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Sarmento, Joaquim Miranda; Renneboog, Luc; Verga-Matos, Pedro

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Measuring highway efficiency by a DEA approach and the Malmquist index

Joaquim Sarmento
ISEG – Lisbon School of Economics & Management, Universidade de Lisboa, Portugal

Luc Renneboog
CentER, Tilburg School of Economics and Management, Tilburg University, the Netherlands

Pedro Verga Matos
ISEG – Lisbon School of Economics & Management, Universidade de Lisboa, Portugal

A growing concern exists regarding the efficiency of public resources spent in transport infrastructures. In this paper, we measure the efficiency of seven highway projects in Portugal over the past decade by means of a data envelopment analysis and the Malmquist productivity and efficiency indices.

We distinguish between technical and technological efficiency and find that most highways face a reduction over time in both types of efficiency. This reduction is mainly due to an increase in operating and maintenance costs, follow-up investments, and a decline in traffic. Some highways only experience a reduction in technological efficiency after a decrease in traffic. They compensate with cost controls and stable investments. While controlling for scale efficiencies, we find a lack of pure technical efficiency in highways that are not subject to a competitive environment, which produces a lack of incentives for better management. Not only does evidence exist of poor management due to a lack of competition, but the increased use of outsourcing also increases inefficiencies. The introduction of tolls and the outburst of the economic crisis in Portugal have substantially reduced traffic that further contributes to inefficiency. The local context, such as highways in low-income areas and rural regions with a lower traffic density, also affects highway performance.

Keywords: DEA; data envelopment analysis; highways; Malmquist index; productivity efficiency; transport.

1. Introduction

Over the last few decades, Portugal has substantially augmented its road infrastructure, in particular its highways. There are several positive externalities from investing in highways, such as reductions in travel time and accidents (Forkenbrock et al., 1997; Debande, 2002; Levkovich et al., 2015). However, in many cases, the required investments and maintenance costs of such projects do not surpass the positive externalities, which raises concerns about efficiency. Highway efficiency does not just boil down to a directly measurable efficiency concept but has a larger impact on regional economic development in terms of trade and mobility (Berechman, 1994; Berechman, Ozmen, & Ozbay, 2006; Nguyen-Hogan, 2015). Although governments have
created most of the highway projects around the world to improve infrastructure, only a few studies have examined their efficiency. The Portuguese experience in which most highways are built and operated by the private sector provides fertile testing ground to explore the factors that can affect efficiency, such as the financial and nonfinancial inputs and outputs, shareholder composition, level of investment, and urban/rural or coastal/interior geography.

We use a data envelopment analysis (DEA) with a Malmquist index to test the efficiency of seven highway projects. Using a non-parametric method such as the DEA has the advantage that one does not need a priori functional form on technology or any restrictive assumption regarding input remuneration (Cooper, Seiford, & Zhu, 2011). The research applies the DEA and the Malmquist index to measure the efficiency of units in a certain year or the change of efficiency over a period of years for a large field of organisations and activities (Vitner, Rozenes, & Spraggett, 2006; Liu, Lu, Lu, & Lin, 2013; Lampe & Hilgers, 2015), such as hospitals or schools (Dharmapala, 2009; Alexander, Haug, & Jaforullah, 2010; Barnum, Walton, Shields, & Schumock, 2011), environmental and energy problems (Zhou, Ang, & Poh, 2008), retail companies (Moreno, 2008; Gupta & Mittal, 2010; Balios, Eriots, Fragoudaki, & Giokas, 2015), seaports (Barros, 2003; Al-Eraqi, 2008; Panayides, Maxoulis, Wang, & Ng, 2009; Barros, Felicio, & Fernandes, 2012), airports (Gillen & Lall, 1997; Fernandes & Pacheco, 2002; Yoshida & Fujimoto, 2004; Barros & Dieke, 2007), and public transport (Husain, Abdullah, & Kuman, 2000; Pina & Torres, 2001; von Hirschhausen & Cullmann, 2010).

The first motivation for this paper is that despite the importance of highways in transport, there is little research on their efficiency, particularly in comparison with other fields of transportation such as seaports and airports. Hence, this paper extends the literature by offering more insight into the determinants of highway performance and efficiency.

Second, our method also provides a contribution to the field. In contrast to previous studies that measure highway performance with the DEA and the stochastic frontier analysis (SFA), we introduce a Malmquist index (to our knowledge, not used before to measure highway efficiency). While both the DEA and SFA are well established (Bogetoft & Otto, 2010; Cooper et al., 2011), the DEA only looks at each year’s efficiency and not the change over time. In contrast, the Malmquist index provides insights into the degree of efficiency and productivity growth (Lee, Leem, Lee, Thales, & Lee, 2011). It also enables a decomposition of productivity by identifying the sources of input and output bias in technological change. We also include in our analysis both financial and nonfinancial inputs (such as operating costs, maintenance costs, investment levels, and the number of employees needed at the maintenance phase) and outputs (revenues, daily traffic). This way we offer an integrated approach on how the different attributes of highway development and maintenance can affect their efficient use.

Third, highways face a problem with efficiency due to the lack of competitive pressure, along with problems of efficiency regarding scale and investments. Having a benchmark of efficiency regarding competitors and managerial procedures is a way to manage their relative performance and therefore is a key issue in highways’ performance. The DEA studies on highways have identified that there is large potential to increase their efficiency, although they fail to bring consensus about how an increase in efficiency can be brought about and how economies of scale can be increased (Amdal, Bårdesen, Johansen, & Welde, 2007; Odeck, 2008). This debate will continue until enough projects detail the entire lifecycle of highway investments and maintenance. Consequently, this paper could be a useful resource for policy-makers and regulators to improve their empirical knowledge on highway performance.

This paper is organized as follows. Section 2 provides a brief review of the literature on the efficiency of highways. Section 3 presents the institutional framework for the Portuguese highway network. Section 4 describes our data and gives a brief account of the method. We present the results in section 5, and the conclusions, limitations and suggestions for future study in section 6.
2. Literature review

2.1 DEA and efficiency studies in transports

The DEA is a useful method for analysing the relative performance within a group of organizations (decision-making units (DMUs)). Building on the seminal work of Farrell (1957), several types of DEA models were developed (for a survey, see Cook & Seiford 2009). Briefly, on distinguishes between CCR DEA models developed by Charnes, Cooper, and Rhodes (1978) that assume constant returns to scale and, the BCC DEA models developed by Banker, Charnes, and Cooper (1984) that adopt variable returns to scale. These models calculate efficiency relative to the DMUs’ observed best performance by considering multiple inputs and outputs.

There are numerous applications of DEA to evaluate efficiency within various fields of transport (for a survey, see Markovits-Somogyi, 2011). For instance, DEA is applied to both infrastructures such as airports (e.g., Suzuki, Nijkamp, Rietveld, & Pels, 2010; Curi, Gitto, & Mancuso, 2010; Suzuki, Nijkamp, Pels, & Rietveld, 2014), railways (e.g., Yu & Lin, 2008; Roets & Christiaens, 2015), and seaports (e.g., Panayides et al., 2009; Odeck & Bråthen, 2012); and operators, such as airline companies (e.g., Chiou & Chen, 2006; Michaelides, Belegri-Roboli, Karlaftis, & Marinos, 2009), road transport firms (e.g., Bhagavath, 2006; Caro-Vela, Paralera, & Contreras, 2013; Jarboui, Pascal, & Younes, 2013; Andrejic, Bojovic, & Kilibarda, 2016), and shipping lines (e.g., Gutiérrez, Lozano, & Furió, 2014). The advantage of the DEA is that no assumptions need to be made about the shape of the efficient frontier or the internal operation of each DMU (Bray, Caggiani, & Ottomanelli, 2015), but the results should still be interpreted with caution due to the methodology’s sensitivity to outliers and measurement errors (Cooper et al., 2011).

The DEA approach is very useful for corporate managers, shareholders, regulators, and policymakers who need quantitative data about the relative performance (Suzuki et al., 2014) in order to create a benchmarking standard. The DEA analysis provides indications for measuring and monitoring efficiency and on how to improve the performance and the ability to reach the efficiency frontier (Andrejic et al., 2016). Research about the efficiency of roads is very important because they are a factor in economic competitiveness and growth in regional development. Further, they have a strategic position in the transport’s value chain with direct and indirect effects on employment; value added; innovation; and global, national, and regional economic growth (Berechman, 1994; Berechman et al., 2006).

2.2 DEA, highways, and efficiency

The DEA’s aim is to calculate an efficiency frontier as the relative performance of different DMUs in terms of distance per unit to the ideal frontier constructed by using observed input and output data (Brebbia, 2014). Bogetoft and Otto (2010) assert that a highway, when considered as a business, contains all the general characteristics of the production systems. Applying the efficiency analysis theory by means of a DEA analysis, researchers can build a model to evaluate alternative schemes and to analyse and diagnose ineffective schemes. In addition, Cooper et al. (2011) stress that highway organisations must rationalise their operating costs to improve the quality of the services they offer. The authors obtain measures of purely technical scale and overall efficiency for both public and private agencies that establishes that the DEA can be used for the evaluation of the relative efficiency of multiple homogeneous DMUs. The authors also state that the advantage of using the DEA framework is that it is capable of handling noneconomic factors, such as the number of accidents, maintenance cost per day, traffic per day, and the average age of the pavement; and it also allows for the measurement of such factors on different scales. Bhagavath (2006) argues that the DEA model is particularly suited for determining the efficiency of highways, as factors such as traffic intensity and safety parameters are an essential part of highway transport.

The objective of highway efficiency models is to meet the largest possible traffic demand and reach the lowest traffic delays with minimal inputs. These models comprise the minimum
number of highway efficiency lanes that requires the lowest level of investment along with the lowest level of operational costs during the lifecycle of the project (Odeck, 2008). The DEA shows that organizations can achieve the most beneficial improvements to operational highway efficiency by reducing the resource consumption levels of individual vehicles, the amount of funds available, the time and space resources of the highway, and the price of environmental pollution, among other resources (Mao, 2010). According to Odeck (2008), the objective of the DEA is to compare the performance of different road networks to provide technical support to policymakers for the choice of actions that need to be implemented to make a highway system efficient. Brebbia (2014) states that the DEA analysis enables highway agencies to calculate a value of the relative efficiency of each highway network on the basis of which networks are ranked, thus distinguishing efficient networks from inefficient ones.

2.3 DEA studies on highway efficiency

Table 1 summarises the studies that exist in this field of research. Academic research has focused on two issues: measuring inefficiencies and the impact of dimensions (economies of scale). Several studies conclude that highways are operating at an inefficient level (Deller & Halstead, 1994; Odeck, 2008; Welde & Odeck, 2011). The studies give different causes for this inefficiency: maintenance costs that are higher than necessary, poor management skills, and a lack of competition.

The debate has also concentrated on the impact of highway dimensions and on whether economies of scale are determinants of highway efficiency. Studies by Amdal et al. (2007) and Odeck (2008) conclude that the highway’s dimension is a critical factor for its efficiency and that increased traffic reduces the unitary operating costs: operating costs vary significantly but larger companies that serve more traffic have lower levels of operating costs per vehicle. This result indicates important and unexploited economies of scale. For Odeck (2008), larger highways (measured by the number of lanes or by the number of kilometres (km)) tend to be more efficient because highways with a longer dimension are able to reduce the unitary fixed costs. The case of the Italian highway concessionaries supports these conclusions, as Benfratello, Iozzi, & Valbonesi (2008) find economies of density and scale using an L-shaped average cost curve over the range of output. In contrast, Welde & Odeck (2011) propound the idea that economies of scale are not always significant in terms of highway efficiency because they are able to present evidence of companies with low traffic levels having efficiency scores of one (the maximum efficiency in the DEA model) or thereabouts. A recent study from Daito and Geiford (2014) assesses US highway efficiency by using a DEA and a stochastic cost frontier method. Private highway projects in the United States showed higher initial costs than non-private projects, although the efficiency scores showed no significant difference between the two groups. This inconsistency between initial costs and technical efficiency scores disclose that complexity involved in private projects was not captured in the efficiency analysis.

To conclude, efficiency measurement and benchmarking in highway transport is an important topic whether one is interested in comparing the efficiency of different highway networks or in learning how to improve their efficiency. The calculation of relative efficiency scores by means of the DEA model generates insights into the performance of highways of various dimensions and localizations, which thus guides the choice of the required actions. The benefit of using the DEA model in this context is that it is free from a priori assumptions on functional forms and is applicable to units, such as highways, that have several outputs (e.g., traffic and revenues). Still, the weakness of the DEA model is that it is sensitive to outliers and can generate multiple best-performers. The research can combine the DEA with other approaches to separate and measure the technological advances that it can use to improve highway efficiency over time.
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3. The Portuguese highway sector experience and SCUTS projects

Regarding the Portuguese experience in the highway sector, two waves of Public Private Partnerships (PPPs) have emerged. The first one comprises seven SCUT highways (where SCUT stands for “Sem Custos para o Utilizador”, which is Portuguese for “without cost to the user”), which are the subject of this study. Since its inception, debate and controversy have existed on whether private concessions are the best option for contracting the construction of highways and whether the private sector has proven to be more efficient. The SCUTs extend over a total of 930 kilometres (see table 2 for more details) and were originally equipped with shadow tolls which mean that the payment to the private sector is at the expense of the public budget in lieu of the users. The second wave of road concessions was launched between 2007 and 2008, when the Portuguese government awarded seven new highway projects to public companies. These projects were completed by 2014 and therefore are not part of this study. Despite the importance of highway investments in Portugal, only a few studies examine the overall PPP experience (de Lemos, Eaton, Betts, & de Almeida, 2004; Monteiro, 2005; Sarmento, 2010; Cruz & Marques, 2011; Sarmento & Renneboog, 2015; Sarmento & Renneboog, 2016), but none on the efficiency of these highway projects.

4. Methodology and data

4.1 Method

In order to assess the efficiency of the Portuguese highways, we use a DEA-BCC model by estimating a productivity Malmquist index (Malmquist, 1953). Our motive for using the BCC model is that unlike the CCR model, BCC allows for variable returns to scale which is important to our particular type of analysis (Banker et al., 1984; Charnes et al., 1978). A detailed description of the model is provided in the appendix.

The Malmquist index measures the productivity changes over a period of several years and is decomposed into changes in efficiency and technology (Lee et al., 2011). The DEA measures the efficiency of each DMU within a group relative to the (observed) more efficient unit within that group (Charnes et al., 1978; Bhagavath, 2006). The more efficient DMU is the one that lies on the efficient frontier and assumes a value of one in the model. The other DMUs have a value between zero and one. A DEA model can be subdivided into an input-oriented model, which minimizes inputs while satisfying at least the given output levels, and an output-oriented model, which maximizes outputs without requiring more of any observed input values (Cook, Tone, & Zhu, 2014). Hence, efficiency is measured in terms of a proportional change in inputs or outputs.

In line with Cooper et al. (2011), we find two types of efficiency in a DEA model: technical and allocative efficiencies. The first type of efficiency signifies that for the current technological level, there is no waste of inputs for a certain level of output. This is the type of efficiency that is directly affected by management or scale; an organization operating at best-practice is said to be 100% technically efficient (Bhagavath, 2006). The allocative efficiency refers to the use of resources so as to maximise profit and utility, specifically, by minimizing the costs for a unit already technically efficient. That is, the inputs should be used in such a way as to reach technical efficiency (i.e., minimum inputs and maximum outputs) but also to minimize costs.

In the DEA, if firms only use one input to produce one output, then the efficiency score is calculated by dividing the value of the input by the value of the output. However, in the case of multiple inputs and outputs, the DEA assumes a linear programming method that
enables the presentation of a single value of efficiency. The DEA makes a combined weight for each unit with an optimal estimation of inputs and outputs. This optimal estimation is constructed as follows: The weights for the inputs and outputs do not have to be identified because they are determined and optimised by the DEA model. The weights used are DMU specific, and during the application of DEA, they are optimised by each DMU to maximise its efficiency rating (for a detailed analysis see Cooper et al., 2007).

Thus, each unit can still be represented in a simple chart with a single input and output value that determines the efficiency frontier. The units in the frontier have a value of one. A DMU is said to be efficient if the ratio of its weighted outputs to its weighted inputs is larger than the similar ratio for every other DMU in the sample. For all other units, the DEA measures the distance of the unit to the frontier, assuming a value between zero and one.

In our DEA analysis, we use the Malmquist index to evaluate the productivity change of each unit between two periods of time (Cooper et al., 2007). This index divides a change in technical efficiency into pure and scale efficiency changes (Malmquist, 1953; Caves, Christensen, & Diewert, 1982). Whereas pure technical efficiency represents the technical efficiency devoid of scale effects in which the efficiency stands entirely under the control of the management (it is also called managerial inefficiency), the latter type of efficiency is a direct consequence of dimension. A unit is scale efficient when its size of operations is optimal: if its size is either reduced or increased, its efficiency drops (Färe, Grosskopf, & Brännlund, 1996; Barros & Dieke, 2008). Scale efficiency has a maximum value of one, which is assumed by the DMU with the most productive scale size.

The Malmquist index decomposes the change in total factor productivity into a change in technical efficiency and technological change, whereby the former is equal to a change in pure technical efficiency multiplied by a change in scale efficiency (Cooper, Seiford, & Tone, 2006). Pure technical efficiency is the impact of management on the company’s efficiency (also called managerial efficiency). Consequently, this part of a company’s increase or decrease in efficiency can be ascribed to the managers and their decisions regarding the level of inputs and outputs and the efficient utilisation of resources. This process can be considered input orientated, which means how much the inputs can be reduced while maintaining the same level of outputs, or output-orientated, and representing how much the outputs can increase by maintaining the same level of inputs. Certain decisions, such as changing the operational process, improving quality, or reducing costs, can lead to better pure technical efficiency.

Scale efficiency regards the (dis)economies of scale of a certain unit. It could either represent economies of scale (i.e., an increasing return to scale (IRS)), due to being at less than optimum size, or diseconomies of scale (i.e., decreasing returns to scale (DRS)), due to being at more than the optimum size (Isik & Hassan, 2003). A reduction in scale efficiency represents the cost of operating at an incorrect scale. However, the consideration that an increased or reduced scale is always influenced by the market is important. Moreover, in the specific cases of highways, there is a clear limitation on the size and scope of the operation, as dimension is a project variable defined and is most difficult to change during the concession period.

Also, highways pose an additional challenge regarding efficiency, as they are indivisible. This indivisibility comes from the fact that for each highway, it is not possible to decompose efficiency in each subsector or sub-lane. However, as mentioned by Suzuki et al. (2010, 2012), the DEA can include inputs that address lumpiness or rigid factors and avoid short-term indivisibility or inertia.

However, large highways, with more kilometres and lanes, should be more efficient due to this scale effect. As economies of scale refer to a situation in which production increases by
some amount, costs increase by a lesser amount. Thus, companies serving a greater number
of lanes and, implicitly, a larger amount of traffic, should be more efficient than others are
(Odeck, 2008).

Technological efficiency is the impact on the increase or decrease of the overall efficiency of
the firm caused by the use of the technology by itself. This efficiency means to produce new
technology that can reduce costs or increase revenues. In the case of highways, several
examples can be given: electronic payment systems, replacing staff with toll-colllecting
machines, or better maintenance materials and systems.

The total factor productivity frontier is de facto a best-practice frontier, and the DEA is also
referred to as ‘balanced benchmarking’ (Sherman & Zhu, 2012). If a change in the distance
to the efficiency frontier relative to the previous year is higher (lower) than one, then a
reduction (increase) in efficiency has occurred. A detailed explanation on how the inputs
and outputs relate to the Malmquist index is discussed in Färe, Grosskopf, Norris, and
Zhang (1994), Isik and Hassan (2003), and Barros et al. (2012).

The linear program software (we used DEA Linear Frontier) takes a three step approach:
first, for each combination of inputs and outputs, an efficiency frontier is generated
consisting of the most efficient units (using a constant return of scale whereby an increase
in inputs results in a proportionate increase in the output levels). Secondly, the Malmquist
index measures the difference of each unit to the efficiency frontier over time: for instance,
as our sample starts in 2003, the efficient frontier is calculated for 2003 and 2004 and the
first value of the Malmquist index for 2004 is the difference in deviations to the efficient
frontier of a unit for 2004 and 2003. The distance function to the efficiency frontier is
calculated as follows (Lee et al, 2011):

\[ MI_{t+1} = \frac{E_{t+1}(x_{t+1}, y_{t+1})}{E_t(x_t, y_t)} \]  

where MI stands for the Malmquist index, and x and y are inputs and outputs, respectively.
Finally, once the Malmquist index is calculated for each year, we take the geometric mean
of the values for each firm. Suppose we have four units using one input and one output for
period t and t+1, and that the units have the following combinations of inputs/outputs: A(0,5,0,5), B(2,2); C(1,2), D(2,1) at t and A1(1,1); B1(2,3), C1(1,3), D1(3,1,5) at t+1. From
Figure 1, we observe that C is the most efficient unit and that B had no efficiency gains
between t and t+1 because its distance to the efficiency frontier has remained equal. (It
should be noted that in this example we have a single input and output case and
consequently the efficiency frontier reduces to a straight line). The detailed linear
programming model is presented in the Appendix A.
Figure 1. The Malmquist index using a constant return of scale DEA model

Note: This figure presents an example of how the Malmquist index is represented by means of the DEA distance function. A, B, C and D represent the input/output efficiency of four firms in year t and A1,B1,C1,D1 in year t+1. The Malmquist index calculates the change in the distance of each unit to the efficiency frontier, regarding period t and t+1. If the unit moves closer to the efficiency frontier, then this represents an increase in efficiency. For each company, we calculate the ratio between the two distance measures at t and t+1.

4.2 Data
We use a balanced data panel that comprises all seven Portuguese companies involved in the first PPP highway wave during the period 2003 to 2012 (see tables 2 and 3). The annual data come from the concessionaries’ annual reports and from the Portuguese highway regulator (InIR – Portuguese for Institute of Road Infrastructure) that also has supervisory responsibilities.

4.3 Input and Outputs variables
One of the most important features of the DEA is that it does not require the specification of a functional form. Therefore, any variable (both input and output variables) can be included in the model without the need to specify functional or parametric relationships (Ozbek et al., 2010). Following the literature cited above, we included in our efficiency models, the following inputs (combining financial and non-financial data):

- O&M (operating and maintenance) costs (including salaries) - which is a key input variable also used in Odeck (2008), Ozbek et al. (2010), and Welde & Odeck (2011). These costs do not just include direct staff costs, but also the costs of outsourcing, along with other costs, such as rents, the costs of collecting tolls, etc. Our sample starts in 2003, as this is the year when these seven highways were fully operational.
- Total assets (i.e., the required investments) - proxies for the capital expenditures needed for each highway in each year. As the sample covers the operational stage, and as during the operational stage, highways tend to carry out some major reparations during the 8th to 12th year period after completion (Ozbek et al., 2010). O&M and major reparations are also affected by factors such as climate, traffic (volume and type), location, and terrain.
- Number of employees (full time equivalents (FTEs)) is relevant - as it show whether the operating firms use both outsourcing and their own personnel resources, which
affects efficiency. It should be noted that a reduction in the number of employees does not necessarily lead to an increase in efficiency, but can simply represent outsourcing to other companies, which affects the O&M costs. Outsourcing is standard practice with private highways, whereby companies reallocate the operational risk to a third party. However, we will show that an increase in efficiency does not result from a reduction of this type of risk.

As outputs, we use the following variables:

- DAT/km - measures the daily average traffic, which is the total traffic on a highway in a year, divided by the number of days (Odeck, 2008, and Welde & Odeck, 2011).
- Revenues - payments from concessionaries, according to contract agreements signed with the Portuguese authorities.

The descriptive statistics of the two output and three input variables are reported in Table 4.

The proportional rule required by the DEA (Cooper et al., 2007) is that the number of observations should be more than three times the sum of inputs and outputs: 63 (7 firms * (10-1) years) is larger than 3 x (2+3) (Cooper et al., 2011).

Exogenous factors that could affect the highway performance include the introduction of tolls in 2010, as these have an impact on the levels of traffic and usage of these highways, and also the Financial Crisis of 2008-2009 (followed by the Recession of 2011-2012). These two events have substantially reduced the economic activity of companies and the disposable income of the Portuguese population, which had a direct impact on the traffic on the highways. Additionally, there has been a significant increase in oil prices between 2004 and 2008, and a subsequent fall in prices induced by the 2008 Financial Crisis. The above affect all highway operations in similar ways. The age of the concession could also play a role, as older infrastructures tend to have higher operating and maintenance costs and occasionally require major repairation work, however, de facto, they do not vary substantially across the sample, as the highways were constructed between 1998 and 2002 and were all operational by 2002.

Additionally, in order to analyse the results provided by the efficiency model, we use the following variables: length, Capex, location, type of district, and type of shareholders. Length is the size of the highway measured in km. The variable Capex by Km is the total capital expenditure (investment) of the project, divided by the length in km, which enables us to compare investments across concessions with different sizes. Location is whether the highway is located on the coast or in the interior of Portugal. Type of district represents whether the highway is predominantly located in an urban or a rural area. Shareholders is defined by whether the majority of the company’s capital is domestic or foreign. For a summary of the characteristics by highway, see Table 2.

The Capex by Km is expected to have a strong impact on efficiency (Chu & Tsai, 2004). The main cost for highways occurs during the construction stage, as the yearly operation and maintenance costs represent only around 1% of total investment (Sarmento, 2010). Location is relevant, as the Portuguese inland is mountainous, faces cold weather (which affects the maintenance costs), and is much less populated than the coastal regions. Furthermore, a highway in an urban area is expected to attract more traffic per km, although its maintenance costs will be higher (as highway maintenance is usually performed without entirely closing the road, and more dense traffic makes the work more complex).

Our data vary across highways: for instance, the Capex by Km varies from 1.69 M € (for the A22, a coastal urban operation in the south of the country) to 6.46 M € (for the GP, a similar
operation – coastal and urban – although it circles around the second largest city in Portugal, with a very high population density). The lengths range from 72 km (the GP) to almost 180 km (the A23 is 178 km and the A25 is 176 km, both are inland rural highways). The average highway stretches for 133 km, with an average Capex by Km of €3.4m. Other performance-related information (revenues, operating and maintenance costs, daily average traffic /km) is presented in Table 3.

Table 2. General characteristics of Portuguese PPP highways

<table>
<thead>
<tr>
<th>SCUT Name</th>
<th>Highway Name</th>
<th>Length (Km)</th>
<th>Capex/Km (million €)</th>
<th>Localisation</th>
<th>Urban</th>
<th>Rural</th>
<th>Main Shareholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>A23</td>
<td>SCUT Beira Interior</td>
<td>178</td>
<td>3.11</td>
<td>I</td>
<td>R</td>
<td></td>
<td>Domestic</td>
</tr>
<tr>
<td>A24</td>
<td>SCUT Interior Norte</td>
<td>155</td>
<td>3.18</td>
<td>I</td>
<td>R</td>
<td></td>
<td>Foreign</td>
</tr>
<tr>
<td>A22</td>
<td>SCUT Algarve</td>
<td>129</td>
<td>1.69</td>
<td>C</td>
<td>U</td>
<td></td>
<td>Foreign</td>
</tr>
<tr>
<td>A17</td>
<td>SCUT Costa de Prata</td>
<td>105</td>
<td>2.79</td>
<td>C</td>
<td>U</td>
<td></td>
<td>Domestic</td>
</tr>
<tr>
<td>GP</td>
<td>SCUT Grande Porto (GP)</td>
<td>72</td>
<td>6.46</td>
<td>C</td>
<td>U</td>
<td></td>
<td>Domestic</td>
</tr>
<tr>
<td>A25</td>
<td>SCUT Beiras litoral e alta</td>
<td>176</td>
<td>3.94</td>
<td>I</td>
<td>R</td>
<td></td>
<td>Domestic</td>
</tr>
<tr>
<td>A27</td>
<td>SCUT do Norte Litoral</td>
<td>115</td>
<td>2.65</td>
<td>C</td>
<td>R</td>
<td></td>
<td>Foreign</td>
</tr>
</tbody>
</table>

Note: I is for highways mainly situated in the interior and C is for those mainly located in the coastal area; U is if the highways are in urban areas, and R is if they are in rural areas.

Table 3. Operational characteristics of Portuguese highways

<table>
<thead>
<tr>
<th>Highway Name</th>
<th>Revenues (000 €)</th>
<th>DAT/KM traffic</th>
<th>O&amp;M Costs (000 €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A23</td>
<td>121,243</td>
<td>9,400</td>
<td>25,442</td>
</tr>
<tr>
<td>A24</td>
<td>90,253</td>
<td>6,685</td>
<td>8,514</td>
</tr>
<tr>
<td>A22</td>
<td>38,592</td>
<td>8,219</td>
<td>4,734</td>
</tr>
<tr>
<td>A17</td>
<td>43,280</td>
<td>19,988</td>
<td>17,015</td>
</tr>
<tr>
<td>GP</td>
<td>78,506</td>
<td>22,151</td>
<td>11,605</td>
</tr>
<tr>
<td>A25</td>
<td>97,147</td>
<td>9,172</td>
<td>18,912</td>
</tr>
<tr>
<td>A27</td>
<td>48,133</td>
<td>20,305</td>
<td>6,951</td>
</tr>
<tr>
<td>Mean</td>
<td>73,879</td>
<td>13,703</td>
<td>13,310</td>
</tr>
<tr>
<td>Median</td>
<td>78,506</td>
<td>9,400</td>
<td>11,605</td>
</tr>
<tr>
<td>St. dev</td>
<td>31,408</td>
<td>6,743</td>
<td>7,441</td>
</tr>
</tbody>
</table>
Table 4. Descriptive Statistics of input and output data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues</td>
<td>In 1,000 Euros at constant prices; 2005=100</td>
<td>2,683</td>
<td>149,222</td>
<td>52,646</td>
<td>41,740</td>
<td>43,147</td>
</tr>
<tr>
<td>DAT/KM real traffic</td>
<td>Daily average traffic by Km (real traffic)</td>
<td>4,257</td>
<td>38,073</td>
<td>18,492</td>
<td>17.202</td>
<td>10,715</td>
</tr>
<tr>
<td>Inputs</td>
<td></td>
<td>4,236</td>
<td>24,943</td>
<td>7,356</td>
<td>5,610</td>
<td>4,525</td>
</tr>
<tr>
<td>O&amp;M Costs</td>
<td>In 1,000 Euros at constant prices 2005</td>
<td>97,009</td>
<td>1,302,098</td>
<td>582,780</td>
<td>507,411</td>
<td>269,229</td>
</tr>
<tr>
<td>Total assets</td>
<td>1,000 Euros at constant prices 2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of ftes (Employees)</td>
<td>Number</td>
<td>2</td>
<td>109</td>
<td>33</td>
<td>19</td>
<td>29</td>
</tr>
</tbody>
</table>

5. Empirical results and discussion

The Malmquist index does not identify the causes of efficiency in the sense that the results only show an increase or decrease in efficiency, but it identifies the inefficient units, either in terms of change in technical efficiency or technological change (Cooper et al., 2011). With that information, we can compare the evolution of either inputs or outputs and how they affect the changes in efficiency. Furthermore, by grouping the different units according to the increase or decrease in the different types of efficiencies, we can assess the possible explanations regarding the units’ characteristics. We consider the following characteristics: (i) the change in the inputs and outputs in each company during this period; (ii) the scale of each highway or its length (in km), as we expect scale efficiency to have a positive impact on overall efficiency; (iii) the investment level as expressed by the Capex/km as a high level of investment is expected to reduce highway efficiency; (iv) the location (inland or coastal) because the Portuguese inland is mountainous which can reduce efficiency, due to higher levels of investment and operational costs; (v) the type of district, as urban areas attract higher traffic density which should increase efficiency; and (vi) the main shareholders: a highway with national shareholders may be more efficient (from a private partner’s perspective), because, due to political connections, they may have been able to attract more favorable contract conditions (regarding firms and political connections see for instance (Chen, Ding, & Kim, 2010) . The average Malmquist indices for each of the toll-free highways are presented in Table 5.

Table 5 (column 1) shows that the total factor productivity change score (which equals the Malmquist index) amounts to 1.2096 which is above one and hence signifies that there was a deterioration (of -0.2096) in highway productivity during this period. The only exception to the overall deterioration is highway A27. The average change in technical efficiency (column 2) amounts to 1.008, which indicates that pure and scale efficiency slightly decreased (the A17 and A25 are mainly responsible for the reduction).
Table 5. Efficiency decomposition for Portuguese PPP highways

<table>
<thead>
<tr>
<th>Highway</th>
<th>(1) Malmquist index</th>
<th>(2) Δ Technical Efficiency</th>
<th>(3) Δ Technical Efficiency</th>
<th>(4) Δ Pure technical efficiency</th>
<th>(5) Δ Scale efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A23</td>
<td>1.161</td>
<td>1.000</td>
<td>1.161</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>A24</td>
<td>1.248</td>
<td>1.000</td>
<td>1.248</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>A22</td>
<td>1.247</td>
<td>1.000</td>
<td>1.247</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>A17</td>
<td>1.339</td>
<td>1.034</td>
<td>1.295</td>
<td>1.007</td>
<td>1.026</td>
</tr>
<tr>
<td>GP</td>
<td>1.233</td>
<td>1.000</td>
<td>1.233</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>A25</td>
<td>1.359</td>
<td>1.021</td>
<td>1.331</td>
<td>1.001</td>
<td>1.021</td>
</tr>
<tr>
<td>A27</td>
<td>0.934</td>
<td>1.000</td>
<td>0.934</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Mean</td>
<td>1.210</td>
<td>1.008</td>
<td>1.200</td>
<td>1.001</td>
<td>1.007</td>
</tr>
<tr>
<td>Median</td>
<td>1.247</td>
<td>1.000</td>
<td>1.247</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.1311</td>
<td>0.0129</td>
<td>0.1217</td>
<td>0.0024</td>
<td>0.0107</td>
</tr>
</tbody>
</table>

Note: This table presents the Malmquist index for the seven highways examined in this study over the period 2003-2012. The index is decomposed in technical efficiency change and technological change ((1)=(2)x(3)). The change in technical efficiency is also dissected into a change in pure technical efficiency and a change in scale efficiency ((2)=(4)x(5)). Source: own calculations.
The average change in the technology (column (3)) amounts to 1.2 and also demonstrates that there was degradation in the technological efficiency, which could mean that investments were scarce over the past decade.

Finally, we observe that the change in pure technical efficiency and the change in the scale efficiency are limited. The former small decline may still be due to the limitations of competition in this sector. The latter indicates that there is no apparent effect of dimension in highways (only A17 and A25 have values slightly different from one where one signifies no change in the efficiency).

When we break down the Malmquist index into an efficiency change and a technological change, we are able to identify three groups of highways. The first category consists of the most inefficient toll-free highways in terms of productivity: their productivity decline is due to the simultaneous deterioration of technological change and technical efficiency, or put differently, the Malmquist index >1, technical efficiency change >1 and technological change >1. Highways A17 and A25 belong to this category and both underwent a substantial increase in the O&M costs and a decrease in the number of employees because the two companies outsourced more of the maintenance and operations. Given that these two highways belong to the same national group, Ascendi, they could be subject to a negative scale efficiency effect, a conclusion reinforced by the fact that the two highways are geographically connected. Also, the value of the assets of these two highway companies increases significantly for both (in terms of additional investments), which were not compensated by higher revenues with a resulting decrease in efficiency. Both highways also suffered from a strong reduction in traffic following the introduction of tolls in 2010.

The second group of highways is characterized by a productivity decline caused by deterioration in technological change (Malmquist index >1 with technical efficiency change =1 and technological change >1) and includes the A22, A23, A24 and GP. The decline in productivity is related to a substantial loss in traffic (almost 50%), but they were able to keep follow-up investments down and to maintain O&M costs at a stable level. The third group of highways with a productivity improvement resulting from technological improvement (Malmquist index less than one, technical efficiency change =1 and technological change <1) only comprises one highway: the A27. Both the O&M costs and the number of employees in this company remained stable, investment was low and, in spite of tolls, traffic did not decline over the sample period.

We also find that for some highways' O&M costs increase that is followed by a significant reduction in the number of employees which indicates that the highway PPPs resorted to more outsourcing. Furthermore, substantial levels of follow-up investment decreased efficiency. However, over the coming years as the investment requirements decline, these highways will most likely augment their efficiency. Further the introduction of tolls (in 2010) along with the economic crisis has led to a substantial reduction in traffic on almost all highways, and hence efficiency.

We rank the seven highways in terms of the efficiency scores (with the most efficient coming first by using the super efficiency concept by Tone, 2001): A27, A23, GP, A22, A24, A17, and A25. This ranking shows that there seem to be no scale effects. In terms of location, highways in coastal areas perform better than the ones in mountainous regions. Also, highways mainly situated in rural areas have a better efficiency score than those located in urban ones, which is related to the fact that O&M costs are higher in urban areas because their maintenance is more complex and costly. The major cause of productivity degradation in (initially) toll-free Portuguese highways is efficiency deterioration.
6. Discussion and conclusions

The main purpose of this research is to analyse the efficiency of seven highway projects in Portugal by using a data envelopment analysis, and the Malmquist productivity and efficiency index to understand the relative efficiency changes. To the best of our knowledge, this study is the first to measure highway performance by means of the Malmquist index that combines a single financial framework and institutional data for each Portuguese concession. For managers, the results offer valuable insights since they identify the relative performance of each concession.

We have estimated the Malmquist input-based index of total factor productivity for seven Portuguese highways over the period 2003 to 2012. A linear programming analysis results in an efficiency frontier - the best-practice benchmark - against which we gauge the efficiency of each highway. We first dissect the productivity change into a change in technical and in technological efficiency. This analysis shows that the average productivity of Portuguese SCUT highways slipped over time. In general, this decrease is mainly caused by a drop in technological efficiency and to a lesser extent to a reduction in technical efficiency. Although the Malmquist index does not identify the causes of each type of (in)efficiency, the identification of poor and strong performers still enables us to delve deeper into the sources of (in)efficiencies. Efficiency change is mainly associated with managerial practices, and technological efficiency is related to new (follow-up) investments and procedures. We find that for most highways, there is some evidence of weak management in terms of Operating and Maintenance costs, possibly due to a lack of competitive pressure. Also, some highways were still, particularly during the first years, making large investments which decreased their efficiency. The substantial reduction in traffic as a consequence of the financial crisis (starting in 2008) and the introduction of electronic tolls (in 2010) resulted in a utilization of the infrastructure below maximum efficiency. It is also important to note that the efficiency performance of each highway is mainly driven by its local context, particularly location and district. Some remote highways are inefficient on account of being located in low-income districts with scant traffic. Other companies suffered from a lack of investment or qualified human resources caused by cost-control policies induced by the financial crises.

The main policy implication of these results is that the Portuguese highways need to increase their efficiency: a new sector framework and new public policies are called for in order to increase competition and attract better management. Furthermore, there is considerable room to increase both technological (e.g: better electronic tolls and payment systems) and technical efficiency. Another policy implication regards the levying of tolls: the introduction of tolls and the financial and economic crises have led to a substantial drop in traffic, reducing highway efficiency. As most of the reduction of highway traffic led to a transfer of traffic to local roads (where drivers do not pay a toll), a new public toll policy addressing this problem is warranted. Reducing prices and providing discounts for frequent highway users could recover some of the lost traffic, which would not only increase revenues but also reduce costs in terms of reduced travel time and fewer traffic accidents. One further issue is that, as the highway companies are not listed, there is no sufficient scrutiny of performance. We have shown that there is room for an upgrade in technical innovation.

Future research is needed to understand the efficiency in highways. First, although this research controls for some of the company’s individual characteristics (e.g., shareholder concentration and type), it is important to further deepen this analysis by identifying more specific information by company (e.g., capital structure, length, type of shareholders, location of the highway, type of tolls among others). Second, since our data refer to a single country, a comparison of findings across different countries may also be useful to assess
the impact of the regulatory contexts. Third, introducing a long-term perspective by means of a larger longitudinal analysis could yield more insight on the whole PPP cycle.

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**References**


Measuring highway efficiency by a DEA approach and the Malmquist index

International Conference on traffic and Transportation Studies. Reston, VA: ASCE Publications.


Appendix A

The Malmquist index hinges on an efficiency methodology that measures the changes over time of productivity of decision making units. These changes are decomposed into technical and technological efficiency changes by means of a Data Envelopment Analysis (DEA) non-parametric approach. As referred to in Fare et al. (1994), the productivity decomposition into technical change and efficiency can be expressed in terms of the distance function (E) represented by the following equations:

\[
\text{Malmquist Index}_t^f = \frac{E^f_t(x^{t+1}; y^{t+1})}{E^f_t(x^t; y^t)}
\]

\[
\text{Malmquist Index}_{t+1}^f = \frac{E^{f+1}_t(x^{t+1}; y^{t+1})}{E^{f+1}_t(x^t; y^t)}
\]

where I denotes the orientation of the Malmquist index model (input or output oriented) and \((x^t, y^t)\) is a production point.

The geometric mean of these two equations is given by:

\[
\text{Malmquist Index}_t^f = (\text{Malmquist Index}_t^f \times \text{Malmquist Index}_{t+1}^f)^{1/2}
\]

\[
= \left( \frac{E^f_t(x^{t+1}; y^{t+1})}{E^f_t(x^t; y^t)} \times \frac{E^{f+1}_t(x^{t+1}; y^{t+1})}{E^{f+1}_t(x^t; y^t)} \right)^{1/2}
\]

The input oriented geometric mean of the Malmquist Index is decomposed using the concept of input oriented technical change (Technological efficiency change - TECH) and input oriented efficiency change (Technical efficiency change - EFFCH) as given by the following equation:

\[
\text{Malmquist Index}_t^f = (\text{EFFCH}_t^f) \times (\text{TECH}_t^f)
\]

\[
= \left( \frac{E^f_t(x^{t+1}; y^{t+1})}{E^f_t(x^t; y^t)} \right)^{1/2} \left( \frac{E^{f+1}_t(x^{t+1}; y^{t+1})}{E^{f+1}_t(x^t; y^t)} \right)^{1/2}
\]

Fare et al. (1994) provide the formal derivation of the model; using a DEA frontier solver we decompose the technological efficiency change into scale efficiency and pure technical efficiency.

Scale efficiency is given by:

\[
\text{Scale efficiency} = \left[ \frac{E^{f+1}_{vrs}(x^{t+1}; y^{t+1})}{E^{f+1}_{crs}(x^t; y^t)} \times \frac{E^{f+1}_{vrs}(x^{t+1}; y^{t+1})}{E^{f+1}_{crs}(x^t; y^t)} \right]^{1/2}
\]

and pure technical efficiency change is presented by:

\[
\text{Pure technical efficiency change} = \frac{E^{f+1}_{vrs}(x^{t+1}; y^{t+1})}{E^{f+1}_{crs}(x^t; y^t)}
\]

Following Cooper et al (2007) and Zhou (2014), the linear programming (LP) approach to calculate the Malmquist Index boils down to the following four LP problems:

Considering a vector of outputs \(Y^t = (y^t_{1j}; \ldots; y^t_{sj})\) produced by each DMU \((j = 1, 2, \ldots, n)\) using a vector of inputs \(X^t = (x^t_{1j}; \ldots; x^t_{mj})\) at each period of time \(t, t = 1, \ldots, T\), the DMU’s efficiency change calculated by the Malmquist Index over the period from \(t\) to \(t+1\) is obtained by:
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i) Comparing the $X_0^t$ to the frontier model at time $t$, calculating $\theta_0^t (X_0^t; Y_0^t)$ in the following input-oriented model:

$$\theta_0^t (X_0^t; Y_0^t) = \min \theta_0$$

subject to:

$$\sum_{j=1}^{n} \lambda_j X_j^t \leq \theta_o X_0^t$$
$$\sum_{j=1}^{n} \lambda_j Y_j^t \geq Y_0^t$$
$$\lambda_j \geq 0; j = 1, \ldots, n$$

where $X_0^t = (X_0^t; \ldots; X_{n_0}^t)$ and $Y_0^t = (Y_0^t; \ldots; Y_{n_0}^t)$ are the input and output vectors of DMU$_0$.

ii) Comparing $X_0^{t+1}$ to the frontier at time $t+1$, calculating $\theta_0^{t+1} (X_0^{t+1}; Y_0^{t+1})$

$$\theta_0^{t+1} (X_0^{t+1}; Y_0^{t+1}) = \min \theta_0$$

subject to:

$$\sum_{j=1}^{n} \lambda_j X_j^{t+1} \leq \theta_o X_0^{t+1}$$
$$\sum_{j=1}^{n} \lambda_j Y_j^{t+1} \geq Y_0^{t+1}$$
$$\lambda_j \geq 0; j = 1, \ldots, n$$

iii) Comparing $X_0^t$ to the frontier at time $t+1$, calculating $\theta_0^{t+1} (X_0^t; Y_0^t)$

$$\theta_0^{t+1} (X_0^t; Y_0^t) = \min \theta_0$$

subject to:

$$\sum_{j=1}^{n} \lambda_j X_j^t \leq \theta_o X_0^t$$
$$\sum_{j=1}^{n} \lambda_j Y_j^t \geq Y_0^t$$
$$\lambda_j \geq 0; j = 1, \ldots, n$$

iv) Comparing $X_0^{t+1}$ to the frontier at time $t$, calculating $\theta_0^t (X_0^{t+1}; Y_0^{t+1})$

$$\theta_0^t (X_0^{t+1}; Y_0^{t+1}) = \min \theta_0$$

subject to:

$$\sum_{j=1}^{n} \lambda_j X_j^t \leq \theta_o X_0^{t+1}$$
$$\sum_{j=1}^{n} \lambda_j Y_j^t \geq Y_0^{t+1}$$
$$\lambda_j \geq 0; j = 1, \ldots, n$$

Consequently, Malmquist Index is defined as:

$$Malmquist\ Index = \left[ \frac{\theta_0^t (X_0^t; Y_0^t)}{\theta_0^{t+1} (X_0^{t+1}; Y_0^{t+1})} \right]^{1/2} \cdot \left[ \frac{\theta_0^{t+1} (X_0^{t+1}; Y_0^{t+1})}{\theta_0^t (X_0^{t+1}; Y_0^{t+1})} \right]^{1/2}$$