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BUSINESS ASSOCIATIONS AND PRIVATE ORDERING

By

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Business Associations and Private Ordering *

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Abstract

We study the capacity of business associations—private, formal, noncommercial organizations designed to promote the common business interests of their members—to support contract enforcement and collective action. Inspired by recent empirical literature, our theoretical framework connects the organizational and institutional features of formal and informal business organization with socioeconomic distance. We show how associations provide value to their members even if members are already embedded in social networks, and which players join an association. We propose explanations for empirical puzzles, put forward novel testable hypotheses, and relate business associations to alternative private ordering institutions.

JEL classification: D02, D71, L14, L31

Keywords: Business Associations, Trade Associations, Economic Governance, Private Ordering, Arbitration

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1 Introduction

A business association is a private, formal, noncommercial organization designed to promote the common business interests of its members.¹ Throughout the last millennium traders have formed associations to represent themselves vis-a-vis other parties and to facilitate collective action. Associations offer members a platform to meet and to exchange views about other industry participants (Doner and Schneider, 2000; Pyle, 2006), to learn about the latest technologies, foreign markets and standardizations (Nugent and Sukiassyan, 2009) and prospective trade partners (Macaulay, 1963; Johnson et al., 2002). Some associations offer their members arbitration services and help to resolve disputes, which mitigates transaction costs (Woodruff, 1998; Pyle, 2005). In supporting honest trade both between members and between members and nonmembers, associations serve as substitutes for ineffective legal systems in developing countries (Kali, 1999) and have provided more effective private legal systems in specific industries, such as the U.S. cotton and diamond trading industries, in developed countries (Bernstein, 1992, 2001).²

These functions are not restricted to modern business associations. At the beginning of the Commercial Revolution in Europe, when long-distance trade started to boom in the tenth and eleventh centuries, the primary function of the first merchant guilds was to protect the property rights of their members vis-a-vis nonmembers (Volckart and Mangels, 1999), in particular vis-a-vis predatory rulers (Greif et al., 1994). Formal associations emerged that “helped long-distance traders solve two fundamental problems of exchange—one hand, protection against crime, warfare, and arbitrary confiscation and, on the other, the enforcement of contracts whenever money or goods changed hands.” (Grafe and Gelderblom, 2010:477). Merchant associations “existed not just in Europe but also in North Africa, the Near East, Central and South America, India and China.” (Ogilvie, 2011:1).³

¹This definition draws on Pyle (2005, 2006) and incorporates most nonprofit professional clubs, trade unions, trade associations, chambers of commerce, industry trade groups, and medieval merchant guilds. It is related to the study of cooperatives, which are often for-profit organizations, though.
²Associations also deal with public authorities and lobby government officials with one voice on behalf of their membership, protect members from illegitimate government interference (Pyle, 2011), and increase the level of trust among members in general (Raiser et al., 2008). By pushing governments for the provision of public goods, associations increase their members’ joint impact on institutional reform (Lambsdorff, 2002).
³Today there are more than 7,800 national associations in the US (National Trade and Professional Associations Directory, 2011), some 750 at the EU-level, and more than 11,700 national associations in the EU (The Directory of Trade and Professional Associations in the European Union, 2004).
Recently, the empirical literature studying the economics of business associations has produced new and intriguing findings. It has also identified several puzzles, which stand to be explained by theory. Business associations are reported to be less valuable for members engaged in local transactions and more valuable in long-distance transactions in Eastern Europe (Pyle, 2005, 2006). Associations are perceived to be less valuable to their members in more competitive industries (Pyle, 2005). Economic history research on merchant guilds and related institutions in the European middle ages has produced empirical findings that are surprisingly similar to those found by studies on modern associations (see Section 2 for details).

What forces drive these results? How do private informal institutions, such as social networks, and formal institutions, such as business associations, both of which transmit relational information about traders’ business conduct, interact? How do these private institutions perform in the shadow of a public court system? Who joins a business association and who does not? What is the role of product market competition and how does the distance between traders influence the choice of the optimal contract enforcement institution?

In comparison to the advances of the empirical literature, the theoretical literature has fallen somewhat behind in answering these questions. In particular, what is lacking is a theoretical framework that connects the organizational and institutional features of formal and informal business organization with the notion of socioeconomic distance, which lies at the heart of several empirical findings, as we will show below.

Dixit (2003b) has laid the foundations for a solution to this problem by introducing a model where the location of the players in socioeconomic space is represented by a circle. We adopt Dixit’s idea of a circle economy, to capture differences in the knowledge and abilities of players who repeatedly face opportunities to transact with new and unfamiliar business partners. In addition to a location on the circle, every player is endowed with an individual level of connectedness, which serves as a proxy for the embeddedness in social networks. Players can choose whether to join an association, or not. We model associations that serve either as a repository for information about the business conduct of members’ trade partners or which offer their members arbitration services in case of disputes with their partners. The value-

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4See Granovetter (1985) for the original conceptualization of embeddedness. Our framework could also contribute to closing a gap that sociologists have accused economists of for a long time, namely that economic models lack social structure and the integration of social relations. See the discussion in Powell (1990) and especially in Nee (2003).

5A related function of associations is to define what the terms of an agreement between two players mean,
generating transaction is a Prisoner’s Dilemma, which can be interpreted in two ways: as a vertical relationship involving a seller and a buyer, where “cooperation” means to keep one’s contractual obligations; and as a horizontal relationship involving two players faced with a collective action problem, where “cooperation” means to forego the short-term gains from free-riding but to contribute to a public or club good, for instance to join in a boycott of a government that did not respect the property rights of another player. The value generated by a certain type of association for a member depends on the range of potential partners for which the association can sustain cooperation in equilibrium, which depends on the individual connectedness.

Our findings show that associations indeed offer value to their members even if these are already connected informally to others via social networks. Associations use a different channel distributing information about players’ behavior than social networks do. However, we find that the value of association membership decreases if transactors are better connected informally. Hence, when endogenizing the players’ membership decisions, we can show that players with low informal connectedness choose to join an association but players with high connectedness do not. This novel result implies that social networks and associations are substitutes with respect to supporting cooperation. Moreover, we find that associations are a hybrid between social networks and public courts, which may be particularly relevant in practice if either of these two institutions does not exist or operate effectively. Depending on the competence of arbitrators, the strength of their ties to state enforcers, and the operation cost of arbitration tribunals, we show when one or the other association function creates more value for members.

The model explains several empirical findings by showing how the value created by associations for its members decreases if competition on product markets is intense or if transactions in an economy take place between local partners. We also show that the primary beneficiaries of associations’ activities are their members but that nonmembers also benefit to some extent because their commitment ability to cooperate increases if their partner is a member.

6 As Bernstein (2001) emphasizes. MacLeod (2007:596) confirms, “the evolution of successful informal agreements depends upon a number of interlocking elements, including a mutual understanding of the events that determine contract breach”. Gibbons and Henderson (2011) call it the clarity problem, which complements the credibility problem that I study here. In the same spirit, Hadfield and Weingast (2011a,b) study the capacity of legal orders to serve as a coordination device among players for which behavior is (not) acceptable. Given its importance and complexity, I defer a formal analysis of the clarity problem in the context of associations to future research.

7 See Dixit (2003b) for the former and Baron (2010) for the latter interpretation.

7 These results underline the positive effects of private ordering institutions for the transactors involved: where noncontractibility or prohibitive transaction costs make court enforcement no available option, private governance
The next section provides an overview over the issues and the related literature. Section 3 introduces the model and analyzes associations serving as information intermediaries and as arbitrators, respectively. Section 4 discusses several technical aspects and delineates the model from other models using Dixit’s circle framework. Section 5 relates associations to alternative private ordering institutions, suggests explanations for some empirical puzzles, and states testable hypotheses. Section 6 concludes. All proofs and supporting calculations are in the Appendix.

2 Business associations and economic governance

The study of economic governance is a central theme of the literature on private ordering. The key questions studied in this literature are, how can opportunistic behavior be avoided in social dilemma situations, where the joint payoffs of the players are maximized under mutual cooperation but it is individually rational to defect and thereby to maximize one’s own payoff at the expense of others? How should institutions be structured such that the incentives of individual players to free-ride on their companions’ efforts are mitigated?

Many institutions exist, both in theory and in practice, that can support cooperation and mitigate free-riding. Masten and Prüfer (2011) propose a classification of such commitment mechanisms, ranging from internal value systems—players cooperate because they like it—to public courts—players cooperate because they want to avoid high penalties and imprisonment. Between these extremes, there are several types of communities, all of which enforce cooperation by threatening defectors with ostracism. Both decentralized social networks and centralized institutions can mitigate Prisoner’s Dilemma problems. See Dixit (2004, 2009) and Williamson (2005) for general overviews of the New Institutional Economics approach to private ordering. The analysis of the normative implications of associations for nonmembers and total welfare is the subject of ongoing research. See Ostrom (1990) or Williamson (2000). Dixit (2009:5) defines the concept of economic governance as “the structure and functioning of the legal and social institutions that support economic activity and economic transactions by protecting property rights, enforcing contracts, and taking collective action to provide physical and organizational infrastructure.”

Dixit (2004), Fafchamps (2004), and Greif (2006) provide instructive overviews.

Communities support cooperation by providing the information channels and incentives for individuals not to transact with defecting others—in contrast to public courts, which have access to the state’s monopoly on coercion and can punish defectors more directly.
2.1 Empirical economic research on associations

We study the capacities of associations to support cooperation in the economy and compare them with the capacities of social networks and courts. The focus is on associations that are independent of state interference and where membership is voluntary.\textsuperscript{11} An important question is, what value do associations offer members if those are already embedded in social networks, an approach that identifies the incentives to join an association and, thereby, endogenizes the set of members. This question has already been studied in empirical work. Survey-based analyses of post-communist countries have shown that small and medium-sized firms join business associations, which comes at a cost, although they are already connected to other industry participants via informal networks (Johnson et al., 2002).\textsuperscript{12}

Pyle (2005) also studies flows of relational information between firms in five Eastern European countries. He underlines that “both manners of information flows—those that are enabled by formal organizations and those that are not—have been recognized for their ability to serve as the basis for relational contracting, to reduce search costs, and to mitigate information asymmetries [..]. However, the economics literature is largely silent as to how the two interact” (548). Pyle finds that associations support information flows in business relations of their members with firms that are located at some distance. In the relationship with local customers, however, associations add only very little value via their function as information repositories. Pyle (2006) confirms that geographic distance is associated with higher dispute-related costs, which are smaller for firms that are members of an association. Recent empirical economic history research about the comparative use of informal and formal commitment mechanisms in the European middle ages finds related results. Studying institutions for the protection of property rights in the Italian Alps between the thirteenth and the nineteenth centuries, Casari (2007) shows that the likelihood of a formal institution’s being established increases with a community’s size, its proximity to other settlements, and the amount of its common resources.

Another important finding of Pyle (2005) is that associations contribute less to the spread of information about contractual disputes when their members’ markets are particularly competitive. Grafe and Gelderblom (2010) obtain a similar result in their study of European merchant

\textsuperscript{11} The former requirement excludes several associations in autocracies, the latter excludes several chambers of commerce in Western democracies, where membership is mandatory for all firms (for instance, in Germany).

\textsuperscript{12} Johnson et al. (2002:230/1) state: "Almost half of the firms we surveyed are members of a trade association. [...] Start-up firms are as likely as privatized firms to be members of trade associations [...], which suggests the services the association offers are valuable.”
guilds: more intense product market competition between mercantile groups and local merchants is associated with a lower degree of control delegation from merchants to guilds, which implies that guild membership had lower value if competition was intense.

Our paper complements these studies as it takes their empirical findings as a starting point and suggests a model that identifies the channels through which associations might offer value even if decentralized social networks among transactors already exist, and how that value depends on the distance between partners and the level of competition on product markets.

2.2 Related theoretical literature

Starting about two decades ago, a literature using game-theoretic tools for the study of private ordering institutions, often complemented by the analysis of historical records and drawing on political science and sociology, has grown. An intensely studied institution to overcome collective action problems if court or other external enforcement mechanisms are not available are decentralized social networks.\textsuperscript{13} Associations with various functions—but all of them private, formal, noncommercial organizations—have also received some scholarly attention.\textsuperscript{14}

A major innovation to this literature was contributed by Dixit (2003b), who put forward the idea of a circle model of the economy, where the distance between two players denotes socioeconomic differences, which can be interpreted widely (1296). This model allows to express the value that an institution generates as the maximum distance between two randomly matched players up to which mutual interaction and cooperation characterizes an equilibrium.\textsuperscript{15}

The circle economy model has been applied widely. Dixit (2003b) compares the scope of cooperation among transactors when making use of a decentralized social network and external enforcement. Leeson (2008b) shows how the scope of cooperation can be increased if transactors’ location on the circle is endogenous. Tabellini (2008) analyzes how values and institutional development interact and evolve over time. Baron (2010) models pro-social preferences that depend on the distance between partners and studies how social label and certification organizations, amongst others, can increase cooperation. Masten and Prüfer (2011) study how the scope of cooperation supported by informal social networks and formal public courts interact.\textsuperscript{16}

\textsuperscript{13}See Kandori (1992), Greif (1993), and Ellison (1994) for early examples.
\textsuperscript{14}See Milgrom et al. (1990), Klein (1992), Greif et al. (1994), Kali (1999), and Dixit (2003a).
\textsuperscript{15}This maximum distance is called the \textit{extent of honesty} (Dixit, 2003b), the \textit{scope of self-regulation} (Baron, 2010), or the \textit{scope of cooperation} (Masten and Prüfer, 2011; this paper), depending on the focus of the authors.
\textsuperscript{16}We postpone a more detailed comparison with the closely related literature to Section 4.
3 The Model

3.1 Matching, trading, and social networks

Consider an economy consisting of a continuum of players uniformly distributed around a circle with a circumference of 2. The mass of players per unit arc length is normalized to one, implying a mass 2 population in the economy. The distance between two players, $X$, measured by the shorter of the two arc lengths between them (hence, $X \leq 1$) can be interpreted as representing differences in any relevant economic or social variables such as technological or resource endowments, knowledge or expertise, or kinship or other social or cultural affinities, as well as geographic location. $X$ affects three considerations in the model, which are specified below: the probability of meeting a given player, the potential gains from interaction, and the probability of receiving information about the previous behavior of other players via social networks.

Every player $i$ is endowed with an individual level of connectedness, $\kappa_i$, which is drawn from a continuous distribution, $z$, over $[0, 1]$, and common knowledge. Let $Z(\kappa_i)$ denote the cumulative distribution function of $z$. $\kappa_i$ determines the probability with which $i$ can send a message about the behavior of his partner along the circle. Abusing notation slightly, we also refer to $\kappa_i$ as the strength of $i$’s social network, or $i$’s embeddedness.

The game is infinitely repeated. Time between periods proceeds in discrete intervals, $t \in \{0, 1, 2, ..., \infty\}$, players live forever and have a uniform per-period discount factor, $\delta \in (0, 1)$. At the beginning of every period, each player is randomly matched with another player at socioeconomic distance $X$ with probability density:

$$\mu = 1 - X$$ (1)

---

17 The general structure of the model follows the infinite-period model by Masten and Prüfer (2011), who build on the two-period model by Dixit (2003b). Section 4 compares our model with these papers.

18 Given that we employ a model with a mass of players, social networks cannot be characterized by a set of specific players linked to each other, unlike in graph theory, where finite sets of discrete players are studied (Goyal, 2007:9). Hence, it would be incorrect to think of a specific network form, where players can send messages with probability 1 to their direct neighbors. Instead, we picture a graph where each player is indirectly linked to every other player and where players with high $\kappa_i$ are more centrally located than those with low $\kappa_i$ and therefore suffer from a lower deterioration of information when sending messages to everyone else.

19 The matching probability between two players is decreasing, that is, there is localized matching. As $\int_{0}^{1} \mu dX = \frac{1}{2}$ (over each arc length), the expected probability for every player $i$ in every period $t$ to be matched with another player is 1.
Every pair of matched players has to decide simultaneously whether or not to transact. Only if both agree to transact, each decides whether to cooperate or defect in the central transaction. The payoff to each player is $a(1 + \theta X)$, where $a$ results from the Prisoner’s Dilemma depicted in Figure 1 and $\theta > 0$. That is, the potential gains from interaction increase in the distance between the partners, $X$, but the rate of increase depends on market conditions, captured by $\theta$. \(^{20}\)

Every player automatically sends a message about the behavior of his partner after the central transaction to the social network, independent of using other enforcement institutions. \(^{21}\) The message space of sender $x$ is that his partner $i \{\text{cooperated, defected, did not interact}\}$. The probability that player $y$ receives $x$’s announcement is:

$$\eta_{x,y} \equiv \kappa_x (1 - |Y - X|), \quad (2)$$

where $|Y - X|$ is the distance between players $y$ and $x$. \(^{22}\) This model is solved as a special case of the information intermediary model in the next subsection.

### 3.2 Business associations as information intermediaries

All players are embedded in social networks, as described above. Additionally, they can become members of a business association that has only one function: to collect information from its members about their partners’ behavior and to distribute this information among its members. \(^{23}\) If player $i$ is a member, his membership status is $M_i = 1$; otherwise $M_i = 0$. The association is a not-for-profit organization that levies a membership fee, $f$, to cover its operating expenses, which are assumed to be constant per member. \(^{24}\) Denote the (endogenous) share of association members in all players in period $t$ by $\sigma_t$ and the common belief about the next period’s membership size by $\sigma_{t+1}$. In each period, the timing of the game is as follows:

- **Stage 0**: Each player can join the information association for the fee $f$.

\(^{20}\)The potential loss from a transaction also increases in $X$, which can be understood as a consequence of the travel cost that are lost if a transaction does not take place cooperatively.

\(^{21}\)That is, players cannot credibly commit not to inform the network about their partner’s behavior.

\(^{22}\)We follow the literature (Kandori, 1992; Kali, 1999; Dixit, 2003b) in assuming that players report truthfully. This assumption is relaxed in Section 4, where the message content is endogenized, allowing for strategic reporting.

\(^{23}\)The function of a centralized information repository is related to credit bureaus and rating agencies.

\(^{24}\)Considering (dis)economies of scale in the production of member services would complicate the analysis without generating substantial insights.
• **Stage 1:** Players are matched according to (1) and learn the location, connectedness, and membership status of their partner. Assume the players $i$ and $x$ are matched at distance $X$. With probability $\eta_{y,i}$, player $i$ receives the message sent by player $y$, who was $x$’s partner in period $t - 1$, about the behavior of $x$ in $t - 1$. If $i$ is an association member, he also obtains a report from the association about the behavior of $x$ in $t - 1$ (only if the association has such information). Partners decide simultaneously whether or not to transact. If either chooses not to transact, both payoffs are zero and the period ends for these players.

• **Stage 2:** If the matched players agree to transact, each decides whether to cooperate or defect and yields the payoff $a(1 + \theta X)$ specified above.

• **Stage 3:** Each player sends a message about the behavior of his partner around the circle. Additionally, association members send a similar report to the association.

We solve this game for a stationary Markov-perfect equilibrium.\textsuperscript{25} To formalize the information a player has about his partner’s previous actions, define $s_{y,t}$ to be player $y$’s state variable before he chooses an action at stage 1 of period $t$: $s_{y,t} = 0$ if player $y$ has received news—via the social network or the association—that his current match $i$ defected in period $t - 1$, or if player $y$ himself defected in period $t - 1$ and his match $i$ learned about it.\textsuperscript{26} Otherwise, $s_{y,t} = 1$.

A strategy for player $i$ in period $t$ is a mapping from his individual connectedness ($\kappa_i$), the match distance ($X$), and his state variable ($s_{i,t}$) to the action set: $\{\text{join, not join the association}\}, \{\text{transact, not transact}\}, \{\text{cooperate, defect}\}$.

**Information Intermediary (I)-Strategy.** Define the following Markov strategy for player $i$:

- In every period $t$, player $i$ joins the association for the cost $f$ if his individual connectedness

\textsuperscript{25}As is usual in infinitely repeated games, this game has multiple equilibria. We focus on one equilibrium in which a business association exists and supports cooperative exchange. For a discussion of the equilibrium concept and uniqueness, see Section 4.

\textsuperscript{26}We assume that a player who defected in a previous period knows whether his new trading partner learned of that defection. Technically, this assumption serves to harmonize the partners’ state variables. If both state variables take on value 0 or both take on value 1, no player has an incentive to deviate from the I-Strategy (see below). Practically, this assumption captures that in communities where all players have some friends a transactor may have heard rumors that his would-be partner is not willing to transact with him—and then can preemptively “reciprocate” by not interacting either. Thereby he complies with the I-Strategy, which is expected by the community. See also footnote 27.
\( \kappa_i \leq \kappa_i^r \), and does not join otherwise.

- In \( t = 1 \), player \( i \) transacts and cooperates with partner \( x \) if the match distance \( X \leq X_i^r \), and does not transact otherwise.

- In every subsequent period \( t \), if player \( i \) is matched to player \( x \) and either the distance \( X > X_i^r \) or \( s_{i,t} = 0 \), then player \( i \) does not transact with \( x \). Otherwise, \( i \) transacts and cooperates with \( x \).

The I-Strategy specifies that players transact cooperatively with their partners if the distance between them is not too large. If either partner deviated from that strategy in \( t - 1 \) by defecting and the other player learns about the defection, the players ought not to transact with each other. In this case not interacting with a formerly defecting partner serves as punishment of the defector. Not interacting is (weakly) incentive compatible for the punisher because the I-Strategy requires a defector who knows that his partner knows about his defection to participate in his own punishment by not transacting as well. Therefore, the would-be punisher would not gain from unilaterally transacting with a defector.\(^{27}\)

Moreover, the I-Strategy specifies that only badly connected players will join the association. In the remainder of this section it is shown that a Markov-perfect equilibrium exists, in which all players play the I-Strategy.

At stage 3 of period \( t \), no player has to make any decision. At stage 2, assume players \( i \) and \( x \) are matched at distance \( X \leq X_i^r \) (where \( X_i^r \) will be specified below). When \( i \) considers whether to cooperate or to cheat, assuming that \( x \) cooperates, it is critical whether \( x \) is an association member, or not. If \( M_x = 0 \), \( i \) knows that the only harm \( x \) could do to him in case \( i \) defects is to send a message to the social network. If \( i \)’s partner in period \( t + 1 \)—call him \( y \)—receives that message, \( s_{y,t+1} = 0 \). Hence, \( y \) would not transact with \( i \), according to the I-Strategy. Therefore, the expected loss \( L \) to player \( i \) from defecting against \( x \) in period \( t \) is

\(^{27}\)This specification of the equilibrium strategy is slightly simpler than reality sometimes is, where cases with two layers of punishment exist. Hadfield and Weingast (2011b:30) explain, based on work by Moore (1985) on 13th century Flemish cloth merchants: “[P]articipation in the injunction not to deal with a merchant who cheated a Flemish merchant was enforced by a provision that punished the non-punisher. [...] That secondary obligation—to refuse to deal with the non-punisher—is also enforced, at least in some measure, by collective punishment.” This is a common structure in repeated games of collective enforcement institutions; see Greif (2006, Appendix C) for more details. It can be interpreted in a way such that there is a slight probability in the background of the model by which not participating in one’s own punishment could be detected by other players. Those players would then be entitled to punish the uncooperative defector by not interacting forever.
the foregone value of transacting and cooperating with $y$ weighted by the probability that $i$ is matched to $y$ in $t + 1$ and the probability that $y$ received news from $x$ that $i$ defected in $t$ (see Figure 2):

\[
L \equiv \delta \int_0^X (1 - Y_1)\kappa_x(1 - (X - Y_1))h(1 + \theta Y_1)dY_1 \\
+ \int_X^1 (1 - Y_2)\kappa_x(1 - (Y_2 - X))h(1 + \theta Y_2)dY_2 \\
+ \int_{1-X}^1 (1 - Y_3)\kappa_x(1 - (2 - X - Y_3))h(1 + \theta Y_3)dY_3 \\
+ \int_0^{1-X} (1 - Y_4)\kappa_x(1 - (Y_4 + X))h(1 + \theta Y_4)dY_4 \\
= \frac{1}{6}\delta\kappa_x h(4 + 4X^3 - 6X^2 + \theta) \quad (3)
\]

Instead, if $M_x = 1$, $x$ has two channels to spread the information about $i$'s defection, the social network and the association. These two information channels can be overlapping. According to (2), the probability with which $y$ receives $x$'s message via the social network is $\kappa_x(1 - |Y - X|)$. Hence, with probability $\sigma_{t+1}\kappa_x(1 - |Y - X|)$, $y$ receives two messages. With probability $(1 - \sigma_{t+1})\kappa_x(1 - |Y - X|)$, $y$ receives $x$'s message only via the informal network. With probability $\sigma_{t+1}(1 - \kappa_x(1 - |Y - X|))$, $y$ receives $x$'s message only via the association and not via the informal network. This latter probability determines the additional expected loss to a defector if his partner is an association member. It follows that, if $M_x = 1$, the expected loss to player $i$ from defection against $x$ in $t$ equals $L + J$, where:

\[
J \equiv \delta \int_0^X (1 - Y_1)\sigma_{t+1}(1 - \kappa_x(1 - (X - Y_1))h(1 + \theta Y_1)dY_1 \\
+ \int_X^1 (1 - Y_2)\sigma_{t+1}(1 - \kappa_x(1 - (Y_2 - X))h(1 + \theta Y_2)dY_2 \\
+ \int_{1-X}^1 (1 - Y_3)\sigma_{t+1}(1 - \kappa_x(1 - (2 - X - Y_3))h(1 + \theta Y_3)dY_3 \\
+ \int_0^{1-X} (1 - Y_4)\sigma_{t+1}(1 - \kappa_x(1 - (Y_4 + X))h(1 + \theta Y_4)dY_4 \\
= \frac{1}{6}\delta\sigma_{t+1} h(2(3 + \theta) - (4 + 4X^3 - 6X^2 + \theta)\kappa_x) \quad (4)
\]

Let $G$ denote a player’s expected per-period gain from mutual cooperation before the identity of his partner is known. If cooperation can be expected for all distances, this implies:28

\[
G = \int_0^1 (1 - X)h(1 + \theta X)dX = \frac{L}{\delta} + \frac{J}{\delta\sigma_{t+1}} = \frac{h}{3}(3 + \theta) \quad (5)
\]

28If cooperation can only be expected along the distances $X \in [0, X^*]$, where $X^* < 1$, then $G = \int_0^{X^*} (1 - X)h(1 + \theta X)dX$. It still holds that $G(X^*) = L(X^*)/\delta + J(X^*)/(\delta\sigma_{t+1}) \forall X^*$. 

11
By the one-stage deviation principle, if player $i$ assumes that his partner $x$ plays the I-Strategy, $i$ will cooperate if and only if:

$$h(1 + \theta X) + \delta(G - M_i f) + \frac{\delta^2}{1 - \delta}(G - M_i f) \geq w(1 + \theta X) + (\delta(G - M_i f) - L - M_x J) + \frac{\delta^2}{1 - \delta}(G - M(x))$$

$$\Leftrightarrow L + M_x J \geq (w - h)(1 + \theta X) \quad (7)$$

The left (right) side of equation (6) is $i$’s expected net present value from cooperating (defecting) in period $t$. The first term on each side is the payoff in $t$, which is larger for defection than for cooperation. The second term is the payoff in $t+1$, which is smaller for defection because $i$’s next match will not interact with a defector if he heard about the defection. The third term, the expected present value of interaction from $t+2$ forward, is equal for both sides. Equation (7) is mathematically identical to (6) and allows us to define the present value of cooperation relative to defection for player $i$ if his partner is $x$ as:

$$V_{I,i} \equiv L(\kappa_x) + M_x J(\kappa_x) - (w - h)(1 + \theta X) \quad (8)$$

$$= \frac{1}{6}h\delta[2(3 + \theta)M_x \sigma_{t+1} + \kappa_x(4 + 4X^3 - 6X^2 + \theta)(1 - M_x \sigma_{t+1})] - (w - h)(1 + \theta X) \quad (9)$$

Performing comparative statics on $V_{I,i}$, $J$, and $L$ yields the following lemma.

**Lemma 1 (Value of cooperation with information intermediary)**

(i) $\frac{\partial L}{\partial \theta} > 0$, $\frac{\partial L}{\partial X} < 0$, $\frac{\partial L}{\partial \kappa_x} > 0$.

(ii) $J > 0 \forall \sigma_{t+1} > 0$, $\frac{\partial J}{\partial \theta} > 0$, $\frac{\partial J}{\partial \sigma_{t+1}} > 0$, $\frac{\partial J}{\partial X} > 0$, $\frac{\partial J}{\partial \kappa_x} < 0$.

(iii) $\frac{\partial V_{I,i}}{\partial X} < 0$, $\frac{\partial V_{I,i}}{\partial \kappa_x} > 0$, $\frac{\partial V_{I,i}}{\partial M_x \sigma_{t+1}} > 0$, $\frac{\partial V_{I,i}}{\partial \theta} > 0$ iff $w < h + \frac{\delta h}{6X}(2 - \kappa_x)M_x \sigma_{t+1} + \kappa_x$.

Lemma 1.(i) describes the marginal incentives to cooperate without association membership. The damage player $i$ expects from defecting against $x$ increases in the baseline value of the transaction ($\theta$) and in the connectedness of $x$ ($\kappa_x$) but decreases in the distance between the partners ($X$). Lemma 1.(ii) states that the additional value of cooperation that player $x$’s association membership creates for his partner $i$ is strictly positive, as long as it can be expected that the association has any members next period. Given that a higher value of cooperation of player $i$ makes $i$ more likely to cooperate, his partner $x$ (the association member) also benefits. Therefore, $J$ can be regarded as a measure for the value of information association membership. $J$ strictly increases in the expected membership size ($\sigma_{t+1}$) and in the baseline value of the transaction ($\theta$). In contrast to $L$, $J$ increases in the distance between transactors ($X$) and decreases in $x$’s connectedness ($\kappa_x$). This implies that the social network induces more cooperation.
between nearby transactors whereas the association induces relatively more cooperation between
distant transactors. The reason for this result lies in the different communication channels of
the two institutions: the social network relies on decentralized word-of-mouth communication,
which is localized, whereas the association collects information in a central hub and distributes
it to members independent of their location. Therefore, the association gets relatively more
valuable in distant relationships.

For a related reason, \( J \) decreases in the individual connectedness of a player’s partner \( (\kappa_x) \).
This means that association membership is less valuable for highly connected players and more
valuable for less connected players. It also implies that, for a given player, association mem-
bership and the embeddedness in social networks (connectedness) are substitutes in increasing
the value of cooperation for their partner—and, by the logic presented above, for themselves.
Rephrased, both enforcement institutions offer players a tool to threaten their partner in case of
defection. If a player is already well equipped with one tool (informal connections), the second
tool (association membership) is less valuable to him.

Lemma 1.(iii) brings the enforcement capabilities of social networks and associations together
with the short-term gains from defection. It shows that the effect of social networks dominates
the effect of associations: the present value of cooperation relative to defection for player \( i \), \( V_{I,i} \)
follows the same comparative statics as \( L \). Naturally, \( V_{I,i} \) decreases in the defection payoff \( (w) \)
and only increases in \( \theta \) if \( w \) is sufficiently small. \( V_{I,i} \) also increases in the expected membership
size \( (\sigma_{t+1}) \)—because more members ruin more business of defectors in the future—and in the
players’ discount factor \( (\delta) \)—because patient players suffer more from future loss.

Call the distance \( X_{I,i}^* \) player \( i \)’s individual scope of cooperation with information inter-
ediary, where \( X_{I,i}^* = \{X|V_{I,i} = 0\} \). The following proposition is proven in the appendix.

**Proposition 1 (Individual scope of cooperation with information intermediary)** If all
other players \(-i\) play the I-Strategy, player \( i \) cooperates at stage 2, if and only if, \( X \leq X_{I,i}^* \).
Otherwise, \( i \) defects.

This proposition holds that it is individually rational for a player to cooperate with his
partner if the two are located sufficiently close to each other in socioeconomic distance. Import-
antly, since \( X_{I,i}^* \) depends on \( V_{I,i} \), which depends on \( L \) and on \( M_x J \), the incentive for a player
to cooperate depends not only on the parameters of the economic environment \( (\theta, w, h) \) and
the distance between partners \( (X) \) but it also increases in the individual connectedness and the
membership status of his partner \( (\kappa_x, M_x) \) and in the expected membership size \( (\sigma_{t+1}) \).
At stage 1 of period \( t \), the matched players might receive information about the former behavior of their partner and have to decide whether to transact with their match, or not. Under which conditions will both players agree to transact with each other? Rephrased, what is the binding scope of cooperation of a partnership, \( X_I^* \), if the individual scopes of cooperation differ for both partners?

Without loss of generality, consider the case where \( \kappa_i > \kappa_x \) and \( M_i = M_x = 0 \). Then \( V_{I,i}(\kappa_x) < V_{I,x}(\kappa_i) \), according to Lemma 1.(iii). Player \( i \) is better connected than player \( x \), which means that \( i \)'s potential to punish \( x \) in case of defection by informing other players via the social network is higher than vice versa. However, because \( x \)'s connectedness is low, the incentive for \( i \) to cooperate is limited. Thus, if \( V_{I,i}(\kappa_x) < 0 \leq V_{I,x}(\kappa_i) \), \( x \) does have an incentive to cooperate but \( i \) does not. However, \( x \) understands \( i \)'s incentive to defect. Therefore, \( x \) will not interact with \( i \). This reasoning also holds if one or both of them are association members. Consequently, if the individual values of cooperation relative to defection differ, the smaller of the two values is decisive for the transaction decisions. The partners transact with each other, if and only if, \( V_I \geq 0 \), where \( V_I = \min \{V_{I,i}(L(\kappa_x), M_x J(\kappa_x)), V_{I,x}(L(\kappa_i), M_i J(\kappa_i))\} \). Hence, in equilibrium the binding scope of cooperation is the smaller one of the individual scopes:

\[
X_I^* = \min \{X_{I,i}^*(L(\kappa_x), M_x J(\kappa_x)), X_{I,x}^*(L(\kappa_i), M_i J(\kappa_i))\}
\]  

These insights are summarized in the following proposition, which nests the scope of cooperation with pure social network enforcement as a special case, in which \( M_i = M_x = 0 \).

**Proposition 2 (The scope of cooperation with information intermediary)** If all other players \( -i \) play the I-Strategy, player \( i \) transacts at stage 1, if and only if, \( X \leq X_I^* \). Otherwise, \( i \) does not transact.

At stage 0 in period \( t \), player \( i \) has to decide whether to join the association for the membership fee \( f \), or not. \( i \) does not know the identity of his partner this period yet, but he knows that his association membership will only create any value for him if it lets him and his partner cooperate this period conditional on they would not have cooperated without \( i \) being a member. This condition equals the probability with which \( M_i \) has an effect on \( X_I^* \), which we denote by \( \beta_I \equiv \text{prob}(X_{I,i}^*(\kappa_x, M_x)) > X_{I,x}^*(\kappa_i, M_i = 0) \). We derive \( \beta_I \) from the model’s fundamentals in the appendix and show that \( \beta_I \) increases in \( \sigma_{t+1} \) but decreases in \( \kappa_i \).

\footnote{If \( i \) becomes a member, \( M_i \) switches from 0 to 1. This can only have an effect if \( X_{I,i}^*(M_i = 0) < X_{I,i}^* \).}
If player $i$’s membership decision influences $X^*_I$, how much is the association membership worth for $i$? In this case, by becoming an association member, $i$ can increase the scope of cooperation from $X^*_NW$ to $X^*_I$, where we define $X^*_NW \equiv X^*_I(\kappa_i, M_i = 0)$, that is, the scope of cooperation without associations, and $X^*_I \equiv X^*_I(\kappa_i, M_i = 1)$, that is the scope of cooperation with $i$ being an association member. Only for matches with partners located at distances $X \in (X^*_NW, X^*_I]$, $i$’s association membership has a positive impact on the scope of cooperation in equilibrium, namely the switch from no transaction to cooperative interaction. For all $X < X^*_NW$, cooperative interaction exists even without association membership, whereas, for all $X > X^*_I$, not transacting is the unique equilibrium with or without $i$ being an association member.

Putting these pieces together, the expected net benefit for $i$ from joining an association that serves as an information intermediary is:

$$B_I \equiv \beta_I \int_{X^*_NW}^{X^*_I} (1 - X)h(1 + \theta X) dX - f$$

(11)

$B_I$ is completely specified depending on exogenous parameters. Player $i$ joins the association, if and only if, $B_I(\kappa_i) \geq 0$.

How many players, and which ones, will join the association, at every point on the circle economy? To study these questions we analyze $B_I$ in more detail and prove the following proposition in the appendix.

**Proposition 3 (Membership decisions with information intermediary)**

(i) Given that $f$ is not prohibitively high, if all other players $-i$ play the $I$-Strategy, player $i$ joins the association, if and only if, $\kappa_i \leq \kappa^*_I$.

(ii) Depending on the membership fee $f$, there are two possible equilibrium membership sizes, characterized by $\kappa^*_I(f)$. For small levels of $f$, there is a unique $\kappa^*_I(f)$. For large levels of $f$, $\kappa^*_I(f) = 0$. In each equilibrium, the share of association members corresponds to the players’ beliefs about the membership size: $\kappa^*_I(f) = \sigma_{t+1} = \sigma_t$.

Proposition 3.(i) answers the question what types of players join the association if there is a membership fee. It builds on the insight that a player with high connectedness, say $i$, has two disadvantages from association membership compared to a player with low connectedness, say $x$. First, for $i$ it is the low connectedness of $x$ that determines the binding scope of cooperation $X^*_I$ in their transaction. By joining an association, $i$ can only further improve his own ability to punish a defecting partner—which is not necessary due to $i$’s high endowed connectedness—but
cannot improve his commitment to cooperate with the less connected partner. Second, even if
i is matched to an even better connected player, say y, and therefore \( \kappa_i \) determines the binding
scope of cooperation, i benefits less from membership than x would benefit if being matched
to y. The reason is that in general the benefit from joining the association comes through the
improved access to all other members. Well connected player i, however, is already connected
to many other players, including some members. Therefore, the additional share of players that
i can inform via the association, on top of the ones in reach of his social network, is smaller
than the additional share of connections that x would enjoy.

Proposition 3.(ii) uses the characteristic that in equilibrium the players’ beliefs about mem-
bership size, which determine the value of membership, have to coincide with the realized,
endogenous membership share. If the membership fee \( f \) is prohibitive, it is unattractive for
everyone to join the association. If the fee is sufficiently small, however, there is unique mem-
bership level \( \kappa_i^* \) at which a common belief of all players about the present and future membership
size leads to a stable share of players becoming and staying members over time (as explained
in part (i), these are the less connected players).  

3.3 Business associations as arbitrators

Next we analyze associations that serve as private arbitrators in disputes involving their mem-
bers. As before, every player is endowed with an individual level of connectedness, \( \kappa_i \). In
contrast to the information association, the arbitrator investigates cases himself if a member
brings a charge against his partner, who can be a member or a nonmember. The association
tells its members not to interact with a player who was found to have defected if that player
refuses to pay a damage payment to his victim. Additionally, the association may be able to
access the coercive enforcement powers of the state in order to make a convict pay damage
payments. This is captured by the probability \( \lambda \in [0, 1] \) with which the loser still has to pay

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30 Note that the result on uniqueness depends on the functional form of the matching probability \( \mu \) assumed
here. For \( E(x) \to 0 \) it is possible that two equilibrium membership sizes exist. See footnote 62 for more details.

31 Many associations have the authority to assess damages. See, for instance, the arbitration rules of The
International Meat Trade Association (http://www.imta-uk.org/terms?start=4) or The International Cotton

32 Ogilvie (2011) describes in detail how medieval merchant guilds could rely on the enforcement support by
local rulers. Bernstein (2001:1737) describes how a modern national arbitration association can draw on public
courts to enforce its decisions, on top of private enforcement. Leeson (2008a:63) writes on international arbitration
and the functioning of the New York Convention (NYC): “Private parties to international commercial contracts
the damage payment even if he is unwilling to under a completely private regime. On the other hand, arbitrators can make mistakes. We assume an arbitrator correctly decides a valid claim with probability \( \tau \in [0, 1] \).\(^{33}\) Moreover, running an arbitration tribunal is costly. Therefore, we assume that the association levies a membership fee, \( F \), to cover its fixed operating cost per member. In each period, the timing of the game is as follows:\(^{34}\)

- **Stage 0:** Each player can join the association for the fee \( F \), as long as he does not have “unpaid damage payments” from period \( t - 1 \). The association announces a rule \( p = p(M_i, M_x, X, \kappa_x) \), according to which it will determine the amount of damage payments if some member \( i \) brings a charge against his partner \( x \) and \( x \) loses the case.

- **Stage 1:** Players are matched according to (1) and learn the location, connectedness, and membership status of their partner. According to (2), they receive messages sent through the social network. If \( i \) is an association member, he obtains a report from the association stating whether his partner \( x \) has “unpaid damage payments” from period \( t - 1 \). Partners decide simultaneously whether or not to transact.

- **Stage 2:** If the matched players transact, each decides whether to cooperate or to defect.

- **Stage 3:** Each player sends a message about the behavior of his partner to the social network. Additionally, association members can bring a charge against their partner to the association’s arbitration tribunal, for a cost \( c \), which covers the marginal cost of the arbitration process. If the arbitrator decides for the plaintiff, he orders the defendant to pay a damage payment \( p \) to the plaintiff. Otherwise, no damages are rewarded.

- **Stage 4:** A losing defendant chooses whether to pay the damage payment to the plaintiff, or not. (Non)payment is observed and recorded by the arbitrator.

Again we solve for a stationary Markov-perfect equilibrium, where some players voluntarily become members. For this section, we redefine player \( y \)’s state variable \( s_{y,t} \) to take the value \( s_{y,t} = 0 \) if player \( y \) has received news via the social network that his current match \( i \) defected agree to have their disputes settled by arbitration associations. Since these associations are private, they cannot formally compel losers to comply with their decisions. However, under the terms of the NYC, winners can have their arbitral decisions enforced by losers’ governments if these governments are members of the convention.”

\(^{33}\)\( \tau \) can be widely interpreted. For instance, \( \tau \) is low if the competence of the arbitrator is low or if the transaction is complex or if the actions in the central transaction are hardly observable ex post.

\(^{34}\)The sequence of this game is influenced by Milgrom et al. (1990). The main assumptions of the arbitrator model reflect the descriptions of the U.S. cotton industry in Bernstein (2001) and in Wall Street Journal (2012).
in period $t - 1$ or if $y$ received news from the association that $i$ has unpaid damage payments from $t - 1$ or if player $y$ himself defected or did not pay a damage payment in period $t - 1$, and his match $i$ learned about it. Otherwise, $s_{y,t} = 1$.

A strategy for player $i$ in period $t$ is a mapping from the match distance ($X$), his individual connectedness ($\kappa_i$), and his state variable ($s_{i,t}$) to the action set: $\{\text{join, not join the association}\}$, $\{\text{transact, not transact}\}$, $\{\text{cooperate, defect}\}$, $\{\text{bring a charge, do not bring a charge}\}$, $\{\text{pay damage payment, do not pay damage payment}\}$.

**Arbitration Association (A)-Strategy.** Define the following Markov strategy for player $i$:

- In every period $t$, player $i$ joins the association for the cost $F$ if his individual connectedness $\kappa_i \leq \kappa^*_A$, and does not join otherwise.

- In $t = 1$, player $i$ transacts and cooperates with partner $x$ if the match distance $X \leq X^*_A$ and if the announced damage payment rule is $p \in [\underline{p}, \overline{p}]$. Otherwise, he does not transact. If $i$ cooperates and his partner defects, $i$ brings a charge. If $i$ defects and his partner $x$ brings a charge, $i$ pays the damage payment.

- In every subsequent period $t > 1$, in addition to the requirements specified above, for $i$ to transact and cooperate with $x$, it must be that $s_{i,t} = 1$.

Related to the I-Strategy, the A-Strategy specifies that only players who are located sufficiently close in socioeconomic distance are to transact with each other. It also holds that those players who are endowed with rather low connectedness, join the association. Importantly, players who ignored the judgement of the arbitrator in the previous period are not eligible for membership. On top, it must be that the damage payment players expect the arbitrator to determine in case of a found defection is neither too low nor too high. We will investigate these final conditions first by proceeding by backward induction.

At stage 4 of the game, if player $i$ was found guilty of unilateral defection by the arbitration tribunal and told to pay a damage payment to his partner, he has to decide whether to submit to the judgement. Assuming that everybody else plays the A-Strategy, $i$ knows that his matching partner in period $t + 1$, say $y$, will not interact with him if $y$ is an association member, which is given with probability $\sigma_{t+1}$. The trade-off faced by $i$ depends on $i$’s own membership status, $M_i$. If $M_i = 1$, according to the solution concept of stationary equilibrium, $i$ also has to join the association in $t + 1$. This requires paying the damage payment in $t$ and the membership fee
F in \(t + 1\). Then \(i\) expects a net present value of 
\[-p + \delta(G - F) - L + \frac{\sigma^2}{1 - \delta} G\]
from cooperation in \(t + 1\). If \(i\) deviates from the A-Strategy and does not follow the arbitrator’s judgement, he still has to pay \(p\) with probability \(\lambda\), due to the associations partial support by state enforcement. However, in this case \(i\) saves on \(F\)— because he cannot renew his membership in \(t + 1\). Hence he expects a net present value of 
\[-\lambda p + \sigma_{t+1}0 + (1 - \sigma_{t+1})(\delta G - L) + \frac{\sigma^2}{1 - \delta} G\]. Rearranging these two terms, it follows that member \(i\) pays the damage payment, if and only if:
\[
p \leq \frac{\sigma_{t+1}(\delta G - L) - \delta F}{1 - \lambda} \equiv \bar{p}_{\text{member}}, \tag{12}\]
where \(\bar{p}_{\text{member}}\) is the maximum damage payment against members.\(^{35}\)

If \(M_i = 0\), in contrast, \(i\) is not interested in association membership in the next period. He expects a net present value of 
\[-p + \delta G - L + \frac{\sigma^2}{1 - \delta} G\] if he pays the damage payment and of 
\[-\lambda p + \sigma_{t+1}0 + (1 - \sigma_{t+1})(\delta G - L) + \frac{\sigma^2}{1 - \delta} G\] if he does not pay. Hence, nonmember \(i\) pays the damage payment, if and only if:
\[
p \leq \frac{\sigma_{t+1}(\delta G - L)}{1 - \lambda} \equiv \bar{p}_{\text{nonmember}} \tag{13}\]

Define \(\bar{p}_{M_i}\) as the maximum effective damage payment against player \(i\), where \(\bar{p}_{M_i}(M_i = 1) = \bar{p}_{\text{member}}\) and \(\bar{p}_{M_i}(M_i = 0) = \bar{p}_{\text{nonmember}}\). Comparing \(\bar{p}_{\text{member}}\) and \(\bar{p}_{\text{nonmember}}\), we obtain the following lemma.

**Lemma 2 (Maximum damage payments)** (i) The maximum effective damage payment is lower for members than for nonmembers (\(\bar{p}_{\text{member}} < \bar{p}_{\text{nonmember}}\)). (ii) The maximum effective damage payment against player \(i\) is decreasing in the connectedness of \(i\)’s victim, member \(x\) (\(\frac{\partial \bar{p}_{M_i}}{\partial \kappa_x} < 0\)) and increasing in the arbitrator’s support by state enforcement (\(\frac{\partial \bar{p}_{M_i}}{\partial \lambda} > 0\)).

Lemma 2.(i) contains the interesting result that, assuming the arbitrator wants to punish a defector as hard as possible, he will determine a higher damage payment for non-members than for members. The reason for this difference is that the arbitrator has to make sure that members who defected find it worthwhile to pay the judgement and the membership fee of the next period. Nonmembers do not have to account for future membership fees, which is why it is rational for them to pay an even higher damage payment than for members to avoid being ostracized by the association.

\(^{35}\)If \(p > \bar{p}_{\text{member}}\), the convicted player refuses to pay. Hence, the arbitration mechanism has no bite and cannot increase cooperation.
Lemma 2.(ii) holds independent of $i$’s membership status because $\frac{\partial G}{\partial \kappa_x} = 0$ but $\frac{\partial L}{\partial \kappa_x} > 0$; see (12), (13), and (5) in the Appendix: defector $i$ expects his highly connected victim $x$ to inform a high share of players via the social network, independent of $x$’s association membership. Thus, the additional threat posed to $i$ if $x$ is an association member is less severe than the additional threat coming from victim $y$ if $\kappa_y < \kappa_x$. Moreover, Lemma 2.(ii) shows that the punishment effectiveness of the arbitrator increases if he can count more on the state’s coercion to enforce his decisions.

At stage 3, every association member has to decide whether to bring a charge against his partner to the arbitration tribunal, or not. The expected payoff to a player from bringing a charge depends on both his behavior and that of his partner in the central transaction. A player who cooperated while his partner defected expects $\tau p - c$ from charging. If he defected while his partner cooperated or if both players cooperated or both defected, bringing a charge leads to an expected payoff of $-c$. It follows that bringing a charge can only be profitable for player $i$ if he cooperated and his partner defected and if:

$$p \geq \frac{c}{\tau} \equiv \bar{p}$$

(14)

The following proposition is a consequence of the previous analysis and does not require a formal proof.

**Proposition 4 (Effectiveness of the arbitrator)** (i) The damage payment determined by an arbitrator against player $i$ is effective if, and only if, $\frac{c}{\tau} \leq \frac{\sigma_{i+1} (5G-L) - M_i \delta F}{1-\lambda}$. If the respective condition, depending on $M_i$, holds, a member whose partner defected brings a charge in equilibrium. (ii) If the arbitrator rules against a defendant $i$, he chooses the maximum effective damage payment: $p^* = p_{M_i}$.

Proposition 4.(i) characterizes the lower and upper constraints faced by the arbitrator if he wants to determine an effective damage payment, that is, to make a judgement that would be followed by the charged player and that would incentivize a member whose partner unilaterally defected to bring a charge. The two intervals specified, depending on $M_i$, are nonempty if the cost of bringing a charge or the membership fee or the connectedness of the victim is low or if the

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36 This model allows only for “type-2” errors of the arbitrator (false negatives).
37 Note that $\bar{p}$ distinguishes between members and non-members because every player can face a judgement of the arbitrator. In contrast, $p$ is only relevant for members because non-members do not have the right to bring a charge to the arbitrator.
probability with which the tribunal correctly decides a valid claim or the expected membership size or the backup of the arbitration decision by the state are high. If the interval is nonempty, it is rational for a victim to bring a charge. Otherwise, getting a judgement that would be paid by the defector would be too expensive for the victim.

Given that \( p \) is determined after the members decide about bringing a charge, the members’ beliefs about \( p \) are important, which are anchored by the arbitrator’s announcement of a damage payment determination rule at stage 0. As we will see below, the effectiveness of the arbitration institution to support cooperative interaction increases in the (expected) damage payment it determines. This explains Proposition 4.(ii).

At stage 2 of the game, if the interval \([\bar{p}, \tilde{p}_M]\) is empty, all players know that the arbitrator is ineffective in equilibrium. Hence, the pure social network case prevails (see Section 3.2, where \( M_i = M_x = 0 \)). For the subsequent analysis, we assume that \( p^* = \tilde{p}_M \) is effective. Consider the case where players \( i \) and \( x \) are matched at distance \( X \leq X_A^* \). Player \( i \)'s incentive compatibility constraint for cooperation, the analogue to (6), is:

\[
\begin{align*}
&h(1 + \theta X) + \delta(G - M_i F) + \frac{\delta^2}{1 - \delta}(G - M_i F) \\
&\geq w(1 + \theta X) - M_x \tau p^* + \delta(G - M_i F) - L(\kappa_x) + \frac{\delta^2}{1 - \delta}(G - M_i F) \\
\Leftrightarrow &\quad L(\kappa_x) + M_x \tau p^* \geq (w - h)(1 + \theta X)
\end{align*}
\]

Equation (15) shows that if player \( i \) defects, he gets the higher payoff \( w(1 + \theta X) \) instead of \( h(1 + \theta X) \) in period \( t \). Then, however, if \( i \)'s partner \( x \) is an association member, \( x \) will bring a charge to the arbitration tribunal, which will rule against \( i \) with probability \( \tau \) and ask \( i \) to pay a damage payment \( p^* \) to \( x \). Given that \( p^* \) is effective, \( i \) will pay. Independent of paying the damage payment, \( i \) expects that \( x \) informs the social network about the defection, which will result in the expected loss \( L \) in period \( t + 1 \). Rewriting equation (15) gives (16), which shows that \( i \)'s incentive to cooperate increases in the expected damage payment and thus explains the arbitrator’s choice to set \( p^* = \tilde{p}_M \) at stage 3. Resubstituting \( p^* \) from (12) and (13) into (16) and rearranging, the present value of cooperation relative to defection for player \( i \) is:

\[
\begin{align*}
V_{A,i} &\equiv L(\kappa_x) + M_x \tau \delta \frac{(\delta G - L(\kappa_x)) - M_i \delta F}{1 - \lambda} - (w - h)(1 + \theta X) \\
&= \frac{1}{6} h \delta \kappa_x (4 + 4X^3 - 6X^2 + \theta) + \frac{M_x \tau \delta \sigma_{i+1} (2(3+\theta) - \kappa_x (4+4X^3 - 6X^2 + \theta))}{6(1-\lambda)} - \frac{\tau M_x M_i \delta F}{(1-\lambda)} - (w - h)(1 + \theta X)8
\end{align*}
\]

Comparative statics on \( V_{A,i} \) are summarized in the following lemma.
Lemma 3 (Value of cooperation with arbitrator)

\[
(i) \quad \frac{\partial V_{A,i}}{\partial \delta_{t+1}} > 0, \quad \frac{\partial V_{A,i}}{\partial M_i F} < 0 \quad (19)
\]

\[
(ii) \quad \frac{\partial V_{A,i}}{\partial X} < 0 \text{ iff } \lambda < 1 - \frac{2h\delta X(1 - X)\tau\kappa_x M_x\sigma_{t+1}}{(w - h)\theta + 2h\delta X(1 - X)\kappa_x} \quad (20)
\]

\[
(iii) \quad \frac{\partial V_{A,i}}{\partial \kappa_x} > 0 \text{ iff } \lambda < 1 - M_x\tau\sigma_{t+1} \quad (21)
\]

\[
(iv) \quad \frac{\partial V_{A,i}}{\partial \theta} > 0 \text{ iff } w < h + \frac{\delta h}{6X} \left( \frac{\tau(2 - \kappa_x)M_x\sigma_{t+1}}{1 - \lambda} + \kappa_x \right) \quad (22)
\]

The value of cooperation increases in the expected number of association members and in the players’ discount factor (Lemma 3.(i)).\textsuperscript{38} There is also a completely a novel insight: the incentive of player \(i\) to cooperate with \(x\) is smaller if \(i\) is an association member himself, in particular if the membership fee is large. The reason for this surprising result is that the damage payment that \(i\) expects to pay if he defects and loses the case is smaller if he is a member. See Lemma 2.(i) and Proposition 4.(ii).

Lemma 3.(ii) is important because \(\frac{\partial V_{A,i}}{\partial X} < 0\) drives the fact that players located close to each other cooperate while players located at a distance do not cooperate and transact—similar to the information association case. \(V_{A,i}\) depends both on \(L\), which is decreasing in \(X\), and on \(\sigma_{t+1}(\delta G - L) = J\) (see (5)), which is increasing in \(X\). The condition in (20) is driven by the fact that the impact of \(J\) is weighted by \(\frac{\tau}{1 - \lambda}\). If this factor is high, the effect of \(J\) outweighs the effects of \(I\) on \(V_{A,i}\). If the arbitrator cannot rely on state enforcement (\(\lambda = 0\)), \(\frac{\partial V_{A,i}}{\partial X} < 0\) for all parameter realizations, just as in the information intermediary model. Because empirical evidence suggests that the state enforcement accessible by arbitrators is positive but modest, for the remainder of the analysis we assume that the condition in (20) holds.\textsuperscript{39}

Qualitatively, the same holds for Lemma 3.(iii): As long as the support of the arbitrator by state enforcement is limited, the value of cooperation increases in the connectedness of a player’s partner, similar to the information association case.

Call the distance \(X_{A,i}^*\) player \(i\)’s individual scope of cooperation with arbitration association, where \(X_{A,i}^* \equiv \{ X | V_{A,i} = 0 \} \).

\textsuperscript{38} Additionally, it is straightforward to show that \(V_{A,i} > 0\) if \(F\) or \(w\) are not prohibitively high.

\textsuperscript{39} Leeson (2008a:61) finds in his empirical study that “state enforcement [as compared to private arbitration enforcement] increases trade between nations by about fifteen to thirtyeight percent”, where fifteen percent refers to cases with only one member state signing the NYC and thirtyeight percent to cases where both parties are NYC members.
Proposition 5 (Individual scope of cooperation with arbitration association) If all other players \(-i\) play the AA strategy, player \(i\) cooperates at stage 2 if and only if \(X \leq X^*_{A,i}\). Otherwise, \(i\) defects.

The proof of Proposition 5 is similar to the proof of Proposition 1 and omitted.

At stage 1 of the game, when the matched partners have to decide about transacting with each other, the analysis follows the information intermediary case. In equilibrium the binding scope of cooperation is the smaller one of the individual scopes:

\[
X^*_A = \min \{X^*_{A,i}(L(\kappa_x), M_x \tau p^*), X^*_{A,x}(L(\kappa_i), M_i \tau p^*)\} \tag{23}
\]

The following proposition, related to Proposition 2, follows.

Proposition 6 (The scope of cooperation with arbitration association) If all other players \(-i\) play the A-strategy, player \(i\) transacts at stage 1 if and only if \(X \leq X^*_A\). Otherwise, \(i\) does not transact.

At stage 0, players decide about membership in the association. The analysis follows the information intermediary case and is conducted in the appendix. The expected net payoff for player \(i\) from joining an association that offers arbitration services is:

\[
B_A \equiv \beta_A \int_{X^*_{NW}}^{X^*_A} (1 - X)h(1 + \theta X)dX - F \tag{24}
\]

The following proposition is proven in the appendix.

Proposition 7 (Membership decisions with arbitrator) (i) Given that \(F\) is not too high, if all other players \(-i\) play the A-strategy, player \(i\) joins the association, if and only if, \(\kappa_i \leq \kappa^*_A\). (ii) Depending on the membership fee \(F\), there are two possible equilibrium membership sizes, characterized by \(\kappa^*_A(F)\). For small levels of \(F\), there are two \(\kappa^*_A(F)\), called \(\kappa^*_{A,1}(F)\) and \(\kappa^*_{A,2}(F)\), \(\kappa^*_{A,1}(F) < \kappa^*_{A,2}(F)\). For large levels of \(F\), \(\kappa^*_A(F) = 0\). (iii) Membership gets more attractive if \(\delta\) or \(\tau\) or \(\lambda\) or \(\theta\) increase.

The intuition of Proposition 7 is related to Proposition 3: players who are badly connected benefit most from association membership. Hence, with rising membership-fees \(F\), well connected players will be the first to find association membership unattractive. The main difference between both propositions stems from the fact that the arbitration association can only add value if the damage payment \(p^*\) determined by the arbitrator is neither too high nor too low.
This leads to the result that the arbitration association has a more restricted set of $\kappa^*$-values for which an equilibrium, characterized by $\kappa^*_A(F)$, exists than for the information intermediary. Moreover, because $B_A$ is hump-shaped in $\kappa^*_A$, independent of the distribution of matches expressed by (1), it has either two equilibrium membership sizes or it cannot exist, due to prohibitive $F$. This is different for the information intermediary, for which we showed a unique equilibrium membership size, based upon the specific matching function (1) assumed here.\(^{40}\)

Proposition 7.(iii) states that the value of membership increases if the players are more patient, if the arbitrator is more competent, if the support by state enforcement is stronger, or if the baseline value of transactions is higher.

Finally, given everybody’s perfect foresight, at stage 0 the arbitrator announces the damage payment determination rule $p = p^*$, as specified in Proposition 4. Announcing a tougher rule, $p > p^*$, would not be credible. Announcing a softer rule, $p < p^*$, would reduce the effectiveness of the arbitrator (see Proposition 4.(ii)).

Comparing the scopes of cooperation of both types of associations, the following proposition is proven in the appendix.

**Proposition 8 (Comparing the scopes of cooperation)** Without loss of generality, consider $X^*_I = X^*_I(\kappa_x, \cdot)$ and $X^*_A = X^*_A(\kappa_x, \cdot)$. The scope of cooperation in the arbitrator case is larger than in the information intermediary case, if and only if,

$$F < \frac{J}{\delta \tau}(\tau + \lambda - 1).$$

This proposition delineates the conditions under which the information intermediary and arbitration functions of associations, respectively, are more valuable for members. $\tau + \lambda > 1$ is a necessary condition for $X^*_A > X^*_I$. That is, only if the arbitrator is sufficiently competent to decide a case correctly, an arbitration association can outperform an information association with respect to supporting cooperative trade. Given that $\tau \in [0, 1]$, an interesting corollary is that $\lambda > 0$ is necessary: if the arbitrator has no access to state enforcement and has to rely completely on private enforcement—namely to exclude defectors from future trade with association members—it cannot be more effective than the information association.\(^{41}\) The

\(^{40}\)These results imply that no general statement is possible about the relative membership sizes $\kappa^*_I(f)$ and $\kappa^*_A(F)$. Multiple equilibria can exist in both cases, which cannot be ranked and interpreted in an economically meaningful (or even testable) way. See Figures 4 and 5 in Appendix A for some illustration.

\(^{41}\)This result gains empirical relevance by recent news from the cotton trader industry. The Wall Street Journal (2012) wrote: “About 520 companies, mostly textile mills in emerging markets such as Asia and Latin America,
latter operates exclusively by private enforcement but saves the cost of arbitration \((c)\) and is easier to set up because the equilibrium membership size \(\kappa_I^*\), unlike \(\kappa_A^*\), is not restricted by the (missing) effectiveness of \(p^*\).

Another condition for \(X_A^* > X_I^*\) is that the arbitration association membership-fee \(F\) must not be too high. \(F\) influences this trade-off but \(f\) does not because the maximum damage payment against a player \(i\) depends on \(i\)'s membership status in the arbitrator case only (see Lemma 2). The larger \(F\), the smaller the maximum effective damage payment the arbitrator can determine against member \(i\). As no damage payments are determined by the information intermediary, \(f\) does not directly influence Proposition 8.\(^{42}\)

Indirectly, however, both \(f\) and \(F\) influence the scope of cooperation implemented by each type of association: the equilibrium membership sizes \(\kappa_I^*(f)\) and \(\kappa_A^*(F)\) depend on membership-fees, that is, on operation costs per member. If these costs change, the equilibrium membership sizes adapt to the change, which influences the value of membership and the scopes of cooperation implemented by each association type. However, it is not possible to state in general in which direction the adjustment process works (see also footnote 40).

4 Discussion

Equilibrium concept, repeated games, and closely related literature: This model builds on Dixit (2003b), who constructs a two-period model, where the Prisoner’s Dilemma payoff from mutual defection (corresponding to \(d\) in Figure 1) is positive.\(^{43}\) He assumes the existence of two behavioral types, Normal (N) and Macchiavellian (M), and specifies that a N type’s payoff is negative either if his partner unilaterally defects (corresponding to \(l\) in Figure 1) or if he is matched to a M type, an “especially skillful cheater” (1299) whose actions are not modeled. Dixit specifies a candidate equilibrium strategy that assumes that a player who are on the ICA [International Cotton Association] default list after refusing to take or deliver cotton they had promised and failed to pay awards issued by arbitrators. [...] Even after winning arbitration cases, companies have struggled to enforce awards in foreign countries. [...] Although arbitrations are by nature private proceedings, arbitration panels in some futures and securities industries publish party names and case summaries and decisions online.”

\(^{42}\)See also Section 5.1 for a comparison of both scopes of cooperation with other enforcement institutions.

\(^{43}\)There are several other important differences. For instance, Dixit—and Baron (2010) and Masten and Prüfer (2011)—assumes exponential functions for the matching probability \(\mu\), the gains from trade \(\alpha(1 + \theta X)\), and information transmission \(\eta\). The linear functions assumed here, however, allow to specify and to analyze many results explicitly (e.g. \(L, J, V\)) without sacrificing any result qualitatively.
receives a message from his current partner’s former partner learns the payoff of the former partner and thereby can condition his strategy on that payoff. He solves for Perfect Bayesian Equilibria.

In Dixit’s model, \( d > 0 \) to incentivize the players to interact at all with an unknown partner. As a result, however, players would always cooperate with another N type because the unique one-shot equilibrium, mutual defection, yields positive payoffs. This is why he introduces the M types: “Without them, there would be no cheating in equilibrium [...]. [T]he N types’ behavior is driven by their fear of being confused with the M types that are known to lurk in the population” (1301).

By contrast, in our model the game is repeated infinitely many times.\(^{44}\) The main reason is that in a \( T \)-period model with private enforcement (where \( T \in \mathbb{N} \)), the unique subgame-perfect equilibrium in \( T \) contains not to interact with each other (in order to avoid the \( d < 0 \) payoff assumed in Figure 1), independent of one’s state variable \( s_y,T \): without a future there is no incentive to cooperate even if the commonly accepted institution asks for it. Hence, in \( T \) the collective punishment mechanism of associations or other private enforcement institutions breaks down. Consequently, in \( T - 1 \) the players have no concerns regarding their reputation in \( T \), and do not interact (and defect) as well. This unraveling proceeds and lets trade break down in all periods. In contrast, modeling an infinitely repeated game allows to assume \( d < 0 \) and to avoid behavioral types. Here players interact with unknown partners if they expect not to be cheated, which depends on the information received about their current partner’s previous action (not a specific payoff). They do not cheat (for \( X \leq X^* \)) because they fear to lose the payoff from mutual cooperation, \( h(1 + \theta X) \), in the next period, not the payoff from mutual defection, \( d(1+\theta X) \), as in Dixit’s model. Thereby, a repeated game allows to incorporate arbitration, damages, and information about whether the trader has paid the assessed damages.

A Perfect Bayesian Equilibrium would have to be conditioned on the complete information that a player has on his partner’s history. In contrast, the concept of stationary Markov-perfectness allows to ignore all details except the current state of a player when choosing an action (Mailath and Samuelson, 2006:177). In line with empirical findings, e.g. Bernstein (1992, 2001) or Greif et al. (1994), the state space is a binary variable here: either a player broke the rules and his new partner heard about it, or not. For tractability, strategies in period \( t \) only

\(^{44}\)Dixit (2011:10) acknowledges that Dixit (2003b) “squeezes the action into two periods. For an infinite-horizon version with some other generalizations and extensions, see Masten and Prüfer (2011).”
depend on behavior in the previous period, \( t - 1 \), but not in earlier periods. This structure is in line with the empirical observation (Ostrom, 1990) and the theoretical finding that optimal punishment periods are finite (Greif, 2006, Appendix C)—although not necessarily at a length of one period.\(^{45}\)

Baron (2010) uses Dixit’s framework but models two types of players’ moral preferences, termed (un)conditional and generalized/limited. The first type specifies whether a player’s utility depends on the behavior he expects from his partner in a Prisoner’s Dilemma. The second specifies whether the socioeconomic distance between players is payoff relevant, or not. Baron provides several models of organizations that can increase the scope of cooperation such as the Fair Labor Association (which can execute a damage against defectors), both nonprofit and for-profit, and pure information associations such as social label organizations and certification organizations. While that paper highlights many important mechanisms, its focus is different than ours. First, because of the assumption of pro-social preferences, Baron can restrict attention to one-period games: if players cooperate because they enjoy it, the more complex dynamics of punishing defectors by excluding them from future trade is not necessary.\(^{46}\) Second, joining an organization in Baron’s model serves as a commitment device: the member exposes himself to harm imposed by the organization in case of detected misbehavior. In our model, in contrast, association membership strengthens the capability of a player to punish other defectors. Third, Baron models the extent of punishment as a choice variable of an organization, which comes at a cost. In my model, in contrast, the punishment capability of an association—\( J(\sigma_{t+1}, \cdot) \) or \( \tau p^*(\sigma_{t+1}, \cdot) \)—depends on the parameters of the economy and on the simultaneous behavior of all other players, expressed by their membership decisions.\(^{47}\)

\(^{45}\)An additional key difference to Dixit (2003b) is that he focuses on the social network model and only offers a reduced form model of external enforcement, where “at a cost \( c \) per unit of arc length along the circle, \textit{any} cheating can be detected and the information is made available to [all] future traders” (1311; emphasis added). Dixit recommends that “external enforcement [...] can be modeled more explicitly” (1312), which is what this paper does, apart from connecting the empirical and theoretical literatures on associations.

\(^{46}\)An exception is Baron’s analysis of “certification organizations”, which are analyzed in a two-period model. He finds that those organizations can mitigate free-riding of less moral players because these transactors want to appear of high moral standards in period one in order to exploit their partners in period two. Referring to the unraveling discussion above, this result would be different in a game with one or with infinitely many periods: only if there are \( T > 1 \) periods and \( T \) is known, players with low moral standards have an incentive to behave well in \( T - 1 \).

\(^{47}\)A minor technical difference, which drives differences in results though, is Baron’s assumption that the players can decide about membership \textit{after} they know the identity of their partner. In my paper, due to the concept of
Masten and Prüfer (2011)(MP), just as this paper, use Dixit’s circle framework in an infinitely repeated game. The main technical difference is that the paper at hand is the first to use linear functions instead of exponential functions for the model fundamentals (see footnote 43 for the implications of this difference). The main economic difference is that MP study the scope of cooperation supported by social networks and public courts. The paper at hand builds on the social network part of MP but models two associations with very different characteristics than social networks on top. Given that all enforcement institutions—networks, information intermediaries, arbitration associations, and courts—are modeled in the same framework, the two papers are complements but their results can be easily compared. See Section 5 for this comparison.

**Equilibrium uniqueness and optimality:** Dixit (2003b:1302) writes: “As usual in such games, there is a multiplicity of equilibria, each sustained by its own expectations. But I shall give this system its best shot by looking at the best possible $X$”. I follow a similar approach to study the (maximum) scope of cooperation under which associations assume information intermediary or arbitration functions in an equilibrium. As is clear from the quotation, introducing behavioral types would not solve the multiple equilibrium issue.

Welfare is created by the support of mutually honest trade in this model. Thus an organization’s performance is measured by the scope of cooperation it supports. Given the two games studied here, and the fact that the enforcement technologies available to both types of associations are limited to ostracism—apart from $\lambda$, which proxies some link to state enforcement/coercion of the arbitration association—it is impossible to construct an equilibrium that creates a scope of cooperation higher than $X^*_I$ or $X^*_A$. See also the discussion about associations and courts in Section 5.

**Strategic reporting and imperfect trustworthiness of messages:** Consider the following variant of our model. After the central transaction each player $i$ endogenously chooses the stationary equilibrium players are de facto asked to join an association forever, or never, independent of a single partner’s characteristics.

48 Consider, for instance, the arbitration association model with prior beliefs $\sigma_{t+1} = 1$. This could drive up membership in $t = 1$ because the expected benefits from membership are high. Depending on parameter realizations, however, this may be no (stationary) equilibrium because $F$ may be higher for well connected players than $B_A$. Hence, these players would not join the association again in $t + 1$, thereby ruling out $\sigma_{t+1} = 1$ as an equilibrium belief.
content of the message sent via the social network about his partner’s behavior and about his own behavior to the social network. As truthful self-reporting of an actual defector leads to an expected payoff loss in the subsequent period, given everybody plays the I/A-strategy, it is dominant for every player to claim that he cooperated, independent of true actions. If player \( y \) receives conflicting messages from the period \( t \)-partners \( i \) and \( x \) about one partner’s behavior, assume that \( y \) will trust the sender located closer to him on the circle.\(^{49}\) Then \( L(\kappa_x, \cdot) \) is reduced to
\[
L_{\text{strat}}(\kappa_x, \cdot) = \delta \left[ \int_{X/2}^{X} (1 - Y_1)\kappa_x(1 - (X - Y_1))h(1 + \theta Y_1)dY_1 + \int_{X}^{1} (1 - Y_2)\kappa_x(1 - (Y_2 - X))h(1 + \theta Y_2)dY_2 \\
+ \int_{1 - X/2}^{1} (1 - Y_3)\kappa_x(1 - (2 - X - Y_3))h(1 + \theta Y_3)dY_3 \right]
\]
\[
= \frac{\delta \kappa_x h}{96} \left[ 8(4 + \theta) + X(16\theta + X(32 - (12 - 11X)\theta) - 12(4 + \theta)) \right] 
\]  

\( L(\kappa_x, \cdot) < L_{\text{strat}}(\kappa_x, \cdot) \) because, if \( i \) defects, fewer other players will believe in victim \( x \)'s message and not interact with \( i \) in period \( t + 1 \) given they are matched with \( i \). Therefore, \( i \) will be less deterred to defect. In a similar fashion, we can express \( J_{\text{strat}}(\kappa_x, \cdot) \) and \( \tau_{p^{*}_{\text{strat}}} \). As a consequence, \( V_{I,\text{strat}} < V_I \) and \( V_{A,\text{strat}} < V_A \). Crucially, however, both values of cooperation still decrease in \( X \) (see Lemmas 1 and 3). Hence, the remaining results of the model do not change.

5 Results, Explanations, and Hypotheses

5.1 Theory: associations and other governance institutions

Based on their empirical findings, Johnson et al. (2002:252) state, “Trade associations providing arbitration services may perform a similar role to the courts. Trade associations may have an additional effect as well, through information services that they provide their members.” If a player is an association member, the value of cooperation for his matched partner is larger than the value with enforcement by social networks only (Lemmas 1 and 3). The intuition is

\(^{49}\)This assumption can be justified by empirical findings showing that, when individuals are socially closer, both trust and trustworthiness rise. See Glaeser et al. (2000) and further literature cited therein. The assumption is intuitive, for instance, by resorting to our interpretation of social distance as the degree of shared knowledge. Then players located close to each other have a high overlap of knowledge sets. Thus, they could check their neighbors’ statements more easily because they understand them better and can detect logical loopholes more easily than the statements of players with very different knowledge sets. Think about communication problems across academic fields or even disciplines.
that centralized organizations such as associations spread news—in the case of an arbitration association even qualified news—about opportunist behavior in the economy and thereby strengthen the collective enforcement mechanism of social networks.

Masten and Prüfer (2011) show that enforcement by social networks and courts is complementary: whereas social networks support cooperation in low-value/short-distance transactions, courts support cooperation in high-value/long-distance transactions. Comparing the scopes of cooperation, this implies that associations are a hybrid between social networks and courts:

\[ X_{NW}^* < \max\{X_I^*, X_A^*\} < X_{Court}^* \]  

(27)

The left inequality builds on Lemmas 1 and 3, the max-condition depends on Proposition 8. The right inequality is rooted in the fact that courts have full access to the coercive powers of the state. Given that they are sufficiently competent, they can enforce transactions with very high value, which coincides with long-distance transactions in this model. In contrast, all (lawful) private organizations, including associations offering arbitration services, are restricted by the economic requirement that the maximum damage payment they decide has to be self-enforcing. If it is too high, convicted players will refuse to pay it.

When should we expect to observe which kind of association? First, given that both types of associations modeled here support cooperation by coordinating a boycott of defectors, it is ineffective for a single association to offer both services, given that the costs of operating them would add up. Comparing both functions from the perspective of a single player, \( i \), the information repository function dominates the arbitration function if \( B_I > B_A \). This boils down to a comparison of the scopes of cooperation and the associated costs, \( f \) and \( F \). Since costs are independent of the players’ individual characteristics, all players unanimously prefer the one or the other association function, depending on Proposition 8.\(^{50}\) Therefore, we can expect associations to adjust the function offered quickly if the parameters of the environment change.

What are the cross effects if several institutions exist? Both social networks and associations rely on ostracism as an enforcement tool. Therefore, as demonstrated above for the case where social networks exist and associations are offered on top, the scope of cooperation strictly increases if an additional institution is available to the players and affordable. This does not hold with respect to communities (including social networks and associations) and courts. As Masten

\(^{50}\)In practice some associations charge different fees according to the revenues of member firms, e.g. in German chambers of commerce. However, it is unclear whether and how revenues are correlated with location or connectedness, the two dimensions of type space in this model.
and Prüfer (2011) demonstrate, whereas the existence of social networks supplements court enforcement, the existence of courts diminishes the effectiveness of social network enforcement. The intuition of the latter effect is that, if court enforcement is available on top of network enforcement, some traders who would have otherwise refused to trade with a defector next period will trade because they know that the court protects them from being cheated. Therefore, the expected network sanction in the next period is lower than it otherwise would be, which diminishes the players’ incentives to cooperate because of fear of ostracism in the current period. Given that associations also mainly rely on ostracism, they suffer from the same crowding out effect if filing a suit with the courts is an option available to the players.

Importantly, existence of an association does not only benefit its members: if a nonmember’s partner joins an association, the nonmember’s individual scope of cooperation increases because now defection is more costly to him (Propositions 1 and 5). This improves the nonmember’s ability to commit to cooperative behavior and, in expectation, increases the scope of cooperation of the entire partnership (Propositions 2 and 6).

5.2 Empirics: explaining puzzles and constructing hypotheses

A key result of this model is that the additional value of association membership decreases if transactors are well connected informally. This result leads to the empirically testable hypothesis that we should observe less associations in small and dense economies such as villages or clans, where most players know each other. The hypothesis is confirmed by Pyle (2005), who reports that business associations are perceived less valuable in local trade in Eastern Europe. Similarly, Casari (2007) finds that informal, decentralized institutions (social networks) were more likely to be used than formal, centralized institutions (so-called “chapters”) by communities that were remote and small in the medieval Italian Alps. The intuition is that enforcement by associations and social networks are substitutes, which work by the same mechanism: ruining a defector’s reputation and, thereby, his business opportunities in the future. If information exchange via social networks is already very effective (high average $\kappa_i$), the additional share of the population that can be informed by an association decreases.

In this model, if $\theta$ decreases, the potential value of any transaction, $a(1 + \theta X)$, decreases. Therefore, $\theta$ can be interpreted as a proxy for the level of competition: the lower $\theta$ is, the higher is the degree of competition.\textsuperscript{51} If $\theta$ decreases, the value of association membership also decreases

\textsuperscript{51}In any imperfectly competitive market with downward-sloping demand, if competition gets more intense, the
(Lemmas 1 and 3). The intuition is that, if $\theta$ is small, the expected damage that a defector who is reported to the association suffers from by losing trade in the next period is relatively low. Consequently, the deterrence effect of joining an association against would-be defectors is rather limited, which explains why fewer members are willing to pay the membership fee. Rephrased, this model suggests the hypothesis that more intense competition leads to less (closely organized) associations. This hypothesis is confirmed by Pyle (2005), who reports that business associations are perceived to be less valuable to members in more competitive industries. Similarly, Grafe and Gelderblom (2010) find that intense competition between medieval mercantile groups and local merchants is associated with lower degrees of control delegation to the groups. Casari (2007) finds that the lower the value of a common resource is (proxied by $\theta$ in our model), the smaller is the potential gain from adopting a formal, centralized institution.

The analysis of endogenous membership decisions has produced two additional testable hypotheses that have not been studied empirically, yet. First, we expect players who are less connected informally to join an association and well connected players not to join.\textsuperscript{52} Second, we expect the average damage payment determined by arbitrators against nonmembers to be higher than the average payment determined against members.\textsuperscript{53} The reason for this difference is not that the arbitrator has a bias against nonmembers but that he has to make sure that members who defected find it worthwhile to pay the judgement \textit{and} the membership fee of the next period. Nonmembers do not have to account for future membership fees and hence would be willing to pay an even higher damage payment to avoid ostracism by the association.\textsuperscript{54}

52In empirical work, individual connectedness could be proxied by the number of years an individual has been working in a certain industry or, on the firm level, by the number of years a firm has been active, controlling for total revenues. Tests should take care of the fact that long-lasting associations may exhibit unobserved internal dynamics that can discriminate against newcomers. Hence, the cleanest test would be to study who joins a newly founded association.

53Bernstein (2001:1727) documents by-laws of a U.S. cotton industry association which explicitly handle arbitration cases involving members and nonmembers.

54For an empirical test, it would be necessary to control for the severity of a transgression, which may be
Another field where the theoretical insights could be applied is the interaction of association membership and political competition. Based on survey data from Russia, Pyle (2011:27) wonders: “it is less than clear why we would not observe higher membership rates in associations if indeed they offer services that secure property rights. Our surveys suggest, after all, that the associations are open and nonexclusive. In some settings, as shown previously, macrolevel political institutions may produce similar outcomes. But to point this out does not clarify why membership rates are not higher in regions with limited political competition. Perhaps the benefits of membership are not widely recognized or understood.” Propositions 3 and 7 suggest an alternative explanation: membership is more attractive for less connected players. If in the regions with less political competition potential members are highly connected to each other on average, for instance because these regions may be located rather remotely, highly connected players may also be expected to be well connected to the autocratic ruler and hence do not need to join an association to protect their property rights. If this holds, the less connected players may share the belief that membership is only attractive for few other players. The association and its members would be trapped in a low $\kappa_I^*$ or $\kappa_{A,1}^*$-equilibrium and remain small.

The model also generates the insight that a larger variance of the distribution of connectedness, $Z$, leads to less network supported cooperation on average. The intuition is that, if $Z$ is more dispersed, the probability that one player in a partnership has a low connectedness ($\kappa_i$) is high. By Propositions 2 and 6, this decreases the binding scope of cooperation of the entire partnership. In turn, it implies that the additional value of formal associations is relatively large. Hence, we obtain the testable hypothesis that in dynamic industries with many entrants (e.g. in the life sciences or Internet technologies/services) many players would join an association, whereas in less dynamic industries with rather homogeneous levels of connectedness the share of association members should be smaller.55

55In empirical work, it would be helpful to obtain data on the main functions of a given association. In the case of dynamic industries, the model predicts that members yield high benefits from association functions that support cooperation in bilateral dealings/honest trade. Contrary, in stable industries with a few long-term players, it might also be that a high share of industry participants are association members—but for collusive reasons rather than to promote bilateral trade, which is already secured by informal ties.
6 Conclusion

Business associations—private, formal, noncommercial organizations designed to promote the common business interests of their members—have assumed many functions throughout history and all over the world. Two functions that are particularly valuable for association members if the public legal system is ineffective or public authorities are even corrupt and exploitative are serving as a repository for member-supplied information about the conduct of their business partners and offering arbitration in the case of disputes involving their members.

Recent empirical research into the functioning of modern business associations and merchant guilds and related institutions in the European middle ages has produced fascinating new results, which stand to be explained by theory. In this paper, we have constructed a theoretical framework that connects the organizational and institutional features of formal and informal business organization with socioeconomic distance. This approach has produced explanations for several empirical puzzles, put forward novel testable hypotheses, and related business associations to alternative governance institutions.

More research, both empirical and theoretical, is needed. In the models of this paper, two partners play the central transaction and a multitude of other players, which can be coordinated by a centralized organization, or not, rests in the back, ready to support the auxiliary transaction (punishment of defectors). Going a step further, several real-world applications introduce a second layer of institutions, where associations are the players in the central transaction and a higher-order association exists to mediate conflicts between the players and coordinate collective action. In the European middle ages, the German Hanse, which was an association of cities dominated by merchant guilds, adopted such a role (Ogilvie, 2011:20). In modern times, the New York (Arbitration) Convention supports the enforcement of arbitration judgments by foreign courts, an important institution of international trade.56 Both applications can be informed by the findings of this paper, but it is unclear how the results change if different governance structures among the players exist, for instance if the second-layer association is dominated by a subset of the players.

An important subject of future research is the interdependence of business associations and the level of political competition. Contributing to this issue, Olson (2000), Frye (2006), and Pyle (2011) represent intriguing studies into the overlap of organizational economics, economic governance, and political economy. A related normative question is, under which circumstances

56See Leeson (2008a) and http://www.newyorkconvention.org/.

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might associations increase the surplus of the players involved and when might there be negative spillover effects on nonmembers that outweigh the positive effects?\textsuperscript{57}

Given that the central transaction in this model is a Prisoner’s Dilemma, the general framework is not restricted to business applications. The results can be applied to other situations as well, as long as the players are connected to each other by some kind of decentralized communications mechanism, and mutual cooperation is efficient but no equilibrium in one-shot interactions. Many social enterprises, which accept members and support the mitigation of societal problems, serve as one example, international organizations that are set up to foster cooperation among their members without a supranational coercive enforcer are another one.\textsuperscript{58} Depending on the specific situation, either the information intermediary or the arbitrator model may be applicable.

Finally, studying all institutions in the typology of commitment mechanisms of Masten and Prüfer (2011) has yet to be completed. Specifically, the analysis of models of “first-party systems” (Dixit, 2009:10), that is, institutions such as social norms that support cooperation by directly affecting the pro-social preferences of players, has just begun.\textsuperscript{59} In addition, those private-ordering institutions that share some characteristics with associations and some with public courts, for instance specialized courts (family and juvenile courts, etc.) and criminal organizations (gangs, Mafia) are understudied. These institutions are both community-embedded, that is, they send and receive informal information about the conduct of players, and have access to coercion to enforce their judgements. We have acknowledged that link by including the parameter $\lambda$ in the arbitration association model above but this is just a starting point. Given the apparent combination of the best of both worlds (of private and public enforcement), why are such “quasi-courts” not more dominant in the economy?

\textsuperscript{57}The former approach is underlined by the private ordering literature, including this paper. The latter approach is followed by several scholars from industrial organization, public choice, and law & economics.

\textsuperscript{58}See http://www.socialenterprise.org.uk/about/about-social-enterprise for an example of the first and http://www.wto.org/ for an example of the second category.

\textsuperscript{59}Tabellini (2008), Dixit (2009), and Baron (2010) are applaudable pioneering papers.
Appendix

A Proofs

A.1 Proof of Proposition 1

To prove Proposition 1 we have to show two characteristics: (i) Monotonicity of $V_{I,i}$ in $X$ and (ii) the conditions under which $V_{I,i} \geq 0$.

On (i): See Lemma 1.(iii). Differentiating $V_{I,i}$ with respect to $X$ shows that

$$\frac{\partial V_{I,i}}{\partial X} = -2\delta h \kappa x (X - X^2)(1 - M_x \sigma_{t+1}) - (w - h) \theta < 0$$

(A.1)

It follows that $V_{I,i}$ is monotonically decreasing in $X$.

On (ii): Setting (9) equal to zero shows that $V_{I,i} \geq 0$ if, and only if:

$$w \leq h + \frac{h\delta}{6(1 + \theta X)} (2(3 + \theta)M_x \sigma_{t+1} + \kappa x (4 + 4X^3 - 6X^2 + \theta)(1 - M_x \sigma_{t+1}))$$

(A.2)

The RHS of (A.2) is strictly positive. Hence, for all supported parameter values (A.2) specifies unambiguously whether $V_{I,i} \geq 0$, or not.

Define $X_{I,i}^* \equiv \{X|V_{I,i} = 0\}$. Because $\frac{\partial V_{I,i}}{\partial X} < 0$, $X_{I,i}^*$ characterizes an upper bound on $i$’s incentive to cooperate in $X$-space.

I have to distinguish among three subcases. First, if (A.2) does not hold at $X = 0$, $V_{I,i} < 0$ for all $X$. Hence, player $i$ has no incentive to cooperate; $X_{I,i}^* < 0$. Second, if (A.2) holds at $X = 1$, $V_{I,i} \geq 0$ for all $X$. Hence, player $i$ has an incentive to cooperate always; $X_{I,i}^* \geq 1$. Third, if (A.2) holds at $X = 0$ but does not hold at $X = 1$, there is a unique $X_{I,i}^* \in [0,1)$, such that $V_{I,i}(X \leq X_{I,i}^*) \geq 0 > V_{I,i}(X > X_{I,i}^*)$. Hence, player $i$ has an incentive to cooperate for $X \leq X_{I,i}^*$ but not for $X > X_{I,i}^*$. Q.E.D.

A.2 Derivation of the probability $\beta_I$

By definition:

$$\beta_I \equiv \text{prob}(X_{I,i}^*(\kappa_x, M_x)) \quad > \quad X_{I,i}^*(\kappa_i, M_i = 0))$$

(A.3)

$$\Leftrightarrow \text{prob}(V_{I,i}^*(\kappa_x, M_x)) \quad > \quad V_{I,i}^*(\kappa_i, M_i = 0))$$

(A.4)

Given the assumptions on functional forms made above, it is possible to specify $X_{I,i}^*$ explicitly. However, the expression is very complex and hard to interpret. Therefore, $X_{I,i}^*$ is only used implicitly here.
From the perspective of $i$ at stage 0, $\kappa_x$ and $M_x$ are unknown. Hence $i$ uses $E(\kappa_x)$ and $E(M_x) = \sigma_t$: the expected equilibrium share of members in the population must correspond to the realized share. Moreover, in a stationary equilibrium, $\sigma_t = \sigma_{t+1}$. Hence, we can rewrite (A.4) as:

$$\text{prob}(L(\kappa_i) < L(E(\kappa_x)) + \sigma_t J(E(\kappa_x)))$$  \hspace{1cm} (A.5)

After substituting (3) and (4) in (A.5) and re-arranging we obtain:

$$\beta_I = \text{prob}(\kappa_i < E(\kappa_x) + \sigma_t^2(\frac{2(3 + \theta)}{4 + 4X^3 - 6X^2 + \theta} - E(\kappa_x)))$$  \hspace{1cm} (A.6)

From (A.6), we conclude that, for $\sigma_t = 0$, $\beta_I = \text{prob}(\kappa_i < \kappa_x) = (1 - Z(\kappa_i))$. For $\sigma_t > 0$, (A.6) shows that $\frac{\partial \beta_I}{\partial \kappa_i} < 0$ and that $\frac{\partial \beta_I}{\partial \sigma_t} > 0$.

A.3 Proof of Proposition 3

(i): Consider equation (11). First, recall from Section A.2 that $\frac{\partial \beta_I}{\partial \kappa_i} < 0$. Second, excluding the effect of $J$ on $X^*_I$ for a second, and recalling that $L$ and $J$ are additively separable, $\frac{\partial X^*_I}{\partial \kappa_i} = \frac{\partial X^*_I}{\partial \kappa_x}$: both boundaries of the interval defined in $B_I$ shift by the same rate if $\kappa_i$ is changed. Now, let us include the effect of $J$ on $X^*_I$. Recall that Lemma 1.(ii) states that $\frac{\partial J(\kappa_i)}{\partial \kappa_x} < 0$. Then, by the logic explained below Lemma 1, $\frac{\partial J(\kappa_i)}{\partial \kappa_i} < 0$. Hence, the interval $(X^*_I - X^*_NW)$ is decreasing in $\kappa_i$. Because the association membership fee $f$ is nondiscriminatory with respect to the connectedness of individuals, by definition, it follows that $B_I$ is monotonically decreasing in $\kappa_i$: $\frac{\partial B_I}{\partial \kappa_i} < 0$. As long as $f$ is not prohibitively high—and hence $B_I \geq 0$ for some players—the marginal member, who has connectedness $\kappa^*_I$, is indifferent between joining and not joining the association: $B_I(\kappa_i = \kappa^*_I) = 0$. It follows that only players with connectedness $\kappa_i \in [0, \kappa^*_I]$ join the association.

(ii): Equilibrium characterization: In a stationary equilibrium of a repeated game, $\sigma_t = \sigma_{t+1}$, by definition. The realized membership share in the total population must be the same in every period and it must correspond to the players’ beliefs about the membership share in the next period. As shown in Section A.2, $\beta_I$ increases in $\sigma_{t+1}$. Moreover, because $J$ increases in $\sigma_{t+1}$, $X^*_I$ also increases in $\sigma_{t+1}$. Therefore, $B_I$ also increases in $\sigma_{t+1}$: from any player’s perspective, if the expected membership size is larger, the value of being able to inform all members about one partner’s behavior increases monotonically.

The fact that $\frac{\partial B_I}{\partial \sigma_{t+1}} > 0$ but $\frac{\partial B_I}{\partial \kappa_i} < 0$ creates a fixed-point problem. An equilibrium is characterized by a player’s level of connectedness $\kappa^*_I = \sigma_{t+1}$, where the player’s belief about

$\text{Note that } \left(\frac{2(3 + \theta)}{4 + 4X^3 - 6X^2 + \theta} - \kappa_x\right) > 0 \forall \kappa_x.$
the membership-size corresponds to his own connectedness and he is indifferent between joining and not joining the association, that is where \( B_I(\kappa^*_I) = 0 \). Then, for every belief \( \sigma_{t+1} \), all players with connectedness \( \kappa_i \leq \kappa^*_I = \sigma_{t+1} \) will join the association, whereas all players with \( \kappa_i > \kappa^*_I = \sigma_{t+1} \) will not join, which confirms the belief that \( \sigma_{t+1} = \kappa^*_I \).

Existence proof: Substituting \( \kappa^*_I \) for \( \kappa_x \) and \( \sigma_{t+1} \) in (4) yields:

\[
J(\kappa^*_I) = \frac{\delta h \kappa^*_I}{6} \left( 2(3 + \theta) - (4 + 4X^3 - 6X^2 + \theta)\kappa^*_I \right)
\]

(A.7)

As the distance \( X \) is unknown to player \( i \) at stage 0, we can use (1) and substitute \( E(X) = 0.5 \) for \( X \). Then we obtain:

\[
\frac{\partial J(\kappa^*_I)}{\partial \kappa^*_I} = \frac{\delta h}{3} (3 + \theta)(1 - \kappa^*_I) > 0
\]

(A.8)

Hence, \( X^*_I \) is also strictly increasing in \( \kappa^*_I \), the interval \( (X^*_I - X^*_NW) \) is strictly increasing in \( \kappa^*_I \), and \( B_I(\kappa^*_I) \) is strictly increasing in \( \kappa^*_I \), for \( \kappa^*_I < 1 \). Moreover, because \( J(\kappa^*_I = 0) = 0 \), \( B_I(\kappa^*_I = 0) = -f \).

Given that \( f > 0 \) but sufficiently small, there is a unique level of \( \kappa^*_I \) for which \( B_I(\kappa^*_I) = 0 \). Hence, there are two possible equilibrium membership sizes, characterized by \( \kappa^*_I(f) \):\(^{62}\)

1. For small levels of \( f \), there is a unique \( \kappa^*_I(f) \).

2. For large levels of \( f \), there is no \( \kappa^*_I(f) \) for which \( B_I(\kappa^*_I) = 0 \). Hence, the association has no members and \( \kappa^*_I(f) = 0 \). Q.E.D.

Illustration: Figure 3 illustrates the dependence of \( B_I \) from \( \kappa_i \) and \( \sigma_{t+1} \). As is visible from equation (4), which shapes \( B_I \), \( J = 0 \) if the players hold the belief \( \sigma_{t+1} = 0 \), independent of \( \kappa_i \). Hence \( B_I(\sigma_{t+1} = 0) = -f \). For all beliefs \( \sigma_{t+1} > 0 \), \( B_I \) is monotonically decreasing in \( \kappa_i \). If \( \sigma_{t+1} = 1 \), \( J > 0 \) for all \( \kappa_i \). If \( f \) is low, this can even lead to incentives for all players to join the association. For higher \( f \) or lower \( \sigma_{t+1} \), there is a unique level of \( \kappa_i \) where \( B_I(\kappa_i = \sigma_{t+1}) = 0 \).

Figure 4 exemplifies the specification of \( \kappa^*_I \). The gross benefit from membership, which follows \( J \), is zero if \( \kappa^*_I = 0 \) (here everybody expects that nobody joins the association; hence nobody joins). \( J(\kappa^*_I) \) is increasing and concave in \( \kappa^*_I \); see (A.8). Therefore, for \( f > 0 \), the net benefit from association membership, \( B_I(\kappa^*_I) \), is negative at \( \kappa^*_I = 0 \). The level of \( \kappa_i \) for which \( B_I(\kappa_i = \sigma_{t+1} = \kappa^*_I) = 0 \) determines the equilibrium membership size \( \kappa^*_I \).

\(^{62}\)Note that this result can differ for different distributions of matching probabilities. If (1) is such that \( E(X) \to 0 \), \( \frac{\partial J(\kappa^*_I)}{\partial \kappa^*_I} > 0 \) for \( \kappa^*_I \geq \frac{3 + \theta}{4 + 4X^3 - 6X^2 + \theta} \). \( J(\kappa^*_I) \) would be hump-shaped in \( \kappa^*_I \). Hence \( B_I(\kappa^*_I) \) would be hump-shaped in \( \kappa^*_I \). In addition to the results on membership sizes with small and large levels of \( f \), there would be a third result, for intermediate levels of \( f \), with two equilibrium membership sizes, \( \kappa^*_I,1 \) and \( \kappa^*_I,2 \).
A.4 Proof of Proposition 7

Preliminaries: As the proof is strongly related to the proof of Proposition 3, we focus on the main differences between the two cases. We first have to specify the dependence of \( \beta_A \), the probability with which \( M_i \) has an effect on \( X^*_A \). By definition:

\[
\beta_A = \text{prob}(X^*_{A,i}(L(\kappa_x), M_x \tau p^*) > X^*_{A,x}(L(\kappa_i), M_i \tau p^*|M_i = 0)) \quad (A.9)
\]

\[
\Leftrightarrow (V^*_{A,i}(L(\kappa_x), M_x \tau p^*) > V^*_{A,x}(L(\kappa_i), M_i \tau p^*|M_i = 0)) \quad (A.10)
\]

\[
\Leftrightarrow \text{prob}(L(\kappa_i) < L(E(\kappa_x)) + M_x \tau \sigma_{t+1} (\delta G - L(E(\kappa_x)))) \quad (A.11)
\]

Recall that \( E(M_x) = \sigma_t \) and \( \sigma_t = \sigma_{t+1} \). Then, using (5) we can rewrite (A.11):

\[
\beta_A = \text{prob}(\kappa_i < \kappa_x (1 - \frac{\sigma_i^2 \tau}{1 - \lambda}) + \frac{\sigma_i^2 \tau}{(1 - \lambda)} \frac{2(3 + \theta)}{(4 + 4X^3 - 6X^2 + \theta)}) \quad (A.12)
\]

It follows that, for \( \sigma_t = 0 \), \( \beta_A = \text{prob}(\kappa_i < \kappa_x) = (1 - Z(\kappa_i)) \). For \( \sigma_t > 0 \), the RHS of (A.12) is positive for all supported parameter values. Hence, \( \frac{\partial \beta_A}{\partial \sigma_t} < 0 \). Moreover, \( \frac{\partial \beta_A}{\partial \sigma_t} > 0 \).

Proof: (i) The proof of Proposition 3.(i) applies if we substitute \( X^*_{A,i} \) for \( X^*_i \), and \( \tau p^*(\cdot) \) for \( J(\cdot) \), \( F \) for \( f \), and \( B_A \) for \( B_i \). As a result, \( B_A \) monotonically increases in the belief \( \sigma_{t+1} \) and monotonically decreases in \( \kappa_i \). As a consequence, only players with low connectedness, \( \kappa_i \in [0, \kappa^*_A] \), join the association, where \( B(\kappa_i = \kappa^*_A) = 0 \).

(ii): Substituting \( \kappa^*_A \) for \( \kappa_x \) and for \( \sigma_{t+1} \) in (12) and using (5) yields:

\[
\tau p^*(\kappa^*_A) = \frac{\tau J(\kappa^*_A) - \delta M_i F}{1 - \lambda} = \tau \delta \left( \frac{1}{\delta} h \kappa^*_A (2(3 + \theta) - \kappa^*_A (4 + 4X^3 - 6X^2 + \theta)) - M_i F \right) \quad (A.13)
\]

We have \( \tau p^*(\kappa^*_A) < 0 \) for \( \kappa^*_A \rightarrow 0 \). Hence, for very low \( \kappa^*_A \), arbitration association membership does not create any value (but would still cost the few members \( F \)). In contrast, \( \tau p^*(\kappa^*_A = 1) > 0 \) for \( F \) sufficiently low. Taking \( \frac{\partial \tau p^*(\kappa^*_A)}{\partial \kappa^*_A} \) shows that \( \tau p^*(\kappa^*_A) \) is hump-shaped in \( \kappa^*_A \). Moreover, according to (14), \( p = \frac{x}{\tau} > 0 \). Hence, the set of \( \kappa^*_A \)-values for which \( p^* \) can sustain an equilibrium based on the A-Strategy is censored at some \( \kappa > 0 \) and some \( \kappa < 1; \kappa \leq \bar{\kappa} \). It follows that \( X^*_A \) is also hump-shaped and censored in \( \kappa^*_A \), the interval \( (X^*_A - X^*_NW) \) is hump-shaped and censored in \( \kappa^*_A \), and \( B_A(\kappa^*_A) \) is hump-shaped and censored in \( \kappa^*_AA \). Figure 5 displays this case. By the intermediate value theorem, it follows that:

1. For small \( F \), there are two \( \kappa^*_A(F) \), denoted by \( \kappa^*_A,1(F) \) and \( \kappa^*_A,2(F) \), \( \kappa^*_A,1(F) < \kappa^*_A,2(F) \).

\(^{63}\)See footnote 61.
2. For large \( F \), there is no \( \kappa^*_A(F) > 0 \) for which \( B_A(\kappa_A) = 0 \). Hence, the association has no members and \( \kappa^*_A(F) = 0 \).

(iii): Membership benefits increase in \( \tau p^*(\kappa^*_A) \). Analyzing (A.13) shows that \( \frac{\partial \tau p^*(\kappa^*_A)}{\partial \kappa_A} > 0, \frac{\partial \tau p^*(\kappa^*_A)}{\partial \lambda} > 0, \frac{\partial \tau p^*(\kappa^*_A)}{\partial \theta} > 0, \frac{\partial \tau p^*(\kappa^*_A)}{\partial \theta} > 0 \). Q.E.D.

A.5 Proof of Proposition 8

Without loss of generality, assume \( X_I^* = X_{I,i}^*(\kappa_x, \cdot) \) and \( X_A^* = X_{A,i}^*(\kappa_x, \cdot) \). Then \( X_I^* < X_A^* \) if, and only if, \( V_I^* < V_A^* \). Assuming \( M_x = 1 \) (otherwise the social network case prevails) the comparison resembles:

\[
J < \frac{\tau(\sigma_{t+1}(\delta G - L) - M_i \delta F)}{1 - \lambda}
\]  

(A.14)

Substituting (5) into (A.14) and differentiating cases, we get that \( X_I^* < X_A^* \) iff:

\[
\tau + \lambda > 1 \quad \text{for} \quad M_i = 0 \]  

(A.15)

\[
F < \frac{J}{\delta \tau}(\tau + \lambda - 1) \quad \text{for} \quad M_i = 1, \]  

(A.16)

Given that the comparison of association functions takes place independent of specific membership decisions, a sufficient condition for \( X_I^* < X_A^* \) is (A.16). Q.E.D.
References


Figure 1: Payoffs in the Prisoner’s Dilemma. Assume $w > h > 0 > d > l$ and $2h > w + l$. 

<table>
<thead>
<tr>
<th>$i / x$</th>
<th>Cooperate</th>
<th>Defect</th>
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<tbody>
<tr>
<td>Cooperate</td>
<td>$h, h$</td>
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<td>Defect</td>
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Figure 2: Possible locations of player $i$’s $t + 1$ match $y$ as compared to $i$’s current match $x$. 
Figure 3: Membership benefit, beliefs, and individual connectedness for intermediate $f$. 
Figure 4: Specification of $\kappa^*_I$ (for small $f$).
Figure 5: Specification of $\kappa_{A,1}^*$ and $\kappa_{A,2}^*$ (for small $F$).