H^*)}{\partial y}.\end{aligned}\tag{8}$$

Furthermore, the value of the firm is

$$F_P^*(y)=\begin{cases}0 & \textit{if } y\leq Y_L^*\\ F_P(y)+\dfrac{(Y_H^*)^{\beta_1}y^{\beta_2}-y^{\beta_1}(Y_H^*)^{\beta_2}}{(Y_H^*)^{\beta_1}(Y_L^*)^{\beta_2}-(Y_L^*)^{\beta_1}(Y_H^*)^{\beta_2}}\Big[-F_P(Y_L^*)\Big] & \\ \quad+\dfrac{y^{\beta_1}(Y_L^*)^{\beta_2}-(Y_L^*)^{\beta_1}y^{\beta_2}}{(Y_H^*)^{\beta_1}(Y_L^*)^{\beta_2}-(Y_L^*)^{\beta_1}(Y_H^*)^{\beta_2}}\Big[F_N^*(Y_H^*)-F_P(Y_H^*)\Big] & \textit{if } Y_L^*<y<Y_H^*\\ F_N^*(y) & \textit{if } y\geq Y_H^*.\end{cases}\tag{9}$$

The proof of this proposition can be found in Appendix A. Note that it is always beneficial to liquidate later under STW than without STW, because $Y_L^* < Y_N^*$

This value function again has an appealing intuitive interpretation. The payoffs for $y \leq Y_L^*$ and $y \geq Y_H^*$ are the payoffs of immediate liquidation and producing at the normal level (including the option value of liquidation), respectively. The value of the firm in the region (Y_L^*, Y_H^*) consists of three parts. The first part is the expected present value of never leaving STW. The second part is the correction for liquidation at Y_L^*, multiplied by the expected discount factor conditional on reaching Y_L^* before Y_H^*. The final part is the correction for returning to normal production at Y_H^*, multiplied by the expected discount factor conditional on reaching Y_H^* before Y_L^*.

2.3 The value of an active firm that has not yet used STW

An active firm that has not used STW yet is confronted with the problem of finding the optimal time at which to exchange the expected present value of current production for the value of a firm that is using STW, F_P^*. That is, the firm solves the optimal stopping problem

$$F^*(y) = \sup_{\tau \in \mathscr{M}} \mathsf{E}_y \left[\int_0^\tau e^{-rt}(Q_N Y_t - c_N)dt + e^{-r\tau} F_P^*(Y_\tau) \right]. \tag{10}$$

Intuitively, this problem should also have a solution that takes the form of a trigger: enter STW as soon as the process $(Y_t)_{t \geq 0}$ reaches some threshold Y^* from above.

In order to prove the existence of such a trigger we need to make an additional assumption that ensures that the expected revenue of STW is sufficiently large. In particular, there must exist states of the economy (i.e. prices y) for which the expected present value of STW relative to normal operation fall sufficiently short of the expected present value of the cost benefits.

Assumption 3 *The expected revenue of STW is sufficiently large. In particular, it holds that*

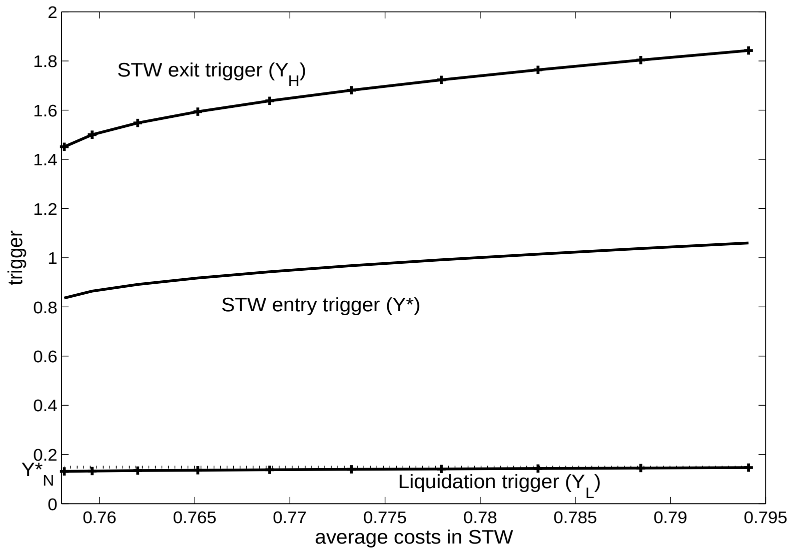
$$\check{Y} \frac{Q_N - Q_P}{r - \mu} > \frac{\beta_1 \beta_2}{(\beta_1 - 1)(\beta_2 - 1)} \frac{c_N - c_P}{r},$$

where

$$\check{Y} = \left[\frac{1 - \beta_2}{\beta_1(\beta_1 - \beta_2)\hat{A}} \frac{Q_N - Q_P}{r - \mu} \right]^{\frac{1}{\beta_1 - 1}},$$

and $\hat{A}$ is as determined by (8) *in Proposition 2.*

Figure 1: Triggers for programme entry, programme exit, and default as a function of average production costs in STW. The base case parameters are $Q_N = 10$, $c_N = 8$, $r = .04$, $\mu = .03$, and $\sigma = .15$. Note that in the base case the average costs of normal production are .8.



Proposition 3 *Suppose that Assumptions 1–3 hold. Then, in addition to the unique triggers Y_L^* and Y_H^*, there exists a unique trigger $Y^* < Y_H^*$ at which it is optimal to adopt STW. Moreover, the value of an active firm that has not yet used STW is*

$$F^*(y) = \begin{cases} F_P^*(y) & \text{if } y \leq Y^* \\ \frac{Q_N y}{r-\mu} - \frac{c_N}{r} + \left(\frac{y}{Y^*}\right)^{\beta_2} \left[F_P^*(Y^*) + \frac{c_N}{r} - \frac{Q_N Y^*}{r-\mu}\right] & \text{if } y > Y^*. \end{cases} \tag{11}$$

The proof of this proposition can be found in Appendix B.

Proposition 3 only makes economic sense if it is optimal to adopt STW before it is optimal to liquidate. This is – indeed – the case:

Lemma 1 *Under the assumptions of Proposition 3 it holds that $Y^* > Y_N^*$.*

The proof of this lemma is in Appendix C.

In Figure 1, the triggers Y_L^*, Y_H^*, and Y^* are plotted for various values of cost reduction. It looks like the liquidation threshold is only marginally influenced by STW. We will see later, however, that the quantitative effect in terms of benefits of STW can be quite large.

3 The Effect of Short-Term Work on Liquidation Probabilities

In this section we compute the probabilities of liquidation of a representative firm over a certain period of time, based on data for several typical STW policies. In order to do so, we need more detail on the firm and how its production technology uses the production factors. Let's assume that the firm uses a fixed amount of capital, K, and a fixed amount of labour, L. The production function of the firm is assumed to be of the Cobb-Douglas type with constant returns to scale, i.e.

$$Q_N = K^{1-\gamma} L^{\gamma}.$$

For simplicity, we assume that the rental rate of capital is constant and equal to ρ, and that the wage rate is constant at w. So, the flow paid to providers of capital (labour) is ρK (wL). Therefore, $c_N = \rho K + wL$. For a given production level Q_N, the firm's (static) profit maximizing capital and labour inputs depend on the parameter γ. In particular,

$$L = \left(\frac{\gamma}{1-\gamma} \frac{\rho}{w} \right)^{1-\gamma} Q_N, \quad \text{and} \quad K = \left(\frac{1-\gamma}{\gamma} \frac{w}{\rho} \right)^{\gamma} Q_N.$$

Throughout this section we assume that K and L are chosen to maximize profits at production level Q_N.

A STW programme is characterized by a number of parameters. First, there is the fraction of worked hours that can be entered into STW, which we denote by α. This implies that

$$Q_P = K^{1-\gamma}[(1-\alpha)L]^{\gamma} = (1-\alpha)^{\gamma} Q_N.$$

Secondly, the programme typically specifies the drop in the wage rate, which we denote by $1-\zeta$. So, for each non-worked hour the employee gets paid a fraction ζ. Thirdly, the government usually only takes on a fraction η of the wage rate (after the reduction has been implemented), leaving the firm to pay a fraction $1-\eta$. So, the cost flow of the firm drops to

$$c_P = \rho K + (1-\alpha)wL + \alpha(1-\zeta)(1-\eta)wL = c_N - \alpha(\zeta + \eta - \zeta\eta)wL.$$

Cahuc and Carcillo (2011) catalogue the wide variety of STW practices in OECD countries. These policies differ in virtually all relevant dimensions such as duration, maximum reduction in the number of hours worked (α), maximum reduction in salary paid ($1-\zeta$) for non-worked hours,

policy	α	ζ	η
Nordic	.63	.78	.17
Anglo-Saxon	.38	.62	.32

Table 1: Three typical STW policies.

Parameter	r	μ	σ	ρ	w	Q_N
Value	.04	.02	.25	.06	.06	10

Table 2: Base case parameter values.

and maximum fraction of salaries paid by the government (η). In order to analyse different real-world policies we group together the Nordic countries and the Anglo-Saxon countries and study the average policies in these groups. We also look at the average policy over all OECD countries that have STW arrangements in place. The data are all obtained from Cahuc and Carcillo (2011). See Table 1 for details.

We consider three types of firms where we differentiate between the labour intensity of the production process. In particular, we study the cases where $\gamma = .25$ (relatively capital intensive), $\gamma = .5$, and $\gamma = .75$ (relatively labour intensive). The other parameters values are given in Table 2. Note that all the STW policies in Table 1 satisfy Assumptions 1–3 with these parameters.

For each firm we assume that $Q_N = 10$. In Table 3 we record the normal costs of production (c_N), the profit maximizing capital and labour input levels (K and L), and production level and costs in STW (Q_P and c_P) under the different policies.

These typical policies lead to different thresholds and, thus, different probabilities of eventual liquidation. The thresholds are reported in Table 4.

Figure 2 shows a sample path for the process $(Y_t)_{t\geq 0}$, which illustrates the goal of STW policies. Here the current state of the economy is fairly low to start with and if no STW were available the firm would liquidate in about 1.8 years' time. With a Nordic style STW programme, the firm, along this particular sample path, would enter STW after approximately 0.2 years, where it remains for approximately 4.7 years. After that period it returns to normal production levels and, crucially, it is still productive after 5 years. Note that after about 2 years the firm almost liquidates, but the economy recovers in time to prevent liquidation from being the optimal choice.

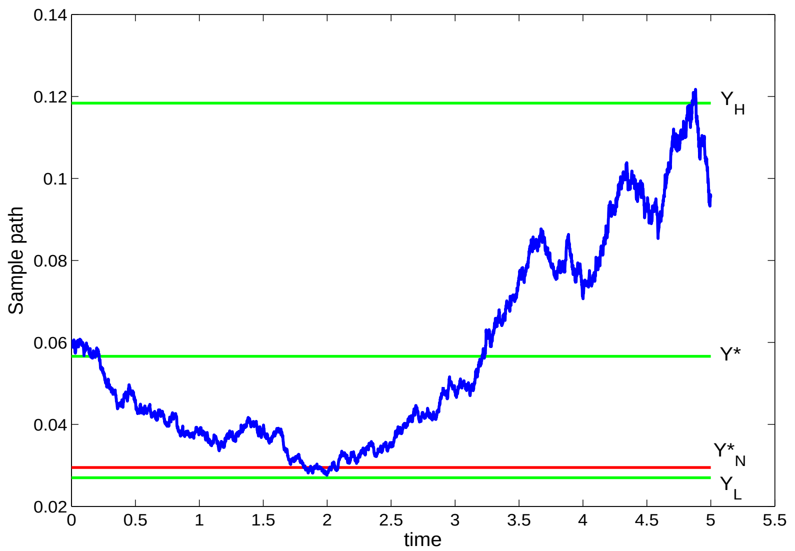
Variable	γ	Nordic	Anglo-Saxon
	.25	13.1607	13.1607
K	.50	10	10
	.75	4.3869	4.3869
	.25	4.3869	4.3869
L	.50	10	10
	.75	13.1607	13.1607
	.25	7.7992	8.8736
Q_P	.50	6.0828	7.8740
	.75	4.7441	6.8970
	.25	1.0529	1.0529
c_N	.50	1.2	1.2
	.75	1.0529	1.0529
	.25	.9173	.9787
c_P	.50	.8910	1.0309
	.75	.6462	.8303

Table 3: Values related to the firm's production process.

Variable	γ	Nordic	Anglo-Saxon
Y_N^*	.25	.0259	.0259
	.50	.0295	.0295
	.75	.0259	.0259
Y^*	.25	.0455	.0480
	.50	.0566	.0572
	.75	.0541	.0524
Y_L^*	.25	.0250	.0254
	.50	.0270	.0282
	.75	.0218	.0240
Y_H^*	.25	.0887	.0966
	.50	.0840	.1190
	.75	.1206	.1127

Table 4: Triggers for various STW policies.

Figure 2: A sample path for the process $(Y_t)_{t\geq 0}$ with triggers based on the base case firm scenario and a Nordic-style STW policy.



In order to judge the efficacy of different STW policies, we could compute the probability that the firm liquidates within, say, T years, as well as the probability that the firm uses the STW measure within T years. Assuming that $Y_0 \equiv y > Y^*$, these probabilities are (cf. Harrison, 1985)

$$\begin{aligned}\mathsf{P}_y\left(\inf_{0\le t\le T} Y_t \le Y_N^*\right) =&\Phi\left(\frac{-\log(y/Y_N^* + (.5\sigma^2-\mu)T)}{\sigma\sqrt{T}}\right)\\ &+\exp\left\{\frac{\sigma^2-2\mu}{\sigma^2}\log\left(\frac{y}{Y_N^*}\right)\right\}\Phi\left(\frac{-\log(y/Y_N^* - (.5\sigma^2-\mu)T)}{\sigma\sqrt{T}}\right),\end{aligned}$$

and

$$\begin{aligned}\mathsf{P}_y\left(\inf_{0\le t\le T} Y_t \le Y^*\right) =&\Phi\left(\frac{-\log(y/Y^* + (.5\sigma^2-\mu)T)}{\sigma\sqrt{T}}\right)\\ &+\exp\left\{\frac{\sigma^2-2\mu}{\sigma^2}\log\left(\frac{y}{Y^*}\right)\right\}\Phi\left(\frac{-\log(y/Y^* - (.5\sigma^2-\mu)T)}{\sigma\sqrt{T}}\right),\end{aligned}$$

respectively, where $\Phi(\cdot)$ is the distribution function of the standard normal distribution.

The probability of liquidation within T years when the firm can use STW can not be computed analytically, because of the multiple ways in which a firm can reach the liquidation threshold. Recall that the two possible liquidation scenarios are:

1. the firm enters STW at Y^* after which Y_L^* is reached before Y_H^*, and
2. the firm enters STW at Y^* after which Y_H^* is reached before Y_L^*, and liquidation then takes place as soon as Y_N^* is reached.

We obtain estimates of this liquidation probability by simulating 50,000 sample paths.

For different values of y and $T = 5$ these liquidation probabilities are reported in Table 5. The probabilities of entering STW are given in Table 6. As starting points we consider the initial states $1.5Y_N^*$ (relatively weak economy), $2Y_N^*$, and $3Y_N^*$ (relatively benign economy), based on the threshold Y_N^* for the case $\gamma = 1/2$.

Note that under the Anglo-Saxon policy it is more likely that a firm enters the STW policy. However, under the Nordic policy the probability of firm liquidation is lower than under the Anglo-Saxon policy.

γ	y	No STW	Nordic	Anglo-Saxon
.25	.0442	.3706	.3374	.3489
	.0589	.1624	.1528	.1520
	.0884	.0346	.0288	.0494
.50	.0442	.5026	.4109	.4481
	.0589	.2428	.1866	.2108
	.0884	.0600	.0406	.0494
.75	.0442	.3706	.2288	.2982
	.0589	.1624	.0880	.1245
	.0884	.0346	.0152	.0239

Table 5: Liquidation probabilities (exact without STW, simulated with STW) for various STW policies.

γ	y	Nordic	Anglo-Saxon
.25	.0442	1	1
	.0589	.6718	.7392
	.0884	.2630	.3055
.50	.0442	1	1
	.0589	.9488	.9620
	.0884	.4594	.4703
.75	.0442	1	1
	.0589	.8922	.8513
	.0884	.4144	.3835

Table 6: Probabilities (exact) of entering STW for various STW policies.

4 Welfare Effects of Short-Term Work

In this section we compare the total value of the firm and the total discounted stream of wages for workers under different STW scenarios. In order to do so, we need more detail on the firm and how its rents are split between the production factors, for which we use the basic set-up used in Section 3.

The surplus created by the firm (profit) is paid to the firm's owners. The rental rate of capital, $\rho > r$, is assumed to take into account the default risk. It is determined at the time that capital K was attracted and, hence, depends on the value of the state variable at that time. If, for example, capital was bought at a time when the value of the state-variable is y, then it is easy to see that

$$\rho = r \left[1 - \left(\frac{y}{Y_N^*} \right)^{\beta_2} \right]^{-1} > r.$$

The welfare effects of STW depend crucially on the way payoffs related to possible future events (like entering STW, exiting STW and returning to normal production, etc.) are discounted. For a firm operating in STW, denote

$$\begin{aligned} \hat{\nu}_y(Y_L^*, Y_H^*) &:= \mathsf{E}_y \left[e^{-r\hat{\tau}(Y_H^*)} \middle| \hat{\tau}(Y_H^*) < \check{\tau}(Y_L^*) \right] \mathsf{P}_y(\hat{\tau}(Y_H^*) < \check{\tau}(Y_L^*)), \quad \text{and} \\ \check{\nu}_y(Y_L^*, Y_H^*) &:= \mathsf{E}_y \left[e^{-r\check{\tau}(Y_L^*)} \middle| \hat{\tau}(Y_H^*) > \check{\tau}(Y_L^*) \right] \mathsf{P}_y(\hat{\tau}(Y_H^*) > \check{\tau}(Y_L^*)). \end{aligned}$$

In Proposition 2 we have already used the fact that (see, for example, Stokey, 2009):

$$\begin{aligned} \hat{\nu}_y(Y_L^*, Y_H^*) &= \frac{y^{\beta_1}(Y_L^*)^{\beta_2} - (Y_L^*)^{\beta_1} y^{\beta_2}}{(Y_H^*)^{\beta_1}(Y_L^*)^{\beta_2} - (Y_L^*)^{\beta_1}(Y_H^*)^{\beta_2}}, \quad \text{and} \\ \check{\nu}_y(Y_L^*, Y_H^*) &= \frac{(Y_H^*)^{\beta_1} y^{\beta_2} - y^{\beta_1}(Y_H^*)^{\beta_2}}{(Y_H^*)^{\beta_1}(Y_L^*)^{\beta_2} - (Y_L^*)^{\beta_1}(Y_H^*)^{\beta_2}}, \end{aligned}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the solutions to (2).

We assume that unemployment incurs a sunk cost equal to a fraction χ of discounted life-time earnings. It has been estimated (Davis and von Wachter, 2011) that $\chi = .11$, going up to $\chi = .19$ in times of recession. Denoting the total expected discounted value to capital, labour, and shareholders by V_k, V_ℓ, and V_s, respectively, we find the following.

Lemma 2 *If the current value of the state-variable is y, then*

$$V_k(y) = \left[1 - \left(\frac{y}{Y_N^*}\right)^{\beta_2}\right] \frac{\rho}{r} K,$$

$$V_\ell(y) = \left[1 - (1+\chi)\left(\frac{y}{Y_N^*}\right)^{\beta_2}\right] \frac{w}{r} L, \quad \textit{and}$$

$$V_s(y) = \left[1 - \left(\frac{y}{Y_N^*}\right)^{\beta_2}\right] \left[\frac{Q_N y}{r-\mu} - \frac{c_N}{r}\right].$$

We can now compute the value for each category (capital, labour, surplus) under an STW programme.

Lemma 3 *If the current value of the state-variable is y, then*

$$V_k^{STW}(y) = \left[1 - \left(\frac{y}{Y_N^*}\right)^{\beta_2} \left(\hat{\nu}_{Y^*}(Y_L^*, Y_H^*) \left(\frac{Y_H^*}{Y_N^*}\right)^{\beta_2} + \check{\nu}_{Y^*}(Y_L^*, Y_H^*)\right)\right] \frac{\rho}{r} K,$$

$$\begin{aligned} V_\ell^{STW}(y) =& \Big\{1 + \left(\frac{y}{Y^*}\right)^{\beta_2} \Big[-1 + (1 - \check{\nu}_{Y^*}(Y_L^*, Y_H^*) - \hat{\nu}_{Y^*}(Y_L^*, Y_H^*))\,(1 - \alpha(1-\zeta)) \\ &+ \hat{\nu}_{Y^*}(Y_L^*, Y_H^*)\Big(1 - \left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}\Big) - \chi\Big(\check{\nu}_{Y^*}(Y_L^*, Y_H^*) + \hat{\nu}_{Y^*}(Y_L^*, Y_H^*)\Big(\frac{Y_H^*}{Y_N^*}\Big)^{\beta_2}\Big)\Big]\Big\} \frac{wL}{r} \end{aligned}$$

$$\begin{aligned} V_s^{STW}(y) =& \Big(1 - \left(\frac{y}{Y^*}\right)^{\beta_2}\Big)\Big(\frac{Q_N y}{r-\mu} - \frac{c_N}{r}\Big) \\ &+ \left(\frac{y}{Y^*}\right)^{\beta_2} (1 - \hat{\nu}_{Y^*}(Y_L^*, Y_H^*) - \check{\nu}_{Y^*}(Y_L^*, Y_H^*))\Big(\frac{Q_P y}{r-\mu} - \frac{c_P}{r}\Big) \\ &+ \left(\frac{y}{Y^*}\right)^{\beta_2} \hat{\nu}_{Y^*}(Y_L^*, Y_H^*) \left(1 - \left(\frac{Y_H^*}{Y_N^*}\right)^{\beta_2}\right) \Big(\frac{Q_N y}{r-\mu} - \frac{c_N}{r}\Big). \end{aligned}$$

The proof can be found in Appendix E.

Typically, nordic STW programmes look more generous in that they allow for a higher reduction in wage costs, lower reductions in salaries paid, and higher fractions of salaries paid for by the government. This would suggest that employees are better off in Nordic countries whereas shareholders are better off in Anglo-Saxon countries. The numerical analysis below shows that this intuition is incorrect.

The values to different stakeholders depend on the current price level in the market. Again we consider the states $1.5Y_N^*$, $2Y_N^*$, and $3Y_N^*$, based on the threshold Y_N^* for the case $\gamma = 1/2$. The value to each stakeholder in the different policies is reported in Tables 7–10.

This numerical analysis indicates, firstly, that the greatest beneficiaries of STW are the providers of capital. The benefit to employees is actually often negative. The intuition for this paradox is

γ	y	No STW	Nordic	Anglo-Saxon
.25	.0442	7.9791	8.2935 (3.94)	8.1727 (2.43)
	.0589	10.8319	11.0660 (2.16)	10.9724 (1.30)
	.0884	13.7182	13.8765 (1.15)	13.8132 (0.69)
.50	.0442	4.8595	5.6241 (15.73)	5.2603 (8.25)
	.0589	7.3190	7.8465 (7.21)	7.5944 (3.76)
	.0884	9.8074	10.1640 (3.63)	9.9936 (1.90)
.75	.0442	2.6597	3.2068 (20.57)	2.9136 (9.55)
	.0589	3.6106	3.9978 (10.72)	3.7907 (4.99)
	.0884	4.5727	4.8344 (5.72)	4.6945 (2.66)

Table 7: Value to capital providers (% change in brackets).

γ	y	No STW	Nordic	Anglo-Saxon
.25	.0442	2.2284	2.1561 (-3.25)	2.0901 (-6.21)
	.0589	3.2840	3.2224 (-1.88)	3.1577 (-3.84)
	.0884	4.3519	4.3103 (-0.96)	4.2666 (-1.96)
.50	.0442	3.7441	4.1049 (9.64)	3.7177 (-0.70)
	.0589	6.4741	6.5215 (0.73)	6.2366 (-3.67)
	.0884	9.2362	9.2683 (0.35)	9.0757 (-1.74)
.75	.0442	6.6853	7.6319 (14.16)	6.7548 (1.04)
	.0589	9.8519	10.2969 (4.52)	9.5610 (-1.38)
	.0884	13.0557	13.3565 (2.30)	12.9637 (-0.70)

Table 8: Value to labour providers, including sunk costs of unemployment (% change in brackets).

γ	y	No STW	Nordic	Anglo-Saxon
.25	.0442	2.6597	2.5759 (-3.15)	2.5143 (-5.47)
	.0589	3.6106	3.5405 (-1.94)	3.4793 (-3.69)
	.0884	4.5727	4.5253 (-1.04)	4.4839 (-1.94)
.50	.0442	4.8595	5.1363 (5.69)	4.7891 (-1.45)
	.0589	7.3190	7.3084 (-0.15)	7.0512 (-3.66)
	.0884	9.8074	9.8002 (-0.07)	9.6264 (-1.85)
.75	.0442	7.9791	8.7452 (9.60)	7.9649 (-0.18)
	.0589	10.8319	11.1492 (2.93)	10.6364 (-1.80)
	.0884	13.7182	13.9327 (1.56)	13.5861 (-0.96)

Table 9: Value to labour providers, excluding sunk costs of unemployment (% change in brackets).

γ	y	No STW	Nordic	Anglo-Saxon
.25	.0442	3.7633	3.9231 (4.25)	3.8687 (2.80)
	.0589	9.1969	9.3183 (1.32)	9.2785 (0.89)
	.0884	21.9765	22.0586 (0.37)	22.0317 (0.25)
.50	.0442	2.4242	2.9588 (22.05)	2.7068 (11.66)
	.0589	7.2904	7.7709 (6.59)	7.5495 (3.55)
	.0884	19.4959	19.8208 (1.67)	19.6711 (0.90)
.75	.0442	3.7633	4.7392 (25.93)	4.2104 (11.88)
	.0589	9.1967	10.0047 (8.78)	9.5610 (3.96)
	.0884	21.9765	22.5226 (2.48)	22.2226 (1.12)

Table 10: Value to shareholders (% change in brackets).

simple: labour is the only production factor that is reduced in STW. By saving on wage costs the firm increases its life-span and, thus, the period of time during which the full costs of capital can be reimbursed.

The occasional negative benefit to labour occurs because the decrease in the present value of sunk costs of unemployment are more than off-set by the decrease in the present value of the reduction in wage income over the STW period. Note that the latter are discounted less than the former.

Secondly, the firm chooses its policy to maximize the value to shareholders, so it is no surprise that this value is positively affected by STW.

Next, all agents are better off in the typical Nordic scenario than the typical Anglo-Saxon scenario. This suggest that more generous STW programmes do not necessarily hurt providers of capital and shareholders. The reason for this might be that, even though firms under a Nordic policy pay more for non-worked hours, they are also allowed to reduce the number of hours worked more.

As a final note, observe that since in the Cobb-Douglas technology capital and labour are interchangeable, an alternative to STW could be to negotiate a "pay holiday" between the firm and the providers of its capital. In such a scenario the roles between capital and labour would be reversed and it would be the employees who are more protected against a drop in the firm's value. Of course, a combination of the two approaches would share the losses more equally.

5 Concluding Remarks

In this paper we studied a firm that can make use of a short-time work programme. Using a real options approach we derived the optimal thresholds for the firm to adopt STW and, once entered, when to exit and revert to normal production levels, or to liquidate. The liquidation threshold is computed both when the firm is using STW and when it is not. We show that a firm that is using STW will liquidate later than a firm that is not using it.

In practice, details of STW are highly variable between (OECD) countries. Our numerical computations are based on three different scenarios. One takes the STW details as they are common in Northern European countries. The second scenario looks at the way STW has been implemented typically in Anglo-Saxon countries. Finally, a third scenario reflects an average of STW programmes across the OECD. We take the point of view of a government and calculate the benefits for society of STW by a dynamic benefit-to-cost ratio and find that the dBCR is typically very small and far below unity. This happens because the benefits of STW accrue later in time than when the costs are incurred and are, thus, discounted more. This shows the importance of a stochastic dynamic approach to analysing such measures.

We also study the value of STW to different stakeholders in the firm. It turns out that employees are actually the worst off. STW is best for capital providers and shareholders. This happens because STW extends the life of the firm and capital providers get remunerated without discount throughout that time. We suggest that a "pay holiday" on interest repayments on capital would lead to similar results as STW but would protect the employee more. This suggests that a balanced approach may provide a better balance between costs and benefits of STW.

We list three possible extensions of this research. First, it would be interesting to make the duration that the firms can use STW time dependent. With such a model one could calculate the optimal duration of the programme both from a firm's and society perspective. Second, one could investigate what the effect would be from the possibility to make use of the programme more than once. Would firms adopt STW earlier? Would they exit earlier? Would this be beneficial for society? Third, it could be interesting to study the effect of STW in a competitive setting. How is the entry threshold of one firm affected by actions of other firms? What if a firm is in competition with another firm that is based in a country where STW measures do not exist? Finally, this paper has not addressed the effect of STW on the labour market. An important question to be asked is whether STW unfairly advantages insiders in firms that have access to STW.

Appendix

A Proof of Proposition 2

1. The optimal stopping problem (4) can be written as

$$F_P^*(y) = \frac{Q_P y}{r-\mu} - \frac{c}{r} + \sup_{\tau} \mathsf{E}_y \left[e^{-r\tau} \max \left\{ G_L(Y_\tau), G_H(Y_\tau) \right\} \right], \tag{12}$$

where

$$G_L(y) = -F_P(y) = \frac{c_P}{r} - \frac{Q_P y}{r-\mu}, \quad \text{and}$$
$$G_H(y) = F_N^*(y) - F_P(y) = \frac{Q_N - Q_P}{r-\mu} y - \frac{c_N - c_P}{r} + \left(\frac{y}{Y_N^*}\right)^{\beta_2} \left[\frac{c_N}{r} - \frac{Q_N Y_N^*}{r-\mu}\right].$$

Note that $G_H(y) > G_L(y)$ if, and only if, $y > Y_N^*$, and that $G_H(Y_N^*) = G_L(Y_N^*)$.

Define the function $G : \mathbb{R}_+ \to \mathbb{R}$ by

$$G(y) = 1_{y \le Y_N^*} G_L(y) + 1_{y > Y_N^*} G_H(y),$$

so that $G = G_L \vee G_H$. Note that G is C^2 on $\mathbb{R}_+ \backslash \{Y_N^*\}$.

2. From Peskir and Shiryaev (2006) it follows that we need to find a function $F_P^* \in C^2$ which dominates G on $\mathbb{R}_+$, and a set $\mathscr{C} \subset \mathbb{R}_+$, that solve the free boundary problem

$$\begin{cases} \mathscr{L} F_P^* - r F_P^* = 0 \quad \text{on } \mathscr{C} \\ F_P^* > G \quad \text{on } \mathscr{C}, \text{ and} \quad F_P^* = G \quad \text{on } \mathbb{R}_+ \backslash \mathscr{C} \\ \frac{\partial F_P^*}{\partial y}_{|\partial \mathscr{C}} = \frac{\partial G}{\partial y}_{|\partial \mathscr{C}}. \end{cases} \tag{13}$$

Here $\mathscr{L}$ denotes the characteristic operator of $(Y_t)_{t \ge 0}$, i.e., for any $\varphi \in C^2$,

$$\mathscr{L}\varphi(y) = \frac{1}{2}\sigma^2 y^2 \varphi''(y) + \mu \varphi'(y).$$

3. On $\mathbb{R}_+$, define the functions $\hat{\varphi} : \mathbb{R}_+ \to \mathbb{R}_+$ and $\check{\varphi} : \mathbb{R}_+ \to \mathbb{R}_+$, by

$$\hat{\varphi}(y) = y^{\beta_1}, \quad \text{and} \quad \check{\varphi}(y) = y^{\beta_2}. \tag{14}$$

Note that $\hat{\varphi}$ and $\check{\varphi}$ are the increasing and decreasing solutions, respectively, to the differential equation $\mathscr{L}\varphi - r\varphi = 0$. So, any solution to $\mathscr{L}\varphi - r\varphi = 0$ is of the form

$$\varphi(y) = \hat{A}\hat{\varphi}(y) + \check{A}\check{\varphi}(y),$$

where $\hat{A}$ and $\check{A}$ are arbitrary constants. Furthermore, it is easily obtained that

$$\hat{\varphi}'(y) = \frac{\beta_1}{y}\hat{\varphi}(y) > 0, \quad \check{\varphi}'(y) = \frac{\beta_2}{y}\check{\varphi}(y) < 0, \quad \text{and}$$
$$\hat{\varphi}''(y) = \frac{\beta_1(\beta_1 - 1)}{y^2}\hat{\varphi}(y) > 0, \quad \check{\varphi}''(y) = \frac{\beta_2(\beta_2 - 1)}{y^2}\check{\varphi}(y) > 0.$$

4. Fix $Y_L \leq Y_N^*$ and define the mapping $y \mapsto V(y; Y_L)$, by

$$V(y; Y_L) = \hat{A}(Y_L)\hat{\varphi}(y) + \check{A}(Y_L)\check{\varphi}(y), \tag{15}$$

where the constants $\hat{A}(Y_L)$ and $\check{A}(Y_L)$ are given by

$$\hat{A}(Y_L) = \frac{\check{\varphi}(Y_L)G_L'(Y_L) - \check{\varphi}'(Y_L)G_L(Y_L)}{\check{\varphi}(Y_L)\hat{\varphi}'(Y_L) - \check{\varphi}'(Y_L)\hat{\varphi}(Y_L)} = \frac{(\beta_2 - 1)Q_P Y_L/(r-\mu) - \beta_2 c_P/r}{(\beta_1 - \beta_2)Y_L^{\beta_1}}, \tag{16}$$

and

$$\check{A}(Y_L) = \frac{\hat{\varphi}'(Y_L)G_L(Y_L) - \hat{\varphi}(Y_L)G_I'(Y_L)}{\check{\varphi}(Y_L)\hat{\varphi}'(Y_L) - \check{\varphi}'(Y_L)\hat{\varphi}(Y_L)} = \frac{(1-\beta_1)Q_P Y_L/(r-\mu) + \beta_1 c_P/r}{(\beta_1 - \beta_2)Y_L^{\beta_2}}. \tag{17}$$

Note that $\mathscr{L}V(y; Y_L) - rV(y; Y_L) = 0$ for all $y \in \mathbb{R}_+$. In addition, the function V satisfies $V(Y_L; Y_L) = G_L(Y_L)$ and $V'(Y_L; Y_L) = G_L'(Y_L)$.

It is easily seen that

$$\check{A}(Y_L) > 0, \quad \text{and} \quad \check{A}'(Y_L) > 0.$$

In addition, Assumption 1 ensures that for all $Y_L \leq Y_N^*$, it holds that

$$\hat{A}(Y_L) > 0, \quad \text{and} \quad \hat{A}'(Y_L) < 0.$$

5. So, the mapping $y \mapsto V(y; Y_N^*)$ is a (strictly) convex function, which satisfies $V(\cdot; Y_N^*) \to \infty$ as $y \to \infty$ or $y \downarrow 0$. Hence, there is a unique point $\hat{Y}_H > Y_N^*$, such that $V'(Y_H; Y_N^*) = G_H'(Y_H)$. This is exactly the value $\hat{Y}_H$ determined by (7) in Assumption 2. This assumption then ensures that $V(Y_H; Y_N^*) < G_H(Y_H)$. Also, since V is more convex than G_H for large y, it holds that $V(y; Y_N^*) > G_H(y)$ for y large enough.

6. Since $\hat{A}(Y_L)$ decreases and $\check{A}(Y_L)$ increases in Y_L, the mapping $y \mapsto V(y; Y_L)$ has the property that for every $y > Y_L$ it holds that $\partial V(y; Y_L)/\partial Y_L < 0$. So, the point $Y_H \in (Y_N^*, \infty)$ where $V'(Y_H; Y_L) = G_H'(Y_H)$ is decreasing in Y_L, as is the value $V(Y_H; Y_L)$. Now decrease Y_L from Y_N^* to 0. There will be a unique Y_L^*, with corresponding Y_H^* at which $V(Y_H^*; Y_L^*) = G_H(Y_H^*)$ and $V'(Y_H^*; Y_L^*) = G_H'(Y_H^*)$.

7. The interval $\mathscr{C} = (Y_L^*, Y_H^*)$ and the proposed function $F_P^* = V(\cdot; Y_L^*)$ together solve the free-boundary problem (13). The fact that Y_L^* and Y_H^* are the unique triggers that make F_P^* a C^1 function on (Y_L^*, Y_H^*) follows by construction.

■

B Proof of Proposition 3

First note that for $y \in [Y_L^*, Y_H^*]$ we can write

$$F_P^*(y) = F_P(y) + Ay^{\beta_1} + By^{\beta_2}.$$

This implies that the foc for maximizing $F^*(\cdot)$ can be written as $g(y) = 0$, where

$$\begin{aligned} g(y) &= -\beta_2[F_P(y) - F_N(y) + Ay^{\beta_1} + By^{\beta_2}] + y[F_P'(y) - F_N'(y) + \beta_1 Ay^{\beta_1-1} + \beta_2 By^{\beta_2-1}] \\ &= -\beta_2[F_P(y) - F_N(y)] + y[F_P'(y) - F_N'(y)] + (\beta_1 - \beta_2)Ay^{\beta_1}. \end{aligned}$$

Since

$$g'(y) = (1-\beta_2)\frac{Q_P - Q_N}{r-\mu} + \beta_1(\beta_1 - \beta_2)Ay^{\beta_1 - 1},$$

and

$$g''(y) = \beta_1(\beta_1 - 1)(\beta_1 - \beta_2)Ay^{\beta_1 - 2} > 0,$$

$g(\cdot)$ is a strictly convex function, which can, therefore, have at most two zeros.

The minimum location of $g(\cdot)$ on $[Y_L^*, Y_H^*]$ can be found analytically:

$$\begin{aligned} g'(y) = 0 \iff y^{\beta_1 - 1} &= \frac{1-\beta_2}{\beta_1(\beta_1-\beta_2)A}\frac{Q_N - Q_P}{r-\mu} = 0 \\ \iff y &= \left[\frac{1-\beta_2}{\beta_1(\beta_1-\beta_2)A}\frac{Q_N - Q_P}{r-\mu}\right]^{\frac{1}{\beta_1 - 1}} \equiv \check{Y}. \end{aligned}$$

We then find that

$$\begin{aligned}
g(\check{Y}) &= (1-\beta_2)\frac{Q_P - Q_N}{r-\mu}\check{Y} + (\beta_1-\beta_2)A\check{Y}^{\beta_1} + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\left[(1-\beta_2)\frac{Q_P - Q_N}{r-\mu} + (\beta_1-\beta_2)A\check{Y}^{\beta_1-1}\right] + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\left[(1-\beta_2)\frac{Q_P - Q_N}{r-\mu} + (\beta_1-\beta_2)A\frac{1-\beta_2}{\beta_1(\beta_1-\beta_2)A}\frac{Q_N - Q_P}{r-\mu}\right] + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\frac{Q_N - Q_P}{r-\mu}\left(-(1-\beta_2) + \frac{1-\beta_2}{\beta_1}\right) + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\frac{Q_N - Q_P}{r-\mu}\frac{(1-\beta_1)(1-\beta_2)}{\beta_1} + \beta_2\frac{c_P - c_N}{r} < 0,
\end{aligned}$$

where the last inequality follows from Assumption 3.

Define

$$f(y) = F_P(y) - F_N(y) + Ay^{\beta_1} + By^{\beta_2}.$$

Since $A(Y_H^*)^{\beta_1} + B(Y_H^*)^{\beta_2} = F_N^*(Y_H^*) - F_P(Y_H^*)$, it holds that

$$\begin{aligned}
f(Y_H^*) &= F_P(Y_H^*) - F_N(Y_H^*) + A(Y_H^*)^{\beta_1} + B(Y_H^*)^{\beta_2} \\
&= F_N^*(Y_H^*) - F_N(Y_H^*) = \left(\frac{Y_H^*}{Y_N^*}\right)^{\beta_2}[-F_N(Y_N^*)],
\end{aligned}$$

and

$$f'(Y_H^*) = \beta_2\left(\frac{Y_H^*}{Y_N^*}\right)^{\beta_2}\frac{1}{Y_H^*}[-F_N(Y_N^*)].$$

We, therefore, find that

$$\begin{aligned}
g(Y_H^*) &= -\beta_2 f(Y_H^*) + Y_H^* f'(Y_H^*) \\
&= (1-\beta_2)\left(\frac{Y_H^*}{Y_N^*}\right)^{\beta_2}[-F_N(Y_N^*)] > 0.
\end{aligned}$$

So, $g(\cdot)$ has a zero at $Y_* \in (\check{Y}, Y_H^*)$, which represents a minimum of $F^*(\cdot)$.

The function $F^*(\cdot)$ has a (unique) maximum at some $Y^* \in (Y_L^*, \check{Y})$ if and only if $g(Y_L^*) > 0$. Note that

$$\begin{aligned}
f(Y_L^*) &= F_P(Y_L^*) - F_N(Y_L^*) + A(Y_L^*)^{\beta_1} + B(Y_L^*)^{\beta_2} \\
&= F_P(Y_L^*) - F_N(Y_L^*) + [-F_P(Y_L^*)] = -F_N(Y_L^*),
\end{aligned}$$

and

$$f'(Y_L^*) = -F_N'(Y_L^*),$$

which implies that

$$g(Y_L^*) = -\beta_2[-F_N(Y_L^*)] + Y_L^*[-F_N'(Y_L^*)].$$

At Y_N^* it holds that

$$-\beta_2[-F_N(Y_N^*)] + Y_N^*[-F_N'(Y_N^*)] = 0,$$

since Y_N^* is a maximum location of $F_N^*(\cdot)$. It, therefore holds that $g(Y_L^*) > 0 \iff Y_L^* < Y_N^*$.

■

C Proof of Lemma 1

Let

$$Y_P^* = \frac{\beta_2}{\beta_2 - 1} \frac{r - \mu}{Q_P} \frac{c_P}{r}.$$

That is, $\check{\tau}(Y_P^*)$ solves the optimal stopping problem

$$G^*(y) = \sup_{\tau \in \mathscr{M}} \mathsf{E}_y \left[\int_0^\tau e^{-rt}(Y_t Q_P - c_P) dt \right].$$

Recall the function g from the proof of Proposition 3. At Y^* it holds that $g(Y^*) = 0$, i.e. that

$$\begin{aligned} &\beta_2[F_P(Y^*) - F_N(Y^*)] = Y^*[F_P'(Y^*) - F_N'(Y^*)] + (\beta_1 - \beta_2)A(Y^*)^{\beta_1} \\ \iff &\beta_2[-F_N(Y^*)] - Y^*[-F_N'(Y^*)] = -[-\beta_2(-F_P(Y^*)) + Y^*(-F_P'(Y^*))] + (\beta_1 - \beta_2)A(Y^*)^{\beta_1}. \end{aligned} \tag{18}$$

Since at Y_P^* it holds that

$$-\beta_2[-F_P(Y_P^*)] + Y_P^*[-F_P'(Y_P^*)] = 0,$$

Y_P^* is a maximum location, and $Y^* > Y_P^*$, we have that the term between square brackets on the right-hand side of (18) is negative. Therefore, the right-hand side of (18) is positive.

This implies that

$$-\beta_2[F_P(Y^*) + F_N(Y^*)] < 0.$$

Since Y_N^* solves

$$-\beta_2[F_P(Y^*) + F_N(Y^*)] = 0,$$

and Y_N^* is a maximum location it, therefore, holds that $Y^* > Y_N^*$.

■

D Proof of Lemma 2

For any Y and $y < Y$, let $\hat{\tau}_y(Y)$ denote the first hitting time from below of Y under P_y. Similarly, for any Y and $y > Y$, let $\check{\tau}_y(Y)$ denote the first hitting time from above of Y under P_y. For any $y > Y_N^*$, we then find that

$$\begin{aligned} V_k(y) =& \mathsf{E}_y \left[\int_0^{\check{\tau}_y(Y_N^*)} e^{-rt} \rho K dt \right] \\ =& \left(1 - \mathsf{E}_y \left[e^{-r\check{\tau}_y(Y_N^*)} \right]\right) \mathsf{E}_y \left[\int_0^{\infty} e^{-rt} \rho K dt \right] \\ =& \left(1 - \left(\frac{y}{Y_N^*}\right)^{\beta_2}\right) \frac{\rho}{r} K. \end{aligned}$$

The other values are computed in the same way, taking into account that the labour factor incurs a sunk cost equal to a fraction χ of life-time discounted expected labour income. That is, the sunk costs equal

$$\begin{aligned} \chi \mathsf{E}_y \left[\int_{\check{\tau}_y(Y_N^*)}^{\infty} e^{-rt} w L dt \right] =& \chi \mathsf{E}_y \left[e^{-r\check{\tau}_y(Y_N^*)} \right] \mathsf{E}_y \left[\int_0^{\infty} w L dt \right] \\ =& \chi \left(\frac{y}{Y_N^*}\right)^{\beta_2} \frac{w}{r} L. \end{aligned}$$

■

E Proof of Lemma 3

The proof follows along similar lines as the proof of the previous lemma, i.e. by carefully discounting expected infinite streams of payoffs. Denote

$$
\begin{aligned}
DK(y) &= \mathsf{E}_y\left[\int_0^\infty e^{-rt}\rho K dt\right] = \frac{\rho}{r}K,\\
DL_N(y) &= \mathsf{E}_y\left[\int_0^\infty e^{-rt}wL dt\right] = \frac{w}{r}L,\\
DL_P(y) &= \mathsf{E}_y\left[\int_0^\infty e^{-rt}[(1-\alpha)wL+\alpha(1-\zeta)wL]dt\right] = \frac{1-\alpha\zeta}{r}wL,\\
DS_N(y) &= \mathsf{E}_y\left[\int_0^\infty e^{-rt}(Q_N Y_t - c_N)dt\right] = \frac{Q_N y}{r-\mu} - \frac{c_N}{r}, \quad \text{and}\\
DS_P(y) &= \mathsf{E}_y\left[\int_0^\infty e^{-rt}(Q_P Y_t - c_P)dt\right] = \frac{Q_P y}{r-\mu} - \frac{c_P}{r}.
\end{aligned}
$$

The providers of capital get paid a stream ρK until the firm liquidates. This happens either if the firms enters the STW programme and then hits the threshold Y_L, or if the firm enters the programme, then hits the threshold Y_H and then hits the threshold Y_N^*. That is,

$$
\begin{aligned}
V_k^{STW}(y) =& \mathsf{E}_y\Big[\int_0^{\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_L)} e^{-rt}\rho K dt \Big| \check{\tau}_{Y^*}(Y_L) < \hat{\tau}_{Y^*}(Y_H)\Big]\mathsf{P}_{Y^*}(\check{\tau}_{Y^*}(Y_L) < \hat{\tau}_{Y^*}(Y_H))\\
&+\mathsf{E}_y\Big[\int_0^{\check{\tau}_y(Y^*)+\hat{\tau}_{Y^*}(Y_H)+\check{\tau}_{Y^*}(Y_N^*)} e^{-rt}\rho K dt \Big| \check{\tau}_{Y^*}(Y_L) > \hat{\tau}_{Y^*}(Y_H)\Big]\mathsf{P}_{Y^*}(\check{\tau}_{Y^*}(Y_L) > \hat{\tau}_{Y^*}(Y_H))\\
=& \frac{\rho}{r}K\left\{1 - \left(\frac{y}{Y^*}\right)^{\beta_2}\left[\check{\nu}_{Y^*}(Y_L,Y_H) + \hat{\nu}_{Y^*}(Y_L,Y_H)\left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}\right]\right\}.
\end{aligned}
$$

The providers of labour receive a flow wL while the firm operates normally and a flow $(1-\alpha+\alpha\zeta)wL = (1-\alpha\zeta)wL$ while the firm operates in STW. Therefore, assuming that $y > Y^*$, it holds that

$$
\begin{aligned}
V_\ell^{STW}(y) =&\mathsf{E}_y\Big[\int_0^{\hat\tau_y(Y^*)} e^{-rt} wLdt\Big] + \mathsf{E}_y\Big[\int_{\hat\tau_y(Y^*)}^{\hat\tau_y(Y^*)+\check\tau_{Y^*}(Y_L)\wedge\hat\tau_{Y^*}(Y_H)} (1-\alpha(1-\zeta))wLdt\Big] \\
&+ \mathsf{E}_y\Big[\int_{\hat\tau_y(Y^*)+\hat\tau_{Y^*}(Y_H)}^{\hat\tau_y(Y^*)+\hat\tau_{Y^*}(Y_H)+\check\tau_{Y_H}(Y_N^*)} e^{-rdt} wLdt\Big|\hat\tau_{Y^*}(Y_H)<\check\tau_{Y^*}(Y_L)\Big]\mathsf{P}_{Y^*}(\hat\tau_{Y^*}(Y_H)<\check\tau_{Y^*}(Y_L)) \\
&- \Big\{\mathsf{E}_y\Big[e^{-r(\hat\tau_y(Y^*)+\hat\tau_{Y^*}(Y_H))}\Big|\hat\tau_{Y^*}(Y_H)<\check\tau_{Y^*}(Y_L)\Big]\mathsf{P}_{Y^*}(\hat\tau_{Y^*}(Y_H)<\check\tau_{Y^*}(Y_L)) \\
&+ \mathsf{E}_y\Big[e^{-r(\hat\tau_y(Y^*)+\check\tau_{Y^*}(Y_L))}\Big|\hat\tau_{Y^*}(Y_H)>\check\tau_{Y^*}(Y_L)\Big]\mathsf{P}_{Y^*}(\hat\tau_{Y^*}(Y_H)>\check\tau_{Y^*}(Y_L))\Big\}\chi\frac{wL}{r} \\
=&\left(1-\left(\frac{y}{Y^*}\right)^{\beta_2}\right)\frac{wL}{r} + \left(\frac{y}{Y^*}\right)^{\beta_2}(1-\check\nu_{Y^*}(Y_L,Y_H)-\hat\nu_{Y^*}(Y_L,Y_H))(1-\alpha(1-\zeta))\frac{wL}{r} \\
&+ \left(\frac{y}{Y^*}\right)^{\beta_2}\hat\nu_{Y^*}(Y_L,Y_H)\left(1-\left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}\right)\frac{wL}{r} \\
&- \chi\left(\check\nu_{Y^*}(Y_L,Y_H)+\hat\nu_{Y^*}(Y_L,Y_H)\left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}\right)\frac{wL}{r},
\end{aligned}
$$

from which the result follows immediately.

Finally, the value accruing to shareholders follows in a similar way and is already discussed in detail in Section 2.

■

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