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A Note on the Characterizations of the Compromise Value

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Abstract: In Borm, Keiding, McLean, Oortwijn and Tijs (1992) the compromise value is introduced as a solution concept on the class of compromise admissible NTU-games. Two characterizations of the compromise value are provided on subclasses of NTU-games.

This note shows that in one of these characterizations the axioms are dependent. As a result of this observation a new characterization of the compromise value is provided. Moreover, it turns out that with a small weakening of the symmetry property the axioms in the original characterization become independent.

Further, it is shown that these characterizations can be extended to a larger class of NTU-games. Finally, all monotonic, Pareto optimal, and covariant values on this class of NTU-games are described.

1 Introduction

Borm, Keiding, McLean, Oortwijn and Tijs (1992) introduced the compromise value as a new solution concept for a large class of NTU-games. The compromise value, by definition, extends the τ-value for TU-games (Tijs (1981)) and the Raiffa–Kalai–Smorodinsky solution (RKS-solution) for bargaining problems (Raiffa (1953), Kalai and Smorodinsky (1975)) to NTU-games. Two characterizations of the compromise value show that also axiomatically the compromise value generalizes the solution concepts mentioned above.

In Section 2 of this note it is shown that in one of the characterizations of the compromise value provided by Borm et al. (1992) the axiom system is dependent. As a result of this observation we obtain a new characterization of the compromise value (for a larger class of games), which is similar to one of the characterizations of the MC-value introduced in Otten, Borm, Peleg and Tijs (1994). Furthermore, we show that by weakening the (strong) symmetry property, the original characterization of the compromise value can be adapted in such a way that the axioms are independent. In the characterizations of the compromise value discussed in Section 2 a non-levelness condition plays a crucial role. Section 3 illustrates that this condition can be weakened in order to obtain a characterization on a larger class of NTU-games. We use a similar technique as Peters and Tijs (1984) who extended Thomson's (1980) axiomatization of the RKS-solution to a larger class of bargaining problems by weakening the non-levelness condition.

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Finally, Section 4 characterizes the set of all monotonic, Pareto optimal, and covariant values on this class of NTU-games using monotonic curve solutions as introduced by Peters and Tijs (1984).

2 The Compromise Value

We start with some definitions. A *non-transferable utility game* or *NTU-game* is a pair (N, V), where N is a finite set of players and V is a map assigning to each coalition $S \in 2^N \setminus \{\emptyset\}$ a subset $V(S)$ of $\mathbb{R}^S$ of *attainable payoff* vectors. We assume that for each $i \in N$ there exists a real number $v(i)$ such that $V(\{i\}) = \{x \in \mathbb{R} \mid x \leq v(i)\}$. Further, we assume that for each $S \in 2^N \setminus \{\emptyset\}$ the following properties hold

(i) $V(S)$ is a non-empty, closed and comprehensive subset of $\mathbb{R}^S$
(ii) $V(S) \cap \{x \in \mathbb{R}^S \mid x_i \geq v(i) \text{ for all } i \in S\}$ is bounded.

An NTU-game (N, V) is often identified with V.
Let V be an NTU-game. For each $S \in 2^N \setminus \{\emptyset\}$, let

$$dom(V(S)) := \{x \in \mathbb{R}^S \mid x < y \text{ for some } y \in V(S)\}$$
$$wdom(V(S)) := \{x \in \mathbb{R}^S \mid x \leq y,\ x \neq y \text{ for some } y \in V(S)\}.$$

The elements of $(w)dom(V(S))$ are (weakly) dominated by the coalition S in the game V. Elements of $V(S) \setminus dom(V(S))$ are called *weakly Pareto optimal* in $V(S)$ and elements of $V(S) \setminus wdom(V(S))$ are called *Pareto optimal* in $V(S)$. The *core of* V, denoted by $C(V)$, consists of all payoff vectors attainable for the grand coalition N which are not dominated by any coalition S.

Let $i \in N$. The *utopia payoff* for player i, $K_i(V)$, is defined by

$$K_i(V) := \sup\{t \in \mathbb{R} \mid \exists_{a \in \mathbb{R}^{N \setminus \{i\}}} : (a, t) \in V(N),\ a \notin dom(V(N \setminus \{i\})),\ a \geq (v(j))_{j \in N \setminus \{i\}}\}.$$

By assumption (ii) in the definition of an NTU-game it follows that $K_i(V) < \infty$. However, it might happen that $K_i(V) = -\infty$. We will restrict ourselves to NTU-games V for which $K_i(V) \in \mathbb{R}$ for all $i \in N$. The vector $K(V) := (K_i(V))_{i \in N}$ is also called the *upper value* of V.

Let $i \in N$ and let $S \in 2^N$ with $i \in S$. The *remainder* of $i \in S$ is given by

$$\rho^V(S, i) := \sup\{t \in \mathbb{R} \mid \exists_{a \in \mathbb{R}^{S \setminus \{i\}}} : (a, t) \in V(S),\ a > (K_j(V))_{j \in S \setminus \{i\}}\}.$$

The *minimal right* of player i is denoted by

$$k_i(V) := \max_{S : i \in S} \rho^V(S, i),$$

and the vector $k(V) := (k_i(V))_{i \in N}$ is also called the *lower value* of V. Note that $k_i(V) \geq v(i)$ for all $i \in N$, but it might happen that $k_i(V) = \infty$ for some $i \in N$. Again, we will restrict ourselves to NTU-games V for which $k(V) \in \mathbb{R}^N$.

The compromise value is defined on the class of compromise admissible NTU-games. An NTU-game V is called *compromise admissible* if

$$k(V) \leq K(V), \text{ and } k(V) \in V(N), K(V) \notin dom(V(N)).$$

It is easy to show that for a compromise admissible game V the assumption $K(V) \notin dom(V(N))$ implies that $K(V) \notin wdom(V(N))$. By C^N we denote the class of all compromise admissible NTU-games with player set N. It is shown by Borm et al. (1992) that an NTU-game with a non-empty core is compromise admissible.

A *value* on C^N is a map $f: C^N \rightarrow \mathbb{R}^N$, which assigns to each $V \in C^N$ a payoff vector. For a compromise admissible NTU-game V the *compromise value* $T(V)$ is defined as the unique vector on the line segment between $k(V)$ and $K(V)$ which lies in $V(N)$ and is nearest to the utopia value $K(V)$, i.e.,

$$T(V) := k(V) + \alpha_V(K(V) - k(V)),$$

where

$$\alpha_V := \max\{\alpha \in [0,1] \mid k(V) + \alpha(K(V) - k(V)) \in V(N)\}.$$

Borm et al. (1992) show that the characterization of the two player RKS-solution by Kalai and Smorodinsky (1975) can be extended in order to provide a characterization of the compromise value. In order to illustrate this result we first need some notation and definitions.

For vectors $x, y \in \mathbb{R}^N$ and a subset $C \subset \mathbb{R}^N$, we define $x * y := (x_i y_i)_{i \in N}$ and $x * C := \{x * c \mid c \in C\}$. For $x \in \mathbb{R}^N$ and $S \in 2^N$, $x_S := (x_i)_{i \in S} \in \mathbb{R}^S$.

Let (N, V) be an NTU-game, $\alpha \in \mathbb{R}^N_{++}$ and $\beta \in \mathbb{R}^N$. The NTU-game $(N, \alpha * V + \beta)$ is defined by

$$(\alpha * V + \beta)(S) := \alpha_S * V(S) + \{\beta_S\} \text{ for all } S \in 2^N.$$

Let $A^N \subset C^N$ and let $f: A^N \rightarrow \mathbb{R}^N$ be a value on A^N.

(i) f is called *Pareto optimal* on A^N if $f(V) \in V(N) \setminus wdom(V(N))$ for all $V \in A^N$.
(ii) f is called *weakly Pareto optimal* on A^N if $f(V) \in V(N) \setminus dom(V(N))$ for all $V \in A^N$.
(iii) f is *symmetric* if $f_i(V) = f_j(V)$ for all $V \in A^N$ and all $i, j \in N$ which are symmetric in V. Here, players $i, j \in N$ are called *symmetric* in V if

  (1) for all $S \subset N \setminus \{i,j\}$, all $x \in V(S \cup \{i\})$ it holds that $y \in V(S \cup \{j\})$, where $y \in \mathbb{R}^{S \cup \{j\}}$ is defined by $y_j = x_i$ and $y_S = x_S$,
  (2) for all $S \subset N$, $i, j \in S$ and all $x \in V(S)$, we have $y \in V(S)$, where $y \in \mathbb{R}^S$ is defined by $y_i = x_j$, $y_j = x_i$ and $y_{S \setminus \{i,j\}} = x_{S \setminus \{i,j\}}$.

(iv) f is *strongly symmetric* on A^N if for all $V \in A^N$ and all $i, j \in N$ with $k_i(V) = k_j(V)$, $K_i(V) = K_j(V)$, we have $f_i(V) = f_j(V)$.
(v) f is *monotonic* on A^N if for all $V, W \in A^N$ with $k(V) = k(W)$, $K(V) = K(W)$ and $V(N) \subset W(N)$ we have $f(V) \leq f(W)$.
(vi) f satisfies *covariance* on A^N if for all $V \in A^N$, all $\alpha \in \mathbb{R}^N_{++}$ and all $\beta \in \mathbb{R}^N$ we have $f(\alpha * V + \beta) = \alpha * f(V) + \beta$.

The properties (i), (ii), (iii) and (vi) are standard, (iv) and (v) are introduced in Borm et al. (1992) as generalizations of well-known axioms which are used to characterize the RKS-solution for bargaining problems. On the class of compromise admissible games the compromise value satisfies all properties mentioned above, except Pareto optimality. This is shown in the following example.

Example 2.1: Let $N := \{1, 2, 3\}$ and define V by

$$V(S) := \{x \in \mathbb{R}^S \mid x \leq 0\} \text{ for all } S \in 2^N \setminus \{\emptyset, N\},$$

$V(N) := compr(conv\{(4, 0, 0), (4, 3, 0), (2, 4, 0), (0, 4, 0), (2, 3, 2), (0, 3, 2), (0, 0, 4)\})$. Here, for a set $C \subset \mathbb{R}^N$, $compr(C)$ denotes the comprehensive hull of C and $conv(C)$ denotes the convex hull of C. The reader easily verifies that $K(V) = (4, 4, 4)$ and $k(V) = (0, 0, 0)$. So, $V \in C^N$ and $T(V) = (2, 2, 2)$. But $(2, 2, 2) \in wdom(V(N))$ since $(2, 3, 2) \in V(N)$. Hence, the compromise value is not Pareto optimal on C^N. Borm et al. (1992) characterize the compromise value on the set $\bar{C}^N \subset C^N$ of all compromise admissible games V satisfying

(A) the boundary of the set $V^*(N) := \{x \in V(N) \mid x \geq k(V)\}$ contains no segments parallel to a coordinate hyperplane, i.e., $V^*(N)$ is *non-level*
(B) $k(V) < K(V)$
(C) $(k_{N \setminus \{i\}}(V), K_i(V)) \in V(N)$ for all $i \in N$
(D) $V(N)$ is convex.

Conditions (A) and (D) are standard conditions used in several characterizations of solution concepts for NTU-games (cf. Aumann (1985)). (B) is a weak condition which excludes games with dummy players.

We now have

Theorem 2.2: (Borm et al. (1992))
The compromise value is the unique value on $\bar{C}^N$ which satisfies weak Pareto optimality, strong symmetry, monotonicity, and covariance.

Of course, in this characterization weak Pareto optimality can be replaced by Pareto optimality since for a game $V \in \bar{C}^N$ all weakly Pareto optimal points in the set $V^*(N)$ are Pareto optimal.

However, in this characterization the monotonicity property is superfluous. This is a consequence of

Theorem 2.3: The compromise value is the unique value on $\bar{C}^N$ which satisfies Pareto optimality, strong symmetry, and covariance.

Proof: Clearly, the compromise value satisfies the properties mentioned above on $\bar{C}^N$. Let $f:\bar{C}^N \to \mathbb{R}^N$ satisfy the three properties, and let $V \in \bar{C}^N$. We show that $f(V) = T(V)$. Let $V' := V - k(V)$. Clearly, $V' \in \bar{C}^N$ and $k(V') = 0$. Moreover by (B), $K(V') = K(V) - k(V) > 0$. Define $\lambda \in \mathbb{R}^N$ by $\lambda_i := (K_i(V'))^{-1}$ for all $i \in N$. Then $\lambda > 0$. Let $W := \lambda * V'$. Then $W \in \bar{C}^N$ and $k(W) = \lambda * k(V') = 0$, $K(W) = \lambda * K(V') = e^N$, where $e^N \in \mathbb{R}^N$ denotes the vector with $e_i^N = 1$ for all $i \in N$. Strong symmetry of f and T implies $f_i(W) = f_j(W)$ for all $i,j \in N$ and $T_i(W) = T_j(W)$ for all $i,j \in N$. From Pareto optimality of f and T it follows that $f(W) = T(W)$. Since $V = K(V') * W + k(V)$ covariance of f and T implies $f(V) = T(V)$. □

Note that in the proof of this theorem we did not use the conditions (C) and (D). So Theorem 2.3 holds on the larger class of compromise admissible NTU-games satisfying (A) and (B).

Theorem 2.3 is similar to one of the characterizations of the MC-value which is introduced in Otten et al. (1994).

In fact, the proof of Theorem 2.2 provided by Borm et al. (1992) shows the following characterization of the compromise value on $\bar{C}^N$ in which strong symmetry is replaced by symmetry.

Theorem 2.4: The compromise value is the unique value on $\bar{C}^N$ which satisfies Pareto optimality, symmetry, monotonicity, and covariance.

The following four examples show that in Theorem 2.4 the properties are independent.

(i) The value $f:\bar{C}^N \to \mathbb{R}^N$ defined by $f(V) := K(V)$ for all $V \in \bar{C}^N$ satisfies symmetry, monotonicity, and covariance, but not Pareto optimality.
(ii) In Section 3 we introduce values corresponding to monotonic curves. These solution concepts satisfy Pareto optimality, monotonicity, and covariance, but not necessarily symmetry.
(iii) The MC-value introduced in Otten et al. (1994) satisfies Pareto optimality, symmetry, and covariance, but not monotonicity.
(iv) Let $f:\bar{C}^N \to \mathbb{R}^N$ be defined as follows: If $V \in \bar{C}^N$ is such that $K(V) > 0$ and $V(N) \cap \mathbb{R}^N_{++} \neq \emptyset$, then $f(V)$ is the unique element of $V(N) \setminus wdom(V(N))$ which belongs to the line segment between 0 and $K(V)$. Otherwise, $f(V) := T(V)$. The value f satisfies Pareto optimality, symmetry, and monotonicity, but not covariance.

3 Characterizations on a Larger Class of NTU-Games

The assumption of non-levelness plays a crucial role in the characterizations of the previous section. We will show that by modifying this assumption one can obtain a characterization of the compromise value on a larger class of compromise admissible NTU-games. This modification is based on Peters and Tijs (1984), who extended Thomson's (1980) characterization of the RKS-solution to a larger class of bargaining problems by weakening the assumption of non-levelness.

We restrict attention to the class $\hat{C}^N$ of all compromise admissible NTU-games V with player set N sastisfying (B)–(D) and, in addition,

(E) for all $x \in V^*(N)$ and all $i \in N$ we have: if $x \in wdom(V(N))$ and $x_i < K_i(V)$, then there exists an $\varepsilon > 0$ such that $x + \varepsilon e^i \in V(N)$.

Here, $e^i \in \mathbb{R}^N$ denotes the vector with $e^i_j = 1$ if $i = j$, and $e^i_j = 0$ otherwise. Clearly, if $V^*(N)$ is non-level, then $V^*(N)$ also satisfies (E).

Note that the NTU-game provided in example 2.1 does not satisfy (E). This is an immediate consequence of the following lemma which shows that the compromise value is Pareto optimal on the class $\hat{C}^N$.

Lemma 3.1: Let $V \in \hat{C}^N$. Then $T(V) \in V(N) \setminus wdom(V(N))$.

Proof: Because of covariance of T attention can be restricted to $V \in \hat{C}^N$ with $k(V) = 0$ and $K(V) = e^N$ (see the proof of Theorem 2.3). So, let $V \in \hat{C}^N$ with $k(V) = 0$ and $K(V) = e^N$. The compromise value of V is an element of the line segment through 0 and e^N. We must prove that $T(V) \in V(N) \setminus wdom(V(N))$. We distinguish two cases.

Obviously, if $T(V) = e^N$, then $T(V) \in V(N) \setminus wdom(V(N))$. Now suppose that $T(V) \neq e^N$ and that $T(V) \in wdom(V(N))$. Then $T(V) < e^N = K(V)$, and so by assumption (E), it follows that for each $i \in N$ there exists an $\varepsilon_i > 0$ such that $T(V) + \varepsilon_i e^i \in V(N)$. Take $\varepsilon := \min\{\varepsilon_i \mid i \in N\}$. By comprehensiveness of $V(N)$ it follows that $T(V) + \varepsilon e^i \in V(N)$ for all $i \in N$. Using convexity of $V(N)$ we obtain that $T(V) + (\varepsilon / |N|) e^N \in V(N)$. Hence, $T(V) \in dom(V(N))$, which contradicts the weak Pareto optimality of T. Hence, $T(V) \in V(N) \setminus wdom(V(N))$. □

Now we can formulate

Theorem 3.2: The compromise value is the unique value on $\hat{C}^N$ which satisfies Pareto optimality, symmetry, monotonicity, and covariance.

Proof: (The proof of this theorem follows the same line as the proof of Theorem 2.2 by Borm et al. (1992).) Clearly, the compromise value satisfies the four properties mentioned above on $\hat{C}^N$. Now let $f: \hat{C}^N \to \mathbb{R}^N$ satisfy the four properties. We prove that $f(V) = T(V)$ for all $V \in \hat{C}^N$.

Because of covariance of f and T it is sufficient to prove that $f(V) = T(V)$ for all $V \in \hat{C}^N$ with $k(V) = 0$ and $K(V) = e^N$ (see the proof of Theorem 2.3). So, let $V \in \hat{C}^N$ with $k(V) = 0$ and $K(V) = e^N$. Then $T(V)$ is an element of the line segment through 0 and e^N. Using the assumptions (C) and (D) we have that $conv\{e^i \mid i \in N\} \subset V(N)$, so $T(V) \geq (1/|N|) e^N$.

Now consider the NTU-game W defined by

$$W(S) := \begin{cases} \{x \in \mathbb{R}^S \mid x \leq 0\} & \text{if } S \in 2^N \backslash \{\varnothing, N\} \\ compr(conv(\{e^i \mid i \in N\} \cup \{T(V)\})) & \text{if } S = N. \end{cases}$$

Obviously, $K(W) = e^N$, and $k(W) = 0$. Hence, $W \in C^N$ and assumption (B)–(D) are satisfied. If $T(V) = e^N$, then $W(N) = compr\{e^N\}$. Otherwise, if $T(V) < e^N$, then $W(N)$ is non-level. In both cases (E) is satisfied, so $W \in \hat{C}^N$. Clearly, $T(W) = T(V)$. Using symmetry of f it follows that $f_i(W) = f_j(W)$ for all $i, j \in N$. So, by Pareto optimality of f and T it follows that $f(W) = T(W)$. Hence, $T(V) = f(W)$. Since, $W(N) \subset V(N)$, $k(V) = k(W)$, and $K(V) = K(W)$, it follows by monotonicity of f that $f(W) \leq f(V)$. Hence, $T(V) \leq f(V)$. But then Pareto optimality of T implies that $T(V) = f(V)$. □

4 The Class of Monotonic, Pareto Optimal and Covariant Values on $\hat{C}^N$

Theorem 3.2 characterizes the compromise value as the unique value on $\hat{C}^N$ which satisfies Pareto optimality, monotonicity, covariance, and symmetry. In this section we drop the symmetry property and characterize all Pareto optimal, monotonic, and covariant solutions on the class $\hat{C}^N$. For this, we use similar techniques as Peters and Tijs (1984) who characterized all Pareto optimal, monotonic, and covariant bargaining solutions on a large class of bargaining problems, using monotonic curve solutions.

Because we consider covariant values on $\hat{C}^N$ attention can be restricted to the class $\hat{C}^N_{0,1}$ of NTU-games $V \in \hat{C}^N$ which satisfy $K(V) = e^N$ and $k(V) = 0$ (cf. the proof of Theorem 3.2).

Using monotonic curves one can define monotonic and Pareto optimal values on the class $\hat{C}^N_{0,1}$.

A *monotonic curve* (Peters and Tijs (1984)) is a map $\gamma : [1, |N|] \to [0, 1]^N$ with

(i) γ is increasing, i.e., $\gamma(s) \geq \gamma(t)$ if $s \geq t$, and
(ii) $\sum_{i \in N} \gamma_i(t) = t$ for all $t \in [1, |N|]$.

Note that (ii) implies that $\gamma(1) \in conv\{e^i \mid i \in N\}$, and $\gamma(|N|) = e^N$. Moreover, it can easily be checked that each monotonic curve is continuous.

Let γ be a monotonic curve. Then γ gives rise to a value f^γ on $\hat{C}^N_{0,1}$ in the following way: For $V \in \hat{C}^N_{0,1}$ define $f^\gamma(V)$ as the unique Pareto optimal point of $V(N)$ lying on the curve $\{\gamma(t) \mid 1 \leq t \leq |N|\}$. It can easily be verified that f^γ is well-defined on $\hat{C}^N_{0,1}$ (cf. Peters and Tijs (1984)). f^γ is called *the value corresponding to the monotonic curve* γ. The reader easily verifies that f^γ is monotonic and Pareto optimal.

Clearly, each f^γ can be extended to a monotonic, Pareto optimal, and covariant value on $\hat{C}^N$ in a unique way.

We now have the following characterization.

Theorem 4.1: Let $f:\hat{C}^N \to \mathbb{R}^N$ be a value on $\hat{C}^N$. Then f satisfies Pareto optimality, monotonicity, and covariance if and only if $f = f^\gamma$ for some monotonic curve $\gamma:[1,|N|] \to [0,1]^N$.

Proof: Clearly, if $f = f^\gamma$ for some monotonic curve γ, then f satisfies the required properties. Conversely, let f satisfy Pareto optimality, monotonicity, and covariance. We construct $\gamma:[1,|N|] \to [0,1]^N$ as follows.

For $t \in [1,|N|]$, let $\gamma(t) := f(V_t)$, where V_t is the NTU-game defined by

$$V_t(S) := \begin{cases} \{x \in \mathbb{R}^S \mid x \leq 0\} & \text{if } S \in 2^N \setminus \{\varnothing, N\} \\ compr(\{x \in \mathbb{R}^N \mid 0 \leq x \leq e^N,\ \sum_{i\in N} x_i \leq t\}) & \text{if } S = N. \end{cases}$$

The reader easily verifies that $K(V_t) = e^N$, $k(V_t) = 0$ and that $V_t \in \hat{C}^N$ for every $t \in [1,|N|]$. Further, by Pareto optimality and monotonicity of f it follows that γ satisfies (i) and (ii). So γ is a monotonic curve. By definition, one has

$$f(V_t) = f^\gamma(V_t) \text{ for all } t \in [1,|N|]. \tag{1}$$

We want to prove that $f = f^\gamma$. In view of covariance of f and f^γ it is sufficient to prove that $f(V) = f^\gamma(V)$ for all $V \in \hat{C}^N$ with $K(V) = e^N$ and $k(V) = 0$.

Let $V \in \hat{C}^N$ satisfy $K(V) = e^N$ and $k(V) = 0$. Let $t := \sum_{i\in N} f_i^\gamma(V)$, and let W be the NTU-game defined by

$$W(S) := \begin{cases} \{x \in \mathbb{R}^S \mid x \leq 0\} & \text{if } S \in 2^N \setminus \{\varnothing, N\} \\ V(N) \cap V_t(N) & \text{if } S = N. \end{cases}$$

Then $W \in \hat{C}^N$ and $K(W) = e^N$ and $k(W) = 0$. Clearly, $f^\gamma(W) = f^\gamma(V) = f^\gamma(V_t)$. Hence, by (1)

$$f^\gamma(V) = f(V_t). \tag{2}$$

Using monotonicity of f, we have $f(W) \leq f(V_t)$, and $f(W) \leq f(V)$, and by Pareto optimality of f it follows that

$$f(W) = f(V_t) = f(V). \tag{3}$$

Combining (2) and (3) we can conclude that $f(V) = f^\gamma(V)$. □

From the proof of Theorem 4.1 it follows that there exists a unique monotonic curve $\gamma^*:[1,|N|] \to [0,1]^N$ such that f^{γ^*} is symmetric, namely,

$$\gamma^*(t) := \frac{t}{|N|} e^N \quad \text{for all } t \in [1,|N|].$$

Clearly, $f^{\gamma^*} = T$, so Theorem 4.1 provides an alternative proof of Theorem 3.2.

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