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Certification and minimum quality standards when some consumers are uninformed[*]

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Abstract

We compare certification to a minimum quality standard (MQS) policy in a duopolistic industry where firms incur quality-dependent fixed costs and only a fraction of consumers observes the quality of the offered goods. Compared to the unregulated outcome, both profits and social welfare would increase if firms could commit to producing a higher quality. An MQS restricts the firms' quality choice and leads to less differentiated goods. This fuels competition and may therefore deter entry. A certification policy, which awards firms with a certificate if the quality of their products exceeds some threshold, does not restrict the firms' quality choice. In contrast to an MQS, certification may lead to more differentiated goods and higher profits. We find that firms are willing to comply with an ambitious certification standard if the share of informed consumers is small. In that case, certification is more effective from a welfare perspective than a minimum quality standard because it is less detrimental to entry.

Keywords: Certification, minimum quality standard, unobservable quality, policy intervention
JEL classification: L15; L13; L51; D82

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1 Introduction

When consumers are ill-informed about the quality of a product, firms may provide less than the socially optimal level of quality.[1] Policy makers have resorted to a variety of instruments to address this problem. Two important such instruments are minimum quality standards and certification. Minimum quality standards (MQS) prevent firms from selling goods whose quality is below a predefined standard. Examples include safety standards as well as occupational licensing for professional services. Certification is a process whereby the government or an independent third party verifies if a product fulfills certain criteria. Products that satisfy these criteria obtain a certificate that is visible to consumers. Certification is used to document, among other things, the security of cars (crash tests, rollover ratings) and the origin of food and wood (organic food certificates, FSC label for timber from sustainable forestry).[2]

It is natural to ask which of these instruments is likely to be most effective in a given context. To investigate this question, we introduce uninformed consumers into the well-known model of Ronnen (1991). We show that certification can outperform minimum quality standards, in the sense that it induces firms to produce qualities that are closer to the socially optimal levels and thus leads to greater welfare. The superior performance of certification arises precisely when the underlying problem these instruments are intended to solve is most severe, namely, when the share of uninformed consumers is large. By contrast, when the share of uninformed consumers is small, the rationale for certification disappears. Minimum quality standards, however, may continue to play a useful role, as Ronnen (1991) established for the limiting case in which all consumers are informed.

Ronnen (1991) studies a duopoly model in which firms play a two-stage game. In stage one, firms decide whether to enter the market and invest in the quality of their product. In stage two, they observe each other's entry and quality choices, and then compete in prices. All consumers observe quality; they have unit demand and differ in their taste for quality. The equilibrium is one of vertical differentiation, with one firm selling high quality at a high price and the other firm selling low quality at a low price. We depart from Ronnen's setup

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[1]Firms respond to this problem in various ways, such as by promising warranties, signaling quality through prices, or developing a reputation for high quality. Research on these market instruments has shown, however, that they may alleviate the problem but rarely eliminate it. See, e.g., Gal-Or (1989) for warranties and Daughety and Reinganum (2008a,b) for price signaling.

[2]A third instrument that is important in practice is mandatory disclosure, whereby firms are obliged to publish standardized and comparable information about their products. The distinction between mandatory disclosure and certification is not always clear-cut. There are two main features that distinguish the two: first, disclosure usually concerns technical information which consumers may not always be able to understand and translate into an assessment of quality, whereas certification usually involves a grading system that is easy to grasp; second, disclosure can in principle give consumers an idea of a product's precise position on the quality scale, whereas certification is coarse (in the sense of dividing the scale into discrete steps) and thus merely allows consumers to identify the interval to which a product belongs. The coarseness of certification is important for the results in this paper.

by introducing a subset of consumers who do not observe quality.

In this setup, absent government intervention, firms' revenues from uninformed consumers do not directly depend on quality: by assumption, uninformed consumers cannot react to changes in quality. When deciding on their investment in quality, firms thus only take into account the revenues from informed consumers. Nevertheless, the uninformed consumers correctly anticipate the firms' quality choices, so that in equilibrium the firms' quality investments determine their revenue from *both* types of consumers. The firms would like to commit to producing higher quality, but have no credible way of doing so. It follows that quality levels are below those that would be optimal for the firms. While the equilibrium continues to feature a vertical-differentiation outcome, both firms' qualities are lower than under full information. We then study the effect of the two policy instruments considered above. Our use of the Ronnen framework, in which an MQS is known to raise welfare when all consumers are informed, gives an MQS its best shot.[3]

Certification offers firms the possibility to commit to a higher level of quality. When the certification threshold is set above the highest quality level that would be offered in an unregulated market, it may induce both firms to raise their quality. With the certificate, a firm can demonstrate that the quality of its product meets at least the certification standard. Therefore, if the standard is not too demanding, one firm will match this level. The other firm differentiates its product by offering low quality, but compared to the unregulated equilibrium, it can raise its quality without compromising the degree of differentiation between the goods (and thus without intensifying price competition). As we show, there exists a certification standard that increases both consumer surplus and industry profit.

When setting the certification standard, the government has to ensure that obtaining the certificate is attractive for producers. The certification threshold has to be set such that the required investment in quality is not too high compared to the expected revenues. The decision to certify a product crucially depends on the profit that a firm expects to earn from selling its product without the certificate. As argued above, the quality level the firms can credibly produce without the certificate (and the resulting profit they can secure) is low when the fraction of informed consumers is small. This leads to the surprising result that if quality is more difficult to discern, a firm is more reliant on certification and tends to comply with a higher certification standard.

The government can exploit this informational problem. Through the strategic use of certification, it can achieve a higher maximum quality than in case the quality is perfectly observable.[4] In our framework, this is welfare-enhancing because even full-information qual-

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[3]A priori, the welfare effect of an MQS is ambiguous; see Leland (1979) and Shapiro (1983).

[4]Indeed, concerning the FSC certificate mentioned above, the timber industry's attempts to install another certification system with a relatively soft standard provide evidence that some firms would prefer less restrictive

ities are lower than the socially optimal ones. The source of this underprovision of quality is twofold. It stems both from the fact that the marginal consumer has a lower taste for quality than the average consumer (see also Spence, 1975), and from the firms' desire to offer differentiated products so as to alleviate price competition. Importantly, our results also apply to other sources of quality underprovision. For example, if underprovision occurs because of positive externalities (e.g., car safety), the government could similarly raise the quality level by resorting to certification if consumers are not fully informed.

The impact of an MQS differs from that of certification. By restricting the admitted quality range and thus the firms' ability to differentiate their goods, an MQS intensifies price competition and requires higher minimum investments in quality. Therefore, adopting an MQS reduces the firms' profits and may deter them from entering. We show that this problem is particularly severe when quality is difficult to observe, so that the firms' ability to credibly produce high quality is limited. By contrast, our analysis emphasizes that suitable certification does not restrict entry. We thus conclude that suitable certification may improve welfare more than an MQS if only few consumers are informed. If instead almost all consumers can observe the actual quality, firms have no need to rely on certification. The welfare gains from adopting a suitable MQS are then higher.

Related Literature. Our model is closely related to the literature on oligopolistic competition and minimum quality standards in markets for vertically differentiated goods. The insight that vertical product differentiation can be used by oligopolistic firms in order to relax price competition is due to Gabszewicz and Thisse (1979) and Shaked and Sutton (1982).[5] Based on this insight, Ronnen (1991) demonstrates that the government can increase welfare by adopting an MQS if improving a product's quality requires fixed investments but no variable costs.[6] Crampes and Hollander (1995) address the same question when firms incur

certification. For a detailed report, see http://www.fern.org/sites/fern.org/files/media/documents/document_1890_1900.pdf. The observation that these attempts have been ineffective so far exemplifies that firms often lack the power to install certification in which consumers have confidence. This in turn allows the government – or institutions which may credibly enforce certification – to step in and manipulate the qualities in the market by setting suitable certification standards. This may also explain why simple threshold schemes are so popular.

Our analysis is based on the assumption that policy interventions are adopted so as to maximize social welfare. Although we speak of governmental interventions for concreteness, our results also apply to non-governmental institutions whose objective is to maximize social welfare. For example, the FSC is a non-governmental, not-for-profit organization that promotes "environmentally appropriate, socially beneficial, and economically viable management of the world's forests" (see http://www.fsc.org/vision_mission.html). This objective could be interpreted as maximizing social welfare.

[5]Wauthy (1996) endogenizes the market coverage.

[6]The same setup, which was originally inspired by Tirole (1988), has also been analyzed by Choi and Shin (1992). Lehmann-Grube (1997) demonstrates that the high quality provider usually earns higher profits and shows that this result survives when firms choose their quality sequentially. In a slightly modified setup, Motta (1993) compares price and quantity competition.

variable costs for quality and obtain the same qualitative effect of introducing an MQS if the costs of quality are convex enough.[7]

Our analysis is also related to the literature on certification. Assuming perfect commitment and treating the seller's quality as exogenously determined, Lizzeri (1999) studies the profit maximizing policy of a monopolistic and of oligopolistic certifiers. Based on a similar framework, Albano and Lizzeri (2001) endogenize the seller's quality choice. Strausz (2005) as well as Mathis, McAndrews, and Rochet (2009) concentrate on the incentives of certifiers to honestly rate the seller's quality. All of these models focus on the behavior of certifiers and consider a rather simple structure of the market for the rated goods.[8] In contrast, we abstract from problems associated with dishonest certification and focus on how costless certification affects the rated firms' behavior in a more complex competitive environment.

The remainder of this paper is organized as follows. Section 2 sets out the model. Section 3 derives the equilibrium prices for given quality choices. Section 4 solves for the equilibrium qualities and the number of active firms in the absence of any intervention. Section 5 investigates the effects of government intervention on the equilibrium quality choices and characterizes the welfare maximizing certification standard and minimum quality standard. It also compares the impact of an MQS to that of certification. Finally, Section 6 concludes. All proofs are relegated to Appendix A.

2 The model

There are two identical potential entrants to the market. Each firm i can offer a single quality $q_i \in [0, \infty)$. The cost of installing the production technology to produce goods of quality q is $C(q)$, where $C(0) = C'(0) = 0$, $C'(q) > 0$ for $q > 0$, $C'' > 0$, and $\lim_{q \to \infty} C'(q) = \infty$. For technical reasons, we also assume $C''' \geq 0$. Once the production technology is in place, actual production is costless (i.e., there are no variable production costs).

There is a mass one of consumers with unit demand. A consumer who buys a product of quality q at price p has utility $\theta q - p$, where θ is the consumer's taste for quality. We assume that θ is uniformly distributed on $[0, 1]$. Consumers differ in their information about quality. A fraction $\alpha \in (0, 1]$ of consumers, which we refer to as *informed*, observe the quality of the products on offer before deciding whether and from which firm to buy. A fraction $1 - \alpha$ of consumers, which we refer to as *uninformed*, do not observe quality before buying.[9] Since

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[7]Kuhn (2007) shows that an MQS may be detrimental in a setup with variable cost of quality if consumers derive some baseline benefits from the consumption of the good that is independent from its quality. Valletti (2000) demonstrates that a mildly restrictive MQS unambiguously reduces total welfare when firms compete in quantities instead of prices.

[8]Similar to our model, Heyes and Maxwell (2004) consider the effect of MQS and certificates, but do not model the marked for the rated goods explicitly.

[9]We assume for simplicity that whether a consumer is informed or uninformed is independent of θ.

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all uninformed consumers have the same information, they form the same belief $\hat{q}_i$ about the expected quality of the good offered by each firm i. All consumers, whether informed or uninformed, can identify by which firm the goods on offer have been produced (perhaps because goods carry the producer's name).

We consider two instruments of government intervention:[10] certification and a minimum quality standard (MQS). Certification entails that the government publicly announces a threshold q^C. Any firm that installs a quality $q \geq q^C$ obtains the certificate, which is observed by all consumers and firms. The government does not disclose the exact quality of the firms that are awarded the certificate. An MQS entails that the government chooses a threshold q^{MQS} below which firms cannot be active. Only firms with a quality $q \geq q^{MQS}$ are allowed to sell their product to consumers. Thus, the difference between certification and an MQS is that under certification, firms whose quality is below the standard are still allowed to be active, whereas with an MQS they are not.

The timing is as follows (see Figure 1). If a certification or MQS policy is in place, the government announces the respective threshold (q^C or q^{MQS}) before the start of the game. At $t = 0$, firms simultaneously decide whether or not to enter the market. Firms that enter also choose the quality of their production technology. If an MQS is in place, they must choose a quality that weakly exceeds q^{MQS}. If a certification policy is in place, all firms that have installed at least q^C obtain the certificate. Each firm in the market learns whether its competitor has entered but does not observe the competitor's quality. Consumers and firms observe which firms have obtained a certificate. Because at this stage firms have the same information about their competitor as the uninformed consumers, we assume that they form the same belief about quality. At $t = 1$, uninformed consumers enter the market. The active firms simultaneously set first-period prices. Based on the prices and on their beliefs, uninformed consumers decide whether and from which seller to buy. At $t = 2$, informed consumers enter the market. Each firm learns its competitor's quality. Firms simultaneously set second-period prices, and the informed consumers decide whether and from which seller to buy. For simplicity, firms do not discount their profits.

The sequential structure of the game captures the idea that early consumers of an experience good have less information about its quality than late consumers, who can learn from the experience of early consumers (e.g., through word of mouth).[11] We assume that all consumers are short lived and have to make their decision in the period they enter the

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[10] We use the term *government* intervention in order to highlight that these instruments are designed to increase social welfare. We abstract from problems such as commitment and honesty, among other things.

[11] The assumption that firms can change prices but not qualities between periods 1 and 2 can be justified by the fact that price changes can be done quickly while a change of the production technology is usually more time-consuming.

Firms decide on entry, observe entry decisions, invest in quality.

Firms set period-1 prices. Uninformed consumers make purchasing decisions.

Firms observe qualities, set period-2 prices. Informed consumers make purchasing decisions.

$t = 0$ $t = 1$ $t = 2$ time

Figure 1: Timing of the game

market, which is exogenously determined.

If a single firm enters, we use the index M. In case two firms enter, we assign the firm that produces a weakly higher quality the index H and the remaining one the index L so that $q_H \geq q_L$. Accordingly, we will refer to firm H as the high-quality producer and to firm L as the low-quality producer. The equilibrium concept we use is Perfect Bayesian Equilibrium. We focus on pure strategy equilibria and impose the refinement that beliefs are *passive*: consumers and competitors do not revise their beliefs about a firm's quality when it charges an unexpected price. We discuss this assumption in detail in the following section.[12] In equilibrium, the beliefs $(\hat{q}_L, \hat{q}_H)$ or $\hat{q}_M$ have to be correct, consumers make the utility maximizing choice, and each firm's strategy maximizes its profits given the beliefs and the equilibrium strategies of the competitor and the consumers.

3 Price Equilibrium

We first examine the equilibrium prices that obtain in period two, when only informed consumers are in the market. The second-period pricing subgame is equivalent to the setup in Ronnen (1991) (where *all* consumers know the quality of the offered products). Let $x \equiv p/q$ denote the quality-deflated or hedonic price. A consumer with taste for quality θ gets positive utility from buying if and only if $\theta q - p \geq 0 \Leftrightarrow \theta \geq x$. Thus, if a single firm has entered, the optimal quality-deflated price is $x_M^* = 1/2$. If both firms have entered, let $r \equiv q_H/q_L$ denote the relative quality of their products. If $q_H > q_L$, a consumer with taste θ prefers H to L if and only if $\theta q_H - p_H \geq \theta q_L - p_L$ or, equivalently, $\theta \geq \frac{p_H - p_L}{q_H - q_L} \equiv \hat{\theta}$. Ronnen (1991) provides a detailed derivation of the following result:

Lemma 1 (Ronnen (1991)). *Suppose that both firms have entered the market with $q_H \geq q_L$ and that consumers observe qualities. In the unique price equilibrium, the quality-deflated prices of firm L and H are $x_L^* = \frac{r-1}{4r-1}$ and $x_H^* = \frac{2(r-1)}{4r-1}$. The indifferent consumer is located at $\hat{\theta}^* = \frac{2r-1}{4r-1}$. Consumers with $\theta \in [0, x_L^*)$ do not buy, those with $\theta \in [x_L^*, \hat{\theta}^*)$ buy from firm L, and those with $\theta \in \left[\hat{\theta}^*, 1\right]$ buy from firm H.*

Thus, for $r > 1$, the equilibrium of the pricing subgame at $t = 2$ is such that the consumers with the lowest taste for quality abstain from buying, the consumers with the highest taste for quality buy the high-quality good, while those with intermediate taste for quality buy the low-quality good. Formally, the equilibrium entails $x_L^* < x_H^* < \hat{\theta}^*$.

From Lemma 1 we can derive the firms' equilibrium revenues per consumer as

$$R^L(q_L, q_H) \equiv q_L x_L^* \left(\hat{\theta}^* - x_L^* \right) = q_L \frac{r(r-1)}{(4r-1)^2} \tag{1}$$

$$R^H(q_L, q_H) \equiv q_H x_H^* \left(1 - \hat{\theta}^* \right) = q_H \frac{4r(r-1)}{(4r-1)^2}. \tag{2}$$

Total revenues from period two are obtained by multiplying these expressions by the share of informed consumers, α. A firm's revenue increases when holding its own quality fixed and raising the degree of disparity r. The reason is that more differentiated products give rise to higher equilibrium prices. If both firms produce the same quality, then the equilibrium prices are zero. Letting subscripts denote partial derivatives, straightforward calculations show that revenues satisfy the following properties, for $i = L, H$:[13]

$$R_{q_i}^i > 0 > R_{q_i,q_i}^i$$
$$R_{q_i,q_j}^i > 0, \quad i \neq j.$$

That is, a firm's revenue is increasing and concave in its own quality, and a firm's marginal revenue is increasing in its competitor's quality (qualities are strategic complements). The intuition for strategic complementarity is the following. As the low-quality firm raises q_L, the high-quality firm has a stronger incentive to raise q_H to escape price competition. As the high-quality firm raises q_H, price competition is alleviated, so the low-quality firm has a stronger incentive to raise q_L to capture a larger share of demand.

We now turn to the equilibrium prices that obtain in period one, when only uninformed consumers are in the market. Note first that period-one prices cannot signal quality. Our setup deliberately rules out all channels through which signaling could be sustained: marginal costs do not depend on quality, as in Daughety and Reinganum (2008b) or Janssen and Roy (2010); firms do not observe each others' quality before setting period-one prices, as in Hertzendorf and Overgaard (2001) or Fluet and Garella (2002); informed consumers are not in the market simultaneously with uninformed consumers, as in Yehezkel (2008). Ruling out signaling considerably simplifies the analysis. It can also be justified on the grounds that we are most worried about those markets where distortions caused by asymmetric information cannot be

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[13]The marginal revenues of the low-quality and high-quality firm with respect to own quality are $R_{q_L}^L = \frac{r^2(4r-7)}{(4r-1)^3}$ and $R_{q_H}^H = \frac{4r(2-3r+4r^2)}{(4r-1)^3}$. They depend on q_L and q_H only through the relative quality r. Formally, marginal revenues are homogeneous of degree zero.

alleviated through signaling: when signaling is possible, the market may be able to solve (or at least mitigate) the underlying information problem without the need for government intervention.

The absence of signaling is why the restriction to equilibria with passive beliefs is reasonable. As is well known, the PBE concept suffers from the problem of multiple equilibria. In our setup, the multiplicity problem is particularly severe: without restrictions on out-of-equilibrium beliefs, (almost) any period-one prices can be supported as an equilibrium.[14] Given that prices cannot signal qualities, however, the most plausible outcome is one in which consumers' beliefs do not depend on period-one prices. Consumers know that prices do not convey any information about quality. Thus, it can be argued that they should not revise their beliefs after observing an unexpected price. This is precisely what occurs under our passive-beliefs assumption.[15] The equilibrium in which beliefs about qualities do not depend on period-one prices is also the only one that satisfies In and Wright's (2011) concept of reordering invariance.[16]

Note that the restriction to passive beliefs does not affect the equilibrium qualities. Since prices do not signal qualities, the actual quality only affects the firms' revenues from informed consumers at $t = 2$. Revenues from uninformed consumers depend on *beliefs* about quality, but not on quality itself.[17] We analyze the choice of quality in the following section.

Since consumers do not revise their belief $\hat{q}_i$ after observing unexpected prices, the equilibrium prices x_i^* at $t = 1$ are determined by the formulas given by Lemma 1 after replacing q_i by $\hat{q}_i$. Similarly, revenues per consumer are given by expressions (1) and (2), replacing q_i by $\hat{q}_i$. Firm i's total revenue for beliefs $(\hat{q}_L, \hat{q}_H)$ and actual qualities (q_L, q_H) is

$$(1 - \alpha) R^i(\hat{q}_L, \hat{q}_H) + \alpha R^i(q_L, q_H), \ i \in \{L, H\}.$$

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[14]It suffices that consumers believe that any firm that deviates has a quality of zero.

[15]Because of our assumption that consumers can identify the producer of a good, their beliefs about the firms' qualities may differ even though the firms are ex-ante identical. In particular, our restriction to passive beliefs implies that if firms produce vertically differentiated goods in equilibrium but the low-quality firm deviates and charges the same price as its competitor, the consumers maintain their beliefs about this firm.

[16]Our setup calls for the firms to make two sequential moves that are not observable to either consumers or competitors (choosing qualities and then choosing period-one prices). Simply put, reordering invariance selects the equilibria that survive a reordering of these moves (i.e., firms choosing period-one prices before choosing qualities). In our game, reordering creates proper subgames (one corresponding to each period-one price), so that subgame perfection can be applied. Since the equilibrium strategy prescribes the same quality whatever the period-one price (see Section 4), a belief that assigns positive probability to any other quality after a deviation is not derived from equilibrium strategies, and can therefore not be part of a reordering-invariant PBE.

[17]The beliefs do affect the equilibrium prices and thus the profits in the second stage, which has an impact on the firms' decision to enter the market and on welfare.

4 Quality choice and entry without intervention

This section examines the equilibrium quality levels in the absence of policy intervention. We first derive the quality choice that maximizes firm L's profit given firm H's quality, q_H, and beliefs $(\hat{q}_L, \hat{q}_H)$, which solves

$$\max_{q_L} \ (1 - \alpha) R^L(\hat{q}_L, \hat{q}_H) + \alpha R^L(q_L, q_H) - C(q_L). \tag{3}$$

Let $b_L(q_H, \alpha)$ denote firm L's restricted best response to quality q_H given α. That is, b_L is the quality q_L that solves (3) subject to $q_L \leq q_H$. The first-order condition

$$\alpha R^L_{q_L}(q_L, q_H) = C'(q_L) \tag{4}$$

uniquely characterizes the best response of firm L whenever $b^L(q_H, \alpha) < q_H$.

The quality choice that maximizes firm H's profit given q_L and beliefs $(\hat{q}_L, \hat{q}_H)$ solves

$$\max_{q_H} \ (1 - \alpha) R^H(\hat{q}_L, \hat{q}_H) + \alpha R^H(q_L, q_H) - C(q_H). \tag{5}$$

Let $b^H(q_L, \alpha)$ denote firm H's restricted best response to quality q_L given α, which is the quality q_H that solves (5) subject to $q_H \geq q_L$. The first-order condition

$$\alpha R^H_{q_H}(\hat{q}_L, q_H) \quad = \quad C'(q_H) \tag{6}$$

uniquely defines the best response of firm H whenever $b^H(q_L, \alpha) > q_L$.[18]

In equilibrium, uninformed consumers correctly anticipate firms' quality choices. Letting $\Pi^i(q_L, q_H)$ denote the profits of firm i when beliefs are correct, we thus have

$$\Pi^i(q_L, q_H) = (1 - \alpha)R^i(q_L, q_H) + \alpha R^i(q_L, q_H) - C(q_i) = R^i(q_L, q_H) - C(q_i).$$

The following Proposition extends Theorem 1 in Ronnen (1991) to the case where $0 < \alpha < 1$.

Proposition 1. *For any $\alpha \in (0, 1]$, there is a unique equilibrium in quality choices, characterized by qualities (q_L^*, q_H^*) that solve conditions (4) and (6). When consumers correctly anticipate that the firms produce at these quality levels, profits are $\Pi^H(q_L^*, q_H^*) > \Pi^L(q_L^*, q_H^*) > 0$. The quality levels q_L^* and q_H^* increase in the fraction of informed consumers α.*

Proposition 1 states that the quality-choice game without government intervention has a unique equilibrium that is characterized by vertical differentiation: firm H best-responds to q_L by offering a quality above q_L, and firm L best-responds to q_H by offering a quality below q_H. This result is illustrated in Figure 2. The proposition also shows that the high-quality firm earns higher profits than the low-quality firm and that both earn positive profits.

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[18]In the proof of Proposition 1 we show that $b^H(\hat{q}_L, \alpha)$ always exists and is bounded above.
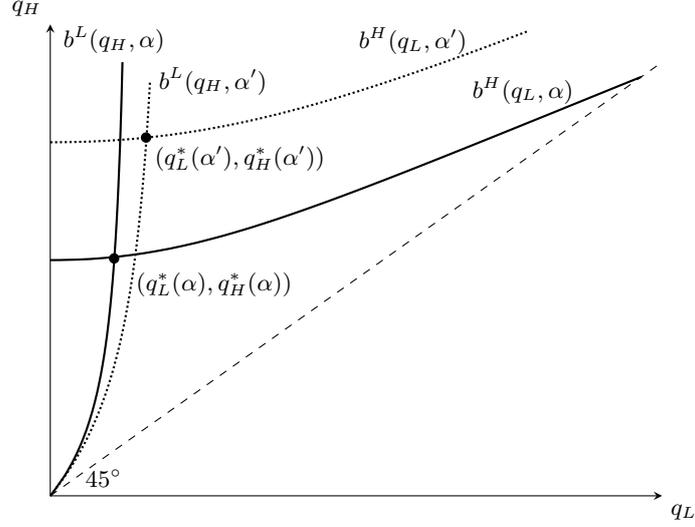
Figure 2: Equilibrium in quality choices: effect of an increase in α ($\alpha' > \alpha$)

To see that the low-quality firm's profit is positive, rewrite the equilibrium profit as $\Pi^{L*} = \int_0^{q_L^*} \left[R_{q_L}^L (q, q_H^*) - C'(q) \right] dq$. The optimality condition (4) implies that the marginal profit with respect to q_L, $R_{q_L}^L (q_L, q_H) - C'(q_L)$, must be non-negative at (q_L^*, q_H^*). The profit of firm L is thus positive, because each firm's marginal profit decreases in the own quality. Since both firms earn positive profits, we may conclude that it is optimal for them to enter the market. The proof of Proposition 1 also establishes that firm H has no incentive to deviate to a quality level below its competitor's. The assumption $C''' \geq 0$ ensures that the low-quality firm has no incentive to deviate to a quality level above its competitor's.

The result that equilibrium qualities increase with α can be explained as follows. Since marginal revenue is downward sloping in the own quality level, conditions (4) and (6) imply that the restricted best responses increase in the fraction of informed consumers when holding the competitor's quality constant. Formally, $b_\alpha^L(q_H, \alpha) \geq 0$ and $b_\alpha^H(q_L, \alpha) \geq 0$ with a strict inequality if $b^L(q_H, \alpha) < q_H$ and $b^H(q_L, \alpha) > q_L$, respectively. Intuitively, an increase in α raises the marginal benefit of producing higher quality because a larger fraction of consumers can react to the increase. This effect is reinforced by strategic complementarity: since $R_{q_i, q_j}^i > 0$, each firm has an incentive to further raise its quality following an increase in its competitor's quality.

Importantly, when $\alpha < 1$, each firm could improve its situation if it were able to commit to producing higher quality: in equilibrium, the beliefs are correct, and the optimality conditions (4) and (6) imply that $\Pi_{q_i}^i(q_L^*, q_H^*) = (1 - \alpha) R_{q_i}^i(q_L^*, q_H^*) > 0$. Thus, if a firm increased its quality slightly *and* consumers adapted their beliefs accordingly, the firm's profit

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would increase. However, the proportion $1 - \alpha$ of consumers does not react to changes of the *actual* quality of a good, so that firms have insufficient incentives to invest in quality. Each firm installs a production technology that maximizes $\alpha R^i(q_L, q_H) - C(q_i)$ instead of the whole profit $R^i(q_L, q_H) - C(q_i)$. In particular, if consumers expected a firm to produce at the profit-maximizing quality level, then this firm would have an incentive to deviate to a lower quality level.[19]

For later reference, we note that a monopolist sets the quality level q_M^* so as to maximize $\alpha R^M(q_M) - C(q_M)$. From $\lim_{q_L \to 0} R^H(q_L, q_M) = R^M(q_M)$ and $R^H_{q_H, q_L} > 0$, it follows that for any $\alpha > 0$, the equilibrium quality level of the high-quality firm exceeds the equilibrium quality level of a monopolist.

In order to assess the scope for government intervention, we now relate the equilibrium qualities to those that maximize social welfare. We define welfare as the difference between the aggregate value of consumption and the cost of supply. Suppose two firms have entered the market. We have

$$W \equiv q_L \int_{x_L^*}^{\hat{\theta}^*} \theta \mathrm{d}\theta + q_H \int_{\hat{\theta}^*}^1 \theta \mathrm{d}\theta - C(q_L) - C(q_H).$$

For any pair of qualities (q_L, q_H) that are correctly anticipated by consumers, social welfare at the ensuing equilibrium prices is

$$W(q_L, q_H) = \frac{q_H \left(12r^2 - r - 2\right)}{2(4r - 1)^2} - C(q_L) - C(q_H). \tag{7}$$

Similarly, revenue and welfare under monopoly are $R^M(q_M) \equiv q_M x_M^* \left(1 - x_M^*\right) = \frac{1}{4} q_M$ and $W^M(q_M) = \frac{3}{8} q_M - C(q_M)$, respectively.[20]

We will refer to a pair of quality levels as *second-best* if they solve

$$\max_{q_L, q_H} W(q_L, q_H).$$

That is, the second-best qualities are those that a social planner who controls qualities but not prices would choose.[21] It is straightforward to verify that the partial derivatives of the welfare function in (7) exceed the respective marginal profits: $W_{q_L} > \Pi_{q_L}^L$ and $W_{q_H} > \Pi_{q_H}^H$.[22] The firms' first order conditions (4) and (6) imply that $\Pi_{q_i}^i(q_L^*, q_H^*) = (1 - \alpha)R_{q_i}^i(q_L^*, q_H^*) \geq 0$ for $i \in \{L, H\}$. Thus, *locally* increasing each firm's quality raises welfare. In addition, as we show in the proof of the following Lemma, $W(q_L, q_H)$ is strictly concave and $W_{q_L, q_H} > 0$. It follows that the second-best qualities exceed the equilibrium qualities.

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[19]A similar result has been discussed by Shapiro (1982) in a monopoly setup.

[20]These formulae obtain from the ones already presented in the limit for $q_L \to 0$.

[21]Clearly, if the planner could also determine the firms' prices, it would be optimal that a single firm serves the whole market at a price of zero. The first best quality level satisfies $C'(q^{FB}) = \int_0^1 \theta \mathrm{d}\theta = \frac{1}{2}$.

[22]See the proof of Lemma 2.

Lemma 2. *For all $\alpha \in (0,1]$, both firms' equilibrium qualities are lower than the second-best qualities: $q_L^* < q_L^{SB}$ and $q_H^* < q_H^{SB}$.*

According to Lemma 2, the equilibrium qualities are always too low from a social point of view. This is true even when consumers are fully informed ($\alpha = 1$). The reason for this result is that in our setup the social incentive to raise quality exceeds the private incentive. Appendix B shows that the difference between the marginal effect of an increase in q_H on welfare and profit can be decomposed into two parts. The first corresponds to the familiar divergence in tastes for quality between the average and marginal consumer, which gives the social planner a stronger incentive to increase quality than the firm. The second is the effect on equilibrium prices. The firm has an incentive to raise quality to soften price competition, which is absent from the planner's considerations. It turns out that in our setup the first effect dominates the second (see the proof of Lemma 2). The presence of uninformed consumers reinforces this effect. While the private incentive to raise quality decreases as the share of informed consumers shrinks, the social incentive remains unchanged. As a result, the problem of underprovision of quality – though present under full information – is most severe when the share of informed consumers is small.

5 The effect of government intervention

5.1 Certification

Under certification, the government announces a certification threshold q^C and awards a quality certificate to all firms whose quality exceeds q^C, without revealing further information about the firms' qualities. Let $s_i \in \{0,1\}$ denote the outcome of certification for firm i, where $s_i = 1$ means a certificate is awarded and $s_i = 0$ means no certificate is awarded to firm i.

Consumers incorporate the additional information made available by the certification system. Since we assume that the government does not make certification mistakes, it is natural to impose that certification restricts out-of-equilibrium beliefs: when a firm deviates and unexpectedly obtains (or unexpectedly does not obtain) a certificate, beliefs should be compatible with the certification outcome. Formally, let $\hat{q}_i(s_i)$ denote the belief about the quality of firm i contingent on the certification outcome s_i. Beliefs are in line with the certification outcome if $\hat{q}_i(1) \in [q^C, \infty)$ and $\hat{q}_i(0) \in [0, q^C)$ for $i \in \{L, H\}$.

In order to avoid implausible equilibria, we restrict attention to equilibria that survive the following natural refinement of out-of-equilibrium beliefs:[23] after an unexpected certification

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[23]For example, the out-of equilibrium belief $\hat{q}_H(0) = 0$ implies a harsh punishment in case firm H is unexpectedly not awarded the certificate. As an implausible consequence, a firm would meet almost any certification threshold as long as it earns non-negative profits to avoid the "stigma" of not getting the certificate.

outcome, consumers believe that firms best-respond to the competitor's quality from the set of qualities compatible with the certification outcome. Thus, if a firm unexpectedly does not obtain the certificate, the belief has to coincide with the profit-maximizing quality within the interval $[0, q^C)$, holding fixed the belief about the rival's quality.[24] Likewise, if a firm unexpectedly obtains the certificate, consumers believe that it produces the profit-maximizing quality level within the interval $[q^C, \infty)$.[25]

In any equilibrium, a firm either produces a quality that satisfies the first-order condition $\alpha R^i_{q_i}(q_L, q_H) = C'(q_i)$ or one that exactly matches q^C. To see that firms produce at no other quality level in equilibrium, suppose that consumers (and the competitor) believed that firm i offers a different quality. Then, it would be profitable for firm i to deviate to some quality close to the anticipated level without changing the certification outcome and thus without changing the uninformed consumers' beliefs. Moreover, there is no equilibrium in which both firms produce goods of quality q^C since Bertrand competition would drive prices down to zero so that the firms would make a loss. Therefore, at most one firm will exactly match the certification threshold.

We restrict attention to a scenario where the government targets the high-quality firm by setting a certification standard which is above firm H's equilibrium quality absent intervention, i.e., $q^C \geq q^*_H$. The rationale for this approach is that when the government targets the low-quality firm, an MQS is more efficient than certification.

In a *certification equilibrium*, both firms enter the market, the high-quality firm exactly matches the certification standard, and the low-quality firm best-responds. Formally, the firms produce the qualities $q^{ce}_H = q^C$ and $q^{ce}_L = b^L(q^C, \alpha)$, where the superscript *ce* indicates the presence of a certification mechanism. Since we focus on certification standards that exceed the unregulated highest quality in the market, it is intuitive that the high quality firm will not produce a quality level that is strictly above q^C. For neither firm to want to deviate, the following conditions must hold:

$$\Pi^L\left(q^{ce}_L, q^C\right) \geq \Pi^H\left(q^C, b^H(q^C, \alpha)\right) \tag{8}$$

$$\Pi^H\left(q^{ce}_L, q^C\right) \geq \max\left\{\Pi^H\left(q^{ce}_L, b^H\left(q^{ce}_L, \alpha\right)\right), \Pi^L\left(b^L(q^{ce}_L, \alpha), q^{ce}_L\right)\right\} \tag{9}$$

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[24]While the profit-maximizing quality within the interval $[0, q^C)$ may not exist for low values of q^C, this theoretical problem does not arise for certification levels that are designed to increase welfare, and can thus be safely ignored.

[25]Define $\varphi(q_1, q_2) \equiv \alpha R^L(q_1, q_2) - C(q_1)$ if $q_1 \leq q_2$ and $\varphi(q_1, q_2) \equiv \alpha R^H(q_2, q_1) - C(q_1)$ if $q_1 > q_2$. Formally, our refinement requires $\hat{q}_i(0) = \min\left\{\arg\max_{q \in [0, q^C)} \varphi(q, q^*_j)\right\}$ and $\hat{q}_i(1) = \min\left\{\arg\max_{q \in [q^C; \infty)} \varphi(q, q^*_j)\right\}$ for $i, j \in \{L, H\}$, $j \neq i$. In our setup this refinement is easy to apply, since the best response of firm i does not depend on the others' belief about this firm's quality $\hat{q}_i$. Moreover, it is appealing because the revised beliefs after an unexpected certification outcome about the deviant's quality coincide with the most profitable deviation. Thus, this refinement leads to "more consistency" off the equilibrium path, as is the case with In and Wright's 2011 concept of reordering invariance.

Condition (8) ensures that the low-quality firm has no incentives to "leapfrog" the high-quality firm by deviating to a quality above q^C.[26] Since $q_L^{ce} = b^L(q^C, \alpha)$ is a restricted best response, firm L has no profitable deviation below q^C. Firm L's most profitable deviation above q^C is $b^H(q^C, \alpha)$. Our refinement on out-of-equilibrium beliefs implies that consumers correctly anticipate that firm L deviated to $b^H(q^C, \alpha)$ when firm L unexpectedly obtains the certificate; thus, this deviation yields $\Pi^H(q^C, b^H(q^C, \alpha))$. This deviation is unprofitable if and only if condition (8) holds.

Similarly, condition (9) ensures that the high-quality producer has no incentive to deviate from q^C. The most profitable deviation is either to best-respond to q_L^{ce} "from above," that is, by choosing $b^H(q_L^{ce}, \alpha)$, or to best-respond "from below," by choosing $b_L(q_L^{ce}, \alpha)$. Our refinement again implies that consumers correctly anticipate the most profitable among those deviations, which yields $\max\left\{\Pi^H\left(q_L^{ce}, b^H\left(q_L^{ce}, \alpha\right)\right), \Pi^L\left(b^L(q_L^{ce}, \alpha), q_L^{ce}\right)\right\}$.

Define a certification threshold q^C as *feasible* if it admits a certification equilibrium. Denote by Q^C the set of feasible certification thresholds. That is, Q^C is the set of all $q^C \geq q_H^*$ satisfying conditions (8) and (9). For all $q^C \in Q^C$, there exists an equilibrium in which $q_H = q^C$ and $q_L = b^L(q^C, \alpha)$.

Lemma 3. *The set of feasible certification thresholds Q^C is nonempty.*

For simplicity, we will assume in what follows that the certification equilibrium is always selected whenever it exists.[27] Whenever $q^C > q_H^*$, the certification equilibrium entails a higher quality for *both* firms than absent intervention. The high-quality firm produces a higher quality in order to match the certification threshold. Anticipating this, the low-quality firm also raises its quality because the quality levels are strategic complements. The fact that firms underprovide quality in the absence of intervention (see Lemma 2) means that certification can raise welfare.

As for the firms' profits, the low-quality firm clearly benefits from less intense competition resulting from more differentiated products. Moreover, it optimally augments the quality of its product which further raises its profit.[28] By contrast, the impact of certification on the

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[26]Note that if $q^C = q_H^*$, then condition (8) is more restrictive than the related "no-leapfrogging" condition in the setup without certification. This is because uninformed consumers will revise their beliefs upon observing that firm L unexpectedly obtains the certificate which makes deviating upwards more profitable now.

[27]There is a second candidate equilibrium in which firms behave as if certification was not available and install the quality levels (q_L^*, q_H^*) defined in Proposition 1. Uniqueness requires

$$\Pi^H(q_L^*, q_H^*) < \Pi^H\left(q_L^*, q^C\right). \tag{10}$$

Condition (10) ensures that (q_L^*, q_H^*) cannot be an equilibrium since H could increase its profit by producing goods of quality q^C and signal this deviation by means of the certificate.

[28]Formally, rewriting the change in firm L's profit yields $\Pi^L(q_L^{ce}, q_H^{ce}) - \Pi^L(q_L^*, q_H^*) = \int_{q_H^*}^{q_H^{ce}} R_{q_H}^L(q_L^*, q)\mathrm{d}q + \int_{q_L^*}^{q_L^{ce}} \Pi_{q_L}^L(q, q_H^{ce})\mathrm{d}q > 0$ which is positive since the integrands of both terms are positive.

profit of the high-quality firm is ambiguous. On the one hand, certification helps the firm to commit to producing a higher quality product, which – according to inequality (9) – increases its profit when holding q_L fixed at $b^L(q^C, \alpha)$. On the other hand, the low-quality firm reacts to firm H's increase to q^C by adjusting its own quality upwards. This reduces the high-quality firm's profit. When the certification standard it set at a high level, so that a large investment is necessary to obtain the certificate, firm H may thus earn less profit in the certification equilibrium than without intervention.

The effect on consumers is a priori less clear-cut as they benefit from higher quality but suffer from higher prices. Specifically, if consumers correctly anticipate the qualities q_L and q_H, then consumer surplus is $CS(q_L, q_H) \equiv \int_{x_L^*}^{\hat{\theta}^*} q_L (\theta - x_L^*) \, d\theta + \int_{\hat{\theta}^*}^{1} q_H (\theta - x_H^*) \, d\theta$. Taking derivatives shows that $CS_{q_L} > 0$ is always true and that $CS_{q_H} > 0$ if and only if $r > \frac{5}{4}$.[29] The optimality condition (4) of firm L implies that $R_{q_L}^L(q_L^*, q_H^*) > 0$ which requires $r^* > \frac{7}{4}$. Hence, certification increases consumer surplus in our setup.[30]

Having seen that welfare-enhancing certification is possible, we now turn to optimal certification. A welfare-maximizing regulator chooses q^C to solve

$$\max_{q^C} \ W(b^L(q^C, \alpha), q^C) \quad \text{subject to} \quad q^C \in Q^C. \tag{11}$$

Among all feasible certification thresholds, the regulator chooses the one that maximizes welfare, given that the high-quality firm exactly matches q^C and the low-quality firm best-responds to q^C.

Our assumption that $C''' \geq 0$ is sufficient for concavity of $W(b^L(q^C, \alpha), q^C)$ in q^C.[31] Therefore, problem (11) has a unique unconstrained maximizer $q^{TB} \equiv \arg\max_q W(b^L(q, \alpha), q)$.[32] Letting $\bar{q} \equiv \max Q^C$, we can state the following result.

Proposition 2. *If α is sufficiently small and $\Pi^M(q^{TB}) > 0$, then $q^{TB} \in Q^C$, and the optimal certification standard is $q^{TB} > q_H^{FI}$. If α is sufficiently large, then $q^{TB} \notin Q^C$, and the optimal certification standard is $\bar{q}$, which solves*

$$\Pi^H\left(b^L(\bar{q}, \alpha), \bar{q}\right) = \Pi^H\left(b^L(\bar{q}, \alpha), b^H(b^L(\bar{q}, \alpha), \alpha)\right).$$

Proposition 2 contains two main results. When the share of informed consumers is small, implementing $q^C = q^{TB}$ is feasible and thus optimal. The condition $\Pi^M(q^{TB}) > 0$ is essentially an assumption on the cost function; it says that a monopolist must be able to profitably

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[29]Formally, $CS = \frac{q_H r(4r+5)}{2(4r-1)^2}$, $CS_{q_L} = \frac{r^2(28r+5)}{2(4r-1)^3}$ and $CS_{q_H} = \frac{r(8r^2-6r-5)}{(4r-1)^3}$.

[30] Note that although aggregate consumer surplus increases, the equilibrium utility of some consumers may shrink.

[31]See the proof of Proposition 2 for details.

[32]The superscript TB ("third best") indicates that in contrast to the second-best quality levels, only the quality of firm H can be chosen by the regulator while firm L chooses a best response.

produce the third-best quality. When the share of informed consumers is large, implementing q^{TB} is no longer feasible. The regulator therefore chooses the largest feasible certification standard, $\bar{q}$.

To understand this result, note first that $q^{TB} > q_H^{FI}$, i.e., the third-best quality exceeds the level that the high-quality firm would choose under full information. Because the second-best qualities are above the unregulated full-information qualities, the third-best quality also is. The second observation is that, for $\alpha = 1$, the highest feasible certification standard is q_H^{FI}. When all consumers observe quality, there is no reason for the high-quality firm to adopt a certificate to signal its quality. It will only adopt the certificate if the threshold is at the level of quality it would have chosen anyway. Taken together, these two facts imply that q^{TB} is not feasible when $\alpha = 1$, and by continuity also in its vicinity.

To see that q^{TB} is feasible when α is small, recall that both firms' best responses tend to zero as the number of informed consumers approaches zero, i.e., $b^i(q, \alpha) \to 0$ as $\alpha \to 0$ for $i = H, L$. Thus, the high-quality firm's profit from foregoing the certificate also tends to zero. By contrast, its profit from obtaining the certificate approaches the monopoly profit, as the low-quality firm produces a good whose quality is close to zero. By assumption, the monopoly profit of producing q^{TB} is positive. Hence, for small α, q^{TB} is in the set of feasible certification thresholds Q^C.

When the share of informed consumers is low, the previous result implies that certification can induce firm H to produce a quality exceeding the full-information level. The firm is willing to install even expensive production technologies to obtain the certificate because its deviation profit is small. Hence, if sufficiently few consumers observe quality, the regulator can exploit the high-quality firm's dependence on certification in order to raise the supplied quality of firm H further towards the socially optimal level than it could by disclosing the exact quality levels on offer.

Corollary. *There exists a range of α over which the optimal certification standard decreases.*

This corollary follows directly from the fact that the optimal certification standard is $q^{TB} > q_H^{FI}$ for small α and $\bar{q}$ for large α, noting that $\bar{q} = q_H^{FI}$ for $\alpha = 1$. It suggests that as the share of informed consumers increases, the regulator may have to reduce the threshold for certification.

5.2 Minimum quality standard

An MQS forces active firms to produce at least a quality of q^{MQS}. Ronnen (1991) shows that when all consumers are informed, introducing a suitable MQS is always welfare enhancing. If both firms enter, an MQS that is set above q_L^* forces firm L to install a higher quality than

it otherwise would. In response, firm H also increases its quality. Ronnen (1991) shows that under an MQS the goods are less differentiated than in the unregulated market, which leads to lower quality-deflated prices and thus to higher participation.

The downside of adopting an MQS is that it may reduce the number of active firms. An MQS introduces a lower bound on the investment in quality that is required to enter the market, and thus plays a role that is similar to an entry cost. An MQS which is set too restrictive does not allow both firms to recoup their initial investments in the production technology. This may result in only one firm entering or no firm at all entering. In order to maintain the competitive pressure on prices, the regulator has to set a rather low MQS so as to ensure that both firms enter. In setting the level of q^{MQS}, the regulator thus faces a tradeoff between improving quality and maintaining price competition.

The negative impact of an MQS on entry is particularly severe if only few consumers observe quality. As discussed in Section 4, the firms then cannot credibly produce high qualities. The equilibrium level of vertical differentiation is low and firms earn little profit. Adopting an MQS in such a situation further squeezes the firms' profits. Even a moderate MQS may deter one of them from entering, so only a relatively low MQS guarantees that both firms enter. It follows that when the share of informed consumers is small the optimal MQS may well be such that only one firm enters. As the following proposition shows, the regulator may prefer a monopoly producing high quality to a duopoly producing low quality.

Proposition 3. *Let $q_M^{SB} \equiv \arg\max_q W^M(q)$. If α is sufficiently small and $\Pi^M\left(q_M^{SB}\right) > 0$, the optimal minimum quality standard is such that only one firm enters, and $q^{MQS} = q_M^{SB}$.*

Proposition 3 extends the analysis in Ronnen (1991) to situations where the number of informed consumers is small. For the reasons discussed above, when $\alpha \to 0$ the highest q^{MQS} that allows two firms to earn non-negative profits is close to zero. Having a single firm enter the market, and controlling the firm's quality through a more ambitious MQS, then yields higher welfare. If $\Pi^M\left(q_M^{SB}\right) > 0$, so that a monopolist can profitably produce the second-best monopoly quality, it is optimal to set $q^{MQS} = q_M^{SB}$.

The following proposition uses these insights together with those in Ronnen (1991) to assess under which conditions an MQS outperforms certification.

Proposition 4. *Suppose $\Pi^M\left(q_M^{SB}\right) > 0$. If α is sufficiently small, optimal certification leads to higher welfare than an optimal MQS. If α is sufficiently large, an optimal MQS leads to higher welfare than optimal certification.*

To see the intuition behind Proposition 4, consider first the case where almost none of the consumers observe quality. By Proposition 3, it is then optimal to set the MQS at

$q^{MQS} = q_M^{SB}$, so that only one firm enters. In this case, certification can do better, since setting $q^C = q_M^{SB}$ will generally induce two firms to enter, with firm H producing $q_H = q_M^{SB}$. Although firm L may credibly produce only at a low quality level, it exerts some competitive pressure on firm H, which keeps the ensuing prices low and thus helps to improve welfare. Since any optimal certification standard q^C must lead to weakly higher welfare than $q^C = q_M^{SB}$, optimal certification performs better than an MQS.

If almost all consumers observe quality, we know from Proposition 2 that the certification threshold has to be set close to the unregulated equilibrium quality level q_H^*. As $\alpha \to 1$, the welfare improvement that certification can bring about vanishes. By contrast, for $\alpha \to 1$, our model converges to that of Ronnen (1991), who has shown that the optimal MQS improves welfare even if all consumers are informed. We thus conclude that an MQS performs better than certification when almost all consumers are informed.

6 Conclusion

This paper has investigated the effects of certification and minimum quality standards when some consumers cannot discern the quality of traded goods. The presence of uninformed consumers reduces firms' incentives to invest in quality. In equilibrium, quality is under-supplied, which reduces both consumer surplus and profits. By certifying their products, firms can demonstrate that their goods are of higher quality than consumers would otherwise expect. We have considered a simple form of government certification whereby firms are awarded a certificate if they produce goods whose quality is above a publicly known threshold. The certification standard must be set low enough to raise firms' profits when making the investment needed to attain the threshold. Nevertheless, certification can induce some firms to raise their quality above the highest level that would be attained if all consumers were informed. When the share of uninformed consumers is large, firms are more reliant on certification. This allows the government to implement an ambitious certification threshold.

We have also compared certification to an MQS. When the share of informed consumers is small, certification may be preferred over an MQS, since the latter potentially deters firms from entering the market. By contrast, when the share of informed consumers is high, an MQS tends to be more effective than certification.

Our results could be extended in several directions by further research. So far, we have analyzed the relative merits of certification and MQS, but have not considered adopting both instruments together. Introducing both instruments would improve the government's ability to manipulate the quality of active firms. In particular, if only few consumers are informed and two firms have entered, then a suitable certification standard allows firm H

to provide high quality goods and the increased differentiation results in higher revenues. Therefore, complementing an MQS with certification may alleviate the entry-deterring effect of a minimum quality standard. Certification may thus be particularly valuable when used in conjunction with an MQS.

The industry's ability to provide a certification system itself is also an interesting point that deserves further attention. We have pointed out that certification standards may increase the profits of all active firms. This suggests that firms may have an interest in building up their own certifying institution whose certification scheme is designed to maximize the industry profit rather than welfare. Why do we observe that firms sometimes rely on government certification? One important issue from which we have abstracted in our model is that a certifier must have correct incentives for designing and enforcing a certification scheme honestly.[33] The recent experiences with private certifiers that have issued inflated ratings for financial products suggest that this requirement is more likely to be satisfied by government certification.[34] Nevertheless, the availability of privately run certification may restrict the government's leeway to manipulate the firms' qualities by means of certification.

Appendix A   Proofs

Proof of Proposition 1

We first show that there is a unique solution (q_L^*, q_H^*) satisfying the necessary conditions (4) and (6). We then show that both firms earn positive payoffs at (q_L^*, q_H^*) and that q_L^* and q_H^* are global best responses when $C''' \geq 0$. Together these properties establish existence and uniqueness of equilibrium. Finally, we show that the equilibrium qualities increase with α.

Let $q^{\max} > 0$ denote the unique solution to the equation $\alpha R_{q_H}^H(q, q) - C'(q) = 0$. Define $B(q, \alpha) \equiv b^L\left(b^H(q, \alpha), \alpha\right) - q$ on $[0, q^{\max}]$. By the properties of b^L and b^H, B is continuous and strictly decreasing in q as $B_q(q) = b_{q_L}^H b_{q_H}^L - 1 < 0$. To see this, note that $R_{q_L}^L$ is homogeneous of degree zero (i.e., $q_L R_{q_L, q_L}^L + q_H R_{q_L, q_H}^L = 0$), implying

$$b_{q_H}^L(q_H, \alpha) = -\frac{R_{q_L, q_H}^L}{R_{q_L, q_L}^L - C''/\alpha} < -\frac{R_{q_L, q_H}^L}{R_{q_L, q_L}^L} = \frac{q_L}{q_H}.$$

Homogeneity of degree zero of $R_{q_H}^H$ similarly implies

$$b_{q_L}^H(q_L, \alpha) = -\frac{R_{q_H, q_L}^H}{R_{q_H, q_H}^H - C''/\alpha} < -\frac{R_{q_H, q_L}^H}{R_{q_H, q_H}^H} = \frac{q_H}{q_L}.$$

It follows that $b_{q_L}^H b_{q_H}^L < r(1/r) = 1$. Moreover, we have $B(0, \alpha) > 0$ and $B(q^{\max}, \alpha) < 0$.

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[33] Gehrig and Jost (1995) studies the incentives of self-regulating organizations to conduct costly monitoring.

[34] See e.g. Mathis, McAndrews, and Rochet (2009).

Thus, there exists a unique q_L^* such that $B(q_L^*) = 0$. By construction, q_L^* and $q_H^* = b^H(q_L^*, \alpha)$ satisfy (4) and (6).

Because q_L is bounded below by zero, $R_{q_L}^L(0, q_H) = 1/16 > 0$ for all $q_H > 0$, and $C'(0) = 0$, the low-quality firm can always guarantee itself a positive payoff when entering, so $\Pi^L(q_L^*, q_H^*) > 0$.

We now show that $\Pi^H(q_L^*, q_H^*) > \Pi^L(q_L^*, q_H^*)$, implying that the high-quality firm earns a positive payoff as well. From (1) and (2), we obtain $R^H(q_L, q_H) - R^L(q_L, q_H) = (q_H - q_L)r/(4r - 1)$. Using the explicit expression for $R_{q_H}^H$ given in Footnote 13, we have

$$\frac{r}{4r - 1} > R_{q_H}^H(q_L, q_H) = \frac{4r(2 - 3r + 4r^2)}{(4r - 1)^3} \Leftrightarrow 4r - 7 > 0.$$

In equilibrium, this inequality must hold because $R_{q_L}^L > 0$ requires $r^* > 7/4$, implying $R^H(q_L^*, q_H^*) - R^L(q_L^*, q_H^*) \geq (q_H^* - q_L^*) R_{q_H}^H(q_L^*, q_H^*)$. We thus have

$$\alpha \left(R^H(q_L^*, q_H^*) - R^L(q_L^*, q_H^*) \right) > \alpha(q_H^* - q_L^*)R_{q_H}^H(q_L^*, q_H^*)$$
$$= (q_H^* - q_L^*)C'(q_H^*) > \int_{q_L^*}^{q_H^*} C'(q)\mathrm{d}q = C(q_H^*) - C(q_L^*),$$

where the equality follows from the first-order condition (6) and the last inequality from the convexity of C. Hence, $R^H(q_L^*, q_H^*) - R^L(q_L^*, q_H^*) > 0$ and $\Pi^H(q_L^*, q_H^*) > \Pi^L(q_L^*, q_H^*)$.

We now check that q_L^* and q_H^* are global best responses when uninformed consumers' beliefs are $\hat{q}_L = q_L^*$ and $\hat{q}_H = q_H^*$. We begin with q_H^*. Clearly, deviating from q_H^* does not affect second-stage revenues, so it suffices to show that

$$\alpha R^H(q_L^*, q_H^*) - C(q_H^*) \geq \max_{q \leq q_L^*} \left[\alpha R^L(q, q_L^*) - C(q) \right].$$

We have

$$\alpha R^H(q_L^*, q_H^*) - C(q_H^*) > \alpha R^L(q_L^*, q_H^*) - C(q_L^*)$$
$$= \max_{q \leq q_H^*} \alpha R^L(q, q_H^*) - C(q) > \max_{q \leq q_L^*} \alpha R^L(q, q_L^*) - C(q),$$

where the first inequality follows from our previous result and the second from the envelope theorem, noting that

$$\frac{\partial}{\partial q_H} \left[\max_{q \leq q_H} \alpha R^L(q, q_H) - C(q) \right] = \alpha R_{q_H}^L > 0.$$

Turning to q_L^*, similarly as above we need to show that

$$\alpha R^L(q_L^*, q_H^*) - C(q_L^*) \geq \max_{q \geq q_H^*} \alpha R^H(q_H^*, q) - C(q).$$
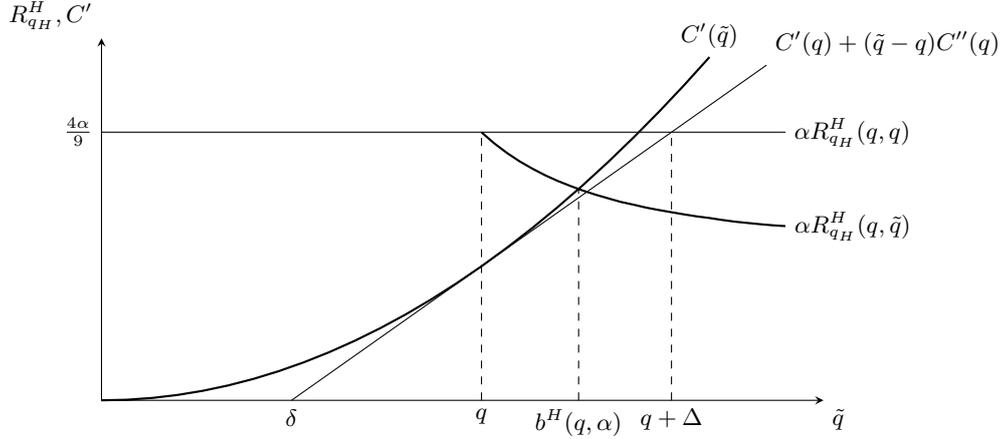
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Figure 3: Proof of Proposition 1

We now establish that $C''' \geq 0$ implies that $\alpha R^H(q_H^*, q) - C(q) < 0$ for all $q \geq q_H^*$, which, together with the fact that $\alpha R^L(q_L^*, q_H^*) - C(q_L^*) > 0$, suffices for the required result. Using $R^H(q, q) = 0$ and concavity in own quality $(R^H_{q_H, q_H} < 0)$, we can write

$$\alpha R^H\left(q, b^H(q, \alpha)\right) - C\left(b^H(q, \alpha)\right) = \int_q^{b^H(q,\alpha)} \left[\alpha R^H_{q_H}(q, \tilde{q}) - C'(\tilde{q})\right] \mathrm{d}\tilde{q} - C(q).$$

The following arguments are illustrated in Figure 3. First, we have

$$\int_q^{b^H(q,\alpha)} \left[\alpha R^H_{q_H}(q, \tilde{q}) - C'(\tilde{q})\right] \mathrm{d}\tilde{q} < \int_q^{q+\Delta} \left[\alpha R^H_{q_H} - \left(C'(q) + (\tilde{q} - q)C''(q)\right)\right] \mathrm{d}\tilde{q}$$

$$= \frac{\left(\alpha R^H_{q_H}(q, q) - C'(q)\right)^2}{2C''(q)},$$

where $\Delta \equiv \left(\alpha R^H_{q_H}(q, q) - C'(q)\right)/C''(q)$ and the inequality is due to the concavity of R^H in q_H and $C''' \geq 0$. Second, we have

$$C(q) = \int_0^q C'(\tilde{q})\mathrm{d}\tilde{q} \geq \int_\delta^q \left[C'(q) - (q - \tilde{q})C''(q)\right] \mathrm{d}\tilde{q} = \frac{(C'(q))^2}{2C''(q)},$$

where $\delta \equiv q - C'(q)/C''(q)$ and the inequality is due to $C''' \geq 0$.

Putting both together yields

$$\alpha R^H\left(q, b^H(q, \alpha)\right) - C\left(b^H(q, \alpha)\right) < \frac{\alpha R^H_{q_H}(q, q)}{2C''(q)} \left(\alpha R^H_{q_H}(q, q) - 2C'(q)\right).$$

Clearly, $\alpha R^H\left(q, b^H(q, \alpha)\right) - C\left(b^H(q, \alpha)\right) < 0$ if $2C'(q) \geq \alpha R^H_{q_H}(q, q)$. Condition (6) implies that $C'(q_H^*) = \alpha R^H_{q_H}(q_L^*, q_H^*) > \alpha/4$ because $R^H_{q_H}$ is bounded below by $\frac{1}{4}$. Hence, $2C'(q_H^*) > \alpha/2 > 4\alpha/9 = \alpha R^H_{q_H}(q, q)$, implying that $\alpha R^H\left(q_H^*, b^H(q_H^*, \alpha)\right) - C\left(b^H(q_H^*, \alpha)\right) < 0$, so deviations to $q \geq q_H^*$ are unprofitable.

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Finally, we establish that q_L^* and q_H^* increase with α. Recall that q_L^* necessarily satisfies $B(q_L^*, \alpha) = 0$. We have $B_q(q, \alpha) < 0$ on the relevant range, and $B_\alpha = \left(b_\alpha^L + b_{q_H}^L\right) b_\alpha^H > 0$, where the arguments are omitted for brevity. By the implicit function theorem, q_L^* thus increases in α. Since $q_H^* = b^H(q_L^*, \alpha)$ and $b_{q_L}^H > 0$, q_H^* also increases in α.

Proof of Lemma 2

We first show that $W_{q_i} > \Pi_{q_i}^i$ and that $\Pi_{q_i}^i(q_L^*, q_H^*) \geq 0$, together implying that $W_{q_i}(q_L^*, q_H^*) > 0$, $i = L, H$. Let $U(q_L, q_H) \equiv W(q_L, q_H) + C(q_L) + C(q_H) = \frac{q_H(12r^2 - r - 2)}{2(4r-1)^2}$ denote welfare gross of investment costs. We have $W_{q_L}(q_L, q_H) = U_{q_L}(q_L, q_H) - C'(q_L) > R_{q_L}^L(q_L, q_H) - C'(q_L) = \Pi_{q_L}^L(q_L, q_H)$ if and only if

$$U_{q_L}(q_L, q_H) > R_{q_L}^L(q_L, q_H) \quad \Longleftrightarrow \quad \frac{r^2(20r - 17)}{2(4r - 1)^3} > \frac{r^2(4r - 7)}{(4r - 1)^3},$$

which is always true for $r > 1$ because $20r - 17 > 2(4r - 7) \Leftrightarrow r \geq 1/4$. We also have $W_{q_H}(q_L, q_H) > \Pi_{q_L}^H(q_L, q_H)$ if and only if

$$U_{q_H}(q_L, q_H) > R_{q_H}^H(q_L, q_H) \quad \Longleftrightarrow \quad \frac{24r^3 - 18r^2 + 5r + 1}{(4r - 1)^3} > \frac{4r(2 - 3r + 4r^2)}{(4r - 1)^3},$$

which holds because $24r^3 - 18r^2 + 5r + 1 > 4r(2 - 3r + 4r^2) \Leftrightarrow 8r^3 - 6r^2 - 3r + 1 = (4r - 1)(2r^2 - r - 1) > 0$ is true for any $r > 1$. From the equilibrium conditions (4) and (6) we have $\Pi_{q_i}^i(q_L^*, q_H^*) = (1 - \alpha)R_{q_i}^i(q_L^*, q_H^*) \geq 0$ and therefore $W_{q_i}(q_L^*, q_H^*) > (1 - \alpha)R_{q_i}^i(q_L^*, q_H^*) \geq 0$.

Next, we establish concavity of U. Differentiating U_{q_L} and U_{q_H} yields

$$U_{q_L, q_L} = -\frac{r^3(4r + 17)}{q_H(4r - 1)^4} < 0$$

$$U_{q_H, q_H} = -\frac{r(4r + 17)}{q_H(4r - 1)^4} < 0$$

$$U_{q_L, q_H} = \frac{r^2(4r + 17)}{q_H(4r - 1)^4} = W_{q_L, q_H} > 0,$$

from which we obtain the determinant of the Hessian of U,

$$U_{q_L, q_L} U_{q_H, q_H} - (U_{q_L, q_H})^2 = 0.$$

Hence, the Hessian of U is negative semi-definite, implying that U is concave. Moreover, since C is strictly convex, W is strictly concave. Combining $W_{q_i}(q_L^*, q_H^*) > 0$ with strict concavity of W and $W_{q_L, q_H} > 0$ yields the claimed result.

Proof of Lemma 3

To establish that Q^C is nonempty, we first prove that condition (8) holds for all $q^C \geq q_M^{FI}$. Then, we show that there exists $q^\dagger \geq q_H^*$ such that condition (9) is satisfied for all $q^C \in [q_H^*, q^\dagger]$. Finally, we show that $\max\{q_M^{FI}, q_H^*\} \leq q^\dagger$, with strict inequality for $\alpha < 1$.

We know from the proof of Proposition 1 that $C''' \geq 0$ implies $\alpha R^H \left(q, b^H \left(q, \alpha \right) \right) - C \left(b^H \left(q, \alpha \right) \right) < 0$ if $2C'(q) \geq \alpha R^H_{q_H} \left(q, q \right) = 4\alpha/9$. Applying this result for $\alpha = 1$, a sufficient condition for deviations from q_L^{ce} to $b^H(q^C, 1)$ to be unprofitable is $2C'(q^C) \geq 4/9$. We have $C'(q_M^{FI}) = R^M_{q_M} \left(q_M^{FI} \right) = 1/4$ and thus $2C'(q_M^{FI}) = 1/2 > 4/9$. Because C' is increasing, the condition is satisfied for all $q^C \geq q_M^{FI}$.

Define $q^\dagger$ as the solution to $q^\dagger = b^H \left(b^L(q^\dagger, \alpha), 1 \right)$. A similar reasoning as in the proof of Proposition 1 implies existence and uniqueness of $q^\dagger$. Condition (9) has two parts. Consider first the inequality $\Pi^H(q_L^{ce}, q^C) \geq \Pi^H(q_L^{ce}, b^H(q_L^{ce}, \alpha))$. For any $q^C \geq q_H^*$, we have $q_H^* \leq b^H(q_L^{ce}, \alpha) \leq q^C$. Thus,

$$\Pi^H(q_L^{ce}, q^C) - \Pi^H(q_L^{ce}, b^H(q_L^{ce}, \alpha)) = \int_{b^H(q_L^{ce}, \alpha)}^{q^C} [R^H_{q_H}(q_L^{ce}, q) - C'(q)] \mathrm{d}q.$$

We have $R^H_{q_H}(q_L^{ce}, q) - C'(q) > 0$ for $q \in [b^H(q_L^{ce}, \alpha), b^H(q_L^{ce}, 1))$. From the definition of $q^\dagger$, it follows that $\Pi^H(q_L^{ce}, q^C) - \Pi^H(q_L^{ce}, b^H(q_L^{ce}, \alpha)) \geq 0$ for all $q^C \in [q_H^*, q^\dagger]$.

Now consider the inequality $\Pi^H(q_L^{ce}, q^C) \geq \Pi^L(b^L(q_L^{ce}, \alpha), q_L^{ce})$. For $q^C \in [q_H^*, q^\dagger]$, there exists $\tilde{\alpha} \leq 1$ such that $b^H(q_L^{ce}, \tilde{\alpha}) = q^C$. This follows from $b^H(q_L^{ce}, \alpha) \leq q^C \leq b^H(q_L^{ce}, 1)$ and continuity of b^H in α. In Proposition 1 we have shown that if $q^C/q_L^{ce} \geq 7/4$ (which is necessarily satisfied since $q_L^{ce} = b^L(q^C, \alpha)$), then $\tilde{\alpha} R^H(q_L^{ce}, q^C) - C \left(q^C \right) > \tilde{\alpha} R^L(q_L^{ce}, q^C) - C(q_L^{ce})$, which implies $R^H(q_L^{ce}, q^C) - C \left(q^C \right) > R^L(q_L^{ce}, q^C) - C(q_L^{ce})$ and thus $\Pi^H(q_L^{ce}, q^C) > \Pi^L(q_L^{ce}, q^C)$. It remains to be shown that $\Pi^L(q_L^{ce}, q^C) \geq \Pi^L(b^L(q_L^{ce}, \alpha), q_L^{ce})$. We have

$$\begin{aligned} \Pi^L(q_L^{ce}, q^C) - \Pi^L(b^L(q_L^{ce}, \alpha), q_L^{ce}) &= \Pi^L(q_L^{ce}, q^C) - \Pi^L(b^L(q_L^{ce}, \alpha), q^C) \\ &\quad + \Pi^L(b^L(q_L^{ce}, \alpha), q^C) - \Pi^L(b^L(q_L^{ce}, \alpha), q_L^{ce}) \\ &= \int_{b^L(q_L^{ce}, \alpha)}^{q_L^{ce}} \Pi^L_{q_L}(q, q^C) \mathrm{d}q + \int_{q_L^{ce}}^{q^C} R^L_{q_H}(b^L(q_L^{ce}, \alpha), q) \mathrm{d}q > 0, \end{aligned}$$

where the inequality follows from both integrands being positive. Hence, for $q^C \in [q_H^*, q^\dagger]$, condition (9) is satisfied.

Finally, $q^\dagger \geq q_H^*$ follows from b^H being increasing in q_L and α as well as b^L being increasing in q_H, while $q^\dagger \geq q_M^{FI}$ follows from $q_M^{FI} = b^H(0, 1)$, $q_L^{ce} > 0$, and b^H being increasing in q_L.

Proof of Proposition 2

We start by showing that $q^{TB} \in Q^C$ for small α. Because $\lim_{\alpha \to 0} b^L(q, \alpha) = 0$ and $\lim_{\alpha \to 0} b^H(q, \alpha) = q$ for any $q \geq 0$, we have

$$\lim_{\alpha \to 0} \Pi^H \left(b^L(q, \alpha), b^H \left(b^L(q, \alpha), \alpha \right) \right) = \Pi^L \left(b^L \left(b^L(q, \alpha), \alpha \right), b^L(q, \alpha) \right) = 0$$

for any $q \geq 0$. By assumption, $\Pi^M(q^{TB}) = \Pi^H(0, q^{TB}) > 0$. Therefore, there exists $\tilde{\alpha} > 0$ such that conditions (8) and (9) are satisfied for all $\alpha \leq \tilde{\alpha}$ when $q^C = q^{TB}$, implying $q^{TB} \in Q^C$.

Since q^{TB} is the unconstrained maximizer of $W(b^L(q^C, \alpha), q^C)$, it is the solution to (11) whenever $\alpha \leq \tilde{\alpha}$.

We now show that $q^{TB} > q_H^{FI}$. Note first that $W((b^L(q^C, \alpha), q^C)$ is concave in q^C when $C''' \geq 0$. In fact, we have

$$\frac{\mathrm{d}^2 W(b^L(q, \alpha), q)}{\mathrm{d}q^2} = W_{q_L, q_L} b_{q_H}^L + W_{q_L} b_{q_H, q_H}^L + W_{q_H, q_H}.$$

The proof of Lemma 2 establishes that $W_{q_L, q_L} < 0$ and $W_{q_H, q_H} < 0$. Moreover, $W_{q_L}(b^L(q^C), q^C) > 0$ over the relevant range of q^C, and $b_{q_H}^L > 0$, so what remains to be shown is that $b_{q_H, q_H}^L \leq 0$. We have

$$b_{q_H, q_H}^L = \frac{\left(R_{q_L, q_L, q_H}^L b_{q_H}^L + R_{q_L, q_H, q_H}^L\right)\left(C''/\alpha - R_{q_L, q_L}^L\right) - R_{q_L, q_H}^L b_{q_H}^L \left(C'''/\alpha - R_{q_L, q_L, q_L}^L\right)}{\left(C''/\alpha - R_{q_L, q_L}^L\right)^2}.$$

Computations yield

$$R_{q_L, q_L, q_L}^L = -\frac{6r^4(20r + 7)}{q_H^2(4r-1)^5} < 0$$

$$R_{q_L, q_L, q_H}^L = \frac{4r^3(16r^2 + 40r + 7)}{q_H^2(4r-1)^5} > 0$$

$$R_{q_L, q_H, q_H}^L = -\frac{2r^2(64r^2 + 100r + 7)}{q_H^2(4r-1)^5} < 0.$$

Because $C''' \geq 0$ by assumption, it follows that $b_{q_H, q_H}^L \leq 0$ if $R_{q_L, q_L, q_H}^L b_{q_H}^L + R_{q_L, q_H, q_H}^L \leq 0$. Using the fact that, as shown in the proof of Lemma 1, $b_{q_H}^L < q_L/q_H$, we obtain

$$R_{q_L, q_L, q_H}^L b_{q_H}^L + R_{q_L, q_H, q_H}^L < \frac{R_{q_L, q_L, q_H}^L}{r} + R_{q_L, q_H, q_H}^L$$

$$= \frac{4r^2(16r^2 + 40r + 7) - 2r^2(64r^2 + 100r + 7)}{q_H^2(4r-1)^5} \leq 0$$

$$\Longleftrightarrow -32r^2 - 20r + 7 \leq 0,$$

which is always satisfied since $r > 1$. Concavity in q^C implies that a sufficient condition for $q^{TB} > q_H^{FI}$ is

$$\left.\frac{\mathrm{d}W(b^L(q^C, \alpha), q^C)}{\mathrm{d}q^C}\right|_{q^C = q_H^{FI}} > R_{q_H}^H(q_L^{FI}, q_H^{FI}) - C'(q_H^{FI}) = 0. \tag{12}$$

We have

$$\left.\frac{\mathrm{d}W(b^L(q^C, \alpha), q^C)}{\mathrm{d}q^C}\right|_{q^C = q_H^{FI}} = b_{q_H}^L(q_H^{FI}, \alpha) W_{q_L}(q_L^{FI}, q_H^{FI}) + W_{q_H}(q_L^{FI}, q_H^{FI}).$$

As we know from the proof of Lemma 2, $W_{q_L}(q_L^{FI}, q_H^{FI}) > 0$, a sufficient condition for (12) is

$$W_{q_H}(q_L^{FI}, q_H^{FI}) + C'(q_H^{FI}) > R_{q_H}^H(q_L^{FI}, q_H^{FI}).$$

We have $W_{q_H}(q_L, q_H) + C'(q_H) = \frac{24r^3 - 18r^2 + 5r + 1}{(4r-1)^3}$, which is bounded below by $3/8$. In any equilibrium with $\alpha = 1$, the first-order condition of the low-quality firm implies $r > 7/4$, so $q_H^{FI} > (7/4)q_L^{FI}$. Because $R_{q_H}^H = \frac{4r(2-3r+4r^2)}{(4r-1)^3}$ is decreasing in r, $R_{q_H}^H(q_L^{FI}, q_H^{FI}) < R_{q_H}^H(q_L, (7/4)q_L) = 7/24 < 3/8 < W_{q_H}(q_L^{FI}, q_H^{FI}) + C'(q_H^{FI})$. Hence, (12) must hold.

Next, we establish that $q^{TB} \notin Q^C$ for large α. Define the highest feasible certification standard as $\bar{q}(\alpha) \equiv \max\{q | \Delta\Pi(q, \alpha) \geq 0\}$, where

$$\Delta\Pi(q, \alpha) \equiv \Pi^H\big(b^L(q, \alpha), q\big) - \Pi^H\big(b^L(q, \alpha), b^H(b^L(q, \alpha), \alpha)\big).$$

Because Π^H is continuous and b^L and b^H are continuous, $\Delta\Pi$ is continuous in q and α. By the properties of b^H (single-valued best response), $\Delta\Pi(q, 1) < 0$ for all $q > q_H^{FI}$, implying $\bar{q}(1) = q_H^{FI}$. Moreover, $\lim_{q \to \infty} \Delta\Pi(q, 1) = -\infty$.

We have shown previously that $q^{TB} > q_H^{FI} = \bar{q}(1)$. Thus, $q^{TB} \notin Q^C$ for $\alpha = 1$. What remains to be shown is that this also holds in the vicinity of $\alpha = 1$. Because of pointwise continuity of $\Delta\Pi$ in α, for any q, $\lim_{\alpha \to 1} \Delta\Pi(q, \alpha) = \Delta\Pi(q, 1)$. For any $\delta > 0$, there is hence some $\epsilon > 0$, such that $\Delta\Pi(q, 1) \leq -\epsilon$ for all $q \geq q_H^{FI} + \delta$. Since $\partial\Delta\Pi/\partial\alpha$ is bounded above, for $\lim_{\alpha \to 1^-} \Delta\Pi(q, 1) < 0$ for all $q \geq q_H^{FI} + \delta$. Therefore, $\bar{q}(\alpha)$ approaches or lies below q_H^{FI} as $\alpha \to 1$. Since $q_H^{FI} < q^{TB}$, $q^{TB} \notin Q^C$ as $\alpha \to 1$.

Proof of Proposition 3

Define $q^{\max}(\alpha)$ as the unique solution to $\alpha R_{q_H}^H(q, q) - C'(q) = 0$, giving the upper bound of the domain on which $b^H(q, \alpha)$ is defined, with $b^H(q^{\max}(\alpha), \alpha) = q^{\max}(\alpha)$. Note that $\Pi^H\big(q^{\max}(\alpha), b^H(q^{\max}(\alpha), \alpha)\big) = \Pi^H\big(q^{\max}(\alpha), q^{\max}(\alpha)\big) = -C(q^{\max}(\alpha)) < 0$. It follows from $R_{q_H}^H(q, q) = 4/9$ that $\lim_{\alpha \to 0} q^{\max}(\alpha) = 0$.

Define the highest MQS that entails an equilibrium in which two firms enter by $\hat{q}(\alpha) \equiv \max\big\{q \geq q_L^* | \Pi^L\big(q, b^H(q, \alpha)\big) \geq 0 \wedge \Pi^H\big(q, b^H(q, \alpha)\big) \geq 0\big\}$. Since Π^L and Π^H are continuous, and since $\Pi^L(q_L^*, q_H^*) > 0$ and $\Pi^H(q_L^*, q_H^*) > 0$ as well as $\Pi^H\big(q^{\max}(\alpha), b^H(q^{\max}(\alpha), \alpha)\big) < 0$, we know that for $\alpha > 0$, $\hat{q}(\alpha)$ exists, is unique, and satisfies $q_L^*(\alpha) < \hat{q}(\alpha) < q^{\max}(\alpha)$. Combining $\lim_{\alpha \to 0} q^{\max}(\alpha) = 0$ and $\hat{q}(\alpha) \geq 0$ yields $\lim_{\alpha \to 0} \hat{q}(\alpha) = 0$.

Next, we show that if α is small enough, then it is optimal to set the MQS such that only one firm enters, i.e., $W^M(q_M^{SB}) > \max_{q^{MQS} \leq \hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS}, \alpha))$. Let $r^{MQS} \equiv \frac{b^H(q^{MQS})}{q^{MQS}}$ and $\psi(r) \equiv \frac{(12r^2 - r - 2)}{2(4r-1)^2}$ so that $W(q_L, q_H) = q_H \psi(q_H/q_L)$. Note that $\max_r \psi(r) = \psi(1) = 1/2$. We now derive an upper bound on the welfare from an MQS that induces both

firms to enter:

$$\max_{q^{MQS} \leq \hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS}, \alpha)) = \max_{q^{MQS} \leq \hat{q}(\alpha)} b^H(q^{MQS}, \alpha)\psi(r^{MQS})$$
$$- C(q^{MQS}) - C(b^H(q^{MQS}, \alpha))$$
$$\leq \max_{q^{MQS} \leq \hat{q}(\alpha)} b^H(q^{MQS}, \alpha) \max_r \psi(r)$$
$$= \max_{q^{MQS} \leq \hat{q}(\alpha)} \frac{b^H(q^{MQS}, \alpha)}{2} \leq \frac{q^{\max}(\alpha)}{2},$$

where the last inequality holds because $b^H(q, \alpha) \leq q^{\max}(\alpha)$. Together with the fact that $q^{\max}(\alpha) \to 0$ as $\alpha \to 0$, this implies

$$\lim_{\alpha \to 0} \max_{q^{MQS} \leq \hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS})) = 0.$$

If only one firm enters, the optimal welfare is $W^M(q_M^{SB}) > 0$ with q_M^{SB} being characterized by $\frac{3}{8}q_M^{SB} = C'(q_M^{SB})$. Thus, $W^M(q_M^{SB}) > \max_{q^{MQS} \leq \hat{q}(\alpha)} W(q^{MQS}, b^H(q^{MQS}))$ for α sufficiently small. By assumption, $\Pi^M(q_M^{SB}) > 0$, so $q^{MQS*} = q_M^{SB}$ makes entry by one firm profitable.

Proof of Proposition 4

We first prove the result for $\alpha \to 0$. Consider the certification level $q^C = q_M^{SB}$. Clearly, $W\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) > \lim_{q_L \to 0} W\left(q_L, q_M^{SB}\right) = W^M(q_M^{SB})$ since $W_{q_L}\left(q, q_M^{SB}\right) > \Pi_{q_L}^L\left(q, q_M^{SB}\right) \geq 0$ for $q \in \left(0, b^L(q_M^{SB}, \alpha)\right)$.

To see that $q^C = q_M^{SB}$ is feasible, note that firm H's profit when producing q^C as $\alpha \to 0$ is $\lim_{\alpha \to 0} \Pi^H\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) = \Pi^M(q_M^{SB})$, which is strictly positive by assumption. Its deviation payoffs are

$$\lim_{\alpha \to 0} \Pi^H\left(b^L(q_M^{SB}, \alpha), b^H\left(b^L(q_M^{SB}, \alpha), \alpha\right)\right) = 0$$
$$\lim_{\alpha \to 0} \Pi^L\left(b^L\left(b^L(q_M^{SB}, \alpha), \alpha\right), b^L(q_M^{SB}, \alpha)\right) = 0.$$

Firm L's payoff when best-responding to q^C from below is $\lim_{\alpha \to 0} \Pi^L\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) = 0$, while its deviation payoff is $\lim_{\alpha \to 0} \Pi^H\left(q_M^{SB}, b^H\left(q_M^{SB}, \alpha\right)\right) = \Pi^H(q_M^{SB}, q_M^{SB}) = -C(q_M^{SB}) < 0$. Therefore, conditions (8) and (9) are satisfied for α sufficiently small. Finally, we have $\max_{q^C \in Q^C} W\left(b^L\left(q^C, \alpha\right), q^C\right) \geq W\left(b^L(q_M^{SB}, \alpha), q_M^{SB}\right) > W^M\left(q_M^{SB}\right)$, establishing the claimed result that certification outperforms an MQS for small α.

The argument for our result that an MQS outperforms certification as $\alpha \to 1$ is that, when all consumers observe quality, certification becomes pointless and cannot improve on the unregulated outcome. Moreover, for $\alpha \to 1$, our model converges to Ronnen (1991), who shows that there exists a welfare-increasing $q^{MQS} > 0$.

Appendix B   Private and social incentives to increase quality

To understand Lemma 2, it is useful to decompose the private and social incentive to increase quality into several parts. We do this here for the high-quality firm; the analysis for the low-quality firm is similar. Rewrite W as

$$W(q_L, q_H) = q_L \int_{x_L^*}^1 \theta \mathrm{d}\theta + (q_H - q_L) \int_{\hat{\theta}^*}^1 \theta \mathrm{d}\theta - C(q_L) - C(q_H).$$

Applying Leibniz' rule and noting that $\int_{\hat{\theta}^*}^1 \theta \mathrm{d}\theta = 1/2 - (\hat{\theta}^*)^2/2 = (1 - \hat{\theta}^*)(1 + \hat{\theta}^*)/2$, the marginal welfare effect of an increase in q_H can be written as

$$W_{q_H} = (1 - \hat{\theta}^*) \frac{1 + \hat{\theta}^*}{2} - \frac{\partial \hat{\theta}^*}{\partial q_H}(q_H - q_L)\hat{\theta}^* - \frac{\partial x_L^*}{\partial q_H} q_L x_L^* - C'(q_H). \tag{13}$$

The first term represents the average high-quality consumer's taste for quality (weighted by the share of consumers buying high quality). The second and third term correspond to the negative effects on welfare that work through the equilibrium prices: an increase in q_H leads to higher prices, leading some consumers to shift from high to low quality and others to leave the market. Formally, we have $\partial \hat{\theta}^*/\partial q_H = 2q_L/(4q_H - q_L)^2 > 0$ and $\partial x_L^*/\partial q_H = 3q_L/(4q_H - q_L)^2 > 0$. The fourth term represents the cost effect.

Now consider the private incentive to raise quality. Writing firm H's revenue per consumer as $R^H(q_L, q_H) = \max_{p_H} p_H (1 - \hat{\theta}(q_L, q_H, p_L, p_H))$ and applying the envelope theorem, we obtain, at $p_L = p_L^* = q_L x_L^*$,

$$R_{q_H}^H = -p_H \left(\frac{\partial \hat{\theta}}{\partial q_H} + \frac{\partial \hat{\theta}}{\partial p_L} \frac{\partial p_L^*}{\partial q_H} \right). \tag{14}$$

From the definition of $\hat{\theta}$, we have $\partial \hat{\theta}/\partial q_H = -\hat{\theta}/(q_H - q_L)$, $\partial \hat{\theta}/\partial p_H = 1/(q_H - q_L)$, and $\partial \hat{\theta}/\partial p_L = -1/(q_H - q_L)$. The first-order condition of firm H's pricing problem gives us $p_H = \frac{1 - \hat{\theta}}{\partial \hat{\theta}/\partial p_H} = (1 - \hat{\theta})(q_H - q_L)$. Replacing these expressions in (14), using $\partial p_L^*/\partial q_H = q_L \partial x_L^*/\partial q_H$, and noting that $\Pi_{q_H}^H = R_{q_H}^H - C'(q_H)$ yields (15). Thus, when all consumers are informed,

$$\Pi_{q_H}^H = (1 - \hat{\theta}^*) \left[\hat{\theta}^* + q_L \frac{\partial x_L^*}{\partial q_H} \right] - C'(q_H). \tag{15}$$

The term in square brackets comprises the marginal consumer's taste for quality, $\hat{\theta}^*$, as well as a competition-relaxing effect of higher quality, which works through the low-quality firm's price, x_L^*. The second term again represents the cost effect.

Subtracting (15) from (13) yields

$$W_{q_H} - \Pi_{q_H}^H = (1 - \hat{\theta}^*) \left[\frac{1 + \hat{\theta}^*}{2} - \hat{\theta}^* \right] - \frac{\mathrm{d}\hat{\theta}^*}{\mathrm{d}q_H}(q_H - q_L)\hat{\theta}^* - \frac{\partial x_L^*}{\partial q_H} q_L (x_L^* + 1 - \hat{\theta}^*). \tag{16}$$

Expression (16) shows that there are several reasons for differences between the social and the private incentive to increase quality. On the one hand, firms care about the marginal consumer whereas the social planner cares about the average consumer, as expressed in the first bracket. Because the average consumer of high quality has a stronger taste for quality than the marginal consumer $((1 + \hat{\theta}^*)/2 > \hat{\theta}^*)$, this tends to give the planner a stronger incentive to increase quality than the firm. On the other hand, an increase in quality leads to higher equilibrium prices. This is detrimental to welfare but beneficial to the firm, and thus tends to give the firm a stronger incentive to raise quality than the planner.

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