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Published in:
Logical Analysis and History of Philosophy

Publication date:
2010

[Link to publication](#)

Citation for published version (APA):
Uckelman, S. L., & Johnston, S. (2010). John Buridan's Sophismata and interval temporal semantics. *Logical Analysis and History of Philosophy*, 13, 133-147.

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John Buridan's *Sophismata* and interval temporal semantics

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June 16, 2010

Abstract

In this paper we look at the suitability of modern interval-based temporal logic for modeling John Buridan's treatment of tensed sentences in his *Sophismata*. Building on the paper [Øhrstrøm 1984], we develop Buridan's analysis of temporal logic, paying particular attention to his notions of negation and the absolute/relative nature of the future and the past. We introduce a number of standard modern propositional interval temporal logics (ITLs) to illustrate where Buridan's interval-based temporal analysis differs from the standard modern approaches. We give formal proofs of some claims in [Øhrstrøm 1984], and sketch how the standard modern systems could be defined in terms of Buridan's proposals, showing that his logic can be taken as more basic.

1 Introduction

The last forty years have seen the development of interval semantics for temporal logics. The stimulus for this development has come from a number of diverse intellectual fields. Philosophically, we can ask whether our analysis of time should be “based on time periods rather than durationless instants” [Burgess 1982, 375]. In linguistics, interval based analyses of some natural language expressions were proposed in [Reichenbach 1966, §51].¹ Interval based temporal logics have also found a number of applications in the fields of artificial intelligence and computer science [Goranko et al. 2004], and they can also be used to provide simpler models of certain problems in physics [van Benthem 1983, 58].

Approaching temporal and tensed statements from an interval-based, as opposed to a point-based, view-point is, however, not new to the 20th century.² An

*The first author was funded by the project “Dialogical Foundations of Semantics” (DiFoS) in the ESF EuroCoRes programme LogICCC (LogICCC-FP004; DN 231-80-002; CN 2008/08314/GW). The second author was funded by the University of Amsterdam, Amsterdam Merit Scholarship.

¹In Reichenbach's three-part analysis of English tenses, the ‘event point’ *E* is not actually a *point* but instead an *interval* or the *duration* of the event being referred to.

²The point-based analysis of temporal statements is, of course, not new to the 20th century either; standard medieval analyses of sophisms involving *incipit* ‘begins’ and *desinit* ‘ceases’ involve point-based structures.

interval-based approach to tensed natural language sentences can be found in the works of John Buridan, a French logician writing in the first half of the 14th century. Buridan’s approach to temporal logic as expressed in his *Sophismata* has been developed previously in [Øhrstrøm 1984]. Øhrstrøm’s paper highlights a number of important, and philosophically interesting, aspects of Buridan’s logic, including Buridan’s definition of truth within a temporal interval [§1] and a distinction Buridan draws between two different kinds of negation [§2].

Øhrstrøm’s paper serves as an excellent starting point for investigating Buridan’s temporal logic, but it is, as the paper itself admits, merely an outline of Buridan’s basic ideas. The paper contains only one reference to Buridan’s actual work, and the logical formalism developed in the paper is done in a fairly informal manner, with minimal reference to other literature on modern interval temporal logics. While the basic ideas are partly expanded upon in [Øhrstrøm et al. 1995], principally in [§§1.3, 1.5], Øhrstrøm’s and Hasle’s discussion there focuses on the material found in the *Sophismata*, without drawing any connections between it and ideas discussed in Buridan’s other works, or between Buridan’s views and those found in modern discussion of interval-based approaches, and their discussion this primarily from a syntactic point of view, without mention of semantics.

Because of Buridan’s idiosyncratic approach to interval-based semantics for temporal logic, and the interesting formal connections between his logic and standard contemporary interval temporal logics, we believe that Buridan’s views deserve a more thorough account, in particular paying attention to the semantic side of modeling Buridan’s approach. Our goal in this paper is to build upon Øhrstrøm’s work to clarify Buridan’s views and to make explicit how his logic compares with contemporary interval temporal logics. First, in §2 we introduce the reader to Buridan’s approach to temporal logic in the *Sophismata*, discussing some specific sophisms to highlight special features of his analysis. In §3, we show how the material in the *Sophismata* connects to related material in Buridan’s other writings, specifically the *Summulae de dialecticae* [Buridan 2001] and *De consequentiis* [Buridan 1976], to place his temporal analysis within the context of his broader logical theory. We give in §4 a brief overview of the development of interval semantics for temporal logic and discuss the various different systems in the current literature so that in §5 we can develop Buridan’s account of temporal logic within the framework of modern interval semantics and apply this formalism to some of the problems Buridan considers. Finally, in §6 we discuss the relevance of Buridan’s logic for modern temporal logic, and give pointers to future work.

2 Interval-based analyses in the *Sophismata*

Buridan’s most important work is the *Summulae de dialectica*, edited in [Buridan n.d.] and translated in [Buridan 2001], written for use as a textbook while he was teaching at the University of Paris between the early 1320s and 1340. He also wrote a *Sophismata*, a work devoted to the analysis and resolution of certain logical puzzles and paradoxes. The *Sophismata* is edited in [Buridan 1977] and translated in [Buridan 2001]. Buridan’s reflections on time and tense are found primarily in the seventh chapter of the *Sophismata*.

Buridan’s account of temporal reasoning in the *Sophismata* is based on rea-

soning within an interval-based logic. The truth of a proposition is analyzed with respect to the ‘present’, but in the *Sophismata*, words such as ‘present’, ‘past’ and ‘future’ all denote intervals of time. The exact length of the various intervals is determined pragmatically. Buridan says in the first sophism of Chapter 7:

Sed tu quaeres quantum est ergo tempus praesens cum non sit instans indivisibile. . . Et ego dico non est nobis determinarum quantum sit tempus praesens quo debeamus uti tanquam praesente. Sed licet nobis uti quanto volumus, vocamus enim istum annum praesentem et hanc diem praesentem et hanc horam praesentem. Et si hac die utamur tanquam praesente, tunc hora prima est et hora meridiei est et hora completorii est, sed successive [Buridan 1977, 113].³

Later he says, *si in aliqua parte temporis praesentis Sortes stat vel est albus vel est mortuus, verum est simpliciter dicere quod ipse stat vel est albus vel est mortuus* [Buridan 1977, 116].⁴ Thus, in his temporal logic, a sentence φ is true within a given interval just in case it is true at some sub-interval of that interval. That is, the proposition ‘Socrates is sitting’ is true just in case there is some part of the present where Socrates is sitting. In other words, the exact range of words like ‘present’ is a conventional issue that is to be taken from the context in which the expression is used.

This definition of truth in an interval gives rise to a number of sophisms that Buridan needs to consider, to ensure that his definition of ‘true in an interval’ is not paradoxical. By considering these two sophisms we will gain a better understanding of Buridan’s conception of the various intervals and how this affects his views of negation.

The first sophism deals with the issue of taking the present interval to be as long or as short as we desire. Let $\varphi :=$ ‘Aristotle argues’ and $\psi :=$ ‘the Antichrist preaches’. Assume, as Buridan did, that the Antichrist will at some point in the future preach, and consider the conjunction $\varphi \wedge \psi$. Is it true? A naïve reaction would be that this conjunction should be seen to be false, since Aristotle is dead, and the Antichrist has yet to preach. However, Buridan says:

Probo quia dictum est quod nobis licitum est uti pro tempore praesenti quantumcumque tempore volumus. . . Respondeo quod sophisma est concedendum in casu in quo tanto tempore volumus uti tanquam praesente quod rationabiliter possumus facere sicut bene arguebatur. Sed quoniam verum est quod nos loquentes communiter de nostris factis quotidianis non solemus uti magno tempore tanquam praesente, sed parvo. . . Tamen ratione utentes et uti volentes tanto tempore tanquam praesente, debemus concedere sophisma et dicere etiam quod Aristotle est mortuus et quod ipse est vivus [Buridan 1977, 117–18].⁵

³“But then you ask: ‘How long is the present time, since there is no indivisible instance?’ . . . And I say that it is not determined for us how much time we ought to use as the present, but we may use as much as we want. For we call this year the present, and this day the present, and this hour the present, and if we use this day as the present, then the first hour is, and noon hour is, and the vesper hour is, but successively” [Buridan 2001, 941–42].

⁴“if during some part of the present time, Socrates is standing or is white or is dead, it is true to say without qualification that he is standing or is white or is dead” [Buridan 2001, 945].

⁵“It has been said that we can use as the present as long a time as we wish. . . I respond

Buridan argues that the conjunction is true as long as the temporal interval for the present is large enough to encompass a time when Aristotle argues and the Antichrist preaches. The oddness of this conclusion is due to the fact that *nos loquentes communiter de nostris factis quotidianus non solemus uti magno tempore tanquam praesente, sed parvo*. In more modern terms, Buridan’s response is that, pragmatically, this sophism represents an abuse of the normal conventions governing the use of the word ‘present’; however when ‘present’ is understood as the sophism requires, the argument is technically true.

The second sophism is the following: First, assume that within some day Socrates sits for a period of time and stands for the rest of it. Let the present range over this day. Now let φ denote the proposition ‘Socrates is sitting’. It follows that φ is true, since there is some sub-interval of the present where Socrates is sitting. However, there is also a sub-interval where φ is false. But then it follows that Socrates is sitting and Socrates isn’t sitting, which is a contradiction (cf. [Buridan 2001, 944]). To this paradox, Buridan replies that

Dico breviter quod haec est vera: ‘Sortes sedet et Sortes stat’ et ‘Sortes est sedens et Sortes est non sedens’ sicut bene probatur [Buridan 1977, 115].⁶

However, from this Buridan argues that the inference to ‘Socrates isn’t sitting’ is false:

‘Quando Sortes stat, ipse non sedet’, quia ibi accipitur ‘quando’ indefinite et sic illa propositio valet istam: ‘In aliquo tempore Sortes stat in quo tempore non sedet’. Et hoc est verum. Sed haec est neganda: ‘Quandocumque Sortes stat, ipse non sedet’, quia valet istam ‘omni tempore in quo Sortes stat in illo non sedet’. Et hoc falsum in casu in quo utimur toto tempore tanquam praesente et quod in prima eius medietate stat et in secunda sed, quia est quoddam tempus in quo sedet et stat. Unde non sequitur: stat ergo non sedet... equus est mortuus ergo non est vivus [Buridan 1977, 116].⁷

Buridan’s response is motivated by how negation distributes a proposition over an interval. The affirmative proposition ‘Socrates is non-sitting’ is true in the present if it is true in some sub-interval of the present, and hence it doesn’t

that the sophism [This conjunction is true: ‘Aristotle argues and Antichrist preaches’] is to be conceded in the case in which we wish to use such a long time as the present and we can reasonably do so, as has been correctly argued. But it is true that when we commonly speak about our everyday dealings, we usually do not use a long time as the present, but a short one... if we follow the argument however, and we wish to use such a long time as the present, we have to concede the sophism and also have to say both that Aristotle is dead, and that he is alive” [Buridan 2001, 942].

⁶ “‘Socrates is sitting and Socrates is standing’, [is true] and so is this: ‘Socrates is sitting and Socrates is non-sitting’ as has been correctly proven” [Buridan 2001, 945].

⁷ “I would easily concede this: ‘when Socrates is standing, he is not sitting’ for ‘when’ is taken here indefinitely, and so this proposition is equivalent to, ‘At some time, Socrates is standing at which he is not sitting’ and this is true. But the following is to be denied: ‘whenever Socrates is standing, he is not sitting’ for this is equivalent to ‘At every time at which Socrates is standing, he is not sitting’ and this is false in a case in which an entire time C would be used as the present, and in the first half of it, he would be standing and in the second he would be sitting, for C would be a time at which he would be sitting and standing. Therefore this is not valid: ‘_____ is standing; therefore _____ is not sitting’... or ‘A horse is dead; therefore it not alive.’” [Buridan 2001, 945].

distribute over the entire present, whereas the negative proposition ‘Socrates isn’t sitting’ distributes over the entire present, in that it is true of an interval if and only if it is true of all sub-intervals. Hence, Buridan can conclude that the inference is invalid:

[I]n affirmativa verbum quantum ad tempus quod consignificat accipitur indefinite. . . Sed in negativa verbum quantum ad tempus consignificatum distribuitur. Ideo si pro aliqua parte temporis praesentis aliquis sedet vel est niger. . . istae sunt falsae, quia ille non sedet vel non est niger. . . Sicut etiam propter divisibilitatem futuri verum est quod A stabit et quod A sedebit, quod erit album et erit nigrum [Buridan 1977, 116].⁸

Thus, Buridan is committed to saying *et dicere etiam quod Aristoteles est mortuus et quod ipse est vivus et ita de Antichristo*.⁹ This demonstrates that Buridan understands his temporal logic as having two different types of negation (cf. [Øhrstrøm 1984, §2]), external negation and internal negation, corresponding to the Aristotelian distinction between sentence negation and term negation. The external negation which occurs in negative propositions such as φ := ‘Socrates isn’t sitting’ ranges over the entire interval: For φ to be true at a given interval I , there must be *no* sub-interval in which Socrates is sitting. In contrast, the internal negation which occurs in affirmative propositions such as ψ := ‘Socrates is non-sitting’, does not distribute: For ψ to be true, there just needs to be some sub-interval at which Socrates isn’t sitting. By drawing this distinction, Buridan is able to block the inference from ‘Socrates is non-sitting’ to ‘It is not the case that Socrates is sitting’.

The second interesting feature of Buridan’s temporal logic is his introduction of both relative and absolute times. In the eighth sophism of Chapter 7, Buridan addresses a problem arising from a claim in Aristotle. Aristotle argues in Physics VI.6.236b32–34 that “whatever moves moved earlier”. However, using Buridan’s temporal analysis, it is possible to construct the following counterargument:

Oppositum arguitur per priorem nostram determinationem quia si utamur hac tota die tanquam praesente, . . . Ponamus ergo quod A nunquam fuit ante istam diem, immo hora prima generabatur, et etiam non erit post istam diem, immo hora vesperorum corrumpetur; et in hora meridiei movetur. Apparet quod A movetur et tamen nunquam movebatur neque movebitur, quia ipsum nunquam fuit et nunquam erit, quia per casum in nullo tempore praeterito fuit, et tamen non fuit nisi in tempore praeterito fuerit. Et eodem modo argueretur si quocumque tempore parvo uteremur tanquam praesente, ponendo quod A praecise in illo parvo tempore moveretur [Buridan 1977, 120].¹⁰

⁸ “[I]n an affirmative [proposition] the verb is taken indefinitely with respect to the consigned time. . . But in a negative [proposition] the verb is distributed with respect to the consigned time; therefore, if during some part of that time someone is sitting or is black. . . the propositions asserting that he is not sitting, or is not black. . . are false. Similarly because of the divisibility of the future it is true that A will be sitting and will be standing, that A will be white and A will be black” [Buridan 2001, 945].

⁹ “both that Aristotle is dead, and that he is alive and the same goes for Antichrist. . .” [Buridan 2001, 946]

¹⁰ “The opposite is argued on the basis of our earlier determination. For if we use this whole

Buridan’s response to this supposed counterexample is to argue that

quod ista nomina ‘praeteritum’ et ‘futurum’ supponentia pro temporibus capi solent aliquando simpliciter et absolute et aliquando respectively. Si ergo capiantur absolute, tunc nulla pars temporis qua utimur tanquam praesente dicenda est praeterita vel futura. . . Aliter accipiuntur ‘praeteritum’ et ‘futurum’ respectively, ita quod praesentis temporis pars prior vocatur praeterita respectu posterioris et pars posterior futura respectu prioris [Buridan 1977, 120].¹¹

Buridan uses this distinction to say that if the past and present are understood absolutely, then Aristotle’s claim is false, and the sophism is correctly proven. However, the relative reading allows for Aristotle’s claim to remain true, since we consider not only the past and present relative to the current day, but we also consider the various parts of the current day.

3 Tensed propositions in Buridan’s other works

While Buridan does discuss time and tense in the *Summulae*, the distinctly interval-based analysis is not explicitly developed. Instead, Buridan discusses a number of questions revolving around the signification of temporal propositions and the supposition of the terms therein. The most explicit discussion of time in the *Summulae* is found in Buridan’s analysis of ‘when’-questions, in his discussion of the Aristotelian categories in treatise 3. He says:

Aliquando etiam respondemus ad ‘quando?’ non per talia aduerbia, sed per nomina declinabilia. . . Et omnia huius modi praedicata pertinent ad hoc praedicamentum, ut ‘hodie’ . . . et ‘heri’ uel ‘cras’ . . . Respondeo quod significant uel connotant certa tempora et distantia eorum ad praesens. . . [Buridan n.d.].¹²

This hardly supports an interval approach to temporal analysis.

De consequentiis contains a more explicit discussion of the analysis of tensed propositions, including their truth conditions, but the analysis is at a fairly superficial level. A present-tensed statement is true *quia qualitercumque ipsa significat ita est, scilicet in re significata in uel in rebus significatis* [Buridan 1976, 17]¹³. A future-tensed sentence, such as *Antichristus praedicabit*, is true *quia ita erit in re sicut propositio significat fore* [Buridan 1976, 17]¹⁴, and similarly

day as the present. . . Let us therefore posit that *A* never existed before today—indeed it is generated in the first hour—and that it will not exist after today, but it is destroyed in the vesper hour and it moves in the noon hour. Then it seems that *A* moves but never moved nor will move for it never was or will be” [Buridan 2001, 949].

¹¹ “the names ‘past’ and ‘present’, which supposit for time, are sometimes taken simply and absolutely, sometimes relatively. If they are taken absolutely then no part of the time that we use as the present should be called past or future. . . In the other way ‘past’ and ‘present’ are taken relatively so that an earlier part of the present time is past relative to a later part [Buridan 2001, 949–50].

¹² “Sometimes we also respond to the question When? not by such adverbs but declinable words. . . And all such predicates pertain to this category, for example ‘today. . . yesterday/tomorrow’ . . . I reply that they signify or connote certain times and their distances from the present. . .” [Buridan 2001, 201].

¹³ “because however it signifies is the case, namely in the thing or things signified.”

¹⁴ “because it will be the case just as the proposition signifies it will be”.

for past tensed sentences. But there is no discussion of what the nature of the present, past, and future are, or how the consequences that this has on the truth conditions for tensed propositions, and our evaluation of them.

We argue that this suggests that Buridan’s temporal analysis in the *Sophismata* is highly pragmatic in nature. Buridan developed this particular analysis of time because it allows him to effectively deal with a number of problems that emerged in the treatment of tensed expressions. This appeal to pragmatic or conventional features of linguistic discourse to provide a basis for a logico-philosophical theory is one of the hall-marks of Buridan’s approach to philosophy and philosophical logic. As Zupko says, “No one before Buridan placed so much of an emphasis on the conventional meaning as a way of both shaping philosophical inquiry and indicating its limits” [Zupko 2003, 22].

Buridan’s temporal logic can be contrasted with various modern approaches which attempt to analyze time as a set of ‘instants’. The choice of an interval-based temporal logic, as opposed to an instant-based one, has important consequences both formally and philosophically, as we will show in this paper (cf. also [Øhrstrøm et al. 1995]). Additionally, from a formal point of view, Buridan’s interval-based approach, as Øhrstrøm notes in [Øhrstrøm 1984, 211], does not require that φ be true at every sub-interval of an interval for it to be true in the interval, in contrast with standard modern approaches to interval semantics for temporal logic. We discuss this and related issues in §4 and §5.

From these observations, it is clear that Buridan’s temporal logic possesses a number of interesting features connected to the pragmatic nature of ‘the present’, the distinction between two types of negation, and the absolute vs. relative nature of time. The various features of Buridan’s logic, namely its definition of truth, the distinction between two types of negation, and the pragmatic features of the logic invite a modern analysis of what exactly is going on. In the next section we introduce modern formal techniques which will allow us to give an analysis of Buridan’s temporal logic in §5.

4 Interval semantics for temporal logics

As we saw in the previous section, the temporal logic that Buridan develops invites formulation using modern interval semantics. In this section we introduce interval semantics, with an eye to capturing the various distinctions that Buridan draws, as well as looking at how his logic relates to modern interval based temporal logics. We will only highlight those temporal logics based on interval semantics which will have an application in the next section. For a more complete overview of modern applications of interval semantics see [Goranko et al. 2004] and the references there.

Given a strict partial order $\mathbb{D} = \langle D, < \rangle$, such as the natural numbers or the real numbers with the usual ordering relation, we define an *interval* in \mathbb{D} as a pair $[d_0, d_1]$ such that $d_0, d_1 \in D$ and $d_0 \leq d_1$ (that is, we restrict our attention to closed intervals). (We will often write I_{d_0, d_1} for the interval $[d_0, d_1]$.) The set of intervals over a partial order \mathbb{D} is designated $\mathbb{I}(\mathbb{D})$. An interval is called *strict* iff $d_1 < d_2$ and a *point* or *instant* iff $d_1 = d_2$. Further, if $d \in [d_1, d_2]$, then $d_1 \leq d \leq d_2$. Given this definition of interval, it is straightforward to define the subset relation:

$$[d_n, d_m] \subseteq [d_k, d_l] \Leftrightarrow \forall d \in [d_n, d_m], \quad d_k \leq d \leq d_l$$

In other words, one interval is contained in another one just in case all of the points in the first interval are also in the second. An *interval structure* is a pair $\mathfrak{D} = \langle \mathbb{D}, \mathbb{I}(\mathbb{D}) \rangle$. There are a number of properties that we might wish to require of our interval structures, such as linearity, density, discreteness, and unboundedness (both above and below); for mathematical definitions of these, see [Goranko et al. 2004, 4–5].

A number of interesting logical systems have been developed to provide analyses of various temporal phenomena. However, for our purposes, we only considered propositional interval temporal logics (ITL). One early ITL was developed in [Burgess 1982]. Burgess takes as his model strict partial orders, and defines strict and non-strict intervals as above.¹⁵ Given this definition of interval, Burgess defines “the natural order relation induced by \leq , namely $[d_0, d_1] \triangleleft [d_2, d_3]$ iff $d_1 \leq d_2$ ” [375].

An interval structure \mathfrak{D} becomes a model \mathfrak{M} with the introduction of a valuation function, V . Burgess considers only valuations that are homogeneous. A valuation function is *homogeneous* if on this valuation all sentences are *homogeneous*. A sentence is *homogeneous* if it is both persistent (or distributive) and cumulative.¹⁶

Definition 4.1. A sentence φ is called *persistent* if it is true over an interval only if it is true in every subinterval.

Definition 4.2. A sentence φ is called *cumulative* if it is true over two overlapping or abutting intervals only if it is true over the sum of the two intervals.

If V is homogeneous, then Burgess extends V to a valuation for complex formulas defined as follows, where a, b, c range over intervals [Burgess 1982, 379]:

$$\begin{aligned} V(\varphi \wedge \psi) &= V(\varphi) \cap V(\psi) \\ V(\neg\varphi) &= \{a : \forall b \subseteq a (b \notin V(\varphi))\} \\ V(G\varphi) &= \{a : \forall b, c ((b \subseteq a \wedge b \triangleleft c) \rightarrow c \in V(\varphi))\} \\ V(H\varphi) &= \{a : \forall b, c ((b \subseteq a \wedge c \triangleleft b) \rightarrow c \in V(\varphi))\} \end{aligned}$$

(The operators F and P are defined as $\neg G \neg$ and $\neg H \neg$, respectively.) From these and the ordering definitions, Burgess introduces an axiomatization for ITL which he proves is sound and complete [Burgess 1982, 380–82]:

- (A1) $G(p \supset q) \supset (Gp \supset Gq)$
- (A2) $P G p \supset p$
- (A3) $G p \supset G G p$
- (A4) $F p \vee F q \supset F(p \vee F q) \wedge F(p \vee q) \wedge F(F p \vee q)$
- (A5) $G p \rightarrow p$

Burgess built his formalization on that given in [Röper 1980], where Röper develops a non-homogeneous temporal logic. A non-homogeneous temporal logic is one in which at least one sentence is not homogeneous. It should be clear

¹⁵Note that Burgess uses the topological expression ‘open interval’ for what we call a strict interval and ‘closed interval’ for a non-strict interval.

¹⁶[Röper 1980, 451]. Röper takes this terminology from Humberstone, whose [Humberstone 1979] formed the basis for [Röper 1980]. These natural language conditions correspond to the formal definitions in [Burgess 1982, 377].

from the preceding section that while sentences in Buridan’s temporal logic are cumulative, they are not persistent, and thus we need a logic which is non-homogeneous. However, as Röper notes [Röper 1980, 459–460], even if we have a non-homogeneous logic, we still may want to be able to form a homogeneous sentences from an arbitrary non-homogeneous sentences. To do this, Röper adds the operation O such that “ $O\varphi$ is true over an interval x iff φ is true for some subinterval z of every subinterval y of x ”, formally:

$$\mathfrak{M}, [d_1, d_2] \models O\varphi \quad \text{iff} \quad \forall [d_3, d_4] \subseteq [d_1, d_2], \exists [d_5, d_6], \mathfrak{M}, [d_5, d_6] \models \varphi$$

As Röper develops his formalism, he notices a ‘problem’ with negation:

[I]t is appropriate to let the negation of a sentence be true for an interval just when that sentence itself is not true for the interval. With negation so defined it can happen that both $O\alpha$ and $O\neg\alpha$ are true over some interval x ; namely in a case in which for every subinterval y of x there is a subinterval z of y for which α is true and also a subinterval z' for which $\neg\alpha$ is true. . . Although there is no actual contradiction involved in the joint truth of $O\alpha$ and $O\neg\alpha$, the motivation for introducing O , as a kind of smoothing out operation, would lead one to regard $O\varphi$ and $O\neg\varphi$ as contraries. Also there seems to be no reason to think that anything that goes on in time and that we might want to describe would have periods of truth and falsehood indefinitely finely intermingled” [Röper 1980, 10].

What Röper has noticed is the same problem that Buridan encountered in sophism 4. Buridan’s analysis of sophisms 4–6 suggests that there are in fact situations where it is desirable for the sentence $O\varphi \wedge O\neg\varphi$ to be true. However, unlike Buridan, Röper dismisses the possibility that there may be a situation where this type of behavior is desirable. He wishes to block the possibility of $O\varphi \wedge O\neg\varphi$ being true, and does so by making the following assumption:

For any interval x there exists a subinterval y such that either α is true for all subintervals of y or α is true for no subinterval of y , where α is any sentence [Röper 1980, 460].

This unfortunate move has the effect of essentially building homogeneity back into the logic, a move which we cannot accept if we wish to accurately model Buridan’s temporal analyses in the *Sophismata*.

We have now seen two simple ITLs, both of which have characteristics which prevent us from using them to model Buridan’s temporal ideas. This is due to the different truth conditions between standard modern ITLs, whose truth conditions do not give the proper truth conditions for the sentence $O\varphi \wedge O\neg\varphi$, which, as we saw in §2, is an important aspect of his treatment of sophism 4, and Buridan’s ITL. This means that we cannot use either of these simple logics as the basis for our formal model of Buridan’s temporal theory. Instead, we modify and extend an ITL called **HS** (for Halpern and Shoham, who first investigated the logic, in [Halpern et al. 1991]). **HS** is the most expressive propositional interval logic with unary modal operators [Goranko et al. 2004, 17], which makes it a natural foundation on which to develop a model for Buridan’s temporal logic. A number of results, including completeness and complexity, for **HS** are known

for **HS** (cf. [Venema 1990]); while we do not show how these results can be adapted to apply to our modified version of **HS** in this paper, we intend to do so in future work.

In **HS**, we have two modal operators $\langle B \rangle$ and $\langle E \rangle$, corresponding to the relations “begins” and “ends”, as well as their inverses, $\langle \bar{B} \rangle$ and $\langle \bar{E} \rangle$. The truth conditions for these operators are defined as follows:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models \langle B \rangle \varphi & \text{ iff } \exists c, a \leq c < b, \mathfrak{M}, I_{a,c} \models \varphi \\ \mathfrak{M}, I_{a,b} \models \langle E \rangle \varphi & \text{ iff } \exists c, a < c \leq b, \mathfrak{M}, I_{c,b} \models \varphi \\ \mathfrak{M}, I_{a,b} \models \langle \bar{B} \rangle \varphi & \text{ iff } \exists c > b, \mathfrak{M}, I_{a,c} \models \varphi \\ \mathfrak{M}, I_{a,b} \models \langle \bar{E} \rangle \varphi & \text{ iff } \exists c < b, \mathfrak{M}, I_{c,b} \models \varphi \end{aligned}$$

In **HS** we can also define a constant π which is true only in point intervals:

$$\mathfrak{M}, I_{a,b} \models \pi \text{ iff } a = b$$

We will use π in the next section when we define the truth conditions of Buridan’s tense operators.

5 Application

As in the previous section, we take as models pairs $\mathfrak{M} = \langle \mathfrak{D}, V \rangle$, where $\mathbb{D} = \langle \mathbb{R}, \leq \rangle$, with \leq being the normal ‘less than’ ordering on \mathbb{R} . We define ‘interval’ and \subseteq as in the previous section, and let $\mathbb{I}(\mathfrak{M})$ be the set of intervals in \mathfrak{M} .

Truth is defined relative to a model and an interval in the model.¹⁷ When no confusion will result, we’ll drop mention of \mathfrak{M} , and speak of the truth of formulas with respect to arbitrary intervals. Additionally, when $\mathfrak{M}, I \models \varphi$ for every $I \in \mathbb{I}(\mathfrak{M})$, we will omit reference to I and just write $\mathfrak{M} \models \varphi$. The truth conditions for the two types of negation used by Buridan are as follows:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models p & \text{ iff } \exists I_{c,d} \subseteq I_{a,b}, I_{c,d} \in V(p) \\ \mathfrak{M}, I_{a,b} \models \bar{p} & \text{ iff } \exists I_{c,d} \subseteq I_{a,b}, I_{c,d} \notin V(p) \\ \mathfrak{M}, I_{a,b} \models \neg p & \text{ iff } \forall I_{c,d} \subseteq I_{a,b}, I_{c,d} \notin V(p) \\ \mathfrak{M}, I_{a,b} \models \neg \bar{p} & \text{ iff } \forall I_{c,d} \subseteq I_{a,b}, I_{c,d} \in V(p) \end{aligned}$$

If we wish to remove the \bar{p} notation, we can do so by introducing a hybrid operator $I_{x,y}$:¹⁸

$$\begin{aligned} \mathfrak{M} \models I_{a,b} p & \text{ iff } \mathfrak{M}, I_{a,b} \models p \\ \mathfrak{M} \models I_{a,b} \neg p & \text{ iff } \mathfrak{M}, I_{a,b} \models \bar{p} \\ \mathfrak{M} \models \neg I_{a,b} p & \text{ iff } \mathfrak{M}, I_{a,b} \models \neg p \\ \mathfrak{M} \models \neg I_{a,b} \neg p & \text{ iff } \mathfrak{M}, I_{a,b} \models \neg \bar{p} \end{aligned}$$

Introducing this operator is acceptable because the distinction between the two types of negation only holds at the atomic level; at the level of more complex

¹⁷Note that doing so assumes that all agents using this model agree on which intervals will count as “present”, “future”, and “past”. In the seventh sophism, Buridan discusses what happens when this is not the case, when, e.g., *Sortes sola hac hora huius diei prima utatur tanquam praesente et Plato utatur tota die tanquam praesente* [Buridan 1977, 119] (“Socrates uses only that first hour of this day as the present, and Plato uses the whole day as the present” [Buridan 2001, 948]).

¹⁸This operator should not be confused with the I operator introduced in [Øhrstrøm 1984], which stands for ‘is’, not ‘interval’, and is intended to represent the verb in a subject predicate sentence.

formulas, the only type of negation is the standard negation. This is because the propositional atoms in our language represent categorical propositions, that is, sentences of the form ‘*Subject is predicate*’, such as ‘Antichrist exists’ or ‘Socrates is sitting’. The internal structure of categorical propositions allows us to make the distinction between ‘It is not the case that Socrates is non-sitting’ and ‘It is the case that Socrates is non-sitting’. However, this distinction is one that only makes sense at the atomic level; it cannot be extended in any natural fashion to complex formulas. This means that extending the truth conditions to cover boolean operations on arbitrary formulas is straightforward and can be done in the expected fashion:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models \neg\varphi & \quad \text{iff} \quad \mathfrak{M}, I_{a,b} \not\models \varphi \\ \mathfrak{M}, I_{a,b} \models \varphi \wedge \psi & \quad \text{iff} \quad \mathfrak{M}, I_{a,b} \models \varphi \text{ and } \mathfrak{M}, I_{a,b} \models \psi \end{aligned}$$

The connectives \vee and \rightarrow are defined in the usual way.

As we discussed in §2, in Buridan’s logic, there are two ways we can speak about the notions ‘past’, ‘present’, and ‘future’. We can take such expressions to be absolute or relative. When we take the terms to be absolute, then future and past are absolutely disjoint from the present, but when we taken them relatively, then parts of the present can be future or past relative to other parts of the present. We distinguish these by introducing the operators F_R and P_R , and F_A and P_A , for the relative and absolute tenses, respectively.

We define F_A and P_A so that the present is not included in the scope:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models F_A \varphi & \quad \text{iff} \quad \exists I_{c,d} \in \mathbb{I}(\mathfrak{M}), I_{a,b} \triangleleft I_{c,d} \text{ and } \mathfrak{M}, I_{c,d} \models \varphi \\ \mathfrak{M}, I_{a,b} \models P_A \varphi & \quad \text{iff} \quad \exists I_{c,d} \in \mathbb{I}(\mathfrak{M}), I_{c,d} \triangleleft I_{a,b} \text{ and } \mathfrak{M}, I_{c,d} \models \varphi \end{aligned}$$

These are the same as the $\langle A \rangle$ ‘after’ and $\langle B \rangle$ ‘before’ operators from [Halpern et al. 1991, 938].

The converse operations G and H are defined in the usual way as $G\varphi := \neg F \neg\varphi$ and $H\varphi := \neg P \neg\varphi$ respectively. This corresponds to the following truth conditions, for the absolute cases:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models G_A \varphi & \quad \text{iff} \quad \forall I_{c,d} \in \mathbb{I}(\mathfrak{M}), \text{ if } I_{a,b} \triangleleft I_{c,d}, \text{ then } \mathfrak{M}, I_{c,d} \models \varphi \\ \mathfrak{M}, I_{a,b} \models H_A \varphi & \quad \text{iff} \quad \forall I_{c,d} \in \mathbb{I}(\mathfrak{M}), \text{ if } I_{c,d} \triangleleft I_{a,b}, \text{ then } \mathfrak{M}, I_{c,d} \models \varphi \end{aligned}$$

The truth conditions for Burgess’s $G\varphi$ can, given the constraint on V discussed in the previous section that Burgess requires, be rewritten so that they are analogous to the above as follows:

$$\mathfrak{M}, I_{a,b} \models G\varphi \quad \text{iff} \quad \forall I_{c,d}, I_{c,d} \subseteq I_{a,b} \text{ if } I_{c,d} \triangleleft I_{e,f} \text{ then } \mathfrak{M}, I_{e,f} \models \varphi$$

From this it is clear that Buridan’s temporal operators differ from Burgess’s by removing the requirement that the right-hand condition hold for every subinterval of $I_{a,b}$. This is in keeping with Buridan’s existential view of truth in an interval, as opposed to the modern view of truth in an interval, which is universal.

Informally, Buridan’s definition of relative past and future means that $I_{a,b} \models P_R \varphi$ if there is some subinterval $I_{c,d} \subseteq I_{a,b}$ and some interval $I_{e,f} \triangleleft I_{c,d}$ such that $e < a$ and $I_{e,f} \models \varphi$. As Øhrstrøm notes [Øhrstrøm 1984, 215], the truth conditions of the relative tense operators can be defined in terms of the absolute ones:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models F_R \varphi & \quad \text{iff} \quad \exists I_{c,d} \subset I_{a,b}, \mathfrak{M}, I_{c,d} \models F_A \varphi \\ \mathfrak{M}, I_{a,b} \models P_R \varphi & \quad \text{iff} \quad \exists I_{c,d} \subset I_{a,b}, \mathfrak{M}, I_{c,d} \models P_A \varphi \end{aligned}$$

More interestingly, we can also define the truth conditions of the relative operators in terms of $\langle \bar{B} \rangle$ and $\langle \bar{E} \rangle$, thus connecting Buridan's logic with that of **HS**:

$$\begin{aligned} \mathfrak{M}, I_{a,b} \models_{\text{FR}} \varphi & \text{ iff } \exists I_{c,d} \subset I_{a,b} \text{ s.t. } \mathfrak{M}, I_{a,c} \not\models \pi \text{ and } \mathfrak{M}, I_{c,d} \models \langle \bar{B} \rangle \varphi \\ \mathfrak{M}, I_{a,b} \models_{\text{PR}} \varphi & \text{ iff } \exists I_{c,d} \subset I_{a,b} \text{ s.t. } \mathfrak{M}, I_{d,b} \not\models \pi \text{ and } \mathfrak{M}, I_{c,d} \models \langle \bar{E} \rangle \varphi \end{aligned}$$

Proving that these definitions are equivalent is straightforward:

Theorem 5.1. Let $I_{a,b} \in \mathbb{I}(\mathfrak{M})$ be arbitrary. Then there is an $I_{c,d} \subset I_{a,b}$, $\mathfrak{M}, I_{c,d} \models \text{F}_A \varphi$ iff there is an $I_{g,h} \subset I_{a,b}$ such that $\mathfrak{M}, I_{a,g} \not\models \pi$ and $\mathfrak{M}, I_{g,h} \models \langle \bar{B} \rangle \varphi$.

Proof. (\Rightarrow) Assume there is $I_{c,d} \subset I_{a,b}$, $\mathfrak{M}, I_{c,d} \models \text{F}_A \varphi$. Since $I_{c,d} \models \text{F}_A \varphi$, we know that there is $I_{e,f} \triangleright I_{c,d}$ such that $I_{e,f} \models \varphi$. Since $I_{c,d} \subset I_{a,b}$, either (1) $a \neq c$ or (2) $b \neq d$.

- (1) Either $d \leq f$ or $d = f$. If $d \leq f$, then the desired $I_{g,h}$ is $I_{c,b}$. It is clear that $I_{a,c} \not\models \pi$. If $d < f$, then $I_{c,b} \models \langle \bar{B} \rangle \varphi$ because $I_{c,f} \models \varphi$, since $I_{e,f} \subseteq I_{c,f}$ and $b < f$. If $b = f$, then pick some $f' > f$, then since $I_{c,f} \subset I_{c,f'}$, the same argument holds with f' substituted for f .
- (2) Either $a \leq c$ or $a = c$. If the former, then the case is as in (1). If $a = c$, then the desired $I_{g,h}$ is $I_{d,b}$, with the argument continuing as in (1).

(\Leftarrow) Assume there is $I_{g,h} \subset I_{a,b}$ such that $\mathfrak{M}, I_{a,g} \not\models \pi$ and $\mathfrak{M}, I_{g,h} \models \langle \bar{B} \rangle \varphi$. This means that there is an $f > h$ such that $I_{g,f} \models \varphi$. If $I_{g,f} \models \varphi$ then there is some $I_{c,d} \subseteq I_{g,f}$ such that $I_{c,d} \models \varphi$. Either (1) $g < c$ or (2) $c = g$.

- (1) Our desired interval is $I_{a,g}$. $I_{a,g} \subset I_{a,b}$ and since $I_{a,g} \triangleleft I_{c,d}$ and $I_{c,d} \models \varphi$, $I_{a,g} \models \text{F}_A \varphi$.
- (2) Since our models are based on \mathbb{R} , and hence are dense, pick some e such that $a < e < g$; then our desired interval is $I_{a,e}$ and the argument continues as in (1).

The proof for P is equivalent. \square

Given these definitions, there are a number of features about Buridan's analysis of temporal logic we can express. First, we can prove the following theorem:

Theorem 5.2. If there is $I_{a,b}, I_{c,d}$ such that $I_{a,b} \models \varphi$ and $I_{c,d} \models \psi$, then $I_{a,d} \models \varphi \wedge \psi$.

Proof. Assume the antecedent. Now, observe that $I_{a,b} \subseteq I_{a,d}$ and $I_{c,d} \subseteq I_{a,d}$, hence, since $I_{a,b} \models \varphi$ and $I_{c,d} \models \psi$ it follows, by the definitions of $I_{a,d}$ and \wedge that $I_{a,d} \models \varphi \wedge \psi$. \square

It follows as an easy corollary that sentences of the form $I_{a,b}(\varphi \wedge \neg\varphi)$ can be consistent in this logic.¹⁹ This corresponds to Buridan's observation that

¹⁹There is an interesting parallel here between Buridan's temporal logic and modern paraconsistent logics (for an overview of paraconsistent logic, see, e.g., [Priest 2002] and [Priest et al. 1989]). In paraconsistent logics the principle of explosion $\models \varphi \wedge \neg\varphi \rightarrow \psi$ is not generally valid. In this temporal logic, the principle of explosion fails, but in a much weaker way. While it is possible for $\models \varphi \wedge \neg\varphi$ to be true, the principle of explosion still holds for claims of the form $\models I_{a,b}\varphi \wedge \neg I_{a,b}\varphi$.

the expression ‘Socrates is sitting and Socrates is non-sitting’ is a consistent expression.

Second, as we noted in §2, Buridan conceives of the past and the future in both an absolute and a relative sense. He needs this distinction in order to discuss sophisms arising from Aristotle’s observation in the *Physics* that ‘all which is moved was moved previously’; he argues that this is false *qua* the absolute past but is true *qua* the past relative to the interval designating the present. We show formally how this can be the case. Let x be an object that never existed before today. Suppose that x comes into being at time a and ceases to be at time b , and that x moves at $t = \frac{1}{2}(a + b)$. Let p denote the sentence ‘ x is moved’, and let the interval $I_{a,b}$ range over a given day. Then the formula $I_{a,b} p \rightarrow P_A p$ is false, since $I_{a,b} \models p$, but for all c , $I_{c,a} \not\models p$, since x does not exist before $I_{a,b}$. However, as Buridan points out, if here we are using the expression ‘was moved previously’ in the relative sense, then the sentence is true:

Proof. $I_{a,b} \models p$ implies that there is an $I_{c,b} \in \mathbb{I}(\mathfrak{M})$ with $c < a$ such that $I_{c,b} \models p$, because $I_{c,t} \models p$ and $I_{c,t} \subseteq I_{c,b}$, and hence $I_{a,b} \models P_R p$. \square

6 Concluding remarks

Buridan’s approach to temporal logic is distinctive for a number of reasons. Buridan’s treatment of time is innovative in that it is strongly influenced by pragmatic considerations deriving from his attempts to analyze various temporal sophisms in an interval-based setting. Formally, the logic is interesting; because of the weakness of Buridan’s truth conditions, the ITL that arises from Buridan’s analysis of time lies in an area of modern ITLs that has not been widely explored.²⁰ There is a simple structural symmetry between the system developed here and the system **HS**, based in the inversion of the truth conditions for truth and false sentences, which lays the basis for fruitful interaction between Buridan’s temporal logic and **HS** in both directions. Looking from **HS** to Buridan’s system, it should be not too difficult to extend the completeness and complexity results for **HS** to Buridan’s logic. In the other direction, the distinction between the relative and absolute tenses can be added to **HS** to give this system a broader range of philosophical application. We hope to pursue both of these strands in the future, along with the point raised in footnote 17, namely that different agents could use different intervals to count as “the present”.

Acknowledgments

We would like to thank Benedikt Löwe, Valentin Goranko, and Peter Øhrstrøm for their useful feedback on an earlier version of this paper. We also gratefully acknowledge the suggestions of the anonymous referees.

²⁰The only reference we found to a modern consideration of truth conditions like Buridan’s is in [van Benthem 1983]. Van Benthem notes that when moving from a point-based structure to a period- (or interval-) based structure, we must ask “should the truth of φ extend downward, to subperiods—or maybe upward, to superperiods? . . . [T]he upward direction seems less relevant; and hence attention will be restricted to the [downward] case” [193–194].

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