Subsidized Capacity Investment under Uncertainty

Wen, Xingang; Hagspiel, V.; Kort, Peter M.

Publication date:
2017

Document Version
Early version, also known as pre-print

CentER

Discussion Paper

No. 2017-043

SUBSIDIZED CAPACITY INVESTMENT UNDER UNCERTAINTY

By

Xingang Wen, Verena Hagspiel,
Peter M. Kort

2 November 2017

ISSN 0924-7815
ISSN 2213-9532

TILBURG UNIVERSITY

Subsidized Capacity Investment under Uncertainty

Xingang Wen *[1], Verena Hagspiel†[2] and Peter M. Kort‡[1,3]

[1]CentER, Department of Econometrics & Operations Research, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands

[2]Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

[3]Department of Economics, University of Antwerp, Prinsstraat 13, 2000 Antwerp 1, Belgium

Abstract

This paper studies how the subsidy support, e.g. price support and reimbursed investment cost support, affects the investment decision of a monopoly firm under uncertainty and analyzes the implications for social welfare. The analytical results show that the unconditional, i.e., subsidy support that is introduced from the beginning, makes the firm invest earlier. Under a linear demand structure, the unconditional subsidy cannot align the firm's investment decision to the social optimal one. However, a conditional subsidy, i.e., subsidy support introduced at the social optimal investment threshold, can align the two decisions. For a non-linear demand structure, it is possible for the unconditional subsidy to make the firm invest according to the social optimum. When the investment decisions are aligned, the firm's investment leads to the first-best outcome.

Keywords: Investment under Uncertainty, Capacity Choice, Welfare Analysis, Linear Demand, Non-linear Demand

JEL classification: D81, L51

1 Introduction

Since the 1970s, many public owned functions and businesses have been decentralized, e.g. postal services, banking, airlines, telecommunication, and public infrastructures. The government owns many resources such as water, land, and mineral. Decentralization means the private firms have the right to invest, produce,

*Corresponding author: Tel. +31 13 466 8312. x.wen@uvt.nl

†verena.hagspiel@iot.ntnu.no

‡kort@uvt.nl

The authors would like to thank Dolf Talman, Bert Willems, Maria Lavrutich, Kuno Huisman, and Cláudia Nunes for their valuable comments.

and make profit from such resources, for example, port investment, agriculture investment, green energy investment, and so on. The privatization is believed to be more efficient and effective in decision making because of the quicker reaction to unanticipated market changes. However, after the privatization, firms prioritize profit maximization and do not consider the social optimal goals when making decisions. This is different from the goal of the social planner, which is to achieve social optimality. For instance, energy producers that use fossil fuel and emit greenhouse gases do not take into account environmental damage (Eichner and Runkel, 2014), whereas the regulator such as the E.U. parliament, has the purpose to fulfill the emissions reduction commitment and encourage the investments of renewable energy. In risky environments it has been argued that a firm tends to postpone investment (see e.g., McDonald and Siegel (1986)). According to Dobbs (2004), the level of investment in capacity might also be constrained by the firm. For example, an electricity producer might hesitate to invest in renewable technology due to high investment costs compared to the fossil fuels. This implies that the energy market has less incentive to deliver the desired level of renewable investment. This difference in objectives and investment strategies between the profit and welfare maximizers poses a coordination problem and requires governmental regulation (Rodrik, 1992).

In a market with uncertain future demand, the firm is constantly forecasting demand and balancing the value of investing now and delaying investments. Thus, the real options approach is used to analyze the investment decisions. Several literatures have studied to use price regulation such as the price cap to regulate the delayed investment under uncertainty. For instance, Dobbs (2004) argues that the first-best outcome cannot be reached as price cap is used for two goals: optimal investment ex-ante and optimal post-investment pricing. Building on Dobbs (2004), Evans and Guthrie (2012) show that the price cap should be lowered under scale economics where grouping investments across time is cost efficient. By contrast, Willems and Zwart (2017) consider constant returns to scale where it is not optimal to group investments. By assuming asymmetric information on investment costs, Willems and Zwart (2017) study the optimal mechanism where a revenue tax increases with the level of the price cap. In this paper, we study the policy instrument of subsidy, rather than price cap.

Subsidy support is a very common policy instrument in the fields of agriculture and green energy. For agriculture in developing economies, there are input subsidies, which are implemented as price subsidies accessible to producers according to Chirwa and Dorward (2013). One example is the Indian fertilizer subsidy in order to encourage the domestic production of fertilizer and to increase its use. To accomplish these two objectives, India introduced the RPS (Retention Price Subsidy) scheme in 1977, where the difference between retail price and retention price (adjusted for freight and dealer's margin) was paid back to the manufactures as a subsidy (Sharma and Thaker, 2010). Under the RPS, the production cost plus 12% profit is covered by the subsidy. Later on, RPS was criticized for being inefficient to motivate the producers to decrease production costs and was replaced by NPS (New Pricing Scheme) in 2003. Under the new system, the producer receives a set amount based on the age of the production plant and the amount of feedstock used.

In the green energy field, the subsidy support can take many forms such as feed-in premiums, reimbursed investment costs, feed-in tariffs, tradable green certificates, and quota obligations. In this research work, two kinds of price support will be discussed: flexible and fixed price support. Under flexible price support, the producer receives a payment proportional to the market price for every product sold to the consumer, like 12% for instance in India's RPS scheme. Under the fixed price support, the producer receives a fixed payment for every product sold to the consumer that is independent of the market price, like the subsidy described in India's NPS scheme. In the green energy field, the fixed price support may take the form as the feed-in premium subsidy.

This research studies how different kinds of subsidy support affect the profit maximizing firm's investment timing and size, and whether it is possible to align the firm's investment decisions to the social optimal ones. Besides the non-linear demand structure used in the price cap literatures, this paper considers also the linear demand structure. More specifically, we consider two kinds of demand shocks for the linear demand: additive (Kulatilaka and Perotti, 1998; Aguerrevere, 2003; Hagspiel et al., 2016) and multiplicative (Grenadier, 2000; Huisman and Kort, 2015) demand shocks. We show that the subsidy policies introduced from the beginning make the firm invest earlier and invest less. Moreover, we find that there exists a conditional subsidy to introduce the subsidy support at the social optimal investment timing to align the firm's optimal investment decision to the social optimal one. For the non-linear demand structure, if the demand is iso-elastic as in Aguerrevere (2009) and Novy-Marx (2007), the influence of the subsidy on the firm's investment decision depends on the subsidy rate. It is possible to align the firm's investment decision to the social optimal decision if subsidies are introduced from the beginning, or at the social optimal investment timing. For both demand structures, the subsidies that align firm's and social optimal investment decision yield the social optimal surplus. To simplify the analysis, we do not consider the efficiency loss in collecting taxes and the allocation of taxation as subsidies.

Several research papers have already shed light on investment decisions under policy schemes in the framework of real options. For the policy scheme that will prevail once being chosen, Pennings (2000) studies the taxation and investment subsidy to stimulate the instant investment, i.e., the waiting time is zero. Hassett and Metcalf (1999) consider the uncertainty in the tax policy, such as the U.S. investment tax credits that have been changed on many occasions since being introduce in 1964, and show that for a relatively low tax rate, more uncertainty in tax policy speeds up irreversible investment because the firm inclines to invest at a low tax rate. This paper focuses on the subsidized investment and the corresponding welfare analysis, rather than on the taxation.

Most of the existing research concerning policy schemes focuses on green investment and takes the subsidy payments as a volatile process. Up to our best knowledge, those papers only study the investment decisions from the perspective of the producer and considers mainly the investment timing. For example, Boomsma et al. (2012) assume that the geometric Brownian motion governs the capital cost, electricity prices, and subsidy payments. The support schemes considered include feed-in tariff, flexible price premium, and renewable energy certificates. The three support schemes differ at how much risk the firm is exposed to the market. This is different from our research, where the price volatility is the only risk in the market. Besides the investment timing, we also consider the influence of subsidies on the firm's optimal investment capacity. Moreover, we study the optimal subsidy schemes to make the firm invest in a social optimal way.

Current literatures on subsidy mainly consider the uncertainty about introduction or retraction of subsidy schemes. Boomsma and Linnerud (2015) also examine how the market risk and the policy risk of retractable support schemes affect the investment timing. They find that the risk of subsidy termination speeds up the investment. This result is also supported by Adkins and Paxson (2015). They provide the intuition that the firm wants to catch the subsidy before it is gone. Similarly, future provision of permanent subsidy delays investment because the firm wants to wait for the subsidy. This influence of subsidy retraction and provision is further studied by Chronopoulos et al. (2016). Besides the investment timing, they also consider the influence of policy uncertainty on the investment capacity/size. They find that the future subsidy retraction lowers the amount of installed capacity, and the future subsidy provision raises the incentive to install a larger capacity. In this research, we also consider both investment timing and capacity. Rather than the policy uncertainty, the focus is on the welfare analysis of the investment subsidy and the optimal subsidy policies to align the firm's investment decision to the social optimal investment decision.

This paper is organized as follows. Section 2 describes the profit and welfare maximizers' investment problems and the subsidy support. In Section 3 we derive the optimal subsidy policy to align the firm's and social optimal investment decisions, and compare the optimal subsidy support schemes. Section 4 studies the optimal subsidy policies and compares them for different demand structures. Section 5 concludes.

2 Model

Consider a continuous-time and one-time irreversible capacity investment problem. Investor needs to decide on the investment timing and investment capacity. There is no depreciation of capacity and no production costs and the marginal cost of investment is constant, $\delta > 0$. Once a capacity K is installed, K will be sold in the market at a price $p(X(t), K)$. $\{X(t)|t \geq 0\}$ is the demand shift parameter and satisfies a geometric Brownian motion,

$$dX(t) = \mu X(t)\,dt + \sigma X(t)\,d\omega(t), \tag{1}$$

in which μ is the drift parameter, $d\omega(t)$ is the increment of a Wiener process, and $\sigma > 0$ is the volatility parameter. The discount rate is r and we assume $r > \mu$. The instant producer surplus is profit flow $p(X(t), K)K$. The instant consumer surplus is denoted as $cs(X(t), K)$. A regulator's objective is to maximize the producer and consumer surplus minus investment costs, i.e.,

$$\max_{T\geq 0, K\geq 0} E\left[\int_{t=T}^{\infty} (p(X(t), K)K + cs(X(t), K)) \exp(-rt)\mathrm{d}t - \delta K \exp(-rT) \,\middle|\, X(0) = X\right]. \tag{2}$$

This yields the social optimal investment decision (X_W^*, K_W^*) with X_W^* the social optimal investment threshold that triggers investing K_W^* once it is reached. Hence, the social optimal investment time T is the first time that the stochastic process, which starts at $X(0)$ at time zero, reaches X_W^*. The profit maximizer, the firm, has the objective to maximize the producer surplus minus investment costs, i.e.,

$$\max_{T\geq 0, K\geq 0} E\left[\int_{t=T}^{\infty} p(X(t),K)K\exp(-rt)\mathrm{d}t - \delta K\exp(-rT)\middle| X(0)=X\right]. \tag{3}$$

The solution gives firm's optimal investment decision as (X^*, K^*). X^* is the optimal investment threshold and triggers the firm to invest K^* once being reached. Comparing (2) and (3), it can be concluded that firm's investment decision is not totally aligned with social optimal investment decision such that $X^* = X_W^*$ and $K^* = K_W^*$ both hold. This distortion implies that firm's optimal investment decision generates externality and does not lead to first-best outcome. In order to align these two investment decisions, the regulator needs to make the firm internalize this externality when deciding on investment. Because the difference between the two objectives, (2) and (3), is consumer surplus, a possible regulation is to propose a contract that specifies a monetary transfer, e.g., a subsidy, to remunerate the firm. Such subsidy scheme can be a subsidy flow $s(X(t),K)$ that satisfies $s(X(t),K) = cs(X(t),K)$, or a lump sum subsidy transfer $s(X,K)$ to the firm when investing at level X with capacity K. Let $S(X,K)$ and $CS(X,K)$ be the discounted expected subsidy and consumer surplus. For both subsidy flow and lump sum subsidy, when firm's investment decision is aligned to the social optimal decision, the following conditions hold,

$$\frac{S(X_W^*, K_W^*)}{\partial X} = \frac{\partial CS(X_W^*, K_W^*)}{\partial X}, \tag{4}$$

$$\frac{S(X_W^*, K_W^*)}{\partial K} = \frac{\partial CS(X_W^*, K_W^*)}{\partial K}. \tag{5}$$

(4) and (5) are straightforward outcomes from the maximization of social surplus and producer surplus when the firm internalizes the subsidy. After subsidy, the producer surplus from profit flow is equal to the producer surplus in (2). The producer surplus from subsidy is equal to subsidy costs. Consumer surplus after subsidy has the same value as in (2). In this way, social surplus reaches the first-best level after subsidy.

Denote a subsidy flow as $s(X(t),K,\tilde{s})$ for given capacity level K and subsidy rate parameter $\tilde{s}\geq 0$. This flow can be implemented in many forms. It can be a flexible price support (a proportional add-on to the market price), i.e., $s(X(t),K,\tilde{s}) = \tilde{s}P(X(t),K)K$, or a fixed price support (a fixed add-on to the market price), i.e., $s(X(t),K,\tilde{s}) = \tilde{s}K$. These two subsidy flows influence firm's profit flow directly because of their relation to market prices. Denote a lump sum subsidy as $s(K,\tilde{s})$. It can be reimbursed investment cost (a one time remuneration transfer as a fraction of investment costs), i.e., $s(K,\tilde{s}) = \tilde{s}\delta K$. Let the expected discounted producer surplus be $V(X,K,\tilde{s})$ for the given geometric Brownian motion level $X(0) = X$ and investment capacity K. The firm's optimal investment decision is $(X^*(\tilde{s}), K^*(\tilde{s}))$ after subsidy $\tilde{s}$. The corresponding expected social surplus is $W(\tilde{s}) = W(X^*(\tilde{s}), K^*(\tilde{s}), \tilde{s})$. When $\tilde{s}^*$ maximizes $W(\tilde{s})$ and yields the firm's optimal decision such that$(X^*(\tilde{s}^*), K^*(\tilde{s}^*)) = (X_W^*, K_W^*)$, then subsidy scheme $\tilde{s}^*$ is optimal. In the following analysis, we focus on the feasibility of these implementations for some specific demand structures.

3 Linear Demand

Let the inverse linear demand function for a given investment capacity $K \geq 0$ be

$$\begin{aligned} p(t) =& \alpha[X(t) - \eta K] + (1-\alpha)[X(t)(1-\eta K)] \\ =& X(t) - \eta K\left(\alpha + (1-\alpha)X(t)\right),\ 0 \leq \alpha \leq 1,\ \eta > 0 \end{aligned}$$

This demand function combines two types of demand shocks: additive demand shocks $X(t) - \eta K$ and multiplicative demand shocks $X(t)(1-\eta K)$. The additive demand shocks have a weight of α. Besides $r > \mu$, it is assumed that $r > 2\mu + \sigma^2$ holds as in Chapters 2 and 3. For additive demand structure, the market size increases when firm waits for a higher demand level to invest. The additive demand structure corresponds to markets where there is no obvious cap on market size. The multiplicative demand structure is restricted by market size, and it corresponds to a market where the amount of potential customers is limited. An example for multiplicative demand structure is the market of agricultural machines, see Boonman (2014), where the amount of acres of farmlands and the number of farmers are limited. This results in an upper bound of demand.

For the given linear demand function, this section first explores the first-best outcome, where the social planner decides about when and how much to invest. This provides a benchmark for the policy regulator to regulate the monopoly firm. Then the firm's optimal investment decision $(X^*(\tilde{s}), K^*(\tilde{s}))$ is analyzed under monetary subsidy. The analysis focuses specifically on the influence of subsidy on the firm's investment decision, which provides insights on the efficiency of subsidy regulation. Moreover, this section discusses the best performance for unconditional subsidy that is implemented from the beginning and for conditional subsidy that is implemented at some specific demand level.

3.1 First-best benchmark

The social planner's maximization problem is described by (2). To get a more specific objective function, we first calculate the discounted consumer and producer surplus separately. For the given level of $X(t)$, the instantaneous consumer surplus is

$$cs(X(t), K) = \int\limits_{X(t)-[\alpha+(1-\alpha)X(t)]\eta K}^{X(t)} \frac{X(t) - p}{\alpha\eta + (1-\alpha)X(t)\eta} \mathrm{d}p = \frac{[\alpha + (1-\alpha)X(t)]\eta K^2}{2}.$$

Given $X(0) = X$, the expected discounted consumer surplus is equal to

$$CS(X, K) = E\left[\int\limits_{t=0}^{\infty} \frac{[\alpha + (1-\alpha)X(t)]\eta K^2}{2} \mathrm{d}t \,\middle|\, X(0) = X\right] = \frac{\eta K^2}{2}\left(\frac{\alpha}{r} + \frac{(1-\alpha)X}{r-\mu}\right).$$

For a given X, the consumer surplus increases with investment capacity K. This is because more capacity yields a lower market price since the firm always produces up to full capacity. For a given amount of investment capacity K, the consumer surplus increases with X. The reason is that a higher level X implies a larger market demand. The highest price that consumers are willing to pay increases. The expected producer surplus is equal to the value of the firm, which is the discounted profit flow minus the investment cost. For a given K, the expected producer surplus is

$$PS(X,K) = \frac{XK}{r-\mu} - \frac{(1-\alpha)X\eta K^2}{r-\mu} - \frac{\alpha\eta K^2}{r} - \delta K.$$

The producer surplus increases with X for given K because a larger demand implies a higher market price level, which increases firm's profit flows. The expected social surplus given at $X(0) = X$ is given by

$$W(X,K) = \frac{XK}{r-\mu} - \frac{(1-\alpha)X\eta K^2}{2(r-\mu)} - \frac{\alpha\eta K^2}{2r} - \delta K.$$

From the discounted social welfare function, we can derive the social optimal investment decision as being summarized in the following proposition. The proof can be found in Appendix.

Proposition 1 *The social optimal investment threshold X_W^* and the social optimal investment capacity K_W^* satisfy the equations*

$$\frac{X}{r-\mu}\frac{\beta-1}{\beta}(2-(1-\alpha)\eta K) - \frac{\alpha\eta K}{r} - 2\delta = 0 \tag{6}$$

and

$$\alpha(1-\alpha)\eta^2K^2 + r\delta(\beta+1)(1-\alpha)\eta K + \alpha(\beta-2)\eta K - 2r\delta = 0, \tag{7}$$

in which

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 2.$$

Next we carry out some further analysis on the social optimal investment decision (X_W^*, K_W^*). First, according to Dixit and Pindyck (1994) it holds that $\frac{\partial\beta}{\partial\sigma} < 0$ and $\frac{\partial\beta}{\partial\mu} < 0$. From equations (6) and (7), it can be derived that $\frac{\partial X_W^*}{\partial\beta} < 0$ and $\frac{\partial K_W^*}{\partial\beta} < 0$. Thus, like the standard real options result for firm's investment decision by Huisman and Kort (2015), we conclude that the increase of uncertainty, that is, a larger value of σ, raises both X_W^* and K_W^*. It implies that the social optimal investment is delayed with a greater volatility, which leads to the adoption of a larger project. This result shows that volatility influences social planner's investment decision in the same way as it influences the firm's investment decision. Moreover, the increase in drift rate parameter, i.e., a larger value of μ, raises X_W^* and K_W^* as well. The implication is that the social planner delays and takes on a larger project upon investment when market grows faster. This is due to the fact that future market demand is taken into consideration when making investment decisions. A faster growing market yields a higher demand in the future. Thus, more capacity is needed to satisfy such demand. It delays investment because of the prolonged waiting for a larger market demand to be reached.

3.2 Subsidized Profit Maximization Investment

As mentioned above, we study monopoly firm's investment decision under subsidy regulation. More specifically, we get the insight of how subsidy influences firm's optimal investment decision, in order to come up with a subsidy that can achieve either the first-best or the second-best outcome. For the given linear demand function and subsidy flow scheme $s(X(t), K, \tilde{s}) = \tilde{s}p(X(t), K)K$, the firm internalizes the subsidy remuneration into the decision making. Substitute the price in firm's objective function (3) with subsidized price $(1+\tilde{s})p(X(t), K)$. For a given capacity K and $X(0) = X$, the firm's value function is equal to

$$V(X, K, \tilde{s}) = (1+\tilde{s})\left[\frac{XK}{r-\mu} - \frac{(1-\alpha)X\eta K^2}{r-\mu} - \frac{\alpha\eta K^2}{r}\right] - \delta K. \tag{8}$$

From this value function of the monopoly firm, we can derive the firm's optimal investment decision and the following proposition.

Proposition 2 *Given subsidy flow $s(X(t), K, \tilde{s}) = \tilde{s}p(X(t), K)K$, the optimal investment threshold $X^*(\tilde{s})$ and investment capacity $K^*(\tilde{s})$ satisfy the equations*

$$X(K) = \frac{\beta(r-\mu)}{(\beta-1)(1-(1-\alpha)\eta K)}\left(\frac{\alpha\eta K}{r} + \frac{\delta}{1+\tilde{s}}\right) \tag{9}$$

and

$$2\alpha(1-\alpha)\eta^2K^2 + \frac{r\delta}{1+\tilde{s}}(\beta+1)(1-\alpha)\eta K + \alpha(\beta-2)\eta K - \frac{r\delta}{1+\tilde{s}} = 0. \tag{10}$$

First note that when there is no subsidy, e.g., $\tilde{s} = 0$, we get the monopoly investment decision $(X^*(0), K^*(0))$. By comparing with the social optimal investment decision (X_W^*, K_W^*), it holds that $X^*(0) = X_W^*$ and $K^*(0) = K_W^*/2$. This indicates that when there is no subsidy under linear demand, the firm and the welfare maximizer have the same investment threshold, but the social optimal investment capacity is twice of the firm's capacity. This result is consistent with the finding by Huisman and Kort (2015), where linear demand is considered as well. We then study the influence of subsidy on firm's optimal investment, which is summarized in the following corollary.

Corollary 1 *Subsidy flow, $s(X(t), K, \tilde{s}) = \tilde{s}p(X(t), K)K$, makes the firm invest earlier and less. Unconditional subsidy cannot align firm's investment decision to the social optimal investment decision.*

Subsidy motivates the firm to invest earlier because monetary transfer increases the firm's expected value, which provides an incentive for the firm to enter the market earlier, when market demand is smaller. This leads to a smaller capacity being invested under subsidy regulation. Because firm's investment capacity without subsidy is already only half of the social optimal capacity, subsidy regulations makes firm invest less than half of the social optimal capacity, and thus deviate from the social optimal decision. So the unconditional subsidy cannot align profit and welfare maximizer's investment decisions. Another insight is that it is difficult to align two decision variables with just one subsidy rate parameter $\tilde{s}$ in unconditional subsidy regulation. This is because both decision variables, $X^*(\tilde{s})$ and $K^*(\tilde{s})$, are changing with $\tilde{s}$. Intuitively, two parameters and a more complicated subsidy regulation scheme will be needed. Next, we check another subsidy flow with one parameter as well, $s(X(t), K, \tilde{s}) = \tilde{s}K$ and $\tilde{s} < r\delta$. This unconditional subsidy regulation makes the firm invest in the way as described by the following proposition. The firm's value function $V(X, K, \tilde{s})$ and the proof of the proposition can be found in the appendix.

Proposition 3 *Subsidy flow $s(X(t), K, \tilde{s}) = \tilde{s}K$ makes the firm invest at threshold $X^*(\tilde{s})$ with capacity $K^*(\tilde{s})$. $X^*(\tilde{s})$ and $K^*(\tilde{s})$ satisfy*

$$X(K) = \frac{\beta(r-\mu)}{(\beta-1)(1-\eta K(1-\alpha))}\left(\frac{\alpha\eta K}{r} - \frac{\tilde{s}}{r} + \delta\right)$$

and

$$2\alpha(1-\alpha)\eta^2K^2 + (\beta+1)(1-\alpha)(r\delta - \tilde{s})\eta K + \alpha(\beta-2)\eta K - (r\delta - \tilde{s}) = 0.$$

Similar to the subsidy flow $\tilde{s}p(X(t), K)K$, we have $\mathrm{d}K^*(\tilde{s})/\mathrm{d}\tilde{s} < 0$ and $\mathrm{d}X^*(\tilde{s})/\mathrm{d}\tilde{s} < 0$, implying subsidy flow $s(X(t), K, \tilde{s}) = \tilde{s}K$ influences firm's investment decision in the same way and cannot achieve the first best if it is unconditional, i.e., implemented at Brownian motion level $X(0) < X^*$. For the lump sum subsidy transfer, it works the same as $s(X(t), K, \tilde{s}) = \tilde{s}K$ as long as its lump sum subsidy rate is $\tilde{s}/(r\delta)$.

3.3 Second-best outcome for unconditional subsidy

Because unconditional subsidy does not yield the first-best outcome, we want to find out the second-best outcome that can be achieved by subsidy regulation. Given the firm has invested at threshold $X^*(\tilde{s})$ with capacity $K^*(\tilde{s})$, the expected social surplus is equal to

$$W(\tilde{s}) = K^*(\tilde{s})\left(\frac{X^*(\tilde{s})}{r-\mu} - \frac{(1-\alpha)X^*(\tilde{s})\eta K^*(\tilde{s})}{2(r-\mu)} - \frac{\alpha\eta K^*(\tilde{s})}{2r} - \delta\right).$$

Because $X^*(\tilde{s}) < X^*(0)$ for $\tilde{s} > 0$, $W(\tilde{s})$ needs to be compared at a predetermined point in time such as $X^*(0)$ with a stochastic discount factor $(X^*(0)/X^*(\tilde{s}))^\beta$.The optimal subsidy rate $\tilde{s}^*$ that yields the second-best outcome satisfies

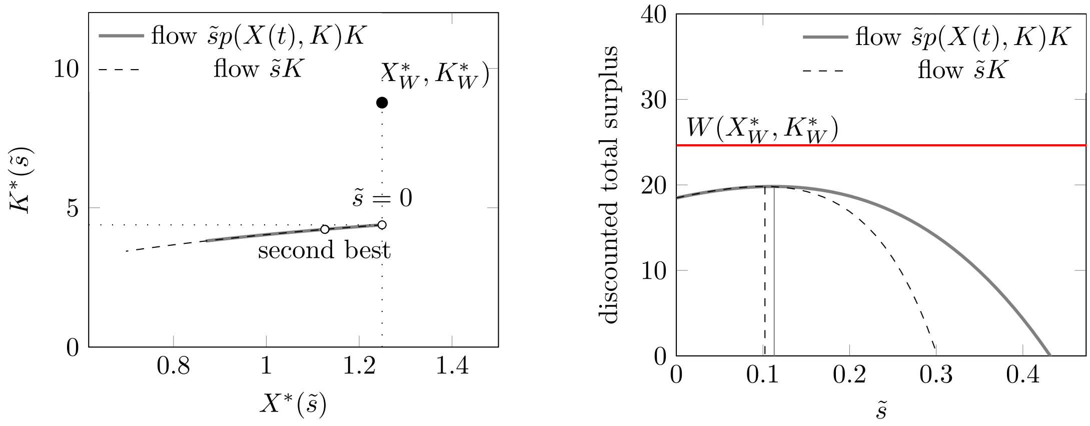
$$\frac{\mathrm{d}}{\mathrm{d}\tilde{s}}\left(\frac{X^*(0)}{X^*(\tilde{s})}\right)^\beta W(\tilde{s})\bigg|_{\tilde{s}=\tilde{s}^*} = \left(\frac{X^*(0)}{X^*(\tilde{s}^*)}\right)^\beta\left(-\frac{\beta W(\tilde{s}^*)}{X^*(\tilde{s}^*)}\frac{\mathrm{d}X^*(\tilde{s})}{\mathrm{d}s}\bigg|_{\tilde{s}=\tilde{s}^*} + \frac{\mathrm{d}W(\tilde{s})}{\mathrm{d}s}\bigg|_{\tilde{s}=\tilde{s}^*}\right) = 0, \tag{11}$$

or $\tilde{s}^* = 0$ if $\left(\frac{X^*(0)}{X^*(\tilde{s})}\right)^\beta W(\tilde{s})$ decreases[1] with $\tilde{s}$. $\tilde{s}^* = 0$ implies that the second-best outcome for unconditional subsidy is to implement no subsidy.

Figure 1 demonstrates the first-best, second-best outcome and the firm's optimal investment threshold $X^*(\tilde{s})$ and investment capacity $K^*(\tilde{s})$. It is shown that without subsidy, i.e., $\tilde{s} = 0$, the firm invests at the social optimal threshold $X^*(0) = X^*_W$ with half of the social optimal capacity $K^*(0) = K^*_W/2$. The half capacity result can be derived by comparing solutions for quadratic equations (7) and (10). For the given parameter values, $K^*(0) = 4.385$ and $K^*_W = 8.770$. This is consistent with the findings by Huisman and Kort (2015). Figure 1a also shows that as $\tilde{s}$ increases, the firm's optimal investment decision $(X^*(\tilde{s}), K^*(\tilde{s}))$ deviates further from the first-best outcome. Moreover, the two subsidy flows $\tilde{s}p(X(t), K)K$ and $\tilde{s}K$ have similar influence on firm's investment decision. We can see this from the overlap of the two curves[2] for

[1] Note that we can rule out the situation where $\frac{\mathrm{d}}{\mathrm{d}\tilde{s}}\left(\frac{X^*(0)}{X^*(\tilde{s})}\right)^\beta W(\tilde{s}) > 0$. This implies an infinite amount of monetary transfer to the firm.

[2] We didn't plot $(X^*(\tilde{s}), K^*(\tilde{s}))$ that generates negative $W^*(\tilde{s})$.



(a) Illustration of $(X^*(\tilde{s}), K^*(\tilde{s}))$.

(b) Illustration of $W(\tilde{s}) = W(X^*(\tilde{s}), K^*(\tilde{s}))$.

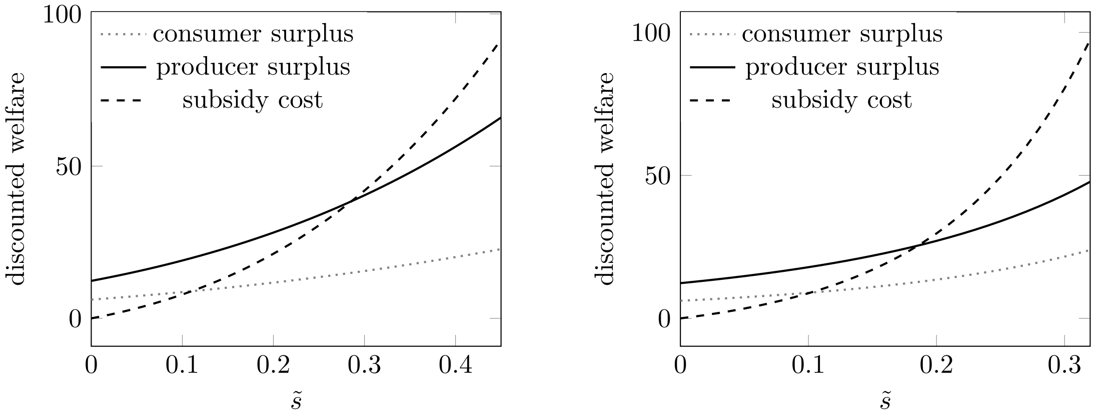
Figure 1: Illustration of $(X^*(\tilde{s}), K^*(\tilde{s}))$, and $W(\tilde{s})$. Parameter values are $\alpha = 0.5$, $\mu = 0.02$, $r = 0.1$, $\sigma = 0.01$, $\eta = 0.05$, $\delta = 10$.

$(X^*(\tilde{s}), K^*(\tilde{s}))$. Besides, the two subsidy flows have the same second-best outcome $(X^*(\tilde{s}^*), K^*(\tilde{s}^*))$ as shown in Figure 1a. This is due to the fact that $W(\tilde{s})$ has the same expression for the two unconditional subsidy flows. However, the second-best outcome is generated by different subsidy rates as illustrated by Figure 1b. The subsidy rate is $\tilde{s}^* = 0.113$ for subsidy flow $\tilde{s}p(X(t), K)K$ and $\tilde{s}^* = 0.102$ for flow $\tilde{s}K$. Figure 1b also illustrates the discounted total surplus generated by the firm's optimal investment decision. Under unconditional subsidy, the social surplus generated by the firm's investment decision is always below the social optimal surplus, implying that unconditional subsidy cannot lead to social optimum. Figure 1b shows that as the subsidy rate goes up, the social surplus first increases and then decreases. Because, the social surplus $W(\tilde{s})$ consists of producer surplus, consumer surplus and subsidy costs, we further analyze this by the illustration of Figure 2.

Figure 2 demonstrates the discounted consumer surplus $\left(\frac{X(0)}{X^*(\tilde{s})}\right)^\beta CS(X^*(\tilde{s}), K^*(\tilde{s}))$, the discounted producer surplus $\left(\frac{X(0)}{X^*(\tilde{s})}\right)^\beta V(X^*(\tilde{s}), K^*(\tilde{s}), \tilde{s})$ and the discounted subsidy cost $\left(\frac{X(0)}{X^*(\tilde{s})}\right)^\beta C(\tilde{s})$ to a predetermined time $X^*(0)$. Note that for subsidy flow $\tilde{s}p(X(t), K)K$, the expected subsidy cost is equal to

$$C(\tilde{s}) = \tilde{s}K^*(\tilde{s}) \left(\frac{X^*(\tilde{s})}{r-\mu} - \frac{(1-\alpha)X^*(\tilde{s})\eta K^*(\tilde{s})}{r-\mu} - \frac{\alpha\eta K^*(\tilde{s})}{r} \right).$$

For subsidy flow $\tilde{s}K$, the expected subsidy cost is $\tilde{s}K^*(\tilde{s})/r$. Figure 2 shows that the discounted consumer surplus, producer surplus and subsidy costs increase with unconditional subsidy rate $\tilde{s}$, despite the fact that $\tilde{s} > 0$ makes the firm invest earlier and less. It is intuitive that the discounted producer surplus increases with the subsidy rate. This is because for two subsidy rates $\tilde{s}_1 > \tilde{s}_2 \geq 0$, the firm can always choose investment decision $(X^*(\tilde{s}_2), K^*(\tilde{s}_2))$ for subsidy rate $\tilde{s}_1$, which would yield a producer surplus that is equal to $PS(X^*(\tilde{s}_2), K^*(\tilde{s}_2)) + \tilde{s}_1 p(X^*(\tilde{s}_2), K^*(\tilde{s}_2))K^*(\tilde{s}_2)$. The fact that the firm chooses investment decision $(X^*(\tilde{s}_1), K^*(\tilde{s}_1))$ implies it generates larger producer surplus. The consumer surplus also increases with



(a) Subsidy flow $\tilde{s}p(X(t), K)K$.

(b) Subsidy flow $\tilde{s}K$.

Figure 2: Illustration of $CS(\tilde{s})$, $PS(\tilde{s})$ and $C(\tilde{s})$. Parameter values are $\alpha = 0.5$, $\mu = 0.02$, $r = 0.1$, $\sigma = 0.01$, $\eta = 0.05$, $\delta = 10$.

$\tilde{s}$ because though the firm's output decreases with a larger $\tilde{s}$, the firm starts production earlier. So the consumption also starts earlier, which is preferred by the consumers and yields a larger consumer surplus. The discounted subsidy cost also increases with $\tilde{s}$. This is intuitive because otherwise the government should provide an infinite subsidy rate given that both producer and consumer surpluses increase with $\tilde{s}$. When $\tilde{s}$ is small, the subsidy cost grows slower than the sum of producer and consumer surplus. This is illustrated by an increasing total surplus in Figure 1b. As $\tilde{s}$ increases, the subsidy cost increases faster than the sum of consumer and producer surplus. This leads to the decrease of total surplus in Figure 1b.

3.4 Optimal conditional subsidy

Though unconditional subsidy does not yield the first-best outcome, it is still possible to align firm's optimal investment decision to the social optimal decision through a conditional subsidy, that is, the subsidy implemented at a specific Brownian motion level. This is due to the same investment threshold of the firm and social planner without subsidy regulation, $X_W^* = X^*(0)$. We take that as one decision variable already being aligned. Then it is only necessary to align the investment capacities by choosing the subsidy rate parameter. The optimal conditional subsidy regulation is given by the following proposition.

Proposition 4 *The optimal conditional subsidy is to introduce subsidy at the social optimal investment threshold X_W^* with the following subsidy rate:*

$$\tilde{s}^* = \begin{cases} \frac{X_W^* - (r-\mu)\delta}{2(r-\mu)\delta - X_W^*} & \text{for subsidy flow } \tilde{s}p(X(t), K)K, \\ \frac{rX_W^* - r(r-\mu)\delta}{r-\mu} & \text{for subsidy flow } \tilde{s}K, \\ \frac{X_W^* - (r-\mu)\delta}{\delta(r-\mu)} & \text{for lump sum subsidy } \tilde{s}K. \end{cases}$$

The optimal conditional subsidy described in this proposition aligns firm's and social planner's investment decision, and generates the first-best outcome. Because there is no asymmetry of information on investment costs, according to Broer and Zwart (2013), a conditional subsidy described in Proposition 4 can be interpreted simply as the regulator tells the monopolist when to invest and how much to invest. The changes in market parameters influence the dynamic optimal subsidy rates, and the influence is summarized in the following corollary.

Corollary 2 *When the volatility rate σ or drift parameter μ increases, the firm needs to be subsidized more in order to invest at the social optimal capacity level.*

This is because an increase in σ or μ makes the social planner invest later and more. In order for the firm to catch up with the social optimal capacity, either to prepare for positive future demand shocks because of larger σ or to satisfy a larger anticipated future market demand growth because of larger market trend μ, more monetary support needs to be transferred to the firm.

4 Non-linear Demand

From previous section, it is now clear that with linear inverse demand function, unconditional subsidy support does not align firm and social planner's investment decision. A possible reason might be the linear demand shocks. In this section, we study nonlinear demand shocks and check the performance of the same subsidy regulations in previous section. Suppose

$$p(t) = X(t)K(t)^{-\gamma}$$

with $0 < \gamma < 1$, and $X(t)$ follows geometric Brownian motion of (1). Investment costs are of the form[3] $\delta_0 + \delta_1 K(t)$ with $\delta_0 \geq 0$ and $\delta_1 > 0$. In the following analysis, we first discuss the first-best outcome and then check whether unconditional subsidy makes the monopolist deviate or converge to the social optimal investment. Later we focus on the optimal subsidy regulation policy.

4.1 First-best benchmark

The producer surplus equals to the value of investment, i.e., expected discounted profit flows after investment minus investment costs. For a given investment capacity K and geometric Brownian motion level $X(0) = X$, the producer surplus at X is given by

$$PS(X,K) = V(X,K) = \frac{XK^{1-\gamma}}{r-\mu} - \delta_0 - \delta_1 K.$$

[3]We take a different cost structure than the linear demand because of two reasons. First reason is that the cost structure δK does not yield any solution for firm's investment decision under non-linear demand. Second reason is that the cost structure $\delta_0 + \delta_1 K$ does not change the main results obtained under linear demand.

For a given level of K, the producer surplus increases with X. The reason is that for a given output K, a larger X implies larger market demand and higher market prices, which makes the firm more profitable and thus generates larger producer surplus. Given the firm produces an output of K, the discounted instantaneous consumer surplus at $X(t)$ is

$$cs(X(t), K) = \int_{p}^{\infty} \left(\frac{X(t)}{p}\right)^{\frac{1}{\gamma}} dp = \frac{\gamma}{1-\gamma} X^{\frac{1}{\gamma}}(t) p^{\frac{\gamma-1}{\gamma}} \Big|_{X(t)K^{-\gamma}}^{\infty} = \frac{\gamma}{1-\gamma} X(t) K^{1-\gamma}.$$

At $X(0) = X$, the expected consumer surplus is

$$CS(X, K) = \frac{\gamma}{1-\gamma} \frac{XK^{1-\gamma}}{r-\mu}.$$

The insight for consumer surplus is the same as that under linear demand. For a given X, the consumer surplus increases with K because more output decreases market prices. For a given K, the consumer surplus increases with X because consumer's willingness to pay increases. The expected social surplus is the sum of producer and consumer surplus and is given by

$$W(X, K) = \frac{1}{1-\gamma} \frac{XK^{1-\gamma}}{r-\mu} - \delta_0 - \delta_1 K.$$

From the social welfare function, we can derive the social optimal investment decision as the first-best benchmark. It is summarized in the following proposition.

Proposition 5 *The social optimal investment threshold is*

$$X_W^* = (r-\mu)\delta_1 \left(\frac{\delta_0 \beta(1-\gamma)}{\delta_1(\beta\gamma - 1)}\right)^{\gamma}$$

and the social optimal investment capacity is

$$K_W^* \equiv K_W^*(X_W^*) = \frac{\delta_0 \beta(1-\gamma)}{\delta_1(\beta\gamma - 1)}.$$

A further analysis on the influence of market volatility yields similar insight as that under linear market demand structure. Because $\partial\beta/\partial\sigma < 0$, $\partial X_W^*/\partial\beta < 0$, and $\partial K_W^*/\partial\beta < 0$, it can be concluded that $\partial X_W^*/\partial\sigma > 0$ and $\partial K_W^*/\partial\sigma > 0$, implying a non-linear demand structure like the iso-elastic demand does not change the standard real option result that a greater volatility delays investment and leads to installing a larger project.

4.2 Subsidized Profit Maximization Investment

In this subsection, subsidy flows $s(X(t), K, \tilde{s}) = \tilde{s}p(X(t), K)K$ and $s(X(t), K, \tilde{s}) = \tilde{s}K$ are considered. The lump sum subsidy transfer $s(K, \tilde{s}) = \tilde{s}(\delta_0 + \delta_1 K)$ is analyzed in more detail than that under the linear demand structure because of the fixed cost, δ_0, from investment. This makes it behave a little differently from the subsidy flow $\tilde{s}K$. The focus of the analysis is on how subsidy influences firm's investment decision. Moreover, it compares the influence of subsidy under non-linear demand with the influence under linear demand.

Proposition 6 *When subsidy flow is $s(X(t), K, \tilde{s}) = \tilde{s}p(X(t), K)K$, firm's optimal investment threshold $X^*(\tilde{s})$ and investment capacity $K^*(\tilde{s})$ are equal to*

$$\begin{aligned} X^*(\tilde{s}) &= \frac{\delta_1(r-\mu)}{(1-\gamma)(1+\tilde{s})}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma}, \\ K^*(\tilde{s}) &= \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}. \end{aligned}$$

When subsidy flow is $s(X(t), K, \tilde{s}) = \tilde{s}K$ and $\tilde{s} < r\delta_1$, firm's optimal investment threshold $X^*(\tilde{s})$ and investment capacity $K^*(\tilde{s})$ are given by

$$\begin{aligned} X^*(\tilde{s}) &= \frac{r-\mu}{1-\gamma}\left(\delta_1 - \frac{\tilde{s}}{r}\right)\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1-\tilde{s}/r)(\beta\gamma-1)}\right)^{\gamma}, \\ K^*(\tilde{s}) &= \frac{\delta_0\beta(1-\gamma)}{(\delta_1-\tilde{s}/r)(\beta\gamma-1)}. \end{aligned}$$

When the lump sum subsidy transfer is $s(K, \tilde{s}) = \tilde{s}(\delta_0 + \delta_1 K)$, firm's optimal investment threshold $X^*(\tilde{s})$ and investment capacity $K^*(\tilde{s})$ are equal to

$$\begin{aligned} X^*(\tilde{s}) &= \frac{\delta_1(r-\mu)(1-\tilde{s})}{1-\gamma}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma}, \\ K^*(\tilde{s}) &= \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}. \end{aligned}$$

By comparing (X_W^*, K_W^*) with $(X^*(0), K^*(0))$, we find that without subsidy regulation, the firm invests later than the social planner but with the social optimal capacity. This is different from the linear demand structure, where the firm invests at the same time as the social planner but with half of the social optimal capacity when $\tilde{s} = 0$. For unconditional subsidy, Proposition 6 shows that subsidy makes the firm invest earlier, the same as the linear demand structure. Another insight of the three subsidy regulations is that subsidy flow $\tilde{s}K$ influences the firm's optimal investment capacity, but subsidy flow $\tilde{s}p(X(t), K)K$ and lump sum subsidy $\tilde{s}(\delta_0 + \delta_1 K)$ do not. This is different from that under linear demand structure, where all the three subsidy regulations make firm invest less.

For the unconditional subsidy flow $\tilde{s}p(X(t), K)K$ and lump sum subsidy $\tilde{s}(\delta_0+\delta_1 K)$, because $X^*(0) > X_W^*$ and subsidy regulation makes firm invest earlier than trigger $X^*(0)$, it is possible to align firm's and social optimal investment threshold by choosing appropriate subsidy rate $\tilde{s}$. This implies that unconditional subsidy can reach the first-best outcome for subsidy flow $\tilde{s}p(X(t), K)K$ and lump sum subsidy $\tilde{s}(\delta_0+\delta_1 K)$. Whereas for subsidy flow $\tilde{s}K$, unconditional subsidy not only makes the firm invest earlier but also makes the firm invest with a capacity that is larger than the social optimal capacity. This implies that the first-best outcome cannot be reached for unconditional subsidy flow $\tilde{s}K$. But a conditional subsidy flow $\tilde{s}K$ can achieve the first-best outcome. This is because subsidy motivates the firm to invest earlier than $X^*(0)$, a conditional subsidy can be implemented such that the firm invests at X_W^* with K_W^* for subsidy flow $\tilde{s}K$. In fact, the optimal conditional subsidy can be implemented for all the three, the same as under the linear demand structure. We summarize the optimal unconditional and conditional subsidy regulations in the following proposition.

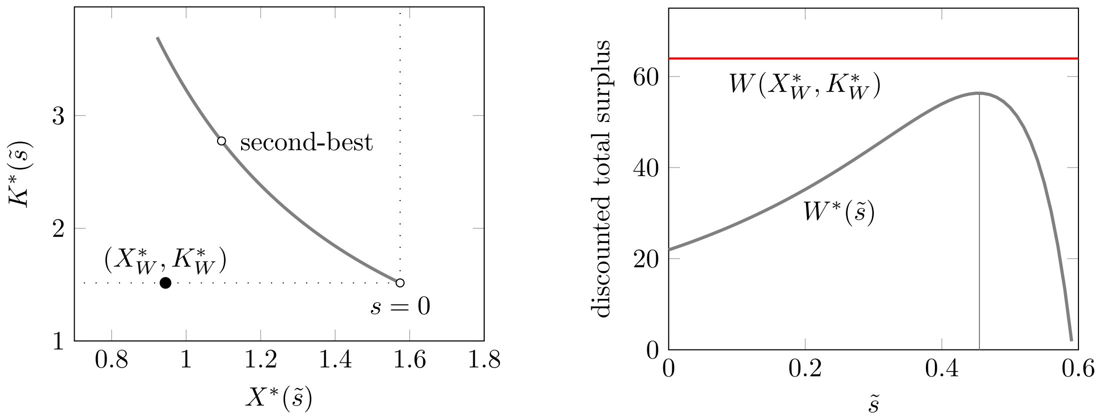
Proposition 7 *Unconditional subsidy regulation implemented at $X(0)$ is optimal for subsidy flow $\tilde{s}p(X(t),K)K$ and lump sum subsidy transfer $\tilde{s}(\delta_0+\delta_1 K)$ if the subsidy rate $\tilde{s}^*$ is equal to,*

$$\tilde{s}^* = \begin{cases} \frac{\gamma}{1-\gamma} & \text{for subsidy flow } \tilde{s}p(X(t),K)K, \\ \gamma & \text{for lump sum subsidy } \tilde{s}(\delta_0+\delta_1 K). \end{cases}$$

Conditional subsidy regulation implemented at X_W^* is optimal if the subsidy rate $\tilde{s}^*$ is given by

$$\tilde{s}^* = \begin{cases} \frac{\gamma}{1-\gamma} & \text{for subsidy flow } \tilde{s}p(X(t),K)K, \\ r\gamma\delta_1 & \text{for subsidy flow } \tilde{s}K, \\ \gamma & \text{for lump sum subsidy } \tilde{s}(\delta_0+\delta_1 K). \end{cases}$$

With the optimal subsidy regulation, the firm's investment decision is aligned to the social optimal decision and leads to the first-best outcome. This result is the same as under the linear demand structure. Recall from the previous section that for unconditional subsidy, the second-best outcome is to implement no subsidy at all. In the following analysis, we check whether this is also true for iso-elastic demand structure. Note that unconditional subsidy flow $\tilde{s}p(X(t),K)K$ and lump sum subsidy $\tilde{s}(\delta_0+\delta_1 K)$ can achieve the first-best outcome. So our focus is on the unconditional subsidy flow $\tilde{s}K$.



(a) Illustration of $(X^*(\tilde{s}), K^*(\tilde{s}))$.

(b) Illustration of $W(\tilde{s}) = W(X^*(\tilde{s}), K^*(\tilde{s}))$.

Figure 3: Illustration of $(X^*(\tilde{s}), K^*(\tilde{s}))$, and $W^*(\tilde{s})$ for subsidy flow $\tilde{s}K$. Parameter values are $\mu = 0.02$, $r = 0.1$, $\sigma = 0.01$, $\gamma = 0.5$, $\delta_0 = 2$, $\delta_1 = 10$.

Figure 3a demonstrates the firm's optimal investment capacity $K^*(\tilde{s})$ and optimal investment threshold $X^*(\tilde{s})$ as functions of subsidy rate $\tilde{s}$. It is clear that when $\tilde{s} = 0$, $K^*(0) = K_W^*$. As $\tilde{s}$ increases, $K^*(\tilde{s})$ deviates from social optimal K_W^*, but $X^*(\tilde{s})$ is getting close to X_W^*. Figure 3b shows the total surplus, discounted to a predetermined time $X^*(0)$, as a function of unconditional subsidy rate. As illustrated, there exists a subsidy rate that generates the highest level of social welfare for unconditional subsidy. The subsidy rate that generates the second-best outcome is $\tilde{s} = 0.455$. As shown in Figure 3b, the total surplus for the second-best outcome is below the social optimal welfare that is also discounted to $X^*(0)$. This result is similar to that under linear demand structure for the unconditional subsidy regulation.

5 Conclusion

This paper analyzes investment decision of a profit maximizer under subsidy regulation and how to align this decision to social optimal decision through optimal subsidy. We show that unconditional subsidy introduced from the beginning accelerates the investment of a monopoly firm. Under linear demand structure, unconditional subsidy regulation cannot align the profit and welfare maximizers' investment decisions. Moreover, it decreases monopolist's optimal investment capacity and results in smaller social surplus. There is conditional subsidy regulation that aligns the firm's investment decision to the social optimal decision. This optimal conditional subsidy requires to introduce subsidy at the social optimal investment threshold. For non-linear iso-elastic demand, depending on the form of subsidy regulations, it is possible to implement unconditional subsidy to align profit maximizing and social optimal investment decisions. The conditional subsidy can also be implemented in a similar way as under linear demand structure. If we dismiss the efficiency loss when collecting and allocating the taxation, the aligned profit maximizer's investment decision can lead to the first-best outcome for both the linear and non-linear market demand.

Appendix

This appendix contains proofs of the propositions for the linear and iso-elastic demand functions.

Proof of Proposition 1 The social optimal investment capacity $K_W(X)$ maximizes $W(X,K)$ and is equal to

$$K_W(X) = \frac{rX - r(r-\mu)\delta}{\eta[(r-\mu)\alpha + r(1-\alpha)X]}, \tag{12}$$

which is equivalent to

$$X = \frac{(r-\mu)(r\delta + \alpha\eta K_W(X))}{r\,(1-(1-\alpha)\eta K_W(X))}.$$

Let the option value before social planner's investment be $A_W X^\beta$. The value matching and smooth pasting at the social optimal investment threshold X_W^* yield

$$\begin{aligned} W(X_W^*, K_W(X_W^*)) &= A_W X_W^{*\beta}, \\ \left.\frac{\partial W(X_W^*, K_W(X_W^*))}{\partial X}\right|_{X=X_W^*} &= \beta A_W X_W^{*\beta-1}. \end{aligned}$$

Then we have the following equation

$$\frac{X_W^*}{r-\mu}\frac{\beta-1}{\beta}\left(2-(1-\alpha)\eta K_W(X_W^*)\right) - \frac{\alpha\eta K_W(X_W^*)}{r} - 2\delta = 0. \tag{13}$$

Combining (12) and (6), we get the social optimal investment capacity K_W^* satisfies the following implicit equation

$$\alpha(1-\alpha)\eta^2K^2 + r\delta(\beta+1)(1-\alpha)\eta K + \alpha(\beta-2)\eta K - 2r\delta = 0. \tag{14}$$

Proof of Proposition 2 For $X(0) = X$, the optimal investment capacity $K(X, \tilde{s})$ maximizes the investment value and thus satisfies the following first order condition

$$\frac{X}{r-\mu} - \frac{\delta}{1+\tilde{s}} = \frac{2\alpha\eta K}{r} + \frac{2(1-\alpha)X\eta K}{r-\mu}.$$

Thus,

$$K(X, \tilde{s}) = \frac{r(1+\tilde{s})X - r(r-\mu)\delta}{2\eta(1+\tilde{s})[(r-\mu)\alpha + r(1-\alpha)X]}. \tag{15}$$

Let the option value before investment be AX^β, $\beta > 2$ from assumptions in the model of additive demand function. From the value matching and smooth pasting at the optimal investment threshold X^*, then

$$\begin{aligned} V(X^*, K(X^*, \tilde{s}), \tilde{s}) &= AX^{*\beta}, \\ \left.\frac{\partial V(X, K(X, \tilde{s}), \tilde{s})}{\partial X}\right|_{X=X^*} &= \beta AX^{*\beta-1}. \end{aligned}$$

This yields that $X(\tilde{s})$ satisfies the following equation

$$\frac{X^*}{r-\mu}\frac{\beta-1}{\beta}\left(1-(1-\alpha)\eta K(X^*)\right) - \frac{\alpha\eta K(X^*)}{r} - \frac{\delta}{1+\tilde{s}} = 0. \tag{16}$$

Solving (15) and (16) yields that the optimal investment capacity $K^*(\tilde{s})$ satisfies the quadratic form

$$2\alpha(1-\alpha)\eta^2K^2 + \frac{r\delta}{1+\tilde{s}}(\beta+1)(1-\alpha)\eta K + \alpha(\beta-2)\eta K - \frac{r\delta}{1+\tilde{s}} = 0. \tag{17}$$

Proof of Corollary 1 Denote $\delta/(1+\tilde{s}) = x$, then from (10), it can be derived that

$$\frac{\mathrm{d}K^*}{\mathrm{d}x}\left(4\alpha(1-\alpha)\eta^2K^* + rx(\beta+1)(1-\alpha)\eta + \alpha(\beta-2)\right) = r\left(1-(\beta+1)(1-\alpha)\eta K^*\right).$$

This implies that $\mathrm{d}K^*/\mathrm{d}x > 0$, i.e., $\mathrm{d}K^*(\tilde{s})/\mathrm{d}\tilde{s} < 0$. So the subsidy in the market motivates the firm to invest less. Moreover, from (15), the profit maximizer's optimal investment threshold is also influenced by the subsidy. It holds that

$$\frac{\mathrm{d}X^*}{\mathrm{d}x}\frac{1-2(1-\alpha)\eta K^*}{r-\mu} = \frac{\mathrm{d}K^*}{\mathrm{d}x}\left(\frac{2(1-\alpha)\eta X^*}{r-\mu} + \frac{2\alpha\eta}{r} + 1\right).$$

This yields that $\mathrm{d}X^*/\mathrm{d}x > 0$, i.e., $\mathrm{d}X^*(\tilde{s})/\mathrm{d}\tilde{s} < 0$. The subsidy also makes the profit maximizer invest earlier.

Proof of Proposition 3 For a given capacity K and $X(0) = X$, the value for the expected discounted profit flow is

$$V(X, K, \tilde{s}) = \frac{XK}{r-\mu} - \eta K^2\left(\frac{\alpha}{r} + \frac{(1-\alpha)X}{r-\mu}\right) + \frac{\tilde{s}K}{r} - \delta K.$$

The optimal capacity for a given X and $\tilde{s}$ maximizes the value of the firm and is given by

$$K(X, \tilde{s}) = \frac{rX + (r-\mu)\tilde{s} - r(r-\mu)\delta}{2\eta\left[(r-\mu)\alpha + r(1-\alpha)X\right]}.$$

For a given capacity size K, by value matching and smooth pasting at the investment threshold, it can be derived that

$$X(K, \tilde{s}) = \frac{\beta(r-\mu)}{\beta - 1} \frac{\alpha\eta K - \tilde{s} + r\delta}{r\left[1 - \eta K(1-\alpha)\right]}.$$

Combining $K(X, \tilde{s})$ and $X(K, \tilde{s})$, we get that the optimal investment capacity $K^*(\tilde{s})$ satisfies the implicit expression

$$2\alpha(1-\alpha)\eta^2 K^2 + (\beta+1)(1-\alpha)(r\delta - \tilde{s})\eta K + \alpha(\beta-2)\eta K - (r\delta - \tilde{s}) = 0.$$

Proof of Proposition 4 For subsidy flow $\tilde{s}p(X(t), K)K$ and $\tilde{s}K$, the optimal subsidy rate can be derived by letting $K_W(X_W) = K(X_W, \tilde{s})$. For lump sum subsidy, the optimal subsidy rate is equal to $\tilde{s}^*/(r\delta)$ given $\tilde{s}^*$ as the optimal subsidy rate for the flow $\tilde{s}K$.

Proof of Corollary 2 Larger σ leads to larger $\tilde{s}^*$ because of $\partial X_W^*/\partial\sigma > 0$ and $\partial\tilde{s}^*/\partial X_W^* > 0$. Thus, it holds that the optimal conditional subsidy rate $\tilde{s}^*$ increases with σ. Next, we check the influence of μ on $\tilde{s}^*$. Recall from previous analysis that $\partial X_W^*/\partial\mu > 0$. Then for the three optimal conditional subsidy rates, we can get the following first order partial derivatives of $\tilde{s}^*$ with respect to μ

$$\frac{\partial \tilde{s}^*}{\partial \mu} = \begin{cases} \frac{\delta}{[2(r-\mu)\delta - X_W^*]^2}\left(X_W^* + (r-\mu)\frac{\partial X_W^*}{\partial \mu}\right) & \text{for flow } \tilde{s}p(X(t),K)K, \\ \frac{r}{(r-\mu)^2}\left(X_W^* + (r-\mu)\frac{\partial X_W^*}{\partial \mu}\right) & \text{for flow } \tilde{s}K, \\ \frac{1}{\delta(r-\mu)^2}\left(X_W^* + (r-\mu)\frac{\partial X_W^*}{\partial \mu}\right) & \text{for lump sum } \tilde{s}K. \end{cases}$$

It can be concluded that $\partial\tilde{s}^*/\partial\mu > 0$.

Proof of Proposition 5 For given X, the investment capacity that maximizes social welfare is equal to

$$K_W(X) = \left(\frac{X}{(r-\mu)\delta_1}\right)^{1/\gamma}.$$

Let the social planner's value before investment be AX^β, then according to value matching and smooth pasting conditions at optimal investment threshold X_W^*, we get

$$\begin{aligned} AX_W^{*\beta} &= \frac{1}{1-\gamma}\frac{X_W^* K_W(X_W^*)^{1-\gamma}}{r-\mu} - \delta_0 - \delta_1 K_W(X_W^*), \\ \beta A X_W^{*\beta-1} &= \frac{1}{1-\gamma}\frac{K_W(X_W^*)^{1-\gamma}}{r-\mu}. \end{aligned}$$

This yields

$$X_W^* = (r-\mu)\delta_1\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^\gamma.$$

The corresponding investment capacity is given by

$$K_W^* \equiv K_W(X_W^*) = \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}.$$

To get the insight of how σ influences X_W^* and K_W^*, we can derive the following first order partial derivatives:

$$\frac{\partial X_W^*}{\partial \beta} = -\frac{\delta_0\gamma(1-\gamma)(r-\mu)}{(\beta\gamma-1)^2}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma-1} < 0,$$

$$\frac{\partial K_W^*}{\partial \beta} = -\frac{\delta_0(1-\gamma)}{\delta_1(\beta\gamma-1)^2} < 0.$$

Proof of Proposition 6 For a given capacity size K and $X(0) = X$, the value of the expected discounted profit flow at X is equal to

$$V(X,K) = \begin{cases} \frac{XK^{1-\gamma}(1+\tilde{s})}{r-\mu} - \delta_0 - \delta_1 K & \text{for subsidy flow } \tilde{s}p(X(t),K)K, \\ \frac{XK^{1-\gamma}}{r-\mu} + \frac{\tilde{s}K}{r} - \delta_0 - \delta_1 K & \text{for subsidy flow } \tilde{s}K, \\ \frac{XK^{1-\gamma}}{r-\mu} - (1-\tilde{s})(\delta_0 + \delta_1 K) & \text{for lump sum subsidy } \tilde{s}(\delta_0 + \delta_1 K). \end{cases}$$

Maximizing $V(X,K)$ with respect to K yields that the optimal capacity for a given X is given by

$$K(X) = \begin{cases} \left(\frac{X(1-\gamma)(1+\tilde{s})}{\delta_1(r-\mu)}\right)^{1/\gamma} & \text{for subsidy flow } \tilde{s}p(X(t),K)K, \\ \left(\frac{X(1-\gamma)}{(r-\mu)(\delta_1-\tilde{s}/r)}\right)^{1/\gamma} & \text{for subsidy flow } \tilde{s}K, \\ \left(\frac{X(1-\gamma)}{(r-\mu)(1-\tilde{s})\delta_1}\right)^{1/\gamma} & \text{for lump sum subsidy } \tilde{s}(\delta_0 + \delta_1 K). \end{cases}$$

Substituting $K(X)$ into $V(X,K)$ gives the expected value as a function of X, i.e., $V(X)$. Let the value before investment threshold X^* be AX^β. Then the value matching and smooth pasting conditions at X^* yield

$$X^*(\tilde{s}) = \begin{cases} \frac{\delta_1(r-\mu)}{(1-\gamma)(1+\tilde{s})}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^\gamma & \text{for subsidy flow } \tilde{s}p(X(t),K)K, \\ \frac{r-\mu}{1-\gamma}(\delta_1 - \tilde{s}/r)\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1-\tilde{s}/r)(\beta\gamma-1)}\right)^\gamma & \text{for subsidy flow } \tilde{s}K, \\ \frac{\delta_1(r-\mu)(1-\tilde{s})}{1-\gamma}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^\gamma & \text{for lump sum subsidy } \tilde{s}(\delta_0 + \delta_1 K). \end{cases}$$

From the optimal investment threshold $X^*(\tilde{s})$, we can get that the optimal investment capacity $K^*(\tilde{s})$ is equal to

$$K^*(\tilde{s}) \equiv K^*(X^*(\tilde{s})) = \begin{cases} \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)} & \text{for subsidy flow } \tilde{s}p(X(t),K)K, \\ \frac{\delta_0\beta(1-\gamma)}{(\delta_1-\tilde{s}/r)(\beta\gamma-1)} & \text{for subsidy flow } \tilde{s}K, \\ \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)} & \text{for lump sum subsidy } \tilde{s}(\delta_0 + \delta_1 K). \end{cases}$$

Proof of Proposition 7 Given in the text.

References

Roger Adkins and Dean Paxson. Subsidies for renewable energy facilities under uncertainty. *The Manchester School*, 2015.

Felipe L. Aguerrevere. Equilibrium investment strategies and output price behavior: A real-options approach. *The Review of Financial Studies*, 16(4):1239–1272, 2003.

Felipe L. Aguerrevere. Real options, product market competition, and asset returns. *The Journal of Finance*, 64(2):957–983, 2009.

Trine K. Boomsma and Kristin Linnerud. Market and policy risk under different renewable electricity support schemes. *Energy*, 89:435–448, 2015.

Trine K. Boomsma, Nigel Meade, and Stein-Erik Fleten. Renewable energy investments under different support schemes: A real options approach. *European Journal of Operational Research*, 220(1):225–237, 2012.

Hendrika J. Boonman. *Strategic real options: Capacity optimization and demand structures.* PhD thesis, 2014.

Peter Broer and Gijsbert Zwart. Optimal regulation of lumpy investments. *Journal of Regulatory Economics*, 44(2):177–196, 2013.

Ephraim Chirwa and Andrew Dorward. *Agricultural input subsidies: The recent Malawi experience.* Oxford University Press, 2013.

Michail Chronopoulos, Verena Hagspiel, and Stein-Erik Fleten. Stepwise green investment under policy uncertainty. *Energy Journal*, 37(4):87 – 108, 2016.

Avinash K. Dixit and Robert S. Pindyck. *Investment under Uncertainty.* Princeton University Press, Princeton, 1994.

Ian M. Dobbs. Intertemporal price cap regulation under uncertainty. *The Economic Journal*, 114(495): 421–440, 2004.

Thomas Eichner and Marco Runkel. Subsidizing renewable energy under capital mobility. *Journal of Public Economics*, 117:50 – 59, 2014.

Lewis Evans and Graeme Guthrie. Price-cap regulation and the scale and timing of investment. *The RAND Journal of Economics*, 43(3):537–561, 2012.

Steven R. Grenadier. Option exercise games: The intersection of real options and game theory. *Journal of Applied Corporate Finance*, 13(2):99–107, 2000.

Verena Hagspiel, Kuno J. M. Huisman, and Peter M. Kort. Volume flexibility and capacity investment under demand uncertainty. *International Journal of Production Economics*, 178:95 – 108, 2016.

Kevin A. Hassett and Gilbert E. Metcalf. Investment with uncertain tax policy: Does random tax policy discourage investment. *The Economic Journal*, 109(457):372–393, 1999.

Kuno J. M. Huisman and Peter M. Kort. Strategic capacity investment under uncertainty. *The RAND Journal of Economics*, 46(2):376–408, 2015.

Nalin Kulatilaka and Enrico C. Perotti. Strategic growth options. *Management Science*, 44(8):1021–1031, 1998.

Robert L. McDonald and Daniel R. Siegel. The value of waiting to invest. *The Quarterly Journal of Economics*, 101(4):707–728, 1986.

Robert Novy-Marx. An equilibrium model of investment under uncertainty. *The Review of Financial Studies*, 20(5):1461, 2007.

Enrico Pennings. Taxes and stimuli of investment under uncertainty. *European Economic Review*, 44(2):383 – 391, 2000.

Dani Rodrik. Political economy and development policy. *European Economic Review*, 36(2):329 – 336, 1992.

Vijay P. Sharma and Hrima Thaker. Fertiliser subsidy in india: Who are the beneficiaries? *Economic and Political Weekly*, 45(12):68–76, 2010.

Bert Willems and Gijsbert Zwart. Optimal regulation of network expansion. *The RAND Journal of Economics*, 2017. (forthcoming).