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**PAYING FOR THE AGEING CRISIS:  
WHO, HOW AND WHEN?**

By

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# Paying for the Ageing Crisis: Who, How and When?

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## Abstract

In many countries population ageing creates an implicit public debt. That is, if policies remain unchanged, the public debt will ultimately become unsustainable. This paper explores the optimal way to achieve debt sustainability. In particular, it asks *when* policy reforms should be made, *how* policies should be changed and *which* generations should make which contributions. As regards timing, we find that policy reform should anticipate future demographic change. As regards policy instruments, we find that optimal policy reform features changes in all available instruments. This implies less consumption of all types of goods; only pure public goods consumption may escape a reduction. The labour supply functions of the young and the old determine the allocation over policy instruments. In particular, the more elastic is the labour supply of the young, the smaller should be the increase in the tax rate on labour income; the more elastic is the labour supply of the old, the larger should be the reduction in transfers to the elderly. As regards generations, we find that the old share relatively little in the fiscal burden; future generations share more or less than the young, depending on future population size. In addition, we find that the change of the public debt is not a given, but a feature of optimal policies. In general,

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optimal policy reform reduces the public debt in anticipation of population ageing, but in a particular case the opposite holds true.

**Keywords:**

Population ageing, Tax smoothing, Public debt

**JEL classification numbers:**

H21, H40, H60

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# 1 Introduction

Population ageing is known to have far-reaching budgetary implications. In industrialized countries populations will become dramatically older in the decades to come: even now, the number of elderly people is projected to more or less double relative to the number of people of working age. Since a large part of the institutions for social security and health care is PAYG-financed, this creates a huge implicit debt that in many cases it not matched by an equally-sized negative statutory debt. Indeed, in many countries the implicit debt and the statutory debt are both positive, implying that without any policy reform, fiscal policies must be judged fiscally unsustainable.

There is a large literature that quantifies the budgetary consequences of ageing. In the early nineties, Blanchard *et al.* (1990) were the first to do so for a large number of industrialized countries. Auerbach *et al.* (1999) did a similar exercise, but more explicitly from the angle of generational accounting. Since then, a number of updates and extensions have been made, both for individual countries (*e.g.* Cardarelli *et al.* (2000), Faruqee and Mühleisen (2001), Congressional Budget Office (2003), Auerbach *et al.* (2004), Van Ewijk *et al.* (2006), Smid *et al.* (2014)) and for groups of countries (Gokhale and Raffelhüschen (2000), Dang *et al.* (2001), Balassone *et al.* (2009), European Commission (2012)). In addition, Börsch-Supan *et al.* (2006) and Fehr *et al.* (2008) consider ageing in different regions in the world and their interactions. Andersen and Pedersen (2006) focus on the impact of economic growth upon fiscal sustainability. Furthermore, some studies allow for a positive impact of the public debt upon the primary balance (Bohn (1998, 2008), Medeiros (2012) and Lukkezen and Rojas-Romagosa (2016)), although one may argue that it is more appropriate to abstract from any future endogenous policy change.

What characterizes the literature is its focus on future projections. These projections imply values for the implicit public debt or, after a translation into annuity values, for the sustainability gap.<sup>1</sup> As stressed by Blanchard *et al.* (1990), sustainability gap is a technical measure without any policy content. Indeed, one can think of a variety of policy reforms that all achieve fiscal sustainability, but are very different in terms of policies, generations affected and the time of policy reform.

This paper starts where the literature on fiscal sustainability ends. It characterizes optimal policy reform as a response to news about future pop-

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<sup>1</sup>The literature also uses tax gap to refer to the annuity value of the public debt. We prefer to use the term sustainability gap in order to signal that there many ways to achieve fiscal sustainability.

ulation ageing. In particular, it analyzes three questions: *when* should the ageing bill be paid, now or in the future? *who* should pay, current working generations, current retired generations or future generations? and *how* should the bill be paid, by tax increases or reductions of transfers, rival public goods or pure public goods? The paper thus focuses explicitly on the optimal time profile of different types of policies and on how different types of policies connect to each other. As far as I know, this paper is the first that characterizes the optimal combination of different types of government policies (*i.e.* tax policies, transfer policies, rival public goods policies and pure public goods policies) and that elaborates the implications for the public debt and the optimal balance between generations.

To be sure, this is not the first paper that focuses on policy reform. Several studies focus on social security reform (*e.g.*, Kotlikoff *et al.* (1999), De Nardi *et al.* (1999), Beetsma *et al.* (2003)), pension reform (Bonenkamp *et al.* (2017)) and health care reform (Roig (2006)). Kotlikoff *i.e.* (2007) and Balassone *et al.* (2009) compare a pre-funding strategy with more gradual fiscal adjustment. Kitao (2014), Kitao (2015) and Bloom *et al.* (2015) compare different types of government policies. None of these studies digs into the question what are the features and effects of optimal policy reform, however. Neither is this paper the first that addresses the question whether optimal policies feature smoothing. Indeed, Barro (1979), Aschauer (1988), Chari *et al.* (1994), Andersen and Dogonowski (2004) and Werning (2007) analyse the issue extensively. This literature does not extend the concept of tax smoothing to other types of government policies however as occurs in the present paper.

The model that this paper constructs is parsimonious. The model is deterministic, contains two overlapping generations of households and describes a small open economy (factor prices are given). Households consume what they earn, but do make a labour supply decision. The government in the model produces rival and pure public consumption goods, spends on transfers to the elderly and levies taxes on the labour income of the youngsters. We distinguish between two types of population ageing. In the longevity boost scenarios the old generation grows in size, whereas in the fertility bust scenarios the young cohort shrinks in size. The two types of scenarios have in common an increase in the old-age dependency ratio (the size of the old cohort relative to that of the young cohort), but differ with respect to population size. We also explore a combined scenario in which the old-age dependency ratio increases, but the population does not change size.

The paper draws three sets of conclusions. First, the government should

act immediately when information about future population ageing arrives. As the ageing of the population can often be anticipated decades ahead, this means that - as we will show - in many cases a policy of debt reduction should be started long before the demographic change will set in. In general, policy reform covers all instruments of economic policies. As we will see, the emergence of an implicit debt in general requires the government to raise tax rates and economize on the spending on rival goods and on transfers to the elderly. Spending on pure public goods should also be considered accordingly but here it is not obvious a priori whether spending should be decreased or increased (or remain unchanged). The effect of the demographic shock in question on population size is of crucial importance in this respect and it is here that the effects of a longevity boost and of a fertility bust will be different. Furthermore, the labour market behaviour of the young and the old is relevant. The paper shows that the more elastic is the labour supply of the young, the less policy reform should rely on increasing tax rates. Similarly, the more elastic the labour supply of the old, the more policies should economize on transfers to the elderly.

Our second set of results regards the balance between generations. The paper finds that all generations will be hurt by the policy reforms that are due to population ageing, but that the generation that is old at the time the information about population ageing arrives will bear a smaller share of the burden. This generation escapes completely the effect of a higher tax burden and partly the effect of a cut in rival public consumption. The relation between the generation who is young at the time of policy reform and future generations depends on the type of demographic shock. If the population expands, future generations benefit more than the young generation from the increase in public goods consumption over time that is due to the demographic shock. Similarly, if the population shrinks, future generations are hurt more than the generation who is young at the time of the policy reform.

Thirdly, we find that in general policy reform reduces the public debt in anticipation of future population ageing. Only in case ageing takes the form of a fertility bust and the consumption of pure public goods is relatively large, the opposite effect may occur: public debt can then be increased in anticipation of higher primary balances which are due to cuts in future pure public consumption. Although this case may be less relevant empirically, we cannot exclude it a priori. This case is also a nice illustration of our finding that the amount of public debt at the end of the planning horizon will in general be different for different types of demographic shocks. Indeed, different scenarios have in common that the public debt ultimately stabilizes,

but not the level at which this occurs. In this respect, our approach differs from the one in Dang *et al.* (2001), in which the public debt to GDP ratio is set to achieve a certain level at some point in time.

The following caveats are in order. Our analysis assumes a small open economy. It is therefore more applicable to one particular country that is hit by an idiosyncratic demographic shock than to a large group of countries that are hit by a common demographic shock and that respond (or anticipate) in a coordinated way. What pleads for the assumption of a small open economy is, first, that demographic projections for different countries differ in the timing and the intensity of population ageing and, second, that the policies investigated here belong to the domain of national authorities. We will discuss the implications of this assumption below.

In addition, the paper assumes that demographic processes are exogenous to economic policies. This assumption is questionable, in particular in the long run. Empirical evidence indicates that the provision of public pensions may reduce fertility (*e.g.*, Cigno *et al.* (2003a) and Zhang and Zhang (2004)) and that transfer policies may have an impact upon fertility (Cigno *et al.* (2003b)). Similarly, empirical evidence indicates that public policies (sewerage, vaccination against infectious diseases, medical-technological innovation) have attributed to the increase in life expectancy during the last few decades (Cutler *et al.* (2006)). Salm (2011) finds that mortality is causally related to pension income. The exogeneity assumption thus biases the impact that policy reform may have on fiscal sustainability. We doubt that these considerations would undermine our proposition that population ageing threatens fiscal sustainability, however.

Furthermore, although our stylized model allows us to study economic policies on a fundamental level, it is less adequate for studying the effects of concrete policy reforms. Indeed, for such kind of policy analysis one would benefit from using models with more generations and a more detailed modelling of policy institutions, like in Kotlikoff *et al.* (1999), De Nardi *et al.* (1999), Beetsma *et al.* (2003) or Fehr *et al.* (2013).

The structure of the rest of the paper is as follows. Section 2 sets the stage by providing information about population ageing. Section 3 constructs our model and section 4 uses it to explore the properties of optimal government policies. Section 5 then defines three stylized demographic shocks and section 6 explores how they change optimal policies. Correspondingly, sections 7 and 8 focus on the implications of these policies for the generational balance and the development of the public debt over time. Section 9 derives the contribution that each of the four policy instruments makes to the closure of the sustainability gap. Finally, section 10 analyzes



extended demographic shocks that generalize the stylized shocks studied earlier, while section 11 ends with concluding remarks.

## 2 The demographic transition

The global features of population ageing are quite well-known. Here, we briefly review the aspects that are most relevant for the analysis in this paper. More concretely, we focus on the universality of demographic trends and their implications for both the future age structure of the population and its size. We restrict the discussion to 42 countries, consisting of the OECD countries and the eight major non-OECD countries (Argentina, Brazil, China, India, Indonesia, Russia, Saudi Arabia and South Africa) on which both the UN and the OECD report (OECD, 2015 and UN, 2015).

### 2.1 History

As regards fertility, OECD (2015) documents the development of the total fertility rate in OECD countries between 1960 and 2013. The OECD average dropped from 3.18 to 1.67. The total fertility rate decreased as well in all of the individual OECD countries in this 53 years period. The same holds true for the non-OECD countries listed above. However, the decline has lost momentum in the last decade or so. In particular, between 2000 and 2013 the OECD average stabilized at the level of 1.67, whereas in countries such as France, Germany and the United Kingdom the total fertility rate showed a (slight) increase.

Developments in life expectancy share with those in fertility that they are structural and apply to many countries in the world. UN (2015) reports the development of life expectancy at birth between 1990-1995 and 2010-2015. The figure for the world average increased about 10 percent in 20 years time, from 64.5 in 1990-1995 to 70.5 in 2010-2015. For 41 out of the 42 OECD and non-OECD countries listed above life expectancy at birth increased in the 20 years period. Only South Africa, in which country life expectancy was already very low in 1990-1995, witnessed a further decline in the following 20 years.

The result of these two demographic forces is a strong increase in old-age dependency ratios. OECD (2015) reports that the OECD weighted average has increased from a level of 13.9 in 1950 to a level of 27.3 in 2015 (the figures for the unweighted average are similar). The same result applies to individual OECD countries: in all of them the old-age dependency ratio increased over this period, although the magnitudes vary quite a lot. Except

for Saudi Arabia, where the old-age dependency ratio is extremely low, the non-OECD countries listed above show the same picture.

The effect of the combination of a drop in fertility and increasing longevity on population growth is ambiguous. Inspection of the figures on population growth learns that populations grew in all of the countries listed above in the 1950-2015 period (UN, 2015). This does not necessarily say that the trend of increasing longevity was the dominant one, for it takes decades before a drop in fertility becomes visible in figures on population growth.

## 2.2 Future

As regards future fertility, the OECD average total fertility rate is expected to increase slightly to 1.85 in 2060. The trend of structural decline that occurred in the last five decades or so is thus expected not to continue. Still, the projected value of 1.85 is below replacement level. Hence, if mortality rates would remain unchanged (and net migration would be zero), the average population would ultimately start to decline.

As regards life expectancy, projections are more in line with historical trends. The world average of life expectancy at birth, which increased from 64.5 in 1990-1995 to 70.5 in 2010-2015, is projected to increase further to 77.1 in 2045-2050 and to 83.2 in 2095-2100. Again, the trend is universal. It applies to all the individual countries listed above, irrespective whether the projection ends in 2045-2050 or in 2095-2100.

The OECD projects the OECD weighted average old-age dependency ratio to almost double in the period 2015-2075, from 27.3 to 54.5 (again, the figures for the unweighted average are similar). Without exception, the old-age dependency ratios are also projected to increase in the 42 countries listed above, although the magnitudes are different. For example, the projected old-age dependency ratio in China increases more than 250 percent, whereas that in Germany increases less than 90 percent.

Interestingly, the picture of future population growth is very different. UN (2015) projects continued growth of the population at the world level, but not in all individual countries. Populations are expected to grow in many countries, especially in the 2015-2050 period. Among them are many European countries, the UK, the US and Indonesia. The highest growth is expected to occur in India, in which country the population is expected to increase from 1311 million people in 2015 to 1659 people in 2100. But there are also many countries that will face a population decline, especially in the 2050-2100 period. Among them again many European countries. More important (in terms of numbers of people) are Japan where the population

is projected to decline from 126 million people in 2015 to 83 million people in 2100, South Korea (from 50 to 38 million people) and Russia (from 143 to 117 million people). The highest number by far applies to China however, in which country the population is expected to decline by almost 400 million people between 2015 and 2100 (from 1376 to 1004 million people).

Summarizing, we see that future projections of the population structure are very different from those of population size. Populations are projected to become older in all of the major industrialized countries. However, depending on the strength of the movements in fertility and mortality, populations are projected to either grow or shrink.

### **3 A model of the ageing problem**

We use two periods to describe the life cycle of households. This is a great simplification, but not uncommon and obvious in terms of our wish to analyze policy reform on a global level. Further, we will assume that both the young and the old generation allocate their time over labour supply and leisure (although not necessarily in equal amounts). This deviates from the assumption commonly made in 2-cohort overlapping-generation models that the young generation works and the old generation is retired. For this paper, such an assumption would be too simple as we want to be able to analyze the labour market effect of an increase of the retirement age.

Another common assumption is that households can save or dissave on capital markets. We assume that only the government can do so and households must consume their disposable income, however. This assumption simplifies the analysis. Moreover, to assume that there is no private saving may be less strange as may seem at first sight. Given that many industrialized countries have public programmes that provide lifelong public pensions, households may face little incentive to change their savings upon a shock in life expectancy or fertility. Below, we will explore the consequences of this assumption in detail.

The two-period setup seems sufficient to model the key features of an ageing society. Indeed, one may interpret the first period of the life cycle as the period in between the age of 20 and the age of 50 and the second period of the life cycle as the period in between the ages of 50 and 80, which corresponds roughly to current economic reality.

Our model further assumes that the government supplies public goods from which both generations benefit, that it makes transfers to the old generation and that it levies taxes on the labour income of the young generation.

Here, transfers include income transfers and transfers in kind like health care and long-term care services. This assumption accords with economic reality in which the bulk of transfers is made to the elderly (that we allocate all transfers to the old generation is obviously an exaggeration) and in which the bulk of taxes is paid by the youngsters (that we allocate all taxes to the youngsters is again an exaggeration). The model also accords with the literature on generational accounting which points out that net transfers from the public sector to the private sector are generally negative in the young stage of life and positive in the old stage in life.

Importantly, we distinguish two types of public goods. Pure public goods conform to the standard Samuelson (1954) definition: they are non-rival and non-excludable. Rival public goods comprise private goods that are publicly provided and public goods that are subject to congestion. As argued in Barro (1990), many public goods such as highways, courts, water and sewer systems, national defense and police, may be subject to congestion: for these goods, population growth diminishes the quality of a given amount of goods.

The model only features a labour income tax. One may ask why the model does not include a consumption tax. In reality, both consumption taxes and taxes on labour income generate a substantial part of total tax revenues. In the model, the two taxes would be rather similar however: apart from generating tax revenues, both would distort the labour-leisure decisions of young and old households. Hence, there is no reason to include both. Obviously, distinguishing between a labour income tax and a consumption tax would be interesting in order to explore intragenerational heterogeneity. However, this paper abstracts from this element. Finally, we can use a symmetry argument to motivate our choice for a labour income tax rather than a consumption tax: transfers in the model flow only to the elderly and taxes are levied only on the youngsters.

### 3.1 Households

The intertemporal utility function of households, denoted  $u$ , consists of four sub-utility functions: one for a composite of private consumption and labour supply when young ( $ucl^y$ ), one for a composite of private consumption and labour supply when old ( $ucl^o$ ), one for rival public consumption ( $ub^r$ ) and one for pure public consumption ( $ub^u$ ). Intertemporal utility for the house-

hold who is born in period  $t$  thus reads as follows:

$$\begin{aligned}
u_t &= ucl^y(c_t^y, l_t^y) + \frac{\zeta_{t+1}}{(1+r)} ucl^o(c_{t+1}^o, l_{t+1}^o) \\
&+ \left[ ub^r(b_t^r) + \frac{\zeta_{t+1}}{(1+r)} ub^r(b_{t+1}^r) \right] \\
&+ \left[ ub^u(b_t^u) + \frac{\zeta_{t+1}}{(1+r)} ub^u(b_{t+1}^u) \right] \tag{1}
\end{aligned}$$

Here,  $c^y$ ,  $c^o$ ,  $l^y$  and  $l^o$  are used to denote private consumption during the first and second period of the life cycle and labour supply during the first and second period of the life cycle respectively.  $b^r$  denotes the volume of rival public goods,  $b^u$  the volume of pure public goods. Both types of goods benefit generations in both stages of their life cycle. As to the (sub-)utility functions, we assume the following. The marginal utilities of the three types of consumption are positive and decreasing. The marginal utilities attached to labour supply are negative. We do not impose that these marginal utilities are decreasing, although we assume that they allow the calculation of a meaningful solution for the household labour-leisure problems. Furthermore, preferences for private consumption goods and labour supply are separable.<sup>2</sup> Finally,  $\zeta_{t+1}$  denotes the survival rate of the cohort that is born in period  $t$  and  $r > 0$  denotes the interest rate which we will take as constant.

We do not distinguish between the individual discount rate and the interest rate. This assumption is quite innocent as we abstract from any private savings in the model. In addition, one could argue that it is appropriate to abstract from private savings only if the individual discount rate and the interest rate are sufficiently close to each other. Our assumption that the two are equal takes this to its extreme.

As mentioned above, we abstract from private savings. Hence, first-period consumption equals after-tax labour income and second-period consumption the sum of labour income and transfers  $p$ ,

$$c_t^y = w_t(1 - \tau_t)l_t^y \tag{2}$$

$$c_t^o = w_t l_t^o + p_t \tag{3}$$

where  $w$  denotes the wage rate and  $\tau$  the rate of labour income taxation.

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<sup>2</sup>Formally,  $\omega_1 > 0$ ,  $\omega_2 < 0$ ,  $\omega_3 < 0$ ,  $\omega_4 = 0$  where  $\omega_1 \in \{\partial ucl^y/\partial c^y, \partial ucl^o/\partial c^o, ub^{r'}, ub^{u'}\}$ ,  $\omega_2 \in \{\partial ucl^y/\partial l^y, \partial ucl^o/\partial l^o\}$ ,  $\omega_3 \in \{\partial^2 ucl^y/(\partial c^y)^2, \partial^2 ucl^o/(\partial c^o)^2, ub^{r''}, ub^{u''}\}$  and  $\omega_4 \in \{\partial^2 ucl^y/\partial c^y \partial l^y, \partial^2 ucl^o/\partial c^o \partial l^o\}$ .

Equations (2) and (3) reflect that transfers are distributed only to the elderly and, similarly, that taxes are paid only by the youngsters. Further, they indicate that we assume that wages earned by the young and the old are the same. This is an innocent simplification, as it would be quite straightforward to generalize the model in this respect. Furthermore, we will assume the wage rate grows at a constant rate  $g \geq 0$  to allow for productivity growth.

Our favorite interpretation of transfers to the elderly is that of income transfers. These income transfers will in general have an effect upon labour supply which our analysis will take into account. For example, a reduction of transfers that results from an increase of the pension eligibility age may increase to some extent the labour supply of old workers. An alternative interpretation would be transfers in kind, but then it would be more difficult to claim that a change in transfers may have labour supply effects.

Both young and old households choose how to divide their time between work and leisure. They thus choose their consumption and labour supply by solving their own maximization problem. For the youngsters, the problem is to maximize  $ucl^y(c_t^y, l_t^y)$ , subject to equation (2). Assuming that the interior solution applies, this implies the following condition:

$$\left(\frac{\partial ucl^y}{\partial c^y}\right)_t w_t(1 - \tau_t) + \left(\frac{\partial ucl^y}{\partial l^y}\right)_t = 0 \quad (4)$$

Similarly, the elderly choose their consumption and labour supply to maximize  $ucl^o(c_t^o, l_t^o)$ , subject to equation (3). Assuming again that the interior solution applies, this implies that

$$\left(\frac{\partial ucl^o}{\partial c^o}\right)_t w_t + \left(\frac{\partial ucl^o}{\partial l^o}\right)_t = 0 \quad (5)$$

### 3.2 The government

The government is assumed to maximize a social welfare function. Social welfare, denoted as  $W$ , adds up the utility functions of all generations involved. Using  $n_t^y$  to denote the size of the generation born in period  $t$  and  $\Delta > 0$  to denote the social discount rate, we have the following:

$$W_t = \sum_{i=t-1}^{\infty} \frac{n_i^y u_i}{(1 + \Delta)^{i-t}} \quad (6)$$

Equation (6) expresses that in determining optimal government policies, the government takes into account the interests of all current and future generations, including those of the currently old.

The maximization problem is subject to the intertemporal government budget constraint. To derive this constraint, we write down the debt accumulation equation. This equation describes how the public debt develops over time given the time pattern of taxes and expenditures. Let us denote the public debt at the end of period  $t-1$  as  $D_{t-1}$ . Further, let us denote expenditure on rival public goods, expenditure on pure public goods, expenditure on transfers and tax revenues during period  $t$  as  $B_t^r$ ,  $B_t^u$ ,  $P_t$  and  $T_t$  respectively. As a convention, we will assume that income flows occur at the beginning of the period to which they refer. The debt accumulation equation can thus be expressed as follows:

$$D_t = (D_{t-1} + B_t^r + B_t^u + P_t - T_t)(1 + r) \quad (7)$$

We impose a solvency condition to the public sector. Technically speaking, the government is required to have eliminated the public debt in present-value terms at the end of her planning horizon:

$$\lim_{N \rightarrow \infty} \frac{D_N}{(1 + r)^N} = 0 \quad (8)$$

Combining the debt accumulation equation (7) and the solvency condition (8), we arrive at the intertemporal government budget constraint:

$$\sum_{i=t}^{\infty} \frac{(T_i - B_i^r - B_i^u - P_i)}{(1 + r)^{i-t}} - D_{t-1} = 0 \quad (9)$$

According to this constraint, the public debt at the beginning of the planning horizon should be redeemed by primary budget surpluses, either in the near or in the distant future.

For future purpose, it is useful to define the implicit public debt. This equals the negative of the first term in equation (9). Using  $G$  to denote the implicit public debt, the formal expression is as follows:

$$G_{t-1} = \sum_{i=t}^{\infty} \frac{(B_i^r + B_i^u + P_i - T_i)}{(1 + r)^{i-t}} \quad (10)$$

The intertemporal government budget constraint in equation (9) thus states that the sum of the explicit and implicit public debt is equal to zero.

For tax revenues, expenditure on transfers, on pure public goods, and on rival public goods, we have the following definitional equations:

$$T_t = \tau_t w_t l_t^y n_t^y \quad (11)$$

$$P_t = p_t n_t^o \quad (12)$$

$$B_t^u = b_t^u \quad (13)$$

$$B_t^r = b_t^r n_t \quad (14)$$

where  $n_t^o$  denotes the size of the old generation and  $n_t \equiv n_t^y + n_t^o$  that of the total population in period  $t$ .

Tax revenues are proportional with the young population (equation (11)) and spending on transfers is proportional with the elderly population (equation (12)). Spending on pure public goods is not related to population size, reflecting the nonrivalrous nature of these goods (equation (13)). Finally, spending on rival public goods relates to the total population (equation (14)).

Throughout the paper, we adopt a normative approach to government policies. We do not regard this approach as superior to the positive approach adopted by, for example, Razin *et al.* (2002). The reason for adopting the normative approach merely relates to the question that this paper addresses: what would be the optimal policy response to news about population ageing? If we were to investigate the question whether it is likely that society will reform optimal policies in the way that our analysis indicates, adopting a positive approach would seem more obvious. To address this question is well beyond the scope of our paper, however.

We can now proceed to define the problem of the government. This is to maximize social welfare, equation (6), subject to the intertemporal government budget constraint, equation (9), using as instruments  $b_i^r$ ,  $b_i^u$ ,  $p_i$  and  $\tau_i$   $i \in [t, \infty)$ . More concisely, the government maximizes the following Lagrangian,

$$\mathcal{L}_t = W_t + \lambda \left[ \sum_{i=t}^{\infty} \frac{(T_i - B_i^r - B_i^u - P_i)}{(1+r)^{i-t}} - D_{t-1} \right] \quad (15)$$

where  $\lambda$  is the Lagrange multiplier.

Below, we will elaborate optimal government policies. In order to close the model, we now describe how the demographic structure of the model evolves through time.

### 3.3 The demographic structure

The demographic structure in this two-period model is fully determined by the structure in the previous period and the fertility and survival rate. More precisely, the size of the young cohort in period  $t$ ,  $n_t^y$ , is determined by the



size of the predeceasing cohort and the fertility rate  $f_t$ :  $n_t^y = f_t n_{t-1}^y$ . The size of the old cohort,  $n_t^o$ , follows from multiplying the size of the same cohort one period earlier with the survival rate  $\zeta_t$ :  $n_t^o = \zeta_t n_{t-1}^o$ . The old-age dependency ratio, defined as  $n_t^o/n_t^y$ , therefore equals the ratio between the survival rate and the fertility rate:  $\zeta_t/f_t$ .

## 4 Optimal policies

This section does two things. First, it explores the general properties of the four types of policies as distinguished in this paper. Second, it makes two simplifying assumptions.

### 4.1 Four flavours of (non-)smoothing policies

We explore the properties of four types of policies: tax policies, transfer policies, rival public goods policies and pure public goods policies. We start with rival public goods policies. Elaboration of the first-order condition for rival public goods consumption in period  $t$  yields the following equation:

$$\partial \mathcal{L}_1 / \partial b_t^r = 0 \quad \longrightarrow \quad \left( \frac{n_t^o \left( \frac{1+\Delta}{1+r} \right) + n_t^y}{n_t} \right) \left( \frac{1+r}{1+\Delta} \right)^{t-1} u b^{r'}(b_t^r) = \lambda \quad (16)$$

Equation (16) shows two things. First, assume there is no population ageing, *i.e.* the cohorts of the young and the old,  $n^y$  and  $n^o$ , are constant through time. Then, it is the relationship between the social discount rate  $\Delta$  and the interest rate  $r$  that determines how consumption of the rival public good should evolve over time. If the social discount rate is higher than the interest rate, the marginal benefit of consumption declines more over time than its price, rendering it optimal to decrease consumption over time. If the social discount rate is lower, the opposite holds true. If the social discount rate and the interest rate coincide, the optimal profile is consumption smoothing. Second, suppose that, due to ageing of the population, the old-age dependency ratio,  $n^o/n^y$ , is larger in some period  $s$  than in other periods. Equation (16) show that this implies relatively high consumption of the rival public good in this period, *i.e.*  $b_s^r > b_{s'}^r$ , where  $s' \neq s$  if the social discount rate is higher than the individual discount rate. The reason is that, if  $\Delta > r$ , the social planner gives a larger weight to the interests of the old generation. If in some period this generation is large relative to the young generation, then it is optimal to have high consumption in that period. If

the social discount rate is lower than the individual discount rate, the reverse holds true:  $b_s^r < b_{s'}^r$ . Note that this result is driven by the size of the old-age dependency ratio. Whether the dependency ratio is high because of an increase in longevity or a bust in fertility is not relevant.

The following equation describes the optimal consumption of pure public goods:

$$\begin{aligned} \partial \mathcal{L}_1 / \partial b_t^u = 0 \quad \longrightarrow \\ \left( n_t^o \left( \frac{1 + \Delta}{1 + r} \right) + n_t^y \right) \left( \frac{1 + r}{1 + \Delta} \right)^{t-1} u b^{u'}(b_t^u) = \lambda \end{aligned} \quad (17)$$

Equation (17) is similar to equation (16) in one respect and different in another one. The similarity concerns the role of the factor  $(1 + r)/(1 + \Delta)$ . Absent population ageing, this factor determines whether consumption of the pure public good should increase or decrease over time. The difference concerns the role of demographics. A high old-age dependency ratio in some period may now imply high consumption or low consumption, depending on the source of the high ratio, a large cohort of elderly or a small cohort of youngsters. Indeed, in the case of pure public goods, it is population size, not population structure, that determines the time profile of consumption. This echoes the peculiar feature of pure public goods: an expansion of the population increases their marginal benefit, but not their marginal cost.

Our third case is that of transfer policies. The equation for optimal transfer policies is a little more complex,

$$\begin{aligned} \partial \mathcal{L}_1 / \partial p_t = 0 \quad \longrightarrow \\ \left( \frac{1 + r}{1 + \Delta} \right)^{t-2} \left[ \left( \frac{\partial ucl^o}{\partial c^o} \right)_t + \left\{ \left( \frac{\partial ucl^o}{\partial c^o} \right)_t w_t + \left( \frac{\partial ucl^o}{\partial l^o} \right)_t \right\} \frac{dl_t^o}{dp_t} \right] = \\ \left( \frac{1 + r}{1 + \Delta} \right)^{t-2} \left[ \left( \frac{\partial ucl^o}{\partial c^o} \right)_t \right] = \lambda \end{aligned} \quad (18)$$

where we have derived the last line by using the optimality condition that describes the optimal allocation of available time over labour and leisure by the old (equation (5)).

Regarding equation (18), three comments are in order. First, similar to the previous cases, the factor  $(1 + r)/(1 + \Delta)$  determines whether transfers should increase or decrease over time. To see how, we have to elaborate marginal utility of private consumption of the old,  $(\partial ucl^o / \partial c^o)_t$ . Recall our assumption that preferences are separable and that this marginal utility function is decreasing in private consumption. If we now also assume that

private consumption is a normal good, *i.e.*  $\partial(w_t l_t^o + p_t)/\partial p_t > 0$ , we have that  $(\partial u c^o/\partial c^o)_t$  is negative in  $p_t$ . Hence, if the social discount rate is higher than the interest rate, it is optimal to let transfers to the elderly decline over time; if the social discount rate is lower than the interest rate, the opposite holds true. Second, as regards the effect of population ageing, the condition for optimal transfers is different from those for rival and pure public goods consumption. In particular, population ageing does not play a role here: the optimal time profile of transfers is independent of population size and population structure as measured by the old-age dependency ratio. The third comment relates to the effect of labour productivity growth. Using the expression for old-age consumption,  $w_t l_t^o + p_t$ , we derive that in case labour supply of the elderly is a non-negative function of the wage rate (the substitution effect is at least as large as the income effect), productivity growth corresponds to a decline of transfers over time. Note that this implies that eventually transfers will turn negative, which means that effectively the elderly will be taxed. How long it will take before transfers turn negative depends on initial transfers and wage income and the labour supply elasticity with respect to the wage rate.

The last case to be considered is that of optimal tax policies. These are described by the following equation,

$$\begin{aligned}
\partial \mathcal{L}_1 / \partial \tau_t = 0 & \longrightarrow \\
\left(\frac{1+r}{1+\Delta}\right)^{t-1} \left[ \left(\frac{\partial u c l^y}{\partial c^y}\right)_t (-w_t l_t^y) + \right. \\
\left. \left\{ \left(\frac{\partial u c l^y}{\partial c^y}\right)_t w_t (1-\tau_t) + \left(\frac{\partial u c l^y}{\partial l^y}\right)_t \frac{d l_t^y}{d \tau_t} \right\} \right] = \\
= -\lambda w_t \left( l_t^y + \tau_t \frac{d l_t^y}{d \tau_t} \right) & \longrightarrow \\
\left(\frac{1+r}{1+\Delta}\right)^{t-1} \left[ \left(\frac{\partial u c l^y}{\partial c^y}\right)_t \right] = \lambda \left( 1 - \epsilon_t^y \frac{\tau_t}{1-\tau_t} \right) & \quad (19)
\end{aligned}$$

where we have derived the last line by using the condition for the optimal allocation of available time over labour and leisure for young households (equation (4)) and the definition of  $\epsilon_t^y$  as the labour supply elasticity with respect to the after-tax wage rate, *i.e.*  $\epsilon_t^y \equiv (d l_t^y / d(w_t(1-\tau_t)))(w_t(1-\tau_t)) / l_t^y$ .

As before, let us focus upon the role of ageing, discounting and labour productivity growth in turn. First, to find the effect of discounting, elaborate  $(\partial u c l^y / \partial c^y)_t$ . Due to separability, this is a function of young-age consumption only,  $w_t(1-\tau_t)l_t^y$ . This young-age consumption is a negative function of the tax rate if  $\epsilon_t^y > -1$ , which we will assume to hold true. Hence, given

that we assume that utility is a concave function of private consumption, we derive that  $(\partial ucl^y / \partial c^y)_t$  is increasing in the tax rate. In order to sign the derivative of  $1 - \epsilon_t^y \tau_t / (1 - \tau_t)$ , we assume the labour supply elasticity is non-negative and constant or close to constant. Under these assumptions,  $1 - \epsilon_t^y \tau_t / (1 - \tau_t)$  is decreasing in the tax rate. Hence, we conclude that under fairly general assumptions the optimal time profile of the tax rate is increasing over time if the social discount rate exceeds the interest rate and decreasing over time if the interest rate exceeds the social discount rate. Second, equation (19) shows that, like in the case of transfers, population ageing does not exert any effect upon the optimal time profile of the tax rate. Thirdly, we derive that, under the same assumptions as those made above, labour productivity growth implies that the optimal tax rate will be increasing over time.

There is no guarantee that the tax rate will lie between 0 and 100 percent. We argue that this is not particularly worrisome, however. As regards the upper limit, if we are willing to make the common assumption  $\lim_{c^y \rightarrow 0} \partial ucl^y / \partial c^y = \infty$ , the tax rate will never achieve a value of 100 percent (although it may get quite close to it). As regards the lower limit, we cannot make a similar statement. Indeed, according to equation (19), the tax rate can become zero or negative at some point in the far future. We do not think this is problematic, however. If the tax rate turns negative, a tax turns into a subsidy and the analysis can be continued as before.

## 4.2 Two simplifying assumptions

Combining the above four first-order conditions, equations (16) to (19), with the intertemporal budget constraint, equation (9), completes our model. It solves for the optimal values of the four policy instruments in all time periods plus the value of the Lagrange multiplier in terms of the parameters and exogenous variables of the model. As it stands, we view the model as too general, however. Hence, we will make two simplifying assumptions.

The first is that the social discount rate equals the interest rate, *i.e.*  $\Delta = r$ . The reason is that we have no information about what would be a realistic value for the social discount rate. One might argue that the social planner will choose the social discount rate such as to obtain consumption smoothing over time. Absent ageing and productivity growth, this would imply a social discount rate equal to the interest rate. Next, one might argue that any other value for the social discount rate than the interest rate is inappropriate as it would imply continuous one-sided redistribution between generations. The argument for pinning the social discount rate down to the

interest rate that we find most convincing is that it is irrelevant for many of our conclusions regarding the impact of demographic shocks. In those instances where it does matter, our conclusions will be biased in a direction that is governed by the relation between the social discount rate and the interest rate. Given that we have no firm idea about the relation between the two, we argue that we do not lose anything by imposing equality between the two.

The second additional assumption is that of a zero rate for labour productivity growth. The reason for making this assumption relates to the latter reason for pinning down the social discount rate to the interest rate: our conclusions regarding the impact of demographic shocks do not relate to the value assumed for labour productivity growth. In the case of productivity growth, the argument is even more powerful. In those cases in which productivity growth is relevant, demographics play no role and in those cases in which demographic factors are relevant, productivity growth does not play a role.

## 5 A permanent longevity boost and a temporary fertility bust

This section explores three demographic shocks: a longevity boost, a fertility bust and a combined shock. All three shocks are stylized by construction: they imply an increase in the old-age dependency ratio in period 2 and an increase, decrease or stabilization of the population in that period. Moreover, in all three cases, the changes that occur in period 2 as to population structure and population size are permanent.

### 5.1 Characteristics of the shocks

In case of a longevity boost, the survival rate  $\zeta$  increases in period 2 and stabilizes at the higher level thereafter. The fertility rate remains unchanged at its initial value, which we have taken to be unity. The longevity boost thus increases the old-age dependency ratio and expands the population in period 2.

In case of a fertility bust, the fertility rate  $f$  falls to a lower level in period 2. This boosts the old-age dependency ratio, but decreases the population. In order to have a stable population in terms of size and structure after period 2, the fertility rate returns to its initial value of unity and the survival rate increases with factor  $1/f_2$  in period 3.

Figures 1 and 2 illustrate. Here, the blocks 'y' and 'o' correspond to the size of the young and old cohort respectively in different time periods. The arrows refer to the changes in fertility and mortality rate that characterize the shocks. The combined shock (not illustrated to save space) is defined as that linear combination of the two shocks that leaves population size unchanged.

[Figure 1 about here.]

[Figure 2 about here.]

One could argue that, taken together, these shocks mimic quite well the ageing of populations across the world. As we described in section 2, old-age dependency ratios are projected to increase everywhere, whereas the projections of population size are more varied. This accords with the three shocks that have in common an increase in the old-age dependency ratio, but differ in terms of population size. On the other hand, one may object that the increase of longevity is not a one-off change, but a more continued process. Similarly, one may object that a permanent fall of the fertility rate to a lower level fits the dynamics of real-world fertility better than the temporary change in fertility that we analyze here. In order to meet these objections, we will analyze a series of longevity increases and a more permanent fall of fertility in section 10. The analysis of the shocks in the present section forms the basis for that analysis.

## 5.2 The impact of shocks to fiscal sustainability

Let us return to the three shocks of this section. These shocks imply a demographic change in period  $t=2$  that becomes known in period  $t=1$ . In all cases, the population stabilizes from period  $t=2$  onwards (in terms of structure and size). Hence, there is nothing in the model that changes after period  $t=2$ . We exploit this characteristic by condensing all periods starting from period  $t=2$  into one "super" period. Let us denote this period henceforth as  $t=2+$ . Note that the  $t=2+$  period is of infinite length, but has, due to discounting, finite weight.

Applying this two-period structure allows us to simplify the intertemporal budget constraint of the government considerably. Indeed, combining equations (9) and (11) to (14) and imposing that period 2 extends into the far future, we arrive at the following version of the government budget

constraint:

$$\begin{aligned}
-D_0 &= \left( n_1 b_1^r + n_{2+} \frac{b_{2+}^r}{r} \right) + \left( b_1^u + \frac{b_{2+}^u}{r} \right) \\
&+ \left( n_1^o p_1 + n_{2+}^o \frac{p_{2+}}{r} \right) - \left( n_1^y \tau_1 w l_1^y + n_{2+}^y \frac{\tau_{2+} w l_{2+}^y}{r} \right) \quad (20)
\end{aligned}$$

The model now consists of nine equations only: the first-order conditions in periods 1 and 2+ and the above version of the intertemporal government budget constraint. It solves for the values of the four policy instruments in periods 1 and 2+ plus the Lagrange multiplier.

We use this model to analyze optimal policy reform in case of a demographic shock. But it is useful to first pose the following question: what do the three shocks imply for the implicit debt if government policies would be left unchanged? That is, how would the implicit public debt change when information about future demographic change became available, but government policies would be left unchanged? To answer this question, we calculate the implicit debt two times: one time under the assumption of initial information about demographics (and initial government policies) and another time under the assumption of new information about demographics (and initial policies). Then, we subtract the former from the latter. Let us use indexes (0) and (1) to refer to initial and new and the  $\Delta$  operator to refer to the difference between the two, *i.e.*  $\Delta(x) \equiv x(1) - x(0)$ . Further, let us for the implicit debt use the notation  $G_t(x, y)$  in which  $x$  refers to demographic information and  $y$  to policies. From this it follows that  $G_0(1, 0) - G_0(0, 0)$  is the ageing-driven change in the implicit debt. For this change, we can elaborate the following expression:

$$\Delta n_{2+} \left( \frac{b_{2+}^r(0)}{r} \right) + \Delta n_{2+}^o \left( \frac{p_{2+}(0)}{r} \right) - \Delta n_{2+}^y \left( \frac{\tau_{2+}(0) w l_{2+}^y(0)}{r} \right) \quad (21)$$

Equation (21) reveals that a longevity boost ( $\Delta n_{2+}^o > 0, \Delta n_{2+}^y = 0$ ) increases the implicit debt unambiguously. For a longevity boost will increase the spending on transfers and on rival goods, whereas it will leave tax revenues unchanged. The case of a combined shock ( $\Delta n_{2+}^o > 0, \Delta n_{2+}^y < 0$ ) is also unambiguous. Indeed, a combined shock features an increase in transfers and a fall of tax revenues. Only the case of a fertility bust ( $\Delta n_{2+}^o = 0, \Delta n_{2+}^y < 0$ ) is theoretically ambiguous. Although now again tax revenues will fall, spending on rival public goods will decrease as well. If the latter change is sufficiently large, a fertility bust will reduce rather than increase the implicit

debt. Empirically, the case in which tax revenues exceed public spending on rival public goods seems most relevant.<sup>3</sup> If we focus on that case, a fertility bust implies an increase in the implicit debt, just like the longevity boost and the combined shock. Then, all three types of demographic change render public finances unsustainable.

In order to restore fiscal sustainability, policies should be changed in order to reduce the implicit public debt back to its initial level. The interesting question is how. The answer will be found by combining the nine equations that make up our model.

## 6 Optimal policy reform

In order to derive the optimal response of government policies to each of the demographic shocks, we proceed as follows. We first combine first-order conditions (16) to (19) in order to eliminate the Lagrange multiplier and impose our simplifying assumptions  $\Delta = r$  and  $g = 0$ . This gives us three optimality conditions that describe the contemporaneous relation between pure public consumption, transfers and the tax rate on the one hand and rival public consumption on the other hand:

$$n_t u b^u(b_t^u) = u b^r(b_t^r) \quad t = 1, 2+ \quad (22)$$

$$\left( \frac{\partial u c l^o}{\partial c^o} \right)_t = u b^r(b_t^r) \quad t = 1, 2+ \quad (23)$$

$$\left( \frac{\partial u c l^y}{\partial c^y} \right)_t / \left( 1 - \epsilon_t^y \frac{\tau_t}{1 - \tau_t} \right) = u b^r(b_t^r) \quad t = 1, 2+ \quad (24)$$

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<sup>3</sup>A rough look at the data tells us that tax revenues exceed the spending by governments on rival public consumption goods. OECD (2017) reports that in 2015 the OECD average for government revenues in terms of GDP was 38.1 percent. The corresponding figure for government spending in terms of GDP was a little higher, 40.9 percent. Now, it is a little ambiguous what spending items should be counted as rival public consumption, but we can circumvent this problem by subtracting from general government spending the spending on social protection and health care. That leaves only  $(1-0.326-0.187)*40.9$  or 19.9 percent of GDP. As the latter figure also includes items like debt service, it serves as an upper bound for the spending on rival public consumption goods. Hence, tax revenues are a factor larger than spending on rival consumption goods.

One may object that we should not account for all tax revenues and that we should also account for the fact that not all transfers accrue to the elderly. To do justice to these objections, we would have to explore in detail how tax revenues and spending relate to the size of the young and old cohorts. Given the prominence of transfers to the elderly and the correlation between income and consumption (thus, the correlation between the revenues from income and consumption taxation) in many industrialized countries, we do not think such an exercise would yield a different result, however.



Next, we totally differentiate these equations. In general, differentiation may not be appropriate since the relations between the various variables are non-linear and the demographic shocks that we study are not marginal, but discrete. However, as I show in the appendix, the coefficients of the differential equations have unambiguous sign. Hence, total differentiation gives correct answers on a qualitative level.

Total differentiation of equations (22), (23) and (24) gives us equations (25), (26) and (27) respectively,

$$db_t^u = \hat{B}_t db_t^r + \hat{C}_t dn_t \quad t = 1, 2+ \quad (25)$$

$$dp_t = \hat{P}_t(\epsilon_t^o) db_t^r \quad t = 1, 2+ \quad (26)$$

$$d\tau_t = \hat{T}_t(\epsilon_t^y) db_t^r \quad t = 1, 2+ \quad (27)$$

where we have relegated the definition of the coefficients  $\hat{B}_t$ ,  $\hat{C}_t$ ,  $\hat{P}_t$  and  $\hat{T}_t$  to the appendix. The appendix shows that each of the coefficients can be signed. In particular,  $\hat{B}_t > 0$ ,  $\hat{C}_t > 0$ ,  $\hat{P}_t > 0$  and  $\hat{T}_t < 0$ .

This system of differential equations is not complete: it consists of three equations and four variables. It can be completed by adding the budget constraint. But before doing so, it is useful to take a look at equations (25) to (27).

First, equation (26) tells us that the change in transfers will always be in the same direction as the change in rival public consumption. This result holds true in both periods and in all three demographic scenarios. Similarly, equation (27) indicates that the change in the tax rate and the change in rival public consumption have opposite sign, in both periods and in all three scenarios. Subsequently, equation (25) tells us that pure public consumption and rival public consumption will change in an equal direction in two cases: in period 1 in each of the three scenarios and in period 2+ in the combined shock scenario. Only in period 2+ of the longevity boost scenario and fertility bust scenario we cannot derive whether the two variables will move in the same or in opposite direction.

Equations (26) and (27) also inform us about the role of labour supply behaviour. I have expressed coefficient  $\hat{P}_t$  in equation (26) as a function of  $\epsilon_t^o$ , the negative of the labour supply elasticity of the old with respect to the transfer, *i.e.*  $\epsilon_t^o \equiv -(dl_t^o/dp_t)/(l_t^o/p_t)$ . As the appendix shows,  $\hat{P}_t(\epsilon_t^o) > 0$ . The economic meaning of this result is that the more elastic is the labour supply of the old, the larger will be the change in transfers relative to other policy instruments. Indeed, the more a cut in transfers to the elderly increases their labour supply, the more effective is the policy

reform: it not only reduces spending on transfers, but also increases the revenues from taxing labour income.

For the young, a similar result holds true. I have expressed coefficient  $\hat{T}_t$  in equation (27) as a function of  $\epsilon_t^y$ , which has been defined in section 4 as the labour supply elasticity of the young with respect to the after-tax wage rate. The appendix demonstrates that  $\hat{T}_t'(\epsilon_t^y) > 0$ . Hence, the more elastic is the labour supply of the young, the smaller will the change in tax rates relative to other policy instruments. Intuitively, the more an increase in tax rates makes the young reduce their labour supply, the less effective is the policy reform as lower labour supply reduces the revenues from labour income taxation.

In order to derive how each of the policy instruments is adjusted as a response to the news about ageing, we now complete the system of differential equations. Total differentiation of the intertemporal budget constraint (equation (20)) gives us a relation between  $db_1^r$ ,  $db_{2+}^r$ ,  $db_1^u$ ,  $db_{2+}^u$ ,  $dp_1$ ,  $dp_{2+}$ ,  $d\tau_1$ ,  $d\tau_{2+}$ ,  $dn_{2+}^y$  and  $dn_{2+}^o$ . Using equations (25), (26) and (27) and the first-order condition with respect to rival public consumption policies, equation (16), we can eliminate all policy variables except  $db_1^r$  (see the appendix for an elaboration). Thus, we end up with a relationship between  $db_1^r$ ,  $dn_{2+}^o$  and  $dn_{2+}^y$ :

$$db_1^r = -\frac{(b_{2+}^r + \hat{C}_{2+} + p_{2+})}{r\hat{E}}dn_{2+}^o - \frac{(b_{2+}^r + \hat{C}_{2+} - \tau_{2+}wl_{2+}^y)}{r\hat{E}}dn_{2+}^y \quad (28)$$

The definition of coefficient  $\hat{E}$  and its relation to the coefficients introduced in equations (25) to (27) can be found in the appendix. The appendix also derives that  $\hat{E} > 0$ . Equation (28) is the basis for the model: combining the results in this equation with equations (25), (26), (27) and (16), we can derive closed-form expressions for all policy variables in the model.

What does this tell us about the effects of a longevity boost and a fertility bust? First of all, optimal policies change immediately upon the arrival of new information about a future demographic change. This may be obvious to many, but a key message for some others: policies should be reformed (long) before the population has started to age.

Next, a longevity boost unambiguously reduces rival public consumption and transfers and unambiguously increases the tax rate, both in period 1 and period 2+. In addition, period-1 pure public consumption is also reduced; only the change of period-2+ pure public goods consumption is ambiguous.

The increase in the shadow price calls for lower period-2+ consumption, whereas the increase of the population in that period has the opposite effect.

The case of a fertility bust is different, even if we assume, as explained in section 5, that a fertility bust will make the public debt unsustainable. First, period-2+ consumption of the pure public good will now definitely be reduced. The increase in the shadow price and the shrinking of the population in period-2+ cooperate to reduce optimal consumption of the pure public good in that period. Second, the effects on rival goods consumption, transfers and the tax rate in periods 1 and 2+ are now ambiguous. Indeed, if the reduction of pure public goods consumption in period 2+ is sufficiently large, there will be room to increase rival goods consumption and transfers and to reduce the tax rate. If the role of pure public goods is less large, a fertility bust will call for lower rival goods consumption, lower transfers and higher tax rates, as in the case of a longevity boost. But even then, the policy implications of a fertility bust will be less dramatic than those of a longevity boost.

Finally, we can use equation (28) also to explore the role of labour supply behaviour. The appendix writes  $\hat{E}$  as a function of the four labour supply elasticities,  $\hat{E} = \hat{E}(\epsilon_1^y, \epsilon_{2+}^y, \epsilon_1^o, \epsilon_{2+}^o)$ , and derives that  $\partial \hat{E} / \partial \epsilon_1^o, \partial \hat{E} / \partial \epsilon_{2+}^o > 0$  and that  $\partial \hat{E} / \partial \epsilon_1^y, \partial \hat{E} / \partial \epsilon_{2+}^y < 0$ . This basically confirms the results derived earlier. The required change in rival public consumption will be unambiguously smaller the more elastic is labour supply of the elderly; it will be unambiguously larger the more elastic is labour supply of the youngsters.

## 7 Evolution of the public debt

This section explores the evolution of the public debt. Note that nowhere we imposed a time path or end value for the public debt. Rather, we must derive it, using the results derived in the previous section for optimal policies.

We adopt a two-step approach. In the first step, we derive a generic formula for the change in the public debt. In the second step, we apply the formula to optimal policies in periods 1 and 2+.

The first step starts from noting that sustainable public finances imply a zero value for the total of explicit and implicit debt:

$$D_t + G_t = 0 \quad \longrightarrow \quad D_t = -G_t \quad (29)$$

Hence, taking first differences yields a simple formula for the evolution of the public debt:

$$\Delta D_t = -\Delta G_t \quad (30)$$

Recall that the implicit debt is defined as the present value of primary deficits,

$$G_{t-1} = F_t + \frac{F_{t+1}}{1+r} + \frac{F_{t+2}}{(1+r)^2} + \dots \quad (31)$$

where  $F_t$  is a shorthand notation for the primary deficit in period  $t$ ,

$$F_t = B_t^r + B_t^u + P_t - T_t \quad (32)$$

Now, combining equations (30) and (31) allows us to write  $\Delta D_t$  as the following function of  $G_{t-1}$  and  $F_t$ :

$$\Delta D_t = -rG_{t-1} + (1+r)F_t \quad (33)$$

The second step is to apply equation (33) to  $t = 1$ . Note that in the case of the shocks studied thus far  $F$  stabilizes at the level it achieves in period  $t = 2+$ . Let us denote this level as  $\bar{F}$ . This then gives a very simple result: the change in the public debt along the optimal path in period 1 is opposite to the change in the primary deficit in period 2+. Formally,

$$\Delta D_1 = -r \left( F_1 + \frac{\bar{F}}{r} \right) + (1+r)F_1 = -(\bar{F} - F_1) \quad (34)$$

As noted above, optimal policies in case of a longevity boost imply an increase of the primary deficit from period 1 to period 2+. The same holds true for the combined shock. Hence, in both cases the public debt falls in period 1. As mentioned earlier, the case of a fertility bust is different. In case of a fertility bust, the change in the primary deficit from period 1 to 2+ can a priori not be signed as the spending on rival and pure public goods falls from period 1 to period 2+. The sign of the change in the public debt is then also ambiguous.

What about  $\Delta D_{2+}$ ? Intuitively, one would expect a zero change in period 2+ as the population structure and size do not change after period 1 in the case of the shocks studied thus far. Applying equation (33) to  $t = 2+$  confirms this intuition:

$$\Delta D_{2+} = -r\bar{F} \left( \frac{1+r}{r} \right) + (1+r)\bar{F} = 0 \quad (35)$$

## 8 The impact on the generational balance

What does optimal policy reform imply for the balance between generations? We will address this question now. In particular, we analyse the generational implications of the four policies in turn.

Let us start with tax policies. As shown, optimal tax policies feature smoothing of the tax rate over time. Consequently, the young and future generation will be hurt to the same extent by optimal tax policies. The currently old are not hurt by tax policy reform. As their income goes untaxed, they escape the higher tax rate. One may disagree with the latter finding as it hinges upon an assumption that is in reality often not true (a zero tax rate on the labour income of the elderly), but this would be an overstatement. The elderly often face lower rates than the youngsters and their labour supply is (much) lower than that of younger cohorts. Generalizing our model in this respect would confirm our finding that tax policy reform will hurt the old less than the other generations.

The case of optimal transfer policies is different. Like optimal tax policies, optimal transfer policies exhibit smoothing. Unlike optimal tax policies, they treat the currently old in the same way as other generations however. Indeed, optimal transfer policy reform achieves perfect balance between all generations involved.

The case of optimal rival public goods policies is similar to that of optimal tax policies. Again, the smoothing principle applies and, again, the effect on the generation that is old at the time of policy reform is smaller than that on other generations. In this case, the old generation faces lower consumption in one stage of her life cycle only.

The case of pure public goods policies is different from the other types of policies as the smoothing principle does not apply. For that reason, the future generation will be hurt more (in case of a fertility bust) or less (in case of a longevity boost) than the generation who is young at the time of policy reform.

## 9 How large is the fiscal impact of the four policy instruments?

A question not addressed thus far concerns the contributions that the different policy instruments make to the closure of the sustainability gap.<sup>4</sup> In particular, how large are the contributions made by the four instruments and how do these relate to the intensity and form of ageing?

In order to explore these questions, we derive an expression for the policy-driven reduction of the implicit debt which, referring to the notation we introduced earlier, we denote as  $G_0(1, 0) - G_0(1, 1)$ . The expression consists

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<sup>4</sup>In this section, we will assume that also a fertility bust implies a sustainability gap.

of four components and can be interpreted as the counterpart of equation (21) for the decomposition of the ageing-driven change in the implicit debt,

$$G_0(1,0) - G_0(1,1) = -N(1)\Delta b^r - \left( \Delta b_1^u + \frac{\Delta b_{2+}^u}{r} \right) - N^o(1)\Delta p + N^y(1)\Delta(\tau w l^y) \quad (36)$$

where we use as shortcuts  $N$ , defined as  $n_1 + n_{2+}/r$  and, similarly,  $N^y$ , defined as  $n_1^y + n_{2+}^y/r$  and  $N^o$ , defined as  $n_1^o + n_{2+}^o/r$ . Furthermore, we have omitted the time indices to the policy variables in case they have identical values in periods 1 and 2+.

Note that the policy-driven reduction in the implicit debt in equation (36) is exactly equal to the ageing-driven increase in the implicit debt, as formulated in equation (21). This can be derived formally by noting that the intertemporal budget constraint implies that  $G_0(0,0) = G_0(1,1) = -D_0$ .

Using equation (36) and the fact that we know the value of  $G_0(1,0) - G_0(1,1)$ , we can now define the contributions that each of the four types of policies make to the reduction of the implicit debt. Let us denote the contributions of rival public consumption policies, pure public consumption policies, transfer policies and tax policies as  $H^r$ ,  $H^u$ ,  $H^p$  and  $H^\tau$  respectively:

$$H^r = \frac{-N(1)\Delta b^r}{G_0(1,0) - G_0(1,1)} \quad (37)$$

$$H^u = \frac{-\Delta b_1^u - \Delta b_{2+}^u/r}{G_0(1,0) - G_0(1,1)} \quad (38)$$

$$H^p = \frac{-N^o(1)\Delta p}{G_0(1,0) - G_0(1,1)} \quad (39)$$

$$H^\tau = \frac{N^y(1)\Delta(\tau w l^y)}{G_0(1,0) - G_0(1,1)} \quad (40)$$

Expressions for  $\Delta p$ ,  $\Delta b^r$  and  $\Delta(\tau w l^y)$  can be obtained by taking the discrete analogues of equations (25) to (27). The implied relation between  $\Delta p$  and  $\Delta b^r$  is independent of demographics (as reflected in equation (26)). The same holds true for the relation between  $\Delta(\tau w l^y)$  and  $\Delta b^r$  (as reflected in equation (27)). Further, note that population ageing as we have defined it, *i.e.* an increase in the old-age dependency ratio that can be accompanied by an increasing, a decreasing or a stable population, increases  $N^o$  relative to  $N$  and decreases  $N^y$  relative to  $N$ . Hence, we derive that the contribution made by transfer policies is relatively large and the contribution made by

tax policies relatively small. In addition, the larger is the increase in the old-age dependency ratio, the larger is the contribution made by transfer policies and the smaller that made by tax policies. Whether the increase in the old-age dependency ratio is due to an expansion of longevity or a fall of fertility is not relevant.

Our model is too general to infer something about the contribution of pure public consumption policies. In order to gain more insight, let us therefore adopt a more specific formulation. In particular, let us assume that the utility functions for the pure and rival public good read as  $ub^r(b_t^r) = (b_t^r)^\phi$  and  $ub^u(b_t^u) = (b_t^u)^\phi$  respectively, with  $0 < \phi < 1$ . Equation (22) now implies that  $b_t^u(j) = b_t^r(j)n_t^{1/(1-\phi)}$  where  $t = 1, 2+$  and  $j = 0, 1$ . If we use this condition to elaborate (the discrete analogue of) equation (25), we can elaborate the expression in equation (38) as follows,

$$H^u = \frac{1}{G_0(1, 0) - G_0(1, 1)} \left( -M(1)\Delta b^r - \frac{b^r(0)n_{2+}(1)^{-\phi/(1-\phi)}}{(1-\phi)r} \Delta n_{2+} \right) \quad (41)$$

where we use as a shortcut  $M(1)$ , defined as  $n_1(1)^{1/(1-\phi)} + n_{2+}(1)^{1/(1-\phi)}/r$ .

For this more specific case we derive that the contribution of spending on the pure public good relative to that of spending on the rival public good falls in case of a longevity boost and increases in case of a fertility bust. This echoes the results derived earlier about the implications of population size for spending on the pure public good.

## 10 A series of boosts in longevity and a permanent fall of fertility

As explained above, the longevity shock that we have studied thus far - a one-time decline of the model's mortality rate - is at odds with the prospect of mortality rates permanently declining. A series of adverse shocks in the mortality rate seems more realistic. Therefore, we now turn to this case. Particularly, we will assume that the mortality rate declines for  $N$  successive periods rather than one period. After these  $N$  periods, the mortality rate stabilizes at the period- $N+1$  level (the first decline occurs in period 2, as in section 5). Basically, this case extends the period of demographic change from 1 to  $N$  periods.

As regards fertility, we follow a similar reasoning. Projections embody more a permanent fall rather than a temporary fall of the fertility rate as assumed thus far. Hence, we will now study the case of a permanent fall of

the fertility rate. Like in the longevity case, this boils down to prolonging the period of the shock that we analyzed before from 1 to  $N$  periods. In particular, we assume that the fertility rate falls in period 2 and remains at the new lower level until period  $N + 1$ . In period  $N + 2$ , the fertility rate returns to its original value, unity, and the mortality rate increases such as to stabilize the population structure from period  $N + 1$  onwards.

[Figure 3 about here.]

[Figure 4 about here.]

## 10.1 Policy implications

Figure 3 and 4 display the projected changes of the population in case of an extended longevity shock and fertility shock. The figures reveal that the extended shocks produce more interesting demographic dynamics than the two stylized shocks studied before. Still, in one important aspect, the implications for optimal policies are not basically different. Again, transfer policies, rival public goods policies and tax policies adhere to the smoothing principle, whereas the profile of pure public goods consumption is driven by that of population size.

In another aspect, there are differences. The extended longevity shock features  $N$  successive increases in old-age dependency ratio and population size. Hence, the implied increase of the implicit debt will be larger than in case of a one-time drop in the mortality rate. The same applies to the changes in instruments that correspond to optimal policies. The period-1 changes in the consumption of rival and pure public goods, transfers to the elderly and the labour income tax rate all magnify the corresponding changes in the case of a stylized shock.

The case of an extended fertility shock is more peculiar. When it comes to population structure, it is similar to the stylized shock. In both cases the old-age dependency ratio increases in period 2 and stabilizes at the higher level thereafter. But, unlike the case of a stylized shock, the case of an extended fertility shock features a series of declines in population size. The extended shock thus exhibits larger declines in tax revenues and spending on rival public goods. In addition, the extended shock also features spending cuts on transfers to the elderly. For, unlike the stylized shock, the fertility bust now also reduces the size of future old-age cohorts. Due to this, it is more difficult to pin down the sign of the effect of demographic change upon the implicit public debt. Depending on the initial composition of the government budget, the implicit debt may still increase, but it may



also remain unchanged or even decrease.<sup>5</sup> However, *if* the fertility shock increases the implicit debt, the four policy instruments will change as before.

We can obtain more insight into these issues by elaborating the generalization of equation (28) for extended shocks. The appendix derives that this generalization looks as follows:

$$\begin{aligned}
db_1^r = & \\
& - \frac{(b_2^r + \hat{C}_2 + p_2)}{(1+r)\hat{E}} dn_2^o - \dots - \frac{(b_N^r + \hat{C}_N + p_N)}{(1+r)^{N-1}\hat{E}} dn_N^o \\
& \quad - \frac{(\bar{b}^r + \bar{C} + \bar{p})}{(1+r)^{N-1}r\hat{E}} d\bar{n}^o \\
& - \frac{(b_2^r + \hat{C}_2 - \tau_2 w l_2^y)}{(1+r)\hat{E}} dn_2^y - \dots - \frac{(b_N^r + \hat{C}_N - \tau_N w l_N^y)}{(1+r)^{N-1}\hat{E}} dn_N^y \\
& \quad - \frac{(\bar{b}^r + \bar{C} - \bar{\tau} w \bar{l}^y)}{(1+r)^{N-1}r\hat{E}} d\bar{n}^y \tag{42}
\end{aligned}$$

Equation (42) reveals that an extended longevity shock is a straightforward generalization of a stylized longevity shock. As the extended shock is characterized by  $d\bar{n}^o > dn_N^o > \dots > dn_2^o > 0$ ,  $d\bar{n}^y = dn_N^y = \dots = dn_2^y = 0$ , equation (42) tells us that, following the arrival of information about an extended longevity shock, optimal policies imply an immediate cut in spending on rival public consumption goods. Equation (42) also shows that the extended fertility shock does not generalize the stylized fertility shock. The extended fertility shock is characterized by  $d\bar{n}^y < dn_N^y < \dots < dn_2^y < 0$ ,  $d\bar{n}^o < dn_N^o < \dots < dn_3^o < 0$ ,  $dn_2^o = 0$ . The sign of the effect upon  $b_1^r$  is now ambiguous. The permanent decline of the fertility rate makes both the younger cohort and the older cohort shrink. Both the impact upon the implicit public debt and upon the period-1 cut in public spending on the rival public good depend on the initial composition of the government budget and cannot unambiguously be signed.

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<sup>5</sup>That the effect of a fertility bust upon the implicit debt may be negative is indeed confirmed by studies like Smid *et al.* (2014) who find that a higher fertility rate will increase the implicit debt. According to this study, the extra tax revenues paid by a new person cannot compensate for the extra government transfers (spending on education, pensions and health care) to this person. This illustrates that in general the effect of a fertility shock upon the implicit debt relates to the composition of the government budget.

## 10.2 Implications for the public debt

Also interesting is what the extended shocks imply for the development of the public debt. Intuitively, as demographic change extends further into the future, there is more time to anticipate future demographic changes and this may affect the time path of the public debt.

To find an expression for the evolution of the public debt, we adopt the expression for the change in the debt as derived in section 7, equation (33). We now impose that the primary deficit stabilizes at the level  $\bar{F}$  in period  $N + 1$ . This then gives the following expression for the change in the public debt in period 1:

$$\begin{aligned}\Delta D_1 &= -r \left( F_1 + \frac{F_2}{1+r} + \dots + \frac{F_N}{(1+r)^{N-1}} + \frac{\bar{F}}{(1+r)^{N-1}r} \right) + (1+r)F_1 \\ &= -(F_2 - F_1) - \frac{1}{1+r}(F_3 - F_2) - \dots - \frac{1}{(1+r)^{N-1}}(\bar{F} - F_N) \quad (43)\end{aligned}$$

This expression for the change in the public debt generalizes that in equation (34). Now the change in the public debt is the opposite of the sum of the discounted changes in the primary deficit in all subsequent periods. As before, as optimal policies imply anticipation to future changes, the change in the debt in period 1 reflects future changes in the primary deficit.

The development of the public debt in subsequent periods can be derived by repeated application of equation (33). For illustration, I give here the expressions for periods 2,  $N$  and  $N+1$  (the latter two mimic earlier results):

$$\Delta D_2 = -(F_3 - F_2) - \frac{1}{1+r}(F_4 - F_3) - \dots - \frac{1}{(1+r)^{N-2}}(\bar{F} - F_N) \quad (44)$$

$$\Delta D_N = -(\bar{F} - F_N) \quad (45)$$

$$\Delta D_{N+1} = 0 \quad (46)$$

These equations demonstrate again that the development of the public debt over time is endogenous and a reflection of optimal policies. Indeed, the behaviour of the public debt will depend on demographic dynamics and the public institutions in the country that is being analyzed. Without making more specific assumptions, it is difficult to tell how the public debt will develop over time.

## 11 Concluding comments

Three assumptions warrant further discussion. The first concerns the absence of a private capital market. The second is the concept of a small open economy. The third is that the population consists of two generations only.

### 11.1 Private capital market

Throughout the paper, we have assumed there is no private capital market. This assumption simplifies our derivations. Moreover, it is not crucial for our analysis. To illustrate, let us review what would happen if we relaxed this assumption.

First, allowing households to save and dissave changes the scope of tax and transfer policies. Indeed, in our model the government uses tax and transfer policies to obtain what households, absent a capital market, cannot achieve: the smoothing of their consumption over the life cycle. If we instead assumed there is a perfect capital market, the government would use transfer and tax policies such as to minimize distortions. This would bring government policies closer to optimal policies. Our focus is not on optimal policies, however. Rather, our focus is on optimal policy reform upon the arrival of news about a future ageing shock. So, the relevant question is what are the implications for optimal policy reform once we allow households to save and borrow on a perfect capital market?

Allowing private savings to respond to policy reform changes two of the differential equations that describe optimal policy reform, namely equations (26) and (27). For details, see the appendix. This appendix shows that allowing for private savings will increase  $\hat{T}_t$  in equation (27). Hence, the government will rely more heavily on tax hikes to tackle the ageing problem. The reason is that now households can adjust their savings to protect their consumption when young from falling too much in case of a tax hike. Similarly,  $\hat{P}_t$  in equation (26) will increase from period  $t = 2$  onwards. The reason is similar. The elderly, when young, anticipate upon the change in transfers when old by adjusting their saving. The government explores this type of self-insurance by relying more on transfer policies. (Obviously, the elderly in period 1 cannot anticipate. In period 1, therefore, equation (26) will remain unchanged.)

Apart from these results, our analysis remains valid. Endogenizing private saving does not change our result that optimal policy reform requires policies in period 1 to anticipate to changes in future periods, that optimal policy reform requires changes in all four instruments and that optimal pol-

icy reform will be different for a longevity boost and a fertility on account of pure public goods.

## 11.2 Small open economy

Up till now, we implicitly assumed that demographic shocks do not change the interest rate (and wage rate). However, simulation models (Miles, 1999; Börsch-Supan *et al.*, 2006) suggest that population ageing will reduce the interest rate (and raise the wage rate). This in itself may change optimal policies. Indeed, given that capital income is relatively important for older generations and labour income is relatively important for younger generations, one would expect that the combination of a lower interest rate and a higher wage rate implies that optimal policy reform puts a heavier load on younger generations.

This brings us to the assumption that the economy under consideration is small. If the economy under consideration, instead, is large, the assumption that the interest rate (and wage rate) is a given may be inadequate. Indeed, policies of public saving (debt reduction) may imply a fall of the interest rate. This in itself will increase the implicit public debt and require further policy adjustments. Hence, policies of public debt reduction may become less attractive when the country in consideration is sufficiently large. I do not expect that this will affect the balance between different policy instruments, but do expect that it will change the profile of optimal policy adjustment over time. This is also the conclusion of earlier contributions to the literature (Cutler *et al.* (1990), Elmendorf and Sheiner (2000)). To investigate this further is a natural topic for further research.

## 11.3 A 2-OLG model

Another assumption of our analysis is that the life cycle of households consists of two periods only. Obviously, this is a caricature of reality, but the question is whether this is relevant for our analysis. Suppose we would assume instead that the life cycle of households consists of 60 or 80 years, as is more common in the literature. How would this change the results of our analysis? The answer is there would not be any substantial changes, except for one point. That is that this modification would increase the number of policy instruments. In particular, optimal policies would feature changes in age-specific taxes and transfers to the elderly. A clear example of why this is relevant can be found in the labour market position of the (young) elderly who are still active on the labour market and the (old) elderly who have fully

retired. The difference in labour market elasticities between the two groups would imply that upon a demographic shock as studied here the transfers to the former group would have to be reduced more than the transfers to the former group. Similarly, also other age groups that feature different labour market elasticities would require different policy changes.

This raises another issue, which is that external factors may hinder optimal policy reform. To return to the example just mentioned, policy instruments often do not differentiate with respect to age in the way that is required by economic analysis. One example that also applies to the stylized model in this paper are restrictions on the development of the public deficit and debt. The Stability and Growth Pact of the EU, for example, imposes restrictions on the deficit to GDP ratio and the debt to GDP ratio that our analysis did not take into account. A third example concerns the uncertainty by which demographic projections are surrounded in reality. This uncertainty may be a reason (and may have been a reason in the past) not to implement radical changes before the ageing of the population has (did) really set in. These examples demonstrate that our analysis does not give answers to all relevant questions and that, in order to complete the picture, more analysis is required.

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## Appendix

### Coefficients of equations (25) to (27): definition

Coefficients  $\hat{B}_t$ ,  $\hat{C}_t$ ,  $\hat{P}_t$  and  $\hat{T}_t$  for  $t = 1, 2+$  in equations (25) to (27) are defined as follows:

$$\hat{B}_t = \frac{ub^{r''}(b_t^r)}{n_t ub^{u''}(b_t^u)} \quad (47)$$

$$\hat{C}_t = \frac{-ub^{u'}(b_t^u)}{n_t ub^{u''}(b_t^u)} \quad (48)$$

$$\hat{P}_t = \frac{ub^{r''}(b_t^r)}{\left(\frac{\partial^2 ucl^o}{(\partial c^o)^2}\right)_t (1 - \epsilon_t^o w l_t^o / p_t)} \quad (49)$$

$$\hat{T}_t = \frac{ub^{r''}(b_t^r) \left(1 - \epsilon_t^y \frac{\tau_t}{1 - \tau_t}\right)}{ub^{r'}(b_t^r) \left(\frac{\epsilon_t^y}{(1 - \tau_t)^2} + \frac{d\epsilon_t^y}{d\tau_t} \frac{\tau_t}{1 - \tau_t}\right) - \left(\frac{\partial^2 ucl^y}{(\partial c^y)^2}\right)_t (1 + \epsilon_t^y) w l_t^y} \quad (50)$$

### Coefficients of equations (25) to (27): sign

Given the properties of the sub-utility functions,  $\hat{B}_t$  and  $\hat{C}_t$  can be shown to be unambiguously positive. For  $\hat{P}_t$  to be positive, we need one additional assumption, namely that  $1 - \epsilon_t^o w l_t^o / p_t > 0$ . This is equivalent to  $dc_t^o / dp_t > 0$ . This means that for the elderly private consumption is a normal good, an assumption that will very likely hold true.

In order to sign  $\hat{T}_t$ , we review three terms in the expression for  $\hat{T}_t$ . The first is  $1 - \epsilon_t^y \tau_t / (1 - \tau_t)$ . This is the effect of a change in the tax rate upon tax revenues. Assumed the economy is on the normal part of the Laffer curve, this term will be positive. The second term to explore is  $1 + \epsilon_t^y$ . We do not want to exclude the case of a negative labour supply elasticity: it is a real possibility that the income effect of a change in the wage rate exceeds the substitution effect. An elasticity smaller than -1 is very implausible, however, so  $1 + \epsilon_1^y$  can safely be assumed to be positive. The third term to consider is the denominator of the expression for  $\hat{T}_t$ . Zero and constant positive values for  $\epsilon_t^y$  render this term positive. It is difficult to exclude a priori that the labour supply elasticity of the youngsters is so negative that it renders the denominator negative, however. In order to gain more insight, let us elaborate a specific case. Suppose that  $ub^{r'}(b_t^r) = (b_t^r)^{\gamma-1}$ ,

$(\partial u c^y / \partial c^y)_t = (c_t^y)^{\gamma-1}$ ,  $c_t^y = b_t^r$  and  $\epsilon_t^y$  independent of  $\tau_t$ . In this specific case, the sign of the denominator of the expression for  $\hat{T}_t$  equals the sign of  $\epsilon_t^y + CRRA(1 - \tau_t)(1 + \epsilon_t^y)$ , where  $CRRA$ , the coefficient of relative risk aversion of the two sub-utility functions, equals  $1 - \gamma$ . Even if we take conservative estimates for  $CRRA$ ,  $\tau_t$  and  $\epsilon_t^y$ , this term will be positive. Suppose for example that  $CRRA = 1$ ,  $\tau_t = 0.50$  and  $\epsilon_t^y = -0.25$ . Then,  $\epsilon_t^y + CRRA(1 - \tau_t)(1 + \epsilon_t^y) = 0.125$ . Likely,  $CRRA$  is higher than 1 and  $\epsilon_t^y$  is less negative than -0.25, whereas in many countries  $\tau_t$  is lower than 50 percent, even if we interpret the tax rate as an indication of all types of taxation rather than taxation of labour income taxation alone. Thus, we must conclude that  $\epsilon_t^y + CRRA(1 - \tau_t)(1 + \epsilon_t^y)$  will likely be higher than 0.125. Hence, we assume henceforth that this term is positive.

### Coefficients of equations (25) to (27): properties

In the main text,  $\hat{P}_t$  and  $\hat{T}_t$  are written as functions of  $\epsilon_t^o$  and  $\epsilon_t^y$  respectively. What can we say about the derivatives? Inspection of equation (49) reveals that  $\hat{P}'_t(\epsilon_t^o) > 0$ . Inspection of equation (50) demonstrates that a higher value for  $\epsilon_t^y$  reduces the absolute value of  $\hat{T}_t(\epsilon_t^y)$ , both on account of a denominator effect and a numerator effect. Since  $\hat{T}_t(\epsilon_t^y) < 0$ , this means that  $\hat{T}'_t(\epsilon_t^y) > 0$ .

### Derivation of equation (28)

Total differentiation of the intertemporal government budget constraint, equation (20), gives the following equation:

$$\begin{aligned}
0 = & \left\{ n_1 db_1^r + \frac{n_{2+}}{r} db_{2+}^r + \frac{b_{2+}^r}{r} dn_{2+} \right\} \\
& + \left\{ db_1^u + \frac{1}{r} db_{2+}^u \right\} + \left\{ n_1^o dp_1 + \frac{n_{2+}^o}{r} dp_{2+} + \frac{p_{2+}}{r} dn_{2+}^o \right\} \\
- & \left\{ n_1^y w l_1^y \left( 1 - \epsilon_1^y \frac{\tau_1}{(1 - \tau_1)} \right) d\tau_1 + \frac{n_{2+}^y w l_{2+}^y}{r} \left( 1 - \epsilon_{2+}^y \frac{\tau_{2+}}{(1 - \tau_{2+})} \right) d\tau_{2+} \right. \\
& \left. + \frac{\tau_{2+} w l_{2+}^y}{r} dn_{2+}^y \right\} \quad (51)
\end{aligned}$$

Upon using equations (25) to (27) for  $t = 1$  and  $t = 2+$ , we can eliminate a number of policy variables,

$$0 = \hat{E}_1 db_1^r + \frac{\hat{E}_{2+}}{r} db_{2+}^r + \frac{(b_{2+}^r + \hat{C}_{2+} + p_{2+})}{r} dn_{2+}^o + \frac{(b_{2+}^r + \hat{C}_{2+} - \tau_{2+} w l_{2+}^y)}{r} dn_{2+}^y \quad (52)$$

where  $\hat{E}_t$  is defined as  $n_t + \hat{B}_t + n_t^o \hat{P}_t - n_t^y w l_t^y (1 - \epsilon_t^y \tau_t / (1 - \tau_t)) \hat{T}_t$  for  $t = 1, 2+$ . Given the assumptions made above,  $\hat{E}_1$  and  $\hat{E}_{2+}$  are positive.

In order to eliminate  $db_{2+}^r$ , we apply equation (16) to  $t = 1$  and to  $t = 2+$ , eliminate the Lagrange multiplier and totally differentiate. This gives the following expression,

$$db_{2+}^r = \hat{I} db_1^r \quad (53)$$

where  $\hat{I}$  equals  $ub^{r''}(b_1^r)/ub^{r''}(b_{2+}^r)$ . Given the properties of the sub-utility functions,  $\hat{I} > 0$ .

Equation (52) can now be further reduced. This gives us equation (28) in the main text, which relates  $db_1^r$  to the demographic shocks,

$$db_1^r = - \left( \frac{b_{2+}^r + \hat{C}_{2+} + p_{2+}}{r \hat{E}} \right) dn_{2+}^o - \left( \frac{b_{2+}^r + \hat{C}_{2+} - \tau_{2+} w l_{2+}^y}{r \hat{E}} \right) dn_{2+}^y \quad (54)$$

where  $\hat{E}$  is defined as  $\hat{E}_1 + \hat{E}_{2+} \hat{I} / r$ . Note that  $\hat{E} > 0$ , given that  $\hat{E}_1, \hat{E}_{2+}, \hat{I} > 0$ .

The role of the labour supply elasticities of the young and the old can be easily derived. Upon using the definitions of  $\hat{E}$ ,  $\hat{E}_1$  and  $\hat{E}_{2+}$  and the results  $\hat{P}_t'(\epsilon_t^o) > 0$  and  $\hat{T}_t'(\epsilon_t^y) > 0$ , we derive that  $\partial \hat{E} / \partial \epsilon_1^o, \partial \hat{E} / \partial \epsilon_{2+}^o > 0$  and  $\partial \hat{E} / \partial \epsilon_1^y, \partial \hat{E} / \partial \epsilon_{2+}^y < 0$ .

## The case of extended shocks

In order to derive equation (42) in the main text, we start formulating the applicable version of the intertemporal government budget constraint. This follows from combining equations (9) and (11) to (14) and imposing that period  $N+1$  extends into the far future. This implies the following version

of the budget constraint,

$$\begin{aligned}
-D_0 = & \\
& \left( n_1 b_1^r + \frac{n_2}{1+r} b_2^r + \dots + \frac{n_N}{(1+r)^{N-1}} b_N^r + \frac{\bar{n}}{(1+r)^{N-1} r} \bar{b}^r \right) + \\
& \left( b_1^u + \frac{1}{1+r} b_2^u + \dots + \frac{1}{(1+r)^{N-1}} b_N^u + \frac{1}{(1+r)^{N-1} r} \bar{b}^u \right) + \quad (55) \\
& \left( n_1^o p_1 + \frac{n_2^o}{1+r} p_2 + \dots + \frac{n_N^o}{(1+r)^{N-1}} p_N + \frac{\bar{n}^o}{(1+r)^{N-1} r} \bar{p} \right) - \\
& \left( n_1^y \tau_1 w l_1^y + \frac{n_2^y}{1+r} \tau_2 w l_2^y + \dots + \frac{n_N^y}{(1+r)^{N-1}} \tau_N w l_N^y + \frac{\bar{n}^y}{(1+r)^{N-1} r} \bar{\tau} w \bar{l}^y \right)
\end{aligned}$$

where we use the notation  $\bar{x}$  to refer to the value of  $x$  from period  $N+1$  onwards.

We now have to make two steps to arrive at equation (42). The first step is to totally differentiate equation (55) and to use equations (25) to (27) to eliminate all policy variables other than  $b^r$ . The second step is to use the counterparts of equation (53) for the case of extended shocks to eliminate all policy variables  $b^r$  except for  $b_1^r$ . These counterpart equations read as follows:

$$\begin{aligned}
db_j^r &= \left( \frac{ub^{r''}(b_1^r)}{ub^{r''}(b_j^r)} \right) db_1^r \equiv \hat{I}_j db_1^r \quad j = 2, \dots, N \\
d\bar{b}^r &= \left( \frac{ub^{r''}(b_1^r)}{ub^{r''}(\bar{b}^r)} \right) db_1^r \equiv \tilde{I} db_1^r \quad (56)
\end{aligned}$$

Now define  $\hat{E}$ ,

$$\hat{E} = \left( \hat{E}_1 + \frac{1}{1+r} \hat{E}_2 \hat{I}_2 + \dots + \frac{1}{(1+r)^{N-1}} \hat{E}_N \hat{I}_N + \frac{1}{(1+r)^{N-1} r} \tilde{E} \tilde{I} \right) \quad (57)$$

where we recall our definition of  $\hat{E}_j$  and apply a similar definition for  $\tilde{E}$ :

$$\begin{aligned}
\hat{E}_j &\equiv n_j + \hat{B}_j + n_j^o \hat{P}_j - n_j^y w l_j^y (1 - \epsilon_j^y \frac{\tau_j}{1 - \tau_j}) \hat{T}_j \\
\tilde{E} &\equiv \bar{n} + \tilde{B} + \bar{n}^o \tilde{P} - \bar{n}^y w \bar{l}^y (1 - \bar{\epsilon}^y \frac{\bar{\tau}}{1 - \bar{\tau}}) \tilde{T} \quad (58)
\end{aligned}$$

Using equation (57), we arrive at equation (42) in the main text.

## The case of endogenous private savings

Allowing households to engage in private savings changes two conditions for optimal policy reform: the relation between the optimal reform of transfer policies and that of rival goods policies (equation (26)) and the relation between the optimal reform of tax policies and that of rival goods policies (equation (27)). We will review these two relations in turn.

In the main text, equation (26) expresses the relation between the optimal reform of transfer policies and that of rival goods policies. It is useful to repeat the relation here in its full form (*i.e.*, invoking equation (49)):

$$\frac{dp_t}{db_t^r} = \frac{ub^{r''}(b_t^r)}{\left(\frac{\partial^2 ucl^o}{(\partial c^o)^2}\right)_t (1 - \epsilon_t^o w_t^o/p_t)} \quad (59)$$

If we now assume that households can save and borrow on the capital market such as to optimize their utility function, the expression for  $dp_t/db_t^r$  changes into the following,

$$\frac{dp_t}{db_t^r} = \frac{ub^{r''}(b_t^r)}{\left(\frac{\partial^2 ucl^o}{(\partial c^o)^2}\right)_t (1 - \epsilon_t^o w_t^o/p_t - \eta_t^o(1+r)s_{t-1}/p_t)} \quad (60)$$

where  $\eta_t^o$  is defined as the negative of the elasticity of private saving with respect to transfers to the elderly, *i.e.*  $\eta_t^o \equiv -(ds_{t-1}/dp_t)(p_t/s_{t-1})$ .

As regards equation (59), we have assumed that  $1 - \epsilon_t^o w_t^o/p_t > 0$ . If this condition were not true, private consumption would be an inferior good, which we regard an implausible assumption. We now make the same assumption in the context of equation (60), which implies that  $1 - \epsilon_t^o w_t^o/p_t - \eta_t^o(1+r)s_{t-1}/p_t > 0$ .

In order to derive the implication for  $dp_t/db_t^r$  of endogenizing private saving, we must make an assumption about  $(\partial^2 ucl^o/(\partial c^o)^2)_t$ . A benchmark assumption is that initially the saving constraint is not binding, so that allowing households to save does not change their initial consumption profile. Then, endogenizing private saving will increase  $dp_t/db_t^r$  by increasing  $\eta_t^o$  from zero to some positive value. Alternatively, we can assume that initially the saving constraint is binding, so that allowing households to save increases their initial saving. This requires the third derivative to be positive, *i.e.*  $(\partial^3 ucl^o/(\partial c^o)^3)_t > 0$ , an assumption that holds true for common utility functions. But then, again, the same result is achieved: endogenizing private saving increases  $dp_t/db_t^r$ .

The second equation to review is that between tax policies and rival goods policies (equation (27)). We repeat it here in its full form (for transparency, we will now regard the elasticity of labour supply of young workers,  $\epsilon^y$ , a constant):

$$\frac{d\tau_t}{db_t^r} = \frac{ub^{r''}(b_t^r) \left(1 - \epsilon^y \frac{\tau_t}{1 - \tau_t}\right)}{ub^{r'}(b_t^r) \left(\frac{\epsilon^y}{(1 - \tau_t)^2}\right) - \left(\frac{\partial^2 ucl^y}{(\partial c^y)^2}\right)_t (1 + \epsilon^y)wl_t^y} \quad (61)$$

Endogenizing private saving implies that equation (61) changes into the following,

$$\frac{d\tau_t}{db_t^r} = \frac{ub^{r''}(b_t^r) \left(1 - \epsilon^y \frac{\tau_t}{1 - \tau_t}\right)}{ub^{r'}(b_t^r) \left(\frac{\epsilon^y}{(1 - \tau_t)^2}\right) - \left(\frac{\partial^2 ucl^y}{(\partial c^y)^2}\right)_t \left((1 + \epsilon^y)wl_t^y - \eta_t^y s_t / (1 - \tau_t)\right)} \quad (62)$$

where  $\eta_t^y$  is defined as the elasticity of private saving with respect to the after-tax wage of young workers, *i.e.*  $\eta_t^y \equiv (ds_t/d(w_t(1 - \tau_t)))(w_t(1 - \tau_t)/s_t)$ .

As regards  $(1 + \epsilon^y)wl_t^y$ , we have assumed before that it is positive. That means, that whatever the labour supply response to a tax hike, a tax hike will decrease the consumption of young workers. Now, we make a similar assumption:  $(1 + \epsilon^y)wl_t^y - \eta_t^y s_t / (1 - \tau_t) > 0$ . This means that although workers may increase their labour supply and also reduce their private savings, a tax hike will always lower the consumption of young workers.

Given this assumption, we derive for equation (62) that  $|d\tau_t/db_t^r| > 0$  if the allowance for private saving does not change the initial consumption profile. If the allowance for saving induces households to save (and if again the third derivative of utility is assumed positive), we achieve the same result.

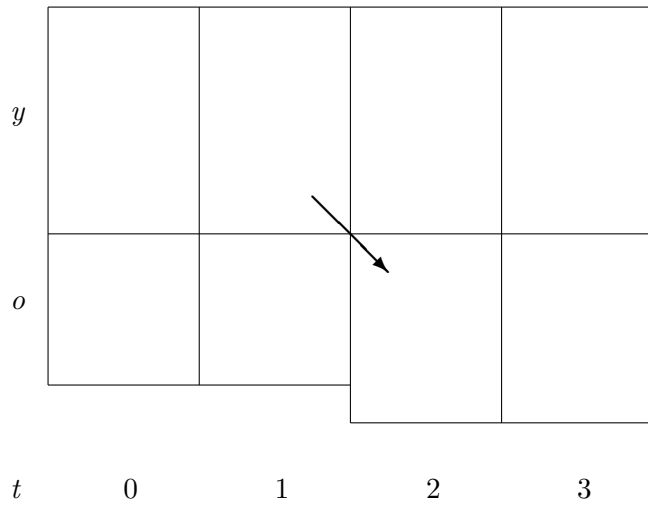


Figure 1: Stylized shock: a permanent longevity boost



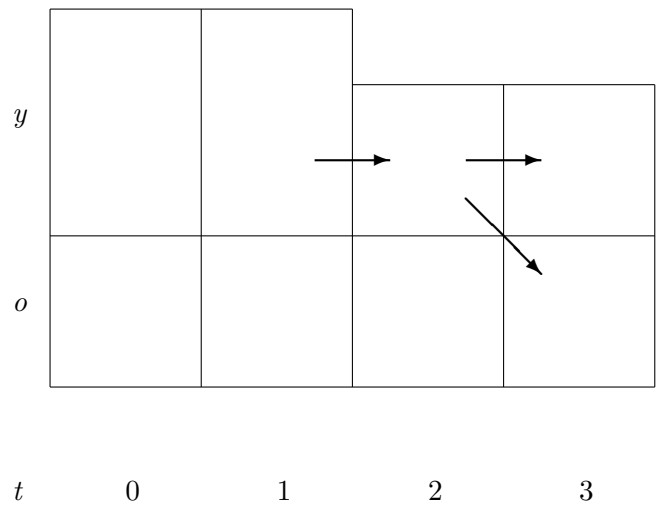


Figure 2: Stylized shock: a temporary fertility bust

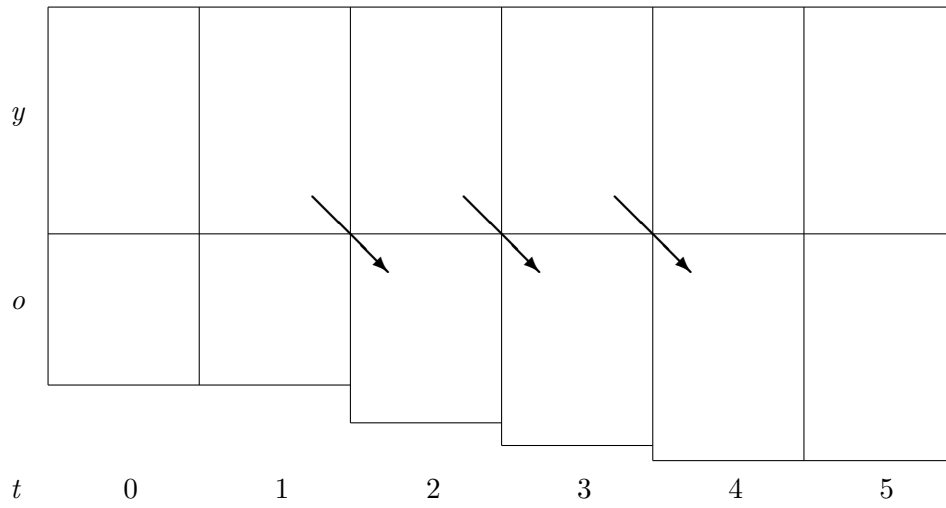


Figure 3: Extended shock: a series of longevity boosts

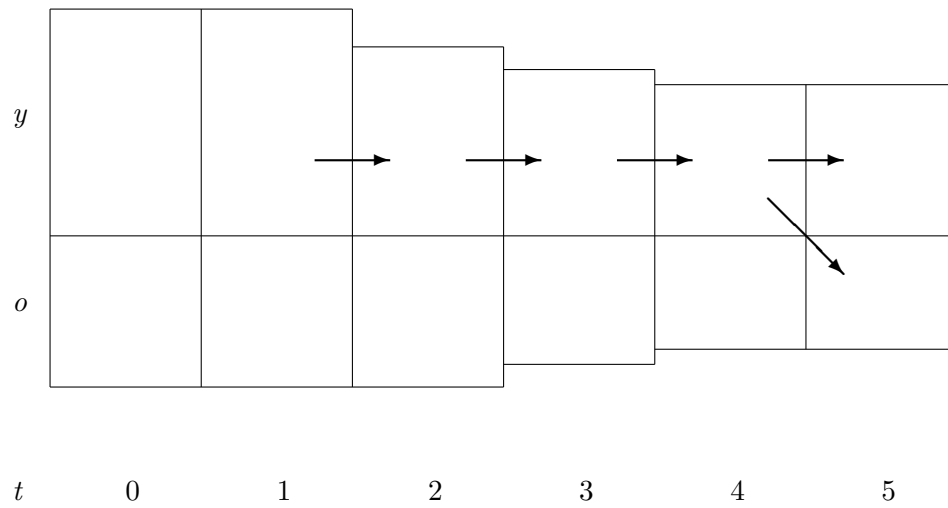


Figure 4: Extended shock: a permanent fertility bust