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Representing Legal Rules in Deontic Logic

Lambèr Royakkers
Representing Legal Rules in Deontic Logic

Proefschrift

ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. L.F.W. de Klerk, in het openbaar te verdedigen ten overstaan van een door het college van dekanen aangewezen commissie in de aula van de Universiteit op maandag 1 april 1996 om 16.15 uur

door

Lambertus Maarten Maria Royakkers

geboren op 4 september 1967 te Heerlen
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I dedicate this book to my mother, whom I cannot thank personally any more: Margriet Royakers-Bertus.
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Chapter 1

Introduction

1.1 Logic and law

Research on the application of computer science to law (legal informatics) is intimately connected with jurimetrics, formal logic, legal theory and the study of positive law (cf. Sartor, 1992). Formal logic comprises the methodology of computer science in general and is also an indispensable instrument of analysis of the interpretation of legal texts and the assessment of the validity of legal reasoning (cf. De Wild, 1979). The close connection of legal informatics and (deontic) logic is most pronounced in the study of expert systems1 and knowledge based systems2 in law.3 The nature of such systems is in part determined by logical formalism used for the representation of the legal rules involved. In practice, it turns out, however, that the results from modern deontic logic concerning the logical structure of legal rules and of legal reasoning are not (explicitly) taken into account in the construction of, e.g., knowledge based systems. Classical two-valued propositional calculus is generally considered strong enough and is supposed to be adequate to cope with the

---

1Expert systems are able to reason with complex rules as well as simple facts and several systems are available which can understand subjects of natural language (cf. Frost, 1986, p. 1).

2A knowledge based system is a set of resources - hardware, software and possibly human - whose collective responsibilities include storing the knowledge base, maintaining security and integrity, and providing users with the required input/output routines, including deductive retrieval facilities, so that the knowledge base can be accessed as required. Unfortunately, at present, there is no widely accepted definition of 'knowledge base' and this term is used by different people to mean different things. We define a knowledge base as a collection of simple facts, such as 'John drives on the motorway' together with general rules, such as 'it is forbidden to drive faster than 120 km/h on motorways' (cf. Frost, 1986, p. 3).

3Knowledge based systems are distinct from expert systems which are most often designed for specific tasks such as mineral prospecting, medical diagnosis, fault-finding and proving mathematical theorems. Knowledge based systems might be used as components in expert systems. However, their use is not limited to this. They can be used as general purpose sophisticated database systems or as components of 'special function' systems such as pattern recognition systems (cf. Frost, 1986, p. 6).
Introduction

problem of the representation of legal rules and legal reasoning (see Koers, Kracht, Smith, Smits and Weusten, 1989). This presupposition, however, can be seriously questioned. At first glance, theory formation in legal informatics is not in line with the state of the art in the field of deontic logic. The question can be asked whether or not there is a correlation between this observation on the one hand and the rather limited usefulness and limited sophistication of most existing expert systems in law on the other hand (cf. Susskind, 1987).

This research starts from the hypothesis that there is a connection between the relative lack of interest in deontic logic (and in legal theory in general) of the builders of knowledge based systems in law on the one hand, and the relative lack of usefulness for legal practice of most expert systems and knowledge based systems as presented in the literature on the other hand. This inquiry intends to improve the usefulness of knowledge based systems in law by taking into account deontic logic in representing legal rules. The scientific interest of this thesis is therefore given, since it tries to apply results from a science of a highly theoretical and abstract nature (such as deontic logic) to an applied science as legal informatics.

In modern logical theory it is generally acknowledged that legal rules and legal reasoning cannot adequately be formalised and assessed in propositional or predicate calculus, and that the system of deontic logic with specific deontic operators is required (see Brouwer, Soeteman and De Wild, 1982). The state of the art in this field is described in handbooks dating already from the seventies (cf. Alchourrón and Bulygin, 1971; Føllesdal and Hilpinen, 1971; Von Kutschera, 1973; Reisinger, 1977). For more recent publications see Martino and Socci Natali, 1986, especially part two; Soeteman, 1989; Brouwer, 1990. A formalisation of deontic notions is required in the cases, when violations must be accounted for (cf. Jones, 1990). To express violations consistently, one has to distinguish what ought to be done and what is the case. If this distinction cannot be made, the concurrent occurrence of a rule and the violation (which then is represented as the negation) of that rule would render the system inconsistent.

In summary, we can say that the role of deontic logic with respect to knowledge based systems in law is both necessary and important. We pose that no appropriate deontic logic is available for legal expert systems and knowledge based systems in law, and that such a logic should be constructed. Our aim is not to construct such an appropriate logic - this would be aiming too high - but to develop a deontic system which will improve the representation of legal texts and of legal knowledge in general, and to formalise adequately legal reasoning by investigating deontic defeasible reasoning and reasoning with normative inconsistent information, such that legal expert systems, or knowledge based systems in

\footnote{For a discussion of the use of deontic logics in representing legal knowledge, see Herrestad (1991).}

\footnote{Legal rules are taken to be inherently subject to exceptions, i.e., they are \textit{defeasible}.}
1.2 Deontic logic

law, can be improved.

The theory that is developed in this thesis is based on two standard deontic logics, one for Ought-to-be statements, which we call 'standard deontic logic' (see subsection 1.2.1), and one for Ought-to-do statements, which we call ‘propositional deontic logic’ (see chapter 2). The difference between Ought-to-be and Ought-to-do is discussed in subsection 1.2.2. In this thesis, we will extend these standard deontic logics with some new concepts. These concepts are:

- (groups of) actors (chapters 3 and 4);
- authorities (chapter 5);
- authority hierarchy (chapter 6);
- defaults (chapter 7).

The latter two concepts relate to the notion of normative inconsistency (see section 1.2.3.).

In this thesis, we will not make a formal comparison between Ought-to-be and Ought-to-do. Both approaches have their own merits. Neither do we aim to construct a combination of the two approaches in this thesis. This would lead us too far away from the main goal of the thesis and, therefore, we leave it for further research.

In the following two sections, we introduce the two main pillars of this thesis. In the next section (standard) deontic logic is introduced, together with some of its main problems. In section 1.3, we present an in-depth discussion of a case of conflicting speed limits on the basis of a judgement by the High Court. This case is a good example of the type of problems that are encountered in representing legal rules. It serves the motivation for the research into new concepts in deontic logic which is pursued in this thesis. In chapter 8, we will analyse and formalise these problems on the basis of the concepts discussed in the other chapters.

1.2 Deontic logic

Deontic logic\(^7\) is a branch of philosophical logic concerning reasoning about norms, or in other words, about normative versus non-normative behaviour. It is the logic of obligations, prohibitions and permissions. As such, it is relevant for the foundations of ethics and law. Deontic logic has been used to analyse the structure of normative law and normative

\(^6\)For a formal approach of the relation between the Ought-to-do and Ought-to-be we refer to d’Altan, Meyer and Wieringa (1993).

\(^7\)The adjective deontic is derived from the Greek word ‘δεοντως’, which means ‘as it should be’.
reasoning in law. In this thesis, we will use deontic logic as a tool for representing legal rules.

Not much is certain in deontic logic, in contrast to, for example, propositional and predicate calculus. There are not many principles, in whatever deontic system, which are undisputed; i.e., which cannot be accepted as rational reconstructions of normative reasoning. There is, nevertheless, a formal system on which several other systems are based, although it has been disputed as a whole as well as with regard to its theorems. Most other systems can be regarded as extensions of standard deontic logic. One may, therefore, to a certain extent rightly speak of a 'standard system of deontic logic'. This is even more justified by the fact that alternative systems have often been developed as a reaction to this system. Every deontic logician has to determine, in one way or another, his attitude towards this standard system. In the following section, we briefly discuss this standard system.

1.2.1 Standard deontic logic

In standard deontic logic (SDL), three deontic operators are used: 'O' (obligatory), 'F' (forbidden) and 'P' (permitted). By connecting propositions to these operators as arguments, well-formed formulas of the system originate from which, by interpretation of the propositions, normative judgements can be formed. E.g., \( O(p) \) means 'it is obligatory that \( p \)'. The deontic operators can be defined in terms of one another. If we take 'O' as a primitive, then the other operators can be defined as follows:

- \( F(p) =_{df} O(\neg p) \);
- \( P(p) =_{df} \neg O(\neg p) \).

Thus, 'it is forbidden that \( p \)' is defined as 'it is obligatory that not-\( p \)', and 'it is permitted that \( p \)' is defined as 'it is not obligatory that not-\( p \).

By standard deontic logic, a modal (Kripke-style) version of the now so-called 'Old System' of Von Wright (1951), we mean the system \( D^9 \) based on propositional logic and axiomatised by the rule of inference:

\[
\begin{align*}
\text{(ROM)} & \quad \frac{E \quad \neg O(p) \quad \neg O(q)}{O(p) \rightarrow O(q)},
\end{align*}
\]

However, as so many subjects in philosophical logic and philosophy in general, the subject was also picked up by computer scientists and AI (artificial intelligence) researchers. Deontic logic proved to be relevant as well for such prosaic matters as authorisation mechanisms, decision support systems, database security rules, fault-tolerant software and database integrity constraints; thus, outside the area of legal analysis and legal automation. A survey of applications can be found in Meyer and Wieringa (1991).

\( \text{System } D^* \) is the smallest normal \( KD \)-system of modal logic (cf. Chellas, 1980).
1.2 Deontic logic

together with the following axiom schemata:10

\[(OC)\] \((O(p) \land O(q)) \rightarrow O(p \land q)\)
\[(ON)\] \(O(p \lor \neg p)\)
\[(OD)\] \(\neg O(p \land \neg p)\)
\[(DF, P)\] \(P(p) \equiv \neg O(\neg p)\)

The semantics of this system can be given using the following model structure \(M = (W, R, V)\) consisting of three elements:

1. the set of possible worlds \(W = \{w_1, w_2, \ldots\}\);
2. the accessibility function \(R \in R\), which takes a world and returns a subset of \(W\): \(R : W \rightarrow 2^W\);
3. a valuation function \(V\), which assigns the values ‘true’ or ‘false’ to a proposition at a world in \(W\).

The intuition behind the function \(R\) is that it yields the deontically ideal worlds relative to a given world. The truth conditions for \(O\) and \(P\) can now be defined as follows:

\[
\mathcal{M}, w \models O(p) \text{ iff } R(w) \subseteq \llbracket p \rrbracket
\]

and

\[
\mathcal{M}, w \models P(p) \text{ iff } R(w) \cap \llbracket p \rrbracket \neq \emptyset
\]

with the function \(\llbracket \cdot \rrbracket \in L \rightarrow 2^W\) and \(L\) the set of well-formed formulas of the propositional calculus.11 Thus, \(O(p)\) holds in \(w\) if and only if \(p\) is true in all ideal worlds with respect to \(w\), and \(P(p)\) holds in \(w\) if and only if there is at least one ideal world with respect to \(w\) in which \(p\) is true.

The following constraint

\[
R(w) \neq \emptyset \text{ for all } w \in W
\]

is added to validate the schema \((OD)\). The truth conditions (1.1) and (1.2) are sufficient to validate the rule and all other schemata. Thus \(D^*\) is sound.12

10Axiom \((ON)\) was rejected by Von Wright (1951, p. 11), since he developed the principle of deontic contingency: ‘A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden’. We have to commit ourselves to this axiom, since otherwise we cannot view deontic logic as a branch of Kripke-style normal modal logic.

11\(\llbracket p \rrbracket = \{w \mid V(w, p) = \text{true}\}\). It is easy to see that the following properties hold: \(\llbracket p \lor q \rrbracket = \llbracket p \rrbracket \cup \llbracket q \rrbracket\), \(\llbracket p \land q \rrbracket = \llbracket p \rrbracket \cap \llbracket q \rrbracket\) and \(\llbracket \neg p \rrbracket = \neg \llbracket p \rrbracket\).

12A system is sound iff for all well-formed formulas \(p\) it holds that if \(\vdash p\) then \(\models p\).
1.2.2 Ought-to-do and Ought-to-be

The expressions in deontic logic are read as 'it is obligatory (forbidden, permitted) that ...' followed by a descriptive sentence, or are read as 'it is obligatory (forbidden, permitted) to ...' followed by a verb (or verb phrase) for a category or type of action or activity.

The reading of the deontic operators with 'that' and 'to', respectively, may be said to answer to two different types of deontic logic. The first type is a logic of that which ought to, may or must not be, and the second a logic of that which ought to, may or must not be done. According to Castañeda (1970, p. 452):

In short, deontic statements divide neatly into: (i) those that involve agents and actions and support imperatives, and (ii) those that involve states of affairs and are agentless and have by themselves nothing to do with imperatives. The former belong to what we used to call the Ought-to-do and the latter to the Ought-to-be.

The difference between Ought-to-do and Ought-to-be matters greatly. Suppose 'p' stands for the description of the act 'to feed the monkeys', then $F(p)$ is read as 'it is forbidden to feed the monkeys'. This rule is broken if someone feeds the monkeys. If 'p', on the other hand, stands for the proposition or the description of the state 'the monkeys are fed', then $F(p)$ is read as 'it is forbidden (to accomplish) that the monkeys are fed', which is a different interpretation. The second prohibition demands more than the first prohibition, since one does not only break the rule if one feeds the monkeys, but also if one neglects to act if another person feeds the monkeys. The norms 'it is forbidden that the monkeys are fed' and 'it is forbidden to feed the monkeys' are not equivalent. Thus, it depends on the interpretation of the norm content one uses, and is especially relevant for the analysis of legal rules, which mostly belong to Ought-to-do and not to Ought-to-be (the 'duties of care' constitute an exception to this). A system expressing Ought-to-do sentences fits better with the common-sense interpretation of legal rules, especially in criminal law\footnote{Criminal law is concerned with behaviour: if an illegal situation is mentioned in the description of an offence, the question is raised as to who created this situation (by action or omission) and who is responsible for continuing the situation. Thus, from the illegal situation a certain type of behaviour is derived, as it were.}, since norms are mostly concerned with behaviour, and are thus essentially related to individuals; they constitute somebody's obligation, permission or prohibition.

Theoretically, it does not matter which type of logic (Ought-to-do or Ought-to-be) one uses for a non-relativised deontic logic and that is why authors often leave this problem out of consideration in their publications. However, in a relativised deontic logic\footnote{Relativised deontic modalities are concerned with what is obligatory for or permitted to an actor or group of actors, as contrasted with what is impersonally obligatory or permitted (see chapters 3 and 4).} the interpretation of the norm content matters greatly, for example with respect to conflicting
1.2 Deontic logic

norms. Suppose ‘p’ is the description of an act, e.g., ‘to feed the monkeys’, then the obligations ‘it is obligatory that the zoo caretaker i feeds the monkeys’ and ‘it is obligatory that visitor j does not feed the monkeys’ are not in conflict. However, if ‘p’ is a proposition or a description of a state of affairs, e.g., ‘the monkeys are fed’, then the obligations ‘it is obligatory that the zoo caretaker i brings about (accomplishes) that the monkeys are fed’ and ‘it is obligatory that visitor j brings about that the monkeys are not fed’ are in conflict.

Bailhache (1981, p. 76), for example, interprets the norm content as the description of a state of affairs:

It is advisable to note that this formula \[ O_y(p) \rightarrow P_z(p) \]

appears paradoxical only if the normative addressee is unduly identified with the subject of proposition p. The formula does not say for example that ‘if y is obliged to go into that house, then z is also permitted to go in’. The formula only corresponds to the following: as soon as an addressee is obliged that such a thing is accomplished, normative coherence makes it necessary that all other individuals are not obliged that this thing is not accomplished (in other words, that they are permitted that it is accomplished).

In SDL, we cannot deal with actions; norms are expressed by applying a sentential operator O to sentence letters p. Now, O(p) cannot be read as ‘it is obligatory to do p’, for then p would not be a sentential letter. In ‘it is obligatory to feed the monkeys’, ‘to feed the monkeys’ is not a sentence. Thus, SDL is inadequate for representing Ought-to-do statements, and thus also inadequate for representing legal rules.

A fairly adequate approach to express Ought-to-do statements is system PD_eL (propositional deontic logic), developed by Meyer (1988, 1989). This system is an extension of SDL using Anderson’s (1967) variant of SDL with propositional dynamic logic. This system will be discussed in chapter 2.

1.2.3 Normative inconsistencies

As stated above, normative inconsistencies play an important role in legal reasoning. In this subsection we take a closer look at these inconsistencies. Let us consider the following

\[ (O_y(p) \rightarrow P_z(p)) \]

Two obligations are in conflict if and only if it is impossible to fulfil both obligations simultaneously.

This formula is read as ‘if it is obligatory for individual y that p, then it is permitted for individual z that p’.

Von Wright (1951) interpreted the norm content as a description of an action (‘act-qualifying properties’), by introducing ‘performance-values’ which are strictly analogous to the truth values in alethic logic and thus present the opportunity to compound actions. E.g., action \( p \land q \) is performed if and only if action p is performed and action q is performed. However, then system SDL as a logic of Ought-to-do is still not satisfactory. The central point of this problem consists of the interpretation of the internal negation. For an analysis of this problem, we refer to Brouwer (1990).
two norms: ‘it is obligatory to feed the monkeys’ and ‘it is forbidden to feed the monkeys’. At first glance, we would say that these norms are conflicting since they cannot be fulfilled simultaneously. This is correct, if we assume that these norms are related to one and the same individual. If we add actors to these statements, we obtain norms which mostly do not conflict: ‘it is obligatory for the zoo caretaker to feed the monkeys’ and ‘it is forbidden for visitors to feed the monkeys’. Thus, by adding actors to a deontic system, some (normative) inconsistencies can be removed. This will be done in chapters 3 and 4. However, contradictory or conflicting norms can still be found - and mostly very frequently - at least in law. Suppose, for example, the norms: ‘it is permitted for the visitors to feed the monkeys’ and ‘it is not permitted for the visitors to feed the animals’. These norms are contradictory. To express this consistently, we can add authorities to the statements (see chapter 5). For example, ‘authority a enacted that it is permitted for the visitors to feed the monkeys’ and ‘authority b enacted that it is not permitted for the visitors to feed the animals’. Now we can consistently express that authorities have enacted contradictory or conflicting norms. However, we have to determine which of the two contradictory or conflicting norms should be followed. Many researchers have suggested to treat these norms by means of defeasible reasoning. In chapter 7, we investigate how to include defeasible reasoning in a deontic logic. Another possibility is to use the authorities to prioritise the norms, by means of an authority hierarchy. This is done in chapter 6.

1.3 Conflicting speed limits

In this section, based on Royakkers (1995), we give an extensive description of an example from the Dutch Traffic Regulation to illustrate the types of problems we try to solve in this thesis.

The amendment of the Dutch Traffic Regulation 1966 (in Dutch: ‘Reglement Verkeersregels en Verkeerstekens 1966’ - RVV 1966) has led to a great deal of largely justified criticism. The Dutch Traffic Regulation 1990 was supposed to become the political showpiece of deregulation. Compared to the 1966 Act, the Dutch Traffic Regulation 1990 showed that a simplification and reduction of regulations in the critical field of road traffic was possible. The aim of the legislature was to increase the credibility of the rules. According to Otte (1993), however, the Dutch Traffic Regulation 1990 turned into a total fiasco as regards simplicity, accessibility and comprehensibility. Also, in a number of articles for the Dutch

\[18\] In the logic of Cuppens (1993), norms are conflicting if an actor has two roles, \(X\) and \(X'\), and is obliged to do something with respect to \(X\) and is forbidden to do the same thing with respect to \(X'\). Cuppens gave the following example: an agent is a Christian and a soldier. He ought not to kill, because he is a Christian, but he ought to kill, because he is a soldier (if he is ordered to kill).

\[19\] The reason for choosing the Dutch Traffic Regulation that this part of Dutch law contains a minimum of fuzzy concepts. Therefore, it seems to lend itself for formalisation.
1.3 Conflicting speed limits

Journal Verkeersrecht, Otte and Simmelink (1993) discussed some of the structural flaws in various rules of the Dutch Traffic Regulation 1990. In one of their so-called 'Kronkels in het RVV 1990' (Twists in the Dutch Traffic Regulation 1990), 'De bijzondere snelheidsmaxima in het RVV 1990' (The special speed limits in the Dutch Traffic Regulation 1990), they question the effectiveness of the speed limit rules. They discuss this on the basis of a case such as the following.

1.3.1 Case

On a national route road A28, within the city limits of Zwolle, a lorry from the firm H.I. drove at a speed of 96 km/h. H.I. was imposed an administrative sanction on the ground of 'a lorry exceeding the speed limit by 15 to 20 km/h'. An appeal was lodged with the public prosecutor and the subdistrict court judge, because H.I. was of the opinion that, on the road in question, traffic signs indicating a speed limit of 100 km/h were in force, and that, therefore, no sanctionable act had been committed, for traffic signs override traffic rules.20

According to Otte and Simmelink, the subdistrict court judge will find it hard to rule in this case, and they advise the following:

Our advice is twofold. The subdistrict court judge has no choice but to pronounce the appeal by the lorry driver valid and to quash the court order by the public prosecutor. When the Dutch Traffic Regulation is evaluated - or sooner - the legislator will have to amend the Dutch Traffic Regulation. (...) Neither is it desirable to amend arts. 21 and 22 of the Dutch Traffic Regulation. The solution should be sought in amending art. 63 of the Dutch Traffic Regulation 1990.

In the proceedings that led to High Court judgment HR 61-93-V,21 the subdistrict court judge ruled differently. His motivation was the following:

In art. 22 of the Dutch Traffic Regulation 1990,22 it is laid down that for lorries the special speed limit of 80 km/h holds. In art. 63 of the Dutch Traffic Regulation 1990, it is laid down that traffic signs override traffic rules, in as far as these rules are incompatible with the signs. From the text it appears that traffic signs only override traffic rules if they are in conflict with the traffic rules.

20 Art. 63 of the Dutch Traffic Regulation 1990: Traffic signs override traffic rules in as far as specific rules are incompatible with specific signs.

21 DD, 94.137.

22 Art. 22 of the Dutch Traffic Regulation 1990: In as far as no lower speed limits have been set in other articles, the following special speed limits hold for the following vehicles: a. for lorries, buses and vehicles with trailers 80 km/h; (...).
This is, according to the subdistrict court judge, not the case here. Traffic sign A1 indeed indicates the speed limit, but as this is not in conflict with the traffic rule as stated in art. 22 of the Dutch Traffic Regulation 1990, the latter rule remains in force. Traffic sign A1 is a regulatory sign (a speed limit of 100 km/h in this case) and does not imply a higher speed limit in force for particular vehicles such as lorries.

H.I. appealed to the court of cassation of this judgement, but the High Court rejected this appeal. In his conclusion, Advocate-General Meijers clearly indicated why, in his opinion, there is no conflict between or incompatibility of traffic sign and traffic rule here.

According to art. 63 of the Dutch Traffic Regulation 1990, the prohibition of art. 22 preamble and sub a, Dutch Traffic Regulation 1990, would, for a lorry driver, be lifted by a traffic sign if that traffic sign were to imply a compulsory minimum speed limit of 81 km/h on that particular section of the road. Only in such a case would there be incompatibility of rule and sign.

1.3.2 Article 63 and incompatibility

In what situations are traffic signs and traffic rules incompatible? To answer this question, we will first take a close look at the notion of 'incompatibility'. Two rules are incompatible if, and only if, they lead to contrary results or conclusions. Let us consider the following example. Car driver $i_1$ is on a major road and approaches a junction, where car driver $i_2$ approaches from the left. According to art. 15 of the Dutch Traffic Regulation 1990, $i_2$ has to give way to $i_1$, who approaches the junction from the right. Also, according to the right-of-way signs (A6 and A9), $i_2$ has to give way to $i_1$. So, it will be clear that there is no incompatibility between traffic rule and traffic signs in this case. However, we do not know exactly on the basis of which $i_2$ has to give way. In the case of an offence in such situations, it does not matter whether the violation of a traffic rule or a traffic sign is held against the suspect. Now, suppose that $i_2$ approaches the junction from the right. On the basis of art. 15 of the Dutch Traffic Regulation 1990, $i_1$ has to yield right of way and, according to the right-of-way signs (A6 and A9), $i_2$ has to yield right of way as well. In a logical sense, there is no incompatibility between signs and rule here; there is a deadlock, however: both drivers have to yield right of way. The relevant rules and signs do not lead to contrary conclusions. However, we may also apply a different interpretation to the situation. On the basis of the principle of trust, we may assume that the obligation of one means the right of the other. 'Trust' here means that a road user can, in principle, expect the other road users to observe the rules (cf. Simmelink, 1995). On the grounds of the rule in art. 15 and the principle of trust, $i_2$ has right of way, and $i_1$ does not; on the grounds of the right-of-way signs and the principle of trust, $i_1$ has right of way, and $i_2$ does not. In this
1.3 Conflicting speed limits

formulation $i_1$ both has right of way and has not, which is clearly a case of incompatibility. So, on the basis of art. 63, $i_2$ has to yield to $i_1$. Strictly speaking (considering the letter of the law), there is no incompatibility; when considering the underlying aims, there is indeed incompatibility between signs and rule.

The necessity of the condition 'in as far as specific rules are incompatible with specific signs' in art. 63 lies in the fact that, should this condition be lacking, a traffic rule could suspend all rules in force at that moment, meaning that no rules are in force as long as there are traffic signs, which is absurd, to say the least. For example, $i_1$ drives his car at the speed of 100 km/h within a built-up area, and is halted by a policeman, because $i_1$ is not permitted to drive faster than 50 km/h. $i_1$ can, however, refer to the fact that there was a traffic sign (for example a 'right-of-way' sign) and that this sign overrides the rule, so that the rule (speed limit of 50 km/h) is no longer in force. In the case mentioned above, this would mean that the lorry driver would be in the right, because the sign overrides the rule, which is thus suspended.

1.3.3 Speed limit

Opinions differ as to the question whether rules and signs pertinent to the speed limit are incompatible or compatible, as can be seen from the motivation by the district court judge, which was not in line with Otte and Simmelink's advice. The explanatory memorandum to the Dutch Traffic Regulation 1990 indicates that there is incompatibility in this case:

The systematic structure of art. 63, according to which traffic signs override traffic rules, implies that a different maximum speed indicated by traffic signs - such as 30 or 70 km/h inside a built-up area - need not be incorporated into the rules of this section.

Let us consider the meaning of traffic sign A1, which indicates that a speed limit of 100 km/h holds for that particular part of a motorway. This only means that each car doing over 100 km/h breaks the law. We obey the law if we drive at a speed of 10 or 20 km/h. But this does not mean that we also observe other possible prohibitions or obligations concerning speed, for example that drivers must adjust their speed to the traffic situation or to weather conditions. Taking weather conditions into account may mean that our maximum speed should be 80 km/h, which implies that, when driving at a speed of 90 km/h, we may observe the former prohibition, but not the latter order. This example shows that the prohibition to drive faster than 100 km/h can only mean that every speed under 100 km/h is permitted in as far as one does not have to take other speed orders or prohibitions into account. The prohibition to drive faster than 100 km/h only means that

\[23\text{Bulletin of Acts and Decrees, 1990, 459, Explanatory memorandum, p. 103.}\]

\[24\text{Hereafter, this traffic sign will be referred to as 'A1 (100).'}\]
we will, in any case, break the law if we drive at a greater speed than 100 km/h. This implies that a traffic sign indicating the prohibition of any speed over 100 km/h is not in conflict with the prohibition in the rule to drive at a greater speed than, for example, 80 km/h. For, if it is not permitted to drive faster than 80 km/h, it is certainly not permitted to drive faster than 100 km/h.

This can be illustrated with a simple mathematical example. Assume the following statement to be correct: ‘x is greater than 0 and x is a natural number’. In a logical sense, this statement is equivalent with the statement ‘x is a natural number’. Thus, the statements ‘x is greater than 0’ and ‘x is a natural number’ are not incompatible. The statement ‘x is greater than 0’ is only limited; ‘x is a natural number’ implies that x is greater than 0. x = 4.5 is in agreement with the former, but not with the latter statement. This means that the two statements are not incompatible, since there are values for x that do lead to a true statement. Two statements are incompatible if no value can be found for x that satisfies both statements, such as ‘x is greater than 0’ and ‘x is smaller than 0’.

The prohibition to exceed the speed limit of 100 km/h clearly does not imply that it is by definition permitted to drive at a speed of 96 km/h: one has to take other speed prohibitions and orders into account, such as art. 25 of the Traffic Act. It would be absurd to stick to this meaning when one considers the following example. There is a traffic jam on the motorway, and a car is driving at a speed of 100 km/h. The driver breaks the law in this situation, and he cannot maintain that it was permitted to drive at a speed of 100 km/h on this section of the road. The High Court and the Advocate-General are in agreement on the meaning of the prohibition to exceed the 100 km per hour limit in the case presented above.

What are the consequences of traffic sign A1 (100) for car drivers? On motorways outside built-up areas, a speed limit of 120 km/h is in force. On the approach of the A1 (100) traffic sign, the maximum speed of 100 km/h holds for car drivers, not, however, because the traffic rule is incompatible with the traffic sign. The speed limit of 120 km/h still holds, because cars are still not permitted to drive faster than 120 km per hour. The prohibition is, however, restricted by another prohibition, namely the prohibition to drive faster than 100 km per hour. The traffic sign has a speed-reducing effect on the rule. For lorry drivers traffic sign A1 (100) has no meaning: for them this sign carries superfluous information, because they are not permitted to drive faster than 80 km/h, so not faster than 100 km/h anyway. The result is that art. 63 of the Dutch Traffic Regulation 1990 can never be applicable as regards speed limits, because speed limits do not conflict.

---

25Art. 25 of the Traffic Act: On the road, it is prohibited to act in such a way that the freedom of traffic is hindered without necessity or that road security is jeopardised or may reasonably be expected to be jeopardised.

26Traffic signs and traffic rules can only be incompatible in situations concerning right of way and changing lanes.
1.3.4 Undesirable consequences

The motivation by the district court judge, which was supported by the High Court, poses problems, however. In many cities, so in built-up areas, there are, for example, A1 traffic signs on circular roads, indicating that it is prohibited to drive faster than 70 km/h. Many drivers will, and justifiably, take this sign to mean that it is permitted to drive at a speed of 70 km/h. This is, however, in disagreement with the motivation by the district court judge and the Advocate-General. According to art. 20 of the Dutch Traffic Regulation 1990, a speed limit of 50 km per hour holds for motor vehicles inside built-up areas. Traffic sign A1 (70) does not imply that on that particular section of the road the minimum speed is 51 km/h. So, there is no conflict here between traffic rules and traffic signs, with the result that traffic sign A1 (70) is totally superfluous and has no other meaning than that it is not permitted to drive at a greater speed than 70 km/h. This we already knew, because it is not permitted to drive faster than 50 km/h anyway. In this case, the district court judge will reason as follows: ‘Traffic sign A1 (70) is a regulatory sign (meaning that it is prohibited to drive faster than 70 km/h) and does not imply that the speed limit in force for motor vehicles in built-up areas is raised.’

There is a similar problem in the case of national routes and motorways inside built-up areas. From the above it follows that it is prohibited to drive faster than 50 km/h on such roads, because they are inside built-up areas. It was, of course, the intention of the legislator to indicate that the prohibition to drive faster than 50 km/h is no longer in force, and that on such roads it is prohibited to drive faster than 100 or 120 km/h, respectively. In the Dutch Traffic Regulation 1990, however, the legislator did not define the speed limit on national routes and motorways inside built-up areas.27

1.3.5 An alternative proposal

In art. 22 of the Dutch Traffic Regulation 1990, the following, superfluous information is given: ‘... in as far as no lower speed limits have been set in other articles’. It would have been sufficient to state: ‘For the following motor vehicles the following speed limits hold: a. for lorries, etc.’ The motor vehicles mentioned will also have to observe lower speed limits, because they belong to the category of motor vehicles.

Section 8 of the Dutch Traffic Regulation 1990 could be amended as follows, which would solve the problems discussed above. The proposed article will thus replace arts. 20, 21 and 22.

The following speed limits hold:

a. for motor vehicles on motorways 120 km/h, on national routes 100 km/h and on other roads 80 km per hour;

27Cf. arts. 20 and 21. The maximum speed limit on motorways has only been set outside built-up areas.
b. for lorries, buses and motor vehicles with trailers 80 km per hour;
c. for mopeds and motorised wheelchairs inside built-up areas 30 km/h and outside built-up areas 40 km/h;
d. for tractors and construction vehicles 25 km/h.

The Dutch Traffic Regulation 1990 distinguishes between driving inside and outside built-up areas. This aspect is largely lost in the alternative proposal. The standard 'it is prohibited to drive faster than 50 km/h' on, for instance, motorways inside built-up areas, is left out. This alternative has two advantages: first, the rules can be applied consistently, and second, the legislator's wish is expressed in a clearer way. The legislator's intention is stated precisely in these rules. In the Dutch Traffic Regulation 1990, this is definitely not the case.

Furthermore, traffic signs A1 will have to be given a different meaning. In my proposal they contain three components:

1. A different speed limit is indicated.

2. A possibly different speed limit is lifted. Traffic sign A1 thus also contains the meaning of traffic sign A2, with respect to a previous, possibly different speed limit.

3. For as long as no different speed limit is indicated, the present speed limit holds. These traffic signs therefore apply to zones.

We have seen that these different speed limits do not conflict with the speed limits according to the traffic rules in the proposed art. 20. The speed limits, as indicated in art. 20, remain in force. In the alternative proposal, different speed limits are provided for by traffic signs A1, and are, by definition, only applicable to those road users for whom this implies a lower speed limit according to art. 20. For road users for whom a speed limit holds according to art. 20, a traffic sign A1 indicating a higher speed limit is superfluous, but no contradicting information.

1.3.6 Emergency service vehicles

There is one case in which art. 63 has a nasty consequence, as Otte and Simmelink (1993) described in their 'Kronkels in het RVV 1990', 'The regulation concerning so-called "emergency service vehicles"', with regard to art. 50 of the Dutch Traffic Regulation 1990.

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28 Only for mopeds and motorised wheelchairs, the distinction between inside and outside built-up areas remains.

29 Traffic sign A2 means: 'end of speed limit'.

30 Art. 50 of the Dutch Traffic Regulation 1990: Road users are obliged to give way to drivers of emergency service vehicles.
They discuss the following situation. A passenger car driving on a major road approaches a junction, and at the same time a police car with flashing light and sirens approaches from the right. On the ground of a traffic rule (art. 50 of the Dutch Traffic Regulation 1990), the driver of the passenger car has to give way to the driver of the police car, but on the ground of the right-of-way signs, the driver of the police car has to give way to the driver of the passenger car. As signs and rule are incompatible in this case, the driver of the passenger car has right of way on the ground of art. 63. Otte and Simmelink, therefore, conclude that the position of emergency service vehicles is not adequately provided for.

It is clearly the intention of art. 50 that police cars, fire engines and ambulances with operating signals should always have right of way, but this cannot be concluded from the systematic structure of the Dutch Traffic Regulation 1990. This problem can easily be solved by replacing art. 50 by:

Yielding right of way by road users to drivers of emergency service vehicles overrides traffic lights and traffic signs and rules that regulate right of way.

This sentence could, systematically, best be incorporated in the section ‘General provisions’ of chapter 3 of the Dutch Traffic Regulation 1990, which also contains art. 63.

1.3.7 Conclusion

We may conclude that rules and signs that provide speed limits can never be in conflict. The ruling of the district court judge in the case discussed is correct, but leads to undesirable situations concerning the speed limit inside built-up areas. As a consequence, the letter of the law with regard to the speed limit is not in agreement with the spirit of the law in the Dutch Traffic Regulation 1990. This does not contribute to the credibility of the law. The reason for this lies in the distinction that is made between driving inside and outside built-up areas, and the lack of clarity about the notion of incompatibility. By means of the solution given above and by clarifying the notion of ‘incompatibility’ the problems concerning speed limits will be solved.

A very undesirable situation also occurs in the application of arts. 50 and 63 Dutch Traffic Regulation 1990, but here too a simple adjustment of art. 50 suffices.

1.4 The structure of this thesis

In this thesis, we will address the problems indicated in the previous section by extending the two standard deontic logics.

In chapter 2, we discuss system $PD_L$, which can formalise Ought-to-do statements and some modifications of this system. In chapters 3 and 4, relativised deontic modalities are investigated in $SDL$ and $PD_L$, respectively. We will see that the addition of actors and
groups of actors gives new expressive power, and the formulas of these relativised deontic logics acquire new meanings, not expressible in SDL and PD_. They are, therefore, subject to new intuitions. The systems developed in chapters 3 and 4 will be extended to authorities in chapter 5. Authorities are responsible for the establishment of norms and supervising the enactment of the norms. With the addition of authorities to a deontic system, we can consistently express conflicting norms enacted by (sets of) authorities. However, this does not determine which norm should be followed in cases of conflicting norms: this will be discussed in chapters 6 and 7. In chapter 6, we introduce the term ‘authority hierarchy’ to overcome this problem. The authorities are used to prioritise the norms they enacted. Chapter 6 also discusses some related issues such as promulgation, derogation and universality. Chapter 7 presents a defeasible deontic reasoning formalism based on preferences, which can deal with conditional norms and inconsistencies between interpretation rules, in contrast to the theory developed in chapter 6, to treat conflicting norms. Finally, in chapter 8, we apply the concepts - investigated in chapters 2 up to and including 7 - to some cases related to the case of ‘conflicting speed limits’, discussed in the present chapter, and some areas for future research are indicated.
Chapter 2

Propositional deontic logic

In this chapter, we will discuss the system $PDeL$ (Propositional Deontic Logic), developed by Meyer, and some modifications to this system. The system will be extended to include actors and authorities, in chapters 4 and 5, respectively.

2.1 Introduction

The system $PDeL$ (Propositional Deontic Logic) was developed by Meyer (1988, 1989). Meyer defined $PDeL$ as a modal logic, following an article by Anderson (1967). The basis for $PDeL$ is the logic framework of (propositional) dynamic logic. The reduction of deontic operators to dynamic ones uses Anderson’s violation atom $V$ to indicate that an action took place that violates one of the deontic constraints, i.e., that the performance of a forbidden action leads to a bad state of affairs. A bad state of affairs can be, e.g., a sanction, a liability to sanction or trouble. What exactly the consequences are of a bad state of affairs is another matter, and depends on the philosophy one adheres to. Our interpretation of the constant $V$ is: the situation is in contravention of the law. Whether it leads to a sanction will be left aside here.

(Propositional) dynamic logic (e.g., Harel, 1984) consists of the normal propositional language extended with modal operator $[\beta]$ for every action $\beta$ in the language. Expression $[\beta]\phi$ means that $\phi$ holds after $\beta$ has been performed. The essential reason to describe deontic logic as a variant of dynamic logic is that now actions and assertions can be strictly separated, because:

- 'an action may change the current situation (world) and an assertion does not' (Meyer, 1988, p. 109);
- 'only assertions can be asserted and actions can be acted or performed. So it is meaningless to state the obligation $O\phi$ of some proposition $\phi$, such as $OO\alpha$, where
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\[ \phi \text{ is taken to be the assertion stating that the action } \alpha \text{ is obligatory} \] (Meyer, 1988, p. 109);

- 'the fact that actions change situations implies some notion of passing time' (Meyer, 1988, p. 109).

We can see \( [\beta] \Phi \) as a refined version of \( \beta \to \Phi \) in traditional deontic logic, the difference being that actions and assertions are strictly separated, and a notion of time-lag is built in. In this approach, \( \beta \) is forbidden \( F(\beta) \) is reduced to a dynamic expression as follows:

\[ F(\beta) \equiv [\beta] V. \]

It is also easy to express the deontic modalities of 'permission' and 'obligation' in dynamic logic, once so-called negated actions (e.g. \( \overline{\beta} \)) have been added to the syntax of actions.

In this chapter, we present a formal introduction to the syntax and semantics of \( PD_e L \): section 2.2 presents the language of actions. The language of \( PD_e L \) assertions and their formal semantics, and the formal system \( PD_e L \) itself are discussed in section 2.3. Section 2.4 discusses the semantics of the deontic assertions starting from some well-known paradoxes. In section 2.5, we present a simplified version of \( PD_e L \) used in this thesis. Finally, we draw some conclusions.

2.2 Actions

Actions are the semantic counterparts of the action expressions used in dynamic logic (and thus used in \( PD_e L \)). The semantics of an action expression is a set of sets of actions. We will come back to this in subsection 2.2.3.

2.2.1 The syntax of actions

We define \( A \) as the set of action symbols. An atomic action is denoted by an underlined action symbol \( (a) \). \( \mathcal{A} \) is the set of atomic actions. Furthermore, we introduce a special action symbol \( \text{skip} \), which is not an element of \( A \) and a special action symbol \( \delta \), which is neither an element of \( A \), and which models failure. Together, they constitute the set of semantic elementary actions.

\[ \text{A deontic system in which time is a central notion is given by Van Eck (1982). The main difference between the temporal treatment in Van Eck's system and } PD_e L \text{ is the definition of the accessibility relation. Van Eck defines this relation between worlds within one time-slice, and in } PD_e L \text{ the relation is defined between worlds with different 'time-stamps'.} \]

\[ \text{The skip symbol, introduced by Dignum (1989, p. 188), is comparable with the } \epsilon \text{ process in process algebra.} \]
2.2 Actions

The set of all action expressions \( \text{Act} \) can now be determined by the following BNF for its elements (\( \beta \))

\[
\beta ::= a | \beta_1 \cup \beta_2 | \beta_1 & \beta_2 | \beta_1 ; \beta_2 | \overline{\beta} | \text{any} | \text{fail} | \text{skip} | \text{change}.
\]

The meaning of \( \beta_1 \cup \beta_2 \) is a choice between \( \beta_1 \) and \( \beta_2 \), \( \beta_1 & \beta_2 \) stands for the simultaneous performance of \( \beta_1 \) and \( \beta_2 \), \( \beta_1 ; \beta_2 \) stands for the sequential composition of \( \beta_1 \) and \( \beta_2 \), and \( \overline{\beta} \) stands for the negation of action expression \( \beta \). The \text{any} action expression indicates a universal or 'don't care which' action. The \text{change} action expression can then be characterised as a 'don't care which, but not the skip' action. The \text{skip} action expression stands for the empty action; that is, the action that has no effect ('does nothing'). Finally, the \text{fail} action expression expresses the action that always fails. After this action, the system stops and nothing can be done any more.

The performance of an atomic action \( a \) (e.g., to walk) involves the performance of a (semantic) elementary action \( a \) (to walk), possibly together with other elementary actions (e.g., to whistle, to look, etc.), and followed by an arbitrary performance of future actions (e.g., to run, to eat, etc.). Thus, we have to be careful not to confuse an atomic action (expression) \( a \) with the semantic elementary action \( a \). The meaning (denotation) of \( a \) only stipulates (specifies) the performance of a corresponding semantic \( a \), but we are free to perform any other set of elementary actions simultaneously with \( a \).

2.2.2 The semantics of action expressions

We give the semantics of action expressions by sets of sequences of what are called synchronicity sets. These sets denote sets of elementary actions that are performed simultaneously. The following definitions formally describe the synchronicity sets, the way in which they can be combined to form sets of sequences of synchronicity sets and the semantic counterparts of the syntactic operators on action expressions.

**Definition 2.2.1**

- Set \( \{ \delta \} \) is a synchronicity set (s-set).
- Set \( \{ \text{skip} \} \) is an s-set.
- Every non-empty subset of \( A \) is an s-set.

We will use \( S, S_1, S_2, \ldots, S', \ldots \) to denote s-sets. The powerset of s-sets with actions in \( A \) will be denoted by \( S \) (the set of non-empty s-sets). In concrete cases, we will write such a set using brackets. Thus, the s-set consisting of the action \( \text{skip} \) is written as \( [\text{skip}] \) and the s-set consisting of actions \( a_1 \) and \( a_2 \) is written as

\[
S = \left[ \begin{array}{c} a_1 \\ a_2 \end{array} \right],
\]
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\[ S = \{ [a_1], [a_2], [a_3], [a_1, a_2], [a_1, a_3], [a_2, a_3], [a_1, a_2, a_3] \}, \]

with \( A = \{ a_1, a_2, a_3 \} \). Note that \( S \) does not contain s-set \([\delta]\) nor s-set \([\text{skip}]\).

Definition 2.2.1 prevents the simultaneous performance of the special actions \( \text{skip} \) and \( \delta \) with other actions, because they are not in \( A \). This is needed because it is, of course, not possible to perform an action and at the same time do nothing, or have a deadlock.

To denote the courses of performances of actions, we use sequences of s-sets, which we shall call synchronicity sequences. These sequences can be finite or infinite, although in practice they will usually be finite. The definition of a sequence is as follows:

**Definition 2.2.2** A synchronicity sequence (s-sequence) is a finite or infinite sequence \( S_1 S_2 \ldots S_n \ldots \) of s-sets \( S_i \).

\( e \) stands for the empty sequence.

Only the last s-set of an s-sequence may be \([\delta]\).

We refer to the number of s-sets in an s-sequence \( t \) as the length of \( t \), denoted by \( l(t) \).

\( l(e) = 0 \). (It is possible that \( l(t) = \infty \).)

We use \( t, t_1, t_2, \ldots t', \ldots \) to denote s-sequences. In a particular case, we can write

\[
t = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix},
\]

with \( l(t) = 3 \).

By defining sequences of s-sets, obviously it must be possible to concatenate these sequences. This is possible by using the \( \circ \) operator, which is defined as follows:

**Definition 2.2.3** Let \( t = S_1 \ldots S_n \) and \( t' = S'_1 \ldots S'_m \ldots \) be two s-sequences (\( t' \) possibly infinite), then

\[
t \circ t' = \begin{cases} t, & \text{if } S_n = [\delta] \\ S_1 \ldots S_n S'_1 \ldots S'_m \ldots & \text{if } S_n \neq [\delta] \end{cases}
\]

If \( t \) is an infinite s-sequence, then \( t \circ t' = t \) for any s-sequence \( t' \).

\( t \circ e = e \circ t = t \).

Note that \([\delta] \circ t = [\delta]\). Since the language of action expressions contains a choice operator introducing non-determinism, we have to consider sets of s-sequences as the semantics of an action expression. Moreover, non-determinism is introduced by using the open specification of actions,
as we have seen in the intended semantics of \( a \). That is, \( a \) indicates a choice between all possible actions in which \( a \) is at least performed. Each s-sequence in such a set stands for a possible choice of a sequence of synchronicity sets. We use \( T, T_1, T_2, \ldots, T', \ldots \) to denote sets of s-sequences.

Domain \( A \) of the action model is now given by:

**Definition 2.2.4** \( A \) is the collection of sets \( T \) consisting of s-sequences.

We also define the length and concatenation of sets of s-sequences.

**Definition 2.2.5** The length of a set of s-sequences \( T \), denoted by \( l(T) \), is defined as follows:

\[
l(T) = \max_{t \in T} (l(t)).
\]

We use \( l(T, T_1) \) to denote that \( l(T) = l(T_1) = 1 \).

**Definition 2.2.6** Let \( T \) and \( T' \) be sets of s-sequences. Then \( T \circ T' \) is defined as the set of s-sequences \( \{ t \circ t' \mid t \in T, t' \in T' \} \).

Note that \( T \circ \{ \varepsilon \} = \{ \varepsilon \} \circ T = T \) and that \( T \circ \{ [\delta] \} = \{ t \circ [\delta] \mid t \in T \} \) and \( \{ [\delta] \} \circ T = \{ [\delta] \} \).

**Example 2.2.7** Let \( S_1, S_2, S_3 \) be s-sets, \( \forall i, j \in \{1, 2, 3\} \land i \neq j \) \((S_i \neq S_j) \land (S_i, S_j \neq [\delta])\), \( T_1 = \{ S_1S_2S_3[\delta], S_1S_2S_3[\delta], S_1S_2S_3[\delta], S_1S_2S_3[\delta] \} \) and \( T_2 = \{ S_1[\delta], S_1S_2 \} \), then

1. \( l(T_1) = 4 \);
2. \( l(T_2) = 2 \);
3. \( T_1 \circ T_2 = \{ S_1S_2S_3[\delta], S_1S_2S_3S_1[\delta], S_1S_2S_3S_1S_2, S_2S_3S_1[\delta], S_2S_3S_1S_2 \} \);
4. \( l(T_1 \circ T_2) = 6 \).

To give the denotation for all action expressions in \( \text{Act} \), we define operations \( \cup, \cap \) and \( \sim \) on domain \( A \), which we will use as semantic counterparts of the syntactic operators \( \cup, \& \) and \( \sim \), respectively, in the language \( \text{Act} \). We first present some definitions that will help in defining these operators. The first definition is of a function \( \text{pref} \) that gives all the prefixes of a given s-sequence.

**Definition 2.2.8**

\[
\text{pref}(t) = \{ t' \mid t' \circ t'' = t \}.
\]

\( \varepsilon \) is an element of the \( \text{pref} \) of any s-sequence.
Example 2.2.9

\[
\text{pref}([a_1][a_2][a_3][a_4]) = \{\epsilon, [a_1], [a_1][a_2], [a_1][a_2][a_3][a_4]\}.
\]

The next function defines the longest common prefix of two s-sequences.

**Definition 2.2.10** Let \( t = S_1...S_n \) and \( t' = S'_1...S'_m \) be two s-sequences. Then \( \text{maxpref}(t, t') \) is the longest s-sequence \( t'' \) such that \( t'' \in \text{pref}(t) \) and \( t'' \in \text{pref}(t') \). (Note that if \( S_1 \neq S'_1 \), \( \text{maxpref}(t, t') = \epsilon \).)

Finally, we define an operator on sets of s-sequences which deletes s-sequences ending in \([\delta]\) if there is another sequence that is the same but with \([\delta]\) replaced by another s-sequence.

**Definition 2.2.11** Let \( T \) be a set of s-sequences. Then

\[
T^\delta = T \setminus \{t | t = t' \circ [\delta] \land \exists t'' \in T \{t'' \neq t \land t' \in \text{pref}(t'')\}\}.
\]

This operator is closely related to what is called failure removal in De Bakker, Kok, Meyer, Olderog and Zucker (1986). The idea is that failure is avoided if possible, i.e., if there is a non-failing alternative. Such an interpretation of non-determinism, where a 'good' alternative is preferred to a 'bad' one, is sometimes called angelic non-determinism (cf. Broy, 1986).

**Example 2.2.12** Using example 2.2.7, it holds that

- \( T_1^\delta = \{S_1S_2S_3S_2, S_2S_2S_3\} \);
- \( T_2^\delta = \{S_1S_2\} \);
- \( (T_1 \circ T_2)^\delta = \{S_1S_2S_3S_2S_1S_2, S_2S_2S_3S_1S_2\} \).

Now we can define the semantic operators on \( A \).

**Definition 2.2.13**

- \( \sqcap \) on s-sequences is defined by:
  
  For s-sequences \( t_1 \) and \( t_2 \):

  \[
  t_1 \sqcap t_2 = \begin{cases} 
  t_1, & \text{if } \text{maxpref}(t_1, t_2) = t_2 \\
  t_2, & \text{if } \text{maxpref}(t_1, t_2) = t_1 \\
  \text{maxpref}(t_1, t_2) \circ [\delta], & \text{otherwise}
  \end{cases}
  \]
• \( \cap \) on sets of s-sequences in \( \mathcal{A} \) is defined by:

For s-sequences \( T_1 \) and \( T_2 \) in \( \mathcal{A} \),

\[
T_1 \cap T_2 = \left( \bigcup_{t_1 \in T_1, t_2 \in T_2} \{ t_1 \cap t_2 \} \right)^{\#}.
\]

Definition 2.2.14 \( \cup \) on sets of s-sequences in \( \mathcal{A} \) is given by:

For \( T_1, T_2 \in \mathcal{A} \),

\[
T_1 \cup T_2 = \left( \left( (T_1 \cup T_2) \setminus (T_1 \cap T_2) \right) \cup (T_1 \cap T_2) \right)^{\#}.
\]

Definition 2.2.15 \( \sim \) is defined as follows:

1. For an s-set \( S \),

\[
\tilde{S} = (S \cup \{ [\text{skip}] \}) \setminus S.
\]

2. For a non-empty s-sequence \( t = S_1 \ldots S_m \ldots \),

\[
\tilde{t} = \{ S_1 \ldots S_{n-1} \tilde{S}_n \mid S_1 \ldots S_{n-1} S_n \in \text{pref}(t) \}.
\]

3. For a non-empty set \( T \in \mathcal{A} \),

\[
\tilde{T} = \cap \{ \tilde{t} \mid t \in T \}.
\]

Note that \( \tilde{S} = [\delta] \).

As was suggested in their notation, \( \cap \) and \( \cup \) resemble ordinary set-theoretical intersection \( \cap \) and union \( \cup \). The only difference is that the former two operators account for the prefixes of the s-sequences involved. Operator \( \cap \) of two argument sets of s-sequences that are not equal, but one is the prefix of the other, yields the one with the longest sequence, and if neither of the s-sequences is a prefix of the other, we take their maximal common prefix, after which we register a conflict between the sequences by putting \([\delta]\) at the end of the maximal common prefix, since the rest of the sequences cannot be performed simultaneously. Operator \( \cup \) of two argument sets of s-sequences, one of which is a proper prefix of the other, yields only the shortest.

The definition of operator \( \sim \) consists of three clauses. Clause (1) is straightforward: if we want to indicate that an s-set \( S \neq [\delta] \) is not involved, we may express this by considering all other s-sets in \( \mathcal{S} \), and if we want to indicate that one does not fail, any s-set is possible. Clause (2) expresses that if we want to say that an s-sequence \( t \) is not involved, we have to consider all s-sequences \( t' \) with \( l(t') \leq l(t) \), where the performance of s-sets according to s-sequence \( t \) are followed up to some moment and then contain a different s-set in \( \mathcal{S} \cup \{ [\text{skip}] \} \). Clause (3) states that the negation of a set \( T \) of s-sequences considers all negations \( \tilde{t} \) of s-sequences \( t \in T \) and takes the \( \cap \)-intersection of these sets.

To obtain a better understanding of these semantic operators on \( \mathcal{A} \), we will give some examples.
Example 2.2.16 Let $A = \{a_1, a_2\}$. Then

1. $\{[a_1]\} \cap \{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\} =$

   $\{(\{[a_1] \cap \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}) \cup (\{[a_1] \cap [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\})\}^\delta = \{[\delta], [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}^\delta =$

   $\{[a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\};$

2. $\{[a_1]\} \cup \{[a_1]\} = (((\{[a_1]\} \setminus \{[a_1]\}) \cup \{[a_1]\})^\delta = \{[a_1]\};$

3. $\{[a_1]\} \cup \{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\} =$

   $((\{[a_1], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}) \setminus (\{[a_1]\} \cap \{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\})) \cup$

   $\{(\{[a_1]\} \cap \{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\})^\delta =$

   $((\{[a_1], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, [a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}) \setminus \{[a_1] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}) \cup \emptyset)^\delta = \{[a_1], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\};$

4. $\{[a_1]\}^- = (S \cup \{\text{skip}\}) \setminus \{[a_1]\} = \{\text{skip}, [a_1], [a_2], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\} \setminus \{[a_1]\} =$

   $\{\text{skip}, [a_2], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\};$

5. $[\delta]^- = S \cup \{\text{skip}\};$

6. $\{[a_1], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}^- = ([a_1])^- \cap \{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\}^- =$

   $\{\text{skip}, [a_2], \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\} \cap \{\text{skip}, [a_1], [a_2]\} =$

   $\{\epsilon \circ [\delta], [\text{skip}], [a_2]\}^\delta = \{\text{skip}, [a_2]\}.$

2.2.3 S-sequence semantics of action expressions

With the above-mentioned definitions, we can now define the semantics of action expressions from Act.

Definition 2.2.17 Semantic function $\llbracket \rrbracket \in \text{Act} \rightarrow A$ is given by:

1. $[a] = \{S \in S | a \in S\};$
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2. \([\beta_1; \beta_2] = [\beta_1] \circ [\beta_2]\);

3. \([\beta_1 \cup \beta_2] = [\beta_1] \cup [\beta_2]\);

4. \([\beta_1 \& \beta_2] = [\beta_1] \cap [\beta_2]\);

5. \([\overline{\beta}] = [\beta]^{-}\);

6. \([\text{skip}] = \{[\text{skip}]\};\)

7. \([\text{fail}] = \{[\delta]\};\)

8. \([\text{any}] = S \cup \{[\text{skip}]\};\)

9. \([\text{change}] = S.\)

Remark. The meaning of action expression \(a\) is expressed by \([a]\): we specify the performance of elementary action \(a\) (simultaneous with some package of actions). Thus, only the performance of \(a\) is determined. Clause (8) expresses that in the denotation of action expression \(\text{any}\) the execution of one arbitrary \(s\)-set in \(S \cup \{[\text{skip}]\}\) is specified.

Example 2.2.18 Let \(A = \{a_1, a_2, a_3\}\). Then

- \([a_1] = \{[a_1], [a_1, a_2], [a_1, a_3], [a_1, a_2, a_3]\};\)

- \([a_2] = \{[a_2], [a_2, a_3], [a_2, a_3, \text{skip}]\};\)

- \([a_1 \& a_2] = \{[a_1], [a_2], [a_1, a_2], [a_2, a_3], [a_1, a_2, a_3]\};\)

- \([a_1 \cup a_2] = \{[a_1, a_2], [a_1, a_3], [a_2, a_3], [a_1, a_2, a_3]\};\)

- \([\text{any}] = \{[\text{skip}], [a_1], [a_2], [a_3], [a_1, a_2], [a_1, a_3], [a_2, a_3]\};\)

- \([a_1 \& a_2]; a_1 \cup a_2] = \{[a_1, a_2], [a_1, a_3], [a_1, a_2, a_3]\};\)

- \([a_1 \& a_2]; a_1 \cdash a_2] = \{[a_1, a_2], [a_1, a_3], [a_1, a_2, a_3]\};\)
In this section, we also introduce an auxiliary notion that we need to be able to refer to the length of action expressions and the equality of action expressions in terms of the length and the equality in the semantics of the action expressions.

**Definition 2.2.19** Let $\beta \in \text{Act}$. The *length* of $\beta$ is defined as $l(\beta) = l([\beta])$.

**Definition 2.2.20** We put $\beta_1 =_A \beta_2$ iff $[\beta_1] = [\beta_2]$.

We can now state the following proposition concerning actions and their relations:

**Proposition 2.2.21**

- **For $A$, the following properties concerning $\cup$ and $\&$ operator hold:**
  1. $\beta \cup \beta =_A \beta$;
  2. $\beta \& \beta =_A \beta$;
  3. $\beta_1 \& \beta_2 =_A \beta_2 \& \beta_1$;
  4. $\beta_1 \cup \beta_2 =_A \beta_2 \cup \beta_1$;
  5. $\beta_1 \& (\beta_2 \cup \beta_3) =_A (\beta_1 \& \beta_2) \cup (\beta_1 \& \beta_3)$;
  6. $\beta_1 \cup (\beta_2 \& \beta_3) =_A (\beta_1 \cup \beta_2) \& (\beta_1 \cup \beta_3)$.

- **$A$ satisfies the following properties regarding $;'':**
  1. $\beta_1 \cup (\beta_1; \beta_2) =_A \beta_1$;
  2. $\beta_1 ; (\beta_2; \beta_3) =_A (\beta_1 ; \beta_2) ; \beta_3$, if $l([\beta_1], [\beta_2])$;
  3. $(\beta_1 ; \beta_2) ; (\beta_3 ; \beta_4) =_A (\beta_1 ; \beta_3) ; (\beta_2 ; \beta_4)$, if $l([\beta_1], [\beta_3])$;
  4. $(\beta ; \beta_1) \cup (\beta ; \beta_2) =_A \beta ; (\beta_1 \cup \beta_2)$;
  5. $(\beta_1 ; \beta) \cup (\beta_2 ; \beta) =_A (\beta_1 \cup \beta_2) ; \beta$, if $l([\beta_1], [\beta_2])$.

- **$A$ satisfies the following properties regarding $\rightarrow$:**
  1. $\overline{\beta} \cup \beta =_A \text{any}$ if $l(\beta) = 1$;
  2. $\overline{\beta} \& \beta =_A \text{fail};$
  3. $\overline{\beta_1} \cup \overline{\beta_2} =_A \overline{\beta_1 \& \beta_2}$;
  4. $\overline{\beta_1} \& \overline{\beta_2} =_A \overline{\beta_1 \cup \beta_2}$;
  5. $\overline{\beta} =_A \beta$;
  6. $\overline{\beta_1} ; \overline{\beta_2} =_A \overline{\beta_1 \cup (\beta_1 \& \beta_2)}$. 

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- For $A$, the following properties concerning the special action expressions hold:

  1. $\beta \cup \text{fail} =_A \beta$;
  2. $\text{any} =_A \text{fail}$;
  3. $\beta \cup \text{any} =_A \text{any}$;
  4. $\text{any} =_A \text{change} \cup \text{skip}$;
  5. $\text{skip} =_A \text{change}$.

**Proof.** Can directly derived from the definitions. See also Meyer (1988) and Dignum (1989).

**Corollary 2.2.22** Let $A_1$ be a subset of $A$ such that for all $T \in A_1$ it holds that $l(T) = 1$, then $(A_1, \cup, \&,-, \text{fail})$ is a Boolean algebra.

Below, we define a normal form for action expressions. This form is needed for some axioms on the postconditions of actions. These axioms only hold if the action expressions are in normal form (cf. Dignum and Meyer, 1990).

**Definition 2.2.23** An action expression $\beta \in \text{Act}$ is said to be in normal form (or to be a normal action) if the following holds:

Every subexpression of $\beta$ of the form $\beta_1 \cup \beta_2$ or $\beta_1 \& \beta_2$ has the property that $\cup\{t_1 \cap t_2 | t_1 \in [\beta_1], t_2 \in [\beta_2] \text{ and } t_1 \neq t_2\} = \emptyset$, i.e., $[\beta_1]$ and $[\beta_2]$ have no intersection consisting of $s$-sequences that do not occur in both sets.

The subclass of $\text{Act}$ consisting of action expressions in normal form will be denoted by $\text{Act}_0$.

The normal form excludes mainly sequential compositions that are a union of two action expressions, one of which is a prefix of the other. The simplest example hereof is $\alpha_1 \cup (\alpha_1; \alpha_2)$. Fortunately, the following theorem holds for $A$:

**Proposition 2.2.24** For any action expression $\beta \in \text{Act}$, an action expression $\beta_1 \in \text{Act}_0$ exists such that $\beta =_A \beta_1$.

**Proof.** This is clear from proposition 2.2.21.

Note that if $\beta_1 \cup \beta_2$ is in normal form, then $[\beta_1 \cup \beta_2] = [\beta_1] \cup [\beta_2]$. And also, if $\beta_1 \& \beta_2$ is in normal form, then $[\beta_1 \& \beta_2] = [\beta_1] \cap [\beta_2]$. 
In this subsection, we discuss some special actions: the positive and negative actions. We will use these special actions in chapter 4. There we will see that it makes a difference whether a group of actors performs a positive or a negative action. A positive action is an action that involves a certain kind of physical activity: a bodily movement and muscular activity, whereas a negative action refers to refraining from a physical activity: an omission (cf. Brouwer, 1990, p. 205). For instance, ‘to move the table’, ‘to close the window’, ‘to walk’, ‘to overtake’, etc., are all positive actions and ‘not move the table’, ‘to be silent’, ‘not walk’, etc., all constitute negative actions. However, the difference between positive and negative actions is not clear cut, because (1) in several cases it is difficult to decide whether an action is positive or negative, and (2) a negative action is a non-action and, at the same time, a ‘mode of action or conduct’.

Omissions are imputed to actors, and have to be taken into account in the dynamic variant of the deontic logic that treats omission as different from merely by not performing something. In PD_vL, we define omissions (negative actions) in terms of not-performing and in terms of the notions of ability and opportunity (cf. Van Linden, Van der Hoek and Meyer, 1996). In this view, someone omitted (neglected) to do something if he could have done it, but did not do it.

**Definition 2.2.25** We define

1. the set Act_p of positive action expressions by the following BNF for its elements (γ):

   \[ γ ::= a|γ_1 \cup γ_2|γ_1 \& γ_2|any|change, \]

   with \( γ_1, γ_2 \in Act_p \), and

2. the set Act_n of negative action expressions by the following BNF for its elements (γ):

   \[ γ ::= a|γ_1 \cup γ_2|γ_1 \& γ_2|fail|skip, \]

   with \( γ_1, γ_2 \in Act_n \).

Note that

- \( Act_p \cup Act_n \subseteq Act \). The converse \((Act_p \cup Act_n \supseteq Act)\) does not hold because, for instance, \( a_1 \& a_2 \in Act \), but is not an element of \( Act_p \cup Act_n \);
- \( Act_p \cap Act_n = \emptyset \);
- \( γ \in Act_n \) iff \( \overline{γ} \in Act_p \). This follows immediately from the definition.
2.2.5 Actions and worlds

We have to relate the action expressions in Act to worlds in which they are performed. Intuitively, the performance of an action in a world yields (a collection of) world(s) in which one arrives after having performed the action. The semantics of action expressions in a certain world can be given informally as follows.

Imagine a world \( w \) in which certain assertions hold. Then, by performing an elementary action \( a \) one moves into a next world \( w' \). In world \( w' \), other assertions may hold than in \( w \), since \( a \) may have changed something. For instance, if in \( w \) the proposition

\[ \phi = 'the window is open' \]

is true and \( a \) is the action 'close the window', then proposition \( \phi \) clearly does not hold any longer in \( w' \), but

\[ \phi' = 'the window is closed' \]

does. We can present this as follows:

\[
\begin{array}{c}
\text{[a]}
\end{array}
\]

\[
\begin{array}{c}
w \quad w' \\
\models \phi & \models \phi'
\end{array}
\]

If one moves into a next world, other assertions may hold, except if one performs the action \( \text{skip} \), because in that case nothing changes. (The action \( \text{skip} \) does not change any assertions in a world.)

\[
\begin{array}{c}
\text{[skip]}
\end{array}
\]

\[
\begin{array}{c}
w \quad w \\
\end{array}
\]

In general, \( \beta \) may lead one into one of several, possible worlds, due to the fact that we have a choice operator \( \cup \) in our language. Thus, \( \beta \) may map \( w \) into a set \( W_{\beta,w} \) of worlds. Schematically:
Now we can give the semantics of the various operators regarding $Act$:

- $(\beta_1; \beta_2)$: this simply stands for performing $\beta_1$ first and $\beta_2$ next. For elementary actions $a_1$ and $a_2$, we can present this as follows:

$$
\begin{array}{c}
\bullet \bullet \\
[ a_1 ] \\
\downarrow \quad \downarrow \\
w \quad w' \\
[ a_2 ] \\
\downarrow \\
w''
\end{array}
$$

In general, however, we get:

$$
\begin{array}{c}
w \\
\bullet \\
[ \beta_1 ] \\
\downarrow \\
W_{\beta_1;\beta_2,w} \\
\quad \downarrow \\
W_{\beta_1,w}
\end{array}
$$

where $W_{\beta_1;\beta_2,w} = \bigcup_{w' \in W_{\beta_1,w}} W_{\beta_2,w'}$.

- $\beta_1 \& \beta_2$: the performance of $\beta_1$ and $\beta_2$ simultaneously. For elementary actions $a_1$ and $a_2$:

$$
\begin{array}{c}
w \\
\bullet \\
[ a_1 ] \\
\downarrow \\
[ a_2 ] \\
\downarrow \\
w' \\
\bullet \\
[ \beta_1 \& \beta_2 ] \\
\downarrow \\
W_{\beta_1 \& \beta_2,w}
\end{array}
$$

In general:
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- $\beta_1 \cup \beta_2$: the performance of $\beta_1$ or $\beta_2$ (or both). For elementary actions $a_1$ and $a_2$ we get:

$$
\begin{array}{c}
\text{w} \\
\ \downarrow \ \\
\{a_1\} \\
\ \downarrow \ \\
\text{w'} \\
\{a_2\} \\
\ \downarrow \ \\
\text{w''} \\
\{a_2\} \\
\ \downarrow \ \\
\text{w'''}
\end{array}
$$

In general:

$$
\begin{array}{c}
\text{w} \\
\ \downarrow \ \\
\text{[\beta_1]} \\
\ \downarrow \ \\
\text{\{\beta_1 & \beta_2\]} \\
\ \downarrow \ \\
\text{W_{\beta_1 \cup \beta_2,w}} \\
\{\beta_2\} \\
\ \downarrow \ \\
\text{w'''}
\end{array}
$$

The semantics are based upon world transformations owing to action expressions $\beta$ in $Act$: given a state $w$, $[\beta](w)$ yields a collection of worlds which one obtains after having performed the action expression $\beta$. Formally, we stipulate that a set $W$ of worlds assigns values to propositional variables and a given function $\rho : S \cup [\delta] \cup [\text{skip}] \rightarrow (W \rightarrow W)$, which for each s-set yields its behaviour in terms of world transitions. Thus, $\rho(S)(w)$ gives the next world, performing all elementary actions in $S$ jointly in world $w$. Function $\rho$ is not further specified here, except for the s-sets $[\delta]$ and $[\text{skip}]$. For these, it stipulates that the failing s-set $[\delta]$ has no successor world and that the s-set $[\text{skip}]$ does not change the successor world.

We choose the s-sets to be deterministic.

**Definition 2.2.26**

- $\rho([\delta])w = \emptyset$
- $\rho([\text{skip}])w = w$
- Function $R(t) \in W \rightarrow W$ is defined inductively by
  - $R(S)(w) = \rho(S)(w)$ for $S \in S \cup [\delta] \cup [\text{skip}]$;
  - $R(t_1 \circ t_2)(w) = R(t_2)(R(t_1)(w))$ for s-sequences $t_1, t_2$;
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\[- R(T)(w) = \{ w' \mid w' = R(t)(w) \text{ for } t \in T \} \text{ for } T \in \mathcal{A}.\]

Note that \( R((t_1 \circ t_2) \circ t_3)(w) = R(t_1 \circ (t_2 \circ t_3))(w), \) since \( R((t_1 \circ t_2) \circ t_3)(w) = R(t_3)(R(t_2)(R(t_1)(w))) = R(t_2 \circ t_3)(R(t_1)(w)) = R(t_1 \circ (t_2 \circ t_3))(w). \)

**Definition 2.2.27** The semantics of action expressions in a certain world can now be given by the function \( \llbracket \cdot \rrbracket_R : \text{Act} \to (W \to \mathcal{P}(W)): \)

\[
\llbracket \beta \rrbracket_R(w) = R(\llbracket \beta \rrbracket)(w).
\]

This function is extended to \( \text{Act} \to (\mathcal{P}(W) \to \mathcal{P}(W)) \) by

\[
\llbracket \beta \rrbracket_R(W') = \bigcup_{w \in W'} \llbracket \beta \rrbracket_R(w),
\]

with \( W' \subseteq W. \)

### 2.3 Deontic assertions

Having given the semantics of action expressions, we can now discuss the language \( \text{Ass} \) of \( PDL. \) The language \( \text{Ass} \) of \( PDL \) consists of assertions concerning action expressions in \( \text{Act}, \) and is given by the BNF:

\[
\Phi ::= \phi | \Phi_1 \wedge \Phi_2 | \Phi_1 \vee \Phi_2 | \Phi_1 \rightarrow \Phi_2 | \neg \Phi | [\beta] \Phi,
\]

where \( \phi \) denotes a propositional variable in \( L, \) being the language of the propositional calculus. Further, we define \( V \) as the propositional variable with liability to sanction as intended meaning. Expression \([\beta] \Phi\) means that \( \Phi \) will hold after \( \beta \) has been performed. \( [\beta] \Phi \) is a refined version of \( \beta \rightarrow \Phi \) in traditional deontic logic, the difference being that now actions and assertions are strictly separated, and a notion of time-lag is built in.\(^3\)

With the language \( \text{Ass}, \) we can formally express when an action is forbidden, permitted or obliged. To express these notions, we use the following abbreviations:

**Definition 2.3.1**

\[ F(\beta) \text{ is an abbreviation of } [\beta]V; \]
\[ P(\beta) \text{ is an abbreviation of } [\neg \beta]V; \]
\[ O(\beta) \text{ is an abbreviation of } [\beta]V. \]

These three abbreviations justify our claim that we are concerned with *deontic logic*. Thus, deontic logic can be reduced to a system of dynamic logic.

\(^3\)[\( [\beta] \Phi \) is known in the realm of programming as the 'weakest precondition' of action expression \( \beta \) with respect to postcondition \( \Phi \).]
Definition 2.3.2 A model $M$ for $PDeL$ is given by

$$M = (A, W, \llbracket \beta \rrbracket_R, \pi),$$

where $A$ is the set of actions, $W$ a set of possible worlds, $\llbracket \beta \rrbracket_R$ a function that associates with action $\beta \in Act$ and world $w$, the set of possible worlds to which the performance of $\beta$ leads, and $\pi$ the usual truth relation between worlds and sentences: $\pi : W \times L \rightarrow \{true, false\}$.

For the semantics of assertions, we employ the semantics $\llbracket \_ \rrbracket_R$ of action expressions:

Definition 2.3.3 For $w \in W$, $\Phi_1, \Phi_2 \in Ass_1$, $w \models \Phi_1 \lor \Phi_2, \ldots, w \models \neg \Phi_1$ are defined as usual.

For the dynamic part, we define

$$w \models [\beta] \Phi \iff \forall w' \in \llbracket \beta \rrbracket_R(w) w' \models \Phi.$$ 

We write $\models \Phi \iff \forall w \in W w \models \Phi$.

This can be represented as:

$$< \beta >$$

is the dual of $[\beta] \Phi$: $w \models < \beta > \Phi \iff w \models \neg [\beta] \neg \Phi$, i.e. $\exists w' \in \llbracket \beta \rrbracket_R(w) w' \models \Phi$.

---

4In words: performing action $\beta$ in a world $w$ necessarily renders proposition $\Phi$ true iff the proposition $\Phi$ holds for all worlds $w'$ after $w$ and performing $\beta$. 
To be able to reason with language $PD_eL$, we introduce the following derivation rules:

**Rules 2.3.4**

- **(N)** if $\vdash \Phi$, then $\vdash [\beta]\Phi$;
- **(S)** $\beta_1 = \mathcal{A} \beta_2 \vdash [\beta_1]\Phi \equiv [\beta_2]\Phi$.

Note that the antecedent of rule **(S)** is not really an expression used in $PD_eL$. It would, however, not be difficult to axiomatise the equations given on the action expressions and to incorporate them into our logic. Without formally performing so, we consider this to be done and will not give all the axioms and derivation rules that are involved in this axiomatisation. We also have the derivation rule modus ponens.

The following axioms hold for $PD_eL$:

**Axiom 2.3.5**

1. all tautologies of the propositional calculus;
2. $[\beta](\Phi_1 \rightarrow \Phi_2) \rightarrow ([\beta]\Phi_1 \rightarrow [\beta]\Phi_2)$;
3. $[\beta_1; \beta_2]\Phi \equiv [\beta_1]([\beta_2]\Phi)$;
4. $[\beta_1 \cup \beta_2]\Phi \equiv [\beta_1]\Phi \land [\beta_2]\Phi$;
5. $[\beta_1]\Phi \lor [\beta_2]\Phi \rightarrow [\beta_1 \& \beta_2]\Phi$;
6. $[\text{fail}]\Phi$;
7. $[\text{skip}]\Phi \equiv \Phi$.

**Some remarks**

1. Axioms 2.3.5.4 and 2.3.5.5 only hold if the actions are in normal form. Axiom 2.3.5.4: we have that $a_1 = \mathcal{A} a_1 \cup (a_1; a_2)$, however, $[a_1]\Phi \neq [a_1]\Phi \land [(a_1; a_2)]\Phi$. Axiom 2.3.5.5: we have that $(a_1; a_2) = \mathcal{A} a_1 \& (a_1; a_2)$, and from $[a_1]\Phi \rightarrow [a_1]\Phi \lor [a_1; a_2]\Phi$ together with 2.3.5.5, we get $[a_1]\Phi \rightarrow [a_1 \& (a_1; a_2)]\Phi$, and $[a_1 \& (a_1; a_2)]\Phi \equiv [a_1][a_2]\Phi$. However, $[a_1][a_2] \neq [a_1][a_2]\Phi$.
2. The soundness of axioms 2.3.5.1 and 2.3.5.6 is obvious; that of 2.3.5.2, 2.5.3.3 and 2.3.5.4 and rule **(N)** is proven as in standard dynamic logic. The soundness of 2.3.5.5 is proven as follows. Suppose we have $w \models [\beta_1]\Phi$, i.e.,

$$\forall w' \in b_1 b(w)w' \models \Phi.$$
2.3 Deontic assertions

\[ [\beta_1 \& \beta_2] = [\beta_1] \cap [\beta_2], \text{ thus } [\beta_1 \& \beta_2] \subseteq [\beta_1] \text{ and, therefore, also } [\beta_1 \& \beta_2]_R(w) \subseteq [\beta_1]_R(w). \text{ Hence, } \forall w' \in [\beta_1 \& \beta_2]_R(w) w' \models \Phi \text{ as well, i.e., } w \models [\beta_1 \& \beta_2] \Phi. \text{ Consequently, } w \models [\beta_1] \Phi \rightarrow [\beta_1 \& \beta_2] \Phi. \text{ Analogously, we can prove that } w \models [\beta_2] \Phi \rightarrow [\beta_1 \& \beta_2] \Phi, \text{ and thus } w \models [\beta_2] \Phi \lor [\beta_2] \Phi \rightarrow [\beta_1 \& \beta_2] \Phi, \text{ which holds for all } w. \text{ Therefore, axiom 2.3.5.5 is sound.}

The soundness of axiom 2.3.5.7 is proven as follows. Suppose we have } w \models [\text{skip}] \Phi, \text{ i.e., } \\
\forall w' \in [\text{skip}]_R(w) w' \models \Phi.

\text{But } [\text{skip}]_R(w) = \{w\}, \text{ hence } w \models \Phi, \text{ which holds for every } w. \text{ Thus, axiom 2.3.5.7 is sound.}

The soundness of rule (S) is proven as follows. Suppose we have } w \models [\beta_1] \Phi, \text{ i.e., } \\
\forall w' \in [\beta_1]_R(w) w' \models \Phi,

\text{then } \forall w' \in [\beta_2]_R(w) w' \models \Phi \text{ holds, i.e., } w \models [\beta_2] \Phi, \text{ since } [\beta_1] = [\beta_2], \text{ thus } [\beta_1]_R(w) = [\beta_2]_R(w) \text{ for all } w \in W. \text{ Consequently, } [\beta_1] \Phi \rightarrow [\beta_2] \Phi. \text{ Analogously, we can prove that } w \models [\beta_2] \Phi \rightarrow [\beta_1] \Phi. \text{ Therefore, } w \models [\beta_1] \Phi \equiv [\beta_2] \Phi, \text{ so rule (S) is sound.}

Thus, } PD_eL \text{ is sound. The natural question arises whether } PD_eL \text{ is complete with respect to the semantics we have defined. The problem of completeness of the system was discussed by Meyer (1989).}

3. Axiom 2.3.5.6 states that if one is asked to perform an impossible action, e.g., } \beta_1 \& \bar{\beta}_1, \text{ then there are no successor worlds and, consequently, assertion } \Phi \text{ is true in all successor worlds (since there are none). With this axiom, it is possible to prove, for example, } \vdash \text{O(any)} (\text{fail} \Phi \rightarrow [\text{fail}] \text{V and } [\text{fail}] \text{V} \rightarrow \text{O(any)}): \text{ 'Existence of an empty normative system'. This theorem is part of the idea of viewing deontic logic as a normal modal logic, which Von Wright rejected by his principle of deontic contingency: 'A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden' (Von Wright, 1951, p. 11).}

4. Rule (N) states that a norm } \Phi \text{ cannot be remitted by any action if } \Phi \text{ can be derived from the system.}

Now we give some theorems of } PD_eL: \text{ }

**Proposition 2.3.6**

1. } \vdash [\beta] \text{true;}

2. } \vdash [\beta](\Phi_1 \& \Phi_2) \equiv [\beta] \Phi_1 \land [\beta] \Phi_2;
3. \( \vdash [\beta] \Phi_1 \vee [\beta] \Phi_2 \rightarrow [\beta](\Phi_1 \vee \Phi_2) \);

4. \( \vdash [\beta_1] \Phi_1 \wedge [\beta_2] \Phi_2 \rightarrow [\beta_1 \& \beta_2](\Phi_1 \wedge \Phi_2) \);

5. \( \vdash [\beta_1] \Phi_1 \wedge [\beta_2] \Phi_2 \rightarrow [\beta_1 \cup \beta_2](\Phi_1 \vee \Phi_2) \);

6. \( \vdash [\beta_1 \cup \beta_2 \cup (\beta_1 \& \beta_2)] \Phi \equiv [\beta_1 \cup \beta_2] \Phi \);

7. \( \vdash F(\beta_1 \cup \beta_2) \equiv F(\beta_1) \wedge F(\beta_2) \);

8. \( \vdash F(\beta_1) \vee F(\beta_2) \rightarrow F(\beta_1 \& \beta_2) \);

9. \( \vdash F(\beta_1; \beta_2) \equiv [\beta_1]F(\beta_2) \);

10. \( \vdash O(\beta_1; \beta_2) \equiv O(\beta_1) \wedge [\beta_1]O(\beta_2) \);

11. \( \vdash O(\beta_1) \vee O(\beta_2) \rightarrow O(\beta_1 \cup \beta_2) \);

12. \( \vdash O(\beta_1 \& \beta_2) \rightarrow O(\beta_1) \wedge O(\beta_2) \);

13. \( \vdash O(\beta_1) \wedge O(\beta_2) \rightarrow O(\beta_1 \& \beta_2) \);

14. \( \vdash P(\beta_1 \cup \beta_2) \equiv P(\beta_1) \vee P(\beta_2) \);

15. \( \vdash P(\beta_1 \& \beta_2) \equiv P(\beta_1) \wedge P(\beta_2) \);

16. \( \vdash P(\beta_1; \beta_2) \equiv < \beta_1 \succ P(\beta_2) \);

17. \( \vdash P(\beta) \equiv < \beta \succ \neg V \);

18. \( \vdash P(\beta) \equiv \neg O(\beta) \);

19. \( \vdash F(\beta_1) \rightarrow F(\beta_1 \& \beta_2) \);

20. \( \vdash O(\beta_1 \& \beta_2) \rightarrow O(\beta_1) \);

21. \( \vdash O(\beta_1) \rightarrow O(\beta_1 \cup \beta_2) \);

22. \( \vdash O(\beta_1 \cup \beta_2) \wedge F(\beta_1) \rightarrow O(\beta_2) \);

23. \( \vdash F(\beta_1 \& \beta_2) \wedge O(\beta_1) \equiv F(\beta_2) \wedge O(\beta_1) \);

24. \( \vdash F(\beta_1 \&(\beta_1; \beta_2)) \equiv F(\beta_1; \beta_2) \);

25. \( \vdash F(\beta_1 \cup (\beta_1 \& \beta_2)) \equiv F(\beta_1) \wedge F(\beta_2) \);

26. \( \vdash O(\beta_1 \cup \beta_1) \equiv O(\beta_1) \);
2.3 Deontic assertions

27. \( \vdash O(\beta_1 \cup \beta_1) \equiv [\text{fail}]V \equiv \text{true} \).


Among these theorems of the system, we find very familiar ones, both evident truths and more controversial assertions. The desirable theorems include assertions such as

(7) Action \( \beta_1 \cup \beta_2 \) is forbidden iff both actions \( \beta_1 \) and \( \beta_2 \) are forbidden.

(9) Action \( \beta_1; \beta_2 \) is forbidden iff \( \beta_2 \) is forbidden if \( \beta_1 \) has already been performed.

(10) Action \( \beta_1; \beta_2 \) is obligatory iff \( \beta_1 \) is obligatory, and \( \beta_2 \) is obligatory once \( \beta_1 \) has been performed.

(13) Action \( \beta_1 \& \beta_2 \) is obligatory if both actions \( \beta_1 \) and \( \beta_2 \) are obligatory.

(17) Action \( \beta \) is permitted iff there is a way of performing \( \beta \) that avoids 'trouble', i.e., iff it is not obligatory to perform not-\( \beta \).

(19) If action \( \beta_1 \) is forbidden, then every simultaneous action together with \( \beta_1 \) is forbidden.

(22) If action \( \beta_1 \cup \beta_2 \) is obligatory and \( \beta_1 \) is forbidden, then action \( \beta_2 \) is obligatory.

Theorems 2.3.6.13, 2.3.6.18 and 2.3.6.27 correspond with axiom schemata \( (OC) \), \( (Df.P) \) and \( (ON) \), respectively, of \( SDL \). Schema

\(~[\text{any}]V,\)

corresponding with the schema \( (OD) \) of \( SDL \), cannot be derived from the rules and axioms of \( PD_eL \). To add this axiom to \( PD_eL \), we have to introduce the following constraint into the semantics:

\( \exists w' \in [\text{any}]_n(w)w' \not\models V \) for all \( w \in W \).

This can be proven as follows. Suppose we have that \( w \models \neg[\text{any}]V \), i.e.,

\( \forall w' \in [\text{any}]_n(w)w' \models V, \)

which is equivalent to \( \exists w' \in [\text{any}]_n(w)w' \not\models V \) for all \( w \in W \).

Using this extra axiom, we are able to derive some more theorems such as

1. \( \vdash \neg O(\beta \& \bar{\beta}) \);
2. \( \vdash O(\beta) \rightarrow P(\beta) \);
3. \( \vdash F(\beta) \rightarrow \neg O(\beta) \);
4. \( \vdash \neg (O(\beta) \land O(\bar{\beta})) \).
At first glance, the axiom does not seem to be controversial, since it merely denies the existence of impossible obligations. However, with the help of this axiom we can now derive the formula

\[ \neg (O(\beta) \land O(\overline{\beta})) \]

which is controversial nowadays (see, e.g., Alchourrón, 1969; Prakken, 1996; Meyer, Dignum and Wieringa, 1996). The main objection to this formula is that it states that there is no conflict of duties, which is clearly not in line with situations in daily life.\(^5\)

It will be obvious that the system \( PDeL \) expresses Ought-to-do sentences. We think that a system expressing Ought-to-do sentences fits better with the common-sense interpretation of legal rules, since norms are essentially related to individuals: it is somebody’s obligation, permission or prohibition (cf. chapter 1). Furthermore, Meyer (1988) and Meyer, Dignum and Wieringa (1994) showed that \( PDeL \) avoids some of the very nasty paradoxes that often appear in other systems, especially in \( SDL \).

### 2.4 Paradoxes and semantics of the deontic operators

The theorems and axioms of \( PDeL \) give rise to a further inspection of the semantics of the deontic operators of \( PDeL \). According to theorem 2.3.6.21, we can derive \( O(\beta_1 \cup \beta_2) \) from \( O(\beta_1) \). This is the well-known Ross (1941) paradox: ‘post the letter’ implies ‘post the letter or burn it’. However, this is not a proper anomaly; only if we make use of the intentional meaning of the word ‘or’, the theorem is contrary to our intuitions. But in our system, we make use of the extensional meaning of the word ‘or’, which we will call the passive choice: the choice between \( \beta_1 \) and \( \beta_2 \) is an underspecification, because it concerns \( \beta_1 \) or \( \beta_2 \), without specifying which. Thus, \( O(\beta_1 \cup \beta_2) \) does not mean that the norm subject is free to choose between \( \beta_1 \) and \( \beta_2 \).

It merely means that the norm-subject is obligated to perform at least one of both acts. In this meaning \([O(\beta_1 \cup \beta_2)]\) follows from \([O(\beta_1)]\), without it

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\(^5\)In spite of this objection, some authors defend this schema for a deontic system. However, it would be naive to think that a normative system is consistent, since we often find inconsistencies among legal rules. Prakken (1996) gives a pragmatic view on the effect of these inconsistencies:

That in one particular situation a rule is dropped to maintain consistency does not mean that it has no binding force at all, since in other, unproblematic situations it can still be applied.

I see no compelling reasons why the binding force of a deontic rule should be equated with its application to every single occasion.

Horty (1994) defends this schema with the argument that the deontic systems are too weak without this schema.
being paradoxical: if one is obliged to perform $[\beta_1]$, then one is also obliged to perform at least one of both $[\beta_1]$ and $[\beta_2]$. By disobeying $[O(\beta_1 \cup \beta_2)]$ thus by performing $[\beta_1 \& \beta_2]$, $[O(\beta_1)]$ is disobeyed as well; therefore it follows from $[O(\beta_1)]$ that $[O(\beta_1 \cup \beta_2)]$ has to be obeyed. (Soeteman, 1989, p. 116)

With regard to the deontic status of $\beta_2$, nothing is said: $\beta_2$ may be obligatory, forbidden or permitted.

Analogously, we can discuss permission $P(\beta)$. $P(\beta)$ means that it is permitted to perform $\beta$ in at least one way. Probably, there are many ways of performing $\beta$, some of which are forbidden; but if $P(\beta)$ holds, not all ways of performing $\beta$ are forbidden. This corresponds with McLaughlin's paradox: 'if one is permitted to walk in a public road and to wear clothes, then one is permitted to walk in a public road' $(P(\beta_1 \& \beta_2) \rightarrow P(\beta_1))$. According to McLaughlin (1955) it is now also permitted to walk in the public road not wearing clothes, but McLaughlin misreads $P(\beta_1)$. This formula does not mean that one is permitted to perform $\beta_1$ in every possible way. We are not permitted to walk in a public road simultaneously disregarding traffic rules, or not wearing clothes.

From the above, it does not follow that operators $O$ and $P$ behave identically within system $PD_eL$. In $PD_eL$, e.g., $P(\beta_1 \& \beta_2) \rightarrow P(\beta_1) \lor P(\beta_2)$ is valid, but $O(\beta_1 \cup \beta_2) \rightarrow O(\beta_1) \lor O(\beta_2)$ is not, and $O(\beta_1) \land O(\beta_2) \rightarrow O(\beta_1 \& \beta_2)$ is valid, but $P(\beta_1) \land P(\beta_2) \rightarrow P(\beta_1 \& \beta_2)$ not. This follows immediately from the semantics.

In contrast with $P(\beta)$ and $O(\beta)$, prohibition $F(\beta)$ means that all ways of performing $\beta$ are forbidden. This follows immediately from $F(\beta) \equiv O(\beta)$. Thus, $F(\beta)$ means that we have to perform $\beta$, and that we are in trouble when performing $\beta$. If, e.g., it is forbidden to drive at a speed of more than 100 km/h, then every speed over 100 km/h is forbidden.

What is important here, is that really undesirable assertions such as $O(\beta_1) \rightarrow O(\beta_1 \rightarrow \beta_2)$, $O(\beta) \rightarrow O(O(\beta))$, $O(O(\beta)) \rightarrow O(\beta)$, $(O(p) \land (p \rightarrow O(q)) \rightarrow O(q))$, $(p \rightarrow q) \rightarrow O(p \rightarrow q)$ (see Hintikka, 1971; Castañeda, 1981; Soeteman, 1989) are either false or nonsensical (not even well-formed) in system $PD_eL$. Also, in some cases the paradox just vanishes. For example, consider Chisholm's (1963) paradox, consisting of the following sentences:

1. It ought to be that a certain man goes to the assistance of his neighbours.
2. It ought to be that if he does go he tells them he is coming.
3. If he does not go, then he ought not to tell them he is coming.
4. He does not go.

Intuitively, this set of sentences is consistent, but it is problematic in $SDL$. In $SDL$, the formalisation of these sentences would be:

\[ ^6 \text{A thorough investigation of this paradox can be found in Smith (1994).} \]
1. $O(p)$;
2. $O(p \rightarrow q)$;
3. $\neg p \rightarrow O(\neg q)$;
4. $\neg p$.

From 1 and 2 we can derive $O(q)$, and from 3 and 4 we can derive $O(\neg q)$. Thus, these sentences are contradictory. However, in $PDeL$ the intention is represented without any problem as far as the first three sentences are concerned. Consider the assertion

$$O(\beta_1) \land [\beta_1]O(\beta_2) \land [\beta_1]O(\beta_2),$$

which is equivalent to

$$O(\beta_1; \beta_2) \land [\beta_1](V \land F(\beta_2)).$$

It is in principle obligatory to do $\beta_1; \beta_2$, but if $\beta_1$ is not done, then, besides already being liable to punishment for not performing $\beta_1$, one is also forbidden to do $\beta_2$. So instead of arriving at an inconsistency we get a meaningful assertion in this case. (Meyer, 1988, p. 120)

### 2.5 Simple $PDeL$

In this thesis, we will use a simplified version of $PDeL$ as the basis for the logic we try to develop. The simplification consists in the restriction to actions: we will not consider sequences of actions. The inclusion of sequences of actions does not lead to semantic difficulties, but complicates the syntax and semantics to the point that it would obscure the issues that we try to clarify in this thesis.

The system Simple $PDeL$ is axiomatised by the following rules of inference

**Rules 2.5.1**

- (N) if $\vdash \Phi$, then $\vdash [\beta]\Phi$;
- (S) $\beta_1 =_A \beta_2 \vdash [\beta_1]\Phi \equiv [\beta_2]\Phi$.

and the axioms

**Axiom 2.5.2**

1. all tautologies of the propositional calculus;
2. $[\beta](\Phi_1 \rightarrow \Phi_2) \rightarrow ([\beta]\Phi_1 \rightarrow [\beta]\Phi_2)$;
2.6 Conclusions

3. \([\beta_1 \cup \beta_2] \Phi \equiv [\beta_1] \Phi \land [\beta_2] \Phi\);

4. \([\beta_1] \Phi \lor [\beta_2] \Phi \rightarrow [\beta_1 \& \beta_2] \Phi\);

5. \([\text{fail}] \Phi\);

6. \([\text{skip}] \Phi \equiv \Phi\).

In the remainder of this thesis, we will refer to this system as \(PDeL\). In chapter 4, we will extend this system to actors and in chapter 5 to authorities. For every action, an actor or a group of actors will be specified performing the action.

## 2.6 Conclusions

The system \(PDeL\) presented in this chapter (together with its semantics) provides a very workable framework for reasoning with Ought-to-do statements, in contrast to \(SDL\), which provides reasoning with Ought-to-be statements. Both systems have their value, since sometimes we need to formalise Ought-to-do statements and sometimes Ought-to-be statements (cf. chapter 1). We stress again, though, that a comparison between \(SDL\) and \(PDeL\) is not a topic of this thesis.

Furthermore, the system \(PDeL\) does not contain the very nasty paradoxes that often appear in other systems in the literature, especially where the connection between actions and assertions is concerned.
Chapter 3

Relativised deontic modalities in $SDL$

Most deontic logics disregard the fact that obligations are thought of as obliging some particular individual. These obligatory acts are impersonal; they relate to one and the same individual all the time. They leave no possibility for directing acts to a particular individual or group of individuals, or take account of the fact that some acts are obligatory for some, but not all, (groups of) individuals. In this chapter$^1$, we discuss formalisations of relativised deontic modalities in standard deontic logic concerning what is obligatory or permitted for individuals or groups of individuals.

3.1 Introduction

We can distinguish three types of participants who play a very important role in a normative system: the authorities, who enact the norms, the addressees, to whom the norms are directed, and the counterparties, who have a ‘right’ against the addressees. In law, the word ‘right’ is often used to designate ‘legal power’, ‘legal claim’ and ‘immunity granted by an authority’ (cf. Hohfeld, 1964).

The authorities are responsible for setting the norms and for supervising the enactment of the norms. In this chapter, we do not discuss the role of the authorities (this will be done in chapter 5). The role of the counterparties is very limited and is discussed at the end of this chapter. The major topic of this chapter and the next one is the notion of addressees. In the sequel, we will call the addressees (groups of) actors or (groups of) individuals.

The notion of (groups of) actors has hardly been formalised (see Bailhache, 1981, 1991; Royakkers, 1993; Wieringa and Meyer, 1993). Most deontic logicians seem to concentrate their interest on one specific actor, and then discuss what his obligations are, whether one obligation follows from another, what is meant when it is said that obligations are consistent, and so on. Thus, the obligatory acts in these deontic logics are ‘impersonal’.

$^1$Some of the ideas in this chapter were presented earlier in Royakkers and Dignum (1995c).
We cannot say that

- acts are obligatory for a particular individual or group of individuals,
  or that

- some acts are obligatory for some, but not all, (groups of) individuals.

Thus, conventional approaches to deontic logic employ impersonal deontic operators. These approaches implicitly refer to one and the same individual all the time, explicit reference being unnecessary (Hintikka, 1971). Sentences like 'it is obligatory for John that the window is closed and it is obligatory for Paul that the light is on' and 'it is obligatory for John that the window is closed and it is obligatory for Paul that the window is open' cannot be expressed within these approaches. They are limited, since they are not able to deal with problems related to (groups of) individuals.

Some researchers tried to include relativised deontic modalities in standard deontic logic (SDL) and to define 'personal' obligations and permissions in terms of the 'impersonal' ones. Relativised deontic modalities are concerned with what is obligatory or permitted for an actor or group of actors, as contrasted with what is only impersonally obligatory or permitted. Deontic operators are relativised to sets of one or more actors. However, we show that all these extensions have some shortcomings. For instance, in the proposals made by Bailhache (1991) and Hilpinen (1973), it is not possible to express consistently normative conflicts of the type 'it is obligatory for \( i_1 \) that \( p \)' and 'it is obligatory for \( i_2 \) that \( \neg p \)' and Hansson (1970) was forced to give up certain basic deontic principles, such as \( O(p) \equiv \neg P(\neg p) \), to construct a model for relativised deontic modalities.

If some form of relativised obligation is introduced, then the impersonal obligation can be interpreted in several ways:

1. It is an obligation for all individuals (general obligation).
2. It is an obligation for some unspecific individual (unspecific obligation).
3. It is an obligation that implies the general obligation (strong obligation).
4. It is an obligation that is implied by the unspecific obligation (weak obligation).

However, Herrestad and Krogh (1995) showed that all these interpretations have to deal with one of the following two problems:

- the problem of interdefinability of non-relativised obligation and permission (i.e., \( O(p) \equiv \neg P(\neg p) \));

\(^2\)We will see that these two problems are related.
3.2 Formalisations of relativised deontic modalities

- the problem of asymmetry (e.g., the general obligation is coupled with the unspecific permission).

We do not think that these two problems are problems of the interpretations, but features of these interpretations. Relativised deontic logic is an extension of SDL; thus, with the new expressive power and the new definitions of the $O$-operator, the formulas acquire new meanings that cannot be expressed in SDL and are, therefore, subject to new intuitions. In this chapter, we investigate these new intuitions for the interpretations mentioned: we formalise some realistic, intuitive notions of these interpretations and analyse the properties that hold for these interpretations.

Thus, in this chapter we add actors and groups of actors to SDL. The concept of collective obligation - an obligation for a group of actors - is rather new, and can be seen as an extension of the relativised deontic modalities concerning what is obligatory or permitted for an actor. The motivation for this extension is that

- there are situations that only can be accomplished by a group;
- we can express group liability (e.g., liability for a trading partnership).

In chapter 1, we saw that in SDL norms are expressed by applying sentential operator $O$ to sentence letters $p$, meaning that $O(p)$ must be read as an Ought-to-be statement. We cannot express Ought-to-do statements in SDL, since in SDL we cannot talk about actions. This means that the relativised obligation $O_i(p)$ will be read as 'it is obligatory for $i$ that $p$' or 'it is obligatory for $i$ to accomplish $p$'. In the following chapter, we will add actors and groups of actors to PDeL, so that we can express Ought-to-do statements such as $O(\beta)$, 'it is obligatory to do $\beta$' and the related relativised obligation $O(i: \beta)$, 'it is obligatory for $i$ to do $\beta$'.

This chapter is structured along the following lines. In section 3.2, some formalisations of relativised deontic modalities are discussed. These formalisations concern connections between impersonal deontic operators and deontic operators relativised to individuals. Section 3.3 introduces the notion of collective obligation. Here, the deontic operators are relativised for groups of actors. Further, the strong and weak obligations are introduced, and we formalise and analyse the properties that hold for these notions. Section 3.4 presents a recapitulation of the relations of all the notions of obligation and permission that are discussed in this chapter. In section 3.5, we briefly discuss the directed obligation on the basis of the correlative terms 'duty' and 'right'. We finish with some conclusions.

### 3.2 Formalisations of relativised deontic modalities

In this section, some existing formalisations of relativised deontic modalities are discussed. In these formalisations, the deontic operators are relativised to individuals: relativised
deontic modalities are concerned with what is forbidden, obligatory or permitted for an individual, as contrasted with what is only impersonally obligatory. Such an extension of deontic logic is concerned with connections between obligations that bind particular individuals, on the one hand, and impersonal obligations, on the other hand. We can distinguish several obligations that bind particular individuals:

- a personal obligation: an obligation for a specific individual;³
- a general obligation: an obligation for all individuals;
- an unspecific obligation: an obligation for some individual.

In order to make a comparison between the approaches, we translate the semantics of the approaches into one and the same semantics. We use the following model structure \( \mathcal{M} = (W, I, R, V) \) for system \( D^* \) consisting of four elements:

1. the set of possible worlds \( W = \{w_1, w_2, \ldots\} \);
2. the non-empty set of individuals \( I = \{i_1, i_2, \ldots, i_n\} \);
3. the set \( R \) of functions: \( \{R_{i_1}, \ldots, R_{i_n}\} \) for each individual \( i_1, i_2, \ldots, i_n \in I \). Thus, \( R = \{R_i \mid i \in I\} \). The function \( R_i \in R \) on \( W \) returns the deontically ideal worlds for individual \( i \) given a world: \( R_i : W \rightarrow 2^W \);
4. a valuation function \( V \), which assigns the value 'true' or 'false' to a proposition at a world in \( W \).

The truth conditions for \( O_i \) and \( P_i \) are defined as follows

\[
\mathcal{M}, w \models O_i(p) \text{ iff } R_i(w) \subseteq \lbrack p \rbrack
\]  

and

\[
\mathcal{M}, w \models P_i(p) \text{ iff } R_i(w) \cap \lbrack p \rbrack \neq \emptyset,
\]

with the following constraint (which gives schema (OD) for \( O_i: \neg O_i(p \land \neg p) \))

\[
R_i(w) \neq \emptyset, \text{ for all } R_i \in R \text{ and for all } w \in W.
\]

From the semantics of \( O_i \) and \( P_i \) it follows that the rule and all schemata for \( O \) of the system \( D^* \) are also valid for \( O_i \). This is intuitively correct, since the impersonal obligation

³We can also define the personal obligation as an obligation for a person that does not apply to another person. This definition is much stronger. Therefore, we call this interpretation of the personal obligation the strict personal obligation.
3.2 Formalisations of relativised deontic modalities

$O(p)$ is read as an obligation related to one and the same individual all the time, which corresponds with $O_i(p)$, i.e., an obligation related to a single individual $i$.

The question arises whether there is a relation between the individuals. According to Hilpinen (1973) and Bailhache (1981, 1991), there is such a relation since they want a coherent system without conflicting obligations, such as $O_i(p) \land O_j(\neg p)$.

Hilpinen expressed this in the following condition:

If a set of propositions $A$ is consistent and

\[ \{O_i(p_1), O_i(p_2), ..., O_i(p_n), ...\} \]
\[ O_i(p_1) \land O_i(p_2) \land ... \land O_i(q) \subseteq A, \]
\[ \text{then } \{p_1, p_2, ..., p_n, ...\} \text{ and } \{q_1, q_2, ...\} \text{ are consistent.} \]

This means that all obligations for all individuals can jointly be realised and that a permission for some individual should not be in conflict with his obligations. Thus, it is not the case that an individual has an obligation that $p$ and that there is an individual for whom $\neg p$ is obligatory. This can be formalised as

\[ \neg (O_i(p) \land \exists j \in I O_j(\neg p)), \]

which is equivalent to

\[ O_i(p) \rightarrow \forall j \in I P_j(p). \]

Bailhache (1981, p. 76) described this as follows:

It is advisable to note that this formula $[O_y(p) \rightarrow P_z(p)]$ appears paradoxical only if the normative addressee is unduly identified with the subject of proposition $p$. The formula does not say for example that 'if $y$ is obliged to go into that house, then $z$ is also permitted to go in'. The formula only corresponds to the following: as soon as an addressee is obliged that such a thing is accomplished, normative coherence makes it necessary that all other individuals are not obliged that this thing is not accomplished (in other words, that they are permitted that it is accomplished).

To accomplish this normative coherence, we have to add the following schema to system $D_i^*$:

\[ \neg (O_i(p) \land O_j(\neg p)), \]  

which is equivalent to

\[ O_i(p) \rightarrow P_j(p). \]

\[ 4 \text{A conflict between two obligations means that it is impossible to fulfil both obligations simultaneously.} \]
However, Bailhache (1991) wanted to obtain a complete, coherent deontic system, i.e., what is obligatory, is permitted for all individuals. Yet, from $O_x(p)$ and $O_y(p)$, we can derive $P_x(p)$ and $P_y(q)$, hence $P_x(p) \land P_y(q)$, but not $P_x(p \land q)$, as required in a complete, coherent deontic system. Therefore, he added a stronger schema to system $D^*_f$:

$$O_{i_1}(p) \land O_{i_2}(q) \land \ldots \land O_{i_k}(v) \rightarrow \forall_{i \in I} P_i(p \land q \land \ldots \land v),$$

which is validated by adding the following constraint

$$\cap_{i \in I} R_i(w) \neq \emptyset, \text{ for all } w \in W. \quad (3.7)$$

This can be proven as follows. Let $O_{i_1}(p) \land O_{i_2}(q) \land \ldots \land O_{i_k}(v)$, then $R_{i_1}(w) \subseteq [p] \land R_{i_2}(w) \subseteq [q] \land \ldots \land R_{i_k}(w) \subseteq [v]$ and thus $\cap_{i \in I} R_i(w) \subseteq [p \land q \land \ldots \land v]$. And since $\cap_{i \in I} R_i(w) \neq \emptyset$, it follows that $\cap_{i \in I} R_i(w) \cap [p \land q \land \ldots \land v] \neq \emptyset$. Hence, $R_i(w) \cap [p \land q \land \ldots \land v] \neq \emptyset$, for all $i \in I$ and for all $w \in W$. Thus, $\forall_{i \in I} P_i(p \land q \land \ldots \land v)$. Nowadays schema (3.4), and so also (3.6), is controversial, because this schema states that there is no conflict of personal duties, which is manifestly not in line with daily life situations, where we often find conflicts between legal rules, moral codes, promises, etc. It removes the possibility of expressing conflicts between personal obligations of different individuals, with the result that the systems of norms must be normative consistent (i.e., without conflicting obligations). That is why we do not enforce principles (3.4) and (3.6).

With the semantics of personal obligation and personal permission, we can formalise the general and unspecific obligations and permissions:

- the general obligation and permission: $\forall_{i \in I} O_i(p)$ and $\forall_{i \in I} P_i(p)$;
- the unspecific obligation and permission: $\exists_{i \in I} O_i(p)$ and $\exists_{i \in I} P_i(p)$.

### 3.2.1 \(O(p)\) as the general or unspecific obligations

In this subsection, we investigate the consequences of some possible definitions for the $O$-operator and the $P$-operator in terms of the $O_i$-operator and the $P_i$-operator. We will formalise some realistic, intuitive notions and analyse the properties that hold for some different proposals. Using the above semantics, the following possibilities are most obvious:

1. $O(p) \equiv \forall_{i \in I} O_i(p)^5$ and $P(p) \equiv \forall_{i \in I} P_i(p)$;
2. $O(p) \equiv \exists_{i \in I} O_i(p)$ and $P(p) \equiv \exists_{i \in I} P_i(p)$;
3. $O(p) \equiv \exists_{i \in I} O_i(p)$ and $P(p) \equiv \forall_{i \in I} P_i(p)$;

---

$^5\forall_{i \in I} O_i(p)$ is an abbreviation of $\forall_{i(I(i) \rightarrow O_i(p))}$, with $I$ the predicate symbol for indicating whether its argument is an element of the set of individuals. Note that $O(p)$ is equivalent to $O_{i_1}(p) \land O_{i_2}(p) \land \ldots \land O_{i_k}$.\]
3.2 Formalisations of relativised deontic modalities

4. \( O(p) \equiv \forall_{i \in I} O_i(p) \) and \( P(p) \equiv \exists_{i \in I} P_i(p) \).

1. \( O(p) \equiv \forall_{i \in I} O_i(p) \) and \( P(p) \equiv \forall_{i \in I} P_i(p) \).

This proposal for the interpretations of \( O(p) \) and \( P(p) \) corresponds with Hansson's (1970) proposal. In his article 'Deontic logic and different levels of generality', Hansson suggested to interpret \( O(p) \) as a general obligation, because

\[
\neg \neg(P) - \text{der} \neg(P) \quad \text{and} \quad P(P) - \text{der} P(P) - \neg\neg(P).
\]

This proposal for the interpretations of \( O(p) \) and \( P(p) \) corresponds with Hansson's (1970) proposal. In his article 'Deontic logic and different levels of generality', Hansson suggested to interpret \( O(p) \) as a general obligation, because

if we say 'it is obligatory to do \( p \)' in a context where there is no tacit reference to a special individual, we often mean 'it is obligatory for everyone to do \( p \)'. (Hansson, 1970, p. 246)

Thus, according to Hansson, \( O(p) \) means 'it is obligatory for everyone to do \( p \)'.

\[
O(p) \equiv \forall_{i \in I} O_i(p).
\]

Surprising is Hansson's definition of \( P(p) \), 'it is permitted for everyone to do \( p \)'.

\[
P(p) \equiv \forall_{i \in I} P_i(p).
\]

A consequence of this definition is that \( P(p) \) is not definable as \( \neg O(\neg p) \), thus

\[
P(p) \neq \neg O(\neg p)
\]

and also

\[
\neg P(\neg p) \neq O(p).
\]

\( \neg P(\neg p) \) means that 'there is an individual \( i \) for whom \( p \) is obligatory' and \( \neg O(\neg p) \) means 'there is an individual \( i \) for whom \( p \) is permitted'. Thus, \( \neg P(\neg p) \) stands for the unspecific obligation and \( \neg O(\neg p) \) for the unspecific permission.\(^6\)

Hansson arranged the different operators in the following schema (figure 3.1), where the arrows indicate provable consequences:

**Figure 3.1**

\[
\begin{align*}
O(p) & \quad \leftrightarrow \quad P(p) \\
\neg P(\neg p) & \quad \leftrightarrow \quad \neg O(\neg p)
\end{align*}
\]

\(^6\) \( \neg P(\neg p) \equiv \forall_{i \in I} P_i(\neg p) \equiv \neg \forall_{i \in I} O_i(p) \equiv \exists_{i \in I} O_i(p) \) and \( \neg O(\neg p) \equiv \forall_{i \in I} O_i(\neg p) \equiv \neg \forall_{i \in I} P_i(p) \equiv \exists_{i \in I} P_i(p) \).
It is easy to see that the following principles, which are valid in SDL, are also valid for this proposal:

(a) $O(p) \land O(q) \equiv O(p \land q)$;
(b) $\neg O(p \land \neg p)$;
(c) $O(p \lor \neg p)$;
(d) $O(p) \rightarrow P(p)$.

In relation to individual obligation and permission, the following principles hold:

(a) $O(p) \rightarrow O_i(p)$;
(b) $P(p) \rightarrow P_i(p)$.

In Kordig's (1975) opinion, non-relativised deontic logic lacks a theoretical niche for the distinction between general obligations and obligations for a person. There is no way of saying that $p$ is obligatory for an individual, and not for everyone. The following formula is not expressible in a non-relativised deontic logic

$$O_i(p) \land \neg O(p),$$

but it is expressible within this proposal. This illuminates supererogation. Supererogation means that there are duties 'far beyond' the 'basic' or 'rock-bottom duties for all'.

2. $O(p) \equiv \exists_{i \in I} O_i(p)$ and $P(p) \equiv \exists_{i \in I} P_i(p)$.

Now, $O(p)$ means 'it is obligatory for someone that $p$' and $P(p)$ 'it is permitted for someone that $p$'. Just like the previous choice, permission $P(p)$ is not definable as $\neg O(\neg p)$. Thus, $P(p) \neq \neg O(\neg p)$. $\neg O(\neg p)$ means that 'it is permitted for everyone that $p$' and $\neg P(\neg p)$ means 'it is obligatory for everyone that $p$'. Thus, $\neg O(\neg p)$ now stands for the general permission and $\neg P(\neg p)$ for the general obligation.

Just like in the previous proposal, we can arrange the different operators in a schema (figure 3.2), where the arrows indicate provable consequences:

**Figure 3.2**

```
\begin{tikzpicture}
  \node (O) at (0,0) {$O(p)$};
  \node (P) at (1.5,0) {$P(p)$};
  \node (Op) at (0,-1) {$O(\neg p)$};
  \node (Pn) at (1.5,-1) {$P(\neg p)$};
  \draw[->] (O) -- (P);
  \draw[->] (O) -- (Op);
  \draw[->] (O) -- (Pn);
  \draw[->] (P) -- (Op);
\end{tikzpicture}
```

---

The proposal of $O(p)$ as an unspecific obligation leads to an additional consequence regardless of the definition of $P(p)$: principle $O(p) \land O(q) \rightarrow O(p \land q)$ (OC) does not hold. This principle, which is equivalent to

$$\exists i \in I O_i(p) \land \exists i \in I O_i(q) \rightarrow \exists i \in I O_i(p \land q),$$

'is clearly counter-intuitive: A janitor might be obliged that the floor in a building is swept clean every morning, and a financial minister might be obliged that the rate of inflation is as low as possible. That there is a person for whom it is obligatory that both the floor is swept and that the inflation rate is as low as possible, we find strange.' (Herrestad and Krogh, 1995, p. 462)

Consequently, we cannot derive principle

$$\neg (O(p) \land O(\neg p)),$$

since it follows from (OC) and (OD). We will denote this principle as $(OD^*)$.

However, Hilpinen (1973) and Bailhache (1981, 1991) want this principle to obtain normative coherence, i.e., if an actor is obliged that $p$, then all actors are permitted that $p$. From $(OD^*)$, which is equivalent to

$$\neg (\exists i \in I O_i(p) \land \exists i \in I O_i(\neg p)),$$

it follows that formula $O_i(p) \land O_i(\neg p)$ is contradictory. The principle is validated by adding the constraint

$$\cap_{i \in I} R_i(w) \neq \emptyset, \text{ for all } w \in W.$$ 

This can be proven as follows. Let $O(p)$ and $O(\neg p)$ hold. Then $\exists i \in I (R_i(w) \subseteq [p])$, say $i_1$, and $\exists i \in I (R_i(w) \subseteq [\neg p])$, say $i_2$. Hence, $R_{i_1}(w) \cap R_{i_2}(w) \subseteq \{ p \} \cap [\neg p] = \emptyset$, and this contradicts that $\cap_{i \in I} R_i(w) \neq \emptyset$, thus $\neg (O(p) \land O(\neg p))$.

This is not what we want, as we already discussed in this section, since it removes the possibility of expressing conflicts between the personal obligations of different individuals.

It is easy to see that the following principles, which are valid in SDL, are also valid for this proposal:

(a) $\neg O(p \land \neg p)$;
(b) $O(p \lor \neg p)$;
(c) $O(p) \rightarrow P(p)$.
In relation to individual obligation and permission, the following principles hold:

1. \( O_i(p) \rightarrow O(p) \);
2. \( P_i(p) \rightarrow P(p) \).

3. \( O(p) \equiv \forall_{i \in I} O_i(p) \) and \( P(p) \equiv \exists_{i \in I} P_i(p) \).

In contrast to the previous two proposals, schema \( (Df.P) \) holds for this proposal:

\[ P(p) \equiv \neg O(\neg p). \]

However, a consequence of this proposal is that we cannot express unspecific obligation and general permission with the help of \( O(p) \) and \( P(p) \). In the two previous proposals, we were able to express the two notions of obligation, i.e., general and unspecific obligation, and their duals, i.e., unspecific and general permission, from non-relativised obligation \( O(p) \) and permission \( P(p) \). Now, we can only express the notion of general obligation, and its dual the unspecific permission.

The following principles, which are valid in \( SDL \), are also valid for this proposal:

1. \( O(p) \land O(q) \equiv O(p \land q) \);
2. \( \neg O(p \land \neg p) \);
3. \( O(p \lor \neg p) \);
4. \( O(p) \rightarrow P(p) \).

In relation to the individual obligation and permission, the following principles hold:

1. \( O(p) \rightarrow O_i(p) \);
2. \( P_i(p) \rightarrow P(p) \).

4. \( O(p) \equiv \exists_{i \in I} O_i(p) \) and \( P(p) \equiv \forall_{i \in I} P_i(p) \).

For this proposal, schema \( (Df.P) \) also holds. However, just like in the previous proposal, we cannot express the two different notions of obligation with non-relativised obligation \( O(p) \). In this case, we can only express the notions of unspecific obligation and general permission.

For the same reasons as in the second proposal, schemata \( (OC) \) and \( (OD^*) \) do not hold. Note that now the principle

\[ O(p) \rightarrow P(p) \]

does not hold, since it is equivalent to \( (OD^*) \), meaning that 'if there is someone for whom it is obligatory that \( p \), then it is permitted for everyone that \( p \)'.

Relativised deontic modalities in \( SDL \)
The following principles, which are valid in SDL, are also valid for this proposal:

(a) $\neg O(p \land \neg p)$;
(b) $O(p \lor \neg p)$.

In relation to the individual obligation and permission, the following principles hold:

(a) $O_i(p) \rightarrow O(p)$;
(b) $P(p) \rightarrow P_i(p)$.

According to Herrestad and Krogh (1995), these four proposals suffer from one of the following two problems:

- the problem of interdefinability;
- the problem of asymmetry.

In the former two proposals, permission $P(p)$ is not definable as $\neg O(\neg p)$. Herrestad and Krogh (1995, p. 461) called this the problem of interdefinability. This is why they rejected these two proposals, for $(Df.P)$ is one of the fundamental elements of deontic logic concerning the relation between obligation and permission. In deontic logic, permission is considered to be the dual of obligation, in the same way as possibility and necessity are in modal logic (cf. Von Wright, 1951).

In the latter two proposals, we cannot express the four notions with non-relativised obligation and permission. That is why Herrestad and Krogh rejected these proposals. About the third proposal they stated:

This suggestion suffers from what we call the problem of asymmetry. It seems like the notion of non-relativised obligation is a strong notion, while the notion of non-relativised permission is a weak notion. (...) These notions are thus impersonal in two different ways. (Herrestad and Krogh, 1995, p. 463)

In the four proposals given, these two problems are related to each other. Since, if we choose for schema $(Df.P)$, i.e., $O(p) \equiv \neg P(\neg p)$, then we meet the problem of asymmetry, and if we choose to give up schema $(Df.P)$, we meet the problem of interdefinability. The problems of asymmetry and interdefinability can be solved by accepting two notions of non-relativised obligation and permission in a system which we will discuss in the next subsection.

We do not view these two problems as real problems. The proposals, with new definitions for the $O$- and $P$-operators and their new expressive power, are extensions of SDL with new meanings, not expressible in SDL, and are subject to new intuitions. It is, therefore, a mistake to read the formulas in these proposals as they are read in SDL, and
it is methodologically strange to reject a proposal just because it does not satisfy some axiom. For instance, for the general obligation and permission it is obvious that they are not interdefinable. However, that is not a problem of the formalisation: it is a \textit{feature} of these notions.

Finally, we summarise the basic principles in \textit{SDL} that also hold for the four proposals in table 3.1.

\textbf{Table 3.1} \textit{The basic principles in SDL and the four proposals}

<table>
<thead>
<tr>
<th></th>
<th>Proposal 1</th>
<th>Proposal 2</th>
<th>Proposal 3</th>
<th>Proposal 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(p) \land O(q) \rightarrow O(p \land q)$</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>$\neg (O(p) \land O(\neg p))$</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>$P(p) \equiv \neg O(\neg p)$</td>
<td>–</td>
<td>–</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$O(p) \rightarrow P(p)$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
</tbody>
</table>

+: the principle is valid.

–: the principle is not valid.

\subsection*{3.2.2 The combined approach}

To solve the 'problem' of interdefinability and the 'problem' of asymmetry simultaneously, we allow two notions of obligation into the system: general obligation $O^+(p)$ with its dual, unspecific permission $P^+(p)$, and unspecific obligation $O^-(p)$ with its dual, general permission $P^-(p)$:

\begin{align*}
O^+(p) & \equiv \forall_{i \in I} O_i(p) \\
P^+(p) & \equiv \exists_{i \in I} P_i(p) \\
O^-(p) & \equiv \exists_{i \in I} O_i(p) \\
P^-(p) & \equiv \forall_{i \in I} P_i(p)
\end{align*}

(3.8) \hspace{2cm} (3.9) \hspace{2cm} (3.10) \hspace{2cm} (3.11)

We do not enforce principle ($OD^*$) for $O^-$, as we stated in the previous subsection. Further, the following four principles, which correspond with the principles in figure 3.1, are valid:

\begin{align*}
O^+(p) & \rightarrow O^-(p) \\
P^-(p) & \rightarrow P^+(p)
\end{align*}

(3.12) \hspace{2cm} (3.13)
3.2 Formalisations of relativised deontic modalities

\[ O^+(p) \rightarrow P^-(p) \]  
(3.14)

\[ O^-(p) \rightarrow P^+(p) \]  
(3.15)

The following figure depicts the logical relations between operators \( O^+, O^-, P^+, P^-, O_i \) and \( P_i \):

**Figure 3.3**

In contrast to this approach, Bailhache (1991) wanted to keep principle \( (OD^*) \) for \( O^- \) to obtain a coherent normative system. This is validated by adding constraint (3.7). Now we can derive some extra principles, such as

\[ O^-(p) \rightarrow P^-(p) \]

and, therefore,

\[ O^-(p) \rightarrow P_i(p) \]

and

\[ O_i(p) \rightarrow P^-(p). \]

The valid properties can be summarised in the following figure:

**Figure 3.4**
3.2.3 Herrestad and Krogh

Herrestad and Krogh (1995) combined Kordig’s (1975) approach and Hilpinen’s (1973) approach. Kordig introduced a stronger notion \((O^\oplus_k)\) of general obligation \(O^+(p)\) and its dual, a weaker notion \((P^\ominus_k)\) of unspecific permission \(P^-(p)\). Hilpinen introduced a weaker notion \((O^\ominus_h)\) of unspecific obligation \(O^-(p)\) and its dual, a stronger notion \((P^\oplus_h)\) of general permission \(P^+(p)\). With the general and unspecific obligations, we can only express obligations for any or some particular individual in a group. Neither Kordig nor Hilpinen exactly defined the stronger \((O^\oplus_k)\) and the weaker \((O^\ominus_h)\) notions of obligation, respectively. However, Herrestad and Krogh suggested that these notions are collective notions: a collective obligation as an obligation that rests on the group, and not on any single individual. They did not offer a (semantic) definition of these collective notions, however. A drawback of this is that the notions of \(O^\oplus_k\) and \(O^\ominus_h\) are vague and, as a consequence difficult to apply.

The following properties hold for these new notions:

\[
O^\oplus_k(p) \rightarrow O^+(p) \tag{3.16}
\]

\[
P^+(p) \rightarrow P^\oplus_k(p) \tag{3.17}
\]

\[
O^-(p) \rightarrow O^\ominus_h(p) \tag{3.18}
\]

\[
P^\ominus_h(p) \rightarrow P^-(p) \tag{3.19}
\]

The truth conditions for \(O^\oplus_k\) and \(P^\ominus_h\) are defined as follows:

\[
\mathcal{M}, w \models O^\oplus_k(p) \text{ iff } R^\oplus_k(w) \subseteq \llbracket p \rrbracket \tag{3.20}
\]

and

\[
\mathcal{M}, w \models P^\ominus_h(p) \text{ iff } R^\ominus_h(w) \cap \llbracket p \rrbracket \neq \emptyset, \tag{3.21}
\]

with the following condition for accessibility function \(R^\oplus_k\):

\[
\bigcup_{i \in I} R_i(w) \subseteq R^\oplus_k(w) \text{ for all } w \in W. \tag{3.22}
\]

The intuition behind function \(R^\oplus_k\) is that it returns at least the union of all ideal worlds of all individuals. This corresponds with Kordig’s proposal. Note that we do not have \(\bigcup_{i \in I} R_i(w) = R^\oplus_k(w)\), since in that case \(O^\oplus_k(p) \equiv O^+(p)\).\(^8\)

\(^8\)Suppose \(\bigcup_{i \in I} R_i(w) = R^\oplus_k(w)\) and \(O^+(p)\) holds, then it follows that \(\forall_{i \in I} R_i(w) \subseteq \llbracket p \rrbracket\), which is equivalent to \(\bigcup_{i \in I} R_i(w) \subseteq \llbracket p \rrbracket\), that \(O^\oplus_k(p)\) holds. The proof of the converse is analogous.
3.2 Formalisations of relativised deontic modalities

From (3.3) and (3.22) it follows that the following constraint is valid:

\[ R_k^O(w) \neq \emptyset \text{ for all } w \in W. \] (3.23)

Thus, schema \((OD^*)\) holds for \(O_k^O\):

\[ \neg (O_k^O(p) \land O_k^O(\neg p)), \] (3.24)

which is equivalent to

\[ O_k^O(p) \rightarrow P_k^O(p). \] (3.25)

The truth conditions for \(O_k^\Theta\) and \(P_k^\Theta\) are defined as follows:

\[ \mathcal{M}, w \models O_k^\Theta(p) \text{ iff } R_k^\Theta(w) \subseteq [p] \] (3.26)

and

\[ \mathcal{M}, w \models P_k^\Theta(p) \text{ iff } R_k^\Theta(w) \cap [p] \neq \emptyset, \] (3.27)

with accessibility function \(R_k^\Theta\):

\[ R_k^\Theta(w) = \cap_{i \in I} R_i(w) \text{ for all } w \in W. \] (3.28)

The intuition behind function \(R_k^\Theta\) is that it returns the shared ideal worlds of all individuals, i.e., the ideal worlds that all individuals have in common. This corresponds with Hilpinen’s proposal.

The difference between Kordig’s theory and Hilpinen’s theory is that Kordig argued that if something is deontically relevant for one individual, it is also considered non-relativised deontically relevant \((R_i(w) \subseteq R_k^O(w))\), whereas Hilpinen claimed that something is deontically relevant for everybody if and only if it is also considered non-relativised deontically relevant \((\cap_{i \in I} R_i(w) = R_k^O(w))\).

Herrestad and Krogh rejected the principle \((OD^*)\) for \(O_k^O\):

\[ \neg (O_k^O(p) \land O_k^O(\neg p)), \] (3.29)

since this would make principle \((OD^*)\) for \(O^-\) derivable: principle (3.29) is equivalent to \(O_k^O(p) \rightarrow P_k^O(p)\), and from (3.18) and (3.19) it follows that \(O^-(p) \rightarrow P^-(p)\). This principle has been rejected, as we discussed in section 3.2. Thus, they did not add principle

\[ R_k^O(w) \neq \emptyset \text{ for all } w \in W. \] (3.30)

Consequently, schema \((OD)\) for \(O_k^O(p)\) is not valid: \(^9\)

\[ \neg O_k^O(p \land \neg p). \] (3.31)

\(^9\)Note that schema \((OD)\) is valid for \(O^-\), however.
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However, schema \((OC)\) is valid: let \(O^\ominus_h(p) \land O^\ominus_h(q)\) hold, then \(R^\ominus_h(w) \subseteq [p]\) and \(R^\ominus_h(w) \subseteq [q]\), hence \(R^\ominus_h(w) \subseteq [p] \land [q] = [p \land q]\). Thus, \(O^\ominus_h(p \land q)\). In subsection 3.2.1, we saw that schema \((OC)\) for \(O^-\) was counter-intuitive and does not hold. The fact that \((OC)\) is not valid for \(O^-\) does not imply that \((OC)\) is necessarily counter-intuitive for \(O^\ominus_h\). In subsection 3.3.2, we shall see that it depends on the interpretation of \(O^\ominus_h\) whether schema \((OC)\) is intuitive or counter-intuitive.

From (3.22) and (3.28) we can derive the constraint
\[
R^\ominus_h(w) \subseteq R^\ominus_k(w) \text{ for all } w \in W,
\]
which provides us with the principle
\[
O^\ominus_k(p) \rightarrow O^\ominus_h(p).
\]

This can be proven as follows. Suppose that \(O^\ominus_k(p)\) holds, then \(R^\ominus_k(w) \subseteq [p]\). Since \(R^\ominus_h(w) \subseteq R^\ominus_k(w)\), it follows that \(R^\ominus_h(w) \subseteq [p]\). Thus, \(O^\ominus_h(p)\).

Consequently, we can derive
\[
P^\ominus_k(p) \rightarrow P^\ominus_h(p).
\]

The problem of accepting \(O^\ominus_k(p) \rightarrow P^\ominus_h(p)\)

Just as where the general obligation implies the general permission (3.14) and the unspecific obligation implies the unspecific permission (3.15), Herrestad and Krogh aimed at holding these principles for \(O^\ominus_k\) and \(O^\ominus_h\):
\[
O^\ominus_k(p) \rightarrow P^\ominus_h(p)
\]

\(O^\ominus_h(p) \rightarrow P^\ominus_k(p)
\]

However, this does not follow from the semantics given so far. That is why Herrestad and Krogh tried to add the constraint:
\[
\text{if } R^\ominus_h(w) \subseteq [p], \text{ then } R^\ominus_h(w) \cap [p] \neq \emptyset \text{ for all } w \in W.
\]

Now principle (3.35) is valid, and by contraposing (3.35) and substituting \(\neg p\) for \(p\), we obtain principle (3.36). However, now it also holds that constraint (3.30) is valid.\(^\text{10}\) This provides principle \((OD^*\) for \(O^\ominus_h\), which implies principle \((OD^*\) for \(O^-\), which Herrestad and Krogh did not want.

\(^\text{10}\) Since \(R^\ominus_k(w) \subseteq [p \lor \neg p]\) is true for all \(w \in W\), then by means of (3.37) and modus ponens \(R^\ominus_h(w) \cap [p \lor \neg p] \neq \emptyset\) is also true for all \(w \in W\), which is equivalent to \(R^\ominus_k(w) \neq \emptyset\) for all \(w \in W\).
According to Herrestad and Krogh, the problem is due to the validity of \((ON)\) for \(O^\oplus\), i.e., \(O^\oplus(p \lor \neg p)\). The solution they provided is to block the inference of \(P^\odot_k(p)\) from \(O^\ominus_k(p)\) when \(p\) is a tautology. Instead of (3.37), they offered the principle\(^{11}\)

\[
(\lozenge p \land \lozenge \neg p) \rightarrow (O^\oplus_k(p) \rightarrow P^\odot_k(p)).
\]  

However, the solution is not quite satisfactory, for the reason that we can derive the following formulas

\[
P^\odot_k(p) \rightarrow (O^\oplus_k(q) \rightarrow P^\odot_k(q)),
\]

because \(R^\ominus_k(w) \neq \emptyset\), if \(R^\oplus_k(w) \cap [p] \neq \emptyset\), and by (3.38) also

\[
O^\oplus_k(p) \rightarrow (O^\oplus_k(q) \rightarrow P^\odot_k(q)),
\]

if \(p\) is no tautology. Thus, if there is a proposition \(p\) (and \(p\) is no tautology) for which \(P^\odot_k(p)\) or \(O^\oplus_k(p)\) is true, schema \((OD^*)\) is valid for \(O^\oplus_k\). We can solve this problem by stating that \(P^\odot_k(p)\) and \(O^\oplus_k(p)\) are always false for every \(p\) which is no tautology. But the question then arises why one introduces, for example, a stronger notion of the obligation where this obligation actually has the value ‘false’.

We do not think that the problem appears to be the validity of \((ON)\) for \(O^\oplus_k\). In subsection 3.3.3, we will provide a much simpler solution, based upon the interpretation of \(O^\oplus_k\) and \(O^\ominus_k\) as collective notions. The collective notions are formalised on the basis of the collective obligation, which we discuss in the following section.

### 3.3 The collective obligation

Obligations can also be aimed at a group, since there are cases in which the ‘addressee’ is not a single individual, but a group.\(^{12}\) For instance, a mother may tell her sons:

‘Boys, the table has to be set.’

This command is given to a group of boys, and it requires that the group accomplish something (i.e., that the table is set). This can be expressed as follows:

\[
O_X(p),
\]

---

\(^{11}\)\(\lozenge\) is the possibility sign. \(\lozenge p\) is true only in case \(p\) is true in some possible world.

\(^{12}\)The necessity to use groups of actors is stated as follows by Bailhache (1991, p. 80): ‘For imagine, for example, that such masons ought to build the foundation of the house, such roofers to assemble its roof, such electricians to set its electrical network, and so on: the firm, as the set of all these workers will have the duty of building the whole house. More generally, if it is obligatory for \(y\) that \(p \rightarrow q\) and obligatory for \(z\) that \(p\), which addressee will be touched by the resulting obligation (that \(q\))? It will be at least the set of \(y\) and \(z\) - without being able to say which individual is concerned.’
where ‘X’ does not refer to an individual, but to a set of individuals, in this case the boys, and ‘p’ to the statement ‘the table is set’. $O_X(p)$ can be read as ‘it is obligatory for X that p’. ‘X’ is used for the expression of a collective agency. We call these obligations collective obligations. Relativised obligation $O_X(p)$ does not mean that this is a restricted general obligation, i.e., that for every actor in X it is obligatory that p, but that X as a group has to accomplish that p. Consider the following example, taken from Rescher (1966, p. 69):

‘John and Paul are obliged that the table is moved across the room.’

If John alone accomplishes that the table is moved across the room, then the group (John and Paul) satisfies the norm, i.e., the obligation that the table is moved across the room. This example expresses a collective obligation, because for example ‘John is obliged that the table is moved across the room’ does not follow. Thus, the obligation does not require that everyone (in this case, John and Paul) in the group takes part in the act to accomplish that p.

In the following subsection, we add groups of actors to the system $SDL$, which can be seen as an extension of the relativised deontic modalities concerning what is obligatory or permitted for an actor. Further, we formalise some realistic, intuitive notions and analyse what properties hold for the relativised deontic modalities concerning groups of actors.

### 3.3.1 The semantics of the collective obligation

In order to discuss the relativised deontic modalities concerning groups of actors in $SDL$, we will use the following semantics. We will use the model structure $\mathcal{M} = (W, \mathcal{P}^+(I), \mathcal{R}_I, V)$ for system $SDL_X$ consisting of four elements:

1. the set of possible worlds $W = \{w_1, w_2, \ldots\}$;
2. the non-empty powerset $\mathcal{P}^+(I)$ of the set of individuals $I = \{i_1, i_2, \ldots\}$;
3. the set of functions $\mathcal{R}_I = \{R_X | X \in \mathcal{P}^+(I)\}$. The function $R_X \in \mathcal{R}_I$ on $W$ returns the deontically ideal worlds for group $X$ given a world: $R_X : W \to 2^W$;
4. a valuation function $V$, which assigns the value ‘true’ or ‘false’ to a proposition at a world in $W$.

We assume that $R_i(w) = R_{\{i\}}(w)$ for all $w \in W$ and $i \in I$.

The truth conditions for collective obligation $O_X$ and collective permission $P_X$ are now defined as follows:

$$\mathcal{M}, w \models O_X(p) \iff R_X(w) \subseteq \llbracket p \rrbracket$$  \hspace{1cm} \text{(3.41)}
3.3 The collective obligation

and

\[ \mathcal{M}, w \models P_X(p) \iff R_X(w) \cap [p] \neq \emptyset. \]  

(3.42)

Schema \((OD)\) holds by adding the following constraint:

\[ R_X(w) \neq \emptyset, \text{ for all } R_X \in \mathcal{R}_I \text{ and for all } w \in W. \]  

(3.43)

Truth conditions (3.41) and (3.42) are sufficient to validate the rule and all other schemata of system \(D^*\) for the collective obligation and permission. At first glance, this seems to be correct, since the impersonal obligation in \(SDL\) is read as an obligation related to one and the same individual or group all the time, which corresponds with \(O_i(p)\) or \(O_X(p)\): an obligation related to a single individual \(i\) or a single group \(X\). However, we shall see that the validity of schema \((OD)\) for \(O_X\) depends on the interpretation of collective obligation \(O_X(p)\).

3.3.2 The interpretation of \(O_X(p)\)

We discuss two possible interpretations of \(O_X(p)\):

1. the strict collective obligation: an obligation aimed strictly at group \(X\);

2. the weak collective obligation: an obligation for some subgroup of \(X\), i.e.,

\[ O_X(p) \equiv \exists Y \in \mathcal{P}^+(X) O_Y(p). \]  

(3.44)

The strict collective obligation

If we interpret \(O_X(p)\) as a strict collective obligation, we can discuss the theory of the relativised deontic modalities concerning groups of actors analogously with the theory concerning actors, since now every group stands on its own, like the actors. In this interpretation of \(O_X(p)\), we consider the groups to be single ‘addressees’. The only difference with the theory concerning actors is the number of addressees. Let \(I = \{i_1, \ldots, i_n\}\), then we have \(n\) addressees for the theory concerning individuals and \(2^n - 1\) addressees for the theory concerning groups of individuals.

A consequence of this interpretation is that there is no relation between the different groups if we accept conflicts between collective obligations for different groups, such as

\[ O_X(p) \land \neg O_Y(p). \]

\(\mathcal{P}^+(X)\) stands for the non-empty powerset of set \(X\).
If we want to obtain a coherent normative system, i.e., every group is permitted what is obligatory, with the formal counterpart

$$O_{X_1}(p) \land O_{X_2}(q) \land \ldots \land O_{X_k}(v) \rightarrow \forall_{X \in P^+(I)} P_X(p \land q \land \ldots \land v),$$  

(3.45)

then there is a relation between the groups, expressed by the following constraint

$$\cap_{X \in P^+(I)} R_X(w) \neq \emptyset, \text{ for all } w \in W.$$  

(3.46)

As we have seen in section 3.2, this removes the possibility of expressing conflicts between collective obligations for different groups of individuals. That is why we do not enforce principle (3.45).

**The weak collective obligation**

Bailhache (1991) stated that if there is a group for which it is obligatory to accomplish $p$, say $X$, and there is a group for which it is obligatory to accomplish $q$, say $Y$, then it is obligatory for at least group $X \cup Y$ to accomplish $p \land q$. This can be formalised as follows

$$O_X(p) \land O_Y(q) \rightarrow O_{X \cup Y}(p \land q),$$  

(3.47)

but this does not follow from the semantics given so far.

To validate principle (3.47), we add an extra condition for accessibility function $R_X$:

$$R_X(w) \subseteq R_Y(w) \text{ if } Y \in \mathcal{P}^+(X) \text{ for all } w \in W.$$  

(3.48)

Thus, if something is deontically relevant to a group $X$, it is also considered deontically relevant to every subgroup of $X$. Note that

$$R_I(w) = \cap_{X \in P^+(I)} R_X(w).$$  

(3.49)

It is easy to show that principle (3.47) is now valid: let $O_X(p) \land O_Y(q)$, then $R_X(w) \subseteq [p]$ and $R_Y(w) \subseteq [q]$. Since $R_{X \cup Y}(w) \subseteq R_X(w)$ and $R_{X \cup Y}(w) \subseteq R_Y(w)$, it holds that $R_{X \cup Y}(w) \subseteq R_X(w) \cap R_Y(w) \subseteq [p] \cap [q] = [p \land q]$. Hence, $O_{X \cup Y}(p \land q)$.

Further, we can derive the formula following immediately from (3.48)

$$O_X(p) \rightarrow O_{X \cup Y}(p).$$  

(3.50)

Thus, if it is obligatory for a group to accomplish something, then it is also obligatory for every superset of that group. And, consequently,

$$P_{X \cup Y}(p) \rightarrow P_X(p).$$  

(3.51)
At first glance, this may seem paradoxical, since it is normally incorrect to say, for instance, that if \( X \) is obliged that \( p \), then \( X \) and \( Y \) are obliged that \( p \). However, we interpret \( O_X(p) \) as an obligation for group \( X \) without being able to say to which subgroup of \( X \) this obligation applies. Thus, if 'it is obligatory for John to accomplish that the window is closed', then 'it is also obligatory for John and Paul to accomplish that the window is closed', meaning that John has to accomplish this, or Paul, or Paul and John together. We can formalise this weak collective obligation as follows:

\[
O_X(p) \equiv \exists y \in \mathcal{P}^+(X) O_Y(p), \tag{3.52}
\]

which is valid by constraint (3.48). Suppose that \( O_X(p) \) holds, then \( R_X(w) \subseteq [p] \). Hence, \( \exists y \in \mathcal{P}^+(X) (R_Y(w) \subseteq [p]) \), namely \( X \), thus \( \exists y \in \mathcal{P}^+(X) O_Y(p) \). Suppose now that \( \exists y \in \mathcal{P}^+(X) O_Y(p) \), then \( \exists y \in \mathcal{P}^+(X) (R_Y(w) \subseteq [p]) \), say \( Z \). Since \( Z \in \mathcal{P}^+(X) \), it follows that \( R_X(w) \subseteq R_Z(w) \). Hence, \( R_X(w) \subseteq [p] \) and thus \( O_X(p) \).

Consequently, by schema \((D f.P)\), it holds that

\[
P_X(p) \equiv \forall y \in \mathcal{P}^+(X) P_Y(p). \tag{3.53}
\]

The 'paradox' involved in (3.50) bears resemblance to the well-known Ross paradox,

\[
O(p) \rightarrow O(p \lor q), \tag{3.54}
\]

which is a valid principle in deontic logic: 'it is obligatory that the letter is mailed' implies that 'it is obligatory that the letter is mailed or burnt' (cf. section 2.4).

With this interpretation of \( O_X(p) \), we have to give up schema \((OD)\) for \( O_X \) if we want to express conflicts between the collective obligations of different groups, such as

\[
O_X(p) \land O_Y(\neg p).
\]

This formula does not hold in the semantics so far given, because \((O_X(p) \land O_Y(\neg p)) \rightarrow (O_X \lor Y(p) \land O_Y(\neg p))\) and \((O_X \lor Y(p) \land O_X \lor Y(\neg p))\) contradicts principle \((OD^*)\). Principle \((OD^*)\) is derived from schemata \((OC)\) and \((OD)\), thus we have to reject one of these schemata. We cannot reject \((OC)\), since this follows immediately from the truth condition for \( O_X \). Furthermore, it would be counter-intuitive to reject this schema. Thus, we reject schema \((OD)\) by giving up constraint (3.43).\(^{14}\) Now, we are able to express conflicts between collective obligations of groups. Since \((OD)\) does not hold, we can also express conflicts between collective obligations for the same group, such as

\[
O_X(p) \land O_X(\neg p).
\]

\(^{14}\)By giving up constraint (3.43), we also have to give up constraint (3.3). Hence, \((O_X)\) and \((OD^*)\) for \( O_X \) are not valid any more.
At first glance, this seems to be a drawback since, if we have a conflict, such as \( O_X(p) \land O_Y(\neg p) \), we can derive anything for the group \( X \cup Y \), for instance \( O_{X \cup Y}(q) \), since \( O_X(p) \land O_Y(\neg p) \rightarrow O_{X \cup Y}(p) \land O_{X \cup Y}(\neg p) \), and from schema \((OC)\) it follows that \( O_{X \cup Y}(p \land \neg p) \). From rule \((ROM)\) it follows that \( O_{X \cup Y}(q \land \neg q) \) and hence, by using once again the rule \((ROM)\), it follows that \( O_{X \cup Y}(q) \). However, this is not a real drawback since, if a group has to accomplish something that is impossible, it is hard to say anything meaningful about that group, so that we can derive anything about the obligations for that group.

### 3.3.3 The strong and weak obligations

With the semantics of the collective obligation and collective permission, we can also formalise the unspecific and general collective obligations and collective permissions. We claim that the general collective obligation and permission are the strongest notions of obligation and permission, and the unspecific collective obligation and permission the weakest notions of obligation and permission. We call the general collective obligation the **strong obligation**, the unspecific collective obligation the **weak obligation**, the general collective permission the **strong permission** and the unspecific collective permission the **weak permission**. We can formalise these notions as follows:

\[
O^\oplus(p) \equiv \forall_{X \in P^+(l)} O_X(p) \tag{3.55}
\]

\[
O^\ominus(p) \equiv \exists_{X \in P^+(l)} O_X(p) \tag{3.56}
\]

\[
P^\oplus(p) \equiv \forall_{X \in P^+(l)} P_X(p) \tag{3.57}
\]

\[
P^\ominus(p) \equiv \exists_{X \in P^+(l)} P_X(p) \tag{3.58}
\]

We define the truth conditions for \( O^\oplus \) and \( P^\oplus \) as follows

\[
\mathcal{M}, w \models O^\oplus(p) \text{ iff } R^\oplus(w) \subseteq \llbracket p \rrbracket \tag{3.59}
\]

and

\[
\mathcal{M}, w \models P^\oplus(p) \text{ iff } R^\oplus(w) \cap \llbracket p \rrbracket \neq \emptyset. \tag{3.60}
\]

To accomplish that these notions are the notions of the strong obligation and the weak permission, we have to restrict accessibility function \( R^\oplus \) as follows:

\[
R^\oplus(w) = \bigcup_{X \in P^+(l)} R_X(w) \text{ for all } w \in W. \tag{3.61}
\]
3.3 The collective obligation

Suppose \( O_\oplus(p) \). This holds iff \( R^\oplus(w) \subseteq \llbracket p \rrbracket \) iff \( \forall x \in \mathcal{P}^+(I) R_X(w) \subseteq \llbracket p \rrbracket \) iff \( \forall x \in \mathcal{P}^+(I) O_X(p) \). Thus, with this restriction of \( R^\oplus \), we accomplish that \( O_\oplus(p) \equiv \forall x \in \mathcal{P}^+(I) O_X(p) \) and, consequently, \( P_\oplus(p) \equiv \exists x \in \mathcal{P}^+(I) P_X(p) \).

Note that the following constraint holds:\(^{15}\)

\[
\cup_{i \in I} R_i(w) \subseteq R^\oplus(w) \text{ for all } w \in W,
\]

and that truth conditions (3.59) and (3.60) are equivalent to

\[
\mathcal{M}, w \models O_\oplus(p) \text{ iff } \forall x \in \mathcal{P}^+(I) (R_X(w) \subseteq \llbracket p \rrbracket)
\]

and

\[
\mathcal{M}, w \models P_\oplus(p) \text{ iff } \exists x \in \mathcal{P}^+(I) (R_X(w) \cap \llbracket p \rrbracket \neq \emptyset).
\]

Schema \((OD)\) becomes valid by adding the following constraint:

\[
R^\oplus(w) \neq \emptyset \text{ for all } w \in W,
\]

which corresponds with \( \exists x \in \mathcal{P}^+(I) (R_X(w) \neq \emptyset) \).

It is easy to see that the following principles are valid:

1. \( O_\oplus(p) \land O_\oplus(q) \rightarrow O_\oplus(p \land q) \);
2. \( O_\oplus(p \lor \neg p) \);
3. \( \neg O_\oplus(p \land \neg p) \);
4. \( P_\oplus(p) \equiv \neg O_\oplus(\neg p) \);
5. \( \neg (O_\oplus(p) \land O_\oplus(\neg p)) \);
6. \( O_\oplus(p) \rightarrow P_\oplus(p) \).

The rule and all the schemata of \( D^* \) are valid for the strong obligation \( O_\oplus \). Note that it does not matter whether we interpret \( O_X(p) \) as the strict collective obligation or as the weak collective obligation, since the strong obligation is an obligation aimed at all groups. Note that the general obligation corresponds with the strong obligation, in the sense that the principles that hold for \( O^+ \) also hold for \( O_\oplus \).

We can define the truth conditions for \( O_\ominus \) and \( P_\ominus \) as follows

\[
\mathcal{M}, w \models O_\ominus(p) \text{ iff } R_\ominus(w) \subseteq \llbracket p \rrbracket
\]

\(^{15}\)We mark again that \( R_i(w) = R_i(I) \) for all \( i \in I \) and for all \( w \in W \).
and

\[ M, w \models P^\Theta(p) \text{ iff } R^\Theta(w) \cap [p] \neq \emptyset. \]  

To accomplish that these notions are the notions of the weak obligation and the strong permission, we restrict accessibility function \( R^\Theta \) as follows

\[ R^\Theta(w) \subseteq \cap \{x \in \mathcal{P}(I) \mid R_X(w) \} \text{ for all } w \in W. \]

However, this is not an accurate definition for the weak obligation as the unspecific collective obligation. Now, we can only derive that \( 3 \times \forall x \neg \text{Et}(p) \rightarrow \neg \text{Et}(p) \). Suppose that \( 3 \times \forall x \text{Et}(p) \). This holds iff \( 3 \times \forall x \neg \text{Et}(p) \), hence \( \cap \{x \in \mathcal{P}(I) \mid R_X(w) \subseteq [p] \} \) and since \( R^\Theta(w) \subseteq \cap \{x \in \mathcal{P}(I) \mid R_X(w) \} \), it follows that \( R^\Theta(w) \subseteq [p] \). Thus, \( O^\Theta(p) \). That is why we define the truth conditions for \( O^\Theta \) and \( P^\Theta \) as follows:

\[ M, w \models O^\Theta(p) \text{ iff } 3 \times \forall x \text{Et}(p) \subseteq [p] \]  

and

\[ M, w \models P^\Theta(p) \text{ iff } \forall x \text{Et}(p) \subseteq [p] \neq \emptyset. \]

For the weak obligation, it does matter how we interpret \( O_X(p) \). As we have seen, there are two possible interpretations for the collective obligation:

1. the strict collective obligation;
2. the weak collective obligation.

1. Suppose that we interpret \( O_X(p) \) as the strict collective obligation. Then schema \((OD)\) for \( O^\Theta \) is validated by adding the constraint

\[ \forall x \in \mathcal{P}(I) \cap R_X(w) \neq \emptyset \text{ for all } w \in W. \]

\[ O^\Theta(p) \land \neg p \) holds iff \( \exists x \in \mathcal{P}(I) \cap R_X(w) \subseteq [p] \land \neg p \) iff \( \forall x \in \mathcal{P}(I) \cap (R_X(w) \subseteq [p] \) iff \( \forall x \in \mathcal{P}(I) \cap (R_X(w) \neq \emptyset). \)

We do not want schema \((OD*)\) for \( O^\Theta \), since it removes the possibility of expressing conflicting collective obligations of different groups. According to the semantics given, this schema does not hold. Thus, principle \( O^\Theta(p) \rightarrow P^\Theta(p) \) does not hold either, which is equivalent to principle \((OD*)\) for \( O^\Theta \).

\[ 16 \text{The strong obligation is an obligation for all groups in } \mathcal{P}(I). \text{ Hence, it does not matter how we interpret the collective obligation in a strong obligation, since for every set of individuals the same obligation holds.} \]

\[ 17 \text{Schema } (OD*) \text{ would be valid by adding the constraint } \cap \{x \in \mathcal{P}(I) \mid R_X(w) \neq \emptyset \text{ for all } w \in W.} \]
3.3 The collective obligation

Since we do not have principle \((OD^*)\) for the weak obligation \(O^\Theta\), schema \((OC)\) does not hold for \(O^\Theta\) either. In subsection 3.2.1, we saw that the formula
\[
(\exists_{i\in I} O_i(p) \land \exists_{i\in I} O_i(q)) \rightarrow \exists_{i\in I} O_i(p \land q)
\]  
was clearly counter-intuitive. That was the reason why we gave up schema \((OC)\) for the unspecific obligation \(O^-\). Analogously, we will reject \((OC)\) for the weak obligation \(O^\Theta\):
\[
(\exists_{x\in \mathcal{P}^+(I)} O_X(p) \land \exists_{x\in \mathcal{P}^+(I)} O_X(q)) \rightarrow \exists_{x\in \mathcal{P}^+(I)} O_X(p \land q),
\]  
since this is also counter-intuitive. Suppose that \((OC)\) would be valid and suppose further that only \(O_X(p)\) and \(O_Y(q)\) hold. Then \(\exists_{x\in \mathcal{P}^+(I)} O_X(p \land q),\) say \(Z\), thus \(O_Z(p \land q)\). Hence, it follows that \(O_Z(p)\) and \(O_Z(q)\) hold. Thus, the obligation to accomplish \(p\) and \(q\), respectively, is not strictly aimed at group \(X\) and \(Y\), respectively, but also at group \(Z\) and this is in contradiction with the interpretation of \(O_X(p)\) as the strict collective obligation.

Note that the weak obligation corresponds with the unspecific obligation in the sense that the principles that hold for \(O^-\) also hold for \(O^\Theta\).

Further, the following four principles are valid if we interpret \(O_X(p)\) as the strict collective obligation:

- (a) \(O^\Theta(p) \rightarrow P^\Theta(p)\);
- (b) \(O^\Theta(p) \rightarrow P^\Theta(p)\);
- (c) \(O^\Theta(p) \rightarrow O^\Theta(p)\);
- (d) \(P^\Theta(p) \rightarrow P^\Theta(p)\).

The former two principles are valid, because of constraint (3.71). The latter two principles are obvious.

The following figure summarises the logical relations between operators \(O^\Theta\), \(O^\Theta\), \(P^\Theta\), \(P^\Theta\), \(O_X\) and \(P_X\):\(^{18}\)

**Figure 3.5**

\[\begin{array}{cccc}
O^\Theta(p) & \rightarrow & P^\Theta(p) \\
\downarrow & & \downarrow \\
O_X(p) & \rightarrow & P_X(p) \\
\downarrow & & \downarrow \\
O^\Theta(p) & \rightarrow & P^\Theta(p)
\end{array}\]

\(^{18}\)Note that this figure corresponds with figure 3.3.
As we have seen in subsection 3.2.3, the theory developed by Herrestad and Krogh suffers from the problem that principle \((OD^*)\) creeps back in for \(O^\Theta_h\) if we accept the principle \(O^\Theta_h(p) \rightarrow P^\Theta_h(p)\). The solution they provide - to block the inference of \(P^\Theta_h(p)\) from \(O^\Theta_h(p)\) when \(p\) is a tautology - was not satisfactory, since we were able to derive the undesirable principles \(P^\Theta_h(p) \rightarrow (O^\Theta_h(q) \rightarrow P^\Theta_h(q))\) and \(O^\Theta_h(p) \rightarrow (O^\Theta_h(q) \rightarrow P^\Theta_h(q))\), if \(p\) is no tautology.

The above consideration is a solution to this problem, since the principle \(O^\Theta(p) \rightarrow P^\Theta(p)\) is valid and principle \((OD^*)\) does not creep back in for \(O^\Theta\). Further, the undesirable principles \(P^\Theta(p) \rightarrow (O^\Theta(q) \rightarrow P^\Theta(q))\) and \(O^\Theta(p) \rightarrow (O^\Theta(q) \rightarrow P^\Theta(q))\) are not valid, which is easy to see from truth conditions (3.69) and (3.70).

This solution consists of substituting the truth condition for \(O^\Theta_h\), \(P^\Theta_h\), \(O^\Theta_h\) and \(P^\Theta_h\) given by Herrestad and Krogh by the truth conditions given in this section, with the consequence that \(O^\Theta\) is defined as the strong obligation and \(O^\Theta\) as the weak obligation. The justification for these definitions is that we claim that the strong obligation is the strongest notion of the obligation and the weak obligation is the weakest notion of the obligation. An advantage of this consideration is that \(O^\Theta\) and \(O^\Theta\) are defined exactly in contrast to Herrestad and Krogh's theory, where the notions of \(O^\Theta_h\) and \(O^\Theta_h\) are vague and, as a consequence, difficult to apply.

The interpretation of \(O_X(p)\) as a strict collective obligation is necessary for this solution, as we shall see in the consideration of the weak collective obligation below.

2. If we interpret \(O_X(p)\) as the weak collective obligation, then schema \((OC)\) for \(O^\Theta\) is valid, since for this interpretation the following principle is valid (which we saw in the previous subsection):

\[
O_X(p) \land O_Y(q) \rightarrow O_{X \cup Y}(p \land q),
\]

because constraint (3.48) holds. Thus, \(O^\Theta(p) \land O^\Theta(q) \rightarrow O^\Theta(p \land q)\) is also valid.

Schema \((OD)\) does not hold for \(O^\Theta\), since \((OD)\) does not hold for the weak collective obligation either (see previous subsection). Suppose \((OD)\) holds for \(O^\Theta\), then \(-O^\Theta(p \land p)\) holds and this is equivalent to \(\forall x \in p+(l) -O_X(p \land -p)\). Hence, this implies that schema \((OD)\) is valid for \(O_X(p)\). Thus, since we do not have schema \((OD)\) for the weak collective obligation, we do not have this schema for the weak obligation either. Consequently, schema \((OD^*)\) does not hold, since it follows from schemata \((OD)\) and \((OC)\).

From (3.52) and (3.56) we can derive the following principles

\[
O_I(p) \equiv O^\Theta(p)
\]  

(3.74)
and, consequently,

\[ P_I(p) \equiv P^\ominus(p). \]  \hspace{1cm} (3.75)

Thus, \( O_I(p) \) coincides with the weak obligation and \( P_I(p) \) with the strong permission.

Now, we only need two different notions to distinguish the weak, collective and strong obligations, since collective obligation \( O_I(p) \) collapses in the weak obligation \( O^\ominus(p) \).

Further, the following two principles are valid

(a) \( O^\oplus(p) \rightarrow O^\ominus(p) \);

(b) \( P^\ominus(p) \rightarrow P^\oplus(p) \).

Principles \( O^\oplus(p) \rightarrow P^\ominus(p) \) and \( O^\ominus(p) \rightarrow P^\oplus(p) \) do not hold, since we do not have constraint (3.43), otherwise \( (OD^*) \) creeps back in for the weak obligation and, in addition for the unspecific obligation. This can easily be shown. Suppose \( O^\oplus(p) \rightarrow P^\ominus(p) \). This holds iff \( \forall x \in p^+(l)(R_X(w) \subseteq \{p\}) \rightarrow \forall x \in p^+(l)(R_X(w) \cap \{p\} \neq \emptyset) \) iff \( \forall x \in p^+(l)(R_X(w) \neq \emptyset) \) iff \( R_I(w) \neq \emptyset \) iff \( \cap x \in p^+(l)R_X(w) \neq \emptyset \). The constraint \( \cap x \in p^+(l)R_X(w) \neq \emptyset \) validates the constraint \( \exists x \in p^+(l)(R_X(w) \subseteq \{p\}) \rightarrow \forall x \in p^+(l)(R_X(w) \cap \{p\} \neq \emptyset) \). Thus, \( O^\ominus(p) \rightarrow P^\ominus(p) \), which is equivalent to principle \( (OD^*) \), creeps back in for the weak obligation.

This corresponds with the problem of accepting \( O^\oplus(p) \rightarrow P^\ominus(p) \) discussed in subsection 3.2.3 and above.

Note that the principle

\[ O^\oplus(p) \rightarrow P^\oplus(p) \]

is valid: 'if it is obligatory for all sets of actors that \( p \), then it is permitted for some sets of actors that \( p \).’ Thus, schema \( (OD^*) \) holds for \( O^\oplus \), because of constraint (3.65) and the truth conditions (3.59) and (3.60), but not for the weak collective obligation \( O_X \), since we do not have constraint (3.43).

The following figure summarises the logical relations between operators \( O^\oplus, O^\ominus, P^\oplus, P^\ominus, O_X \) and \( P_X \):

**Figure 3.6**
3.4 The notions of obligation

In this chapter, we distinguished six notions of obligations:

1. the personal obligation: $O_i(p)$;
2. the general obligation: $O^+(p)$;
3. the unspecific obligation: $O^-(p)$;
4. the collective obligation: $O_X(p)$;
5. the strong obligation: $O^\oplus(p)$;
6. the weak obligation: $O^\ominus(p)$,

with the corresponding notions of permission.

In this section, we summarise the relations between these notions. In the previous subsection, we saw that the validity of some relations, such as $O^\oplus(p) \rightarrow P^\ominus(p)$, depends on the interpretation of $O_X(p)$. First, we will give the relations between the different notions if we interpret $O_X(p)$ as the strict collective obligation.

The valid properties of the notions, with $O_X(p)$ as the strict collective obligation, are summarised in the following figure:

**Figure 3.7**

If we consider $O_X(p)$ to be the weak obligation, we do not need six but five different notions of obligation, since the weak collective obligation $O_I(p)$ collapses in the weak obligation $O^\ominus(p)$. Further, the principles $O^\oplus(p) \rightarrow P^\ominus(p)$ and $O^\ominus(p) \rightarrow P^\oplus(p)$ do not hold in this case, as we already discussed in the previous subsection.

The valid properties of the notions, with $O_X(p)$ as the weak collective obligation, are summarised in the following figure:
3.5 The directed obligation

In this section, we briefly discuss the addition of counterparties to the system $SDL_X$. The counterparty is an individual or a group of individuals that has a 'right' against the addressee, that has a 'duty' (an obligation). 'Right' and 'duty' are correlative terms, i.e., when a right is invaded, a duty is violated.

Several authors (Kanger, 1971, 1985; Lindahl, 1977; Makinson, 1986; Herrestad and Krogh, 1995) analysed or described the types of rights relationships between the addressees (or bearers) and counterparties from the classic work by Hohfeld (1964). Here, no attempt is made to examine the approaches taken by these authors; we limit ourselves to a suggestion of how counterparties can be indexed in the system $SDL_X$.

The directed obligation $yO_x(p)$ can be read as 'it is obligatory for $X$ towards $Y$ that $p$', with $X, Y \in P^+(I)$. An example of a directed obligation is that John is obliged towards Paul that he shall stay away from Paul's land. In other words, Paul has a right against John that the latter will stay away from Paul's land, the correlative (and equivalent) being that John is under a duty toward Paul to stay away from the place (cf. Hohfeld, 1964).

We will use the model structure $\mathcal{M} = (W, P^+(I), R^I, V)$ for system $SDL_{XY}$ consisting of four elements:

1. the set of possible worlds $W = \{w_1, w_2, \ldots\};$

2. the non-empty powerset $P^+(I)$ of the set of individuals $I = \{i_1, i_2, \ldots\};$

3. the set of functions $R^I = \{R_{XY} \mid X, Y \in P^+(I)\}$. The function $R_{XY} \in R^I$ on $W$ returns the deontically ideal worlds for group $X$ towards $Y$ given a world: $R_{XY} : W \rightarrow 2^W$;

4. a valuation function $V$, which assigns the value 'true' or 'false' to a proposition at a world in $W$. 

---

**Figure 3.8**

![Diagram of directed obligations](image_url)
The truth conditions for directed obligation $\gamma O_X$ and collective permission $\gamma P_X$ are now defined as follows:

$$M, w \models \gamma O_X(p) \iff R_{XY}(w) \subseteq [p]$$  \hspace{1cm} (3.76)

and

$$M, w \models \gamma P_X(p) \iff R_{XY}(w) \cap [p] \neq \emptyset.$$  \hspace{1cm} (3.77)

Schema (OD) holds by adding the following constraint:

$$R_{XY}(w) \neq \emptyset, \text{ for all } R_{XY} \in \mathcal{R}'_i.$$  \hspace{1cm} (3.78)

Truth conditions (3.76) and (3.77) are sufficient to validate the rule and all other schemata of system $D^*$ for the directed obligation and permission. The addition of constraint (3.78) depends on the interpretation of directed obligation $\gamma O_X(p)$. As we have seen in subsection 3.3.2, we have to give up schema (OD) for $\gamma O_X$ if we add the condition for accessibility function $R_{XY}$:

$$R_{XY}(w) \subseteq R_{ZY}(w) \text{ if } Z \in \mathcal{P}^+(X) \text{ for all } w \in W.$$  \hspace{1cm} (3.79)

Thus, we can derive the formula

$$\gamma O_X(p) \equiv \exists_{Z \in \mathcal{P}^+(X)} \gamma O_Z(p),$$

corresponding with the weak collective obligation.

Just like the two possibilities of interpretations of $O_X(p)$, we can distinguish two interpretations of the directed obligation $\gamma O_X(p)$ with respect to the counterparty:

1. the strict directive obligation: an obligation for $X$ strictly towards group $Y$;
2. the weak directed obligation: an obligation for $X$ towards some subgroup of $Y$, i.e.,

$$\gamma O_X(p) \equiv \exists_{Z \in \mathcal{P}^+(Y)} Z O_X(p).$$  \hspace{1cm} (3.80)

To validate principle (3.80), we have to add an extra condition to accessibility function $R_{XY}$:

$$R_{XY}(w) \subseteq R_{XZ}(w) \text{ if } Z \in \mathcal{P}^+(Y) \text{ for all } w \in W.$$  \hspace{1cm} (3.81)

Thus, if it is obligatory for $X$ towards $Y$ that something will be accomplished, then $X$ is also obliged to accomplish the same thing towards every superset of $Y$. This bears resemblance to the paradoxical character of the weak collective obligation (see subsection 3.3.2).
3.6 Conclusions

From the above consideration we showed that there are several interpretations of the directed obligation $O_X(p)$, depending on whether one adds constraints (3.79) and (3.81). As with the addition of (groups of) actors to SDL, the addition of counterparties leads to new deontic operators with a new meaning and therefore subject to new intuitions. From the above definitions, it can be seen that these changes are similar to those effected by the addition of actors. Theoretically, an analysis of the new intuitions would therefore be (for a large part) a repetition of the previous sections. We will therefore abstain from such an analysis at this place and just conclude that it is possible to extend $SDL_X$ with counterparties and that the same type of choices with respect to counterparties can be made as were made with respect to actors.

3.6 Conclusions

The result of the presentations of formalisations of relativised deontic modalities is a distinction between several levels of notions of obligation and permission. The first level is the level of the personal notions: $O_i$ and $P_i$. The second level is the level of the general and unspecific notions: $O^+$ and $P^-$, and $O^-$ and $P^+$, respectively. The third level is the level of the collective notions: $O_X(p)$ and $P_X(p)$, and the last level we discussed is the level of strong and weak notions: $O^\Theta$ and $P^\Theta$, and $O^-\Theta$ and $P^-\Theta$, respectively. The relations between these four levels are summarised in figures 3.7 and 3.8.

The extension by including relativised deontic modalities in SDL provides us with the possibility of expressing, for example, that an act is obligatory for an individual, but not for everyone:

$$O_i(p) \land \neg \forall_{j \in I} O_j(p),$$

which is not expressible within a non-relativised deontic logic.

A problem arose in Herrestad and Krogh's theory when they added the principle $O^\Theta_h(p) \rightarrow P^\Theta_h(p)$ to their system, since this gave principle $(OD^*)$ for $O^\Theta_h$ and for $O^-$. They proposed a restricted bridge principle. However, this solution is not quite satisfactory. Instead, we suggest to define $O^\Theta(p)$ as the strong obligation and $O^-\Theta(p)$ as the weak obligation, based on the collective obligation $O_X(p)$ interpreted as the strict collective obligation. We have shown that this smoothly solves the problem without undesirable consequences. The interpretation of $O_X(p)$ as the strict collective obligation is essential since the problem is not solved if we interpret $O_X(p)$ as the weak collective obligation.

This distinction between the interpretations of $O_X(p)$ is important since it acquires different meanings of and intuitions about collective obligations.

Furthermore, we have shown how system $SDL_X$ can be extended to counterparties, and formalised the various interpretations of the directed obligation.
Concluding, we may state that we presented a description of relativised deontic operators for Ought-to-be statements. This approach avoids the problems encountered in other approaches. In the next chapter, we will give a description of relativised deontic operators pertaining to Ought-to-do statements.
Chapter 4

Relativised deontic modalities in $PDeL$

This chapter presents the inclusion of relativised deontic modalities in $PDeL$ concerning individuals and groups of individuals.

4.1 Introduction

A normative rule is aimed at actors, who are expected to follow the norms specified by the (relevant) authority. The addition of actors to $PDeL$ allows us to express who has the responsibility for performing an action. For instance, the applicability of the norm 'it is obligatory to perform action $\beta (O(\beta))$' depends on the individual for whom the norm is meant. According to the Dutch Traffic Regulation 1990, the speed limit for mopeds is 30 km/h within built-up areas and 50 km/h for motor vehicles. Therefore, the norm $O(\beta)$, where $\beta$ is the action 'driving slower than 30 km/h' is aimed at moped riders and not at motor-vehicle drivers. Thus, moped riders can be fined for speeding with respect to this norm, but on the strength of this norm motor-vehicle drivers cannot.

It is obvious that an action cannot be performed by itself; it has to be performed by an actor or a group of actors. We will take an event to be an action initiated by an actor or a group of actors. We distinguish two kinds of events:

1. individual events, i.e., actions initiated by individuals;
2. collective events, i.e., actions initiated by groups of actors.

With the help of these events, we extend $PDeL$ with actors and groups of actors. The concept of collective events is new, and can be seen as a modification and an extension of the actor logic $L(Sig_{Dyn})$ developed by Wieringa and Meyer (1993).

1Some of the ideas in this chapter were presented earlier in Royakkers and Dignum (1995a, 1995b).
This chapter mainly deals with a semantic and an axiomatic study of extensions of $PD_eL$, taking into consideration actors and groups of actors on the basis of dynamic logic. These extensions of $PD_eL$ enables us to distinguish strong, general, personal, group, weak and unspecific obligations (the same types of obligations as in chapter 3), and to formalise and analyse the relations between these different notions of obligation.

The organisation of this chapter is as follows. Section 4.2 deals with the semantics of individual events and the extension of $PD_eL$ concerning the addition of actors on the basis of individual events. In section 4.3, we discuss the semantics of collective events and the extension of $PD_eL$ concerning the addition of groups of actors on the basis of collective events. Section 4.4 presents the distinction between the different notions of obligation and their relations and analyses the properties of some notions of obligation. In section 4.5, we discuss the satisfaction of norms by groups. In the final section, we provide some conclusions.

4.2 The extension of $PD_eL$ with actors

In this section, we discuss the extension of $PD_eL$ with actors on the basis of individual event expressions. First, we discuss the semantics of individual event expressions.

4.2.1 The semantics of individual event expressions

The non-empty set of actors who initiate the actions is denoted by $I$. An atomic individual event is an action initiated by an actor. (For short, we will write 'events' instead of 'individual events' in this section.) An atomic event is denoted by an actor $i \in I$ and an action $\beta$ as follows: $i: \beta$. We define $Evt$ as the set of atomic events.\(^2\)

The set of all event expressions $Evt$ can now be determined by the following BNF for its elements ($\alpha$):

$$\alpha ::= i: \beta|\alpha_1 \cup^* \alpha_2|\alpha_1 \& \alpha_2|\alpha_1 \& \alpha_2|\alpha_1,$$

where $i \in I$, $\alpha_1, \alpha_2 \in Evt$ and $\beta \in Act$.

The meaning of $\alpha_1 \cup^* \alpha_2$ is a choice between $\alpha_1$ and $\alpha_2$. Furthermore, $\alpha_1 \& \alpha_2$ stands for the simultaneous performance of $\alpha_1$ and $\alpha_2$, and $\overline{\alpha}$ stands for the negation of event expression $\alpha$. The event expression $i: \text{change}$ means that $i$ performs a universal ('do not care which') action. The event expression $i: \text{any}$ means that $i$ performs a universal ('do not care which') action. The $i: \text{change}$ can then be characterised as the performance of a 'do not care which, but not the skip' action by $i$. The event expression $i: \text{skip}$ stands for $i$ 'performing' the empty action, i.e., the event that has no effect ('does nothing'). Finally, the event expression $i: \text{fail}$ means that $i$ performs an action that always fails. After this event, the system stops and nothing can be done any more. Note that the meanings of

\(^2\)Note that an action $\beta$ in an atomic event is not necessarily an atomic action.
4.2 The extension of $PD_eL$ with actors

these event expressions correspond with the meanings of the action expressions discussed in chapter 2, except that actors are added here.

Remarks.

1. Wieringa and Meyer (1993) make a distinction between two kinds of choices:

   - the active choice: the choice labelled by an actor who makes the choice. They write $i : (\beta_1 + \beta_2)$ for the active choice made by actor $i$;
   - the passive choice: the choice between $\beta_1$ and $\beta_2$ is an underspecification, because it is either action $\beta_1$ or action $\beta_2$, but it is not specified which. They write $+$ for passive choice.

   They show that active and passive choices differ. For example, the passive choice is associative ($i : ((\beta_1 + \beta_2) + \beta_3) = i : (\beta_1 + (\beta_2 + \beta_3))$) and the active choice is not. In our language of $E vt$, the choice $i : \beta_1 + \beta_2$ stands for the passive choice.

2. The performance of an atomic event, e.g., $i : a_1 \& a_2$ (e.g., to walk & to whistle) involves the performance of the (semantical) elementary actions $a_1$ (to walk) and $a_2$ (to whistle), possibly together with other elementary actions (e.g., to look, to cross the street, etc.). The meaning of $i : a_1 \& a_2$ only stipulates the performance of actions $a_1$ and $a_2$ (corresponding semantical $a_1$ and $a_2$), but $i$ is free to perform any other set of elementary actions simultaneous with $a_1$ and $a_2$.

Formally, we give the semantics of event expressions by means of synchronicity sets, similar to the ones defined in chapter 2. These synchronicity sets denote performances of `packages' of (semantical) elementary actions that have to be performed simultaneously by the same actor.

Definition 4.2.1

1. Set $\{\delta\}$ is a synchronicity set ($s$-set).

2. Set $\{i : skip\}$ is an $s$-set.

3. Every pair of a non-empty subset of $A$ and an individual in $I$ is an $s$-set.

We use $S, S_1, S_2, ...$ for $s$-sets. The set of all $s$-sets, except $\{\delta\}$, will be denoted by $S^*$. In concrete cases, we write an $s$-set using brackets. For instance, let $A = \{a_1, a_2, a_3\}$ and $I = \{i_1, i_2\}$, then $s$-set $S$ consisting of the atomic actions $a_1$ and $a_2$ performed by $i_1$ is written as

$$S = [i_1 : A']$$

or

$$S = \begin{bmatrix} i_1 & a_1 \\ & a_2 \end{bmatrix}.$$
with $A' = \{a_1, a_2\}$.

The s-set consisting of event $i_1 : \text{skip}$ is written as $[i_1 : \text{skip}]$. Thus, $S^* = \{[i : A'] \mid i \in I, A' \subseteq A, A' \neq \emptyset\} \cup \{[i : \text{skip}] \mid i \in I\}$. For the above example, $S^* =$

$$
\{[i_1 : \text{skip}], [i_1 : a_1], [i_1 : a_2], [i_1 : a_3], [i_1 : a_1], [i_1 : a_2], [i_1 : a_3], [i_2 : a_2], [i_2 : a_3], [i_2 : a_2], [i_2 : a_3], [i_2 : a_2], [i_2 : a_3], [i_3 : a_3], [i_3 : a_3]
$$

We use $S^*_i$ to denote the set of all s-sets, except s-set $[\delta]$, with $i$ as the individual, thus $S^*_i = \{[i : A'] \mid A' \subseteq A, A' \neq \emptyset\} \cup \{[i : \text{skip}]\}$.

**Definition 4.2.2** Let $I = \{i_1, i_2, \ldots, i_n\}$, then $\mathcal{T}$ is defined as the set of the elements of the indirect product of $S^*_i, S^*_i, \ldots, S^*_i$ and $S^*_n$. We denote this as $\mathcal{T} = \text{def } \times_{i \in I} S^*_i$. An element of $\mathcal{T}$ is called a step. We use the letter $t$ with possible marks for steps. We use $t_i$ for the set of actions in $A$ in the s-set of $t$ which has $i$ as the actor.

Thus, a step $t$ of $\mathcal{T}$ is an n-tuple. The order of the s-sets in a step is irrelevant. For instance, let $A = \{a_1, a_2, a_3\}$ and $I = \{i_1, i_2, i_3\}$, then the triple $t$ consisting of s-sets $[i_1 : a_1], [i_2 : a_1, a_3]$, and $[i_3 : a_2]$ is written as

$$
\begin{bmatrix}
[1] \\
[2] \\
[3]
\end{bmatrix}
$$

and this is equal to the step

$$
\begin{bmatrix}
[1] \\
[2] \\
[3]
\end{bmatrix}, \begin{bmatrix}
[1] \\
[2] \\
[3]
\end{bmatrix}, \begin{bmatrix}
[1] \\
[2] \\
[3]
\end{bmatrix}
$$

with, e.g., $t_{i_2} = \{a_1, a_3\}$. Note that the equality can be formally realised by defining an equivalence relation between steps. Here, it suffices to give this informal account.

A step denotes a deterministic sets of actions for all actors simultaneously. It shows for each actor what he will do next. The usefulness of the $\text{skip}$ operator is that we can now say that in a certain situation some actors do nothing or do not affect the situation. Without the $\text{skip}$ operator, we would have to indicate for each moment what actions are performed
4.2 The extension of PD_eL with actors

by each actor, even though some actors do not affect the situation. This leads to a very artificial and counter-intuitive specification, because it is unlikely that every actor does something all the time. We will use skip to indicate that actions performed by some actor have no influence on the situation.

Since our language of events contains a non-deterministic (choice) operator \( \cup^* \), we have to consider sets of steps \( t \in T \). We use \( T, T_1, T_2, \ldots \) to denote sets of steps.

Our event domain \( M_T \) is now given by:

**Definition 4.2.3** \( M_T \) is the collection of sets \( T \) consisting of steps.

Note that \( M_T \) is the powerset of \( T \), normally denoted by \( 2^T \) or \( \mathcal{P}(T) \). We will use the operations \( \sqcup \), \( \sqcap \) and \( \neg \) (operator for the set-theoretic complement) on the domain \( M_T \) as semantical counterparts of the syntactical operators \( \cup^* \), \( \&^* \) and \( \neg \), respectively, in our language \( Evt \) of event expressions. Before we give these definitions, we define a handy operator on sets of steps:

**Definition 4.2.4** Let \( T \) be a set of steps, then

\[
T^S = \begin{cases} 
T \setminus \{[\delta]\} & \text{if } \exists t \in T \neq [\delta] \\
\{[\delta]\} & \text{otherwise.}
\end{cases}
\]

Now we can give the semantical operators on \( M_T \). For the simultaneous operator \( \&^* \), we use a set intersection \( \sqcap \), which is almost the same as the normal set intersection.\(^3\)

**Definition 4.2.5** For \( T_1, T_2 \in M_T \):

\[
T_1 \sqcap T_2 = \begin{cases} 
T_1 \cap T_2 & \text{if } T_1 \cap T_2 \neq \emptyset \\
\{[\delta]\} & \text{otherwise.}
\end{cases}
\]

Operator \( \sqcup \) on sets of steps is defined as follows:

**Definition 4.2.6** For \( T_1, T_2 \in M_T \):

\[
T_1 \sqcup T_2 = (T_1 \cup T_2)^S.
\]

Thus, the choice between two sets of steps is the union of those two sets minus \( \{[\delta]\} \), unless the union does not contain anything else.

\(^3\)Note that the definitions are simpler than in chapter 2, because of the fact that no \( s \)-sequences are allowed.
The definition of $\neg$ is given as follows:

**Definition 4.2.7**

1. For a step $t$,
   \[ t^- = T \setminus \{t\}. \]
2. For a non-empty set $T \in M_T$,
   \[ T^- = \cap \{t^- | t \in T\}. \]

Thus, for a step $t$ the negation just yields the set-theoretic complement of $\{t\}$ with respect to $T$. For the negation of a set of steps $T$, we take the intersection of the sets of the negation of all the steps contained in $T$. Note that $T \subseteq T$ and that $T \in M_T$. Now we can define the semantics of event expressions:

**Definition 4.2.8** The semantic function $[\ ] : Evt \to M_T$, with $a, \beta_1, \beta_2 \in Act$, $i \in I$ and $\alpha, \alpha_1, \alpha_2 \in Evt$, is given by:

1. $[i : a] = \{t \in T | a \in t_i\}$;
2. $[i : \beta_1 \cup \beta_2] = [i : \beta_1] \cup [i : \beta_2]$;
3. $[i : \beta_1 \& \beta_2] = [i : \beta_1] \cap [i : \beta_2]$;
4. $[i : \beta_1^+] = [i : \beta_1]$;
5. $[\alpha_1 \cup^* \alpha_2] = [\alpha_1] \cup [\alpha_2]$;
6. $[\alpha_1 \&^* \alpha_2] = [\alpha_1] \cap [\alpha_2]$;
7. $[\alpha]^- = [\alpha]$;
8. $[i : \text{fail}] = \{[\delta]\}$;
9. $[i : \text{any}] = T$;
10. $[i : \text{skip}] = \{t \in T | t_i = \{\text{skip}\}\}$;
11. $[i : \text{change}] = T \setminus [i : \text{skip}]$. 
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Remarks

1. \([i : a]\) expresses the meaning of event expression \( i : a \): we specify the performance of the elementary action \( a \) (simultaneous with some package of actions) by \( i \) and simultaneous with the performance of undetermined actions by the other individuals. Thus, only the performance of \( a \) by \( i \) is determined. \([i : \text{any}]\) stands for the set of all steps in \( T \).

Suppose \( \alpha \) is event expression \( i_1 : a \cup^* i_2 : b \), then \([\alpha]\) is the set of all steps containing an \( s \)-sets with elementary action \( a \) performed by \( i_1 \) or \( b \) performed by \( i_2 \). In the same way, \([i_1 : a \&^* i_2 : b]\) is the set of all steps consisting of elementary action \( a \) performed by \( i_1 \) and \( b \) performed by \( i_2 \).

2. Definition 4.2.8.7 defines negation as a complement operator. \( \overline{\alpha} \) means: '\( \alpha \) does not occur, and it is not specified what does occur.' This is similar to the meaning of negation in propositional logic. Wieringa and Meyer (1993) call this interpretation of event negation passive event negation.

3. From definitions 4.2.8.4 and 4.2.8.7 it now follows that \([i : \overline{\beta}] = [i : \overline{\beta}]\). Thus, the negation of event expression \( i : \beta \) stands for not-performing action expression \( \beta \) by actor \( i \). Wieringa and Meyer (1993) call this interpretation of event negation active event negation.\(^4\) They make a distinction between active event negation in a local way and in a global way. In a local way, it can be interpreted as the statement that \( i \) does something other than \( \beta \), and in a global way it can be interpreted as the statement that another actor performs \( \beta \). We use the local active event negation, because we can derive \([i : \text{fail}] = [i : \text{any}]\), and this is only reasonable if we assume local active event negation.\(^5\)

4. Above we noted that event expression \( i : \beta_1 \cup^* \beta_2 \) stands for the passive choice. This is expressed formally in definition 4.2.8.2.

Example. Let \( A = \{a_1, a_2\} \) and \( I = \{i_1, i_2\} \). Then,

- \([i_1 : a_1] = \{(i_1 : a_1), [i_2 : a_1], [i_1 : a_1], [i_2 : a_2], [i_1 : a_1], [i_2 : \text{skip}]\}\)

\(^4\)Note that we do not make a distinction between active and passive event negation.

\(^5\)The local interpretation of event negation is enforced by the axiom

\([i : a][i : \overline{a}] = [i : \text{any}]\).

The global interpretation of event negation cannot be axiomatised in the example specification, for it requires a constant that denotes the process "any actor does something." (Wieringa and Meyer, 1993, p. 305)
Relativised deontic modalities in $P_{D, L}$

\[
\left(\left[ i_1 : a_1 \right], \left[ i_2 : a_2 \right] \right), \left(\left[ i_1 : a_1 \right], \left[ i_2 : a_1 \right] \right), \left(\left[ i_1 : a_1 \right], \left[ i_2 : a_2 \right] \right),
\left(\left[ i_1 : a_1 \right], \left[ i_2 : a_2 \right] \right), \left(\left[ i_1 : a_1 \right], \left[ i_2 : \text{skip} \right] \right)
\]

- $\left[ i_1 : \overline{a_1} \right] = \left[ i_1 : a_1 \right] =
\left\{\left(\left[ i_1 : a_2 \right], \left[ i_2 : a_1 \right] \right), \left(\left[ i_1 : a_2 \right], \left[ i_2 : a_2 \right] \right), \left(\left[ i_1 : a_2 \right], \left[ i_2 : \text{skip} \right] \right),
\left(\left[ i_1 : \text{skip} \right], \left[ i_2 : a_1 \right] \right), \left(\left[ i_1 : \text{skip} \right], \left[ i_2 : a_2 \right] \right), \left(\left[ i_1 : \text{skip} \right], \left[ i_2 : \text{skip} \right] \right),
\right\}$

- $\left[ i_1 : a_1 \&^{*} i_1 : a_2 \right] = \left[ i_1 : a_1 \& a_2 \right] =
\left\{\left(\left[ i_1 : a_1 \right], \left[ i_2 : a_1 \right] \right), \left(\left[ i_1 : a_2 \right], \left[ i_2 : a_2 \right] \right), \left(\left[ i_1 : a_2 \right], \left[ i_2 : \text{skip} \right] \right),
\right\}$

- $\left[ i_1 : a_1 \cup^{*} i_1 : a_2 \right] = \left[ i_1 : a_1 \right] \cup \left[ i_1 : a_2 \right] = \left[ i_1 : \text{change} \right]$

- $\left[ i_1 : a_1 \cup^{*} i_2 : a_2 \right] = T \setminus \left[ i_1 : a_1 \cup^{*} i_1 : a_2 \right] = \left[ i_1 : a_1 \right] \cup \left[ i_2 : a_2 \right] =
\left\{\left(\left[ i_1 : a_2 \right], \left[ i_2 : a_1 \right] \right), \left(\left[ i_1 : a_2 \right], \left[ i_2 : \text{skip} \right] \right),
\left(\left[ i_1 : \text{skip} \right], \left[ i_2 : \text{skip} \right] \right), \left(\left[ i_1 : \text{skip} \right], \left[ i_2 : a_1 \right] \right) \right\}$

- $\left[ i_1 : a_1 \&^{*} i_2 : a_2 \right] = T \setminus \left[ i_1 : a_1 \&^{*} i_2 : a_2 \right] = \left[ i_1 : a_1 \right] \cup \left[ i_2 : a_2 \right] = \left[ i_1 : \overline{a_1} \right] \cup \left[ i_2 : \overline{a_2} \right].$

Note that $\left[ i_1 : a_1 \&^{*} i_1 : \overline{a_1} \right] = \left[ i_1 : \text{fail} \right] = \{\delta\}$ and that $\left[ i_1 : a_1 \&^{*} i_2 : \overline{a_1} \right] \neq \{\delta\}.$

In this section, we also introduce an auxiliary notion to be used in the sequel:

**Definition 4.2.9** We put $\alpha_1 = \tau \alpha_2$ iff $[\alpha_1] = [\alpha_2].$

The following proposition can easily be derived from the semantic function:

**Proposition 4.2.10**

1. For $M_T$, the following properties concerning operators $\cup^{*}$ and $\&^{*}$ hold:

   (a) $\alpha \cup^{*} \alpha = \tau \alpha;$
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\(\alpha \& \alpha = T \alpha;\)
\(\alpha_1 \cup^* \alpha_2 = T \alpha_2 \cup^* \alpha_1;\)
\(\alpha_1 \& \alpha_2 = T \alpha_2 \& \alpha_1;\)
\(\alpha_1 \& \alpha_2 = T (\alpha_1 \& \alpha_2) \cup^* (\alpha_1 \& \alpha_3);\)
\(\alpha_1 \cup^* (\alpha_2 \& \alpha_3) = T (\alpha_1 \cup^* \alpha_2) \& (\alpha_1 \cup^* \alpha_3).\)

2. $MT$ satisfies the following properties regarding $^*$:

- $\alpha \& \alpha = T \alpha$;
- $\alpha \cup^* \alpha = T \alpha$;
- $\alpha \cup^* \alpha = T \alpha_1 \& \alpha_2$;
- $\alpha_1 \cup^* \alpha_2 = T \alpha_1 \& \alpha_2$;
- $\alpha \& \alpha = T \alpha$.

3. $MT$ satisfies the following properties regarding the actions performed by an actor $i$:

- $i : \beta_1 \cup^* i : \beta_2 = T i : \beta_1 \cup \beta_2$;
- $i : \beta_1 \& i : \beta_2 = T i : \beta_1 \& \beta_2$;
- $i : \beta = T i : \beta$.

4. $MT$ satisfies the following properties concerning the special events:

- $\alpha \cup^* i : \text{fail} = T \alpha$;
- $\alpha \& i : \text{fail} = T \alpha$;
- $(i : \text{change}) \cup^* (i : \text{skip}) = T i : \text{any}$;
- $(i : \text{change}) \& (i : \text{skip}) = T i : \text{fail}$;
- $i : \text{skip} = T i : \text{change}$;
- $i : \text{any} = T i : \text{fail}$.

**Proposition 4.2.11** $(MT, \cup^*, \&^*, \text{fail})$ is a Boolean algebra.

**Proof.** Follows directly from the equations mentioned above.

**Remarks**

1. The choice has the usual properties of choice in process algebra: it is commutative, associative and idempotent. The simultaneous event also has these properties. It can easily be proven from the equations mentioned above.
2. In the models presented by Wieringa and Meyer (1993), the formulas $i: \beta \cup^* i: \text{fail} = i: \beta$ and $i: \beta \&^* i: \text{fail} = i: \text{fail}$ are omitted from the specification. In their models, these two formulas are not valid. They argue that $i: \beta \cup^* i: \text{fail} = i: \beta$ is only valid if $i: \beta$ is a choice over a set of options that includes $i: \text{fail}$, which, according to Meyer and Wieringa, is highly questionable. The formula $i: \beta \&^* i: \text{fail} = i: \text{fail}$ is not valid, for the following reason: $i: \beta \&^* i: \text{fail}$ does not denote anything at all, because it means that $i$ performs $\beta$ and does nothing at the same time, and is, therefore, not equal to $i: \text{fail}$, which means that $i$ fails but that other actors can still perform actions.

In our model of the semantics of events expression, these formulas are valid. The event expression $i: \text{fail}$ is the event that always fails. After this event, the system stops and nothing can be done any more by anybody (there is no successor state). Thus, if $i$ fails, then there are no actors that still perform actions. Thus, in our model it holds that $i_1: \text{fail} = i_2: \text{fail}$, which does not hold in the Wieringa and Meyer's models. In an analogous way, we can defend the validity of the formula $i: \beta \cup^* i: \text{fail} = i: \beta$.

The interpretation of the action expression $\text{fail}$ in Wieringa and Meyer (1993) corresponds more or less with the action skip used here. There, the action expression $\text{fail}$ is interpreted as 'to do nothing' and the event expression $i: \text{fail}$ as 'i fails but the rest of the world can still continue performing events' (Wieringa and Meyer, 1993, p. 306), which can be interpreted as the event $i: \text{skip}$: 'i does not affect the situation, but the rest of the world can still do so.' Also, in our system the formulas $i: \beta \cup^* i: \text{skip} \neq i: \beta$ and $i: \beta \&^* i: \text{skip} \neq i: \text{skip}$. The different interpretation of the action expression $\text{fail}$ is the reason why Wieringa and Meyer omitted the two formulas mentioned above from the specification; it is also the reason why we add these formulas to our model.

### 4.2.2 $PD_eL(Evt)$

Now we extend the system $PD_eL$ with actors by changing the language $Ass$ of $PD_eL$. The language $Ass$ consists of assertions pertaining to actions (and actions that are composed sequentially). The expressions of $PD_eL(Evt)$ are assertions concerning events in $Evt$. An expression $\Phi$ of $PD_eL(Evt)$ is a norm formulation in Ruiter's (1989) terminology.

**Definition 4.2.12** The language $Ass^*$ of $PD_eL(Evt)$ consists of assertions ($\Phi$) concerning events in $Evt$:

$$\Phi ::= \phi \mid \neg \Phi \mid \Phi_1 \vee \Phi_2 \mid \Phi_1 \land \Phi_2 \mid \Phi_1 \rightarrow \Phi_2 \mid [\alpha] \Phi,$$

with $\phi \in L$, $\Phi_1, \Phi_2 \in Ass^*$ and $\alpha \in Evt$. 


4.2 The extension of $PD_eL$ with actors

As in chapter 2, the deontic operators are defined as follows: $F(\alpha) \equiv [\alpha]V$, $O(\alpha) \equiv F(\bar{\alpha})$ and $P(\alpha) \equiv \neg F(\alpha)$, with the special propositional symbol $V$, which means that the situation is in contravention of the law. Thus, $\alpha$ is forbidden ($F(\alpha)$) is reduced to a dynamic expression as follows:

$$F(\alpha) \equiv [\alpha]V.$$ 

$[\alpha]$ is interpreted as a modal operator of the necessity ($\Box$) in a Kripke structure induced by the performance of events.

Definition 4.2.13 A model $M$ for $PD_eL(Evt)$ is given by

$$M = (I, A, W, [\alpha]_R, \pi),$$

where $I$ is the set of actors, $A$ the set of actions, $W$ a set of possible worlds, $[\alpha]_R$ a function that associates with event $\alpha \in Evt$ and world $w$, the set of possible worlds to which the performance of $\alpha$ leads, and $\pi$ the usual truth relation between worlds and propositions:

$$\pi : W \times L \rightarrow \{true, false\}.$$ 

We stipulate that we have a set $W$ of worlds assigning values to propositional variables and a given function $\rho : T \rightarrow (W \rightarrow W)$, which for each step yields its behaviour in terms of world transitions. Thus, $\rho(t)(w)$ gives the next world when each individual $i$ performs all the semantical actions in $t_i$ simultaneously in world $w$. ($t_i$ is the set of actions in $A$ in the $s$-set of $t$ which has $i$ as the actor.)

Definition 4.2.14

- $\rho([\delta])(w) = \emptyset$.
- For a step $t$, the function $R(t) \in W \rightarrow W$ is defined by
  $$R(t)(w) = \rho(t)(w).$$
- For a set $T$ of steps,
  $$R(T)(w) = \{w' \mid w' = R(t)(w) \text{ for } t \in T\}.$$ 

Definition 4.2.15 Function $[\ ]_R : Evt \rightarrow (W \rightarrow 2^W)$ is defined as

$$[\alpha]_R(w) = R([\alpha])(w).$$

The truth definitions are standard. For dynamic formulas they are as follows:
Definition 4.2.16

\[ w \models [\alpha]\Phi \text{ iff } \forall w' \in [\alpha]_{R(w)}w' \models \Phi, \]

i.e., sentence \([\alpha]\Phi\) is true in \(w\) iff \(\Phi\) holds in every world accessible from \(w\) by the performance of \(\alpha\).

For assertions, we can now give the formal system \(PD_{e}(Evt)\) that is sound with respect to the given semantics:

**Axiom 4.2.17**

1. all tautologies of the propositional calculus;
2. \([\alpha](\neg \neg z \rightarrow \neg \neg z) \Rightarrow ([\alpha]z \rightarrow [\alpha]z)\);
3. \([\alpha_1 \cup_0 \alpha_2]z \equiv [\alpha_1]z \land [\alpha_2]z\);
4. \([\alpha_1]z \lor [\alpha_2]z \Rightarrow [\alpha_1]z \land [\alpha_2]z\);
5. \([i : \text{fail}]z\);
6. \([i : \text{skip}]z \equiv z\).

**Rules 4.2.18**

\((MP)\)

\[ \Phi_1 \Rightarrow \Phi, \Phi_1 \]

\[ \Phi_2 \]

\((R)\)

\[ \alpha_1 =_0 \alpha_2 \]

\[ [\alpha_1]z \equiv [\alpha_2]z \]

\((N)\)

\[ z \]

\[ [\alpha]z \]

Note that we do not have the schema \(\neg[i : \text{any}]z\) corresponding with the schema \((OD)\) of \(SDL^6\). Using the definitions of deontic operators, the following proposition can be derived immediately.

**Proposition 4.2.19** Let \(\beta, \beta_1\) and \(\beta_2\) be events, then all the theorems given in proposition 2.3.6 hold for \(PD_{e}(Evt)\).
4.2.3 The general and unspecific obligations

In the previous subsection, we formalised the concept of actors in \( PD_eL(Evt) \), which allows us to specify which actors are permitted or obliged to perform an action. In this subsection an analysis is made of the (restricted) general and (restricted) unspecific obligations and permissions, which gives us new expressible power, and their relations.

With the help of the personal obligation and permission, we can formalise the general and unspecific obligations and permissions analogous to the previous chapter:

- the general obligation \( O^+(\beta) \) and permission \( P^-(\beta) \):

\[
O^+(\beta) \equiv \forall_{i \in I} O(i : \beta) \quad \text{and} \quad P^-(\beta) \equiv \forall_{i \in I} P(i : \beta).
\]

- the unspecific obligation \( O^-(\beta) \) and permission \( P^+(\beta) \):

\[
O^-(\beta) \equiv \exists_{i \in I} O(i : \beta) \quad \text{and} \quad P^+(\beta) \equiv \exists_{i \in I} P(i : \beta).
\]

The restricted general obligation \( O^+_X(\beta) \) is defined as

\[
O^+_X(\beta) \equiv \forall_{i \in X} O(i : \beta),
\]

where \( X \subseteq I \), and the restricted unspecific obligation \( O^-_X(\beta) \) as

\[
O^-_X(\beta) \equiv \exists_{i \in X} O(i : \beta),
\]

where \( X \subseteq I \). Analogously, we define the restricted general and unspecific permissions:

\[
P^+_X(\beta) \equiv \exists_{i \in X} P(i : \beta)
\]

and

\[
P^-_X(\beta) \equiv \forall_{i \in X} P(i : \beta).
\]

The general obligations

The best known norms are the legal norms. These norms are mostly aimed at individuals belonging to a particular group. Most of the normative rules are restricted general norms, because these rules are aimed not at all actors but at actors of a particular group. For instance, the various sections of the Dutch traffic regulations are aimed at specific groups of road users, such as the group of cyclists, motor-vehicle drivers, etc. For instance:

**Article 20.** Within built-up areas the following speed limits hold:

---

\(^7\)Note that the following equivalence holds: \( \forall_{i \in I} O(i : \beta) \equiv \forall_{i \in I} O(i : \beta). \)

\(^8\)Note that the following equivalence holds: \( \exists_{i \in I} O(i : \beta) \equiv \forall_{i \in I} O(i : \beta). \)
Relativised deontic modalities in \( P_{DL} \)

- for motor vehicles: 50 km/h;
- for mopeds and motorised wheelchairs: 30 km/h.

Thus, for every individual in the group of motor-vehicle drivers it is forbidden to drive faster than 50 km/h (within built-up areas). This prohibition is a \textit{general} prohibition with respect to the group of motor-vehicle drivers: it is a prohibition in force for everybody in the group of the motor-vehicle drivers. This can be formalised as follows:

\[ \forall_{i \in M} F(i : \beta), \]

where \( M \subseteq I \) denotes the group of motor-vehicle drivers and \( \beta \) the action 'to drive faster than 50 km/h'. Thus, a motor-vehicle driver satisfies this norm if he does not perform action \( \beta \). A restricted general obligation is aimed at individuals in a certain group and allows us to express that each individual in that group separately has the responsibility to satisfy the norm. Thus, a general obligation implies a personal obligation. Let \( i_1 \in I \) and \( \beta \in \text{Act} \), then

\[ O^+(\beta) \equiv \forall_{i \in I} O(i : \beta) \text{ and } \forall_{i \in I} O(i : \beta) \rightarrow O(i_1 : \beta). \]

The unspecific obligation

The opposite of the general obligation is the \textit{unspecific} obligation. This is an obligation for a non-specified individual. It is an obligation in force for at least one of the actors in the group. We can denote this by the existential quantifier \( \exists \). For instance, \( \exists_{i \in U} F(i : \beta_1) \), where \( U \subseteq I \) denotes the group of road users and \( \beta_1 \) the action 'to drive faster than 30 km/h', is a prohibition in force, e.g., for moped riders and not, e.g., for motor-vehicle drivers.

Note that the (restricted) general obligation implies the (restricted) unspecific obligation:

\[ O^+_X(\beta) \rightarrow O^-_X(\beta), \]

where \( X \subseteq I \).

A restricted unspecific obligation is aimed at individuals in a certain group, and allows us to express that some individual in that group has the responsibility to satisfy the norm, without specifying whom, e.g., the last person to leave the room has to turn off the light. Thus, a personal obligation implies an impersonal obligation:

\[ O(i_1 : \beta) \rightarrow O^-(\beta). \]

Now we are also able to express that an action \( \beta \) is obligatory for a particular person \( i_1 \), but not for all individuals:

\[ O(i_1 : \beta) \land \forall_{i \in I} O(i : \beta), \]

which is equivalent to

\[ O(i_1 : \beta) \land P^+(\beta). \]
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The strict personal obligation

Finally, we define the strict personal obligation. This is an obligation only for a specific individual; thus, no other individual has that obligation. An example of a strict personal obligation can be a decision (an order which is not of a general nature, including refusal of an application for such an order), or an obligation for the prime minister. A strict personal obligation for $i_1$ can be formalised as follows:

$$O(i_1 : \beta) \land \forall i \in \Lambda \backslash \{i_1\} \neg O(i : \beta).$$

4.2.4 The relations between the notions of obligation and permission

In subsection 3.2.2, we gave the relations between the general, personal and unspecific notions of obligation and permission. The results were summarised in figures 3.3 and 3.4. In $PDL_c$, the following two principles are valid:

$$O^+(\beta) \rightarrow O^-(\beta) \quad (4.1)$$

and

$$P^-(\beta) \rightarrow P^+(\beta). \quad (4.2)$$

The principles relating to a notion of obligation and a notion of permission are not valid, i.e., $O^+(\beta) \rightarrow P^-(\beta)$, $O^-(\beta) \rightarrow P^+(\beta)$, $O^+(\beta) \rightarrow P^+(\beta)$ and $O^-(\beta) \rightarrow P^-(\beta)$, since we do not have schema $\neg[i : \text{any}]\bigwedge$ corresponding with schema $(OD)$ of $D_i$.

The obligations ‘it is obligatory for the zoo caretaker to feed the monkeys’ and ‘it is obligatory for the visitors not to feed the monkeys’, with $\beta$ the action ‘to feed the monkeys’, are intuitively not in conflict. In general, obligations $O(i : \beta)$ and $O(j : \beta)$ are never in conflict (if $i \neq j$), in contrast to obligations $O_i(p)$ and $O_j(\neg p)$ in the extension of $SDL$ concerning the addition of actors (see previous chapter). Obligations $O_i(p)$ and $O_j(\neg p)$ are always in conflict, since ‘$p$’ is a description of a state of affairs, for instance ‘that the monkeys are fed’. Then, $i$ has to accomplish that the monkeys are fed and $j$ has to accomplish that the monkeys are not fed, i.e., the opposite of what $i$ has to accomplish. Thus, principle $\neg(O_i(p) \land O_j(\neg p))$ and thus also principle $\neg(O^-(p) \land O^-(\neg p))$ are defensible, if one wants to obtain a coherent normative system, i.e., without conflicting obligations, as was considered desirable by Bailhache (1991) and Hilpinen (1973).

The valid principles between the various notions of the obligation and permission can be summarised in the following figure.

---

9See section 1:3 of the General Administrative Law Act (‘Algemene Wet Bestuursrecht’).
4.2.5 The directed obligation

Actions performed by an individual, can be directed towards another individual or group of individuals (directed actions) or can be independent of another individual or group of individuals (undirected actions). Some examples of directed actions are 'to give right of way', 'to overtake', 'to pay', etc. It matters greatly to whom such an action is directed. For instance, according to the Dutch traffic regulation it is obligatory for motor-vehicle drivers to give right of way to motor-vehicle drivers from the right, but they are not obliged to give right of way to cyclists from the right. Examples of undirected actions are 'to drive at a speed of 50 km/h', 'to turn left' and 'to park'.

In this subsection, we give a formalisation of directed actions in our semantics of actions. Let \( a_1 \) be a directed action (e.g., 'to give right of way') and \( I = \{i_1, \ldots, i_n\} \), then \( a_1 \) will be defined as

\[
a_1 := a_1(i_1) \cup a_1(i_2) \cup \ldots \cup a_1(i_n).
\]

The domain of the action \( a_1 \) is \( I \). \( a_1(i_k) \) will be read as 'to give right of way to \( i_k \)'. Note that \( a_1(i_k) \) is a directed atomic action and \( a_1 \) is not.

However, some actions are directed not only to individuals, but can also be directed to, for example, a union, a firm, a group of individuals, etc. Suppose \( a \) stands for the action 'to pay'. This action can be directed towards individuals, but also towards a union, a firm, a collective group of individuals, etc. The argument of \( a \) can be an individual in \( I \), but also a firm or union. Let \( F \) be a firm, then \( i : a(F) \) means that \( i \) pays to firm \( F \).

Thus, we can extend the language \( Act \) by extending the domains of directed actions. The domain of a directed action depends, first, on the action itself, in relation to its object. For example, you cannot overtake a firm. Thus, sometimes the domain of a directed action will be restricted only to individuals or only to unions, etc. And, second, it depends on the individual or group of individuals that performs the directed action. For example, an individual cannot overtake himself. Thus, for every directed action we have to determine its domain.

With the help of these directed actions, we can express directed obligations. The
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directed obligation

$$O(i : a(j))$$

is read as 'individual $i$ is obliged to give right of way to $j$', with $a$ the directed action 'to give right of way'. Thus, individual $i$ (the bearer) has a duty and individual $j$ (the counterparty) has a right (cf. Hohfeld, 1964).

4.3 The extension of $PDeL$ with groups of actors

In this section, we discuss the extension of $PDeL$ concerning the addition of groups of actors on the basis of collective event expressions. First, we discuss the semantics of collective event expressions.

4.3.1 The semantics of collective event expressions

It is important to note that collective event $X : \beta$ is not an event that is performed if and only if every actor in group $X$ performs the action $\beta$. A group event $X : \beta$ can be performed if only a subgroup of $X$ (maybe one person or the whole group together) has performed the action $\beta$. Thus, we can express, on the one hand, that group $X$ as a whole has the responsibility to perform action $\beta$, even if the action is performed by a subgroup (the electricians) of $X$ (the firm). On the other hand, we can model the fact that if the group as a whole performs an action, none of its members performs that action, for instance, when a construction company builds a house.

In the discussion of the semantics of collective event expressions, i.e., actions performed by a group of actors, we see that most definitions are extensions of the definitions of subsection 4.2.1.

The set of all event expressions $Evt'$ can be determined by the following BNF for its elements ($\alpha$):

$$\alpha ::= X : \beta | \alpha_1 \cup' \alpha_2 \cap' \alpha_2 | \alpha, \quad X \in \mathcal{P}^+(I),$$

where $\mathcal{P}^+(I)$ the non-empty powerset of $I$, $\alpha_1, \alpha_2 \in Evt'$ and $\beta \in Act$.

Definition 4.3.1

1. Set $\{\delta\}$ is a synchronicity set (s-set).
2. Set $\{X : \text{skip}\}$ is an s-set, with $X$ a non-empty subset of $I$.
3. Every pair of a non-empty subset of $A$ and a non-empty subset of $I$ is an s-set.
Relativised deontic modalities in $PD_{xL}$

We use $S, S_1, S_2, \ldots$ for s-sets. The set of all s-sets, except $\{\emptyset\}$, will be denoted by $S'$.

Let $X \in P^+(I)$ and $A = \{a_1, a_2\}$, then s-set $S$ consisting of elementary actions $a_1$ and $a_2$ performed by set $X$ is written as

$$S = [X : A'], \text{ or } S = \begin{bmatrix} X : a_1 \\ a_2 \end{bmatrix},$$

with $A' = \{a_1, a_2\}$. Thus, $S' = \{[X : A'] | A' \subseteq A, A' \neq \emptyset, X \in P^+(I)\} \cup \{[X : \text{skip}] | X \in P^+(I)\}$. Let $X$ be the set $\{i_1, i_2\}$, then $S'$ is

$$\{\begin{bmatrix} \{i_1\} : a_1 \\ \{i_2\} : a_2 \end{bmatrix}, \begin{bmatrix} \{i_1\} : \text{skip} \\ \{i_2\} : a_2 \end{bmatrix}, \begin{bmatrix} \{i_1\} : a_1 \\ \{i_2\} : a_2 \end{bmatrix}, \begin{bmatrix} \{i_1\} : \text{skip} \\ \{i_2\} : a_2 \end{bmatrix} \}.$$

Further, we use $S'_X$ to denote the set of all s-sets with $X$ in $P^+(I)$, thus $S'_{\{i_1, i_2\}} = \{\begin{bmatrix} \{i_1\} : A' \end{bmatrix} | A' \subseteq A, A' \neq \emptyset\}$. Note that $S^* \subseteq S'$, if we assume that $S'_t = S'_{\{i\}}$.

It is clear that a group can only do something if some members of the group do something, and it is plausible to assume that the elementary actions of a group are constituted, in some way or other, by the elementary actions of the members of the group. For instance, if a member performs an elementary action $a$, the group also performs this action. However, the group is not restricted to performing the elementary actions that are performed by the individual members of the group, since a group can perform an elementary action that cannot be performed by a single individual. For instance, ‘to lift a stone of 500 lbs’. This relationship between the group and the members of the group with respect to their performances of elementary actions manifests itself in the following definition.

**Definition 4.3.2** Let $I = \{i_1, i_2, \ldots, i_n\}$, then $T'$ will be defined as

$$T' = \{x \in P^+(I)S'_X \mid \forall X, Y \in P^+(I) Y \subseteq X \rightarrow t_Y \subseteq t_X, \forall X, Y \in P^+(I)t_Y = \{\text{skip}\} \rightarrow t_{X \cup Y} = t_X\},$$

with $t_X$ the set of actions in s-set $[X : t_X]$, a coefficient of step $t$ corresponding with $X$. An element of $T'$ will be called a step. We use the letter 't' with possible marks for steps.

A step $t$ of $T'$ is a $2^n - 1$-tuple. The order of the s-sets in a step is again irrelevant. $T'$ is not just the indirect product of all the sets $S'_X$, with $X \in P^+(I)$, but a subset of this indirect product. $T'$ has been restricted for the following two reasons:
1. Let \( t \) be a step in \( T' \) containing s-set \([Y : t_Y]\) with \( t_Y = \{a_1, \ldots, a_m\} \), a subset of \( A \). Thus, group \( Y \) simultaneously performs the actions \( a_1, \ldots, a_m \). Then all supersets of \( Y \) also perform these actions, possibly together with other actions in \( A \), because of the fact that \( Y \) is a subgroup of \( X \). (\( X \) performs the actions in \( t_Y \) if and only if some subset of \( X \) performs these actions.)

**Example 4.3.3** Let \( A = \{a_1, a_2\} \) and \( I = \{i_1, i_2\} \), then
\[
\left( \left\{ i_1 \right\} : a_1 \right), \left\{ i_2 \right\} : a_2 \right), \left\{ i_1, i_2 \right\} : a_1 \right) \notin T'.
\]
For instance, if John posted the letter, then also the group John and Paul posted the letter, because John is a subgroup of the group consisting of John and Paul.

2. Let \( t \) be a step in \( T' \) containing the s-set \([Y : \text{skip}]\). This means that \( Y \) does not affect the situation. Thus, for every real superset \( X \) of \( Y \) it follows that the groups \( X \) and \( X \setminus Y \) perform the same actions.

**Example 4.3.4** Let \( A = \{a_1, a_2\} \) and \( I = \{i_1, i_2\} \), then
\[
\left( \left\{ i_1 \right\} : \text{skip} \right), \left\{ i_2 \right\} : a_1 \right), \left\{ i_1, i_2 \right\} : a_1 \right) \notin T'.
\]
**Example 4.3.5** Let \( A = \{a_1, a_2\} \) and \( I = \{i_1, i_2\} \), then \( T' = \)

\[
\{ \left( \left\{ i_1 \right\} : \text{skip} \right), \left\{ i_2 \right\} : \text{skip} \right), \left\{ i_1, i_2 \right\} : \text{skip} \right) ,
\left( \left\{ i_1 \right\} : \text{skip} \right), \left\{ i_2 \right\} : a_1 \right), \left\{ i_1, i_2 \right\} : a_1 \right) ,
\left( \left\{ i_1 \right\} : \text{skip} \right), \left\{ i_2 \right\} : a_2 \right), \left\{ i_1, i_2 \right\} : a_2 \right) ,
\left( \left\{ i_1 \right\} : \text{skip} \right), \left\{ i_2 \right\} : a_2 \right), \left\{ i_1, i_2 \right\} : a_2 \right) ,
\left( \left\{ i_1 \right\} : a_1 \right), \left\{ i_2 \right\} : \text{skip} \right), \left\{ i_1, i_2 \right\} : a_1 \right) ,
\left( \left\{ i_1 \right\} : a_1 \right), \left\{ i_2 \right\} : a_1 \right), \left\{ i_1, i_2 \right\} : a_1 \right) ,
\left( \left\{ i_1 \right\} : a_2 \right), \left\{ i_2 \right\} : a_2 \right), \left\{ i_1, i_2 \right\} : a_2 \right) ,
\left( \left\{ i_1 \right\} : a_2 \right), \left\{ i_2 \right\} : a_2 \right), \left\{ i_1, i_2 \right\} : a_2 \right) .
\]
Since our language of event expressions contains a non-deterministic (choice) operator $\cup'$, we have to consider sets of steps $t \in \mathcal{T}'$. We use $T, T_1, T_2, \ldots$ to denote sets of steps.

Our event model $M_{\mathcal{T}'}$ is now given by:

**Definition 4.3.6** $M_{\mathcal{T}'}$ is the collection of sets $T$ consisting of steps.

Note that $M_{\mathcal{T}'}$ is the powerset of $\mathcal{T}'$, normally denoted by $2^{\mathcal{T}'}$ or $\mathcal{P}(\mathcal{T}')$. We shall use the operations $\cup, \cap$ and $\neg$ (operator for the set-theoretic complement) on the domain $M_{\mathcal{T}'}$ as semantical counterparts of the syntactical operators $\cup', \&'$ and $\neg$, respectively, in our language $Evt'$ of event expressions. Before we give these definitions, we define a handy operator on the sets of steps:

**Definition 4.3.7** Let $T$ be a set of steps, then

$$T^\delta = \begin{cases} T \setminus \{[\delta]\} & \text{if } \exists t \in T \neq [\delta] \\ \{[\delta]\} & \text{otherwise.} \end{cases}$$
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Now we can give the semantical operators on $MT_{\epsilon}$. For the simultaneous operator $\&'$, we use a set intersection $\cap$, which is almost the same as the normal set intersection.

**Definition 4.3.8** For $T_1, T_2 \in M_{T_{\epsilon}}$,

$$T_1 \cap T_2 = \begin{cases} T_1 \cap T_2 & \text{if } T_1 \cap T_2 \neq \emptyset \\ \{[\delta]\} & \text{otherwise.} \end{cases}$$

Operator $\cup$ on sets of steps is defined as follows:

**Definition 4.3.9** For $T_1, T_2 \in M_{T_{\epsilon}}$,

$$T_1 \cup T_2 = (T_1 \cup T_2)^\delta.$$  

Thus, the choice between two sets of steps is the union of those two sets minus $\{[\delta]\}$, unless the union does not contain anything else.

The definition of "-' is given as follows:

**Definition 4.3.10**

1. For a step $t$, 
   $$t^- = T' \setminus \{t\}.$$  
2. For a non-empty set $T \in M_{T_{\epsilon}}$, 
   $$T^- = \cap \{t^- | t \in T\}.$$  

Thus, for a step $t$ the negation just yields the set-theoretic complement of $\{t\}$ with respect to $T'$. For the negation of a set of steps $T$, we take the intersection of the sets of the negation of all the steps contained in $T$. Note that $T \subseteq T'$ and that $T' \in M_{T_{\epsilon}}$.

Now we can define the semantics of event expressions in $Evt'$:

**Definition 4.3.11** The semantic function $[\cdot] : Evt' \rightarrow M_{T_{\epsilon}}$, with $\beta, \beta_1, \beta_2 \in \text{Act}$, $X \in \mathcal{P}^+(I)$ and $\alpha, \alpha_1, \alpha_2 \in Evt'$, is given by:

1. $[X: a] = \{t \in T' | a \in t_X\}$;
2. $[X: \beta_1 \cup \beta_2] = [X: \beta_1] \cup [X: \beta_2]$;
3. $[X: \beta_1 \& \beta_2] = [X: \beta_1] \cap [X: \beta_2]$;
4. $[X: \beta'] = [X: \beta]$;
5. $[\alpha_1 \cup' \alpha_2] = [\alpha_1] \cup [\alpha_2]$;
6. $[\alpha_1 & \alpha_2] = [\alpha_1] \cap [\alpha_2]$;

7. $[\overline{\alpha}] = [\alpha]^*$;

8. $[X : \text{fail}] = \{[\delta]\}$;

9. $[X : \text{any}] = T'$;

10. $[X : \text{skip}] = \{t \in T' | \forall y \in \mathcal{P}^+(X) t_y = \{\text{skip}\}\}$;

11. $[X : \text{change}] = T' \setminus [X : \text{skip}]$.

Remarks

1. $[X : a]$ expresses the meaning of event expression $X : a$: it is the set of all steps in $T'$ which contains the s-set $[X : t_x]$ with $a \in t_x$: $X$ performs the elementary action $a$ possibly together with other elementary actions.

2. $[\alpha]^*$ stands for the set-theoretic complement of $[\alpha]$ with respect to $T'$. $[X : \overline{\alpha}]$ is the set of all steps in $T'$ which contains the s-set $[X : t_x]$ with $a \notin t_x$. Thus, no subgroup of $X$ performs action $a$.

3. $[X : \text{any}]$ stands for the set of all steps in $T'$.

4. $[X : \text{skip}]$ is the set of all steps, such that for every non-empty subset $Y$ of $X$ it follows that $[Y : \text{skip}]$ is an s-set in these steps. If a situation is not affected by group $X$, then it is also not affected by a subgroup of $X$.

To understand the semantical function, we provide some examples.

Example 4.3.12 Let $A = \{a_1, a_2\}$ and $I = \{i_1, i_2\}$, then

- $[[i_1] \cdot a_1] = \{t \mid t_{(i_1)} \in \{a_1\}, \forall x \in \mathcal{P}^+(I), x \neq (i_1) t_x \in \mathcal{P}^+(A)\} \cap T'$;

- $[[i_1, i_2] \cdot \overline{a_1}] = \{[i_1, i_2] \cdot a_1\} = T' \setminus \{[i_1, i_2] \cdot a_1\} =
  \{t \mid t_{\{i_1, i_2\}} \in \{\text{skip}\}, a_2\} \cup \forall x \in \mathcal{P}^+(I), x \neq (i_1, i_2) t_x \in \mathcal{P}^+(A)\} \cap T'$;

- $[[i_1, i_2] \cdot \text{skip}] = \{t \mid t_{\{i_1, i_2\}} = \{\text{skip}\}, \forall x \in \mathcal{P}^+(I), x \neq (i_1, i_2) t_x \in \mathcal{P}^+(A)\} \cap T' =
  \{t \mid \forall x \in \mathcal{P}^+(I) t_x = \{\text{skip}\}\}$. 

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- $[\{i_1\} : a_1 \cup \{i_2\} : a_2] = T' \setminus [\{i_1\} : a_1 \cup \{i_2\} : a_2] = [\{i_1\} : a_1] \cap [\{i_2\} : a_2] = \{t \mid t(i_1) \in \{\text{skip}\}, \{a_2\}, \forall x \in \mathcal{P}^+(I), x \neq (i_1) \in \mathcal{P}(A)\} \cap \{t \mid t(i_2) \in \{\text{skip}\}, \{a_1\}, \forall x \in \mathcal{P}^+(I), x \neq (i_2) \in \mathcal{P}(A)\} \cap T' = \{t \mid t(i_1) \in \{\text{skip}\}, \{a_2\}, t(i_2) \in \{\text{skip}\}, \{a_1\}, t(i_1, i_2) \in \mathcal{P}(A)\} \cap T'.

Note that every time we take the intersection with $T'$ to ensure that the constraints defined on steps are enforced.

Note that $[i_1 : \beta]$ is not equal to $[[i_1] : \beta]$. $i_1 : \beta$ is an individual event expression, and the semantic function for individual event expressions (definition 4.2.8) differs from the semantic function for group event expressions (definition 4.3.11).

**Proposition 4.3.13** Let $I$ be the set of actors, $X, Y \in \mathcal{P}^+(I)$ and $\beta_1, \beta_2 \in \text{Act}$. Then

1. $[X : \beta_1 \& \beta_2] \subseteq [X : \beta_1]$;
2. $[X : \beta_1] \subseteq [X : \beta_1 \cup \beta_2]$;
3. $[X : \beta_1 \&' X : \beta_1] = \{[\delta]\}$;
4. $\forall x, y \{[X : \text{fail}] = [Y : \text{fail}]\}$;
5. $\forall x, y \{[X : \text{any}] = [Y : \text{any}]\}$.

**Proof**

1. $[X : \beta_1 \&' \beta_2] = [X : \beta_1] \cap [X : \beta_2] \subseteq [X : \beta_1]$.
2. $[X : \beta_1] \subseteq [X : \beta_1] \cup [X : \beta_2] = [X : \beta_1 \cup \beta_2]$.
3. $[X : \beta_1 \&' X : \beta_1] = [X : \beta_1] \cap [X : \beta_1] = [X : \beta_1] \cap [X : \beta_1] = \{[\delta]\}$.
4. Follows immediately from 4.3.11.8.
5. Follows immediately from 4.3.11.9.

If some actors in a group $X$ perform action $a$, and other actors in $X$ do not perform $a$, then this does not mean that $X$ performs action $a \& \bar{a}$. $X$ performs action $a$ if and only if some subgroup of $X$ performs $a$, and $X$ does not perform $a$ if and only if no subgroup of $X$ (thus, also no actor of $X$) performs action $a$. Thus, a group $X$, just like individuals, cannot perform an impossible action $\beta \& \bar{\beta}$. Event $X : \beta \& \bar{\beta}$ means that the action $\beta \& \bar{\beta}$ performed by $X$ always fails.

In this section, we also introduce an auxiliary notion that we will use in the sequel:
Definition 4.3.14 We put $\alpha_1 = T_\alpha \alpha_2$ iff $[\alpha_1] = [\alpha_2]$.

Proposition 4.3.15 All the equations of proposition 4.2.10 hold for $M_T$, with $X : \beta$ instead of $i : \beta$, $X \in P^+(I)$ and $\alpha_1, \alpha_2 \in Evt'$.

Proposition 4.3.16 $(M_T', \cup', \&', \sim, X : \text{fail})$ is a Boolean algebra.

In chapter 2, we made a distinction between positive and negative action expressions. We saw that a positive action expression expresses an action that involves a certain form of physical activity, and a negative action means not carrying out that physical activity: an omission (see subsection 2.2.4). This distinction carries much weight with collective event expressions, since a group performs a positive action $\gamma$ if and only if a subgroup of that group performs action $\gamma$ and a group 'performs' a negative action $\bar{\gamma}$ if and only if every subgroup of that group does not perform $\gamma$.

The following principles are valid for a special kind of collective event expressions: positive or negative actions performed by groups of actors.

Proposition 4.3.17 Let $I$ be the set of actors, $X, Y \in P^+(I)$ and $\gamma \in Act_p$. Then

1. $Y \in P^+(X) \rightarrow [Y : \gamma] \subseteq [X : \gamma]$;
2. $Y \in P^+(X) \rightarrow [X : \bar{\gamma}] \subseteq [Y : \bar{\gamma}]$;
3. $[X \cap Y : \gamma] \subseteq [X : \gamma] \cap [Y : \gamma]$, with $X \cap Y \neq \emptyset$;
4. $[X : \gamma] \cup [Y : \gamma] \subseteq [X \cup Y : \gamma]$;
5. $[X \cup Y : \bar{\gamma}] \subseteq [X : \bar{\gamma} \&' Y : \bar{\gamma}]$;
6. If $Y \in P^+(X)$ then $[X : \bar{\gamma} \&' Y : \gamma] = \{\delta\}$.

Proof

1. Let $Y \in P^+(X)$ and $t \in [Y : \gamma]$. Then $t$ contains an s-set $[Y : t_Y]$, with $t_Y \subseteq A$. By definition 4.3.2, the s-set with group $X$ also contains $t_Y$, possibly together with other actions in $A$, because $\gamma$ does not contain negative actions. Thus, $t \in [X : \gamma]$. Hence, $Y \in P^+(X) \rightarrow [Y : \gamma] \subseteq [X : \gamma]$.

2. Let $Y \in P^+(X)$. Then, by 4.3.17.1, $[Y : \gamma] \subseteq [X : \gamma]$. This is equivalent to $T' \backslash [X : \gamma] \subseteq T' \backslash [Y : \gamma]$. Hence, by 4.3.11.4 and 4.3.11.7, $[X : \bar{\gamma}] \subseteq [Y : \bar{\gamma}]$. 

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3. Let $t \in [X \cap Y : \gamma]$. Then, $t$ contains an s-set $[X \cap Y : t_{X \cap Y}]$, with $t_{X \cap Y} \subseteq A$. By definition 4.3.2, the s-sets with group $X$ and group $Y$ also contain $t_{X \cap Y}$, possibly together with other actions in $A$, because $\gamma$ does not contain negative actions. Thus, $t \in [X : \gamma] \cap [Y : \gamma]$. Hence, $[X \cap Y : \gamma] \subseteq [X : \gamma] \cap [Y : \gamma]$. The converse does not hold. Suppose $I = \{i_1, i_2, i_3\}$ and $A = \{a_1, a_2\}$, then step $t$:

$$([\{i_1\} : a_1], [\{i_2\} : a_2], [\{i_3\} : a_1], [\{i_1, i_2\} : a_1])$$

is an element of $[[\{i_1, i_2\} : a_1] \cap [\{i_2, i_3\} : a_1]]$, but not an element of $[[\{i_1, i_2\} \cap \{i_2, i_3\} : a_1] = [[\{i_2\} : a_1]]$, because $a_1 \not\in t_{(i_2)}$.

4. Let $t \in [X : \gamma] \cup [Y : \gamma]$. Then $t$ contains an s-set $[X : A']$ or $[Y : A']$, with $A' \subseteq A$. By definition 4.3.2, the s-sets with group $X \cup Y$ also contain $A'$, possibly together with other actions in $A$, because $\gamma$ does not contain negative actions. Thus, $t \in [X \cup Y : \gamma]$. Hence, $[X : \gamma] \cup [Y : \gamma] \subseteq [X \cup Y : \gamma]$.

5. By 4.3.17.4, 4.3.11.4 and 4.3.11.7 it follows that $[X \cup Y : \gamma] = [X \cup Y : \gamma] \subseteq [X : \gamma] \cup [Y : \gamma]$. This is equal to $[X : \gamma] \cap [Y : \gamma] = [X : \gamma] \cap [Y : \gamma] = [X : \gamma \& Y : \gamma]$.

6. Let $Y \in \mathcal{P}^+(X)$. Then $[[X : \gamma \&' Y : \gamma]] = [X : \gamma] \cap [Y : \gamma] \subseteq [Y : \gamma] \cap [Y : \gamma] = [Y : \gamma] \cap [Y : \gamma] = \{[\delta]\}$.

4.3.2 $PD_eL(Evt')$

At this point, we are able to extend the system $PD_eL$ with groups of actors, by changing the language $Ass$ of $PD_eL$. The language $Ass'$ consists of assertions concerning actions. The expressions of $PD_eL(Evt')$ are assertions concerning events in $Evt'$. An expression $\Phi$ of $PD_eL(Evt')$ is a norm formulation in Ruiter’s (1989) terminology.

Definition 4.3.18 The language $Ass'$ of $PD_eL(Evt')$ consists of assertions ($\Phi$) concerning events in $Evt'$:

$$\Phi ::= \phi \mid \neg \Phi \mid \Phi_1 \lor \Phi_2 \mid \Phi_1 \land \Phi_2 \mid \Phi_1 \rightarrow \Phi_2 \mid [\alpha]\Phi,$$

with $\phi \in L$, $\Phi, \Phi_1, \Phi_2 \in Ass'$ and $\alpha \in Evt'$. 
Definition 4.3.19 A model $M$ for $PD_L(Evt')$ is given by

$$M = \langle P^+(I), A, W, [\alpha]_R, \pi \rangle,$$

where $P^+(I)$ is the non-empty powerset of the set $I$ of actors, $A$ the set of actions, $W$ a set of possible worlds, $[\alpha]_R$ a function that associates with event $\alpha \in Evt'$ and world $w$, the set of possible worlds to which the performance of $\alpha$ leads, and $\pi$ the usual truth relation between worlds and propositions.

We stipulate that we have a set $W$ of worlds assigning values to propositional variables and a given function $\rho : T' \to (W \to W)$, which for each step yields its behaviour in terms of world transitions. Thus, $\rho(t)(w)$ gives the next world when each group $X$ performs all the semantical actions in $t_X$ simultaneously in world $w$. ($t_X$ is the set of actions in the coefficient of $t$ which has $X$ as its group.)

Definition 4.3.20

- $\rho([\delta])(w) = \emptyset$.
- For a step $t$, the function $R(t) \in W \to W$ is defined by
  $$R(t)(w) = \rho(t)(w).$$
- For a set $T$ of steps,
  $$R(T)(w) = \{ w' \mid w' = R(t)(w) \text{ for } t \in T \}.$$

Definition 4.3.21 The function $[\ ] : Evt' \to (W \to \text{Pow}(W))$ is defined as

$$[\alpha]_R(w) = R([\alpha])(w).$$

The truth definitions are standard. For dynamic formulas they are as follows:

Definition 4.3.22

$$w \models [\alpha] \Phi \iff \forall w' \in [\alpha]_R(w) w' \models \Phi,$$

i.e., sentence $[\alpha] \Phi$ is true in $w$ iff $\Phi$ holds in every world accessible from $w$ by performing $\alpha$.

For assertions, we can give the formal system $PD_L(Evt')$, which is sound with respect to the given semantics:
Axiom 4.3.23

1. all tautologies of the propositional calculus;
2. \([\alpha](\Phi_1 \rightarrow \Phi_2) \rightarrow ([\alpha]\Phi_1 \rightarrow [\alpha_2]\Phi_2);\)
3. \([\alpha_1 \cup' \alpha_2]\Phi \equiv [\alpha_1]\Phi \land [\alpha_2]\Phi;\)
4. \([\alpha_1]\Phi \lor [\alpha_2]\Phi \rightarrow [\alpha_1&'\alpha_2]\Phi;\)
5. \([X : \text{fail}]\Phi;\)
6. \([X : \text{skip}]\Phi \equiv \Phi.\)

Rules 4.3.24

(MP) \[
\frac{\Phi_1 \rightarrow \Phi_2, \Phi_1}{\Phi_2}
\]

(R) \[
\frac{\alpha_1 = \alpha_2}{[\alpha_1]\Phi \equiv [\alpha_2]\Phi}
\]

(N) \[
\frac{\Phi}{[\alpha]\Phi}
\]

Using the definitions of the deontic operators, the following proposition follows immediately.

**Proposition 4.3.25** Let \(\beta, \beta_1, \beta_2 \in \text{Evt}', then all the theorems given in proposition 2.3.6 hold for \(PD_eL(\text{Evt}').\)

Finally, we introduce the notion of involvement between events in \(\text{Evt}':\)

**Definition 4.3.26** Let \(\alpha_1, \alpha_2 \in \text{Evt}', then we define \(\alpha_1 \text{ involves } \alpha_2' \) \((\alpha_1 \triangleright \alpha_2)\) as follows:

\[\alpha_1 \triangleright \alpha_2 \text{ iff } [\alpha_1] \subseteq [\alpha_2].\]

**Corollary 4.3.27** Let \(\alpha_1, \alpha_2 \in \text{Evt}', then we have

\[\alpha_1 \triangleright \alpha_2 \rightarrow ([\alpha_2]\text{V} \rightarrow [\alpha_1]\text{V}).\]
4.4 The strong and weak obligations

We take as a primitive notion the relativised obligation $O(X : \beta)$, which means that 'group $X$ is obliged to perform $\beta$'. In this section, we show that it is possible to define all sorts of notions of collective and individual obligation in terms of $O(X : \beta)$, and discuss the relation between all these notions of obligation and permission. With the help of group obligation $O(X : \beta)$, we can distinguish three notions of obligations:

- the **strong** obligation $(O^\#(\beta))$: for every set in $P^+(I)$ it is obligatory to perform $\beta$: $\forall_{X \in P^+(I)} O(X : \beta)$\(^{10}\), with $I$ the set of actors;

- the **weak** obligation $(O^o(\beta))$: there is a set in $P^+(I)$ for which it is obligatory to perform $\beta$: $\exists_{X \in P^+(I)} O(X : \beta)$\(^{11}\);

- the **group** obligation: for the group $X$ it is obligatory to perform $\beta$: $O(X : \beta)$.

It is clear that the strong obligation implies group obligation, and that group obligation in turn implies the weak obligation.

In comparison with the strong and the weak obligations, the restricted strong and weak obligations are restricted to a set in $P^+(I)$ instead of to $I$ itself. Let $X \in P^+(I)$, then $O^\#_X(\beta)$ (equivalent to $\forall_{Y \in P^+(I)} O(Y : \beta)$) is an example of a restricted strong obligation.

4.4.1 The relations between group and individual norms

Most existing formalisations of relativised deontic modalities are relativised to *actors* and not to groups of actors. In section 4.2.3, we distinguished, analogous to the collective obligations, three notions of obligations:

- the **general** obligation $(O^+(\beta))$: for all actors it is obligatory to perform $\beta$;

- the **personal** obligation $(O(i : \beta))$: for $i$ it is obligatory to perform $\beta$;

- the **unspecific** obligation $(O^-(\beta))$: for some person it is obligatory to perform $\beta$.

It is obvious that we have a distinction between the notions of obligation with the personal obligation as the primitive notion and the notions of obligation with the group obligation as the primitive notion. The notions of strong, weak and group obligations enable us to analyse obligations concerning *groups of actors*: collective obligations, and the notions of general, unspecific and personal obligations enable us to analyse obligations concerning *actors*: individual obligations.

\(^{10}\)Note that the following equivalence holds: $\forall_{X \in P^+(I)} O(X : \beta) \equiv \bigwedge_{X \in P^+(I)} O(X : \beta)$.

\(^{11}\)Note that the following equivalence holds: $\exists_{X \in P^+(I)} O(X : \beta) \equiv \bigvee_{X \in P^+(I)} O(X : \beta)$. 
4.4 The strong and weak obligations

However, we can formalise the general, personal and unspecific obligations in terms of group obligation $O(\{i\} : \beta)$. Group obligation $O(\{i\} : \beta)$ can be considered to be an individual obligation. Therefore, we can formalise

- the general obligation $(O^{+}(\beta))$ as
  \[ \forall_{i \in I} O(\{i\} : \beta) \]

- the personal obligation as
  \[ O(\{i\} : \beta) \]

- the unspecific obligation $(O^{-}(\beta))$ as
  \[ \exists_{i \in I} O(\{i\} : \beta) \]

With the help of the two new notions $O^{+}$ and $O^{-}$ we can define the different notions of collective obligation (i.e., strong, weak and group obligations) and individual obligation (i.e., general, unspecific and personal obligations) in the same semantical model.

Note that $O(\{i\} : \beta)$ is not the same as $O(i : \beta)$, since they differ semantically. $O(i : \beta)$ is an assertion concerning an event in $Evt$ and $O(\{i\} : \beta)$ is an assertion concerning an event in $Evt'$. However, both obligations express the same: 'An obligation for $i$ to perform the action $\beta$.' The only difference in interpretation is that in $O(\{i\} : \beta)$ individual $i$ is considered to be a group of one individual and in $O(i : \beta)$ to be just an individual.

Now, the following two relations hold:

- The strong obligation implies the general obligation:
  \[ O^{\Theta}(\beta) \rightarrow O^{+}(\beta). \]

- The unspecific obligation implies the weak obligation:
  \[ O^{-}(\beta) \rightarrow O^{\Theta}(\beta). \]

If something is obligatory for all groups of a set of actors (the strong obligation), then it is also obligatory for all groups consisting of one actor, i.e., the general obligation, and if something is obligatory for a group consisting of one actor, thus for an actor, then it is also obligatory for some group of actors (the unspecific obligation).

In contrast to the two relations mentioned above, the relation between the group obligation and the personal obligation is not fixed. However, in certain cases the relation is determined by the kind of action (i.e., a positive or negative action), which we will discuss in the next section.
Figure 4.2 represents all these notions of obligation:\(^{12}\)

**Figure 4.2**

\[
\begin{array}{ccc}
\text{strong obligation} & O^{\Theta}(\beta) & \rightarrow & O^+\beta) & \rightarrow & \text{general obligation} \\
\text{group obligation} & O(X : \beta) & \rightarrow & O(\{i\} : \beta) & \rightarrow & \text{personal obligation} \\
\text{weak obligation} & O^{\Theta}(\beta) & \rightarrow & O^{-}\beta) & \rightarrow & \text{unspecific obligation}
\end{array}
\]

In chapter 3, we discussed the problem of the interdefinability of obligation and permission. This problem does not pose itself for the group obligation and permission, since we have \(O(X : \beta) \equiv \neg P(X : \bar{\beta})\). We call \(P(X : \beta)\) the dual of \(O(X : \beta)\): the principle of interdefinability (cf. Herrestad and Krogh, 1995). Also, for the strong and weak obligations we do not have this problem; however, the (restricted) weak permission is coupled with the (restricted) strong obligation, and the (restricted) strong permission with the (restricted) weak obligation. This is called the asymmetry between permission and obligation:

- \(O^{\Theta}(\beta) \equiv \neg P^{\Theta}(\bar{\beta})\): 'for every group it is obligatory to perform \(\beta'\) is equivalent to 'there is no group for which it is permitted not to perform \(\beta';\)

- \(O^{\Theta}(\beta) \equiv \neg P^{\Theta}(\bar{\beta})\): 'there is a group for which it is obligatory to perform \(\beta'\) is equivalent to 'it is not permitted for all groups not to perform \(\beta'.\)

The valid relations between the different notions of obligation and permission can be summarised in the following figure, which corresponds with figure 3.8:\(^{15}\)

---

\(^{12}\)The arrows indicate provable consequences.

\(^{13}\)\(O^{\Theta}(\beta) \equiv \forall x \in \mathcal{P} + (1) O(X : \beta) \equiv \forall x \in \mathcal{P} + (1) \neg P(X : \bar{\beta}) \equiv \exists x \in \mathcal{P} + (1) P(X : \bar{\beta}) \equiv \neg P^{\Theta}(\bar{\beta}).\)

\(^{14}\)\(O^{\Theta}(\beta) \equiv \exists x \in \mathcal{P} + (1) O(X : \beta) \equiv \exists x \in \mathcal{P} + (1) \neg P(X : \bar{\beta}) \equiv \forall x \in \mathcal{P} + (1) P(X : \bar{\beta}) \equiv \neg P^{\Theta}(\bar{\beta}).\)

\(^{15}\)Note that in figure 3.8 principle \(O^{\Theta}(p) \rightarrow P^{\Theta}(p)\) is valid, and that the corresponding principle \(O^{\Theta}(\beta) \rightarrow P^{\Theta}(\beta)\) is not valid in \(PDeL(\mathcal{E} \omega')\).
4.5 When does a group satisfy a norm?

In this chapter, we have formalised the concept of groups of actors ($PD_eL(Evt')$) in dynamic deontic logic, which allows us to specify that a group is obliged, prohibited or permitted to perform an action. The addition of groups of actors in a deontic system gives us the opportunity to express which group has the responsibility to perform an action. In this section, we investigate when a group of actors satisfies a norm.

We consider sets of actors. Suppose that four friends go to a restaurant, and after their meal they (for short, group $X$) have to pay 50 dollars. This event ‘$X$: to pay 50 dollars’ is performed if and only if some subgroup of $X$ pays 50 dollars. Thus, this obligation is fulfilled if and only if a subgroup of $X$ pays 50 dollars. Not everyone has to pay 50 dollars, to satisfy the norm ‘it is obligatory for $X$ to pay 50 dollars’. Even if no one pays the 50 dollars, it is possible that the event will be performed, for example, if everyone pays 12.50 dollars. Thus, $O(X : \beta)$ is not equivalent to $\forall i \in X O(\{i\} : \beta)$ (the restricted general obligation), with $\beta$ the action ‘to pay 50 dollars’.

Suppose that for group $X$ it is forbidden to steal 50 dollars. Group $X$ satisfies this norm if the event is not performed. Thus, no subgroup of $X$ may steal 50 dollars to satisfy the norm.

In this example, we see that sometimes a group $X$ fulfils obligation $O(X : \beta)$ if and only if every subgroup has to perform action $\beta$, and sometimes it is sufficient that some subgroup performs action $\beta$ to fulfil the obligation. In the former case, it concerns a negative action, and in the latter, a positive action:

1. A group $X$ ‘performs’ negative action $\neg \gamma$, i.e., does not perform action $\gamma$, if and only if every subgroup of $X$ does not perform action $\gamma$. Thus, $X$ is obliged not to perform $\gamma$ if and only if every subgroup of $X$ is obliged not to perform $\gamma$. The formal counterpart of this is:

$$O(X : \neg \gamma) \equiv \forall Y \in P^+(X) O(Y : \neg \gamma).$$
This can be proven as follows. Suppose that \( \forall_{Y \in P^+(X)} O(Y : \gamma) \). \( X \) is a subset of \( X \), hence \( \forall_{Y \in P^+(X)} O(X : \gamma) \rightarrow O(X : \gamma) \). Suppose now that \( O(X : \gamma) \) and \( Y \in P^+(X) \). Then, it follows by 4.3.17.1 that \( [Y : \gamma] \subseteq [X : \gamma] \), and by 4.3.26 and 4.3.27 that \( [X : \gamma]V \rightarrow [Y : \gamma]V \). This holds for every set \( Y \) in \( P^+(X) \), thus \( [X : \gamma]V \rightarrow \forall_{Y \in P^+(X)} [Y : \gamma]V \). Hence, \( O(X : \gamma) \rightarrow \forall_{Y \in P^+(X)} O(X : \gamma) \).

2. A group \( X \) performs positive action \( \gamma \) if and only if a subgroup of \( X \) performs action \( \gamma \). Thus, \( X \) is obliged to perform action \( \gamma \) if and only if some subgroup of \( X \) is obliged to perform action \( \gamma \). The formal counterpart of this is:

\[
O(X : \gamma) \equiv \exists_{Y \in P^+(X)} O(Y : \gamma).
\]

This can be proven analogously as above.

Now it is easy to see that the following proposition holds:

**Proposition 4.5.1** Let \( X \in P^+(I) \) and \( \gamma \in \text{Act}_p \), then

1. \( O(I : \gamma) \equiv O^\emptyset(\gamma) \);
2. \( O(I : \gamma) \equiv O^\emptyset(\gamma) \).

Thus, the strong obligation collapses in group obligation \( O(I : \beta) \), if \( \beta \) is a negative action, and the weak obligation collapses in group obligation \( O(I : \beta) \), if \( \beta \) is a positive action.

Further, the following two relations hold between group obligation \( O(X : \beta) \) and personal obligation \( O(\{i\} : \beta) \):

**Proposition 4.5.2** Let \( i \in X \in P^+(I) \). Then,

\[
O(X : \beta) \rightarrow O(\{i\} : \beta) \text{ iff } \beta \in \text{Act}_n \vee X = \{i\};
\]

\[
O(\{i\} : \beta) \rightarrow O(X : \beta) \text{ iff } \beta \in \text{Act}_p \vee X = \{i\}.
\]

If it is obligatory for \( X \) not to perform a positive action, then for every subgroup of \( X \) it is forbidden to perform that action, thus also for every actor of \( X \). If it is obligatory for an actor \( i \in X \) to perform a positive action, then it is also obligatory for every set with \( i \) as an element to perform that action, thus also \( X \).

Finally, we present a proposition that enables us to obtain a better understanding of group events in \( PD_eL(Evt') \), with respect to positive and negative actions.

\[16\] Note that formula \( O(X : \gamma) \rightarrow O(X \cup Y : \gamma) \) is valid, which seems paradoxical, more or less in the same way as Ross's paradox. See also section 3.3.2.
Proposition 4.5.3 Let \( X, Y \in \mathcal{P}^+(I) \) and \( \gamma_1, \gamma_2 \in \text{Act}_p \), then

1. \( O(X \cup Y : \gamma_1) \rightarrow O(X : \gamma_1) \);
2. \( O(X : \gamma_1) \rightarrow O(X \cup Y : \gamma_1) \);
3. \( O(X : \gamma_1 \cup Y : \gamma_1) \rightarrow O(X \cap Y : \gamma_1) \), with \( X \cap Y \neq \emptyset \);
4. \( O(X : \gamma_1 \cup Y : \gamma_1) \rightarrow O(X \cup Y : \gamma_1) \);
5. \( O(X : \gamma_1 \cup Y : \gamma_2) \rightarrow O(X \cap Y : \gamma_1 \& \gamma_2) \), with \( X \cap Y \neq \emptyset \);
6. \( O(X : \gamma_1 \cup Y : \gamma_2) \rightarrow O(X \cup Y : \gamma_1 \& \gamma_2) \).

Proof

1. \( O(X \cup Y : \gamma_1) \equiv [X \cup Y : \gamma_1] V, [X \cup Y : \gamma_1] V \rightarrow [X : \gamma_1 \cup Y : \gamma_1] V \) (by 4.3.17, 4.3.26 and 4.3.27), \( [X : \gamma_1 \cup Y : \gamma_1] V \equiv [X : \gamma_1] V \land [Y : \gamma_1] V = O(X : \gamma_1) \land O(Y : \gamma_1) \), and \( O(X : \gamma_1) \land O(Y : \gamma_1) \rightarrow O(X : \gamma_1) \). Hence, by modus ponens it follows that \( O(X \cup Y : \gamma_1) \rightarrow O(X : \gamma_1) \).

2. \( O(X : \gamma_1) \equiv [X : \gamma_1] V, [X : \gamma_1] V \rightarrow [X : \gamma_1 \& Y : \gamma_1] V, [X : \gamma_1 \& Y : \gamma_1] V \rightarrow [X \cup Y : \gamma_1] V, [X \cup Y : \gamma_1] V \equiv O(X \cup Y : \gamma_1) \). Hence, by modus ponens it follows that \( O(X : \gamma_1) \rightarrow O(X \cup Y : \gamma_1) \).

3. \( O(X : \gamma_1) \lor O(Y : \gamma_1) \equiv [X : \gamma_1] V \lor [Y : \gamma_1] V, [X : \gamma_1] V \lor [Y : \gamma_1] V \rightarrow [X : \gamma_1 \& Y : \gamma_1] V, [X : \gamma_1 \& Y : \gamma_1] V \rightarrow [X \cap Y : \gamma_1] V, [X \cap Y : \gamma_1] V \equiv O(X \cap Y : \gamma_1) \). Hence, by modus ponens it follows that \( O(X : \gamma_1 \cup Y : \gamma_1) \rightarrow O(X \cup Y : \gamma_1) \).

4. \( O(X : \gamma_1) \land O(Y : \gamma_1) \equiv [X : \gamma_1] V \land [Y : \gamma_1] V \equiv [X : \gamma_1 \cup Y : \gamma_1] V, [X : \gamma_1 \cup Y : \gamma_1] V \rightarrow [X \cap Y : \gamma_1] V, [X \cap Y : \gamma_1] V \equiv O(X \cap Y : \gamma_1) \). Hence, by modus ponens it follows that \( O(X : \gamma_1 \cup Y : \gamma_1) \rightarrow O(X \cup Y : \gamma_1) \).

5. \( O(X : \gamma_1 \& Y : \gamma_2) \equiv O(X : \gamma_1) \land O(Y : \gamma_2), O(Y : \gamma_2) \rightarrow O(X \cap Y : \gamma_1 \& \gamma_2), O(X \cap Y : \gamma_1 \& \gamma_2) \equiv O(X \cap Y : \gamma_1 \& \gamma_2) \). Hence, by modus ponens it follows that \( O(X : \gamma_1 \& Y : \gamma_2) \rightarrow O(X \cup Y : \gamma_1 \& \gamma_2) \).

6. \( O(X : \gamma_1 \& Y : \gamma_2) \equiv O(X : \gamma_1) \land O(Y : \gamma_2), O(X : \gamma_1) \land O(Y : \gamma_2) \rightarrow O(X \cup Y : \gamma_1 \& \gamma_2), O(X \cup Y : \gamma_1 \& \gamma_2) \equiv O(X \cup Y : \gamma_1 \& \gamma_2) \). Hence, by modus ponens it follows that \( O(X : \gamma_1 \& Y : \gamma_2) \rightarrow O(X \cup Y : \gamma_1 \& \gamma_2) \).
The formulas in proposition 4.5.3 include the following assertions:

1. If a negative action is obligatory for a group, then it is also obligatory for every subgroup.

2. If a positive action is obligatory for a group, then it is also obligatory for every superset of that group.

3. If it is obligatory that a group ‘performs’ a negative action or that another group ‘performs’ that action, then that action is obligatory for the intersection of both groups.

4. If it is obligatory that a group performs a positive action or that another group performs that action, then that action is obligatory for the union of both groups.

5. If it is obligatory that a group ‘performs’ a negative action and another group ‘performs’ another negative action, then it is obligatory for the intersection of the groups to ‘perform’ both actions.

6. If it is obligatory that a group performs a positive action and another group performs another positive action, then it is obligatory for the union of the groups to perform both actions.

### 4.6 Evaluation and conclusions

In this chapter, we formalised the concept of (groups of) actors in $PD_{e}L$, a deontic system as a variant of dynamic logic. The concept of groups of actors in $PD_{e}L$ is new. With the addition of groups of actors, we are able to express who (a group or an actor) has the responsibility for performing certain actions, and when a group or an actor satisfies a norm.

To express the notions of individual obligation (i.e., the general, personal and unspecific obligations) we first used individual events. However, we showed that these notions can also be expressed using collective events. This enabled us to express all the different notions of the individual and collective obligations in the same semantical model. Further, we investigated the relations between these different notions of obligation, analogous to the relations discussed in chapter 3.

With the introduction of collective events, it is now possible to make a distinction between actions performed by all members of a group $X$ and actions performed by a group $X$ as a whole. The type of action is very important to indicate when a group fulfils an obligation; e.g., if a group $X$ has to perform a positive action, then some subset of that
group has to perform that action, and if a group has to ‘perform’ a negative action, then every subset of that group has to ‘perform’ that action.

Finally, we show how the concepts developed in this, and the previous, chapter can help to formalise and analyse the following two judgements. The first judgement concerns an individual obligation and the second, in my opinion, a weak collective obligation.

The first case reads as follows. According to the ‘Reglement van Politie voor de scheepvaart op de Merwede’ (Police regulations concerning navigation on the river Merwede), it is prohibited for more than three boats to be moored next to one another breadthways in the river outside harbours.

The High Court specified this norm as follows:17

It is prohibited to perform an action resulting in an unwanted situation. Such a situation was created by the fourth captain, who moored his boat last.

The second case reads as follows. Under the Dutch Road Traffic Act, it was prohibited for three cyclists to cycle next to one another. In its 1948 ruling, the High Court considered all three cyclists to have broken the law, because each one of them was able to bring an end to the situation.18

In the case of the four boats, the person who creates the illegal situation is obliged to bring an end to that situation. We can formalise this by using the system $SDL_X$:

$$O_{i_4}(\neg p),$$

with $p$ ‘more than three boats are moored next to one another breadthways in the river outside the harbour’ and $i_4$ the fourth captain, who moored his boat last. Thus, here we are concerned with a personal obligation.

In the case of the three cyclists, all three are obliged to bring an end to the illegal situation:

$$\forall i \in X O_i(\neg q),$$

with $X$ the set of the three cyclists and $q$ ‘more than two cyclists are cycling next to one another’. Here we are concerned with a restricted general obligation. This is very strange, since if one of the cyclists ends the situation, the other cyclists are released from their obligation. A better formalisation would be that they, as a group, are obliged to bring an end to the situation, so

$$O_X(\neg q).$$

Although all three are considered to have broken the law, this does not alter the fact that they, as a group, are responsible for bringing an end to the illegal situation. Note that we

17HR, 19-1-1931, NJ 1931, 1455.
18HR, 9-3-1948, NJ 1948, 370.
are here concerned with a weak collective obligation, not with a strict collective obligation. Suppose, for example, that four cyclists are cycling next to one another. In this case, none of them individually can bring an end to the illegal situation, and the general obligation

$$\forall_{i \in Y} O_i(q),$$

with $Y$ the set of four cyclists, would be a void obligation, because each cyclist is not able to bring an end to the illegal situation. The individual is not able to end that situation, the group, however, is:

$$O_Y(\neg q),$$

with $Y$ the group of four cyclists.

However, the formalisation is not really satisfactory, since it does not fit into our system of criminal law. The law is geared to unlawful behaviour and illegal situations. Criminal law is concerned with behaviour: if an illegal situation is mentioned in the description of an offence, the question is raised as to who created this situation (by action or by omission) and who is responsible for continuing the situation (omission). From the illegal situation a certain type of behaviour is derived, as it were.

The derivation of an action or a set of actions from a situation is not a clear-cut matter. Take, for instance, obligation $O_X(\neg q)$: 'it is obligatory for group $X$ that no more than two cyclists cycle next to one another.' Now, we want to derive an Ought-to-do statement from this Ought-to-be statement. However, one can think of many actions to be taken by group $X$ that would end the illegal situation.

Many human actions are described by their results, something on which von Wright in particular has focussed. Thus, according to the suggestion mentioned, in the case of John opening the window, John selects and runs a routine such that at the end of that routine the window is open; in the case of John eating (all of) an apple, John selects and runs a routine at the end of that routine John has eaten an apple; etc. Thus there is a large class of actions of the type 'doing $A$', where $A$ is a proposition expressing a state-of-affairs. (Segerberg, 1989, p. 327)

In his article 'Bringing it about', Segerberg (1989) introduced an operator $\delta$ such that '$\delta(p)$' will carry the informal meaning of 'bringing about that $p$' or 'doing $p$'. Thus, $\delta(p)$ is the action with $p$ as the result (or state of affairs). In general, $\delta(p)$ is a choice between several actions $\beta_1, \ldots, \beta_n$, which all bring about that $p$. Thus, $\delta(p)$ is equal to $\beta_1 \cup \beta_2 \cup \ldots \cup \beta_n$.

With the help of the operator '$\delta$', we can smoothly formalise the obligations belonging to the two cases:

\[\text{For the formal representation of } \delta(p) \text{ we refer to Segerberg (1989).}\]
1. in the case of the four boats:
   \[ O(i_4 : \delta(\neg p)) \].

2. in the case of the three cyclists:
   \[ O(X : \delta(\neg q)) \].

From the above consideration we can conclude that it is important to distinguish Ought-to-do statements (which may be interpreted as expressing imperatives of the form 'an addressee ought to perform an action') and Ought-to-be statements (which express a desired state of affairs without necessarily mentioning addressees and actions bearing relations with that state of affairs). There are situations in which we would like to formalise norms as Ought-to-be statements, and there are situations in which we would like to formalise norms as Ought-to-do statements. In the case of the Penal Code, the formalisation of obligations as Ought-to-do statements fits better than as Ought-to-be statements. Thus, with regard to the Penal Code, \( PD_4 L(Evt') \) is to be preferred to \( SDL_X \).
Chapter 5

Deontic systems and authorities

This chapter\(^1\), presents the addition of (sets of) authorities in SDL\(_X\) and PD\(_x\)L(Evt') such that we can consistently express normative inconsistencies, i.e., conflicting norms enacted by (sets of) authorities.

5.1 Introduction

In the previous two chapters, we added actors to the norms, in order to be able to specify to whom the norms pertain. The addition of actors already reduces the number of inconsistencies normally found in formal representations of legal code. Still some 'inconsistencies' remain, one of the main reasons being the fact that norms are enacted by different authorities in a legal system. If the authorities are not accounted for in the logical system, some formulas might be inconsistent, whereas if authorities are added, they would not. This is identical to the following example expressing the change from propositional logic to predicate logic: \( p \land \neg p \equiv \text{false} \), but \( p(a) \land \neg p(b) \) is not necessarily false, with variables \( a \) and \( b \) (e.g., being authorities).

In order to deal with this type of inconsistencies, we add (sets of) authorities enacting the norms to the deontic systems SDL\(_X\) and PD\(_x\)L(Evt'), which were discussed in the previous two chapters. The normative authorities play an important role in a normative system since they are responsible for establishing the norms and for supervising the enactment of the norms.

First, corresponding with Bailhache's (1981, 1991) theory, we develop a theory for a coherent deontic system, i.e., a normative agreement between all (sets of) authorities. A drawback of this theory, however, inherent to the purpose of this theory, is that we cannot express conflicts between enacted norms. These conflicts exist since norms come into being and cease to exist in complex ways, involving different authorities at different

\(^1\)Some of the ideas in this chapter were presented earlier in Royakkers and Dignum (1994).
times in different places (cf. Prakken, 1993). Second, we show that this theory has to deal with serious problems concerning the power of expressibility. For instance, in this theory we cannot express that a permission has been enacted by a set of authorities.

To overcome these problems and especially to express normative inconsistencies in a consistent way, we modify the theory. A consequence of this modification is that the theory becomes more powerful, certain formulas acquire new meanings differing from Bailhache's theory. These new meanings are discussed formally for systems $D_A$ and $PDL^A(Evt')$.

The organisation of this chapter is as follows: in section 5.2, we extend $SDL_X$ to sets of authorities on the basis of Bailhache's theory. This theory excludes the possibility of expressing normative inconsistency, which is discussed in section 5.3. Section 5.4 presents system $D_A$, a modification and extension of the system discussed in the previous section, which enables us to express conflicting norms enacted by sets of authorities. The addition of sets of authorities to system $PD_eL(Evt')$ is discussed in section 5.5. In section 5.6 we introduce the term 'normative system' in relation to the norms enacted by the authorities. In the last section, we draw some conclusions.

5.2 Authorities in $SDL_X$

In this section, we discuss the addition of sets of authorities on the basis of Bailhache's (1981,1991) theory. Bailhache wanted to obtain a deontic coherent system, i.e., a normative agreement between all sets of authorities. This is accomplished by avoiding any conflict between the obligations enacted by the sets of authorities. It is necessary and sufficient for each set of authorities not to forbid - in other words to permit - what a set of authorities makes obligatory.

To add authorities to system $SDL_X$, we have to introduce the set $NA$ of authorities. Let $a \in NA$, then $O_X^a(p)$ can be read as 'a makes it obligatory for $X$ that $p$'. If there is a normative agreement between all the authorities, then this means that norm $O_X(p)$ holds.

Now it seems natural to make a reduction in statement $O_X(p)$ of $SDL_X$ to statement $O_X^a(p)$: norm $O_X(p)$ holds if and only if there is an authority which enacted that norm. This can be expressed as

$$O_X(p) =_{def} \exists a \in NA O_X^a(p). \tag{5.1}$$

We refer to the two directions of the equivalence of (5.1) as $(5.1)^{-}$ and $(5.1)^{+}$. There is an argument against the intuitive validity of $(5.1)^{-}$.

- Implication $(5.1)^{+}$ asserts that if an authority enacts a norm $O_X(p)$, that norm holds. In a coherent deontic system this seems very plausible.
5.2 Authorities in SDLX

- Implication (5.1)$^{-}$ asserts that if norm $O_X(p)$ holds, then an authority enacted that norm. This is simply not true. As a counterexample, consider obligation $O_X(p \land q)$, which is equivalent to $O_X(p) \land O_X(q)$. Suppose that $O_X(p)$ was enacted by authority $a$ and $O_X(q)$ by authority $b$, then norm $O_X(p \land q)$ was not enacted by a single authority, but by the two authorities $a$ and $b$, in other words by a set of authorities. Thus, the formula

$$ \exists a \in NA O_X^a(p) \land \exists a \in NA O_X^a(q) \rightarrow \exists a \in NA O_X^a(p \land q) \quad (5.2) $$

is counter-intuitive and it would be strange if it were valid in a formal system (cf. Hansson, 1970, p. 244), in contrast with the formula in SDLX:

$$ O_X(p) \land O_X(q) \rightarrow O_X(p \land q). \quad (5.3) $$

Hence, a norm that holds is not necessarily enacted by one authority.

Thus, the definition for $O_X(p)$ as $\exists a \in NA O_X^a(p)$ is inadequate. This above problem can be solved by using sets of authorities. This leads to an extension of system SDLX to SDL$^A$, where the norms are enacted by sets of authorities. Let $A$ be a subset of $NA$, then $O_X^A(p)$ will be read as ‘$A$ makes it obligatory for $X$ that $p$’.

The system SDL$^A$ is given by the following rules and axioms:

$$(RO^A M) \quad \frac{P \rightarrow q}{O_X^A(p) \rightarrow O_X^A(q)}$$

$$(RO^A M2) \quad \frac{B \in P^+(A)}{O_X^A(p) \rightarrow O_X^B(p)}$$

$$(RO^A M3) \quad \frac{X \in P^+(Y)}{O_X^A(p) \rightarrow O_X^B(p)}$$

together with the following axiom schemata

$$(O^A C) \quad (O_X^A(p) \land O_X^A(q)) \rightarrow O_X^A(p \land q)$$

$$(O^A N) \quad O_X^A(p \lor \neg p)$$

$$(O^A D) \quad \neg O_X^A(p \land \neg p)$$

$$(Df. P^A) \quad P_X^A(p) \equiv \neg O_X^A(\neg p)$$

For the semantical interpretation of this system SDL$^A$ we use the following model structure $M = (W, R, P^+(NA), P^+(I), V)$, with $R$ the set of functions $\{R_X^{A_1}, R_X^{A_2}, \ldots\}$ for each set of authorities $A_1, A_2, \ldots \in P^+(NA)$ and for all $X \in P^+(I)$, and the non-empty powerset $P^+(NA)$ of the set of authorities $NA = \{a_1, a_2, \ldots\}$. The function $R_X^A \in R$ on $W$, which returns the deontically ideal worlds for set $A$ of authorities and for set $X$ of individuals given a world: $R_X^A : W \rightarrow 2^W$.

The truth conditions for $O_X^A$ and $P_X^A$ are defined as follows:

$$ M, w \models O_X^A(p) \text{ iff } R_X^A(w) \subseteq [p] \quad (5.4) $$
and
\[ M, w \models P^A_X(p) \text{ iff } R^A_X(w) \cap \{p\} \neq \emptyset. \] (5.5)

The following additional constraint gives schema \( O^A \). Let \( R^A_X \in \mathcal{R} \), then
\[ R^A_X(w) \neq \emptyset \text{ for all } R^A_X \in \mathcal{R} \text{ and for all } w \in W. \] (5.6)

The constraints

\[
\text{if } R^A_X, R^B_X \in \mathcal{R} \text{ and } B \in \mathcal{P}^+(A), \text{ then } R^A_X(w) \subseteq R^B_X(w) \text{ for all } w \in W
\] (5.7)

and

\[
\text{if } R^A_X, R^A_Y \in \mathcal{R} \text{ and } X \in \mathcal{P}^+(Y), \text{ then } R^A_Y(w) \subseteq R^A_X(w) \text{ for all } w \in W
\] (5.8)

validate rules \((RO^A M2)\) and \((RO^A M3)\), respectively.

Rule \((RO^A M2)\) states that if a set of authorities is included in another set, every obligation enacted by the former is also an obligation enacted by the latter. Rule \((RO^A M3)\) states that if a set of authorities makes it obligatory for a set \( X \) of individuals that \( p \), then this set of authorities makes it also obligatory for every superset of \( X \) that \( p \). Now it follows that\(^2\)
\[ O^A_X(p) \equiv \exists_{Y \in \mathcal{P}^+(X)} O^A_Y(p). \]

Thus, the collective obligations enacted by sets of authorities are interpreted as weak collective obligations (see section 3.3.2). In the sequel of this thesis, we will interpret the collective obligation as the weak collective obligation.

From constraint (5.7) we can derive the constraint
\[ \text{if } R^A_X \in \mathcal{R}, \text{ then } \cap_{B \in \mathcal{P}^+(A)} R^B_X(w) = R^A_X(w) \text{ for all } w \in W. \] (5.9)

Hence, \( \forall_{A,B \in \mathcal{P}^+(NA)} (R^A_X(w) \cap R^B_X(w) \neq \emptyset) \), since \( \emptyset \neq R^{NA}_X(w) = \cap_{A \in \mathcal{P}^+(NA)} R_X^{NA}(w) \subseteq R^A_X(w) \cap R^B_X(w). \)

**Proposition 5.2.1** Let \( X \in \mathcal{P}^+(I) \) and \( A, B, C \in \mathcal{P}^+(NA) \). Then,

1. \( O^A_X(p) \rightarrow P^B_X(p) \);
2. \( O^A_X(p) \land O^B_X(q) \rightarrow O^A_X(p \land q) \);
3. \( O^A_X(p) \equiv \exists_{B \in \mathcal{P}^+(A)} O^B_X(p) \);

\(^2\)Suppose that \( O^A_X(p) \) holds, then there is a subset \( Y \) of \( X \), such that \( O^A_Y(p) \) holds, namely \( X \). Thus, \( \exists_{Y \in \mathcal{P}^+(X)} O^A_Y(p) \). Now suppose that \( \exists_{Y \in \mathcal{P}^+(X)} O^A_Y(p) \), say \( Z \). Since \( Z \in \mathcal{P}^+(X) \), it holds by rule \((RO^A M3)\) that \( O^A_X(p) \).
5.2 Authorities in $SDL_x$

4. $P_X^A(p) \equiv \forall_{B \in P^+(A)} P_X^B(p)$;

5. $O_X^A(p) \land O_X^B(q) \rightarrow P_X^C(p \land q)$.

Proof

1. Suppose that $O_X^A(p)$ holds. It follows from rule $(RO^AM2)$ that $O_X^A(p) \rightarrow O_X^{A \cup B}(p)$. From $(O^A D)$ and $(O^A C)$ it follows that $O_X^{A \cup B}(p) \rightarrow P_X^{A \cup B}(p)$. On the basis of $(Df.P^A)$ and $(RO^AM2)$ it holds that $P_X^{A \cup B}(p) \rightarrow P_X^B(p)$. Hence, by modus ponens, it follows that $O_X^A(p) \rightarrow P_X^B(p)$.

2. Suppose that $O_X^A(p) \land O_X^B(q)$ holds. Then, by $(RO^AM2)$ $O_X^{A \cup B}(p) \land O_X^{A \cup B}(q)$ holds. From $(O^A C)$ it now follows that $O_X^{A \cup B}(p \land q)$.

3. Suppose that $O_X^A(p)$ holds, then there is a subset $B$ of $A$, such that $O_X^B(p)$ holds, namely $A$. Thus, $\exists_{B \in P^+(A)} O_X^B(p)$. Now suppose that $\exists_{B \in P^+(A)} O_X^B(p)$, say $C$. Since $C \in P^+(A)$, it holds by rule $(RO^AM2)$ that $O_X^A(p)$.

4. Follows immediately from 5.2.1.3 and $(Df.P^A)$.

5. From 5.2.1.2 it follows that $O_X^A(p) \land O_X^B(q) \rightarrow O_X^{A \cup B}(p \land q)$, and from 5.2.1.1 it follows that $O_X^{A \cup B}(p \land q) \rightarrow P_X^C(p \land q)$. Thus, by modus ponens $O_X^A(p) \land O_X^B(p) \rightarrow P_X^C(p \land q)$.

Proposition 5.2.1.1 expresses the normative agreement between all the possible couples of sets of authorities. Proposition 5.2.1.5 expresses that we have a complete deontic coherent system, i.e., a normative agreement between all sets of authorities (cf. Bailhache, 1991). Deontic (or normative) coherence makes it necessary that it is not obligatory for all sets of actors that some thing is accomplished as soon as it is obligatory for some set of actors that this thing is accomplished. Thus, if sets of authorities enacted that $p$ and $q$ are obligatory for a certain set of individuals, then every set of authorities has to respect this and should permit that $p \land q$ for that set of individuals. The question arises how we have to interpret the words 'should permit'. It would be very strange to interpret this as 'enacted the permission', since this is counter-intuitive with the fact that there is a normative agreement between all the sets of authorities. Consider, for example, the formula

$$O_X^A(p) \land P_X^B(\neg p)$$

(5.10)

and $A \not\subseteq B$. This formula is not contradictory in the system $SDL^A$. However, if we interpret $P_X^B(\neg p)$ as 'A enacted that it is permitted for $X$ that $\neg p$', then formula (5.10) is countere-intuitive for a deontic coherent system. It expresses that $A$ enacted that $O_X(p)$ and $B$ enacted that $P_X(\neg p)$. Now, there is no normative agreement between these two sets of authorities, since the enacted norms are conflicting: $O_X(p) \land P_X(\neg p) \equiv \neg P_X(\neg p) \land P_X(\neg p)$.
We have to interpret ‘should permit’ as ‘has not enacted that it is forbidden’. Now the formula (5.10) is not counter-intuitive. It only expresses that \( A \) enacted \( O_X(p) \) and that \( B \) did not enact \( F_X(\neg p) \), i.e., \( O_X(p) \). Thus, \( P_X^A(p) \) has to be read as ‘\( A \) did not enact that it is forbidden for \( X \) that \( p \)’.

### 5.2.1 The relation between SDL\(_X\) and SDL\(_A\)

Now that we have accomplished a normative agreement between all sets of authorities, we can attempt a reduction in statements of SDL\(_X\) to statements of SDL\(_A\): norm \( O_X(p) \) holds if and only if there is a set of authorities that enacted that norm. This can be expressed as

\[
O_X(p) = \text{def} \exists_{A \in \mathcal{P}^+(NA)} O_X^A(p),
\]

and so we can say that norm \( P_X(p) \) holds if and only if every set of authorities did not prohibit that \( p \), which corresponds with \( P_X(p) \equiv \neg O_X(\neg p) \). Thus,

\[
P_X(p) = \text{def} \forall_{A \in \mathcal{P}^+(NA)} P_X^A(p).
\]

Note that \( \exists_{A \in \mathcal{P}^+(NA)} O_X^A(p) \) is equivalent to \( O_X^{NA}(p) \) and that \( \forall_{A \in \mathcal{P}^+(NA)} P_X^A(p) \) is equivalent to \( P_X^{NA}(p) \). Thus, \( O_X(p) \equiv O_X^{NA}(p) \) and \( P_X(p) \equiv P_X^{NA}(p) \).

- Implication (5.11)** states that if there is a norm \( O_X(p) \) that holds, then there is a set of authorities that enacted that norm. This seems correct, since a norm can only hold if it has been enacted, and a norm can only be enacted by authorities.

- Implication (5.11)** states that if a set of authorities enacted a norm \( O_X(p) \), this norm holds. This also seems correct in a coherent deontic system.

The semantics can easily cope with both cases when we add the constraint

\[
R_X(w) = R_X^{NA}(w) \text{ for all } w \in W \text{ and for all } X \in \mathcal{P}^+(I).
\]

This can be proven as follows. Suppose \( O_X(p) \), hence \( R_X(w) \subseteq [p] \), which is equivalent to \( R_X^{NA}(w) \subseteq [p] \). Consequently, \( \exists_{A \in \mathcal{P}^+(NA)} (R_X^A(w) \subseteq [p]) \), so \( \exists_{A \in \mathcal{P}^+(NA)} O_X^A(p) \). Now suppose \( \exists_{A \in \mathcal{P}^+(NA)} O_X^A(p) \), then \( \exists_{A \in \mathcal{P}^+(NA)} (R_X^A(w) \subseteq [p]) \), say that this set of authorities is \( B \), thus \( R_X^B(w) \subseteq [p] \). Since \( R_X(w) = R_X^{NA}(w) \subseteq R_X^B(w) \), it follows that \( R_X(w) \subseteq [p] \), thus \( O_X(p) \). The validity of (5.12) follows immediately from \( P_X(p) \equiv \neg O_X(\neg p) \) and the proof above.
5.3 Normative inconsistencies

The relations between statements $O_X(p)$ and $P_X(p)$ of $SDL_X$ and the statements $O^A_X(p)$ and $P^A_X(p)$ can be summarised in the following figure:

**Figure 5.1**

At first glance, this theory based on Bailhache (1981, 1991), seems a good approach for a coherent deontic system, i.e., a normative agreement between all the sets of authorities. However, this approach has two serious drawbacks. We cannot express that

1. a set of authorities enacts a permission. Only obligations (and thus also prohibitions) can be enacted by the sets of authorities. Conversely, we can not express that a set of authorities does not enact an obligation, but only permissions;

2. a set of authorities enacts a combination of collective obligations, e.g., $O_X(p) \lor O_X(q)$. The sets of authorities can only enact an atomic collective obligation ($O_X(p)$).

Another drawback - although, this drawback is inherent to the developed theory - is that we cannot express normative conflicts, i.e., inconsistencies between enacted norms.

5.3 Normative inconsistencies

In the previous section, we developed a coherent deontic system: a deontic system with a normative agreement between all sets of authorities. However, it is a fact that authorities enact rules that conflict with norms enacted by other authorities; this happens very frequently. The conflicts arise when these norms become part of the same normative system, not if they belong to different systems.

Before we develop a theory that can establish a normative agreement between authorities with several ranks of authorities - in spite of the normative inconsistencies between the norms enacted by the authorities - in chapters 6 and 7, we have to develop a theory to allow normative inconsistencies in our system, such as

$$O^A_X(p) \land F^B_X(p).$$  \hspace{1cm} (5.14)

In $SDL^A$, this formula is contradictory, since $O^A_X(p) \land F^B_X(p) \rightarrow O^{A\cup B}_X(p) \land O^{A\cup B}_X(\neg p)$, and $O^{A\cup B}_X(p) \land O^{A\cup B}_X(\neg p) \rightarrow O^{A\cup B}_X(p) \land \neg O^{A\cup B}_X(p)$. 

To deal with this, we can choose to give up rule (RO$^A$M2) or schema (OA$^A$D) of system SDL$^A$.

- If we choose to give up rule (RO$^A$M2), we can express formula (5.14) if $A \neq B$. However, then we cannot derive formula

$$O^A_X(p) \land O^B_X(q) \rightarrow O^{A\cup B}_X(p \land q), \quad (5.15)$$

which is counter-intuitive. Suppose $NA = \{a, b\}$; $a$ has only enacted $O_X(p)$ and $b$ only $O_X(q)$. Then, norm $O_X(p \land q)$ intuitively holds, thus the norm has to be enacted. The only possibility is that the norm was enacted by the set $\{a, b\}$. Thus, we cannot give up rule (RO$^A$M2).

Furthermore, we would not be able to derive formula

$$O^A_X(p) \land F^A_X(p), \quad (5.16)$$

since this is blocked by axiom (OA$^A$D). A set of authorities can enact conflicting rules; for example, let $A = \{a, b\}$ and $a$ enacted $O_X(p)$ and $F_X(q)$, and $b$ enacted $O_X(p \rightarrow q)$, then $A$ enacts $O_X(q)$ as well as $F_X(q)$. This cannot be expressed if we give up rule (RO$^A$M2). Thus, we cannot give up rule (RO$^A$M2).

- It would be preferable to give up axiom (OA$^A$D). (OA$^A$D) removes the possibility of expressing conflicts between norms enacted by sets of authorities, with the result that the deontic system is coherent. By eliminating axiom (OA$^A$D) from SDL$^A$, we can express formulas (5.14), (5.15) and (5.16).

However, this is not a real solution, since we cannot consistently express that, for example, an authority $a$ enacted $O_X(p)$ and an authority $b \neg O_X(p)$. Then, we can derive $O^A_X(p)$ and $\neg O^A_X(p)$, with $A = \{a, b\}$, and $O^A_X(p) \land \neg O^A_X(p)$ is false.

To express these normative conflicts between enacted norms, we choose for another approach. Instead of $A$ modifying $O_X(p)$, we choose to treat $A$ as a modal operator. Writing $A : O_X(p)$, and instead of $\neg O^A_X(p)$ we write $A : \neg O_X(p)$. This seemingly small change has large consequences. The system becomes more powerful, because we can express

- that a set $A$ of authorities does not enact a norm, for example, $O_X(p)$:

$$\neg(A : O_X(p));$$

- that a set $A$ of authorities enacted a combination of norms, for example, $O_X(p) \lor O_X(q)$:

$$A : (O_X(p) \lor O_X(q));$$
5.4 The system \( D^*_A \)

- normative conflicts, for example:

\[
A : O_X(p) \land A : \neg O_X(p).
\]

Thus, this new approach acquires new meanings, not expressible in \( SDL^A \), and are therefore subject to new intuitions, which we analyse in the following section.

5.4 The system \( D^*_A \)

Let \( A \in \mathcal{P}^+(NA) \) and \( \Theta \) a well-formed formula (wff) of \( SDL_X \), then \( A : \Theta \) has to be read as ‘\( A \) makes that \( \Theta \)’ or ‘\( A \) enacted \( \Theta \)’. \( \neg (A : \Theta) \) has to be read as ‘\( A \) did not enact \( \Theta \)’. Thus, \( \neg (A : O_X(p)) \) states that ‘the set \( A \) of authorities did not enact that it is obligatory for the set \( X \) of actors that \( p \)’, which cannot be expressed in the system \( SDL^A \).

The system \( D^*_A \), which we discuss in this section, is an extension of \( SDL^A \) in three ways, considering:

1. the possibility of expressing conflicting norms;
2. the possibility of expressing that norms, especially obligations, are not enacted by a set of authorities;
3. the possibility of expressing that a set of authorities enacts permissions and combinations of norms.

The language \( \mathcal{D} \) of \( D^*_A \) consists of assertions concerning the enactment of norms by sets of authorities and can be determined by the following BNF for its elements (\( \Phi \)):

\[
\Phi ::= A : \Theta \mid \neg \Phi \mid \Phi_1 \land \Phi_2 \mid \Phi_1 \lor \Phi_2 \mid \Phi_1 \to \Phi_2,
\]

with \( A \in \mathcal{P}^+(NA) \), \( \Phi, \Phi_1, \Phi_2 \in \mathcal{D} \) and \( \Theta \) a wff of \( SDL_X \).

The system \( D^*_A \) is given by the following axioms and rules:

**Axiom 5.4.1**

1. all tautologies of the propositional calculus;
2. \( A : \Theta_1 \land A : \Theta_2 \rightarrow A : (\Theta_1 \land \Theta_2) \).

**Rules 5.4.2**

R1

\[
\Phi_1 \rightarrow \Phi_2, \Phi_1 \\
\Phi_2
\]
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R2

\[ B \in \mathcal{P}^+(A) \quad \Rightarrow \quad B : \Theta_1 \rightarrow A : \Theta_1 \]

R3

\[ \Theta_1 \rightarrow_{SDL_X} \Theta_2 \quad \Rightarrow \quad A : \Theta_1 \rightarrow A : \Theta_2 , \]

with \( \Theta_1 \) and \( \Theta_2 \) wffs of \( SDL_X \).

For the semantical interpretation of this system \( D^*_A \), we use the following model structure \( \mathcal{M} = (W, \mathcal{R}_I, \mathcal{R}_N, \mathcal{P}^+(NA), \mathcal{P}^+(I), V) \), with \( \mathcal{R}_N \), the set of functions \( \{R^A_1, R^A_2, \ldots\} \) for each set of authorities \( A_1, A_2, \ldots \in \mathcal{P}^+(NA) \). The function \( R^A \in \mathcal{R}_N \) on \( W \), which returns the ideal worlds for set \( A \) of authorities given a world: \( R^A : W \rightarrow 2^W \).

The sentence 'A enacted \( O_X(p) \)' does not say that the norm \( O_X(p) \) is valid (is the case), in the sense that \( O_X(p) \) is applicable - instead, it says that \( O_X(p) \) is valid in a world which is ideal for \( A \) (see chapter 6). Thus, the statement 'A enacted that \( p \) is obligatory for \( X \)' describes some idealised world for \( A \) and not the actual world, since the norm can be overruled by, for example, a norm enacted by a superior authority or a norm enacted at a later point in time. In other words, a norm that is enacted by a set \( A \) of authorities in the actual world is valid in an ideal world for \( A \).

The truth condition for \( A : \Theta \), with \( \Theta \) a wff of \( SDL_X \), is defined as follows:

\[
\mathcal{M}, w \models A : \Theta \iff R^A(w) \subseteq [\Theta], \tag{5.17}
\]

and \( R^A(w) \subseteq [\Theta] \) is defined as follows:

- if \( \Theta \equiv O_X(p) \), then

\[
R^A(w) \subseteq [\Theta] \iff \forall w' \in R^A(w) (R_X(w') \subseteq [p]);
\]

- if \( \Theta \equiv P_X(p) \), then

\[
R^A(w) \subseteq \Theta \iff \forall w' \in R^A(w) (R_X(w') \cap [(p)] \neq \emptyset);
\]

- if \( \Theta \equiv \Theta_1 \land \Theta_2 \), then

\[
R^A(w) \subseteq [\Theta] \iff R^A(w) \subseteq [\Theta_1] \land R^A(w) \subseteq [\Theta_2],
\]

with \( \Theta_1, \Theta_2 \) wffs of \( SDL_X \);
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- if $\Theta \equiv \Theta_1 \lor \Theta_2$, then
  \[ R^A(w) \subseteq [\Theta] \text{ iff } R^A(w) \subseteq [\Theta_1] \cup [\Theta_2], \]
  with $\Theta_1, \Theta_2$ wffs of $SDL_X$;

- if $\Theta \equiv \neg \Theta_1$, then
  \[ R^A(w) \subseteq [\Theta] \text{ iff } R^A(w) \subseteq [\neg \Theta_1] \text{ iff } R^A(w) \cap [\Theta_1] = \emptyset, \]
  with $\Theta_1$ a wff of $SDL_X$.

The constraint
\[ R^A(w) \subseteq R^B(w) \text{ for all } w \in W, B \subseteq A \text{ and } A, B \in \mathcal{P}^+(NA), \]
validates rule $R2$. From constraint (5.18), it follows that
\[ R^A(w) = \cap_{B \in \mathcal{P}^+(A)} R^B(w). \]

Rule $R2$ states that if a set of authorities is included in another set, every norm enacted by the former is also a norm enacted by the latter. Note that in $SDLA$ - expressed by rule $(RO^A M2)$ - this only holds for the obligation and not for the permission, since the meaning of $P^A_X(p)$ does not correspond with the meaning of $A : P_X(p)$: in contrast to $P^A_X(p)$ meaning that $A$ does not enact that $p$ is forbidden, $A : P_X(p)$ means that $A$ enacted that $p$ is permitted for the group $X$ of actors. Thus, $A : P_X(p) \rightarrow \forall_{B \in \mathcal{P}^+(A)} B : P_X(p)$ does not hold in contrast to the formula in $SDLA$: $P^A_X(p) \rightarrow \forall_{B \in \mathcal{P}^+(A)} P^B_X(p)$.

Furthermore, the formula $\exists_{A \in \mathcal{P}^+(NA)} A : O_X(p) \rightarrow \forall_{B \in \mathcal{P}^+(NA)} B : P_X(p)$ does not hold, in contrast to the corresponding formula in $SDLA$: $\exists_{A \in \mathcal{P}^+(NA)} O^A_X(p) \rightarrow \forall_{B \in \mathcal{P}^+(NA)} P^B_X(p)$. This last formula is used to obtain a coherent deontic system, which removes the possibility of expressing conflicting norms enacted by the authorities.

For the axioms and for rule $R2$ it does not matter whether we interpret $O_X(p)$ as a strict collective obligation or as a weak collective obligation (see subsection 3.3.2). However, for rule $R3$ it does matter which interpretation we use, since, for example, formula $A : O_X(p) \rightarrow A : O_{X \cup Y}(p)$ is valid for the weak collective obligation, but not for the strict collective obligation. As we already mentioned, we treat the collective obligation in the sequel as the weak collective obligation.

### 5.4.1 Conflicting norms or normative agreement

#### Conflicting norms

With the help of this new approach, we can simply express conflicting norms enacted by authorities. Suppose $a$ enacted $O_X(p)$ and $b \neg O_X(p)$, then we can derive $A : O_X(p)$ and
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$A : \neg O_X(p)$, with $A = \{a, b\}$. Thus, in contrast to the system $SDL^A$, we can express that a set of authorities enacted conflicting norms, for example,

$$A : O_X(p) \land A : \neg O_X(p).$$

(5.20)

This is the reason why we do not add the constraint

$$\forall_{A \in \mathcal{P}^+(NA)} R^A(w) \neq \emptyset \text{ for all } w \in W,$$

(5.21)

since this constraint would make (5.20) contradictory.\(^4\)

Normative agreement

Constraint (5.21) would validate principle

$$A : \Theta \rightarrow \neg (B : \neg \Theta),$$

(5.22)

meaning that if set $A$ of authorities enacted the norm $\Theta$, then no set of authorities did not enact the negation of that norm, i.e., $\neg \Theta$. This can be proven as follows. Suppose $A : \Theta$, then $R^A(w) \subseteq [\Theta]$. Then, by constraint (5.18), it follows that $R^{NA}(w) \subseteq [\Theta]$. Hence, $R^{NA}(w) \cap [\Theta] \neq \emptyset$. From constraint (5.19) it follows that $\cap_{B \in \mathcal{P}^+(NA)} R^B(w) \cap [\Theta] \neq \emptyset$, hence $\forall_{B \in \mathcal{P}^+(NA)} (R^B(w) \subseteq [\neg \Theta])$. Thus, $\neg (B : \neg \Theta)$ for all $B \in \mathcal{P}^+(NA)$.

Principle (5.22) expresses the normative agreement between all sets of authorities. Thus, we obtain complete deontic coherence if we add constraint (5.21) to our semantics, which validates principle (5.22).

If we compare system $D^*_A$ including axiom (5.22) with $SDL^A$ (corresponding to Bailhache’s theory), we can conclude that both theories obtain a complete deontic coherent system; system $D^*_A$ including axiom (5.22) is more powerful, however, because we can express for each norm (or combination of norms) whether it was enacted or was not enacted by sets of authorities.

5.4.2 Some properties of $D^*_A$

The following proposition shows a number of properties for $D^*_A$, part of which follows from the properties of $SDL_X$.

**Proposition 5.4.3** Let $A, B \in \mathcal{P}^+(NA)$ and $X, Y \in \mathcal{P}^+(I)$. Then,

1. $A : (\Theta_1 \land \Theta_2) \rightarrow (A : \Theta_1 \land A : \Theta_2)$;
2. $(A : \Theta_1 \lor A : \Theta_2) \rightarrow A : (\Theta_1 \lor \Theta_2)$;

\(^4\)Suppose that $R^A(w) \neq \emptyset$. $A : O_X(p) \land A : \neg O_X(p)$ holds iff $R^A(w) \subseteq [O_X(p)] \cap [\neg O_X(p)] = \emptyset$, which is in contradiction with the assumption that $R^A(w) \neq \emptyset$.\)
5.4 The system $D_A^*$

3. $A : \Theta \equiv \exists_{B \in \mathcal{P}^+(A)} B : \Theta$;

4. $\neg(A \cup B : \Theta) \rightarrow \neg(A : \Theta \lor B : \Theta)$;

5. $\neg(A : \Theta \land \neg(A \cup B : \Theta))$

6. $A : O_X(p) \rightarrow A : O_{X \cup Y}(p)$;

7. $A : O_X(p) \land A : O_Y(p) \rightarrow A : O_{X \cup Y}(p)$;

8. $A : O_X(p) \lor A : O_X(q) \rightarrow A : O_X(p \lor q)$;

9. $A : P_X(p \land q) \rightarrow A : P_X(p) \land A : P_X(q)$;

10. $A : O_X(p) \land B : O_Y(q) \rightarrow A \cup B : O_{X \cup Y}(p \land q)$.

Proof

1. Since $\Theta_1 \land \Theta_2 \rightarrow \Theta_1$ and $\Theta_1 \land \Theta_2 \rightarrow \Theta_2$, it follows from rule R3 that $A : (\Theta_1 \land \Theta_2) \rightarrow A : \Theta_1$ and $A : (\Theta_1 \land \Theta_2) \rightarrow A : \Theta_2$. Hence, $A : (\Theta_1 \land \Theta_2) \rightarrow A : \Theta_1 \land A : \Theta_2$.

2. Since $\Theta_1 \rightarrow \Theta_1 \lor \Theta_2$ and $\Theta_2 \rightarrow \Theta_1 \lor \Theta_2$, it follows from rule R3 that $A : \Theta_1 \rightarrow A : (\Theta_1 \lor \Theta_2)$ and $A : \Theta_2 \rightarrow A : (\Theta_1 \lor \Theta_2)$. Hence, $(A : \Theta_1 \lor A : \Theta_2) \rightarrow A : (\Theta_1 \lor \Theta_2)$.

3. Suppose that $A : \Theta$ holds, then there is a subset $B$ of $A$, such that $B : \Theta$, namely $A$, thus $\exists_{B \in \mathcal{P}^+(A)} B : \Theta$. Suppose now that $\exists_{B \in \mathcal{P}^+(A)} B : \Theta$, say $C$, hence $C : \Theta$. Since $C \in \mathcal{P}^+(A)$, it holds by rule R2 that $A : \Theta$.

4. By rule R2 and contraposing, we obtain $\neg(A \cup B : \Theta) \rightarrow \neg(A : \Theta)$ and $\neg(A \cup B : \Theta) \rightarrow \neg(B : \Theta)$, thus $\neg(A \cup B : \Theta) \rightarrow \neg(A : \Theta) \land \neg(B : \Theta)$, which is equivalent to $\neg(A \cup B : \Theta) \rightarrow \neg(A : \Theta \lor B : \Theta)$.

5. Suppose $A : \Theta \land \neg(A \cup B : \Theta)$ holds, then it follows from R2 that $A \cup B : \Theta \land \neg(A \cup B : \Theta)$ also holds, which is a contradiction. Hence, $\neg(A : \Theta \land \neg(A \cup B : \Theta))$ is true.

6. It holds that $O_X(p) \rightarrow O_{X \cup Y}(p)$. Hence, by R3 it follows that $A : O_X(p) \rightarrow A : O_{X \cup Y}(p)$.

7. Follows immediately from 5.4.3.6.

8. By rule R3 it follows that $A : O_X(p) \rightarrow A : O_X(p \lor q)$ and that $A : O_X(q) \rightarrow A : O_X(p \lor q)$. Thus, $A : O_X(p) \lor A : O_X(q) \rightarrow A : O_X(p \land q)$.

9. Since $P_X(p \land q) \rightarrow P_X(p) \land P_X(q)$, it follows from R3 that $A : P_X(p \land q) \rightarrow A : P_X(p) \land P_X(q)$, and from 5.4.3.1 that $A : P_X(p) \land A : P_X(q)$.
10. Suppose that \( A : O_X(p) \land B : O_Y(q) \) holds, then by R2 and R3 it follows that \( A \cup B : O_{X \cup Y}(p) \land A \cup B : O_{X \cup Y}(q) \). By axiom 5.4.1.2 it now follows that \( A \cup B : (O_{X \cup Y}(p) \land O_{X \cup Y}(q)) \) and by R3 that \( A \cup B : (O_{X \cup Y}(p \land q)) \).

The converse of 5.4.3.4 does not hold. Suppose \( A : O_X(p) \) and \( B : O_X(q) \) hold, and suppose further that \( \neg (A : O_X(p \land q)) \) and \( \neg (B : O_X(p \land q)) \) also hold, then we can derive \( A \cup B : O_X(p \land q) \), thus formula \( \neg (A : O_X(p \land q)) \land \neg (B : O_X(p \land q)) \rightarrow \neg (A \cup B : O_X(p \land q)) \) is not valid. In general, we can say that if a set of authorities did not enact a norm \( \Theta \), then no subset of that set of authorities enacted \( \Theta \).

Proposition 5.4.3.3 states that a set of authorities enacted norm \( \Theta \) iff a subset of that set of authorities enacted that norm. Proposition 5.4.3.4 states that if a set of authorities did not enact norm \( \Theta \), then no subset of that set enacted that norm. Proposition 5.4.3.5 is equivalent to

\[
A : \Theta \rightarrow A \cup B : \Theta.
\]

In general, this expresses that if a set of authorities enacted a norm, then all supersets of that set of authorities enacted that norm.

The major point of the theory discussed in this section is that we can consistently express conflicting duties with norms expressing Ought-to-be statements in system \( D^*_A \). Thus, we cannot say that a norm, say \( O_X(p) \), is valid, in the sense of being applicable (see chapter 6), if there is a set \( A \) of authorities that enacted that norm, i.e., \( A : O_X(p) \). This is only possible in a normative consistent system. In the next chapter we will see how \( O_X(p) \) can be constructed on the basis of \( A : O_X(p) \) by using a hierarchy on authorities. Before we do that, we show how \( PD_eL(\text{Evt'}) \) can be extended to sets of authorities in the same way as \( SDL_X \).

### 5.5 Authorities in \( PD_eL(\text{Evt'}) \)

In this section, we extend system \( PD_eL(\text{Evt'}) \) to system \( PD_eL^A(\text{Evt'}) \) with sets of authorities. This extension corresponds with the extension of system \( SDL_X \) to system \( D^*_A \). The extension to \( PD_eL^A(\text{Evt'}) \) mainly consists of changing the language \( \text{Ass}' \) of \( PD_eL(\text{Evt'}) \) into the language \( \text{Ass}^A \). The language \( \text{Ass}^A \) of \( PD_eL^A(\text{Evt'}) \) consists of assertions concerning the enactment of norms by sets of authorities.

**Definition 5.5.1** The set of assertions in the language \( \text{Ass}^A \) can be determined by the following BNF for its elements (\( \Psi \)):

\[
\Psi ::= A : \Theta | \neg \Psi | \Psi_1 \lor \Psi_2 | \Psi_1 \land \Psi_2 | \Psi_1 \rightarrow \Psi_2,
\]

with \( A \in \mathcal{P}^+(NA) \), \( \Psi, \Psi_1, \Psi_2 \in \text{Ass}^A \), \( \Theta \in \text{Ass}' \).
5.5 Authorities in \(PD_e L(Evt')\)

\[A: \Theta\] states that set \(A\) of normative authorities enacted norm \(\Theta\). The formula \(A: \Theta\) is a norm formulation in the terminology of Ruiter (1989).

Now we can give the formal system \(PD_e L^A(Evt')\) for assertions in \(Ass^A\), which is sound with respect to the semantics given: \(^5\)

**Axiom 5.5.2**

1. all tautologies of the propositional calculus;
2. \(A: \Theta_1 \land A: \Theta_2 \rightarrow A: (\Theta_1 \land \Theta_2)\).

**Rules 5.5.3**

\((MP)\)

\[
\frac{\Psi_1 \rightarrow \Psi_2, \Psi_1}{\Psi_2}
\]

\((R1)^6\)

\[
A: \Theta_1 \rightarrow A: \Theta_2, \text{ with } \Theta_1, \Theta_2 \in Ass'
\]

\((R2)\)

\[
A \in P^+(B), A: \Theta \rightarrow B: \Theta', \text{ with } \Theta \in Ass'
\]

For the semantical interpretation of the assertions in \(Ass^A\) we use the following model structure \(M = (W, A, R_{NA}, P^+(NA), P^+(I), [\alpha]_R, \pi)\), with \(R_{NA}\), the set of functions \(\{R_{A_1}, R_{A_2}, \ldots\}\) for each set of authorities \(A_1, A_2, \ldots \in P^+(NA)\). The function \(R^A \in R_{NA}\) on \(W\), which returns the ideal worlds for set \(A\) of authorities given a world: \(R^A : W \rightarrow 2^W\). This corresponds with the model structure of the system \(D^*_A\).

The statement '\(A\) enacted \(\Theta\) (\(A: \Theta\)) describes some idealised world for \(A\) and not the actual world, since the norm \(\Theta\) can be overruled. In other words, a norm that was enacted by a set \(A\) of authorities in the actual world is valid in an ideal world for \(A\).

The truth condition for \(A: \Theta\), with \(\Theta \in Ass'\) is defined as follows:

\[M, w \models A: \Theta \text{ iff } R^A(w) \subseteq [\Theta], \quad (5.23)\]

and \(R^A(w) \subseteq [\Theta]\) is defined as follows:

- if \(\Theta \equiv [\alpha]V\), then

\[R^A(w) \subseteq [\Theta] \text{ iff } \forall w' \in R^A(w) ([\alpha]_R(w') \subseteq [V]);\]
• if $\Theta \equiv \neg[\alpha]V$, then

$$R^A(w) \subseteq \Theta \text{ iff } \forall_{\omega' \in R^A(w)}([\alpha]_{R}(w') \cap [V] \neq \emptyset)$$

• if $\Theta \equiv \Theta_1 \land \Theta_2$, then

$$R^A(w) \subseteq \Theta \text{ iff } R^A(w) \subseteq [\Theta_1] \text{ and } R^A(w) \subseteq [\Theta_2],$$

with $\Theta_1, \Theta_2 \in \text{Ass'}$;

• if $\Theta \equiv \Theta_1 \lor \Theta_2$, then

$$R^A(w) \subseteq \Theta \text{ iff } R^A(w) \subseteq [\Theta_1 \cup \Theta_2],$$

with $\Theta_1, \Theta_2 \in \text{Ass'}$;

• if $\Theta \equiv \neg \Theta_1$, then

$$R^A(w) \subseteq \Theta \text{ iff } R^A(w) \subseteq [\neg \Theta_1] \text{ iff } R^A(w) \cap [\Theta_1] = \emptyset,$$

with $\Theta_1 \in \text{Ass'}$.

The constraint

$$R^A(w) \subseteq R^B(w) \text{ for all } w \in W, B \subseteq A \text{ and } A, B \in \mathcal{P}^+(NA) \tag{5.24}$$

validates rule (R2). From constraint (5.24), it follows that

$$R^A(w) = \cap_{B \in \mathcal{P}^+(A)} R^B(w).$$

Some remarks

1. The axioms of $PD_\alpha L^A(Evt')$ correspond with the axioms of $D_A^\ast$. Rules (R1) and (R2) correspond with rules $R2$ and $R3$ of $D_A^\ast$.

2. Since we do not have the constraint

$$\forall_{A \in \mathcal{P}^+(NA)}(R^A(w) \neq \emptyset) \text{ for all } w \in W \tag{5.25}$$

principle

$$A : \Theta \rightarrow \neg(B : \neg \Theta) \tag{5.26}$$

is not valid, which is equivalent to

$$\neg(A : \Theta \land B : \neg \Theta). \tag{5.27}$$
5.5 Authorities in $PD_e L(A(Evt'))$

This principle would remove the possibility of expressing conflicting norms enacted by sets of authorities. Since this principle is not valid, we can consistently express conflicting norms enacted by the same set of authorities or by different sets of authorities. For example,

$$A : O(X : \beta) \land A : \neg O(X : \beta),$$

expresses that $A$ enacted that $X$ is obliged to do $\beta$, as well as that $X$ is not obliged to do $\beta$.

Now we give some properties of system $PD_e L^A(Evt')$. Note that the properties concerning the relation between sets of authorities correspond with the properties of $D_\Lambda^A$.

**Proposition 5.5.4**

1. $A : (\Theta_1 \land \Theta_2) \rightarrow (A : \Theta_1 \land A : \Theta_2)$;
2. $(A : \Theta_1 \lor A : \Theta_2) \rightarrow A : (\Theta_1 \lor \Theta_2)$;
3. $A : \Theta \equiv \exists_{B \in P^+(A)} B : \Theta$;
4. \(\neg (A \cup B : \Theta) \rightarrow \neg (A : \Theta \lor (B : \Theta));
5. \(\neg (A : \Theta \land \neg (A \cup B : \Theta))
6. $A : O(X : \beta) \rightarrow A : O(X \cup Y : \beta)$;
7. $A : O(X : \beta) \land A : O(Y : \beta) \rightarrow A : O(X \cup Y : \beta)$;
8. $A : O(X : \beta_1) \lor A : O(X : \beta_2) \rightarrow A : O(X : \beta_1 \cup \beta_2)$;
9. $A : P(X : \beta_1 \& \beta_2) \rightarrow A : P(X : \beta_1) \land A : P(X : \beta)$;
10. $A : O(\alpha_1) \land B : O(\alpha_2) \rightarrow (A \cup B) : O(\alpha_1 \land \alpha_2)$.

**Proof.** The proofs of propositions 5.5.4.1 up to and including 5.5.4.9 are analogous to the proofs of propositions 5.4.3.1 up to and including 5.4.3.9, respectively. Proposition 5.5.4.10 can be proven as follows. Suppose that $A : O(\alpha_1) \land B : O(\alpha_2)$, then $A \cup B : O(\alpha_1) \land A \cup B : O(\alpha_2)$ follows by $R2$. Hence, by axiom 5.5.2.2 it follows that $A \cup B : O(\alpha_1) \land O(\alpha_2)$, which is equivalent to $A \cup B : O(\alpha_1 \land \alpha_2)$, because $O(\alpha_1) \land O(\alpha_2) \equiv_{(PD_e L(A(Evt')))} O(\alpha_1 \land \alpha_2)$.

With systems $D_\Lambda^A$ and $PD_e L^A(Evt)$ we can formalise that individual authorities or sets of authorities have enacted a certain norm. Furthermore, we are able to formalise consistently that (sets of) authorities enacted conflicting norms.

With the help of the norms explicitly enacted by the individual authorities, we can define the term 'normative system', which we discuss in the following section, and will be used in the next chapter.
5.6 Normative systems

A normative system is defined as all the norms enacted by competent normative authorities, and all the logical consequences of these norms. The set of competent normative authorities will be denoted by $NA$.

**Definition 5.6.1** Let $N$ be a set of norms. Then, we define $Cn(N)$ as the set of norms that includes $N$ and all its consequences. Set $N$ is called the basis of system $Cn(N)$.

Thus, $Cn(N)$ is the transitive closure of $N$ with respect to the derivation ($\vdash$).

**Definition 5.6.2** Let $a$ be a normative authority ($a \in NA$). Then, $N_a$ is defined as the set of norms explicitly enacted by authority $a$. $Cn(N_a)$ is called the individual normative system enacted by authority $a$.

As a matter of fact, set $N_a$ will always be finite, because it originates in a finite number of legislative acts. The individual normative system as a whole, i.e., the set $Cn(N_a)$, is infinite, however. Note that $Cn(N_a) \neq Cn(Cn(N_a))$.

If we only consider single authorities, it is obvious that we obtain a normative system consisting of the rules of the individual normative systems. Suppose that $A = \{a_1, a_2\}$, and that $a_1$ enacted $O(\alpha_1)$ and $a_2$ $O(\alpha_2)$. So, $O(\alpha_1)$ and $O(\alpha_2)$ are elements in the system obtained: $Cn(N_{a_1}) \cup Cn(N_{a_2})$. However, $O(\alpha_1 \& \alpha_2)$ is not an element, because $N_{a_1} \not\vdash O(\alpha_1 \& \alpha_2)$ and $N_{a_2} \not\vdash O(\alpha_1 \& \alpha_2)$. We can remedy this situation by considering sets of authorities. Let $N_A$ be a set of rules explicitly enacted by set $A$ of authorities: $N_A = N_{a_1} \cup N_{a_2}$. Thus, we get system $Cn(N_{a_1} \cup N_{a_2})$. Now we can not only derive the rules from the individual normative systems, but also all the rules that are logical consequences of $N_A$, because $Cn(N_{a_1}) \cup Cn(N_{a_2}) \subseteq Cn(N_{a_1} \cup N_{a_2})$.

**Definition 5.6.3** Let $A \in \mathcal{P}^+(NA)$ and $A = \{a_1, \ldots, a_n\}$. Then we define $Cn(N_A)$ as the normative system induced by the set of norms enacted by the set $A$ of normative authorities. $N_A$ is the set of all the norms explicitly enacted by the set $A$ of authorities:

$$N_A = N_{a_1} \cup \ldots \cup N_{a_n}.$$ 

It is often the case that two authorities enacted two contradictory norms. The consequence is that the normative system - with these two rules - loses its meaning in a logical sense, since in case of inconsistency everything can be deduced. In the next chapter, we will introduce the term 'authority hierarchy' to overcome this consequence, by ordering the norms at the basis of the hierarchical relations between the authorities, so that we can determine, for example, which of the two conflicting norms actually holds.
5.7 Conclusions

In this chapter, we extended $SDL_X$ and $PD_{e,L}(Evt')$ with sets of authorities, which enables us to express who enacted a certain norm. Furthermore, it enables us to treat several classical issues: hierarchical norms (see chapter 6), completeness of a legal system (universality, see chapter 6) and normative inconsistencies (i.e., conflicting norms enacted by sets of authorities).

First, we discussed system $SDLA$, based upon Bailhache's theory. This theory was developed to obtain a deontic coherent system. Inherent to this theory is that we cannot express conflicts between enacted norms. A major innovation of this theory is the use of sets of authorities. Sets of authorities are needed to determine the consequences of obligations, enacted by a combination of individual authorities.

However, we have seen that this theory has some serious drawbacks. We cannot express that

1. a set of authorities enacts a permission. Only obligations can be enacted by the sets of authorities. Conversely, we cannot express that a set of authorities does not enact an obligation, only permissions;

2. a set of authorities enacted a combination of norms, for example, $O_X(p) \lor O_X(q)$. In this approach, sets of authorities can only enact atomic collective obligations ($O_X(p)$).

To deal with these drawbacks we have chosen for another approach: instead of the set $A$ of authorities modifying a norm $\Theta$, the set $A$ is treated as a modal operator. With the help of this (seemingly small) change, the drawbacks disappear in system $D^*_A$ (extension of $SDL_X$) and system $PD_{e,L}^A(Evt')$ (extension of $PD_{e,L}(Evt')$).

Furthermore, in both systems we can consistently formalise normative inconsistencies, i.e., conflicting norms enacted by sets of authorities. Consider the following example (see subsection 1.3.2): car driver $i_1$ is on a major road and approaches a junction, where car driver $i_2$ approaches from the right. According to art. 15 of the Dutch Traffic Regulation 1990, $i_1$ has to give way to $i_2$, who approaches the junction from the right. Also, according to the right-of-way signs (A6 and A9), $i_2$ has to give way to $i_1$. On the ground of the principle of trust, $i_2$ does not have to give right of way according to art. 15, and on the ground of the traffic signs, $i_2$ has to give right of way. In $PD_{e,L}(Evt')$,\(^7\) we cannot consistently express this, since this is formalised as follows:

$$\neg O(\{i_2\} : \beta) \land O(\{i_2\} : \beta),$$

\(^7\)Note that we use system $PD_{e,L}(Evt')$ instead of $SDL_X$, since the obligations considered are Ought-to-do statements.
where $\beta$ stands for the action 'to give right of way'. This formula is a contradiction. However, in $PD_e L^A(Evt')$ we can express this as follows:

$$A : \neg O({i_2} : \beta) \land A : O({i_2} : \beta),$$

where $A$ is the set of authorities that enacted the articles in the Dutch Traffic Regulation 1990. This formula expresses, in a consistent way, that the authorities enacted two conflicting norms. Which norm should be followed will be a topic of discussion in the next two chapters.

The axioms, rules and propositions from $D_A^E$ and $PD_e L^A(Evt')$ correspond, since the addition of sets of authorities is independent of the kind of norms (Ought-to-be statements or Ought-to-do statements) they enact. Now that we can express who enacted a norm, we can define a hierarchy of norms - to deal with normative inconsistencies - on the basis of the competence of the (sets of) authorities that enacted the norms. This will be done in the next chapter.
Chapter 6

Deontic inconsistency and universality

When we consider the norms in a legal code, we often find inconsistencies between them. A major cause for these inconsistencies is the fact that norms are enacted by different competent authorities in a legal system. In this chapter, we describe how these inconsistencies can be handled with the help of an authority hierarchy.

Furthermore, we discuss two basic types of legislative acts of the normative authorities: promulgation and derogation, with respect to the authority hierarchy. Finally, we discuss the concept of universality for legal norms.

6.1 Introduction

Law changes - the 'dynamic' character of law - as new norms are incorporated by competent normative authorities, and existing norms are repealed and so removed from the legal order. It is our task here to identify of all the norms that were enacted and the validity of these norms at a certain moment. The norms which are used to express and illustrate the issues discussed in this chapter, are norm expressions of $PDACL(Evt')$.\(^1\)

The term 'validity', as it is used in legal discourse, is ambiguous. It is possible to distinguish several meanings in which a legal norm can be said to be valid. In this chapter, we are concerned with two such meanings (two concepts of validity): membership and applicability. Both of them play a central role in law and in legal theories (cf. Bulygin, 1982).

- A norm can be said to be valid in the sense that it belongs to or is a member of a legal system. 'Membership' is a descriptive concept, because the sentence 'O(α) is valid'

\(^1\)We might also have used norm expressions of $DACL_A$; however, we have chosen for norm expressions of $PDACL(Evt')$ for no particular reason.
Deontic inconsistency and universality

is a descriptive proposition, not a norm. There are various criteria for membership, but we restrict ourselves to the criterion of competence of the authority that has created the norm: a norm is valid if it has been issued (enacted, promulgated) by a competent authority.

- A norm is often also said to be valid in the sense that it is obligatory or has a 'binding force'. 'Applicability' is also a descriptive concept, for to say that a norm is valid in this sense is not to give a prescription, but to state that there is a prescription according to which the norm must be applied. Here the sentence 'O(α) is valid' is again a proposition, though referring to a norm.\

In this chapter, we investigate the relation between these two concepts on the basis of the hierarchical structure of the legal system and two types of legislative acts: promulgation and derogation. Law changes by promulgation and derogation: norms are incorporated by promulgation and norms are repealed by derogation.

When we consider the norms in a legal system, we can often discern some kind of hierarchy among the norms: some are regarded as more basic than others.

It may be determined in part by considerations arising from the text of the regulations themselves, such as the existence of cross-references from one to another; and it may also be determined in part by factors of a more extrinsic kind, such as the powers and competences of the issuing bodies, dates of promulgation and amendment, and the degree of specificity or generality of the regulations made. (Alchourrón and Makinson, 1981, p. 125)

With the help of such a hierarchy of norms, we can overcome some logical imperfections in the legal code, especially conflicting norms. In this chapter, we define the hierarchy of norms on the basis of the competences of the normative authorities that enact the norms, following an authority hierarchy.

If authority a has enacted norm O(α) and authority b norm O(β), we say that the two norms are conflicting, since event α is obligatory and at the same time forbidden. This is the 'classical' notion of normative inconsistency. Two authorities promulgating (enacting) two contradictory or conflicting norms is an extremely frequent phenomenon, at least in certain areas like law (see Alchourrón and Bulygin, 1981). The conflict arises when the norms become members of the same normative system, not if they belong to different systems. Such a system loses its meaning in a logical sense in the case of inconsistency: everything

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2A norm has a 'binding force' may also mean that there is a prescription to obey and apply the norm. Then we have a normative concept of validity: 'a norm is valid' is to describe that it should be obeyed and applied; so, in this sense, 'O(α) is valid' is not a proposition, but a prescription, i.e., a norm.
can be deduced and, in particular, all obligations, permissions, etc., are deducible (*ex falso sequitur quodlibet*).

In this chapter, we develop a theory to overcome some normative inconsistencies among the norms promulgated (enacted) by the authorities with respect to their hierarchical order, and to determine which norms, members of a given legal system, are applicable; i.e., which norms must be obeyed and applied.

Another issue we discuss is the possibility of a universal legal system. A universal system is a normative-consistent system, such that all events are prescribed, prohibited or permitted in that system.

The organisation of this chapter is as follows. In section 6.2, we determine the hierarchical structure of a legal system on the basis of sets of normative authorities to overcome some normative conflicts between enacted norms. Section 6.3 discusses two basic types of legislative acts of the normative authorities: promulgation and derogation, with respect to the authority hierarchy. Section 6.4 presents (postulated) universality. In the last section we draw some conclusions.

### 6.2 Authority hierarchy

The sources of law give rise not only to norms, but also to hierarchical criteria that determine the relative importance of various norms that form part of a legal order. The identification of the material of which a legal system is composed must include a series of relations determining the relative weight of norms. The clearest example is the criterion based on the level of the authority that enacted the norm: for example, a constitutional rule is hierarchically superior to a rule enacted by the ordinary legislature, and the latter, in turn, is superior to a rule enacted by a city council. This criterion is called the ‘Lex Superior’. The ‘Lex Superior’ principle is based on the general hierarchy of a legal system: normative authority is divided along the lines of the hierarchical structure of the normative system; authorities with a lower rank of authority have to respect what was enacted by an authority with a higher rank.

There are more criteria or rules of preference to resolve normative inconsistencies, such as ‘Lex Specialis’ and ‘Lex Posterior’. So, the law is not merely a set of norms, but a hierarchical system. Each of these principles determines different hierarchical relations between norms and their role is to give a solution in those situations in which one has to choose from among different norms. For convenience, we will restrict ourselves to the ‘Lex Superior’ principle, but the theory can easily be extended with other rules of preference.

In this section, we develop a deontic system with normative agreement between the
sets of authorities in the same normative system with respect to their hierarchical order, to
overcome some normative inconsistencies between the norms enacted by sets of authorities.
The authorities will be ordered in a hierarchical system, which helps to determine which
norm should be followed in cases of deontic inconsistencies. The authorities are thus used
to prioritise the norms they enacted.\footnote{This gives a priority logic in the tradition of Brewka (1991). See also Prakken (1993).} The hierarchy of authorities does not resolve all the
inconsistencies; norms with the same priority may still be inconsistent. In such a case, the
judge or lawyer may resolve these inconsistencies with the help of interpretative strategies,
e.g., the grammatical and the teleological strategies. The theory of authority hierarchy
does not solve the latter problem, but indicates and locates this problem.

First, we will introduce a partial ordering of the authorities:

**Definition 6.2.1** Let \( a, b \in NA \). We define

- \( a \sim b \) iff \( a \) and \( b \) have the same rank of authority;
- \( a \succ b \) iff \( a \) has a higher rank of authority than \( b \);
- \( a \succeq b \) iff \( a \sim b \lor a \succ b \).

**Definition 6.2.2** Let \( A_i, A_j \subseteq NA \). We define

- \( A_i \succ A_j \) iff \( \forall a \in A_i, \forall b \in A_j (a > b) \);
- \( A_i \sim A_j \) iff \( \forall a \in A_i, \forall b \in A_j (a \sim b) \);
- \( A_i \succeq A_j \) iff \( \forall a \in A_i, \exists b \in A_j (a \succ b \lor a \sim b) \).

**Proposition 6.2.3** \( \succeq \) over \( 2^{NA} \) is transitive and reflexive, but in general not symmetric.
\( \succ \) over \( 2^{NA} \) is transitive and asymmetric, and thus a strict partial ordering of \( 2^{NA} \).

**Definition 6.2.4** Let \( A \subseteq NA \). Then, \( NA \) is defined as the set of norms enacted by the
authorities in set \( A \) (cf. definition 5.6.3).

**Definition 6.2.5** Let \( A_1, A_2, \ldots, A_n \subseteq NA \). Then, \( D(A_1, A_2, \ldots, A_n) \) is an authority
hierarchy iff

\[
\forall i < j (A_i > A_j) \land \forall i, \forall a, b \in A_i (a \sim b).
\]

In \( D(A_1, \ldots, A_n) \), \( A_1 \) is the set of authorithies with the highest ranking authority and \( A_n \) the
set with the lowest ranking authority. The ordering of authorities in an authority hierarchy
is a total ordering: for every two authorities \( a \) and \( b \) in \( \bigcup_{i=1}^{n} A_i \) it follows exclusively that
\( a \succ b, b \succ a \) or \( a \sim b \).
Example 6.2.6 Let \( a, b, c, d, e \in NA \) and \( a \rightarrow b, b \rightarrow c, c \rightarrow d \) and \( d \rightarrow e \). Then, \( D(\{a, b\}, \{c, d\}, \{e\}) \) is an authority hierarchy.

Definition 6.2.7 Let \( D(A_1, A_2, \ldots, A_n) \) be an authority hierarchy, \( A \subseteq \bigcup_{i=1}^{n} A_i \), and let \( k = \max\{i \mid A \cap A_i \neq \emptyset\} \). Then we define \( S(N_A) \) as the set of norms enacted by all the authorities superior or equal to the authorities of \( A \):

\[
S(N_A) = \bigcup_{i=1}^{k} N_{A_i}.
\]

Note that \( \forall_{A' \subseteq A} S(N_A) = S(N_{A'}) \).

Example 6.2.8 Consider example 6.2.6, and suppose that \( a \) enacted \( O(\alpha_1) \), \( b \) enacted \( P(\alpha_2) \) and \( O(\alpha_3) \), \( c \) enacted \( O(\alpha_4) \), \( d \) enacted \( P(\alpha_5) \) and \( \neg O(\alpha_3) \) and \( e \) enacted \( P(\alpha_3) \). Then, we obtain, for example,

1. \( S(N_{\{a\}} = S(N_{\{b\}} = S(N_{\{a,b\}} = \{O(\alpha_1), P(\alpha_2), O(\alpha_3)\};\)

2. \( S(N_{\{c\}} = S(N_{\{d\}} = S(N_{\{c,d\}} = S(N_{\{a,c\}} = S(N_{\{a,d\}} = S(N_{\{b,c\}} = S(N_{\{b,d\}} = S(N_{\{a,b,c,d\}} = \{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4), P(\alpha_5), \neg O(\alpha_3)\};\)

3. \( S(N_{\{e\}} = S(N_{\{a,b,c,d,e\}} = \{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4), P(\alpha_5), \neg O(\alpha_3), P(\alpha_3)\};\)

4. \( S(N_{\{a\}} \subseteq S(N_{\{c\}} \subseteq S(N_{\{e\}}).\)

To overcome normative inconsistencies among the enacted norms in a normative system, we have to remove at least one of the norms. This can also be achieved by more generous subtractions, such as removing all the norms. However, this is absurd. What is wanted is to remove a minimum of norms, on the basis of the authority hierarchy, compatible with the requirement that the resulting set of norms is normative-consistent. Before we define this formally, we give a definition of normative-consistency (\(N\)-consistency).

Definition 6.2.9 Let \( N \) be a set of norms. Then,

\[
N \text{ is } N\text{-consistent iff } \exists_{\alpha \in Evl}((N \vdash O(\alpha)) \land (N \vdash O(\overline{\alpha}))).
\]
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Note that if \( N \) is \( N \)-consistent, then \( N \) is also consistent, since if \( N \) is inconsistent, everything can be deduced; thus, also \( O(\alpha) \) and \( O(\overline{\alpha}) \). The converse does not hold. Suppose that \( N = \{O(\alpha), O(\overline{\alpha})\} \), then \( N \) is consistent, but not \( N \)-consistent.\(^6\)

By means of \( N \)-consistency, we can define a maximal \( N \)-consistent system for a given authority hierarchy as follows:

**Definition 6.2.10** Let \( D(A_1, A_2, \ldots, A_n) \) be an authority hierarchy and \( A \subseteq \bigcup_{i=1}^n A_i \). Then, the maximal \( N \)-consistent system \( Sys(D(A_1, A_2, \ldots, A_n)) \) is defined as \( S(N_A) \), such that

- \( S(N_A) \) is \( N \)-consistent;
- \( \exists A' \subseteq \bigcup_{i=1}^n A_i \) \( (A \supset A' \land S(N_{A'}) \) is \( N \)-consistent).

System \( Sys(D(A_1, A_2, \ldots, A_n)) \) is based upon the individual normative systems of the authorities in the authority hierarchy, with respect to their ranks of authority. However, this definition does not give acceptable results, because if two 'higher' norms conflict, no norms enacted by lower ranked authorities can be determined.\(^7\)

**Example 6.2.11** Suppose that \( a \) enacted \( O(X : \beta_1) \) and \( O(Y : \beta_2) \), that \( b \) enacted \( O(X : \beta_1) \) and \( O(Y : \beta_2) \), and that \( c \) enacted \( P(Z : \beta_3) \). Then,

1. \( Sys(D(\{a, b\}, \{c\})) = \emptyset \);
2. \( Sys(D(\{a\}, \{b\}, \{c\})) = \{O(X : \beta_1), O(Y : \beta_2)\}; \)
3. \( Sys(D(\{a, c\}, \{b\})) = Sys(D(\{a\}, \{c\}, \{b\})) = \{O(X : \beta_1), O(Y : \beta_2), P(Z : \beta_3)\}. \)

We can solve this problem by removing only the norms that are responsible for the normative conflicts. Therefore, we change the definition of maximal \( N \)-consistent system. A set is maximal \( N \)-consistent with respect to system \( S(N_A) \) by removing only the norms in \( S(N_A) \) which are responsible for the normative conflicts, depending on the authority hierarchy. If two norms, enacted by authorities with different ranks of authority, are conflicting, then

\(^6\)These terms, i.e., \( N \)-consistency and consistency, are equivalent if we add schema \( \neg [\alpha \cup' \overline{\alpha}] V \) to system \( PD_e L(Evt') \) by introducing the following constraint on the semantics:

\[ \exists w' \in [\alpha' \cup' \overline{\alpha}]_n(w) w' \neq V \text{ for all } w \in W. \]

Note that this schema corresponds with schema \((OD)\) of SDL.

Suppose \( N \) is \( N \)-inconsistent. Then there is an event, say \( \alpha \) in \( Evt' \), such that \( N \models O(\alpha) \) and \( N \models O(\overline{\alpha}) \). From the schema \( \neg [\alpha \cup' \overline{\alpha}] V \) we can derive principle \( O(\alpha) \rightarrow \neg O(\overline{\alpha}) \). Hence, it follows that \( N \models O(\overline{\alpha}) \) and \( N \models \neg O(\overline{\alpha}) \). Thus, \( N \) is inconsistent if \( N \) is \( N \)-inconsistent. Hence, \( N \) is consistent if and only if \( N \) is \( N \)-consistent.

\(^7\)In the study by Alchourrón and Makinson (1981) the same problem arose. Alchourrón and Makinson suggested that an additional notion of relevance is necessary, but they gave no formal definition.
the norm enacted by the authority with the lower rank will be removed, and if two norms enacted by authorities with the same rank are conflicting, then we have to choose which norm will be removed.

**Definition 6.2.12** Let \( D(A_1, A_2, \ldots, A_n) \) be an authority hierarchy and \( A \subseteq \bigcup_{i=1}^{n} A_i \), with the lowest ranking authority coming from \( A_k \). Then, \( D \) is a maximal \( N \)-consistent set of \( S(N_A) \) iff

\[
\begin{align*}
1. & \ D \subseteq S(N_A); \\
2. & \ D \text{ is } N\text{-consistent}; \\
3. & \ \neg \exists r \in S(N_A) \setminus D \left( \{r\} \cup D \text{ is } N\text{-consistent} \right); \\
4. & \ \exists D' \in MC(S(N_{A_{k-1}})) (D' \subseteq D).
\end{align*}
\]

\( MC(S(N_A)) \) is defined as the set of all the maximal \( N \)-consistent sets of \( S(N_A) \).

The first three conditions are obvious and are needed to express that \( D \) is \( N \)-consistent and maximal with respect to \( S(N_A) \). Without the fourth condition, the maximal \( N \)-consistent sets of \( S(N_A) \) would be independent of the given authority hierarchy.

**Example 6.2.13** Let \( D(\{a\}, \{b\}) \) be an authority hierarchy, \( N_{\{a\}} = \{O(\alpha_1), F(\alpha_2)\} \) and \( N_{\{b\}} = \{P(\alpha_2), P(\alpha_3)\} \). Then,

\[
\{O(\alpha_1), F(\alpha_2), P(\alpha_3)\}
\]

is the only maximal \( N \)-consistent set of \( S(N_{\{a,b\}}) \). The set \( \{O(\alpha_1), P(\alpha_2), P(\alpha_3)\} \) is not an element of \( MC(S(N_{\{a,b\}})) \), since it does not satisfy the fourth condition of definition 6.2.12: \( \{O(\alpha_1), F(\alpha_2)\} \) is the only maximal \( N \)-consistent set of \( S(N_{\{a\}}) \), but is not a subset of set \( \{O(\alpha_1), P(\alpha_2), P(\alpha_3)\} \).

**Corollary 6.2.14** Let \( D(A_1, A_2, \ldots, A_n) \) be an authority hierarchy and \( A \subseteq \bigcup_{i=1}^{n} A_i \). Then,

1. \( S(N_A) \) is the only maximal \( N \)-consistent set of \( S(N_A) \) iff \( S(N_A) \) is \( N \)-consistent;
2. there are at least two maximal, consistent sets of \( S(N_A) \) iff \( S(N_A) \) is \( N \)-inconsistent.

**Example 6.2.15** Let \( D(\{a, b\}) \) be an authority hierarchy, \( N_{\{a\}} = \{O(\alpha_1), F(\alpha_2)\} \) and \( N_{\{b\}} = \{P(\alpha_2), P(\alpha_3)\} \). Then, we have two maximal \( N \)-consistent sets of \( N_{\{a,b\}} \):

\[
MC(S(N_{\{a,b\}})) = \{\{O(\alpha_1), F(\alpha_2), P(\alpha_3)\}, \{O(\alpha_1), P(\alpha_2), P(\alpha_3)\}\}.
\]
There is no logical criterion according to which we can decide which of these two sets should be used. This we call logical indeterminacy (cf. Navarro and Redondo, 1990). The decision is a matter of interpretation by the judge or lawyer.

In the introduction we already mentioned that law changes by promulgation and derogation. In the following section, we analyse the consequences of these two legislative acts of the authorities for the validity of the norms, in the sense of membership and applicability, on the basis of the theory developed in this section.

6.3 The promulgation and derogation of norms

Two basic types of legislative acts can be distinguished: promulgation and derogation. Frequently, promulgation has been referred to as the introduction of a norm into a legal system, and derogation as the removal of a norm from a legal system. We will see that there is more to it than that by investigating what the consequences are for a legal system when an authority enacts or derogates a norm.

According to Alchourrón (1982), an important feature of promulgation is that the resulting system is perfectly identified and that this process differs from derogation since the result of derogation is not always determinate: there is not always one, unique system that can be identified as the result of a process of derogation. We will see that the result of promulgation is not always determinate either, in contrast to Alchourrón’s statement.

Before we discuss the two legislative acts, we make some assumptions to be used in the sequel. We assume in this section that $N$ is a maximal $N$-consistent set of $S(NA)$ with respect to the given authority hierarchy $D(A_1, A_2, \ldots, A_n)$. Further, $Cn(N)$ is the set of norms that embraces $N$ and all its consequences (cf. section 5.6).

6.3.1 Promulgation

Legal norms take effect by the promulgation of laws. In the Netherlands, this is done by publication in the Bulletin of Acts and Decrees. At the same time, the date of the law’s taking effect is determined. Businesses and institutions have their own procedures for the promulgation of current standards. Ethical norms (for example, one should not kill a human being) are not promulgated as such. By way of political discussions, such norms will be reflected in the legislations of most states. Matters concerning the principles and developments of ethical norms are discussed in the philosophical literature.

In the promulgation of a norm $r$ to an $N$-consistent system $Cn(N)$, two cases can be distinguished:

1. system $Cn(N \cup \{r\})$ is $N$-consistent;
2. system $Cn(N \cup \{r\})$ is $N$-inconsistent.
6.3 The promulgation and derogation of norms

System \( Cn(N \cup \{r\}) \) is \( N \)-consistent

In the case of \( N \)-consistency, two subcases can be distinguished:

1. \( Cn(\{r\}) \subseteq Cn(N) \): this we call \( N \)-consistent redundant promulgation. The promulgated norm was already deducible and is, therefore, redundant. In other words, norm \( r \) was already applicable in system \( Cn(N) \). Norm \( r \) becomes a new member of the system if \( r \) has not been enacted explicitly by the authorities before the promulgation. This may be important. Suppose \( r \) was not a member of the system, but deduced from norm \( p \). If \( p \) is derogated, then \( r \) is no longer deduced. Thus, it matters greatly that \( r \) becomes a member of the system. Anyway, \( r \) becomes a member of the individual normative system of the authority that promulgated norm \( r \). Further, it can restrict the competence of inferior authorities to derogate norm \( r \).

2. \( Cn(\{r\}) \not\subseteq Cn(N) \): this we call \( N \)-consistent material promulgation. The system changes because the number of deducible norms increases. In this resulting system, the new norms are not only the ones pertaining to this norm and its consequences, but also those that can be derived from the promulgated norm and the rest of the already existing norms. From this viewpoint of a formal reconstruction, this has consequences for the sum of the two following sets:

- the existing (explicitly enacted) norms \( N \) and
- the explicitly promulgated norm \( r \).

Thus, the act of promulgation of \( r \) leads from system \( Cn(N) \) to system \( Cn(N \cup \{r\}) \), and not to system \( Cn(N) \cup Cn(\{r\}) \).\(^8\) In this case, norm \( r \) is a new member of the system and is applicable.

We illustrate such cases of promulgation in the following example (for norm expressions of \( PDeL(Evt') \)).

Example 6.3.1 Let
- \( N_{\{a_1\}} = \{O(\alpha_1), P(\alpha_2)\} \);
- \( N_{\{a_2\}} = \{O(\alpha_3), O(\alpha_4 \cup' \bar{\alpha_1})\} \);
- \( N_{\{a_3\}} = \{O(\bar{\alpha_2})\} \).

We define the following authority hierarchy: \( D(\{a_1\}, \{a_2\}, \{a_3\}) \), thus \( A_1 = \{a_1\} \), \( A_2 = \{a_2\} \) and \( A_3 = \{a_3\} \). Then there is one maximal \( N \)-consistent set \( N \) for the given authority hierarchy:

\[ N = \{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4 \cup' \bar{\alpha_1})\} \].

\(^8\)Note that \( Cn(N) \cup Cn(\{r\}) \not\subseteq Cn(N \cup \{r\}) \).
Let us consider the following two cases:

1. authority $a_2$ promulgates norm $O(\alpha_1 \&' \alpha_3)$;
2. authority $a_1$ promulgates norm $O(\alpha_5)$.

1. If $a_2$ promulgates $O(\alpha_1 \&' \alpha_3)$, then $Cn(N \cup \{O(\alpha_1 \&' \alpha_3)\}) = Cn(N)$, since $N \models O(\alpha_1 \&' \alpha_3)$. Thus, the promulgation of norm $O(\alpha_1 \&' \alpha_3)$ by authority $a_2$ is a redundant promulgation.

2. If $a_1$ promulgates $O(\alpha_5)$, then $Cn(N \cup \{O(\alpha_5)\})$ is $N$-consistent and $Cn(\{O(\alpha_5)\}) \not\subseteq Cn(N)$. Thus, the promulgation of norm $O(\alpha_5)$ by authority $a_1$ is a material promulgation: norm $O(\alpha_5)$ is a new member and also applicable. Further, it holds that $Cn(\{O(\alpha_5)\}) \cup Cn(N) \subset Cn(N \cup \{O(\alpha_5)\})$, since $O(\alpha_3 \&' \alpha_5)$ is an element of $Cn(N \cup \{O(\alpha_5)\})$ and not of $Cn(N) \cup Cn(\{O(\alpha_5)\})$.

**System $Cn(N \cup \{r\})$ is $N$-inconsistent**

In the case of $N$-inconsistency, two subcases can be distinguished:

1. $\{N\} = MC(S(N_{An}) \cup \{r\})$: this we call $N$-inconsistent redundant promulgation. The negation of the promulgated norm is deducible from a set of norms enacted by a set of authorities with higher ranks of authority than the authority promulgating norm $r$. Norm $r$ becomes a member of the system, but is not applicable.

2. $\{N\} \neq MC(S(N_{Aa}) \cup \{r\})$: this we call $N$-inconsistent material promulgation. In this case, there is no set of authorities with higher ranks of authority enacting the negation of the promulgated norm than the authority promulgating norm $\{r\}$. This case can again be subdivided into two cases:

   - $N \not\in MC(S(N_{Aa}) \cup \{r\})$: the negation of the promulgated norm is only deducible minimally from sets of norms enacted by sets $A$ of authorities with lower ranks of authority than authority $a$ promulgating norm $r$, i.e., $\{a\} \not\mathrel{\supseteq} A$. Norm $r$ becomes a member and is also applicable.
   - $N \in MC(S(N_{Aa}) \cup \{r\})$: the negation of the promulgated norm is deducible minimally from a set of norms enacted by a set $A$ of authorities with an authority with the same rank of authority as the authority promulgating norm $r$. Norm $r$ becomes a new member of the system if the norm was not enacted before the promulgation, and is applicable in some, but not all, maximal $N$-consistent sets in $MC(S(N_{Aa}) \cup \{r\})$. Anyway, norm $r$ becomes a member of the individual normative system of the authority that promulgated norm $r$. 
6.3 The promulgation and derogation of norms

We illustrate this with the following example.

Example 6.3.2 Let
- \(N_{\{a_1\}} = \{O(a_1), P(a_2)\}\);
- \(N_{\{a_2\}} = \{O(a_3), O(\alpha_4 \cup \alpha_2), O(\bar{\alpha}_2)\}\);
- \(N_{\{a_3\}} = \{\neg P(\alpha_3)\}\).

We define the following authority hierarchy: \(D(\{a_1\}, \{a_2\}, \{a_3\})\), thus \(A_1 = \{a_1\}, A_2 = \{a_2\}\) and \(A_3 = \{a_3\}\). Then there is one maximal \(N\)-consistent set \(N\) for the given authority hierarchy:

\[N = \{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4 \cup \alpha_2)\}\].

Let us consider the following three cases:

1. authority \(a_2\) promulgates norm \(\neg P(\alpha_2)\);
2. authority \(a_1\) promulgates norm \(\neg O(\alpha_3)\);
3. authority \(a_2\) promulgates norm \(O(\bar{\alpha}_4)\).

1. If authority \(a_2\) promulgates \(\neg P(\alpha_2)\), then \(\text{Cn}(N \cup \{\neg P(\alpha_2)\})\) is inconsistent, thus also \(N\)-inconsistent. There is a set \(A\) of authorities with \(A \succ \{a_2\}\), namely the set \(\{a_1\}\) that enacted \(P(\alpha_2)\). Thus, the promulgation of norm \(\neg P(\alpha_2)\) by authority \(a_2\) is an \(N\)-inconsistent redundant promulgation. Note that \(\neg P(\alpha_2)\) is now a member of the system, however.

2. If authority \(a_1\) promulgates \(\neg O(\alpha_3)\), then \(\text{Cn}(N \cup \{\neg O(\alpha_3)\})\) is inconsistent. There is no set \(A\) of authorities with \(A \succ \{a_1\}\), such that \(A : O(\alpha_3)\), thus the promulgation is an \(N\)-inconsistent material promulgation. Thus, we have to determine the maximal \(N\)-consistent sets of set \(S(N_{A_3}) \cup \{\neg O(\alpha_3)\}\) for the given authority hierarchy:

\[\text{MC}(S(N_{A_3}) \cup \{\neg O(\alpha_3)\}) = \{\neg O(\alpha_3), O(\alpha_1), P(\alpha_2), O(\alpha_4 \cup \alpha_2), \neg P(\alpha_3)\}\].

The consequences of this promulgation are:

(a) the membership of \(\neg O(\alpha_3)\);
(b) the applicability of \(\neg O(\alpha_3)\);
(c) norm \(O(\alpha_3)\) is not applicable any more;
(d) norm \(\neg P(\alpha_3)\) becomes applicable.

Note that \(N \notin \text{MC}(S(N_{A_3}) \cup \{\neg O(\alpha_3)\})\).
3. If authority $a_2$ promulgates $O(\alpha_4)$, then $Cn(N \cup \{O(\alpha_4)\})$ is inconsistent. There is no set $A$ of authorities with $A \succ \{a_2\}$, such that $A : \neg O(\alpha_4)$, thus the promulgation is an $N$-inconsistent material promulgation. We have to determine the maximal $N$-consistent sets of set $S(N_{A_3}) \cup \{O(\alpha_4)\}$ for the given authority hierarchy:

$$MC(S(N_{A_3}) \cup \{O(\alpha_4)\}) = \{\{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4 \cup \alpha_2)\},$$

$$\{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4)\}\}.$$ 

Norm $O(\alpha_4)$ is now a new member of the system and is applicable in one of the two maximal $N$-consistent sets. Note that $N$ is still a maximal $N$-consistent set, though not the only one.

The promulgation by authority $a_2$ of this last example shows that the result of a promulgation, in contrast to Alchourrón's statement, is not determinate since there is not one unique system that can be identified as the result of a process of promulgation. Note that the result of promulgation of a norm to a system $Cn(N)$ is determinate if system $Cn(N \cup \{r\})$ is $N$-consistent.

### 6.3.2 Derogation

One important characteristic of legal rules is their persistence over time. This means, among other things, that if a norm belongs to a system $S$, then it belongs to all successive systems of $S$, until it is removed from one of them. It is, therefore, possible to ask: when is it true that a norm $r$ has been removed from a system $S$? The following answers can be given:

1. when a competent authority derogates $r$;

2. when $r$ is only a logical consequence of a set $\{r_1, r_2, \ldots, r_n\}$ of norms, and one of these norms has been removed from the system;

3. when norm $r$ has restricted the temporal extension of its validity;

4. according to some theories, whether a norm is a member of a legal system not only depends on the actions of the normative authorities, but also on the acts of the addressees of the norm; in this sense, an ineffective norm does not belong to a legal system:

... the effectiveness of a norm consists in the fact that it is actually observed by and large ... Effectiveness is a condition for validity to the extent that

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a single norm and a whole normative order lose their validity - cease to be valid - if they lose their effectiveness or the possibility of effectiveness. (Kelsen, 1991, p. 139)

We do not consider cases 3 and 4, because our purpose here is to look at legislation as a technique for the introduction and removal of legal rules.

Derogation is a common technique for the removal of legal rules from a legal system. A normative authority may decide that a norm \( r \) has to be removed, and in order to obtain this result, the authority decides on a derogatory disposition \( d \), i.e., a non-normative disposition which has the function of indicating a set of legal dispositions that must leave the system. Thus, the objects of dispositions of type \( d \) can be rules, definitions, etc. In this chapter, we consider only the derogation of norms. The decision on a derogatory disposition can be seen as the result of a specific normative act: the rejection of a norm (cf. Navarro, 1993). Therefore, the derogation process begins with the promulgation of some disposition \( d \). In other words, the process of removing a norm \( r \) begins with introducing a disposition \( d \). The promulgation of \( d \) by some authority \( a \) is important, because it prevents authorities inferior to \( a \) from introducing, for example, the norm being the object of rejection. We formalise the enactment of a derogatory disposition, i.e., the rejection of a norm \( r \), by enacting the negation of that norm (i.e., \( \neg r \)), on the understanding that this last norm does not become a member of the individual normative system of the authority that decided on the derogatory disposition (if it was not a member of that system already). Such a norm we call a disposition object.

We distinguish two cases of derogation of a norm \( r \) of a \( N \)-consistent system \( C_n(N) \):

1. \( r \notin C_n(N) \): this we call derogation in advance. No norm in \( N \) is removed; however, a consequence of this derogation is the restriction of the competence of inferior authorities.

2. \( r \in C_n(N) \): in this case, two cases can be distinguished:

   (a) \( \{N\} = MC(S(N_{A_n}) \cup \{\neg r\}) \): this we call redundant derogation. Norm \( r \) is deducible from a set of norms enacted by a set of authorities with higher ranks of authority than the one enacting disposition object \( \neg r \).

   (b) \( \{N\} \neq MC(S(N_{A_n}) \cup \{\neg r\}) \): this we call material derogation. In this case, there is no set of authorities with higher ranks of authority enacting norm \( r \) than the authority enacting disposition object \( \neg r \). This case can again be subdivided into two cases:

   - \( N \notin MC(S(N_{A_n}) \cup \{\neg r\}) \): Norm \( r \) is only deducible minimally from sets of norms enacted by sets \( A \) of authorities with lower ranks of authority than authority \( a \) enacting disposition object \( \neg r \), i.e., \( \{a\} \succ A \). Note that norm
\( \neg r \) does not become a member of the system if it was not enacted before the enactment of disposition object \( \neg r \), and that norm \( r \) is not a member nor applicable any more.

- \( N \in MC(S(N_{A_n}) \cup \{\neg r\}) \): Norm \( r \) is only deducible minimally from sets of norms enacted by sets \( A \) of authorities with an authority with the same rank of authority as the authority enacting disposition object \( \neg r \). Norm \( r \) will be rejected in some, but not all, maximal \( N \)-consistent sets in \( MC(S(N_{A_n}) \cup \{\neg r\}) \).

In the case of material derogation, the question may be raised what norms should be removed from set \( N \) in order that norm \( r \) is not deducible any more. What is wanted is to remove a minimum of norms in \( N \) compatible with the requirement that norm \( r \) cannot be derived.\(^{10}\) This allows us to formulate the following adequacy conditions for the process of derogation:

1. the derogated norm \( r \) cannot be derived in the resulting systems, and
2. one should not eliminate more norms in \( N \) than is strictly necessary in order to achieve the first condition.

We can accomplish this by determining the maximal \( N \)-consistent sets of \( S(N_{A_n}) \cup \{\neg r\} \). The resulting systems are these maximal \( N \)-consistent sets without norm \( \neg r \), if \( \neg r \notin S(N_{A_n}) \).

Remarks

1. It is possible to obtain more than one resulting system after a material derogation. Such a derogation, which gives rise to more than one resulting system, will be called indeterminate derogation (cf. Alchourrón, 1982).

2. The derogation of a norm \( r \) corresponds to the promulgation of \( \neg r \) with the exception that \( \neg r \) does not become a member of the resulting system if it was not enacted before the enactment of disposition object \( \neg r \).

Example 6.3.3 Let

- \( N_{(a_1)} = \{O(\alpha_1), P(\alpha_2)\} \);
- \( N_{(a_2)} = \{O(\alpha_3), O(\alpha_4 \cup \bar{\alpha_1})\} \);
- \( N_{(a_3)} = \{O(\bar{\alpha_2})\} \);
- \( N_{(a_4)} = \{P(\alpha_5)\} \).

\(^{10}\)This is a process of shrinking or contracting a theory to eliminate a norm. For a formal approach of this process, with the notions of safe contraction and partial meet contraction, we refer to Alchourrón and Makinson (1982) and Alchourrón, Gärdenfors and Makinson (1985).
We define the following authority hierarchy: \( D(\{a_1\}, \{a_2, a_3\}, \{a_4\}) \), thus \( A_1 = \{a_1\}, A_2 = \{a_2, a_3\} \) and \( A_3 = \{a_4\} \). Then there is one maximal \( N \)-consistent set \( N \) for the given authority hierarchy:

\[
N = \{O(\alpha_1), P(\alpha_2), O(\alpha_3), O(\alpha_4 \cup \overline{\alpha_1}), P(\alpha_5)\}.
\]

Let us consider the following four cases:

1. Authority \( a_1 \) enacts disposition object \( \neg O(\alpha_6) \);
2. Authority \( a_4 \) enacts disposition object \( \neg O(\alpha_4) \);
3. Authority \( a_1 \) enacts disposition object \( \neg O(\alpha_4) \);
4. Authority \( a_3 \) enacts disposition object \( \neg O(\alpha_3) \).

1. Authority \( a_1 \) enacts disposition object \( \neg O(\alpha_6) \). However, the negation of this norm, i.e., \( O(\alpha_3) \), is not an element of \( C_n(N) \). Thus, this is a derogation in advance. It restricts the competence of authorities \( a_2, a_3 \) and \( a_4 \). For example, \( a_3 \) can promulgate \( O(\alpha_6) \), but this norm will not belong to the resulting system since the norm has not been promulgated by a 'competent' authority, i.e., an authority with a rank equal to or higher than authority \( a_1 \). Thus, derogation by authority \( a_1 \) restricts the competence of authority \( a_3 \).

2. Authority \( a_4 \) enacts disposition object \( \neg O(\alpha_4) \). Norm \( O(\alpha_4) \in C_n(N) \), since \( \{a_1\} \cap \{a_2\} \cap \{\alpha_4 \cup \overline{\alpha_1}\} = \{a_1, a_2\} : O(\alpha_4) \). This is a redundant derogation, since norm \( O(\alpha_4) \) is enacted by a set of authorities with higher ranks of authority than the one enacting disposition object \( \neg O(\alpha_4) \). Thus, \( O(\alpha_4) \) is still a member of the system and applicable.

3. Authority \( a_1 \) enacts disposition object \( \neg O(\alpha_4) \). Norm \( O(\alpha_4) \in C_n(N) \). This is a material derogation since there is no set of norms enacting by a set of authorities with higher ranks of authority than the one enacting disposition object \( \neg O(\alpha_4) \). Further, it holds that

\[
MC(S(N_{A_1}) \cup \{\neg O(\alpha_4)\}) = \{O(\alpha_1), P(\alpha_2), O(\alpha_3), \neg O(\alpha_4), P(\alpha_5)\}.
\]

Thus, \( N \notin MC(S(N_{A_3}) \cup \{\neg O(\alpha_4)\}) \). Note that \( \neg O(\alpha_4) \) will not belong to the resulting system, i.e., the set

\[
\{O(\alpha_1), P(\alpha_2), O(\alpha_3), P(\alpha_5)\}.
\]
The consequences of this derogation are:

(a) norm $O(a_4 \cup' \alpha_1)$ will be removed from the normative system; thus, is not applicable any more;

(b) it restricts the competence of authorities $a_2, a_3$ and $a_4$, since it prevents these authorities, for example, from introducing the derogated norm.

4. Authority $a_3$ enacts disposition object $\neg O(\alpha_3)$. Norm $O(\alpha_3) \in Cn(N)$. This is a material derogation since norm $O(\alpha_3)$ is only enacted minimally by authority $a_2$ with the same rank as the one enacting disposition object $\neg O(\alpha_3)$. Thus, norm $O(\alpha_3)$ will be rejected in some, but not all, maximal $N$-consistent sets in $MC(S(N_{A_3}) \cup \{-O(\alpha_3)\})$:

$$\{N, \{O(\alpha_1), P(\alpha_2), \neg O(\alpha_3), O(a_4 \cup' \alpha_1), P(\alpha_5)\}\}$$

Note that $N$ in an element of $MC(S(N_{A_3}) \cup \{-O(\alpha_3)\})$. The resulting systems are $N$ and $\{O(\alpha_1), P(\alpha_2), O(a_4 \cup' \alpha_1), P(\alpha_5)\}$.

6.3.3 Evaluation

The relationship between 'promulgation entrance' of norms into a system and 'derogation departure' of norms from a system is only one aspect of the process of introduction and removal of norms. On the one hand, through the promulgation of a norm $r$, we not only introduce $r$ into a system, but also restrict the competence of inferior authorities, i.e., authorities with a lower rank of authority than the authority promulgating norm $r$, as regards the set of norms which enters the system as a consequence of the promulgation of $r$. On the other hand, the derogation of a norm usually takes place through the promulgation of a derogatory disposition $d$. The promulgation of $d$ modifies the normative system in two ways:

1. a disposition enters the system;

2. the set of norms indicated in $d$ is removed from it.

Besides modifying the system, the derogation of a set of norms has the function of preventing any authority inferior to the one enacting disposition $d$ from introducing the derogated norm(s) into the system. Therefore, derogation and promulgation have two features in common:

1. the modification of the applicable norms;

2. the restriction of the competence of inferior authorities.
6.4 Universality

Promulgation and derogation of norms are structurally similar processes: both operations may result in norms being introduced into or removed from a normative system.

Sometimes the promulgation or derogation of a norm leads to an undecided situation with respect to the norms of a normative system $N$.

Example 6.3.4 Let
- $N_{a_1} = \{P(a_2)\}$
- $N_{a_2} = \{O(\overline{a_1}), O(a_1 \cup \overline{a_3})\}$

We define the following authority hierarchy: $D(\{a_1\}, \{a_2\})$. Then there is one maximal $N$-consistent set $N$ for the given authority hierarchy:

$$N = \{P(a_2), O(\overline{a_1}), O(a_1 \cup \overline{a_3})\}.$$

Suppose authority $a_1$ promulgates norm $O(a_3)$ or enacts disposition object $\neg O(\overline{a_3})$. Rule $O(\overline{a_3})$ is an element of $Cn(N)$. In both cases, one of the two norms enacted by $a_2$ becomes inapplicable; however, which one is undetermined. This is called the 'logische Unbestimmtheit des Normensystems' or logical indeterminacy.

In the following section, we discuss the notion of universality. A universal system is a normative-consistent system, such that all events are prescribed, prohibited or permitted. Thus, for every event there is a norm enacted by a set of authorities, which is applicable.

6.4 Universality

The question whether universality can be reached can be formulated as follows: is it possible to set norms for all events such that the obtained set of norms is $N$-consistent? In other words: can all events be prescribed, prohibited or permitted in a normative-consistent way?

Of course, the set of norms concerned has to be consistent. In the case of an inconsistent set, the question of universality would lose its meaning: everything would be deducible and, in particular, every norm that prescribes, prohibits or permits an event would be deducible. In order to deal with the question of universality, some concepts need to be introduced. We have to keep in mind that universality concerns three sets (cf. Sarlemijn, 1985):

1. the set of norms that prescribes, prohibits or permits events;
2. the set of events ($Evt'$);
3. the set of events that are actually regulated ($Evt^*$).

\[\text{Bulygin, 1976, p. 619.}\]
A system $N$ is universal if there is a norm for every event $\alpha \in \text{Evt}'$, that is $\text{Evt}^* = \text{Evt}'$ and if $N$ is $N$-consistent. Usually, $\text{Evt}^*$ is only a real subset: $\text{Evt}^* \subset \text{Evt}'$. To give a more precise formulation:

**Definition 6.4.1** $N$ is universal iff for every $\alpha \in \text{Evt}'$ it holds (exclusively) that:
- $O(\alpha) \in Cn(N)$, or
- $F(\alpha) \in Cn(N)$, or
- $(P(\overline{\alpha}) \land P(\alpha)) \in Cn(N)$.

From this definition it follows that a universal set is $N$-consistent.

**Proposition 6.4.2** If $N$ is universal, then $N$ is $N$-consistent.

**Proof.** Assume that $N$ is $N$-inconsistent. Then it holds for an event $\alpha$ that $N \vdash O(\alpha)$ and $N \vdash O(\overline{\alpha})$. Hence, $O(\alpha) \in Cn(N)$ and $F(\alpha) \in Cn(N)$. Thus, $N$ is not universal.

**Proposition 6.4.3** Every $N$-consistent set $N$ can be extended to a universal set $N^*$.

The extension is based on the countability of the formulas. The notion of countability does not establish the precise order of the formulas. It has been avoided, however, that both $O(\alpha)$ and $O(\overline{\alpha})$ occur in the universal set. From the above it also is clear that this possibility is purely a logical one, and in practice may entail a very difficult (promulgation) process (for example, in legal practice or in the practice of making a traffic system universal). The contents of $N^*$ have indeed not been determined unambiguously. This appears from the nature of the construction process of $N^*$. On the basis of purely logical considerations, a certain order in counting cannot be enforced. We call this the logical indeterminacy (cf. Navarro and Redondo, 1990). But in practice, that is where important decisions will be made as to the norms that are to be promulgated or not.

From this it appears that, starting from a certain set of norms, many different universal sets can be obtained. In this sense, the concept is ambiguous. However, it has to be stated that a universal set cannot be constructed in practice,\(^\text{12}\)

- because of the incredibly great number of events in $\text{Evt}'$;
- because some events do not need to be regulated according to a legislator;
- because of the creation of ever more new actions, and so also new - and thus not yet regulated - events.

As a result, all events that have been regulated by law constitute a subset of the set of events that are possible: $\text{Evt}^* \subset \text{Evt}'$.

6.4 Universality

6.4.1 Postulated universality

As we have seen above, the introduction of a universal system of norms meets with practical difficulties. These difficulties are, in fact, well known, and one tries to get round them by formulating general closure rules such as:

- All acts that are not forbidden, are permitted.
- All acts that are not expressly permitted, are forbidden.

The former corresponds with the legal expression *nulla poena sine lege*, stated in article 1 sub 1 of the Dutch Penal Code. The latter is used in the rules of certain games (such as chess, checkers and football).

To show that the general rules mentioned above are not altogether unproblematic, we concentrate on the former rule. This rule is in conformity with 'whatever is not forbidden, is permitted'. In what way is this rule interpreted? The obvious thing would be to accept the following principle:

\[ \neg F(a) \rightarrow P(a). \]

This principle seems to be in accordance with what is meant, but does not add anything, because it is already incorporated in our system. If interpreted in this way, the rule mentioned does not guarantee universality, for axioms, theorems and definitions have to be applicable to non-universal systems. The principle

'what is forbidden is permitted' addresses the judge and forbids him to extend the whole of legal prohibitions on the grounds of his own or someone else's political or moral conviction. (Soeteman, 1989, p. 189)

This interpretation can be formulated as follows.\(^\text{13}\) Let \( N \) be a \( N \)-consistent set of norms, then,

\[ \text{if } F(\alpha) \notin Cn(N), \text{ then } P(\alpha) \in Cn(N). \]

This means that it is forbidden for a judge to prohibit events which have not been prohibited up to today. Thus, this rule is regarded as a norm of judicial decision-making (cf. Soeteman, 1989).

The rule mentioned above, which is supposed to guarantee universality, also poses a problem of an entirely different nature. If we interpret the rule to mean that every event

\(^{13}\text{We can also formalise this in terms of authorities. Let } NA \text{ be the set of authorities enacting the norms of a certain legal system. Then we can formalise 'what is forbidden, is permitted' as follows. If the set of authorities has not enacted that } \alpha \text{ is forbidden, then } \alpha \text{ is permitted:} \]

\[ \neg (NA : F(\alpha)) \rightarrow NA : P(\alpha). \]
is permitted unless it is explicitly or implicitly prohibited, then we encourage people to find loopholes in the law. Yet, this kind of practice is considered to be against the spirit of the law. It cannot be the intention of legal principles (such as the rule mentioned above) to encourage acting against the spirit of the law (the danger of postulated universality). For example, in 1935 the owner of a taxi company hid illegal pamphlets in the car of his competitor after he had heard that the latter had to go to Germany to pick up a ‘fare’. The man notified German customs, hoping that his competitor would be caught and convicted. But German customs informed their Dutch colleagues, who prevented the man from crossing the border with the pamphlets. On the basis of article 328bis of the Dutch Penal Code, the former entrepreneur was convicted of ‘unfair competition’. The man lodged an appeal on the basis of the consideration that the act had nothing to do with competition (which involves clients). The High Court’s reply is interesting:

The fraudulent practice does not have to involve a potential client. It is shown clearly by the suspect’s proven ingenuity that a rule stated in general terms has the advantage that types of unfair competition not thought of when the law came into effect, can still be dealt with under criminal law.

Cases that were not thought of by members of government or parliament when the law was created can still be dealt with under that law on the basis of the consideration that they are determined by the spirit of a certain (criminal) law. From this it appears that the general principle which states that everything is permitted that is not prohibited, is limited by the rule that it is not permitted to violate the ‘spirit’ of the (criminal) law.

In the construction of universality, it appears that the interpretation of norms plays an important role. The conclusion that ‘this act is not explicitly provided for by the legislator’ can be drawn prematurely. We have to look for the cause ‘in den semantischen Eigenschaften der Sprache (aktuelle und potentielle Vagheit der Begriffe)’ (Alchourrón and Bulygin, 1977, p. 25). Then the set of possible instances of a concept is not determined in advance of the application of that concept. Only a judge can decide whether an object is an instance of a certain concept or not, e.g., is a ‘surfboard’ an instance of the concept ‘boat’?

In the philosophical literature (see Alchourrón and Bulygin, 1971, especially chapter 7), it has been mentioned that universal systems of norms restrict the freedom and responsibility of the individual. Do individuals still possess the personal intellectual freedom to develop systems of norms for themselves on the basis of which they can responsibly act

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14I am in debt to Prof. dr. A. Sarlemijn for pointing out the two judgements discussed in this subsection.
15HR, 24-6-1935, NJ 1936, 91.
16This principle can, in the context of criminal law, be associated with the principle of no punishment without preceding statutory law.
17In English: ‘in the semantic properties of language (topical and potential vagueness of concepts).’
in a certain manner? In our opinion, this question springs from an incorrect application of the notion of universality. Not universality of a legal system restricts the freedom and responsibility of individuals, but the extensions of obligations and prohibitions to that legal system. Permissions do not restrict personal intellectual freedom.

In a legal sense, it is manifest that the legislator leaves room for an individual system of norms. Consider the following example. In 1943, M.M.V. abused a mentally handicapped girl. He was convicted of ‘having intercourse with someone who is incapacitated’ (article 243, Penal Code). But the High Court later acquitted the man on the grounds of the following consideration:\(^{19}\)

An offence that formerly was not provided for in an article can later be incorporated because the meaning of a legal term or concept has changed in the course of time. In this way, a judge can pass a verdict that takes full account of the societal convictions about the punishability of behaviour. This is not the case here. The Penal Code has been with us for over sixty years; it has frequently been revised, and those involved in its revision were fully aware of the fact that ‘incapacitation’ only refers to the physical absence of power. Those involved in the revision of the Penal Code saw no reason to modify the article in this respect.

The reasoning is clear: the legislator has frequently revised formulations in the law. It can be assumed that the legislator knew that this particular rule only referred to physical powerlessness. Yet, the legislator did not change the formulation. Let us, therefore, assume that the legislator did not want to include non-physical powerlessness (mental powerlessness) in the rules of this law. To ensure that the above example be well understood, we should like to point to the following particulars that are important in the judgement thereof.

1. In criminal law, it indeed holds strictly that the law should not be extended on the basis of the reasoning that the legislator was not concerned with this type of cases; should he have done so, he would have reasoned analogously. This approach is not pursued here.

2. In the former case (the owner of the taxi company), one starts from the assumption that the legislator wanted to state explicit rules for this case as well.

\(^{18}\)A good example of this can be found in the replacement of the Dutch Traffic Regulation 1966 by the Dutch Traffic Regulation 1990. In the explanatory memorandum it is stated: ‘Because of the unequivocal formulation of many rules for relatively uncomplicated situations the road user is deprived of part of his responsibility. There is no need for him to think; the government has done that for him.’ (Dutch Traffic Regulation 1990, explanatory memorandum, p. II-B-7).

\(^{19}\)HR, 21-6-1943, NJ 1943, 559.
In the former case, we have to do with analogous reasoning.\textsuperscript{20} In the latter case, we have to do with reasoning from the spirit of the law.

The philosophical assumption that universality hinders an individual's norms awareness appears not to hold in practice: according to the High Court's judgement, abusing a mentally handicapped woman is permitted in the sense that it is not a punishable act. It will, however, be clear that personal, normative judgements may differ.

6.5 Conclusions

A normative system \( Cn(N) \) can be (normative-)inconsistent since norms are enacted, for example, by different authorities. To obtain a maximal normative-consistent system of a normative system \( Cn(N) \), we introduced an ordering of the authorities with respect to their competence, in other words, an authority hierarchy. For example, a norm \( O(\alpha) \), enacted by an authority \( a \), is not applicable if there is a set \( A \) of authorities all with higher ranks of authority than \( a \) which enacted \( O(\overline{\alpha}) \) or \( \neg O(\alpha) \).

The theory of the authority hierarchy provides a very workable framework for reasoning with other orderings; for instance, \( D(A_1, \ldots, A_n) \) can be interpreted as a specificity hierarchy. \( A_k \) stands for the rank of specificity of a certain norm. For example, a norm with rank \( A_1 \) of specificity has a higher priority than a norm with rank \( A_2 \) of specificity. This can easily be applied to a deontic system, in this chapter system \( PD_e L(Evt') \). For example, suppose \( A_1 \) stands for the rank of specificity of traffic signs, and \( A_2 \) stands for the rank of specificity of traffic rules. By art. 63 of the Dutch Traffic Regulation 1990, traffic signs override traffic rules in as far as specific rules are incompatible with specific signs.\textsuperscript{21}

Thus, we can state that \( A_1 \succ A_2 \). Consider the following example (see section 5.7): car driver \( i_1 \) is on a major road and approaches a junction, where car driver \( i_2 \) approaches from the right. On the ground of art. 15 of the Dutch Traffic Regulation 1990, \( i_1 \) has to give way to \( i_2 \), who approaches the junction from the right. But, on the grounds of the right-of-way signs (\( A_6 \) and \( A_9 \)) and the principle of trust, \( i_1 \) does not have to give way to \( i_1 \). We can formalise this as follows:

\[
A_1 : \neg O(\{i_1\} : \beta) \land A_2 : O(\{i_1\} : \beta),
\]

where \( \beta \) stands for the action 'to give right of way'. Thus, we can consistently express these conflicting norms with the addition of specificity. Further, norm \( \neg O(\{i_1\} : \beta) \) is applicable - in contrast to norm \( O(\{i_1\} : \beta) \), since norm \( \neg O(\{i_1\} : \beta) \) has a higher rank of specificity than norm \( O(\{i_1\} : \beta) \).

\textsuperscript{20}The analogous reasoning is as follows: 'Just as the legislator wanted to set rules against the abuse of physically impaired people, he also wanted to set rules against the abuse of the mentally handicapped.' Such analogous extensions in law are not applied in criminal law.

\textsuperscript{21}The expressing of such a meta-statement needs further research (cf. Prakken, 1993).
6.5 Conclusions

A normative system changes by promulgation (the introduction of a norm into a legal system) and derogation (the removal of a norm from a legal system), i.e., the dynamic character of law. We have seen that these two legislative acts by authorities have two features in common: the modification of the applicable norms and the restriction of the competence of inferior authorities. Furthermore, we have seen that not only derogation but also promulgation may lead to logical indeterminacy, in contrast to Alchourrón’s (1982) statement.

Finally, we discussed (postulated) universality, and the way in which universality can be reached by a closure rule, such as *nulla poena sine lege*, expressed in art. 16 of the Dutch Constitution and art. 1 sub 1 of the Dutch Penal Code. We argued that postulated universality does not restrict individual freedom and responsibility: the individual’s norms awareness is restricted by the extension of the whole of legal prohibitions.

In this chapter, we have seen how we can deal with normative inconsistencies on the basis of an authority hierarchy. A drawback of our approach is that we can only deal with norms and, more precisely, with unconditional norms. We cannot deal with, for example, inconsistencies between ‘classification rules’ (interpretation rules). To deal with such inconsistencies, all sorts of consistency-based approaches have been developed, such as the non-monotonic logic of McDermott and Doyle (1980) and the default logic of Reiter (1980, 1987). In the following chapter, we will develop a defeasible deontic reasoning formalism based on preferences, which can deal with conditional norms and inconsistencies between interpretation rules.
Chapter 7

Defeasible reasoning with legal rules

Over the last few years, several defeasible deontic reasoning formalisms have been developed to solve the problem of deontic inconsistency. However, these formalisms cannot deal with some very common forms of deontic reasoning since, e.g., their expressiveness is restricted. In this chapter\(^1\), we establish a priority hierarchy of legal rules to solve the problem of deontic conflicts, and we present a mechanism to reason about nonmonotonicity of legal rules over the priority hierarchy. The theory presented here is based on default logic, and is a modification and extension of Prakken's argumentation framework. It adequately deals with some shortcomings of other defeasible deontic reasoning approaches.

7.1 Introduction

Logical analysis of reasoning with inconsistent rules is a very relevant area for AI-and-Law research, since rules used in the legal domain are often conflicting. 'Prioritised' rules received attention in the research on the formalisation of nonmonotonic reasoning, particularly as a way of modelling the choice criterion in dealing with exceptions (cf. Poole, 1988; Shoham, 1988; Brewka, 1991; Prakken, 1993).

To deal with the inconsistencies, various sorts of consistency-based approaches have been developed, such as McDermott and Doyle's (1980) nonmonotonic logic and Reiter's (1980) default logic. But these approaches fail to reason about conflicting norms since they are all based on non-modal logics. As a way of solving the problems of deontic conflicts, forms of defeasible reasoning (cf. Pollock, 1987) have been adopted, which provide a mechanism to establish preference hierarchies of norms and to select a more applicable norm from among conflicting ones in a specific situation (cf. Alchourrón and Makinson, 1981; Royakkers and Dignum, 1994). The existing formalisations of defeasible deontic reasoning approaches (Horty, 1994; Tan and Van der Torre, 1994; Ryu, 1995) cannot deal

\(^1\)Some of these ideas in this chapter were presented earlier in Royakkers and Dignum (1996).
with several highly common forms of deontic logic (cf. Prakken, 1993).

A first problem is the absence of the notion of permission in certain approaches (Horty, 1994; Tan and Van der Torre, 1994). In these approaches, the conditional obligation is represented as \( O(p\mid q) \): ‘\( p \) is obligatory in case of \( q \)’, and is treated as a normal default; it can be read as a Reiter default \( q : p\neg p \): a non-deontic Reiter default. Inherent in this treatment is the absence of a reasonable Reiter default for the negated obligation (permission).

Another problem is the defeasibility of only a single opposing statement in some approaches (Horty, 1994; Prakken, 1994; Tan and Van der Torre, 1994). In these approaches, only couples of statements are considered to check inconsistencies. For instance, take the three statements \( O(a), O(\neg a \lor b) \) and \( \neg O(b) \). No single statement is in conflict with the other single statements. However, the group of statements \( O(a) \) and \( O(\neg a \lor b) \) implies \( O(b) \) in standard deontic logic, which is in conflict with statement \( \neg O(b) \).

The third problem is that most approaches (Horty, 1994; Tan and Van der Torre, 1994; Ryu, 1995) can only deal with defeasible conditionals that are deontic. But deontic defaults are not the only defaults in legal reasoning. Consider the deontic default \( a : O(b)/O(b) \). With this default, it is very often the case that \( a \) is derived by another default rule, e.g. \( c : a\sim a \), which is called a ‘classification rule’ or an ‘interpretation rule’. In the legal domain, it is accepted that these rules are also defeasible (cf. Hart, 1961). Prakken showed this by extending Hart’s standard example on a park regulation that forbids vehicles to enter the park:

> Not only this rule itself may turn out to be defeasible, for example, if the vehicle is an ambulance, but also rules on when something counts as a vehicle may be defeasible: imagine that a court says that objects on wheels that are meant for normal transport are vehicles: then roller skates used by people on their way to the office might be recognised as an exception. (Prakken, 1996)

In this chapter, we develop a theory of defeasible deontic reasoning which adequately deals with the above-mentioned problems. The theory is an extension and modification of the argumentation framework in default logic developed by Prakken (1993), and Prakken and Sartor (1995). Further, our theory is an extension of Dung’s (1993) theory, which only considers argumentation frameworks with one kind of conflict between arguments.

The structure of this chapter is as follows. In section 7.2, we give the representation of legal rules, which we subdivide into rules and conditional norms. Section 7.3 discusses the argumentation framework for rules. The argumentation framework for norms depending on rules selected from the argumentation framework for rules is discussed in section 7.4. Here, we concentrate on defeasibility and violation. We end this chapter with some conclusions.
7.2 Legal rules: rules and conditional norms

The fundamental logical structure of legal knowledge gives rise to the nonmonotonicity of legal reasoning (Reiter, 1980; Delgrande, 1988): the consequences that may follow from a set of legal and factual premises can be invalidated by further information. This means that rules can be ‘defeated’ by other or new rules and facts. The principal idea of this chapter, which goes back to Rescher (1964), is to allow the rules to be ordered and to use this ordering in such a way that conflicts can be solved in a logical argumentation framework using nonmonotonic logic. Such an ordering can often be discerned when considering the rules in a legal code. The ‘Lex Superior’ principle, for instance, is based on the general hierarchy of a legal system; the rules are divided along the lines of the hierarchical structure of the normative system. Rules with a lower rank of priority have to respect the consequences that follow from a higher ranked rule (see chapter 6). To describe the ordering between the formulas, we use the following notation. Let \( \preceq \) and \( \succeq \) be legal rules, then \( x \preceq y \) means that \( y \) is preferred to \( x \); \( x \sim y \) is an abbreviation of \( x \preceq y \) and \( y \preceq x \); and \( x \prec y \) is an abbreviation of \( x \preceq y \) and \( y \not\preceq x \). The ordering relation \( \preceq \) is reflexive and transitive.

Legal rules, or in any case most of them, subordinate a legal effect to a legal condition. By legal effect we mean every qualification generated by a legal norm: the ascription of deontic or normative modalities, status, professional titles, other legal qualities of persons and things. By legal condition we mean every condition to which a legal effect is subordinated. The legal rules are represented as conditional statements of the type

\[
a_1 \land a_2 \land \ldots \land a_n \Rightarrow \theta,
\]

where \( \theta \) is the legal effect and \( a_1, a_2, \ldots, a_n \) are the elements of the antecedent: the conjunction of literals, representing the legal condition. If \( \theta \) is a norm: an obligation \((O(\phi))\) or a permission \((P(\phi))\), with \( \phi \) a formula of the propositional logic, then the conditional statement is called a conditional norm. Thus, the conditional obligation is represented as \( a_1 \land a_2 \land \ldots \land a_n \Rightarrow O(p) \), instead of \( O(p/a_1 \land a_2 \ldots \land a_n) \) as in Horty (1994) and Tan and Van der Torre (1994). If \( \theta \) is a literal, then the statement is called a rule.

The statement \( A \Rightarrow B \) has to be interpreted as a normal default according to Reiter’s (1980, 1987) theory, \( A : B/B \): ‘If \( A \), and it can be consistently assumed \( B \), then we can infer \( B \).’ This means that \( \Rightarrow \) is not interpreted as the material implication, but as an inference rule that can be defeated. From \( A \) and \( A \Rightarrow B \), we can infer \( B \) unless \( \neg B \) can be proven. This representation corresponds to the formalisations usually proposed by legal theory and legal logic (cf. Sartor, 1993).

\(^2\)A literal is any atomic propositional formula and any negation of an atomic propositional formula.

\(^3\)For a discussion of the problems of the formalisation of the conditional norms, we refer to Alchourrón (1986).
In our theory, we distinguish between rules and norms for the following reasons:

- Rules cannot be violated.
- The defeasibility of rules is different from the defeasibility of norms (cf. definition 7.3.5 and definition 7.4.8), which is the most important difference.

The most important thing about the difference between rules and norms is not what differences there are, but simply that there are differences. This is why we discuss our theory on different levels: first, on the level of rules (section 7.3), and second, on the level of norms based on a given set of rules (section 7.4).

The set of rules is denoted by \( W \) and the set of conditional norms by \( \Delta \). Furthermore, we have a factual sentence \( F \) representing the factual situation, which consists of background knowledge and contingent facts. The background knowledge consists of necessary conditions, for example, a human being is mortal. A set of conditional norms, a set of rules and a factual sentence is called a deontic context.

**Definition 7.2.1** A deontic context \( T = (\Delta, W, F, \preceq) \) consists of a set \( \Delta \) of conditional norms, a set \( W \) of rules, a factual propositional sentence \( F \): the conjunction of background knowledge \( F_b \) and contingent facts \( F_c \), and an ordering \( \preceq \) over rules and conditional norms.

### 7.3 Rules

Facts (formalised by the sentence \( F \)) can contain material implications. Rules, however, are represented by normal defaults written as a conditional statement of the type \( a_1 \wedge a_2 \wedge \ldots \wedge a_n \Rightarrow \theta \), with \( \theta \) the legal effect formalised by a literal. Our theory of defeasible reasoning for rules is based on four notions:

- the notion of argument (definition 7.3.4);
- the notion of defeating (definition 7.3.5);
- the notion of defeasibility chain (definition 7.3.7);
- the notion of justified, defensible and overruled arguments (definition 7.3.9).

At the end of this section, we define maximal-coherent argument sets of rules that we use for the notion of the applicability of norms and the violation of obligations in section 7.4.

Before we discuss the notion of argument, we give three definitions which we will use in the sequel.
Definition 7.3.1 Let $F$ be the factual propositional sentence, $V$ be a set of rules and $r$ a literal, then $V$ explains $r$ $(V \cup \{F\} \models r)$ iff

$$r \in \bigcup_{i=0}^{\infty} G_i,$$

with

$$G_0 = \{r \mid \{F\} \models r\} \text{ and } G_{i+1} = G_i \cup \{r \mid \exists a_1 \land a_2 \land \ldots a_n \Rightarrow r \in V(G_i \cup \{F\} \models a_i)\},$$

for all $i \in \{0, 1, 2, \ldots\}$.

Intuitively, explaining is the same as logical consequence, except that now we deal with defaults and not with implications.

Definition 7.3.2 Let $V$ be a set of rules, then the consequences of $V \cup \{F\}$ $(Cons(V \cup \{F\}))$ is defined as

$$Cons(V \cup \{F\}) := \{r \mid r \text{ is a literal and } V \cup \{F\} \models r\}.$$ 

Thus, the $Cons$ relation is a transitive closure of the explaining. It gives the set of all the literals that can be consistently derived from $V$ and $\{F\}$.

Definition 7.3.3 Let $V$ be a set of rules. Then $V \cup \{F\}$ is coherent iff

$$\neg \exists r \text{ is a literal } (r \in Cons(V \cup \{F\}) \land \neg r \in Cons(V \cup \{F\})).$$

The notion of argument can now be defined as follows:

Definition 7.3.4 Let $M \subseteq W$, $\phi$ a literal and $M \cup \{F\}$ coherent. Then $M$ explains $\phi$ minimally iff

- $\{F\} \cup M \models \phi$ and
- $\neg \exists_{\phi_1 \in M}(\{F\} \cup M \setminus \{\phi_1\}) \models \phi$.

We call $M$ a minimally explaining set or an argument. The set of all arguments is denoted as $\mathcal{M}$. The $\phi$-relevant set of $W$, denoted by $[\phi]\mathcal{M}$, is the set of all arguments in $\mathcal{M}$ that explain $\phi$ minimally.

$M_1$ is a subargument of $M$ iff $M_1 \subseteq M$ and $M_1$ is an argument. If there is an argument for $\phi$, thus $[\phi]\mathcal{M} \neq \emptyset$, then $\phi$ is called an outcome.

Our definition of 'defeat' is based on the idea that, in order to defeat an argument, a counterargument can point its attack at the argument itself, but also at one of its proper subarguments, since an argument cannot be stronger than its weakest link (cf. Prakken and Sartor, 1995).
Definition 7.3.5 Let $M_1 \in [\phi]M$ and $M_2 \in [\phi']M$. Then $M_1$ is defeated by $M_2$ ($M_1 \prec^* M_2$) iff

$$\exists \phi_2 \in M_1 \exists \phi_4 \in M_2 \{\phi_3 \Rightarrow \phi_4 \Rightarrow \phi_1 \Rightarrow \phi_2\} \text{ and } \{\phi_2\} \cup \{\phi_4\} \cup \{F\} \text{ is inconsistent.}$$

Thus, an argument $M_2$ defeats an argument $M_1$ iff $M_1$ and $M_2$ have contradictory conclusions $\phi_2$ and $\phi_4$ with respect to the factual sentence $F$, and the rule $\phi_3 \Rightarrow \phi_4 \in M_2$ (responsible for the conflict) does not have a lower priority than the rule $\phi_1 \Rightarrow \phi_2 \in M_1$. Note that $\{\phi_2\} \cup \{F\}$ and $\{\phi_4\} \cup \{F\}$ are consistent, which directly follows from definition 7.3.4.

Relation $\prec^*$ is not transitive and not asymmetric. It is possible that $M_1 \prec^* M_2$ and $M_2 \prec^* M_1$ both hold. The following example illustrates this point:

Example 7.3.6

(1) $a \Rightarrow b$

(2) $c \Rightarrow \neg a$

(3) $d \Rightarrow a$

(4) $b \Rightarrow \neg c$

(5) $e \Rightarrow c$

$F: f \land (f \rightarrow d) \land e$

with (5) $\prec^*$ (4) $\prec^*$ (3) $\prec^*$ (2) $\prec^*$ (1).

Let $M_1 = \{e \Rightarrow c, c \Rightarrow \neg a\}$ and $M_2 = \{d \Rightarrow a, a \Rightarrow b, b \Rightarrow \neg c\}$. Then $M_1 \prec^* M_2$, since $e \Rightarrow c \prec^* b \Rightarrow \neg c$, and $M_2 \prec^* M_1$, since $d \Rightarrow a \prec^* c \Rightarrow \neg a$.

Definition 7.3.7 A defeasibility chain is a sequence of arguments in $M$:

$M_1 \prec^* M_2 \prec^* \ldots \prec^* M_n$

with the following conditions:

- $\forall_{k,l=1,2,\ldots,n:k<l} M_k \not\subseteq M_l$;
- $\neg \exists_{M_{n+1} \in M} \{(M_1, \ldots, M_n \not\subseteq M_{n+1}) \land (M_n \prec^* M_{n+1})\}$.

We define $Ch(M)$ as the set of all defeasibility chains of arguments in $M$.

The first condition ensures that cycles in defeasibility chains are avoided. Suppose that $M_1 \prec^* M_2$ and $M_2 \prec^* M_3$, with $M_1 \subset M_3$. We would thus end up with the endless chain $M_1 \prec^* M_2 \prec^* M_3 \prec^* M_2 \prec^* M_3 \ldots$. This would also be accomplished by 'Ʌ' instead of 'Ʌ'. The reason why indeed we need 'Ʌ' is to satisfy the weakest link principle, which will become clear in example 7.3.17.
7.3 Rules

The second condition provides that a chain stops if there is no 'stronger' argument than the last argument in the chain. Take the example above, then

\[ Ch(M) = \{ \{d \Rightarrow a, a \Rightarrow b\} \prec^* \{e \Rightarrow c, c \Rightarrow \neg a\}, \]
\[ \{d \Rightarrow a, a \Rightarrow b, b \Rightarrow \neg c\} \prec^* \{e \Rightarrow c, c \Rightarrow \neg a\}, \]
\[ \{e \Rightarrow c\} \prec^* \{d \Rightarrow a, a \Rightarrow b, b \Rightarrow \neg c\}, \]
\[ \{d \Rightarrow a\} \prec^* \{e \Rightarrow c, c \Rightarrow \neg a\}, \]
\[ \{e \Rightarrow c, c \Rightarrow \neg a\} \prec^* \{d \Rightarrow a, a \Rightarrow b, b \Rightarrow \neg c\}. \]

Definition 7.3.8 \( Ch(M) \) is the set of all defeasibility chains in \( Ch(M) \) starting with \( M \).

The defeasibility chains in \( Ch(M) \) take the set of all possible arguments and their mutual relations of defeat as inputs. They produce a distinction between in three types of argument:\(^4\)

1. justified arguments;
2. overruled arguments;
3. defensible arguments.

A justified argument is a 'winning' argument. Such an argument can be defeated by another argument, but that argument will be overruled. An overruled argument is a 'losing' argument. A defensible argument is an argument that is neither justified nor overruled. In other words, an 'undeciding' argument.

Definition 7.3.9 Let \( M \in M \). Then

- \( M \) is a justified argument iff \( Ch(M) = \{M\} \) or for all chains \( M \prec^* M_1 \prec^* \ldots \prec^* M_n \in Ch(M) \) it holds that
  \[ n \text{ is even } \land \neg \exists_{M' \in M} M_n \prec^* M' \]
  \[ \forall_k \text{ is even}(M_k \text{ is a justified argument assuming } M \text{ is a justified argument}). \]

- \( M \) is an overruled argument iff there is a chain \( M \prec^* M_1 \prec^* \ldots \prec^* M_n \in Ch(M) \) with
  \[ n \text{ is odd } \land \neg \exists_{M' \in M} M_n \prec^* M'. \]

\(^4\)The terms justified, overruled and defensible arguments were introduced by Prakken and Sartor (1995).
• *M* is a defensible argument iff *M* is neither a justified argument nor an overruled argument.

Note that *M*ₙ in the chains of definition 7.3.9 is a justified argument, since \(Ch(Mₙ) = \{Mₙ\}\), which is equivalent to \(\exists M' ∈ M Mₙ \nless M'\). The condition in the definition of a justified argument *M* that all arguments *M*ₖ with *k* are even are justified assuming *M* is a justified argument is necessary, since the justification of *M*ₖ can depend on *M* (see example 7.3.11). In other words, ‘if it cannot be proven whether *M*ₖ is justified or not, then *M*ₖ should be justified assuming that *M* is justified.’

Let \(M \nless M_₁ \nless \ldots \nless M_n\) be a chain in \(Ch(M)\), then we call the arguments *M*ᵢ with *i* is odd *odd arguments*, and the arguments *M* and *M*ᵢ with *i* is even *even arguments*.

In a defeasibility chain \(M \nless M_₁ \nless \ldots \nless M_n\) with *n* is even (odd) and \(Ch(M_n) = \{M_n\}\), we stipulate that the odd (even) arguments are the attacked arguments and the even (odd) arguments the non-attacked arguments. The chain ends with *M*ₙ, so *M*ₙ₋₁ is an attacked argument, because it is defeated by a non-attacked argument. *M*ₙ₋₂ is not attacked, because it is defeated by an attacked argument (*M*ₙ₋₁), and so on. For example, *M* is overruled if *M*₁ is a non-attacked argument and this follows if *n* is odd. The following example clarifies the above definition:

**Example 7.3.10**

\[(1)\] \(a \Rightarrow b\)
\[(2)\] \(c ⇒ ¬b\)
\[(3)\] \(d ⇒ ¬e\)
\[(4)\] \(e ⇒ ¬f\)
\[(5)\] \(b ⇒ e\)
\[(6)\] \(h ⇒ b\)
\[(7)\] \(f ⇒ ¬d\)
\[(8)\] \(i ⇒ f\)
\[(9)\] \(j ⇒ d\)

\(F: a ∧ c ∧ h ∧ i ∧ j\)

with \((9) \nless (8) \nless \ldots \nless (1)\).

Let

\(M_₁ = \{a \Rightarrow b\}\)
\(M_₂ = \{c ⇒ ¬b\}\)
\(M_₃ = \{d ⇒ ¬e, j ⇒ d\}\)
\(M_₄ = \{e ⇒ ¬f, b ⇒ e, h ⇒ b\}\)
\(M_₅ = \{f ⇒ ¬d, i ⇒ f\}\)
7.3 Rules

1. \(Ch(M_1) = \{ M_1 \}\), thus \(M_1\) is a justified argument.

2. \(Ch(M_2) = \{ M_2 \rightarrow M_1 \}\), thus \(M_2\) is an overruled argument, since the chain contains an even number of arguments and \(Ch(M_1) = \{ M_1 \}\).

3. \(Ch(M_3) = \{ M_3 \rightarrow M_5 \rightarrow M_4 \rightarrow M_2 \rightarrow M_1 \}\). \(M_3\) is not overruled, since otherwise the chain contains an even number of arguments. It depends on argument \(M_4\) whether \(M_3\) is justified or defensible.

4. \(Ch(M_4) = \{ M_4 \rightarrow M_2 \rightarrow M_1, M_4 \rightarrow M_3 \rightarrow M_5 \}\). \(M_4\) is not a justified argument, since \(Ch(M_5) \neq \{ M_5 \}\). \(M_4\) is not an overruled argument, since there is no chain in \(Ch(M_4)\) with an even number of arguments. Thus, \(M_4\) is a defensible argument. Hence, from the chain in \(Ch(M_3)\) it also follows that \(M_3\) is a defensible argument.

5. \(Ch(M_5) = \{ M_5 \rightarrow M_4 \rightarrow M_2 \rightarrow M_1, M_5 \rightarrow M_4 \rightarrow M_3 \}\). \(M_5\) is an overruled argument, since the chain \(M_5 \rightarrow M_4 \rightarrow M_2 \rightarrow M_1\) contains an even number of arguments and \(Ch(M_1) = \{ M_1 \}\).

In definition 7.3.9 of a justified argument \(M\), the even arguments have to be justified assuming \(M\) is justified. The condition 'assuming \(M\) is justified' is necessary since otherwise we cannot determine whether an argument is justified or defensible in certain cases. Consider the next example.

Example 7.3.11

\[
\begin{align*}
(1) & \quad i \Rightarrow \neg d \\
(2) & \quad b \Rightarrow \neg h \\
(3) & \quad d \Rightarrow \neg b \\
(4) & \quad f \Rightarrow \neg d \\
(5) & \quad a \Rightarrow b \\
(6) & \quad c \Rightarrow d \\
(7) & \quad j \Rightarrow \neg h \\
(8) & \quad e \Rightarrow f \\
(9) & \quad g \Rightarrow h \\
(10) & \quad F: a \land c \land e \land g \land i \land j \\
\end{align*}
\]

with \(10 \prec 9 \prec \ldots \prec 1\).

Let \(M_1 = \{ c \Rightarrow d, d \Rightarrow \neg b \}\);
\[ M_2 = \{ e \Rightarrow f, f \Rightarrow \neg d \}; \]
\[ M_3 = \{ g \Rightarrow h, h \Rightarrow \neg f \}; \]
\[ M_4 = \{ a \Rightarrow b, b \Rightarrow \neg h \}; \]
\[ M_5 = \{ i \Rightarrow \neg d \}; \]
\[ M_6 = \{ j \Rightarrow \neg h \}. \]

Suppose that we state that the even arguments have to be justified instead of the even arguments have to be justified assuming \( M \) is justified in the definition 7.3.9 of a justified argument \( M \). Then, we cannot determine whether \( M_2 \) and \( M_4 \) are justified or defensible arguments. The set of defeasibility chains starting with \( M_4 \) is

\[ Ch(M_4) = \{ M_4 \leftarrow^* M_1 \leftarrow^* M_2 \leftarrow^* M_3 \leftarrow^* M_6, M_4 \leftarrow^* M_1 \leftarrow^* M_5 \}. \]

\( M_4 \) is not an overruled argument, since both chains contain an odd number of arguments. Suppose \( M_4 \) is a justified argument, then the following three conditions have to be satisfied:

1. \( Ch(M_5) = \{ M_5 \}; \)
2. \( Ch(M_6) = \{ M_6 \}; \)
3. \( M_2 \) is a justified argument.

The first two conditions are satisfied. For the third condition, we have to analyse the set of defeasibility chains starting with \( M_2 \):

\[ Ch(M_2) = \{ M_2 \leftarrow^* M_3 \leftarrow^* M_4 \leftarrow^* M_1 \leftarrow^* M_5, M_2 \leftarrow^* M_3 \leftarrow^* M_6 \}. \]

\( M_2 \) is not an overruled argument, since both chains contain an odd number of arguments. Suppose \( M_2 \) is a justified argument, then the following three conditions have to be satisfied:

1. \( Ch(M_5) = \{ M_5 \}; \)
2. \( Ch(M_6) = \{ M_6 \}; \)
3. \( M_4 \) is a justified argument.

Again, the first two conditions are satisfied. Thus, whether \( M_2 \) is a justified argument or not, depends on the 'status' of argument \( M_4 \), and vice versa. Hence, we cannot find out whether \( M_2 \) and \( M_4 \) are justified or defensible arguments. To overcome this problem, we have set the condition that the even arguments in the chains are justified assuming \( M \) is justified:

According to definition 7.3.9, argument \( M_4 \) is justified if

1. \( Ch(M_5) = \{ M_5 \}; \)
2. $Ch(M_6) = \{M_6\}$

3. $M_2$ is justified assuming that $M_4$ is justified.

Argument $M_2$ is justified assuming $M_4$ is justified, since the three conditions mentioned above, i.e., $Ch(M_5) = \{M_5\}$, $Ch(M_6) = \{M_6\}$ and $M_4$ is a justified argument, are now satisfied. Thus, $M_2$ is a justified argument assuming that $M_4$ is justified. Hence, $M_4$ is a justified argument.

Analogously, we can show that $M_2$ is also a justified argument. Intuitively, this is correct: argument $M_4$ ($M_2$) is only defeated by argument $M_1$ ($M_3$), and in turn, $M_1$ ($M_3$) is defeated by argument $M_6$ ($M_6$), which is not defeated. Thus, $M_1$ ($M_3$) is an overruled argument, and cannot invalidate an argument.

**Proposition 7.3.12**

1. Let $M$ be a justified argument. Then all odd arguments in the chains of $Ch(M)$ are overruled arguments.

2. Let $M$ be an overruled argument. Then there is a chain in $Ch(M)$ where all even arguments are overruled or defensible.

**Proof**

1. Let $M$ be a justified argument. Then for all chains $M \prec \cdots \prec M_n$ in $Ch(M)$, it holds that $n$ is even and $\exists_{M'} M_n \prec M'$. Let $M_k$ with $k$ is odd be an argument of a chain $M \prec \cdots \prec M_n$ in $Ch(M)$. Then this chain without the first $k$ arguments, thus $M_k \prec \cdots \prec M_n$, is a chain in $Ch(M_k)$ which satisfies the conditions of an overruled argument $M_k$, since the chain contains an even number of arguments and $Ch(M_n) = M_n$. Thus, all odd arguments are overruled arguments.

2. Let $M$ be an overruled argument. Then there is a chain $M \prec \cdots \prec M_n$ with $n$ is odd and $\exists_{M'} M_n \prec M'$. Suppose an even argument $M_k$ is justified, then the chain $M \prec \cdots \prec M_n$ without the first $k$ arguments, i.e., $M_k \prec \cdots \prec M_n$ is a chain in $Ch(M_k)$ and does not satisfy the conditions for a justified argument $M_k$, since the chain contains an even number of arguments. Thus, the even arguments in such chains in $Ch(M)$ are overruled or defensible.

The condition that all even arguments in the chains of $Ch(M)$ with $M$ being a justified argument have to be justified arguments, is necessary, since otherwise we obtain some undesirable results:
Example 7.3.13

(1)  \( a \Rightarrow \neg b \)
(2)  \( c \Rightarrow a \)
(3)  \( b \Rightarrow \neg d \)
(4)  \( e \Rightarrow b \)
(5)  \( \neg g \Rightarrow \neg f \)
(6)  \( d \Rightarrow \neg h \)
(7)  \( f \Rightarrow d \)
(8)  \( i \Rightarrow f \)
(9)  \( h \Rightarrow \neg j \)
(10) \( k \Rightarrow h \)
(11) \( j \Rightarrow \neg g \)
(12) \( l \Rightarrow j \)

\[ F: c \land e \land i \land k \land l \]

with \((12) \preceq (11) \preceq \ldots \preceq (1)\).

Let \( M_1 = \{ a \Rightarrow \neg b, c \Rightarrow a \} \);
\( M_2 = \{ b \Rightarrow \neg d, e \Rightarrow b \} \);
\( M_3 = \{ \neg g \Rightarrow \neg f, j \Rightarrow \neg g, l \Rightarrow j \} \);
\( M_4 = \{ d \Rightarrow \neg h, f \Rightarrow d, i \Rightarrow f \} \);
\( M_5 = \{ h \Rightarrow \neg j, k \Rightarrow h \} \).

\( Ch(M_3) = \{ M_3 \prec^* M_5 \prec^* M_4 \prec^* M_2 \prec^* M_1 \} \) and \( Ch(M_1) = \{ M_1 \} \), thus without the condition that all even arguments have to be justified arguments, \( M_3 \) would be a justified argument instead of a defensible argument, since \( M_4 \) is not a justified argument, but defensible.\(^5\) Suppose \( M_3 \) is justified. The outcomes of argument \( M_4 \) and its subarguments are \( \neg h, \neg d \) and \( f \), and the outcome of \( M_3 \) is \( \neg f \). Thus, if we decide to use argument \( M_4 \), there is a conflict with the ‘winning’ (justified) argument \( M_3 \); we cannot use \( M_4 \), since we need the outcome \( f \) to derive \( \neg h \) (the outcome of \( M_4 \)), which is in conflict with \( f \), the outcome of the justified argument \( M_3 \). Thus, it is counter-intuitive to call \( M_4 \) defensible and \( M_3 \) justified. However, \( M_3 \) is a defensible argument. Now we have to decide which of the two arguments will be used: both arguments are defensible.

\(^5\) Argument \( M_4 \) is defensible: the set of defeasibility chains starting with \( M_4 \) is \( Ch(M_4) = \{ M_4 \prec^* M_2 \prec^* M_1, M_4 \prec^* M_5 \prec^* M_3 \} \). Since \( M_5 \) is an overruled argument (the chain \( M_5 \prec^* M_4 \prec^* M_2 \prec^* M_1 \) satisfies the conditions for the overruled argument \( M_5 \)), the chain \( M_4 \prec^* M_3 \prec^* M_5 \) does not satisfy the conditions for a justified argument \( M_4 \). Thus, \( M_4 \) is not a justified argument. \( M_4 \) is not an overruled argument either, since neither of the two chains in \( Ch(M_4) \) satisfies the conditions for an overruled argument \( M_4 \). Thus, \( M_4 \) is a defensible argument.
7.3 Rules

Corollary 7.3.14

1. A justified argument can only be defeated by an overruled argument.
2. If there is no justified argument, then there is no overruled argument.
3. There is no justified argument iff all arguments are defensible.
4. There is a justified argument iff there is a defeasibility chain with one argument.

Proof

1. Let \( M \) be a justified argument, and defeated by \( M_1 \). We have to prove that \( M_1 \) is an overruled argument. For all chains \( M \prec^* M_1 \ldots \prec^* M_n \) in \( Ch(M) \) it holds that \( n \) is even and \( Ch(M_n) = \{ M_n \} \). For all these chains without the first argument \( M \), i.e., \( M_1 \prec^* \ldots \prec^* M_n \), it holds that they are elements of \( Ch(M) \) and satisfy the conditions of an overruled argument. Thus, \( M_1 \) is an overruled argument.

2. Suppose that there is no justified argument. Then there is no chain \( M \prec^* M_1 \prec^* \ldots \prec^* M_n \) in \( Ch(M) \) with \( Ch(M_n) = \{ M_n \} \). Hence, there is no overruled argument. The converse does not hold. For example, let \( W = \{ a \Rightarrow b \} \) and \( a \) a fact. Then the only argument is \{ \( a \Rightarrow b \) \}, which is justified.

3. Suppose that there is no justified argument. Then there is no overruled argument, thus all arguments are defensible. Evidently, if all arguments are defensible, then there are no justified arguments.

4. Suppose that \( M \) is a justified argument. Then for all chains \( M \prec^* M_1 \prec^* \ldots \prec^* M_n \) in \( Ch(M) \) it holds that \( Ch(M_n) = \{ M_n \} \). Thus, there is a defeasibility chain with one argument.
   If there is a chain with one argument, say \( M \), then \( M \) is a justified argument. Hence, there is a justified argument.

The converse of 7.3.14.1 does not hold: an overruled argument need not be defeated by a justified argument.

Example 7.3.15

(1) \( a \Rightarrow b \)
(2) \( c \Rightarrow d \)
(3) \( \neg b \Rightarrow \neg e \)
(4) \( f \Rightarrow \neg b \)
(5) \( e \Rightarrow \neg d \)
(6) \( g \Rightarrow e \)
(7) \( h \Rightarrow d \)
F: \( a \land c \land f \land g \land h \)

with (7) \( \prec (6) \prec \ldots \prec (1) \).

Let \( M_1 = \{ a \Rightarrow b \} \);
\( M_2 = \{ c \Rightarrow d \} \);
\( M_3 = \{ \neg b \Rightarrow \neg e, f \Rightarrow \neg b \} \);
\( M_4 = \{ g \Rightarrow e, e \Rightarrow \neg d \} \);
\( M_5 = \{ h \Rightarrow d \} \).
Then \( Ch(M_5) = \{ M_5 \prec M_4 \prec M_3 \prec M_1, M_5 \prec M_4 \prec M_2 \} \). \( M_5 \) is an overruled argument, since chain \( M_5 \prec M_4 \prec M_3 \prec M_1 \) contains an even number of arguments and \( Ch(M_1) = \{ M_1 \} \). Further, \( M_5 \) is only defeated by argument \( M_4 \), which is an overruled argument, since \( M_4 \prec M_2 \in Ch(M_4) \) and \( Ch(M_2) = \{ M_2 \} \). Thus, an overruled argument can be defeated by an overruled argument.

The following proposition shows that definition 7.3.9 satisfies the weakest link principle.

**Proposition 7.3.16** All subarguments of a justified argument are justified arguments.

**Proof.** Suppose that \( M \) is a justified argument and \( M' \) is a subargument of \( M \). We have to prove that \( M' \) is a justified argument, and without loss of generality we assume that \( M' \) is a 'largest' subargument of \( M \), i.e., \( \neg \exists M'' \in M, M' \subset M'' \subset M \). For if this argument is justified, we can repeat this process for \( M' \) to prove that all subarguments of \( M' \) are justified. Suppose that \( M' \) is not a justified argument, then \( M' \) is an overruled or a defensible argument.

- Suppose that \( M' \) is an overruled argument. Then there is a chain \( M' \prec M_1 \prec \ldots \prec M_n \) with \( Ch(M_n) = \{ M_n \} \), and \( n \) is odd. However, then the chain \( M \prec M_1 \prec \ldots \prec M_n \) is a chain of \( Ch(M) \), which is in contradiction with the assumption that \( M \) is a justified argument.

- Suppose now that \( M' \) is a defensible argument. Then there is a chain \( M' \prec M_1 \prec \ldots \prec M_n \) and \( Ch(M_n) \neq \{ M_n \} \). Now, the chain \( M \prec M_1 \prec \ldots \prec M_n \) is not an element of \( Ch(M) \), but part of a chain in \( Ch(M) \). \( M_n \) can only be followed by \( M' \), thus \( M \prec M_1 \prec \ldots \prec M_n \prec M' \prec M_1 \prec \ldots \prec M_i \), with \( Ch(M_i) = \{ M_i \} \). However, now it follows that \( M' \) is an overruled argument (if it is an odd argument in the chain (proposition 7.3.12.1)) or a justified argument (if it is an even argument (definition 7.3.9)), which is in contradiction with the assumption that \( M' \) is defensible.

Thus, \( M' \) is a justified argument.

In definition 7.3.7, we chose operator '\( \prec \)' instead of '\( \not\prec \)' , otherwise the weakest link principle is not satisfied. The following example illustrates this point:
Example 7.3.17

(1) \( d \Rightarrow \neg e \)
(2) \( e \Rightarrow \neg b \)
(3) \( a \Rightarrow b \)
(4) \( b \Rightarrow \neg c \)
(5) \( c \Rightarrow e \)
(6) \( f \Rightarrow c \)

\( F: \ a \land d \land f \)

with (6) \( \prec (5) \prec \ldots \prec (1) \).

Let \( M_1 = \{d \Rightarrow \neg e\} \); \n\( M_2 = \{e \Rightarrow \neg b, c \Rightarrow e, f \Rightarrow c\} \);
\( M_3 = \{a \Rightarrow b, b \Rightarrow \neg c\} \); \n\( M_4 = \{a \Rightarrow b\} \).

Then, \( Ch(M_3) = \{M_3 \prec \ast M_2 \prec \ast M_1\} \) and \( Ch(M_1) = \{M_1\} \), thus \( M_1 \) and \( M_3 \) are justified arguments. \( M_4 \) is a subset of the justified argument \( M_3 \), thus also justified (proposition 7.3.16).

Suppose now that we use operator ‘\( \neq \)’ instead of ‘\( \prec \)’, then subargument \( M_4 \) of \( M_3 \) is not justified, since the chain \( M_4 \prec \ast M_2 \prec \ast M_3 \) would be an element of \( Ch(M_4) \) and \( Ch(M_3) \neq \{M_3\} \). Thus, the weakest link principle is not satisfied if we use operator ‘\( \neq \)’.

The following three problems form the main problems in the literature on defeasible arguments. We will show that these problems can be adequately dealt with in the theory as it follows from definition 7.3.9.

Example 7.3.18 The intermediate conclusion

(1) \( a \Rightarrow b \)
(2) \( c \Rightarrow \neg b \)
(3) \( d \Rightarrow a \)

\( F: \ c \land d \)

with (3) \( \prec (2) \prec (1) \).

Here, a conflict arises between rules (1) and (2). By definition 7.3.5, the choice is made between the rules which are certain to be in conflict with each other. Rule (3) with intermediate conclusion \( a \), necessary to derive outcome \( b \), is irrelevant to the conflict.

The minimally explaining sets (arguments) are \( M_1 = \{d \Rightarrow a, a \Rightarrow b\} \), \( M_2 = \{c \Rightarrow \neg b\} \) and \( M_3 = \{d \Rightarrow a\} \). The sets of defeasibility chains are \( Ch(M_1) = \{M_1\} \), \( Ch(M_2) = \{M_2 \prec \ast M_1\} \) and \( Ch(M_3) = \{M_3\} \). \( M_1 \) and \( M_3 \) are justified, since they are not defeated by any argument. \( M_2 \) is overruled, because it is defeated by justified argument \( M_1 \). Thus, \( a \) and \( b \) are the outcomes.
Example 7.3.19 Iterated conflicts

(1) \( c \Rightarrow a \)
(2) \( d \Rightarrow \neg a \)
(3) \( \neg a \Rightarrow \neg b \)
(4) \( a \Rightarrow b \)

\( F: c \land d \)

with (4) \( \prec (3) \prec (2) \prec (1) \).

Here, a conflict arises between rules (1) and (2) and between rules (3) and (4). Rules (1) and (2) have intermediate conclusions \( a \) and \( \neg a \) for their final conclusions \( \neg b \) and \( b \). This type of problem is called iterated conflicts: conflicts on both intermediate and final conclusions.

The minimally explaining sets are \( M_1 = \{ c \Rightarrow a \} \), \( M_2 = \{ d \Rightarrow \neg a \} \), \( M_3 = \{ d \Rightarrow \neg a, \neg a \Rightarrow \neg b \} \) and \( M_4 = \{ c \Rightarrow a, a \Rightarrow b \} \). The sets of defeasibility chains are \( CH(M_1) = \{ M_1 \} \), \( CH(M_2) = \{ M_2 \prec M_1 \} \), \( CH(M_3) = \{ M_3 \prec M_1 \} \) and \( CH(M_4) = \{ M_4 \prec M_3 \prec M_1 \} \). \( M_1 \) and \( M_4 \) are justified arguments. \( M_2 \) and \( M_3 \) are overruled arguments, because they are defeated by justified argument \( M_1 \): \( a \) and \( b \) are the outcomes. Note that \( M_4 \) is defeated by \( M_3 \), though not overruled, since \( M_3 \) is an overruled argument (defeated by \( M_1 \)).

Example 7.3.20 Circular conflicts

(1) \( a \Rightarrow b \)
(2) \( c \Rightarrow \neg b \)
(3) \( b \Rightarrow \neg a \)
(4) \( d \Rightarrow a \)
(5) \( d \Rightarrow b \)

\( F: c \land d \)

with (5) \( \prec (4) \prec (3) \prec (2) \prec (1) \).

Here, we have conflicts between rules (1) and (2), rules (3) and (4) and rules (2) and (5). The applicability of rule (1) depends on rule (4) with intermediate conclusion \( a \). Rule (4) is in conflict with a higher rule, (3), (iterated conflict), and the applicability of (3) depends on (5), which is in conflict with (2). Rule (2) is in conflict with rule (1), and the applicability of (1) depends on rule (4), and so on. This will never stop, therefore we call this the problem of circular conflicts.

The minimally explaining sets are \( M_1 = \{ d \Rightarrow a, a \Rightarrow b \} \), \( M_2 = \{ c \Rightarrow \neg b \} \), \( M_3 = \{ d \Rightarrow b, b \Rightarrow \neg a \} \), \( M_4 = \{ d \Rightarrow a \} \) and \( M_5 = \{ d \Rightarrow b \} \). The sets of defeasibility chains
are \( Ch(M_1) = \{ M_1 \rightarrow M_3 \rightarrow M_2 \} \), \( Ch(M_2) = \{ M_2 \rightarrow M_1 \rightarrow M_3 \} \), \( Ch(M_3) = \{ M_3 \rightarrow M_2 \rightarrow M_1 \} \), \( Ch(M_4) = \{ M_4 \rightarrow M_3 \rightarrow M_2 \} \) and \( Ch(M_5) = \{ M_5 \rightarrow M_2 \rightarrow M_1 \} \). All the arguments are defensible since there is no justified argument. Thus, \( b, \neg b, \neg a \) and \( a \) are the outcomes.

**Definition 7.3.21** Let \( M^* \) be the set of all defensible and justified arguments in \( M \). Then \( w \) is a maximal-coherent argument set iff \( w \) is the set \( \{ M_1, M_2, \ldots, M_n \} \) of arguments in \( M^* \), such that

- \( Cons(M_1 \cup M_2 \cup \ldots \cup M_n \cup \{ F \}) \) is consistent and
- \( \neg \exists_{M \in M^* \setminus w}(Cons(M_1 \cup M_2 \cup \ldots \cup M_n \cup \{ F \} \cup M) \) is consistent).

\( W \) is defined as the set of all maximal-coherent argument sets \( w \subseteq M \). \( Out(w) \) is defined as the set of all outcomes of the arguments \( M \in w \).

Consider example 7.3.20. Then \( W = \{ w_1, w_2, w_3 \} \) with \( w_1 = \{ M_1, M_4, M_5 \} \), \( w_2 = \{ M_2, M_4 \} \) and \( w_3 = \{ M_3, M_5 \} \). It follows that \( Out(w_1) = \{ a, b \} \), \( Out(w_2) = \{ a, \neg b \} \) and \( Out(w_3) = \{ \neg a, b \} \). There is no \( w \in W \) with \( Out(w) = \{ \neg a, b \} \), since \( M_3 \) is the argument for \( \neg a \) and \( M_3 \cup \{ F \} \models b \). Thus, if \( \neg a \in Out(w) \), then \( b \) must also be an element in \( Out(w) \), since otherwise we cannot derive \( \neg a \): the literal \( b \) is necessary to derive \( \neg a \). In other words, if an argument \( M \) is a subset of a maximal-coherent argument set \( w \), then all outcomes of the subarguments of \( M \) are elements of the set \( Out(w) \).

**Corollary 7.3.22** Let \( w \in W \). Then for all justified arguments \( M \) in \( M \) it holds that \( M \in w \).

The following definition is needed for the definition of arguments of norms in the next section.

**Definition 7.3.23** Let \( W \) be the set of rules, \( w \in W \) and \( w = \{ M_1, M_2, \ldots, M_n \} \), then \( W(w) \) is defined as the maximal-coherent set of rules in \( W \) with respect to \( w \) and \( F \). Let \( \phi \Rightarrow \phi_1 \in W \), then

\[ \phi \Rightarrow \phi_1 \in W(w) \iff \{ \phi \Rightarrow \phi_1 \} \cup M_1 \cup M_2 \cup \ldots \cup M_n \cup \{ F \} \text{ is coherent.} \]

Note that all rules in the arguments in \( w \) are in \( W(w) \).

**Example 7.3.24**

(1) \( c \Rightarrow a \)

(2) \( d \Rightarrow \neg a \)

(3) \( d \Rightarrow b \)

(4) \( \neg c \Rightarrow \neg b \)

\( F: c \land d \)

with (4) \( \prec (3) \prec (2) \prec (1) \).
Then $w = \{\{c \Rightarrow a\}, \{d \Rightarrow b\}\}$ and $W(w) = \{c \Rightarrow a, \neg c \Rightarrow \neg b, d \Rightarrow b\}$. Rule $d \Rightarrow \neg a$ is not an element of $W(w)$, since the rule is incoherent with $\{c \Rightarrow a\} \cup \{d \Rightarrow b\} \cup \{F\}$. Furthermore, note that $\neg c \Rightarrow \neg b$ is an element of $W(w)$, though not of an argument in $w$. We will see in the following section that such a rule can be applicable for the derivation of a certain norm.

### 7.4 Norms

Defeasible deontic reasoning is based on five notions: the notion of applicable norms based on a set $w \in W$ (definition 7.4.1) and the same four notions of defeasible reasoning for rules (definitions 7.4.3, 7.4.8, 7.4.10 and 7.4.12).

We also define maximal-coherent argument sets for norms that we will use for the definition of violated obligations in subsection 7.4.2. At the end of this section, we give an example of defeasible deontic reasoning with violated norms.

**Definition 7.4.1** Let $w \in W$, $w = \{M_1, M_2, \ldots, M_n\}$ and $a_1 \land a_2 \land \ldots \land a_n \Rightarrow \psi$ be a conditional norm in $\Delta$. Then

$$\psi \in \Delta(w, F) \text{ iff } \forall i \in \{1, 2, \ldots, n\} M_1 \cup M_2 \cup \ldots \cup M_n \cup \{F\} \models a_i.$$ 

$\psi$ is called an applicable norm in $\Delta$ with respect to $w$ and $F$. Thus, $\Delta(w, F)$ is the set of all applicable norms in $\Delta$ with respect to $w$ and $F$.

**Example 7.4.2**

$W: (1) \ a \Rightarrow b \quad \Delta: \ (1') \ b \Rightarrow O(\neg a) \quad F: \ a \land e \land h$

$\quad (2) \ b \Rightarrow c \quad \ (2') \ d \land h \Rightarrow O(i \lor l)$

$\quad (3) \ e \Rightarrow d \quad \ (3') \ g \Rightarrow O(b)$

$\quad (4) \ f \Rightarrow g$

$\quad (5) \ h \Rightarrow \neg c$

with $(5) \prec (4) \prec (3) \prec (2) \prec (1)$ and $(3') \prec (2') \prec (1')$.

The arguments are:

$M_1 = \{a \Rightarrow b\}$;

$M_2 = \{a \Rightarrow b, b \Rightarrow c\}$;

$M_3 = \{e \Rightarrow d\}$;

$M_4 = \{h \Rightarrow \neg c\}$.

It is easy to see that $M_1, M_2$ and $M_3$ are justified arguments, since they are not defeated by an argument. $M_4$ is defeated by $M_2$, thus $M_4$ is an overruled argument. There is one maximal-coherent argument set $w$: $\{M_1, M_2, M_3\}$. Now we can give the applicable norms:
1. \( O(\neg a) \in \Delta(w, F) \), since \( M_1 \cup M_2 \cup M_3 \cup \{F\} \models b \);  
2. \( O(i \lor l) \in \Delta(w, F) \), since \( M_1 \cup M_2 \cup M_3 \cup \{F\} \models d \) and \( M_1 \cup M_2 \cup M_3 \cup \{F\} \models h \);  
3. \( O(b) \notin \Delta(w, F) \), since \( M_1 \cup M_2 \cup M_3 \cup \{F\} \models g \).

Thus, \( \Delta(w, F) = \{O(\neg a), O(i \lor l)\} \).

For \( \Delta(w, F) \), we use SDL with the axiom schemata and inference rule given in chapter 1. Furthermore, we add the following inference rule to SDL:\(^6\)

\[(ROM2)\]

\[
O(p_1), O(p_2), \ldots, O(p_n), p_1 \land p_2 \land \ldots \land p_n \Rightarrow q
\]

\[O(q)\]

Analogous with the definition of arguments for rules, we define arguments for norms.

**Definition 7.4.3** Let \( N \subseteq \Delta(w, F) \), \( \psi \) a norm and \( N \cup \{F_b\} \cup W(w) \) consistent. Then \( N \) explains \( \psi \) minimally iff

\[
\begin{align*}
&\bullet \{F_b\} \cup W(w) \cup N \models \psi \text{ and} \\
&\bullet \neg \exists_{\psi \in N} \{F_b\} \cup W(w) \cup N \setminus \{\psi_1\} \models \psi.
\end{align*}
\]

We call \( N \) a minimally explaining set or an argument. The set of all arguments will be denoted as \( N(w, F) \). The \( \psi \)-relevant set of \( \Delta(w, F) \), denoted by \([\psi]N(w, F)\) is the set of all arguments that explain \( \psi \) minimally.

Let \( N \subseteq N(w, F) \). If there is an argument for \( \psi \), thus \([\psi]N \neq \emptyset\), then \( \psi \) is called an outcome.

We do not use \( F \), but \( F_b \) in this definition, because otherwise we would derive ridiculous conclusions. Consider the following example.

**Example 7.4.4**

(1) \( O(a) \): It is obligatory to go to school.  
(2) \( O(b) \): It is obligatory to behave.  
\( F = F_c: a \land \neg b: \text{You go to school and you do not behave, with (2) } < (1) \).

Now we can derive \( O(\neg b) \) from \( O(a) \) and \( F_c \), because \( \{F_c\} \models a \rightarrow \neg b \) and by \( (ROM) \) \( O(a) \rightarrow O(\neg b) \). Thus, if we use \( F \) instead of \( F_b \), then \( \{O(a)\} \) would be an argument for the outcome \( O(\neg b) \), which is not a desirable result.

\(^6\)For a brief discussion of this rule, see section 7.5.
Definition 7.4.5 Let $N \subseteq \mathcal{N}(w, F)$ and $N_1, N_2 \in N$. Then $N_1$ and $N_2$ are in conflict iff

$$\{F_b\} \cup W(w) \cup N_1 \vdash \psi \text{ and } \{F_b\} \cup W(w) \cup N_2 \vdash \neg \psi.$$ 

Example 7.4.6

(1) $O(a \lor b)$
(2) $O(\neg a)$
(3) $O(c \lor \neg b)$
(4) $O(\neg c)$
with (4) $\prec$ (3) $\prec$ (2) $\prec$ (1).

Let $N_1 = \{O(a \lor b), O(c \lor \neg b), O(\neg c)\}$ and $N_2 = \{O(\neg a)\}$ be arguments in $N$. Then $N_1$ is an argument for $O(a)$ and $N_2$ an argument for $\neg O(a)$, since $O(\neg a) \rightarrow \neg O(a)$ is deduced by axioms (OD) and (OC). Thus, $N_1$ and $N_2$ are in conflict.

Example 7.4.7

(1) $O(a)$
(2) $O(\neg b)$
(2) $\neg (1)$ and $W(w) = \{a \Rightarrow b\}$.

Let $N_3 = \{O(a)\}$ and $N_4 = \{O(\neg b)\}$ be arguments in $N$. Then $N_3$ is an argument for $O(b)$, since $\{a \Rightarrow b\} \cup N_3 \vdash O(b)$ and $N_4$ is an argument for $\neg O(b)$. Thus, $N_3$ and $N_4$ are in conflict.

### 7.4.1 Defeasibility

Because the arguments in $N(w, F)$ can be in conflict, we resolve these deontic conflicts by adopting defeasible reasoning. The defeasibility of norms in $\Delta(w, F)$ determines the validity of these norms. The validity depends on the rules in $w$. For instance, let $\Delta(w, F) = \{O(a), O(b)\}$, $O(b) \prec O(a)$ and $w = \{a \Rightarrow \neg b\}$. Then $\{O(a)\}$ is an argument for $\neg O(b)$, since $w \cup \{O(a)\} \vdash \neg O(b)$. Now we say that $\{O(a)\}$ defeats $\{O(b)\}$, and that $O(b)$ is not valid. Suppose that $w = \emptyset$, then $O(a)$ and $O(b)$ are both valid, because $\{O(a)\}$ and $\{O(b)\}$ are not in conflict.

Definition 7.4.8 Let $N_1$ and $N_2$ be arguments. Then $N_1$ is defeated by $N_2$ ($N_1 \prec^* N_2$) iff $N_1$ and $N_2$ are in conflict and

$$\exists \psi_1 \in N_1 \forall \psi_2 \in N_2 \psi_1 \preceq \psi_2.$$ 

The main difference between this definition and definition 7.3.5 is that here we look at the statements of the two arguments with the lowest priority, and in definition 7.3.5 we
looked at the statements of the two arguments with conflicting conclusions. Note that it is possible that $N_1 \prec^* N_2$ and $N_2 \prec^* N_1$ both hold, but only if the statements with the lowest priority of the two arguments have the same priority. $\prec^*$ is not transitive and not asymmetric.

**Example 7.4.9** Consider example 7.4.6. $N_1$ is defeated by $N_2$, since $O(\neg c)$ has a lower priority than all the norms in $N_2$: $O(\neg c) \preceq O(\neg a)$.

Now consider example 7.4.7. $N_4$ is defeated by $N_3$, since $O(\neg b) \preceq O(a)$. If (1) $\sim$ (2), then $N_3 \prec^* N_4$ and $N_4 \prec^* N_3$.

The following three definitions for arguments in $\mathcal{N}$ concerning defeasibility chains, $Ch(\mathcal{N})$ and justified, defensible and overruled arguments are exactly the same as the definitions for arguments in $\mathcal{N}$ (cf. definitions 7.3.7, 7.3.8 and 7.3.9).

**Definition 7.4.10** Let $\mathcal{N} \subseteq \mathcal{N}(w, F)$. A defeasibility chain is a sequence of arguments in $\mathcal{N}$: $N_1 \prec^* N_2 \prec^* \ldots \prec^* N_n$ with the following conditions:

- $\forall k, l = 1, 2, \ldots, n \wedge k < n N_k \not\subseteq N_l$;
- $\exists N_{n+1} \in \mathcal{N} \forall (N_1, \ldots, N_n \not\subseteq N_{n+1}) \rightarrow (N_n \prec^* N_{n+1})$.

We define $Ch(\mathcal{N})$ as the set of all defeasibility chains in $\mathcal{N}$.

**Definition 7.4.11** Let $\mathcal{N} \subseteq \mathcal{N}(w, F)$. Then $Ch(\mathcal{N})$ is the set of all defeasibility chains in $Ch(\mathcal{N})$ starting with $N$.

**Definition 7.4.12** Let $N \in \mathcal{N}(w, F)$. Then

- $N$ is a justified argument iff $Ch(\mathcal{N}) = \{N\}$ or for all chains $N \prec^* N_1 \prec^* \ldots \prec^* N_n \in Ch(\mathcal{N})$ it holds that

$$n \text{ is even } \wedge \neg \exists N' \in \mathcal{N} N_n \prec^* N' \wedge \forall_k \text{ is even}(N_k \text{ is a justified argument assuming } N \text{ is a justified argument}).$$

- $N$ is an overruled argument iff there is a chain $N \prec^* N_1 \prec^* \ldots \prec^* N_n \in Ch(\mathcal{N})$

$$n \text{ is odd } \wedge \neg \exists N' \in \mathcal{N} N_n \prec^* N'.$$

- $N$ is a defensible argument iff $N$ is neither a justified argument nor an overruled argument.

Note that $\neg \exists N' \in \mathcal{N} N_n \prec^* N'$ is equivalent to $Ch(\mathcal{N}) = \{N_n\}$. 
Example 7.4.13 Consider example 7.4.6. Let \( N = \{N_1, N_2, \ldots, N_6\} \), with
\[
\begin{align*}
N_1 &= \{O(a \lor b), O(c \lor \neg b), O(\neg c)\} \in [O(a)]N \\
N_2 &= \{O(\neg a)\} \in [\neg O(a)]N \\
N_3 &= \{O(a \lor b), O(\neg a)\} \in [O(b)]N \\
N_4 &= \{O(c \lor \neg b), O(\neg c)\} \in [\neg O(b)]N \\
N_5 &= \{O(a \lor b), O(c \lor \neg b), O(\neg a)\} \in [O(c)]N \\
N_6 &= \{O(\neg c)\} \in [\neg O(c)]N
\end{align*}
\]

Argument \( N_1 \) is defeated by justified argument \( N_2 \), since \( O(\neg c) \prec O(\neg a) \) and \( Ch(N_2) = \{N_2\} \), thus \( N_1 \) is an overruled argument. \( N_3 \) and \( N_5 \) are justified arguments, since \( Ch(N_3) = \{N_3\} \) and \( Ch(N_5) = \{N_5\} \). \( N_4 \) and \( N_6 \) are overruled arguments, since \( N_4 \prec^* N_5 \in Ch(N_4) \) and \( N_6 \prec^* N_5 \in Ch(N_6) \).

If \( (4) \prec (3) \prec (2) \prec (1) \), then \( N_5 \) and \( N_6 \) are defensible arguments, since \( Ch(N_5) = \{N_5 \prec^* N_6\} \) and \( Ch(N_6) = \{N_6 \prec^* N_5\} \).

Example 7.4.14 Consider example 7.4.7. \( N_4 \) is defeated by justified argument \( N_3 \) and \( Ch(N_3) = \{N_3\} \), thus \( N_4 \) is an overruled argument.

Definition 7.4.15 Let \( N \subseteq N(w, F) \) and \( N^* \) be the set of all defensible and justified arguments in \( N \). Then \( o \) is a maximal-consistent argument set iff \( o \) is the set \( \{N_1, N_2, \ldots, N_n\} \) of arguments in \( N^* \), such that

- \( N_1 \cup N_2 \cup \ldots \cup N_n \cup \{F_b\} \cup W(w) \) is consistent and
- \( \neg \exists_{N \in N^*} \forall_o (N_1 \cup N_2 \cup \ldots \cup N_n \cup \{F_b\} \cup W(w) \cup N \) is consistent).}

\( O(N) \) is defined as the set of all maximal-consistent argument sets \( o \). \( \text{Out}(o) \) is defined as the set of all outcomes of the arguments \( N \in o \).

Corollary 7.4.16 Let \( o \in O(N) \). Then for all justified arguments \( N \) in \( N \) it holds that \( N \in o \).

Example 7.4.17 Consider example 7.4.13 with the justified arguments \( N_2, N_3 \) and \( N_5 \). There is one maximal-consistent argument set \( o: o = \{N_2, N_3, N_5\} \).

If \( (4) \prec (3) \prec (2) \prec (1) \), then \( N_5 \) and \( N_6 \) are defensible (see example 7.4.13). Then there are two maximal-consistent argument sets: \( o_1 = \{N_2, N_3, N_5\} \) and \( o_2 = \{N_2, N_3, N_6\} \), thus \( O(N) = \{o_1, o_2\} \).
7.4 Norms

7.4.2 Violation

An obligation is violated if and only if the obligation is not fulfilled. In SDL, we can represent the violated obligation by $O(p) \land \neg p$. We define the violated obligation in our theory analogously.

**Definition 7.4.18** Let $\mathcal{N} \subseteq \mathcal{N}(w, F)$, $o \in O(\mathcal{N})$, $o = \{N_1, \ldots, N_n\}$, $w = \{M_1, \ldots, M_l\}$ and $N_1 \cup N_2 \cup \ldots \cup N_n \cup W(w) \cup \{F_1\} \vdash O(\phi)$. The norm $O(\phi)$ is violated iff $M_1 \cup M_2 \cup \ldots \cup M_l \cup \{F\} \models \neg \phi$. The set of violated norms will be denoted as $V(o)$.

**Example 7.4.19** Let $o = \{\{O(a)\}, \{O(b)\}\}$, $W(w) = \{a \Rightarrow c\}$, $\{F_b\} = w = \emptyset$ and $F_c: \neg b \land \neg c$. Then $O(c) \in V(o)$, since $\{O(a)\} \cup \{O(b)\} \cup W(w) \vdash O(c)$ and $\{F\} \vdash \neg c$ and $O(b) \in V(o)$, since $\{O(a)\} \cup \{O(b)\} \cup W(w) \vdash O(b)$ and $\{F\} \vdash \neg b$.

**Definition 7.4.20** Let $\mathcal{N} \subseteq \mathcal{N}(w, F)$, $o \in O(\mathcal{N})$ and $V(o)$ be the set of violated norms. Then $N \in o$ is a violated argument iff

$$\exists \psi \in V(o) N \cup W(w) \cup \{F_b\} \vdash \psi.$$  

$\mathcal{N}(V(o))$ is the set of all the violated arguments with respect to $V(o)$.

**Example 7.4.21** Consider example 7.4.6:

1. $O(a \lor b)$
2. $O(\neg a)$
3. $O(c \lor \neg b)$
4. $O(\neg c)$

with (4) $\prec$ (3) $\prec$ (2) $\prec$ (1), $W(w) = \{F_b\} = \emptyset$ and $F_c: \neg c$. Suppose further (see example 7.4.13) that $\mathcal{N} = \{N_1, N_2, \ldots, N_6\}$, with

$$
\begin{align*}
N_1 & = \{O(a \lor b), O(c \lor \neg b), O(\neg c)\} \in [O(a)]\mathcal{N} \\
N_2 & = \{O(\neg a)\} \in [\neg O(a)]\mathcal{N} \\
N_3 & = \{O(a \lor b), O(\neg a)\} \in [O(b)]\mathcal{N} \\
N_4 & = \{O(c \lor \neg b), O(\neg c)\} \in [\neg O(b)]\mathcal{N} \\
N_5 & = \{O(a \lor b), O(c \lor \neg b), O(\neg a)\} \in [O(c)]\mathcal{N} \\
N_6 & = \{O(\neg c)\} \in [\neg O(c)]\mathcal{N}.
\end{align*}
$$

From examples 7.4.13 and 7.4.17 it follows that $o = \{N_2, N_3, N_5\}$. $O(c) \in V(o)$, since $N_5 \not\vdash O(c)$ and $\{F\} \not\vdash \neg c$. Therefore, $N_5$ is a violated argument. $N_2$ and $N_3$ are no violated arguments, since $N_2 \not\vdash O(c)$ and $N_3 \not\vdash O(c)$.

**Definition 7.4.22** Let $\mathcal{N} \subseteq \mathcal{N}(w, F)$ and $\psi_1, \ldots, \psi_n \in \Delta(w, F)$. Then

$$\mathcal{N} \setminus \{\psi_1, \ldots, \psi_n\} = \bigcup\{N | N \in \mathcal{N} \land \forall i=1,\ldots,n \psi_i \not\in N\}.$$
Thus \( N \setminus \star \{ \psi_1, \ldots, \psi_n \} \) is the set of arguments in \( N \) without the arguments containing a norm of the set \( \{ \psi_1, \ldots, \psi_n \} \).

**Example 7.4.23** A programme committee requires that conference submissions (papers) are sent in by mail. However, if your paper is not sent by mail, then your paper should be sent by fax. And if no faxmachine is available, one should try to send it by e-mail. The facts are that the paper is sent by fax and if you send your paper by fax, then you do not send the paper by e-mail. The formalisation is as follows:

\[
W: (1) \text{email} \Rightarrow \neg \text{mail} \quad \Delta: (1') O(\text{mail}) \quad F_c: \text{fax}
\]
\[
(2) \text{fax} \Rightarrow \neg \text{mail} \quad (2') \neg \text{mail} \Rightarrow O(\text{fax}) \quad F_b: \text{fax} \rightarrow \neg \text{email}
\]
\[
(3') \neg \text{fax} \Rightarrow O(\text{email})
\]

with \( (2) \prec (1) \) and \( (3') \prec (2') \prec (1') \).

Let us consider this example. If the paper is sent by mail, then there is no violation; this is the best situation. However, the norm \( O(\text{mail}) \) is violated, because the paper is not sent by mail. Then the second-best situation is that your paper is sent by fax. We can formalise this process as follows:

\[
\begin{align*}
Out(w) &= \{ \neg \text{mail} \}, W(w) = \{ \text{fax} \Rightarrow \neg \text{mail}, \text{email} \Rightarrow \neg \text{mail} \} \\
\Delta(w, F) &= \{ O(\text{mail}), O(\text{fax}) \} \\
N(w, F) &= \{ N_1, N_2 \}, \text{ with } N_1 = \{ O(\text{mail}) \} \text{ and } N_2 = \{ O(\text{fax}) \}.
\end{align*}
\]

Note that \( O(\text{email}) \) is not an element of \( \Delta(w, F) \), since it is not an applicable norm: \( w \cup \{ F \} \not\models \neg \text{fax} \).

\( N_1 \) is a justified argument, since \( Ch(N_1) = \{ N_1 \} \), and \( N_2 \) is an overruled argument, since it is defeated by \( N_1 \). \( N_1 \) and \( N_2 \) are in conflict, since \( \{ F_b \} \cup W(w) \cup N_1 \vdash O(\text{mail}) \) and \( \{ F_b \} \cup W(w) \cup N_2 \vdash \neg O(\text{mail}) \) and \( O(\text{fax}) \prec O(\text{mail}) \). Thus, \( o = \{ N_1 \} \).

The norm \( O(\text{mail}) \) has a higher priority than the norm \( O(\text{fax}) \). However, the norm \( O(\text{mail}) \) is violated:

\[
V(o) = \{ O(\text{mail}) \} \text{ and } (N(w, F))(V(o)) = \{ N_1 \}.
\]

Now we consider the norms again without the violated norm, since they can become valid because of the violation of the norm \( O(\text{mail}) \). Let \( N' = N(w, F) \setminus \star \{ O(\text{mail}) \} \). Then \( N = \{ N_2 \} \). Argument \( N_2 \) is a justified argument. Let \( o' \in O(N) \), then it follows that \( o' = \{ N_2 \} \). \( N_2 \) (with \( O(\text{fax}) \) as an outcome) is a justified and non-violated argument with respect to the violation of the norm \( O(\text{mail}) \). Thus, sending in your paper by fax is the second-best situation.
7.5 Conclusions

In this chapter, we presented a theory of defeasible deontic reasoning dealing with some very common problems of other approaches:

- defeasibility between groups of conditional norms;
- the combination of defeasibility of rules and norms;
- the absence of the notion of permission.

However, we do not claim that our theory is completely free from the above-mentioned problems. Adopting defeasible reasoning for non-deontic constraints (rules) is not trivial, like the study of the validity of deducing \( O(b) \) from \( O(a) \) and \( a \Rightarrow b \). This corresponds with the choice of rule (ROM2):

\[
\frac{O(p_1), O(p_2), \ldots, O(p_n), p_1 \land p_2 \land \ldots \land p_n \Rightarrow q}{O(q)}
\]

However, it is also possible to add the default rule schema (RK)

\[
(p_1 \land p_2 \land \ldots \land p_n \Rightarrow q) \rightarrow (O(p_1) \land O(p_2) \land \ldots O(p_n) \Rightarrow O(q)),
\]

which also allows defaults between norms.

The consequence of the use of rule (ROM2) is that norms are dependent on rules and that rules are not dependent on norms. This means that rules have a higher priority than norms. Thus, an argument of rules cannot be defeated by an argument of norms. If we had not made this distinction between rules and conditional norms (i.e., if we had opted for (RK) instead of (ROM2)), we would have got, for example, the following situation:

\[
W \cup \Delta: \begin{array}{ll}
1) & O(a) \quad F: \emptyset \\
2) & O(b) \\
3) & a \Rightarrow \neg b \\
\end{array}
\]

with \( (3) \prec (2) \prec (1) \).

From \( O(a) \) and \( a \Rightarrow \neg b \) we can deduce \( O(\neg b) \). Thus, \( \{O(a), a \Rightarrow \neg b\} \) would be an argument for \( O(\neg b) \). Thus, the set \( \{O(a), O(b), a \Rightarrow \neg b\} \) is incoherent. We cannot deduce \( \neg(a \Rightarrow \neg b) \) from \( O(a) \) and \( O(b) \) with the (ROM2) rule. However, with (RK) we can derive \( \neg(a \Rightarrow \neg b) \) from \( O(a) \) and \( O(b) \):

- \( O(a) \land O(b) \rightarrow O(a) \land \neg O(\neg b); \)
- \( O(a) \land \neg O(\neg b) \rightarrow \neg(O(a) \lor O(\neg b)); \)
- \( \neg(O(a) \lor O(\neg b)) \rightarrow \neg(O(a) \rightarrow O(b)); \)
- \( \neg(O(a) \rightarrow O(b)) \rightarrow \neg(a \Rightarrow \neg b). \)

Therefore, \( \{O(a), O(b)\} \) is an argument for \( \neg(a \Rightarrow \neg b) \).\footnote{As a consequence, we allow negations of default rules. It is not trivial to decide on the meaning of these formulas.}
The advantage of replacing rule (ROM2) by a stronger rule is that rules and conditional norms do not have to be separated in the arguments, and that, by definition, rules do not have a higher priority than conditional norms, which is a consequence from the theory presented in this chapter. At first glance, this seems a good concept for solving the problem of the separation of rules and norms. However, this concept gives rise to some serious problems since the definition of defeating arguments for rules is different from the definition of norms. In the definition of defeating arguments for rules, we only look at single statements with conflicting final conclusions, whereas in the definition of norms we not only look at final conclusions but also at groups of statements deriving conflicting conclusions. Furthermore, the applicability of a conditional norm depends on the facts and the rules. This means that we have to separate rules and conditional norms. We leave this issue of the separation of arguments for rules and conditional norms in deontic reasoning as a future research topic.

Most deontic defeasible reasoning approaches are based on specificity considering the amount of relevant information or supporting evidence. Our approach is based on the more general idea of priority (authority). Other defeasibility criteria can easily be converted to some form of defeasibility on the basis of priority.
Chapter 8

Conclusion

Assuming that logic, especially deontic logic, can provide foundations for the construction of legal expert systems and knowledge based systems in law, the purpose of this thesis was to give a logical analysis of representation of legal rules and legal reasoning. We investigated three concepts with respect to deontic logic:

- the addition of (groups) of actors;
- the addition of authorities;
- reasoning with conflicting rules.

These concepts are necessary to deal with the problems discussed on the basis of a case of conflicting speed limits in chapter 1. Therefore, they are also necessary ingredients for any formalism that functions as a foundation for legal expert systems and knowledge based systems in law.

A first aim has been to clarify the addition of (groups of) actors and authorities in deontic logic in order to represent legal rules. This showed that the extensions of deontic logic have more expressive power and that formulas acquire new meanings, not expressible in SDL and PD_eL. A second aim has been to reason with conflicting legal rules. We investigated two reasoning approaches of this: first, the priority of the norms with the help of an authority hierarchy to determine which norm should be followed in cases of conflict, and second, defeasible deontic reasoning, i.e., reasoning with legal rules which are implicitly subject to exceptions, and reasoning with inconsistent information. Both aims have been reached, as we will summarise in this chapter. We can safely conclude that we can improve the representation of legal texts and of legal knowledge in general, and that we can formalise adequately legal reasoning by investigating deontic defeasible reasoning and reasoning with normative inconsistent information. Therefore, such an improvement and a formalisation form a basis for the improvement of legal expert systems and knowledge based systems in law.
This research was meant to be a contribution to developments initiated by others, partly by applying these developments to the legal domain and partly by adding something new to the developments themselves.

8.1 Summary

In chapter 2, we started with the presentation of $PDeL$ (propositional deontic logic). We saw that this system provides a very workable framework for reasoning with Ought-to-do statements, in contrast to $SDL$ (standard deontic logic), which enables us to reason with Ought-to-be statements.

In chapters 3 and 4, we presented formalisations of relativised deontic modalities. The result was that we distinguished several levels of the notions of obligation and permission. The first level was the level of the personal notions, the second level was the level of the general and unspecific notions, the third level was the level of the collective notions, and the fourth level was the level of strong and weak notions. The relations between these four levels were investigated and summarised in figures 3.7, 3.8, and 4.3.

The addition of (groups of) actors in $SDL$ resulted in a description of relativised deontic operators for Ought-to-be statements. Our approach avoided problems encountered in other approaches, especially, in the approach taken by Herrestad and Krogh. We showed that the problem in this approach could be smoothly solved by introducing the strong and weak obligations, on the basis of collective obligations. Interpreting the collective obligation as the strict collective obligation was essential, since the problem was not solved when interpreting the collective obligation as the weak collective obligation.

In the extension of $PDeL$ with groups of actors, we saw that the type of action is very important to indicate when a group fulfils an obligation; e.g., if a group has to perform a positive action, then some subset of that group has to perform that action, and if a group has to 'perform' a negative action, then every subset of that group has to perform that action.

The difference between the extensions of the systems $SDL$ and $PDeL$ with groups of actors is that in the extension of $SDL$ we are able to express that a group of actors or an actor has to accomplish a certain situation, and that in the extension of $PDeL$ we are able to express that a group of actors or an actor has to perform a certain action. We saw that in the case of the Penal Code, the formalisation of obligations as Ought-to-do statements fitted better than Ought-to-be statements. Thus, with regard to the Penal Code, the extension of $PDeL$ is to be preferred.

In chapter 5, we extended the system discussed in the chapters 3 and 4 with sets of authorities, which enabled us to express who enacted a certain norm. Furthermore, it enabled us to treat several classical issues: hierarchical norms, completeness of a legal
system and normative inconsistencies.

The theory we presented was a modification of Bailhache's theory. This theory was developed to obtain a deontic coherent system. Inherent to this theory is that we cannot express conflicts between enacted norms. However, we showed that this theory has some serious drawbacks, e.g., that we cannot express that an authority enacted a permission. To deal with these drawbacks, we chose for another approach: instead of the set $A$ of authorities modifying a norm, the set $A$ is treated as a modal operator. With the help of this change, the drawbacks disappeared from our theory. Furthermore, we saw that we could consistently formalise normative inconsistencies, i.e., conflicting norms enacted by sets of authorities.

In chapter 6, we saw that a normative system changes by promulgation (the introduction of a norm into a legal system) and derogation (the removal of a norm from a legal system), i.e., the dynamic character of law. These two legislative acts by authorities have two features in common: the modification of the norms applicable and the restriction of the competence of inferior authorities. Furthermore, we showed that not only derogation but also promulgation can lead to logical indeterminacy, in contrast to Alchourrón's statement. Since normative systems change by promulgation and derogation, normative inconsistencies may exist. To deal with these normative inconsistencies, we developed a theory on the basis of an ordering of the authorities with respect to their competence, in other words, an authority hierarchy. A drawback of this approach is that we can only deal with norms and, more precisely, with unconditional norms.

Finally, (postulated) universality was discussed in chapter 5, and the way in which universality can be reached by a closure rule, such as *nulla poena sine lege*. We argued that postulated universality does not restrict individual freedom and responsibility: the individual's norms awareness is restricted by the extension of the whole of legal prohibitions.

In chapter 7, we presented a theory of defeasible deontic reasoning dealing with some very common problems of other approaches: defeasibility between *groups of conditional norms*; the combination of defeasibility of rules and norms; the absence of the notion of permission. In this approach, defeasibility occurs in two phases. The first phase concerns the defeasibility of rules, and the second phase concerns the defeasibility of norms with respect to the first phase. A consequence of this approach is that norms are dependent on rules and that rules are not dependent on norms. This means that rules have a higher priority than norms.

### 8.2 The Dutch Traffic Regulation 1990 revisited

In this section, we formalise some cases related to the case concerning speed limits discussed in chapter 1. Before we present the cases, we will give the formal representation of arts.
20, 21 and 22 of the Dutch Traffic Regulation 1990 concerning speed limit. We use the following abbreviations:

- \( Q_1(i) \): \( i \) is within built-up areas
- \( Q_2(i) \): \( i \) is outside built-up areas
- \( Q_3(i) \): \( i \) is on motorways
- \( Q_4(i) \): \( i \) is on national routes

\( a_l \): to drive \( l \) km/h, for \( l = 0, 1, 2, \ldots, 300 \)

\( b_i \): \( a_{i+1} \cup a_{i+2} \cup \ldots \cup a_{300} \)

\( c(i) \): to give right of way to \( i \).

\( M \): the group of motor-vehicle drivers
\( C_1 \): the group of moped drivers
\( C_2 \): the group of motorised wheelchair drivers
\( E_1 \): the group of lorry drivers
\( E_2 \): the group of bus drivers
\( E_3 \): the group of tractor drivers
\( E_4 \): the group of construction vehicle drivers
\( E_5 \): the group of motor vehicles with trailers drivers

The formalisation of the articles is as follows:

**Article 20.** Within built-up areas the following speed limits hold:
- for motor vehicles 50 km/h;
- for mopeds and motorised wheelchairs 30 km/h.

\[ \forall i \in M (Q_1(i) \Rightarrow F(i : b_{50})) \]

\[ \forall i \in C_1 \cup C_2 (Q_1(i) \Rightarrow F(i : b_{30})) \]

**Article 21.** Outside built-up areas the following speed limits hold:

a. for motor vehicles on motorways 120 km/h, on national routes 100 km/h and on other roads 80 km/h;

b. for mopeds and motorised wheelchairs 40 km/h.

\[ \forall i \in M ((Q_2(i) \land Q_3(i)) \Rightarrow F(i : b_{120})) \]

\[ \forall i \in M ((Q_2(i) \land Q_4(i)) \Rightarrow F(i : b_{100})) \]

\[ \forall i \in M ((Q_2(i) \land \neg(Q_3(i) \lor Q_4(i))) \Rightarrow F(i : b_{80})) \]

\[ \forall i \in C_1 \cup C_2 (Q_2(i) \Rightarrow F(i : b_{40})) \]

**Article 22.** Assuming that, in accordance with other sections, no lower speed limit holds, the following speed limits hold for the following vehicles:
8.2 The Dutch Traffic Regulation 1990 revisited

a. for lorries, buses and motor vehicles with trailers 80 km/h;
b. for tractors and construction vehicles 25 km/h.

$$\forall i \in E_1 \cup E_2 \cup E_3 \ F(i : b_{80})$$

$$\forall i \in E_3 \cup E_1 \ F(i : b_{25}).$$

Speed limit

The main case in chapter 1 was as follows. On a national route, a lorry driver drove at a speed of 96 km/h. The lorry driver was imposed an administrative sanction on the ground of 'a lorry exceeding the speed limit by 15 to 20 km/h'. However, the lorry driver was of the opinion that, on the road in question, traffic signs indicating a speed limit of 100 km/h were in force, and that, therefore, no sanctionable act had been committed, for traffic signs override traffic rules according to art. 63 of the Dutch Traffic Regulation 1990.

In art. 22 of the Dutch Traffic Regulation 1990, it is laid down that for lorries the special speed limit of 80 km/h holds, which can be formalised as follows:

$$\forall i \in E_1 \ F(i : b_{80}).$$

According to traffic sign A1(100), it is forbidden to drive faster than 100 km/h:

$$\forall i \in M \ F(i : b_{100}).$$

Let $i_1$ be the lorry driver. Then, the following statement holds

$$F(i_1 : b_{80}) \land F(i_1 : b_{100}),$$

since $i_1 \in E_1$ and $i_1 \in M$. Note that $E_1 \subseteq M$. This formula is equivalent to

$$F(i_1 : b_{80});$$

thus, the two prohibitions are not in conflict. Hence, art. 63 is not applicable in this case, since traffic signs override traffic rules in as far as these rules are incompatible with the signs.

However, as we have seen in chapter 1, the letter of the law with regard to the speed limit is not in agreement with the spirit of the law in the Dutch Traffic Regulation 1990. In built-up areas, there are, for example, A1(70) traffic signs on circular roads, indicating that it is prohibited to drive faster than 70 km/h:

$$\forall i \in M \ F(i : b_{70}).$$

---

1 $F(i_1 : b_{80}) \land F(i_1 : b_{100}) \equiv F(i_1 : a_{a1} \cup \ldots \cup a_{a300}) \land F(i_1 : a_{a101} \cup \ldots \cup a_{a300}) \equiv F(i_1 : a_{a81}) \land \ldots \land F(i_1 : a_{a300}) \land F(i_1 : a_{101}) \land \ldots \land F(i_1 : a_{300}) \equiv F(i_1 : a_{81}) \land \ldots \land F(i_1 : a_{300}) \equiv F(i_1 : b_{80}).$
Many drivers will, and justifiably, take this sign to mean that it is permitted to drive at a speed of 70 km per hour. This is, however, in disagreement with the motivation by the district court judge and the Advocate-General. According to art. 20 of the Dutch Traffic Regulation 1990, a speed limit of 50 km/h holds for motor vehicles inside built-up areas. Thus, the following norm holds

$$\forall_{i \in M} F(i : b_{50})$$

In this case, the following combination of the two general prohibitions holds:

$$\forall_{i \in M} F(i : b_{70}) \land \forall_{i \in M} F(i : b_{50}),$$

which is equivalent to

$$\forall_{i \in M} F(i : b_{50})$$

In chapter 1, we solved this problem by replacing articles 20, 21 and 22 by the following article

The following speed limits hold:

a. for motor vehicles on motorways 120 km/h, on national routes 100 km/h and on other roads 80 km/h;

b. for lorries, buses and motor vehicles with trailers 80 km/h;

c. for mopeds and motorised wheelchairs inside built-up areas 30 km/h and outside built-up areas 40 km/h;

d. for tractors and construction vehicles 25 km/h.

Thus, the speed limit of 50 km/h for motor vehicles inside built-up areas is removed, and speed has to be regulated by means of A1 signs. Then, the above case can be formalised as follows:

$$\forall_{i \in M} F(i : b_{80}) \land \forall_{i \in M} F(i : b_{70}),$$

which is equivalent to

$$\forall_{i \in M} F(i : b_{70}).$$

This alternative has two advantages: first, the rules can be applied consistently, and second, the legislator's wish is expressed in a clearer way. The legislator's intention is stated precisely in these rules. In the Dutch Traffic Regulation 1990, this is definitely not the case.

**Right of way**

In chapter 1, we saw that traffic signs and traffic rules can be incompatible in cases concerning 'right of way'. We gave the following example. Car driver $i_1$ is on a major road and approaches a junction, where car driver $i_2$ approaches from the right.
8.2 The Dutch Traffic Regulation 1990 revisited

On the grounds of the rule in art. 15, \( i_1 \) has to give right of way to \( i_2 \); on the grounds of the right-of-way signs (A6 and A9) and the principle of trust, \( i_1 \) does not have to give right of way to \( i_2 \). In this formulation, \( i_1 \) both has to give right of way and does not have to give right of way, which is clearly a case of incompatibility, which can be expressed as follows on the basis of the principle of trust:

\[
O(i_1 : c(i_2)) \land \neg O(i_1 : \underline{c(i_2)}).
\]

It is obvious, that here we have an inconsistency. We can express this consistently with the addition of articles and signs, in the same way as the addition of authorities in chapter 5:

\[
\text{art.15} : O(i_1 : \underline{c(i_2)}) \land \text{sign(A6/9)} : \neg O(i_1 : \underline{c(i_2)}),
\]

meaning that according to art. 15, \( i_1 \) is obliged to give right of way to \( i_2 \), and according to signs A6 and A9, \( i_1 \) is not obliged to give right of way to \( i_2 \). In this formalisation, there is clearly a case of incompatibility, thus art. 63 is applicable. Since signs A6 and A9 have a higher priority than art. 15, the norm \( \neg O(i_1 : c(i_2)) \) is followed in this situation.

Just as we define an authority hierarchy, we can define an article/sign hierarchy, i.e., an ordering of articles and signs. This is not only a hierarchy concerning two sets, the set of articles and the set of traffic signs; articles are also ordered among themselves. For example, according to art. 15.1 a driver has to give right of way to another driver who approaches the junction from the right, and according to art. 15.2 a driver on a dirt road has to give right of way to another driver on a paver road. Art. 15.2 has a higher priority than art. 15.1 in case of incompatibility.

Emergency service vehicles

As we saw in chapter 1, there is one case in which art. 63 has a nasty consequence. A passenger car driving on a major road approaches a junction, and at the same time a police car with flashing light and sirens approaches from the left. On the ground of art. 50, driver \( i_1 \) of the passenger car has to give way to driver \( i_2 \) of the police car, but on the ground of art. 15 and right-of-way signs A6 and A9, \( i_2 \) has to give way to \( i_1 \). This can be formalised as follows on the basis of the principle of trust:

\[
O(i_1 : \underline{c(i_2)}) \land \neg O(i_1 : \underline{c(i_2)}).
\]

It is obvious, that we have an inconsistency; however, this can be consistently expressed with the addition of articles and signs:

\[
\text{art.50} : O(i_1 : \underline{c(i_2)}) \land \text{art.15} : \neg O(i_1 : \underline{c(i_2)}) \land \text{sign(A6/9)} : \neg O(i_1 : \underline{c(i_2)}).
\]

Art. 50 has a higher priority than art. 15, and signs A6 and A9 have a higher priority than art. 50. Hence, on the ground of art. 63, the norm \( \neg O(i_1 : c(i_2)) \) is chosen. We solved this problem by replacing art. 50 by:
Yielding right of way by road users to drivers of emergency service vehicles overrides traffic lights and traffic signs and rules that regulate right of way.

Then, the above case can be formalised as follows:

\[
\text{e.s.v.} : O(i_1 : c(i_2)) \land \text{art.15} : \neg O(i_1 : c(i_2)) \land \text{sign(A6/9)} : \neg O(i_1 : c(i_2)).
\]

Now, norm \(O(i_1 : c(i_2))\) is chosen, since emergency service vehicles (e.s.v.) have a higher priority than signs and articles.

The above cases show that we can smoothly represent articles in the Dutch Traffic Regulation 1990 in our 'extended' deontic logic. Using this representation, we can analyse legal reasoning with conflicting norms or inconsistent information.

### 8.3 Further research

This thesis invites further research on several areas:

- The incorporation and implementation of the theories discussed in this thesis, in knowledge based systems.

- The representation of meta-statements, such as art. 63 of the Dutch Traffic Regulation. These statements are not explicitly expressed in our representation of legal rules.

- The representation of open-textured concepts. Open texture is one of the ways in which a judge deals with unforeseen change, and is essential to facilitate legal change. However, this is one of the hardest problem of representation.

- The role of the individuals in a group with respect to collective obligations are indeterminate; every individual plays the same part in the group. In reality, there is more structure in a group, like goalkeeper, defenders, midfield players and attackers in a football team, but they all have the same goal, i.e., to win.

- The combination of Ought-to-do and Ought-to-be statements. There are situations in which we would like to relate the two Oughts with each other (see d’Altan, Meyer and Wieringa, 1993).
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Samenvatting

Deontische logica is van groot belang voor het formaliseren van juridische regels. Deontische logica is te omschrijven als de bestudering van de formele structuur van normatief taalgebruik. Met andere woorden, deontische logica is de analyse van redeneerregels met uitspraken waarin begrippen als ‘verplicht’, ‘verboden’ en ‘toegestaan’ voorkomen. Er bestaan veel verschillende systemen van deontische logica, omdat er geen overeenstemming bestaat onder logici. Er bestaat bijvoorbeeld geen overeenstemming over de axiomatische opbouw van een deontische logica en over de semantische status van de uitspraken waar deontische operatoren in voorkomen.

In dit onderzoek zijn twee ‘standaard systemen van deontische logica’ behandeld: *standaard deontische logica* (SDL) voor ‘Ought-to-be’ beweringen en *propositionele deontische logica* (PDeL) voor ‘Ought-to-do’ beweringen. Het verschil tussen beide systemen is, dat een formule als $O(p)$ in het eerste systeem gelezen wordt als ‘Het is verplicht zo te handelen dat de propositie $p$ waar wordt’, bijvoorbeeld ‘Het is verplicht zo te handelen dat er niet gerookt wordt’, en in het tweede systeem als ‘Het is verplicht handeling $p$ uit te voeren’, bijvoorbeeld ‘Het is verplicht niet te roken’.

Dit proefschrift, getiteld ‘Representation of Legal Rules in Deontic Logic’, heeft als uitgangspunt dat binnen juridische kennissystemen het gebruik van niet-standaardlogica’s, eventueel in combinatie met elkaar, een verbetering kan zijn ten opzichte van het gebruik van de standaardlogica. Onder juridische kennissystemen worden met name computerprogramma’s verstaan die tot doel hebben rechtsvragen op te lossen. In dit proefschrift komt deontische logica, een niet-standaardlogica, aan de orde. Deze is onmisbaar voor de formalisering van juridische regels. Een geschikte deontische logica is echter nog niet voorhanden en zeker geen niet-monotone variant hiervan, waarmee we kunnen redeneren onder voorbehoud van uitzonderingen. Dit komt veelvuldig voor in het recht. Dit onderzoek levert een bijdrage aan een verdere ontwikkeling van een deontisch systeem voor het recht.

De deontische logica wordt uitgebreid door middel van

- het toevoegen van (groepen) actoren;
- het toevoegen van autoriteiten.

Actoren zijn individuen waarop de normen van toepassing zijn. Autoriteiten zijn individuen
die de normen bepalen. Verder hebben we twee benaderingen gegeven om te redeneren met inconsistente informatie: een benadering met behulp van een autoriteiten-hiërarchie en één met behulp van niet-monotoon redeneren.

In hoofdstuk 1 worden de doelstelling en uitgangspunten van het onderzoek geschetst. Een centraal uitgangspunt is dat deontische logica bruikbaar is voor de analyse van juridische kennisystemen. Een ander belangrijk uitgangspunt is dat de analyse niet in direct programmeerbare theorieën behoeft uit te monden. Dit onderzoek gaat hier noodzakelijkerwijs aan vooraf: een analyse op formeel beschrijvingsniveau die geschikt is als beoordelingsmaatstaf voor geïmplementeerde kennisystemen.

In hoofdstuk 1 wordt tevens uitvoerig ingegaan op een arrest betreffende de notie van onverenigbaarheid in art. 63 van het RVV 1990. Deze casus dient als ondersteuning voor het formaliseren en als voorbeeld van de manier waarop de betreffende problemen met behulp van de deontische logica kunnen worden uitgedrukt en opgelost.

Hoofdstuk 2 behandelt het systeem propositionele deontische logica (PDeL). Daarin wordt de deontische logica gereduceerd tot een variant van de dynamische logica. Dynamische logica is een systeem waarin we eigenschappen kunnen uitdrukken zoals: 'Als men een actie uitvoert in de huidige situatie, dan zal na de uitvoering van die actie een bepaalde bewering mogelijk (of noodzakelijk) gelden'. Bijvoorbeeld, als het raam open staat en men doet het raam dicht, dan zal na die actie de bewering 'Het raam is dicht' gelden en niet meer de bewering 'Het raam staat open'. Dynamische logica kan beschouwd worden als een zwakke modale logica met toegevoegde axioma's voor het gedrag van de verschillende acties die strikt gescheiden worden van de beweringen in het systeem. Hier blijkt dat deze laatste eigenschap van de syntax ons behoedt voor intuïtief tegenstrijdige beweringen, die vaak in de literatuur over deontische logica's te vinden zijn.

In de hoofdstukken 3 en 4 worden formalisaties van relatieve deontische modaliteiten gegeven. Relatieve deontische modaliteiten hebben betrekking op wat verplicht, verboden of toegestaan is voor een individu of groep individuen. Hierdoor ontstaat de mogelijkheid om weer te geven voor wie een bepaalde norm geldt, bijvoorbeeld 'Het is verboden voor Jan om te roken'. In tegenstelling tot wat 'niet persoonlijk' verplicht, verboden of toegestaan is, bijvoorbeeld 'Het is verboden om te roken'. Het resultaat is dat we vier verschillende noties kunnen onderscheiden van verplichtingen, verboden en permissies:

1. de individuele notie ('Het is verboden voor Jan om te roken');
2. de generale notie ('Voor iedereen is het verboden om te roken') en haar duale notie de onbepaalde notie ('Er is iemand waarvoor het verboden is om te roken');
3. de collectieve notie ('Voor Jan en Piet is het verplicht om de tafel te verplaatsen');
4. de sterke notie ('Voor iedere groep individuen is het verboden om de tafel te verplaatsen') en haar duale notie de zwakke notie ('Er is een groep individuen waarvoor
het verboden is om de tafel te verplaatsen").

De relaties tussen deze vier noties zijn uitgebreid onderzocht in de twee hoofdstukken.

Met name, het idee van de collectieve notie is nieuw. Verplichtingen berusten niet altijd op individuen, maar kunnen ook gericht zijn op groepen individuen. Bijvoorbeeld, ‘Jan en Piet zijn verplicht de tafel te dekken’. Deze verplichting is niet gericht op elk individu in de groep (in dit geval de groep Jan en Piet). Met andere woorden, het is geen verplichting voor zowel Jan als Piet om de tafel te dekken. Als alleen Jan de tafel dekt is aan de verplichting voldaan.

Bij de uitbreiding van \( PDL_\varepsilon \) met groepen individuen is het van belang welke type actie (positief of negatief) aan de orde is om aan te geven wanneer een groep aan een verplichting voldoet. Een positieve actie is een actie die enige vorm van lichamelijke activiteit met zich meebrengt. Als een groep verplicht is om de tafel te verzetten (een positieve actie), dan voldoet de groep aan de verplichting als een subgroep die tafel verzet. Als een groep verplicht is de tafel te laten staan (een negatieve actie), dan voldoet de groep aan deze verplichting als elke subgroep de tafel laat staan. De norm wordt al geschonden als een individu uit de groep de tafel verzet.

Er bestaat een wezenlijk verschil tussen de uitbreiding van de systemen \( SDL \) en \( PDL_\varepsilon \) met groepen individuen. In de uitbreiding van \( SDL \) zijn we in staat uit te drukken dat een individu of groep individuen verplicht is om een bepaalde situatie te bewerkstelligen. In de uitbreiding van \( PDL_\varepsilon \) kan worden uitgedrukt dat een individu of groep individuen verplicht is een bepaalde actie uit te voeren. Wat betreft het strafrecht is de uitbreiding van \( PDL_\varepsilon \) te prefereren, omdat het daar steeds om gedragingen gaat. Wanneer in een delictsomschrijving een onwettige toestand wordt genoemd, vraagt men zich af wie die toestand in het leven heeft geroepen (of laten ontstaan) en wie die toestand heeft laten voortbestaan. De onwettige toestand wordt dus herleid tot een gedraging.

In hoofdstuk 5 breiden we de systemen, die behandeld zijn in de vorige twee hoofdstukken, uit met autoriteiten, zodat de mogelijkheid ontstaat om uit te drukken wie een bepaalde norm heeft bepaald. Op basis hiervan worden verschillende onderwerpen behandeld, zoals hiërarchische normen, (gepostuleerde) universaliteit en normatieve inconsistenties.

De theorie die besproken wordt, is een uitbreiding en wijziging van de theorie van Bailhache. Deze theorie was ontwikkeld om een deontisch coherent systeem - dat wil zeggen ‘Wat verplicht is, is toegestaan voor iedereen’ - te krijgen. We zien echter dat deze theorie enkele serieuze bezwaren met zich brengt: (1) alleen verplichtingen en geen permissies kunnen bepaald worden door autoriteiten. Het is dus onmogelijk om uit te drukken dat een autoriteit bepaalt dat het toegestaan is om te parkeren; (2) autoriteiten kunnen alleen een atomaire collectieve verplichting bepalen en geen combinaties van verplichtingen. Het is onmogelijk om uit te drukken dat een autoriteit bepaalt dat het verplicht is rechts te houden en verplicht is om je snelheid aan te passen; en (3) normatieve inconsistenties kunnen niet
worden uitgedrukt. Met normatieve inconsistentie wordt bedoeld dat in een bepaald geval twee tegenstrijdige regels van toepassing zijn. Een oorzaak hiervan is bijvoorbeeld dat de wetgever geen uniek orgaan is: de regelgevende bevoegdheid is verdeeld over een groot aantal instanties, waardoor conflicten tussen regels kunnen ontstaan. Het laatste bezwaar is inherent aan een deontisch coherent systeem. Deze drie bezwaren verdwijnen in onze theorie.

In hoofdstuk 6 zien we dat een normatief systeem verandert door promulgatie (het introduceren van normen) en derogatie (het verwijderen van normen): het dynamische karakter van het recht. Deze twee wetgevende activiteiten, uitgaande van autoriteiten, hebben twee gemeenschappelijke eigenschappen: de wijziging van daadwerkelijk geldende normen en de beïnvloeding van de competentie van ‘lagere’ autoriteiten. Bovendien wordt getoond dat niet alleen derogatie maar ook promulgatie kan leiden tot logische onbepaaldheid. Onder logische onbepaaldheid verstaan we dat we logisch gezien niet kunnen bepalen welke norm impliciet moet worden afgeschaft. Door het dynamische karakter van het recht kunnen normatieve inconsistenties ontstaan. Om deze inconsistenties het hoofd te bieden, wordt een theorie ontwikkeld op basis van een ordening van autoriteiten met betrekking tot hun competentie, met andere woorden een autoriteiten-hierarchie. Een nadeel van deze theorie is dat alleen ongeconditioneerde normen worden behandeld.

Tot slot bespreken we (gepostuleerde) universaliteit van normen en hoe deze bereikt kan worden. We zien dat de invoering van een universeel normensysteem op praktische moeilijkheden stuit. Deze moeilijkheden kunnen eenvoudig omzeild worden door de invoering van algemene sluitingsregels, zoals nulla poena sine lege (‘geen straf zonder voorafgaande strafbepaling’). Hierdoor ontstaan echter geheel andere problemen, zoals het bevorderen van het zogenaamde zwemmen door de mazen van de wet. Verder blijkt dat niet de gepostuleerde universaliteit de individuele vrijheid en verantwoordelijkheid aantast, maar de uitbreiding van verplichtingen en verboden.

In hoofdstuk 7 wordt een theorie van een deontische variant van niet-monotoon redeneren ontworpen, die enkele problemen van bestaande theorieën omzeilt. Bijvoorbeeld, de nieuwe theorie bevat de toevoeging van een Reiter default voor de negatie van de verplichting (permissie) en kan niet alleen niet-monotoon redeneren met normen, maar ook met interpretatie-regels.

Hoofdstuk 8 bevat de resultaten van het onderzoek. Het gaat te ver om te stellen dat de resultaten van dit onderzoek onmiddellijk een optimale vorm bieden om juridische teksten te reprenteren, ofschoon dat op onderdelen niet is uitgesloten. De omzetting van theoretische kennis in praktische regels voor de ontwerper van een kennisysteem vergt een grondige uiteenzetting en afweging van de waarden en doeleinden waarop die regels berusten en dikwijls ook kennis van empirische verbanden. Het is evenwel onze overtuiging dat dit onderzoek, als analytische studie, een bijdrage levert aan het inzicht in de wijze waarop juridische regels kunnen worden geregistreerd en hoe met deze regels kan worden
geredeneerd.
Curriculum Vitae
