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*Published in:*  
Journal of Population Economics

*Publication date:*  
1997

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*  
Canton, E. J. F., & Meijdam, A. C. (1997). Altruism and the macroeconomic effects of demographic changes. *Journal of Population Economics*, 10(3), 317-334.

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# Altruism and the macroeconomic effects of demographic changes

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Received April 17, 1996/Accepted December 10, 1996

**Abstract.** In this paper we show that the macroeconomic effects of demographic changes strongly depend on the degree of altruism and on the specification of the intertemporal utility function. We allow for agents either to be altruistic in the sense of Barro (1974) or non-altruistic. In the latter case, generations are heterogeneous like in the “unloved children” model of Weil (1989). In the former case, where the model is a standard Ramsey model with identical agents, we distinguish a Millian and a Benthamite intertemporal utility function. For each of these models, we study the effects of an anticipated and unanticipated permanent decline in population growth as well as the consequences of a baby-boom/baby-bust scenario.

**JEL classification:** D 60, E 32, J 10

**Key words:** Demographic changes, altruistic and non-altruistic agents

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## 1. Introduction

The baby boom in the sixties and the current ageing process in most Western countries have brought about a lively debate among economists about

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All correspondence to Lex Meijdam. We would like to thank Alessandro Cigno, Gianluca Femminis, Henri de Groot, Eline van der Heijden, Martijn van de Ven, an anonymous referee, and conference participants at the Tenth Annual Meeting of the European Society for Population Economics for helpful suggestions. Erik Canton acknowledges the Rens-Holle Association for providing a travel grant. This paper is part of Erik Canton’s doctoral dissertation on “Economic Growth and Business Cycles”, written at Tilburg University, Tilburg, The Netherlands. *Responsible editor:* Alessandro Cigno.

the effects of these demographic changes. A seminal paper in this respect is Cutler et al. (1990), describing the economic consequences of the expected ageing of the American population using a neoclassical growth model with varying relative sizes of self-supporting and dependent populations. Another example of this literature is Yoo (1994), who compares the effects of a baby-boom/baby-bust in the dependency-ratio model of Cutler et al., in a standard neoclassical growth model, and in a Diamond overlapping-generations model. His conclusion is that the different models lead to results that are qualitatively the same.

There are, however, two important points that should be taken into account when analyzing the macroeconomic effects of demographic changes. Firstly, models with intergenerational altruism generate different responses to demographic shocks than models with selfish agents. Secondly, if there is intergenerational altruism in the sense of Barro (1974), one needs to distinguish between a Millian and a Benthamite intertemporal utility function. In the literature, different assumptions are used without discussing their implications. The results of Cutler et al., for example, crucially depend on the assumption of a Benthamite intertemporal utility function. There is, however, no compelling economical argument to prefer this function above a so-called Millian utility function.<sup>1</sup> And the conclusions of Yoo hinge on the fact that he uses a Millian utility function in case of the neoclassical growth model and the overlapping-generations model whereas he uses a Benthamite utility function in case of the dependency-ratio model.

It is remarkable to notice that the literature on the economic effects of demographic transitions largely ignored the consequences of distinguishing a Millian from a Benthamite utility function, whereas the literature on endogenous fertility and optimal population size extensively discusses the implications of using either criterion function. For instance, Nerlove (1988) shows that maximizing a Benthamite intergenerational social welfare function always leads to a larger population than maximizing a Millian welfare function. However, in the context of a simple model of endogenous growth, Palivos and Yip (1993) show that the Benthamite criterion leads to a smaller population size and higher output growth. Other examples include Becker and Barro (1988), Cigno (1993), and Nerlove et al. (1982, 1985).

The hypotheses of altruism and non-altruism are empirically investigated by Cigno and Rosati (1992, 1996). Cigno and Rosati (1992) study saving and fertility behaviour in Italy and their empirical findings support the hypothesis that individual behaviour is motivated by self-interest rather than intergenerational altruism. Similar findings are obtained in a more recent study for Germany, Italy, United Kingdom, and United States (Cigno and Rosati 1996).

In this paper we show that the macroeconomic effects of demographic changes strongly depend on how altruism is modelled. We employ a general specification of intertemporal utility, allowing us to study the Millian and the Benthamite utility functions as special cases. Moreover, we allow for agents either to be altruistic in the sense of Barro (1974) or non-altruistic. In the former case, the model is a standard Ramsey model where all agents are identical. The latter case leads to a model with heterogeneous generations like the "unloved children" model of Weil (1989). For each of these models, we study the effects of demographic shocks in the long-run as well as in the short-run. The long-run results are based on a comparative statics

analysis. Analytical solutions for the short-run effects are derived by the method of comparative dynamics (Judd 1982). Two demographic scenarios are studied: an anticipated and unanticipated permanent decrease in population growth (to mimic the ageing process) as well as a temporary increase in population growth (to mimic a baby-boom/baby-bust).

The paper proceeds along the following lines. In Sect. 2 we introduce the model. The model's steady-state is derived in Sect. 3. Some comparative statics results are discussed in the same section. Adjustment trajectories in an ageing scenario and a baby-boom/baby-bust scenario are studied in Sect. 4. Finally, Sect. 5 briefly concludes.

## 2. The model

The production process is described by the production function  $y = f(k)$ , where  $f$  satisfies the usual conditions and  $y$  and  $k$  denote per capita aggregate output and capital respectively. Denoting the interest rate on capital by  $r$  and wage income per worker by  $w$ , competitive markets require that  $r = f'(k) - \delta$  and  $w = y - f'(k)k$  where  $\delta$  is capital depreciation. On the household side, labour is homogeneous and labour supply is inelastic and equal to population size  $L$ , growing at an exogenous rate  $n$ .

The model allows for three different settings. In the first and the second setting, there is altruism in the sense of Barro (1974). In that case, there is a fixed number of dynasties. Each new born child is linked through intergenerational transfers to a pre-existing dynasty. The dynasties thus grow at the same rate as the population. In this interpretation, the model is a Ramsey-model with homogeneous agents. The difference between the first and the second setting is the form of the utility function. In the Ramsey-Benthamite variant of the model, the utility of a representative member of the dynasty is weighted by the size of the dynasty. In the Ramsey-Millian variant, this is not the case.

In the third variant of the model there is no altruism and generations are heterogeneous (cf. Weil 1989). Children who are born do not belong to any pre-existing dynasty. Each "unloved" child initiates a dynasty by itself. The rate at which new dynasties are created is thereby equal to the rate of population growth.<sup>2</sup> To be able to present these three variants in one model, we assume that instantaneous utility of a member of a dynasty is a CRRA function of his consumption level  $\hat{c}$  (we use a hat  $\hat{\cdot}$  to distinguish quantities per member of a dynasty from per capita aggregate variables which are different if agents are heterogeneous). The Ramsey model with altruistic agents can easily be studied for more general utility functions (see Appendix 2), but the Weil model with heterogeneous agents becomes analytically untractable in that case. Intertemporal utility of a dynasty  $\hat{U}$  is defined as the discounted flow of instantaneous utilities, weighted by  $L^e$  and discounted at the rate of time preference  $\rho$ :

$$\hat{U}(0) = \int_0^{\infty} \frac{\hat{c}(t)^{1-\theta}}{1-\theta} L(t)^e e^{-\rho t} dt. \quad (1)$$

In other words, utils are discounted at an effective rate  $\rho - \varepsilon n$ ,  $0 \leq \varepsilon \leq 1$ . If  $\varepsilon = 1$ , utility is weighted by the size of the dynasty and we have a Benthamite intertemporal utility function. If  $\varepsilon = 0$  we have a Millian intertemporal utility function.

Note that, although Eq. (1) is usually considered as a social welfare function, we interpret this expression as an intertemporal utility function. In other words, our analysis is positive rather than normative. However, as markets are efficient, there is no reason for government intervention in the Ramsey version of the model with homogeneous agents. In case of heterogeneous agents, government intervention might be desirable for equity reasons. Here we abstract from this possibility. However, Calvo and Obstfeld (1988) introduced a government that maximizes a social welfare function (being the discounted sum of the lifetime utilities of all generations) in a similar type of model without population growth. They show that this effectively leads to a Ramsey economy if the government is able to use time and age dependent lump sum taxes.<sup>3</sup> Unfortunately, the correspondence of these two models breaks down if population growth is not constant.<sup>4</sup>

Denoting the connectedness of generations through interdynasty transfers by  $\lambda$ , we can formulate the budget constraint as:

$$\frac{d\hat{k}}{dt} = (r - \lambda n)\hat{k} + w - \hat{c}. \quad (2)$$

Notice that wage income per worker is identical for all generations, i.e.  $w = \hat{w}$ . If  $\lambda = 1$  transfers are given to new generations so that their wealth equals that of existing generations. In that case, the model is a Ramsey model with homogeneous agents. If  $\lambda = 0$  there is no altruism and we have the Weil (1989) model of unloved children where generations are heterogeneous.

We can define  $r - \lambda n$  as the effective interest rate, and per capita human wealth as

$$h = \int_0^{\infty} w(t) e^{-\int_0^t (r(s) - \lambda n) ds} dt. \quad (3)$$

Straightforward optimization and aggregation gives that per capita aggregate consumption is a fraction  $\phi$  of per capita total wealth,

$$c = \phi(k + h), \quad (4)$$

where the propensity to consume out of total wealth,  $\phi$ , is given by (cf. Barro and Sala-i-Martin 1995):

$$\frac{1}{\phi} = \int_0^{\infty} e^{\int_0^t [(r - \lambda n)(1 - \theta) / \theta - (\rho - \varepsilon n) / \theta] ds} dt. \quad (5)$$

The dynamic properties of the system in per capita aggregate quantities can be summarized by the following laws of motion

$$\dot{c}/c = \theta^{-1} [r - \lambda n - \rho + \varepsilon n] - (1 - \lambda) nk/(k + h) \quad (6)$$

$$\dot{k}/k = r - n + w/k - c/k \quad (7)$$

$$\dot{h}/h = r - \lambda n - w/h. \quad (8)$$

Equation (7) is the economy's resource constraint. The  $n$ -term in this equation indicates capital dilution. Equation (8) follows from differentiating (3) w.r.t. time. Equation (6) follows from differentiating (4) and (5), and substituting (7) and (8).

### 3. Steady-state and comparative statics

In the absence of technical progress, per capita quantities are constant in the steady-state:  $\dot{c}/c = \dot{k}/k = \dot{h}/h = 0$ . To be able to express the steady-state of our general model in closed form, we assume a Cobb-Douglas production function:  $f(k) = k^a$ , where  $a$  is the production elasticity of capital. (The analytical results derived below can easily be proved for a general form of the production function; see Appendix 2 and 3.) In that case we can describe the steady-state solution by:

$$\frac{k^*}{h^*} = \frac{a - \delta \kappa^*}{1 - a}; \quad k^* = \kappa^{*\frac{1}{1-a}}; \quad c^* = (1/\kappa^* - \delta - n) k^*, \quad (9)$$

where  $\kappa^*$  denotes the steady-state capital output ratio. In a Ramsey model ( $\lambda = 1$ ) we have  $\kappa^* = \frac{a}{\rho + n(1-\varepsilon) + \delta}$ ; in a model with heterogeneous agents ( $\lambda = 0, \varepsilon = 0$ ), we have  $\kappa^* = \frac{-\beta_2 - \sqrt{\beta_2^2 - 4\beta_1 a}}{2\beta_1}$  where  $\beta_1 = \delta(n\theta + \delta + \rho)$ ;  $\beta_2 = -(a+1)\delta - \rho - na\theta$ , cf. Appendix 1. (In Appendix 1 we also show the uniqueness of this solution.) We now analyze the long-run effects of demographic shocks in the three versions of the general model presented above. (Appendix 2 presents comparative statics results for the Ramsey model, Appendix 3 for the Weil model.)

Firstly, if  $\lambda = 1$  and  $\varepsilon = 1$ , i.e. transfers are given to new generations so that their wealth equals that of existing generations and utility is weighted by population size, we have a standard *Ramsey* model with a *Benthamite* utility function. Comparative statics learns that in this case  $\frac{dc^*}{dn} < 0$  and  $\frac{dk^*}{dn} = 0$ . So ageing, i.e. a decrease in population growth, increases per capita consumption in the long-run, but does not affect the capital-labour ratio. The latter result is due to the fact that the difference between the effective rate of time preference and the effective rate of return on capital, which determines savings, is not affected by ageing in this case. The long-run effect of ageing is indicated graphically in Fig. 1. Note that the  $\frac{\dot{c}}{c}$ -locus is

vertical in this case. The intersection with the  $\frac{\dot{k}}{k}$ -locus gives the steady-state for the Ramsey-Benthamite economy (point B). A decrease in  $n$  now shifts the  $\frac{\dot{k}}{k}$ -locus upward but does not affect the  $\frac{\dot{c}}{c}$ -locus, leading to a new steady-state B'.

Secondly, if  $\lambda = 1$  and  $\varepsilon = 0$ , we have the dynastic Ramsey model with a Millian utility function, i.e. utility is not weighted by population size. Notice that the effective rate of time preference in this case is larger than in a Benthamite economy. Consequently, the capital intensity is lower than in case of a Benthamite economy (see the solution for  $\kappa^*$  above). As it can easily be shown that the steady-state of the Benthamite economy is dynamically efficient, this implies that consumption will be lower too in a Millian economy. Comparative statics shows that in this case  $\frac{dc^*}{dn} < 0$  and  $\frac{dk^*}{dn} < 0$ . So, in a Ramsey-Millian economy, ageing in the long-run not only increases per capita consumption, but also leads to a higher capital intensity. The reason for this is that the effective rate of time preference is not affected by ageing in a Millian economy while the effective interest rate rises, which stimulates savings. For a graphical illustration see Fig. 2. The  $\frac{\dot{k}}{k}$ -locus is the same as in the Benthamite economy. The  $\frac{\dot{c}}{c}$ -locus is vertical again but lies to the left of that of a Benthamite economy. Point M is the initial steady-state of the Millian economy. A decline in population growth now not only shifts the  $\frac{\dot{k}}{k}$ -locus upward, but also shifts the  $\frac{\dot{c}}{c}$ -locus to the right leading to the new steady-state M'.

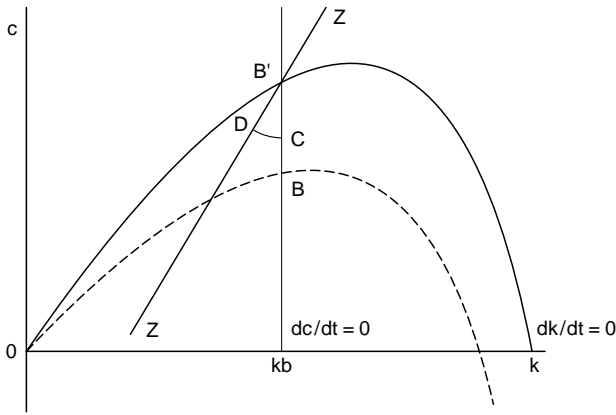
Thirdly, if  $\lambda = 0$  and  $\varepsilon = 0$ , i.e. there is no altruism and utility is not weighted by population size, we have the Weil (1989) model of unloved children. There are no transfers to new generations in this case. Consequently, the generations' wealth, and therefore their consumption levels too, will differ. In this case, the model presented above describes the evolution of average per capita variables. The  $\frac{\dot{k}}{k}$ -locus is the same as for the Ramsey models. The  $\frac{\dot{c}}{c}$ -locus is now upward sloping with a vertical asymptote. It can easily be derived from the results above that the steady-state (point W in Fig. 3) lies between the steady-states of the Ramsey-Millian and the Ramsey-Benthamite economies iff

$$\theta < \frac{1}{a} \frac{\rho + n + (1 - \alpha)\delta}{\rho + n}. \tag{10}$$

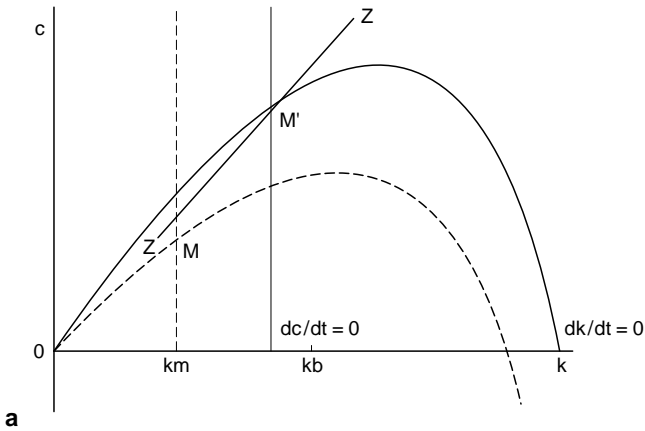
This condition holds for plausible parameter values. A decline in population growth now not only shifts the  $\frac{\dot{k}}{k}$ -locus upwards, but also shifts the  $\frac{\dot{c}}{c}$ -locus downward. Consequently the capital intensity will increase and consumption rises in the new equilibrium (W'), i.e.  $\frac{dc^*}{dn} < 0$  and  $\frac{dk^*}{dn} < 0$ . It should be noted that this variant of our model captures the most important element of overlapping-generations models, namely the absence of altruism.<sup>5</sup>

#### 4. Demographic changes

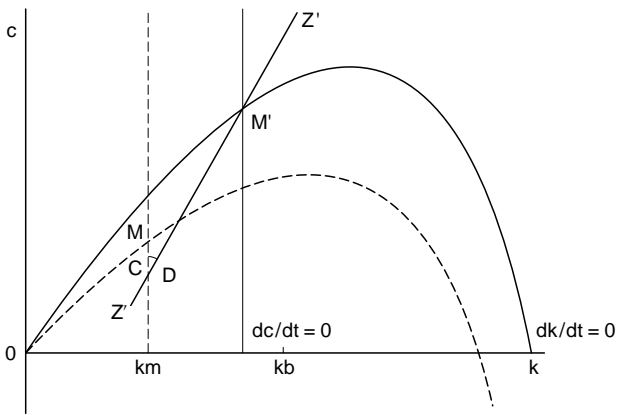
In this section we evaluate the short-run effect of demographic changes in the three different variants of the model presented above. Several techni-



**Fig. 1.** A decline in population growth in the Ramsey-Benthamite economy



**a**



**b**

**Fig. 2a, b.** A decline in population growth in the Ramsey-Millian economy with a relatively flat stable manifold. **b** A decline in population growth in the Ramsey-Millian economy with a relatively steep stable manifold



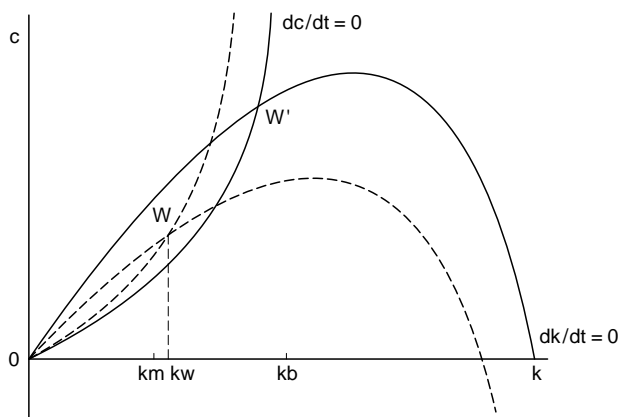


Fig. 3. A decline in population growth in the Weil economy

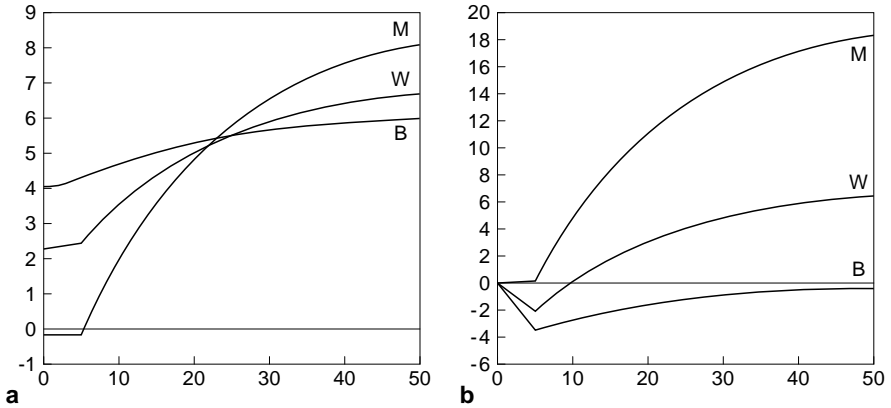
ques have been proposed in the literature to study adjustment trajectories when the economic system is hit by shocks. Mankiw (1987) and Ihoro (1990) for example, evaluate the transition dynamic adjustment path graphically. In this section we use this graphical technique to illustrate the effects of ageing (cf. Figs. 1–3). However, using this graphical analysis, nothing can be said a priori in the Ramsey-Millian economy and the Weil economy where the initial response depends on the steepness of the stable manifold. Therefore we solve for the fixed point problem analytically, using the method of comparative dynamics as described in Judd (1982). (Appendix 2 contains details of this methodology for the Ramsey model, Appendix 3 for the Weil model.)

Finally, in addition to the analytical approach (the method of comparative dynamics) and the graphical technique, we present the consequences of demographic changes by numerical simulation experiments using the method of multiple shooting (see Ascher et al. 1988). The following parameter set pertains to these experiments:  $a=0.36$ ;  $\delta=0.05$ ;  $\theta=2$ ;  $\rho=0.03$ ;  $n=0.01$ . The reason to adopt the method of multiple shooting instead of the more well-known time-elimination method for nonlinear systems by Barro and Sala-i-Martin (1995) is that this time-elimination technique cannot solve the system's dynamic response in case of anticipated perturbations.

Two demographic scenarios, both starting in the steady-state, are analyzed: an *ageing* scenario, i.e. a permanent decrease in population growth, and a *baby-boom/baby-bust* scenario, i.e. an unanticipated increase in population growth which is thought to be permanent, followed by an unanticipated return to the original low rate of population growth.

### Ageing

In case of the *Ramsey-Benthamite* economy, the steady-state for a lower rate of population growth lies vertically above the old steady-state. Therefore, if the decrease in population growth is unexpected, the economy immediately jumps to its new steady-state (from B to B' in Fig. 1). If the decrease in population growth is anticipated, consumption also jumps upward



**Fig. 4a, b.** Macroeconomic consequences of an anticipated permanent decline in population growth. **a** Consumption, **b** capital. *Note:* B, M, W denote the Ramsey-Benthamite, the Ramsey-Millian, and the Weil economy. Numbers on the vertical axis are %-deviations from the original steady-state. Time is on the horizontal axis. The impulse is a permanent 0.01 decrease in  $n$  from  $t=5$  onwards

at the announcement date. However, this jump is smaller, viz. from B to C in Fig. 1. It places the economy on an unstable arm of the system with the old parameter values. At the moment of the parameter shock ( $t=5$ ), the economy arrives at the stable arm (point D in Fig. 1) of the system with the new parameter values, leading to the new steady-state (B' in Fig. 1).

This dynamic adjustment process is shown in Fig. 4. The consumers anticipate on the future increase in consumption possibilities (enabled by the lower rate of capital dilution when population growth falls) by immediately increasing consumption. Consequently, savings fall and the capital stock starts declining until the decrease in  $n$  actually takes place. Then less saving is needed in order to compensate for capital dilution and  $k$  starts rising again, gradually approaching the steady-state level. After its initial jump, consumption also gradually grows to its new steady-state value.

In case of the *Ramsey-Millian* economy, the consequences of ageing are completely different. The economy jumps to the stable arm leading to the new steady-state (M' in Fig. 2) if the decline in population growth comes unexpected, since the economy cannot immediately jump to its new steady-state. The slope of the stable arm depends on the parameter values. We consider two possibilities, viz. a relatively flat stable arm ZZ in Fig. 2a and a relatively steep stable arm Z'Z' in Fig. 2b. In the former case, consumption initially jumps upward, whereas consumption falls in the latter case. So, in case of a Millian utility function, consumption can either rise or fall initially in response to an unanticipated negative population growth shock. Using comparative dynamics (see Judd 1982) it can be shown (cf. Appendix 2) that the initial change in  $c$  is positive iff

$$\theta > \frac{(1 - \varepsilon) c^*}{\mu k^*}, \tag{11}$$

where  $\mu$  is the positive eigenvalue of the system's Jacobian matrix. This condition is not satisfied in our numerical simulations (cf. Table A2.1 in Appendix 2), so that  $c$  will go down initially. The same condition applies if the ageing is anticipated, in which case the system follows trajectory MCDM', illustrated in Fig. 2b when the stable manifold is relatively steep.

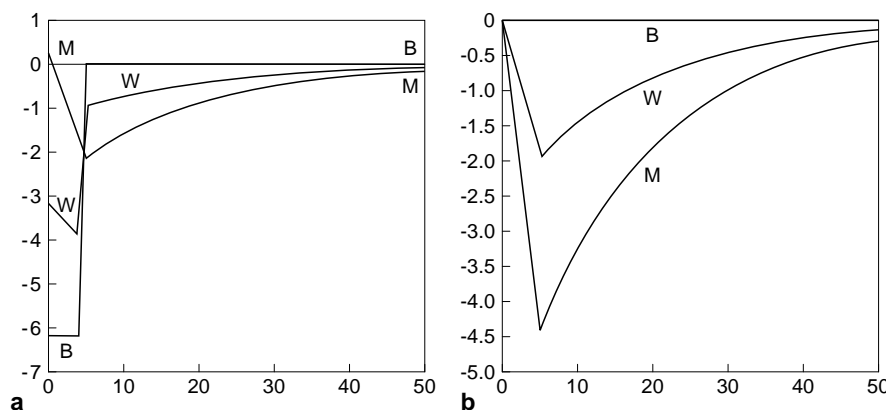
So, in our numerical simulations consumption will initially decrease in case of an anticipated decline in population growth. However, the initial change in  $c$  is smaller than in case of an unanticipated change. As in the Benthamite economy, the system moves along an unstable manifold to its new stable arm, on which it arrives at the date of the shock. Figure 4 presents the adjustment process. Consumption can be seen to stay below its old steady-state value until the time of the demographic shock after which it starts increasing. Consequently, the capital-labour ratio increases monotonically. So in this case, consumers anticipate on the rise in the effective rate of return on capital (relative to the effective rate of time preference) after the demographic shock by immediately raising savings.

The effects of ageing in the setting of the *Weil* model resemble to a certain extent the effects in a Ramsey-Millian economy. Again, an unexpected decline in population growth causes consumption to jump to the stable arm leading to the new steady-state ( $W'$  in Fig. 3). In Appendix 3 we solve the fixed point problem in the *Weil* model analytically for a logarithmic utility function. It is shown that the initial effect on consumption of a decrease in population growth is positive if  $\mu > \rho + npk^*/c^*$ , where  $\mu$  is the positive eigenvalue of the Jacobian matrix. This condition does not allow a general answer to the question whether the initial change in consumption due to a decrease in population growth is positive or negative. Consumption jumps upward in our numerical experiments. (Sensitivity analysis demonstrates that this result is quite robust with respect to parameter changes.) The same result applies if the ageing is anticipated. Figure 4 presents the adjustment process for this case. Consumption immediately jumps upward and starts to increase until the new steady-state is reached. Due to the initial increase in consumption, the capital-labour ratio goes down in the short-term before increasing monotonically to its new equilibrium level.

Hence, the results in a non-altruistic model are on average in between the opposite results for the two Ramsey models. The results for the different generations may be quite different, however. The initial increase in consumption is due to the fact that human wealth jumps upward by more than 6%. As the youngest generations do only possess human wealth, their increase in consumption will be much larger than the average increase. The wealth of older generations, on the other hand, consists for a relatively large part of financial wealth. As the value of financial capital does not change initially, this implies that their initial increase in consumption will be much lower than the average increase.

### *Baby-boom/baby-bust*

In the first variant of the model, the *Ramsey-Benthamite* economy, the steady-state capital intensity is invariant to changes in population growth. Therefore, a baby-boom/baby-bust scenario will not induce transition dynamics. If population growth unanticipatedly increases, consumption imme-



**Fig. 5 a, b.** Macroeconomic consequences of an unanticipated rise in population growth, followed by an unanticipated return to the original population growth rate. **a** Consumption, **b** capital. *Note:* B, M, W denote the Ramsey-Benthamite, the Ramsey-Millian, and the Weil economy. Numbers on the vertical axis are %-deviations from the original steady-state. Time is on the horizontal axis. The impulse is a 0.01 increase in  $n$  from  $t=0$  to  $t=5$

diately jumps to a lower level whereas the capital stock is unaffected. If the baby boom unexpectedly comes to an end, a reverse process instantaneously restores the prior equilibrium: consumption jumps upward. This dynamic adjustment process is shown in Fig. 5.

In the *Ramsey-Millian* case the impact of the baby-boom/baby-bust scenario is different. Again, Fig. 5 shows the adjustment process. A permanent increase in population growth then lowers the consumption and capital intensity in the long-run. However, when this baby-boom is not expected, consumption initially jumps upwards, which causes a fast decrease in the capital stock. As a consequence of this, consumption also starts to fall. The unanticipated return to the original population growth rate, in turn, leads to rising consumption levels again. Due to the lower level of capital dilution this can go along with a gradual increase in capital intensity, until the original value is restored in the long-run.

We finally consider the baby-boom/baby-bust scenario in the setting of the *Weil* economy. As in the *Ramsey-Millian* world, an unexpected increase in population growth induces transition dynamics, which is illustrated in Fig. 5. At the beginning of the baby-boom, consumption immediately falls downward and the capital stock starts to decline until the baby-boom unexpectedly comes to an end. At that moment, consumption jumps upward and almost reaches its original level again. From that time on, consumption and capital monotonically increase, eventually settling down in the original equilibrium.

## 5. Conclusion

There is a large and fast growing literature on the macroeconomic effects of demographic changes as e.g. ageing or the sudden ending of a baby-

**Table 1.** The effects of demographic changes on the capital stock

	Ageing	Baby-boom/baby-bust
Ramsey + Benthamite	Temporary recession	Status quo
Ramsey + Millian	Permanent boom	Temporary deep recession
Weil	Temporary recession followed by permanent boom	Temporary recession

boom. A central question in this literature is whether the optimal response to such shocks is to anticipate on the future capital abundance and to increase consumption, or to increase savings. In this paper it is shown that the optimal reaction to demographic changes strongly depends on the intertemporal utility function used as well as on the question whether agents are altruistic or not, a point that has largely been neglected in the literature so far.

Table 1 summarizes the results of our numerical simulation experiments. As shown in this table, the results of demographic shocks in case of a Ramsey model with a Millian utility function may be exactly opposite to the result when a Benthamite utility function is employed. In the latter case, for instance, ageing leads to an increase in consumption which causes a temporary decrease in the capital stock. This behaviour corresponds to the optimal reaction to ageing for the United States presented by Cutler et al. (1990). If, however, in an otherwise identical Ramsey model a Millian utility function is used, the optimal reaction to the same impulse may be to initially decrease consumption and to increase capital accumulation, leading to a permanent boom. The results in a non-altruistic model are on average in between the opposite results for the two Ramsey models.

## Appendix 1

### *Derivation of steady-state in the Weil model*

Recall that in the Weil economy we have  $\lambda = \varepsilon = 0$ . In steady-state (8) implies  $r^* = w^*/h^*$ , or (by the assumption of competitive markets)

$$ay^*/k^* - \delta = (1 - a)y^*/h^*. \quad A1 \leftarrow$$

Denoting  $k/h$  by  $\zeta$  and  $k/y$  by  $\kappa$ , one can find from A1.1 that

$$\zeta^* = \frac{a - \delta\kappa^*}{1 - a}. \quad A2 \leftarrow$$

In steady-state Eq. (6) reads as

$$\theta^{-1} [r - \rho] = nk/(k + h). \quad A3 \leftarrow$$

Or,

$$\theta^{-1} [a/\kappa^* - \delta - \rho] = n\zeta^*/(\zeta^* + 1). \quad (A4)$$

After some manipulations one gets

$$\kappa_{1,2}^* = \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 4\beta_1 a}}{2\beta_1}, \quad (A5)$$

where  $\beta_1 = \delta(n\theta + \delta + \rho)$ ;  $\beta_2 = -(a + 1)\delta - \rho - na\theta$ . To ensure that  $\zeta^* > 0$ , we require that  $\kappa^* < \kappa_{crit} = a/\delta$ . Then, from  $\kappa = k/y = k^{1-a}$ , we find

$$k^* = \kappa^{*\frac{1}{1-a}}. \quad (A6)$$

Equation 7 boils in steady-state down to

$$r^* - n + w^*/k^* - c^*/k^* = 0. \quad (A7)$$

Using that  $r = a/\kappa$  and  $w/k = (1 - a)/\kappa$ , finally gives

$$c^* = (1/\kappa - \delta - n)k^*. \quad (A8)$$

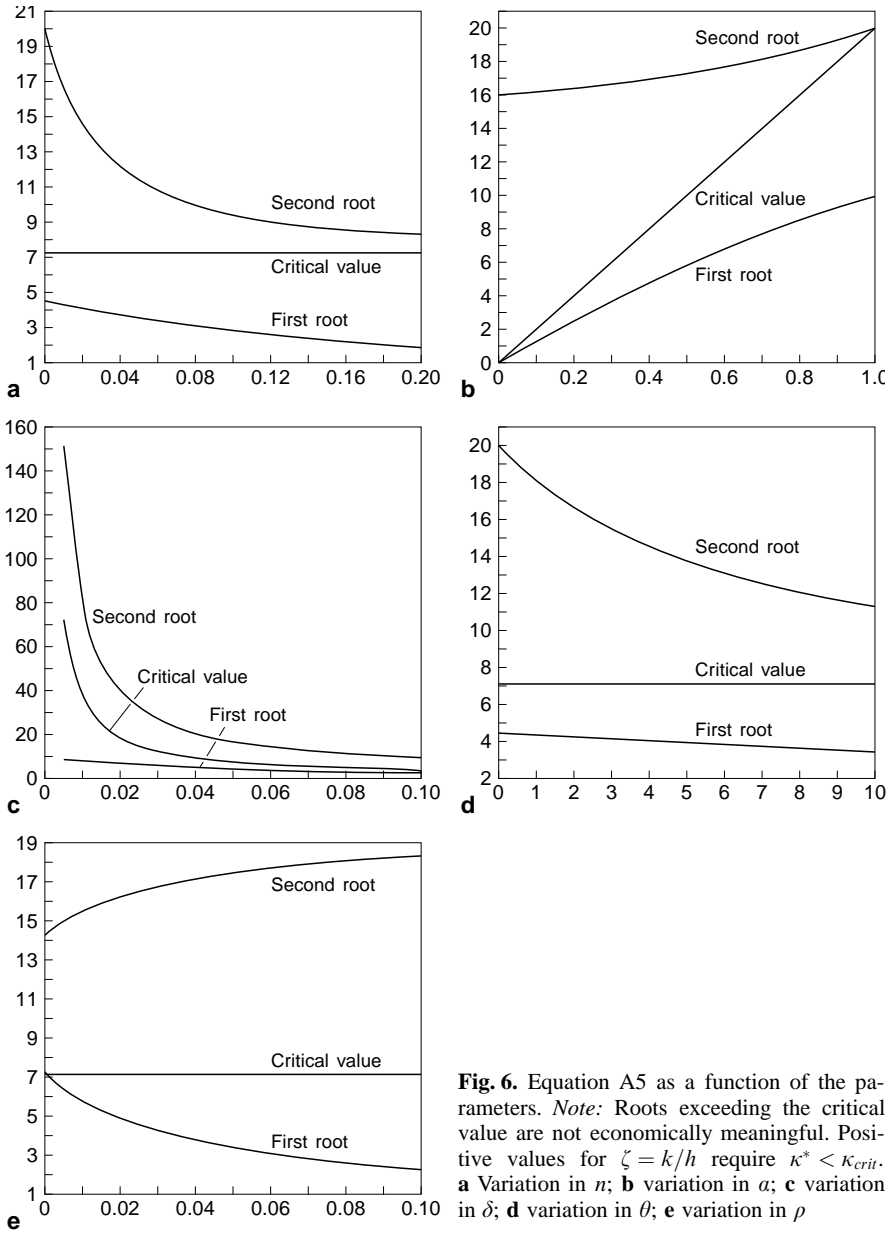
*Numerical example.* Let  $a=0.36$ ;  $\delta=0.05$ ;  $\theta=2$ ;  $\rho=0.03$ ;  $n=0.01$ . The solution is  $\kappa=4.30$  ( $< \kappa_{crit}=7.2$ );  $k^*/h^*=0.23$ ;  $k^*=9.77$ ;  $c^*=1.69$ . In Fig. 6, we show that the largest solution of (A5) exceeds the critical value for the economically plausible parameter domain. Therefore, in the text (see p. 321) we only consider the smallest root.

## Appendix 2

### *Comparative statics and comparative dynamics in the Ramsey model*

In this appendix we present an analytical solution for the long-run as well as for the short-run effects of a change in population growth in the Ramsey model (i.e.  $\lambda=1$ ) for a general utility function  $u(c)$  ( $u'(c) > 0$ ,  $u''(c) < 0$ ) and a general constant-returns-to-scale production function  $f(k)$  ( $f'(k) > 0$ ,  $f''(k) < 0$ ). The short-run solution is derived by the method of comparative dynamics (cf. Judd 1982).

Suppose that the pattern of population growth over time is given by  $n_t = n^* + \gamma z_t$ , where vector  $z_t$  denotes the time pattern of the perturbation in population growth, and scalar  $\gamma$  denotes the magnitude. We now loglinearize (6) and (7) around the steady state  $(c^*, k^*)$  by taking the derivative with respect to  $\gamma$ :



**Fig. 6.** Equation A5 as a function of the parameters. *Note:* Roots exceeding the critical value are not economically meaningful. Positive values for  $\zeta = k/h$  require  $\kappa^* < \kappa_{crit}$ . **a** Variation in  $n$ ; **b** variation in  $a$ ; **c** variation in  $\delta$ ; **d** variation in  $\theta$ ; **e** variation in  $\rho$

$$\frac{d[d \log(c)/dt]}{d\gamma} = \frac{k^* f''(k^*)}{\theta} \frac{d \log(k)}{d\gamma} - \frac{1 - \varepsilon}{\theta} z_t \tag{A9}$$

$$\frac{d[d \log(k)/dt]}{d\gamma} = -\frac{c^*}{k^*} \frac{d \log(c)}{d\gamma} + (\rho - \varepsilon n) \frac{d \log(k)}{d\gamma} - z_t \tag{A10}$$

where  $\theta = \frac{-c^* u''(c^*)}{u'(c^*)}$ . Or, in matrix notation,

**Table 2.** Critical value of the rate of risk aversion ( $\varepsilon=0$ )

$a$	$\delta$	$\rho$	$\theta_{crit}$
0.36	0.05	0.03	2.17
0.25	0.05	0.03	3.08
0.50	0.05	0.03	1.60
0.36	0.02	0.03	2.00
0.36	0.08	0.03	2.28
0.36	0.05	0.01	2.45
0.36	0.05	0.05	2.04

$$\begin{bmatrix} \frac{d [d \log (c) / dt]}{d \gamma} \\ \frac{d [d \log (k) / dt]}{d \gamma} \end{bmatrix} = J \begin{bmatrix} \frac{d \log (c)}{d \gamma} \\ \frac{d \log (k)}{d \gamma} \end{bmatrix} - \begin{bmatrix} 1 - \varepsilon \\ \theta \\ 1 \end{bmatrix} z_t,$$

$$J = \begin{bmatrix} 0 & \frac{k^* f''(k^*)}{\theta} \\ -\frac{c^*}{k^*} & \rho - \varepsilon n \end{bmatrix} \tag{A11}$$

where  $J$  is the Jacobian matrix. It can easily be checked that the determinant of this matrix is negative, that is, that the system is saddlepoint stable.

The comparative statics results can now be derived from:

$$\begin{bmatrix} \frac{d \log (c^*)}{d \gamma} \\ \frac{d \log (k^*)}{d \gamma} \end{bmatrix} = J^{-1} \begin{bmatrix} 1 - \varepsilon \\ \theta \\ 1 \end{bmatrix} z^* \tag{A12}$$

This gives the result that a decrease in population growth ( $z^* < 0$ ) leads to an increase in consumption in the long-run. The effect on the capital-labour ratio depends on  $\varepsilon$ . In case of a Ramsey-Benthamite economy ( $\varepsilon=1$ )  $k^*$  is not affected while it increases in a Ramsey-Millian economy ( $\varepsilon=0$ ).

The initial jump in consumption induced by  $\gamma$  follows from (cf. Judd 1982, Eq. (7)).

$$\frac{d \log (c_0)}{d \gamma} = \left[ \frac{1 - \varepsilon}{\theta} - \frac{\mu k^*}{c^*} \right] \tilde{z}_t(\mu), \quad \tilde{z}_t(\mu) = \int_0^\infty z_t e^{-\mu t} dt. \tag{A13}$$

where  $\mu$  is the positive eigenvalue of the Jacobian matrix and  $\tilde{z}_t(\mu)$  is the Laplace transform of  $z_t$ .

It follows that the initial effect of a decrease in population growth ( $\tilde{z}_t < 0$ ) is positive if  $\theta > \frac{(1-\varepsilon)c^*}{\mu k^*}$ . This condition does not always have to hold. It holds, however, if  $\varepsilon = 1$ . That is, in case of a Ramsey-Benthamite economy, ageing increases consumption in the short-run. In a Ramsey-Millian economy ( $\varepsilon = 0$ ), the initial effect on consumption depends on the



rate of risk aversion. Consider the simple case of a Cobb-Douglas production function  $f(k) = k^a$  and CRRA utility. Table 2 presents some numbers on  $\theta_{crit}$ : the critical value of the rate of risk aversion. Consumption will initially increase in response to a negative population growth shock when the actual value of  $\theta$  exceeds the critical value (consumption smoothing becomes more important for higher values of  $\theta$ ).

### Appendix 3

#### *Comparative statics and comparative dynamics in the Weil model*

In this appendix we present an analytical solution for the long-run as well as for the short-run effects of a change in population growth in the Weil-model (i.e.  $\lambda = \varepsilon = 0$ ) for a logarithmic utility function  $u(c) = \log(c)$  and a general constant-returns-to-scale production function  $f(k)$  ( $f'(k) > 0$ ,  $f''(k) < 0$ ). The short-run solution is derived by the method of comparative dynamics (cf. Judd 1982).

Suppose that the pattern of population growth over time is given by  $n_t = n^* + \gamma z_t$ , where vector  $z_t$  denotes the time pattern of the perturbation in population growth, and scalar  $\gamma$  denotes the magnitude. We now substitute (4) in (6) with  $\phi = \rho$  for  $\theta = 1$  and loglinearize (6) and (7) around the steady state  $(c^*, k^*)$  by taking the derivative with respect to  $\gamma$ :

$$\frac{d[d \log(c)/dt]}{d\gamma} = \frac{n \rho k^*}{c^*} \frac{d \log(c)}{d\gamma} + \left[ k^* f''(k^*) - \frac{n \rho k^*}{c^*} \right] \cdot \frac{d \log(k)}{d\gamma} - \frac{\rho k^*}{c^*} z_t \tag{A14}$$

$$\frac{d[d \log(k)/dt]}{d\gamma} = -\frac{c^*}{k^*} \frac{d \log(c)}{d\gamma} + \left[ f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} \right] \cdot \frac{d \log(k)}{d\gamma} - z_t. \tag{A15}$$

Or, in matrix notation,

$$\begin{bmatrix} \frac{d[d \log(c)/dt]}{d\gamma} \\ \frac{d[d \log(k)/dt]}{d\gamma} \end{bmatrix} = J \begin{bmatrix} \frac{d \log(c)}{d\gamma} \\ \frac{d \log(k)}{d\gamma} \end{bmatrix} - \begin{bmatrix} \frac{\rho k^*}{c^*} \\ 1 \end{bmatrix} z_t,$$

$$J = \begin{bmatrix} \frac{n \rho k^*}{c^*} & k^* f''(k^*) - \frac{n \rho k^*}{c^*} \\ -\frac{c^*}{k^*} & f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} \end{bmatrix} \tag{A16}$$

where  $J$  is the Jacobian matrix. It can easily be checked that the determinant of this matrix is negative, that is, that the system is saddlepoint stable.

The comparative statics results can now be derived from:

$$\begin{bmatrix} \frac{d \log(c^*)}{d\gamma} \\ \frac{d \log(k^*)}{d\gamma} \end{bmatrix} = J^{-1} \begin{bmatrix} \rho k^* \\ c^* \\ 1 \end{bmatrix} z^* \quad (17)$$

This gives the result that a decrease in population growth ( $z^* < 0$ ) increases consumption as well as the capital-labour ratio in the long-run. (It should be noted that  $f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} = r^* - n > 0$ .) The initial jump in consumption induced by  $\gamma$  follows from (cf. Judd 1982, Eq. (7))

$$\frac{d \log(c_0)}{d\gamma} = \left[ \frac{\rho k^*}{c^*} - \frac{\mu - n \rho k^* / c^*}{c^* / k^*} \right] \tilde{z}_t(\mu), \quad \tilde{z}_t(\mu) = \int_0^\infty z_t e^{-\mu t} dt, \quad (18)$$

where  $\mu$  is the positive eigenvalue of the Jacobian matrix and  $\tilde{z}_t(\mu)$  is the Laplace transform of  $z_t$ . So the initial effect of a decrease in population growth ( $\tilde{z}_t < 0$ ) is positive if  $\mu > \rho + n \rho k^* / c^*$ . This condition does not allow a general answer to the question whether the initial change in  $c$  due to a decrease in population growth is positive or negative.

## Endnotes

- <sup>1</sup> For an ethical discussion on this topic, we refer to Sidgwick (1874) and Sumner (1978).
- <sup>2</sup> The fourth possibility, the combination of non-altruistic agents and Benthamite utility, i.e. weighting future utility by population size, is logically inconsistent.
- <sup>3</sup> In the model used by Calvo and Obstfeld, the size of the population is constant. The model can easily be extended so as to allow for constant population growth. In that case, depending on whether the generations' lifetime utility is weighted by their size or not, we get the results for a Ramsey-Benthamite or a Ramsey-Millian economy.
- <sup>4</sup> Meijdam and Verbon (1997) analyze the effects of population growth shocks in a discrete time overlapping-generations model with a government maximizing the discounted sum of lifetime utilities.
- <sup>5</sup> In discrete time overlapping-generations models, like the well-known Diamond model (Diamond 1965), the steady-state may be dynamically inefficient so that an increase in capital intensity may, *ceteris paribus*, lead to a fall in consumption. In our model, however, this is not possible. But dynamic inefficiency can easily be allowed for by assuming that labour productivity declines with age (see Blanchard and Fischer 1989, p. 119–120).

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