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Abstract

I study the capacity of business associations—private, formal, noncommercial organizations designed to promote the common business interests of their members—to support contract enforcement and collective action. Inspired by recent empirical literature, my theoretical framework connects the organizational and institutional features of formal and informal business organization with socioeconomic distance. I show how associations provide value to their members even if members are already embedded in social networks, and which players join an association. I propose explanations for empirical puzzles, put forward novel testable hypotheses, and relate business associations to alternative private ordering institutions.

JEL classification: D02, D71, L14, L31
Keywords: Business Associations, Trade Associations, Economic Governance, Private Ordering, Arbitration, Merchant Guilds

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1 Introduction

A business association is a private, formal, noncommercial organization designed to promote the common business interests of its members.\footnote{This definition draws on Pyle (2005, 2006) and incorporates most nonprofit professional clubs, trade unions, trade associations, chambers of commerce, industry trade groups, and medieval merchant guilds. It is related to the study of cooperatives, which are often for-profit organizations, though.} Throughout the last millennium traders have formed associations to represent themselves vis-a-vis other parties and to facilitate collective action. Associations offer members a platform to meet and to exchange views about other industry participants (Doner and Schneider, 2000; Pyle, 2006), to learn about the latest technologies, foreign markets and standardizations (Nugent and Suklassyan, 2009) and prospective trade partners (Macaulay, 1963; Johnson et al., 2002). Some associations offer their members arbitration services and help to resolve disputes, which mitigates transaction costs (Woodruff, 1998; Pyle, 2005). In supporting honest trade both between members and between members and nonmembers, associations serve as substitutes for ineffective legal systems in developing countries (Kali, 1999) and have provided more effective private legal systems in specific industries, such as the U.S. cotton and diamond trading industries, in developed countries (Bernstein, 1992, 2001).\footnote{Associations also deal with public authorities and lobby government officials with one voice on behalf of their membership, protect members from illegitimate government interference (Pyle, 2011), and increase the level of trust among members in general (Raiser et al., 2008). By pushing governments for the provision of public goods, associations increase their members’ joint impact on institutional reform (Lambsdorff, 2002).}

These functions are not restricted to modern business associations. At the beginning of the Commercial Revolution in Europe, when long-distance trade started to boom in the tenth and eleventh centuries, the primary function of the first merchant guilds was to protect the property rights of their members vis-a-vis nonmembers (Volckart and Mangels, 1999), in particular vis-a-vis predatory rulers (Greif et al., 1994). Formal associations emerged that “helped long-distance traders solve two fundamental problems of exchange—on the one hand, protection against crime, warfare, and arbitrary confiscation and, on the other, the enforcement of contracts whenever money or goods changed hands.” (Grafe and Gelderblom, 2010:477). Merchant associations “existed not just in Europe but also in North Africa, the Near East, Central and South America, India and China.” (Ogilvie, 2011:1).\footnote{Today there are more than 7,800 national associations in the US (National Trade and Professional Associations Directory, 2011), some 750 at the EU-level, and more than 11,700 national associations in the EU (The Directory of Trade and Professional Associations in the European Union, 2004).}

Recently, the empirical literature studying the economics of business associations has produced new and intriguing findings. It has also identified several puzzles, which stand to be
explained by theory. For instance, business associations are reported to be less valuable for members engaged in local transactions and more valuable in long-distance transactions in Eastern Europe (Pyle, 2005, 2006). Associations are perceived to be less valuable to their members in more competitive industries (Pyle, 2005). Economic history research on merchant guilds and related institutions in the European middle ages has produced empirical findings that are surprisingly similar to those found by studies on modern associations (see Section 2 for details).

What forces drive these results? How do private informal institutions, such as social networks, and formal institutions, such as business associations, both of which transmit relational information about traders’ business conduct, interact? How do these private institutions perform in the shadow of a public court system? Who joins a business association and who does not? What is the role of product market competition and how does the distance between traders influence the choice of the optimal contract enforcement institution?

In comparison to the advances of the empirical literature, the theoretical literature has fallen somewhat behind in answering these questions. In particular, what is lacking is a theoretical framework that connects the organizational and institutional features of formal and informal business organization with the notion of socioeconomic distance, which lies at the heart of several empirical findings, as I will show below.

Dixit (2003b) has laid the foundations for a solution to this problem by introducing a model where the location of the players in socioeconomic space is represented by a circle. I adopt Dixit’s idea of a circle economy, to capture differences in the knowledge and abilities of players who repeatedly face opportunities to transact with new and unfamiliar business partners. In addition to a location on the circle, every player is endowed with an individual level of connectedness, which serves as a proxy for the embeddedness in social networks. Players can choose whether to join an association, or not. I model associations that serve either as a repository for information about the business conduct of members’ trade partners or which offer their members arbitration services in case of disputes with their partners. The value-generating transaction is a Prisoner’s

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4See Granovetter (1985) for the original conceptualization of embeddedness. My framework could also contribute to closing a gap that sociologists have accused economists of for a long time, namely that economic models lack social structure and the integration of social relations. See the discussion in Powell (1990) and especially in Nee (2003).

5A related function of associations is to define what the terms of an agreement between two players mean, as Bernstein (2001) emphasizes. MacLeod (2007:596) confirms, “the evolution of successful informal agreements depends upon a number of interlocking elements, including a mutual understanding of the events that determine contract breach”. Gibbons and Henderson (2011) call it the clarity problem, which complements the credibility problem that I study here. In the same spirit, Hadfield and Weingast (2011a,b) study the capacity of legal orders to serve as a coordination device among players for which behavior is (not) acceptable. Given its importance and
Dilemma, which can be interpreted in two ways: as a vertical relationship involving a seller and a buyer, where “cooperation” means to keep one’s contractual obligations; and as a horizontal relationship involving two players faced with a collective action problem, where “cooperation” means to forego the short-term gains from free-riding but to contribute to a public or club good, for instance to join in a boycott of a government that did not respect the property rights of another player.\(^6\) The value generated by a certain type of association for a member depends on the range of potential partners for which the association can sustain cooperation in equilibrium, which depends on the individual connectedness.

My main findings show that associations indeed offer value to their members even if these are already connected informally to others via social networks. Associations use a different channel distributing information about players’ behavior than social networks. However, I find that the value of association membership decreases if transactors are better connected informally. I also endogenize the players’ membership decisions and show that players with low informal connectedness choose to join an association but players with high connectedness do not. This implies that social networks and associations are substitutes with respect to supporting cooperation. Moreover, I find that associations are a hybrid between social networks and public courts, which may be particularly relevant in practice if either of these two institutions does not exist or operate effectively. Depending on the competence of arbitration tribunals relative to the credibility of the unverified information distributed by information intermediaries, I show when one or the other association function creates more value for members.

My model explains several empirical findings by showing how the value created by associations for its members decreases if competition on product markets is intense or if transactions in an economy take place between local partners. I also show that the primary beneficiaries of associations’ activities are their members but that nonmembers also benefit to some extent because their commitment ability to cooperate increases if their partner is a member.\(^7\)

The next section provides an overview over the issues and the related literature. Section 3 introduces the model and analyzes associations serving as information intermediaries and as arbitrators, respectively. Section 4 relates associations to alternative private ordering institutions, suggests explanations for some empirical puzzles, and states testable hypotheses. Section complexity, I defer a formal analysis of the clarity problem in the context of associations to future research.

\(^6\)See Dixit (2003b) for the former and Baron (2010) for the latter interpretation.

\(^7\)These results underline the positive effects of private ordering institutions for the transactors involved: where noncontractibility or prohibitive transaction costs make court enforcement no available option, private governance institutions can mitigate Prisoner’s Dilemma problems. See Dixit (2004, 2009) and Williamson (2005) for general overviews of the New Institutional Economics approach to private ordering. The analysis of the normative implications of associations for nonmembers and total welfare is the subject of ongoing research.
2 Business associations and economic governance

The study of economic governance issues is a central theme of the literature on private ordering. The key questions posed in this literature are, how can opportunistic behavior be avoided in Prisoner’s Dilemma situations, where the joint payoffs of the players are maximized under mutual cooperation but it is individually rational to defect and thereby to maximize one’s own payoff at the expense of others? How should institutions be structured such that the incentives of individual players to free-ride on their companions’ efforts are mitigated?

Many institutions exist, both in theory and in practice, that can support cooperation and mitigate free-riding. Masten and Prüfer (2011) propose a classification of such commitment mechanism, ranging from internal value systems—players cooperate because they like it—to public courts—players cooperate because they want to avoid high penalties and imprisonment. Between these extremes, there are several types of communities, all of which enforce cooperation by threatening defectors with ostracism. Both decentralized social networks and centralized associations with different functions are classified as communities.

2.1 Empirical economic research on associations

In this paper, I study the relative capacities of associations to support cooperation among their members and compare them with the capacities of social networks and courts. More specifically, I focus on associations that are independent of state interference and where membership is voluntary. A key question is, what value do associations offer members if those are already embedded in social networks, an approach that identifies the incentives to join an association and, thereby, makes the set of members endogenous. This question has already been studied in empirical work. Survey-based analyses of post-communist countries, for instance, have shown

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8See Ostrom (1990), Williamson (2000), or Dixit (2009). Dixit (2009:5) defines the concept of economic governance as “the structure and functioning of the legal and social institutions that support economic activity and economic transactions by protecting property rights, enforcing contracts, and taking collective action to provide physical and organizational infrastructure.”

9Dixit (2004), Fafchamps (2004), and Greif (2006) provide instructive overviews.

10Communities support cooperation by providing the information channels and incentives for individuals not to transact with defecting others—in contrast to public courts, which have access to the state’s monopoly on coercion and can punish defectors more directly.

11The former requirement excludes several associations in autocracies, the latter excludes several chambers of commerce in Western democracies, where membership is mandatory for all firms (for instance, in Germany).
that small and medium-sized firms join business associations, which comes at a cost, although they are already connected to other industry participants via informal networks (Johnson et al., 2002).\textsuperscript{12}

Pyle (2005) also studies flows of relational information between firms in five Eastern European countries. He underlines that “both manners of information flows—those that are enabled by formal organizations and those that are not—have been recognized for their ability to serve as the basis for relational contracting, to reduce search costs, and to mitigate information asymmetries […]. However, the economics literature is largely silent as to how the two interact” (548). He finds that associations support information flows in business relations of their members with firms that are located at some distance. In the relationship with local customers, however, associations add only very little value via their function as information repositories. Pyle (2006) confirms that geographic distance is associated with higher dispute-related costs, which are smaller for firms that are members of an association. Recent empirical economic history research about the comparative use of informal and formal commitment mechanisms in the European middle ages finds related results. Studying institutions for the protection of property rights in the Italian Alps between the thirteenth and the nineteenth centuries, Casari (2007) shows that the likelihood of a formal institution’s being established increases with a community’s size, its proximity to other settlements, and the amount of its common resources.

Another important finding of Pyle (2005) is that associations contribute less to the spread of information about contractual disputes when their members’ markets are particularly competitive. Grafe and Gelderblom (2010) obtain a similar result in their study of European merchant guilds: more intense product market competition between mercantile groups and local merchants is associated with a lower degree of control delegation from merchants to guilds, which implies that guild membership has lower value if competition is intense.

My paper complements these studies in the sense that it takes their empirical findings as a starting point and suggests a model that identifies the channels through which associations might offer value even if decentralized social networks among transactors already exist, and how that value depends on the distance between partners and the level of competition on product markets.

\textsuperscript{12}Johnson et al. (2002:230/1) state: ”Almost half of the firms we surveyed are members of a trade association. […] Start-up firms are as likely as privatized firms to be members of trade associations […], which suggests the services the association offers are valuable.”
2.2 Related theoretical literature

Starting about two decades ago, a literature using game-theoretic tools for the study of private ordering institutions, often complemented by the analysis of historical records and drawing on methodology from political science and sociology, has grown. An intensely studied institution to overcome collective action problems if court or other external enforcement mechanisms are not available are decentralized social networks.\textsuperscript{13} Associations with various functions—but all of them private, formal, noncommercial organizations—have also received some scholarly attention.\textsuperscript{14}

A major innovation to this literature was contributed by Dixit (2003b), who put forward the idea of a circle model of the economy, where the distance between two players denotes socioeconomic differences, which can be interpreted widely (1296). This model allows to express the value that a certain institution generates for the players as the maximum distance between two randomly matched players up to which mutual interaction and cooperation characterizes an equilibrium.\textsuperscript{15}

The circle economy model has been applied widely. Dixit (2003b) compares the scope of cooperation among transactors when making use of a decentralized social network and external enforcement. Leeson (2008b) shows how the scope of cooperation can be increased if transactors’ location on the circle is endogenous. Tabellini (2008) analyzes how values and institutional development interact and evolve over time. Baron (2010) models pro-social preferences that depend on the distance between partners and studies how social label and certification organizations, amongst others, can increase cooperation. My paper complements Baron’s in the sense that I analyze how certain private organizations can increase cooperation if the players have standard preferences and by endogenizing players’ membership decisions in associations. Masten and Prüfer (2011) study how the scope of cooperation supported by informal social networks and formal public courts interact. I complement that paper by studying communities that are hybrids of social networks and courts.

\textsuperscript{13}See Kandori (1992), Greif (1993), and Ellison (1994) for early examples.

\textsuperscript{14}See Milgrom et al. (1990), Klein (1992), Greif et al. (1994), Kali (1999), and Dixit (2003a).

\textsuperscript{15}This maximum distance is called the extent of honesty (Dixit, 2003b), the scope of self-regulation (Baron, 2010), or the scope of cooperation (Masten and Prüfer, 2011; this paper), depending on the focus of the authors.
3 The Model

3.1 Matching, trading, and social networks

My analysis of the capacities of business associations to sustain cooperation draws on Dixit (2003b). Consider an economy consisting of a continuum of players uniformly distributed around a circle with a circumference of 2. The mass of players per unit arc length is normalized to one, implying a mass 2 population in the economy. The distance between two players, \( X \), measured by the shorter of the two arc lengths between them (hence, \( X \leq 1 \)) can be interpreted as representing differences in any relevant economic or social variables such as technological or resource endowments, knowledge or expertise, or kinship or other social or cultural affinities, as well as geographic location. \( X \) affects three considerations in the model, which are specified below: the probability of meeting a given player, the potential gains from interaction, and the probability of receiving information about the previous behavior of other players via social networks.

Every player \( i \) is endowed with an individual level of connectedness, \( \kappa_i \), which is drawn from a continuous distribution, \( z \), over \([0, 1]\), and common knowledge. Let \( Z(\kappa_i) \) denote the cumulative distribution function of \( z \). \( \kappa_i \) determines the probability with which \( i \) can send a message about the behavior of his partner along the circle. Abusing notation slightly, I will also refer to \( \kappa_i \) as the strength of \( i \)'s social network, or \( i \)'s embeddedness.\(^{17}\)

I model an infinitely repeated game, where the time between periods proceeds in discrete intervals, \( t \in \{0, 1, 2, \ldots, \infty\} \), players live forever and have a uniform per-period discount factor, \( \delta \in (0, 1) \). At the beginning of every period, each player is randomly matched with another player at socioeconomic distance \( X \) with probability density: \(^{18}\)

\[
\mu \equiv \frac{e^{-X}}{2(1 - e^{-1})} \tag{1}
\]

Every pair of matched players has to decide simultaneously whether or not to transact. Only

---

\(^{16}\)The general structure of my model follows the infinite-period model by Masten and Prüfer (2011), who build on the two-period model by Dixit (2003b). These papers contain a comprehensive discussion of the main model assumptions. Appendix A compares my model with Dixit’s.

\(^{17}\)Given that I employ a model with a mass of players, I cannot characterize social networks by a set of specific players linked to each other, unlike in graph theory where finite sets of discrete players are modeled (Goyal, 2007:9). As will be clear below, the only important characteristic of social networks used here is the relative strength of the players’ embeddedness, \( \kappa_i \) vs. \( \kappa_x \), which enables them to threaten their interaction partners to inform more or less other players about the partner’s behavior.

\(^{18}\)The matching probability between two players is exponentially decreasing, that is, there is localized matching. \( \int_{0}^{1} \mu dX = \frac{1}{2} \) (over each arc length), that is, the expected probability for every player \( i \) in every period \( t \) to be matched with another player is 1.
if both agree to transact, each decides whether to cooperate or defect in the central transaction. The payoff to each player is $ae^{\theta X}$, where $a$ results from the Prisoner’s Dilemma depicted in Figure 1 and $\theta > 0$. That is, the potential gains from interaction increase in the distance between the partners, $X$, but the rate of increase depends on market conditions, captured by $\theta$.\footnote{The potential loss from a transaction also increases in $X$, which can be understood as a consequence of the travel cost that are lost if a transaction does not take place cooperatively.}

<table>
<thead>
<tr>
<th>$i / x$</th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>$h, h$</td>
<td>$l, w$</td>
</tr>
<tr>
<td>Defect</td>
<td>$w, l$</td>
<td>$d, d$</td>
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Figure 1: Payoffs in the Prisoner’s Dilemma. Assume $w > h > 0 > d > l$ and $2h > w + l$.

I assume that every player is embedded in a social network and automatically sends a message, about the behavior of his partner, after the central transaction, independent of using other enforcement institutions.\footnote{Rephrased, I assume that players cannot credibly commit not to inform the network about their partner’s behavior. As is common in the literature (Kandori, 1992; Kali, 1999; Dixit, 2003b), I also assume that reporting via the social network, which encompasses a player’s friends and close business contacts, is truthful. This assumption is relaxed in Section 3.2. See also Footnote 22.} Specifically, I assume that every player reports the identity of his partner and the partner’s behavior chosen from the message space \{cooperated, defected, did not interact\}. The probability that player $y$ receives player $x$’s announcement is:

$$\eta_{x,y} \equiv \kappa_x e^{-|Y-X|},$$

(2)

where $|Y - X|$ is the distance between players $y$ and $x$. I solve this model as a special case of the information intermediary model in the next subsection.

### 3.2 Business associations as information intermediaries

All players are embedded in social networks, as described above. Additionally, they can become members of a business association that has only one function: to collect information from its members about their partners’ behavior and to distribute this information among its members.\footnote{The function of a centralized information repository is related to credit bureaus and rating agencies.} As the information intermediary does not verify the content of the messages sent by its members, the members discount the credibility of the information they receive by $\lambda < 1$. In contrast, information received via the social network is not discounted.\footnote{This assumption serves to normalize beliefs. An alternative specification would be to place credibility $\lambda_{NW}$ and $\lambda_{II}$ on messages being received through either information channel, with $\lambda_{NW} > \lambda_{II}$. It captures that people} If player $i$ is a member, his
membership status is $M_i = 1$; otherwise $M_i = 0$. The association is a not-for-profit organization that levies a membership fee, $f$, to cover its operating expenses. Denote the (endogenous) share of association members in all members in period $t$ by $\sigma_t$ and the common belief about the next period’s membership size by $\sigma_{t+1}$. In each period, the timing of the game is as follows.

- **Stage 0**: Each player can join the information association for the fee $f$.

- **Stage 1**: Players are matched according to (1) and learn the location, connectedness, and membership status of their partner. Assume the players $i$ and $x$ are matched at distance $X$. With probability $\eta_{y,i}$, player $i$ receives the message sent by player $y$, who was $x$’s partner in period $t - 1$, about the behavior of $x$ in $t - 1$. If $i$ is an association member, he also obtains a report from the association about the behavior of $x$ in $t - 1$ (only if the association has such information). Partners decide simultaneously whether or not to transact. If either chooses not to transact, both payoffs are zero and the period ends for these players.

- **Stage 2**: If the matched players agree to transact, each decides whether to cooperate or defect and yields the payoff $ae^{8X}$ specified above.

- **Stage 3**: Each player sends a message about the behavior of his partner around the circle. Additionally, association members send a similar report to the association.

I solve this game for a stationary Markov-perfect equilibrium. To formalize the information a player has about his partner’s previous actions, I define $s_{y,t}$ to be player $y$’s state variable before he chooses an action at stage 1 of period $t$: $s_{y,t} = 0$ if player $y$ has received news—via the social network or the association—that his current match $i$ defected in period $t - 1$, or if player $y$ himself defected in period $t - 1$ and his match $i$ learned about it. Otherwise, $s_{y,t} = 1$.

A strategy for player $i$ in period $t$ is a mapping from his individual connectedness ($\kappa_i$), the match distance ($X$), and his state variable ($s_{i,t}$) to the action set: $\{\text{join, not join the association}\}, \{\text{transact, not transact}\}, \{\text{cooperate, defect}\}$.

**Information Intermediary (II) Strategy.** Define the following Markov strategy for player $i$:

tend to believe their friends, whom they can hold accountable for, more than senders whom they have no personal relationship to.

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23 As is usual in infinitely repeated games, this game has multiple equilibria. I focus on one equilibrium in which a business association exists and supports cooperative exchange. For a discussion of the equilibrium concept and uniqueness, see Appendix A.

24 For the purposes of this model, I assume that players who defected in a previous period know whether their new trading partners learned of that defection.
• In every period $t$, player $i$ joins the association for the cost $f$ if his individual connectedness $\kappa_i \leq \kappa^*_{II}$ and does not join otherwise.

• In $t = 1$, player $i$ transacts and cooperates with partner $x$ if the match distance $X \leq X^*_{II}$, and does not transact otherwise.

• In every subsequent period $t$, if player $i$ is matched to player $x$ and either the distance $X > X^*_{II}$ or $s_{i,t} = 0$, then player $i$ does not transact with $x$. Otherwise, $i$ transacts and cooperates with $x$.

The II Strategy specifies that players transact cooperatively with their partners if the distance between them is not too large. If either partner deviated from that strategy in $t − 1$ by defecting and the other player learns about the defection, the players ought not to transact with each other. In this case not interacting with a formerly defecting partner serves as punishment of the defector. Not interacting is (weakly) incentive compatible for the punisher because the II Strategy requires a defector who knows that his partner knows about his defection to participate in his own punishment by not transacting as well. Therefore, the would-be punisher would not gain from unilaterally transacting with a defector.\(^{25}\) Moreover, the II Strategy specifies that only those players will join the association who are less well connected to others individually. In the remainder of this section I will show that a Markov-perfect equilibrium exists, in which all players play the II Strategy.

At stage 3 of period $t$, no player has to make any decision. At stage 2, assume players $i$ and $x$ are matched at distance $X \leq X^*_{II}$ (where $X^*_{II}$ will be specified below). When $i$ considers whether to cooperate or to cheat, assuming that $x$ cooperates, it is critical whether $x$ is an association member, or not. If $M_x = 0$, $i$ knows that the only harm $x$ could do to him in case $i$ defects is to send a message to the social network. If $i$’s partner in period $t+1$—call him $y$—receives that message, $s_{y,t+1} = 0$. Hence, $y$ would not transact with $i$, according to the II Strategy. Therefore, the expected loss $L$ to player $i$ from defecting against $x$ in $t$ is the foregone value of transacting and cooperating with $y$ weighted by the probability that $i$ is matched to $y$.

\(^{25}\)This specification of the equilibrium strategy is slightly simpler than reality sometimes is, where cases with two layers of punishment exist. Hadfield and Weingast (2011b:30) explain, based on work by Moore (1985) on 13th century Flemish cloth merchants: “[P]articipation in the injunction not to deal with a merchant who cheated a Flemish merchant was enforced by a provision that punished the non-punisher. […] That secondary obligation—to refuse to deal with the non-punisher—is also enforced, at least in some measure, by collective punishment.” See Greif (2006, Appendix C) for more theoretical work on second-order punishment.
in \( t + 1 \) and the probability that \( y \) received news from \( x \) that \( i \) defected in \( t \), or (see Figure 2):

\[
L \equiv \delta \left[ \int_0^X \frac{e^{-Y_1}}{2(1-e^{-1})} \kappa_x e^{-(X-Y_1)} h e^{\theta Y_1} dY_1 + \int_1^X \frac{e^{-Y_2}}{2(1-e^{-1})} \kappa_x e^{-(Y_2-X)} h e^{\theta Y_2} dY_2 
+ \int_{1-X}^1 \frac{e^{-Y_3}}{2(1-e^{-1})} \kappa_x e^{-(2-X-Y_3)} h e^{\theta Y_3} dY_3 + \int_0^{1-X} \frac{e^{-Y_4}}{2(1-e^{-1})} \kappa_x e^{-(X+Y_4)} h e^{\theta Y_4} dY_4 \right] \quad (3)
\]

Figure 2: Possible locations of player \( i \)'s \( t + 1 \) match \( y \) as compared to \( i \)'s current match \( x \).

Instead, if \( M_x = 1 \), \( x \) has two channels to spread the information about \( i \)'s defection, the social network and the association. These two information channels can be overlapping. (2) specifies that the probability with which \( y \) receives \( x \)'s message via the social network is \( \kappa_x e^{-|Y-X|} \). Hence, with probability \( \sigma_{t+1} \kappa_x e^{-|Y-X|} \), \( y \) receives two messages. With probability \( (1-\sigma_{t+1}) \kappa_x e^{-|Y-X|} \), \( y \) receives \( x \)'s message only via the informal network. With probability \( \sigma_{t+1}(1-\kappa_x e^{-|Y-X|}) \), \( y \) receives \( x \)'s message only via the association and not via the informal network. This latter probability determines the additional expected loss to a defector if his partner is an association member. It follows that, if \( M_x = 1 \), the expected loss to player \( i \) from defecting against \( x \) in \( t \) equals \( L + \lambda I \), where:

\[
I \equiv \delta \left[ \int_0^X \frac{e^{-Y_1}}{2(1-e^{-1})} \sigma_{t+1}(1-\kappa_x e^{-(X-Y_1)}) h e^{\theta Y_1} dY_1 + \int_1^X \frac{e^{-Y_2}}{2(1-e^{-1})} \sigma_{t+1}(1-\kappa_x e^{-(Y_2-X)}) h e^{\theta Y_2} dY_2 
+ \int_{1-X}^1 \frac{e^{-Y_3}}{2(1-e^{-1})} \sigma_{t+1}(1-\kappa_x e^{-(2-X-Y_3)}) h e^{\theta Y_3} dY_3 + \int_0^{1-X} \frac{e^{-Y_4}}{2(1-e^{-1})} \sigma_{t+1}(1-\kappa_x e^{-(X+Y_4)}) h e^{\theta Y_4} dY_4 \right] \quad (4)
\]

Let \( G \) denote a player’s expected per-period gain from mutual cooperation before the identity of his partner is known. By the one-stage deviation principle, if player \( i \) assumes that his partner
x plays the II Strategy, i will cooperate if and only if:

$$he^{\theta x} + \delta(G - M_i f) + \frac{\delta^2}{1 - \delta}(G - M_i f) \geq \theta e^{\theta x} + (\delta(G - M_i f) - L - M_x \lambda I) + \frac{\delta^2}{1 - \delta}(G - M[\theta])$$

$$\Leftrightarrow L + M_x \lambda I \geq (w - h)e^{\theta x}$$ (6)

The left (right) side of equation (5) is i’s expected net present value from cooperating (defecting) in period t. The first term on each side is the payoff in t, which is larger for defection than for cooperation. The second term is the payoff in t+1, which is smaller for defection because i’s next match will not interact with a defector if he heard about the defection. The third term, the expected present value of interaction from t + 2 forward, is equal for both sides. Equation (6) is mathematically identical to (5) and allows us to define the present value of cooperation relative to defection for player i if his partner is x as:

$$V_{II,i} \equiv L(\kappa_x) + M_x \lambda I(\kappa_x, \sigma_{t+1}) - (w - h)e^{\theta x}$$ (7)

Performing comparative statics on $$V_{II,i}, I, \text{and } L$$ yields the following lemma.

**Lemma 1 (Value of cooperation with information intermediary)**

(i) $$I > 0 \forall \sigma_{t+1} > 0.$$ (ii) $$\frac{\partial I}{\partial \theta} > 0, \frac{\partial I}{\partial \sigma_{t+1}} > 0, \frac{\partial I}{\partial \kappa_x} < 0.$$ (iii) $$\frac{\partial L}{\partial \kappa_x} > 0, \frac{\partial V_{II,i}}{\partial \kappa_x} > 0.$$

Lemma 1.(i) states that the additional value of cooperation that a player’s association membership creates for his partner (on top of the value created by social networks) is strictly positive, as long as it can be expected that the association has any members next period. Given that a higher value of cooperation of one player makes this player more likely to cooperate, his partner (the association member) also benefits. Therefore, I can be regarded as a measure for the *value of information association membership*.

Lemma 1.(ii) complements (i) by stating that the value of association membership increases in the base value of the transaction and in the expected number of members. More importantly, this value decreases in the individual connectedness of a player’s partner. This means that association membership is less valuable for highly connected players and more valuable for less connected players. It also implies that, for one player, association membership and the embeddedness in social networks (connectedness) are substitutes in increasing the value of cooperation for their partner —and, by the logic presented above, for themselves. Rephased, both enforcement institutions offer players a tool to threaten their partner in case of defection. If a player is already well equipped with one tool (informal connections), the second tool (association membership) is less valuable to him.

Analogous to the value of association membership, Lemma 1.(iii) states that the value of the social network for a player ($L$) increases in his partner’s individual connectedness. Therefore,
the comparative statics effects of $\kappa_x$ on $I$ and $L$ are contrary. However, the second effect dominates the first: the present value of cooperation relative to defection for player $i$, $V_{II,i}$, which depends both on $L$ and on $I$, increases in player $x$’s connectedness, independent of $x$’s association membership.

Call the distance $X_{II,i}$ player $i$’s individual scope of cooperation with information intermediary. I prove the following proposition in the appendix.

**Proposition 1 (Individual scope of cooperation with information intermediary)** If all other players $-i$ play the II Strategy, player $i$ cooperates at stage 2 if and only if $X \leq X_{II,i}^*$. Otherwise, $i$ defects.

This proposition holds that it is individually rational for a player to cooperate with his partner if the two are located sufficiently close to each other in socioeconomic distance. Importantly, since $X_{II,i}^*$ depends on $V_{II,i}$, which depends on $L(\kappa_x)$ and on $M_x \lambda I(\kappa_x, \sigma_{t+1})$, the incentive for a player to cooperate depends not only on the parameters of the economic environment ($\theta, w, h, \lambda$) and the distance between partners ($X$) but it also increases in the individual connectedness and the membership status of his partner ($\kappa_x, M_x$) and in the expected membership size ($\sigma_{t+1}$).

At stage 1 of period $t$, the matched players might receive information about the former behavior of their partner and have to decide whether to transact with their match, or not. Under which conditions will both players agree to transact with each other? Rephrased, what is the binding scope of cooperation of a partnership, $X_{II}^*$, if the individual scopes of cooperation differ for both partners?

Consider the case where $\kappa_i > \kappa_x$ and $M_i = M_x = 0$. Then $V_{II,i}(\kappa_x) < V_{II,x}(\kappa_i)$, according to Lemma 1.(iii). Player $i$ is better connected than player $x$, which means that $i$’s potential to punish $x$ in case of defection by informing other players via the social network is higher than vice versa. However, because $x$’s connectedness is low, the incentive for $i$ to cooperate is limited. Thus, if $V_{II,i}(\kappa_x) < 0 \leq V_{II,x}(\kappa_i)$, $x$ does have an incentive to cooperate but $i$ does not. However, if $x$ understands $i$’s incentive to defect, $x$ will not interact with $i$. This reasoning also holds if one or both of them are association members. Consequently, if the individual values of cooperation relative to defection differ, the smaller of the two values is decisive in making transacting with the partner rational. The partners transact with each other if and only if $V_{II} \geq 0$, where $V_{II} = \min\{V_{II,i}(L(\kappa_x), M_x \lambda I(\kappa_x, \sigma_{t+1})), V_{II,x}(L(\kappa_i), M_i \lambda I(\kappa_i, \sigma_{t+1}))\}$. Hence, in equilibrium the binding scope of cooperation is the smaller one of the individual scopes:

$$X_{II}^* = \min\{X_{II,i}^*(L(\kappa_x), M_x \lambda I(\kappa_x, \sigma_{t+1})), X_{II,x}^*(L(\kappa_i), M_i \lambda I(\kappa_i, \sigma_{t+1}))\} \quad (8)$$
I summarize these insights in the following proposition, which nests the scope of cooperation with pure social network enforcement as a special case, in which $M_i = M_x = 0$.

**Proposition 2 (The scope of cooperation with information intermediary)** If all other players $-i$ play the II Strategy, player $i$ transacts at stage 1 if and only if $X \leq X^*_II$. Otherwise, $i$ does not transact.

At stage 0 in period $t$, player $i$ has to decide whether to join the association for the membership fee $f$, or not. $i$ does not know the identity of his partner this period yet, but he knows that his association membership will only create any value for him if it lets him and his partner cooperate this period conditional on they would not have cooperated without $i$ being a member. This condition equals the probability with which $M_i$ has an effect on $X^*_II$, which I denote by $\beta_{II} \equiv \text{prob}(X^*_II,L(\kappa_i,M_i=0)) > X^*_II,L(\kappa_i,M_i=1))|M_i = 0).$ 26 I derive $\beta_{II}$ from the model’s fundamentals in the appendix and show that $\beta_{II}$ increases in $\sigma_{t+1}$ and decreases in $\kappa_i$ and in $E(\kappa_x)$.

Given that player $i$’s membership decision influences $X^*_II$, how much is the association membership worth for $i$? In this case, by becoming an association member, $i$ can increase the scope of cooperation from $X^*_NW,x \equiv X^*_II,x(L(\kappa_i,M_i=0))$ to $X^*_II,x \equiv X^*_II,x(L(\kappa_i),M_i = 1,1(\kappa_i,\sigma_{t+1}))$. Only for matches with partners located at distances $X \in [X^*_NW,x,X^*_II,x]$, $i$’s association membership has a positive impact in equilibrium, the switch from no transaction to cooperative interaction. For all $X < X^*_NW,x$, cooperative interaction exists even without association membership, whereas, for all $X > X^*_II,x$, not transacting is the unique equilibrium with or without an association.

Putting these pieces together, the expected net payoff for $i$ from joining an association that serves as an information intermediary is:

$$B_{II} \equiv \beta_{II} \int_{X^*_NW,x}^{X^*_II,x} \frac{e^{-X}}{2(1 - e^{-1})} h e^{\theta X} dX - f$$

(9) $B_{II}$ is completely specified depending on exogenous parameters. Player $i$ joins the association if and only if $B_{II}(\kappa_i) \geq 0$.

How many players, and which ones, will join the association, at every point on the circle economy? To study these questions I analyze $B_{II}$ in more detail and prove the following proposition in the appendix.

**Proposition 3 (Membership decisions with information intermediary)** (i) Given that $f$ is not prohibitively high, if all other players $-i$ play the II Strategy, player $i$ joins the association if and only if $\kappa_i \leq \kappa^*_II$. 

26 If $i$ becomes a member, $M_i$ switches from 0 to 1. This can only have an effect if $X^*_II,x(M_i = 0) < X^*_II,x$. 

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(ii) Depending on the membership fee $f$, there are three possible equilibrium membership sizes, characterized by $\kappa^*_I(f)$. For small levels of $f$, there is a unique $\kappa^*_I(f)$. For intermediate levels of $f$, there are two $\kappa^*_I(f)$—denoted by $\kappa^*_{I,1}$ and $\kappa^*_{I,2}$; $\kappa^*_{I,1} < \kappa^*_{I,2}$. For large levels of $f$, $\kappa^*_I(f) = 0$. In each equilibrium, the share of association members corresponds to the players’ beliefs about the membership size: $\kappa^*_I(f) = \sigma_{t+1} = \sigma_t$.

Proposition 3.(i) answers the question what types of players join the association if there is a membership fee. It builds on the insight that players with high connectedness have two disadvantages from association membership compared to players with low connectedness. First, for these well connected players, it is often the low connectedness of their partners that determines the binding scope of cooperation in their transaction. By joining an association, they can only further improve their own ability to punish a defecting partner—which is not necessary due to their high endowed connectedness—but they cannot improve their commitment to cooperate with their less connected partner. Second, even if a well connected player is matched to another player who is also well connected (or who improved his de facto connectedness by joining the association) and, therefore, the well connected player’s connectedness determines the binding scope of cooperation, he benefits less from membership than a less connected player who is matched to the same partner. The reason is that in general the benefit from joining the association comes through the improved access to all other members. A well connected player, however, is already connected to many other players, including some members. Therefore, the additional share of players that he can inform via the association, on top of the ones in reach of his social network, is smaller than the additional share of connections that less connected players would enjoy.

Proposition 3.(ii) builds on the insight that in equilibrium the players’ beliefs about membership size, which determine the level to which a partner’s membership can deter a player’s defection, have to coincide with the realized, endogenous membership share. Depending on the membership fee, there are no, one, or two membership levels at which a common belief of all players about the present and future membership size leads to a stable and constant share of players becoming members (as explained in part (i), these are the less connected players).

3.3 Business associations as arbitrators

Next we analyze associations that serve as private arbitrators in disputes involving their members. As before, every player is endowed with an individual level of connectedness, $\kappa_i$. The arbitrator investigates cases himself if a member brings a charge against his partner, who can be a member or a nonmember. The association tells its members not to interact with a player
who was found to have defected if that player refuses to pay a damage payment to his victim. As a consequence of qualifying the messages sent by members through the arbitration tribunal’s investigation, the credibility of messages is not discounted, i.e. \( \lambda_{AA} = 1 \).\(^{27}\) Since running an arbitration tribunal is costly, the association levies a membership fee, \( F > f \), to cover its operating expenses. In each period, the timing of the game is as follows.\(^{28}\)

- **Stage 0:** Each player can join the association for the fee \( F \), as long as he does not have “unpaid damage payments” from period \( t - 1 \). The association announces a rule \( d = d(M_i, M_x, X, \kappa_i, \kappa_x) \), according to which it will determine the amount of damage payments if some member \( i \) brings a charge against his partner \( x \) and the arbitrator rules against \( x \).

- **Stage 1:** Players are matched according to (1) and learn the location, connectedness, and membership status of their partner. According to (2), they receive messages sent through the social network. If \( i \) is an association member, he obtains a report from the association stating whether his partner \( x \) has “unpaid damage payments” from period \( t - 1 \). Partners decide simultaneously whether or not to transact.

- **Stage 2:** If the matched players agree to transact, each decides whether to cooperate or to defect.

- **Stage 3:** Each player sends a message about the behavior of his partner around the circle. Additionally, association members can bring a charge against their partner to the association’s arbitration tribunal, for a cost \( c_A \). The arbitrator studies the case and correctly decides a valid claim with probability \( \tau_A \in [0, 1] \).\(^{29}\) If the arbitrator decides for the plaintiff, he orders the defendant to pay a damage payment \( d \) to the plaintiff. Otherwise, no damages are rewarded.

- **Stage 4:** A convicted defendant chooses whether to pay the damage payment to the plaintiff, or not. (Non)payment is observed and recorded by the arbitrator.

Again, I solve this game for a stationary Markov-perfect equilibrium. For this section, I redefine player \( y \)'s state variable \( s_{y,t} \) to take the value \( s_{y,t} = 0 \) if player \( y \) has received news via the social network that his current match \( i \) defected in period \( t - 1 \) or if \( y \) received news from

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\(^{27}\)Again, this is a simplifying assumption instead of assuming \( \lambda_{AA} > \lambda_{II} \), which leads to equivalent results.

\(^{28}\)The sequence of this game is influenced by Milgrom et al. (1990). The main assumptions of the arbitrator model reflect the descriptions of the U.S. cotton industry in Bernstein (2001).

\(^{29}\)\( \tau_A \) can be widely interpreted. For instance, \( \tau_A \) is low if the competence of the arbitrator is low or if the transaction is complex or if the actions in the central transaction are hardly observable ex post.
the association that \( i \) has unpaid damage payments from \( t - 1 \) or if player \( y \) himself defected or did not pay a damage payment in period \( t - 1 \) and his match \( i \) learned about it. Otherwise, \( s_{y,t} = 1. \)

A strategy for player \( i \) in period \( t \) is a mapping from the match distance \( (X) \), his individual connectedness \( (\kappa_i) \), and his state variable \( (s_{i,t}) \) to the action set: \{[join, not join the association], [transact, not transact], [cooperate, defect], [bring a charge, do not bring a charge], [pay damage payment, do not pay damage payment]\}).

**Arbitration Association (AA) Strategy.** Define the following Markov strategy for player \( i \):

- In every period \( t \), player \( i \) joins the association for the cost \( F \) if his individual connectedness \( \kappa_i \leq \kappa_{AA}^* \), and does not join otherwise.

- In \( t = 1 \), player \( i \) transacts and cooperates with partner \( x \) if the match distance \( X \leq X_{AA}^* \) and if the expected damage payment in case of a default is \( d \in [d, \bar{d}] \). Otherwise, he does not transact. If \( i \) cooperates and his partner defects, \( i \) brings a charge. If \( i \) defects and his partner \( x \) brings a charge, \( i \) pays the damage payment.

- In every subsequent period \( t > 1 \), in addition to the requirements specified above, for \( i \) to transact and cooperate with \( x \), it must be that \( s_{i,t} = 1. \)

Related to the Information Intermediary Strategy, the AA Strategy specifies that only players who are located sufficiently close to each other in socioeconomic distance are to transact with each other. It also holds that those players who are endowed with rather low connectedness, join the association. Importantly, players who ignored the judgement of the arbitrator in the previous period are not eligible for membership. On top, it must be that the damage payment players expect the arbitrator to determine in case of a found defection is neither too low nor too high. I will investigate these final conditions first by proceeding by backward induction.

At stage 4 of the game, if player \( i \) was found guilty of unilateral defection by the arbitration tribunal and sentenced to pay a damage payment to his partner, he has to decide whether to submit to the judgement. Assuming that everybody else plays the AA Strategy, \( i \) knows that his matching partner in period \( t + 1 \), say \( y \), will not interact with him if \( y \) is an association member, which is given with probability \( \sigma_{t+1} \). The trade-off faced by \( i \) depends on \( i \)'s own membership status, \( M_i \).

If \( M_i = 1 \), according to the solution concept of stationary equilibrium, \( i \) also has to join the association in \( t + 1 \). This requires paying the damage payment in \( t \) and the membership fee \( F \) in \( t + 1 \). Then \( i \) expects a net present value of \(-d + \delta(G - F) - L + \frac{\delta^2}{1 - \delta}G \) from cooperation
in \( t + 1 \).\(^{30}\) If \( i \) deviates from the AA Strategy and does not pay \( d \)—and consequently also saves on \( F \) because he cannot renew his membership in \( t + 1 \)—he expects a net present value of \( \sigma_{t+1}0 + (1 - \sigma_{t+1})(\delta G - L) + \frac{\delta^2}{1-\delta} G \). Rearranging these two terms, it follows that \( i \) pays the damage payment if and only if:

\[
 d \leq \sigma_{t+1}(\delta G - L) - \delta F \equiv \bar{d}_{\text{member}},
\]

where \( \bar{d}_{\text{member}} \) is the maximum self-enforcing damage payment for members.\(^{31}\)

If \( M_i = 0 \), in contrast, \( i \) is not interested in association membership in the next period. It follows that \( i \) expects a net present value of \( -d + \delta G - L + \frac{\delta^2}{1-\delta} G \) if he pays the damage payment and of \( \sigma_{t+1}0 + (1 - \sigma_{t+1})(\delta G - L) + \frac{\delta^2}{1-\delta} G \) if he does not pay. Hence, \( i \) pays the damage payment if and only if:

\[
 d \leq \sigma_{t+1}(\delta G - L) \equiv \bar{d}_{\text{non-member}}
\]

Comparing \( \bar{d}_{\text{member}} \) and \( \bar{d}_{\text{non-member}} \), I obtain the following lemma.

**Lemma 2 (Maximum damage payments)** (i) The maximum self-enforcing damage payment is lower for members than for nonmembers (\( \bar{d}_{\text{member}} < \bar{d}_{\text{non-member}} \)). (ii) The maximum self-enforcing damage payment against player \( i \) is decreasing in the connectedness of the victim, member \( x \) (\( \frac{\partial \bar{d}_{\text{li}}}{\partial \kappa_x} < 0 \)).

Lemma 2.(i) contains the interesting result that, assuming the arbitrator wants to punish a defector as hard as possible, he will determine a higher damage payment for nonmembers than for members. The reason for this difference is that the arbitrator has to make sure that members who defected find it worthwhile to pay the judgement and the membership fee of the next period. Nonmembers do not have to account for future membership fees, which is why it is rational for them to pay an even higher damage payment than for members to avoid being ostracized by the association.

Lemma 2.(ii) is related to Lemma 1.(ii) and holds independent of \( i \)'s membership status because \( \frac{\partial G}{\partial \kappa_x} = 0 \) but \( \frac{\partial L}{\partial \kappa_x} > 0 \); see (10), (11), and (B.11) in the Appendix: a defector expects a highly connected victim to reach a high share of players via the social network and thereby to ruin many potential business opportunities of the defector in the future, independent of the victim’s membership status. Thus, the additional threat posed to the defector if the victim is

\[^{30}\text{Recall my assumption that } i \text{'s cheated partner in } t \text{, say } x, \text{ would still inform the social network about } i \text{'s defection, independent of his behavior with respect to the damage payment. Hence, here } L = L(\kappa_x).\]

\[^{31}\text{If } d > \bar{d}_{\text{member}}, \text{ the convicted player refuses to pay. Hence, the arbitration mechanism has no bite and cannot increase cooperation.}\]
an association member is less severe than the additional threat coming from a less connected victim.

At stage 3, every association member has to decide whether to bring a charge against his partner to the arbitration tribunal, or not. Given my assumptions, the expected payoff to a player from bringing a charge depends on both his behavior and that of his partner in the central transaction. A player who cooperated while his partner defected expects \( \tau_A d - c_A \) from charging. If he defected while his partner cooperated or if both players cooperated or both defected, bringing a charge leads to an expected payoff of \(-c_A\).\(^{32}\) It follows that bringing a charge can only be profitable for player \( i \) if he cooperated and his partner defected and if:\(^{33}\)

\[
d \geq \frac{c_A}{\tau_A} = d
\]

The following proposition is a consequence of my previous analysis and does not require a formal proof.

**Proposition 4 (Effectiveness of the arbitrator)** (i) The damage payment determined by an arbitrator against player \( i \) is effective if, and only if, \( \frac{c_A}{\tau_A} \leq \sigma_{i+1}(\delta G - L) - M_i \delta F \). If the respective condition holds, a member whose partner defected unilaterally brings a charge in equilibrium. (ii) If the arbitrator rules against a defendant \( i \), he chooses the maximum self-enforcing damage payment as equilibrium damage payment (\( d^* = \bar{d}_{M_i} \)).

Proposition 4.(i) characterizes the lower and upper constraints faced by the arbitrator if he wants to determine an effective damage payment, that is, to make a judgement that would be voluntarily followed by the charged player and that would incentivize a member whose partner unilaterally defected to bring a charge. The two intervals specified, depending on \( M_i \), are nonempty if the cost of bringing a charge or the membership fee or the connectedness of the victim is low or if the probability with which the tribunal correctly decides a valid claim or the expected membership size are high. If the interval is nonempty, it is rational for a victim to bring a charge. Otherwise, getting a judgement that would be paid by the defector would be too expensive for the victim.

Given that \( d \) is determined after the members decide about bringing a charge, the members’ beliefs about \( d \) are important, which are anchored by the arbitrator’s announcement of a damage

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\(^{32}\)This model allows only for “type-2” errors of the arbitrator (false negatives). The victim of a defector could bring a charge, which would lead to an expected payoff of \(-\tau_A d \) for the defector—but this decision rests with the victim.

\(^{33}\)Note that \( \bar{d} \) distinguishes between members and nonmembers because every player can face a judgement of the arbitrator. In contrast, \( d \) is only relevant for members because nonmembers do not have the right to bring a charge to the arbitrator.
payment determination rule at stage 0. As we will see below, the effectiveness of the arbitration institution to support cooperative interaction increases in the (expected) damage payment it determines. This explains Proposition 4.(ii).

At stage 2 of the game, if the interval $[d, \bar{d}_M]$ is empty, all players know that the arbitrator is ineffective in equilibrium. Hence, the pure social network case prevails (see Section 3.2, where $M_i = M_x = 0$). For the subsequent analysis I assume that $d^* = \bar{d}_M$ is effective.\(^{34}\) Consider the case where players $i$ and $x$ are matched at distance $X \leq X_{AA}^*$. Player $i$’s incentive compatibility constraint for cooperation, the analogue to (5), is:

$$he^{\theta X} + \delta(G - M_i F) + \frac{\delta^2}{1 - \delta} (G - M_i F) \geq we^{\theta X} - M_x \tau_A d^* + \delta(G - M_i F) - L(\kappa_x) + \frac{\delta^2}{1 - \delta} (G - M_i F)$$

\[\Leftrightarrow L(\kappa_x) + M_x \tau_A d^* \geq (w - h)e^{\theta X} \tag{14}\]

Equation (13) builds on the idea that if player $i$ defects, he gets the higher payoff $we^{\theta X}$ instead of $he^{\theta X}$ in period $t$. Then, however, if his partner $x$ is an association member, he will bring a charge to the arbitration tribunal, which will rule against $i$ with probability $\tau_A$ and ask $i$ to pay a damage payment $d^*$ to $x$. Given that $d^*$ is effective, $i$ will pay. Independent of paying the damage payment, $i$ expects that $x$ informs the social network about the defection, which will result in the expected loss $L$ in period $t + 1$. Rewriting equation (13), I obtain (14), which shows that $i$’s incentive to cooperate increases in the expected damage payment $d$ and thus explains the arbitrators choice to set $d = d^*$ at stage 3. Resubstituting $d^*$ from (10) and (11) into (14) and rearranging, the present value of cooperation relative to defection for player $i$ is:

$$V_{AA,i} \equiv L(\kappa_x) + M_x \tau_A (\sigma_{t+1}(\delta G - L(\kappa_x)) - M_i \delta F) - (w - h)e^{\theta X} \tag{15}$$

Performing comparative statics on $V_{AA,i}$, I obtain the following lemma.

**Lemma 3 (Value of cooperation with arbitrator)** (i) $\tau_A d^* > 0 \forall \sigma_{t+1} > 0$, $\frac{\partial \tau_A d^*}{\partial \sigma_{t+1}} > 0$, $\frac{\partial \tau_A d^*}{\partial M_i F} > 0$. (ii) $\frac{\partial V_{AA,i}}{\partial \kappa_x} < 0$, $\frac{\partial V_{AA,i}}{\partial \sigma_{t+1}} < 0 \forall \tau_A \sigma_{t+1} < 1$. (iii) $\frac{\partial V_{AA,i}}{\partial M_i F} < 0$, $\frac{\partial V_{AA,i}}{\partial M_i F} < 0$.

Lemma 3.(i) states that the additional value of cooperation for player $i$ that comes via the association membership of his partner $x$ is strictly positive as long as the association is expected to have any members next period. $\tau_A d^*$ can serve as a measure of the value of arbitration association membership, which is increasing in the base value of any transaction, the expected membership size, and the competence of the arbitration tribunal to correctly judge

\[34\text{Recall that, at stage 2, the matched partners know each others’ membership status and connectedness, on top of all other parameters. Hence, they can calculate whether }d^*(M_i, \kappa_x) \text{ is effective, or not.}\]
a valid charge. Lemma 3.(ii) shows that this value is decreasing in the victim’s connectedness but that the total value of cooperation as compared to defection is still increasing in that connectedness. Whereas all of these results are economically in line with the results of the information intermediary case (see Lemma 1), Lemma 3.(iii) introduces a completely a novel insight: both the value of association membership and the incentive of $i$ to cooperate with $x$ are smaller if $i$ is an association member himself (and if the membership fee is large). The reason for this surprising result is that the damage payment that $i$ expects to pay if he defects and loses the case is smaller if he is a member (see Lemma 2.(i)).

Call the distance $X^*_A,i$ player $i$’s indi\(\text{vidual scope of cooperation with arbitration association,}\) which is specified in the following proposition. The corresponding proof is similar to the proof of Proposition 1 and omitted.

**Proposition 5 (Individual scope of cooperation with arbitration association)** If all other players $-i$ play the AA strategy, player $i$ cooperates at stage 2 if and only if $X \leq X^*_A,i$. Otherwise, $i$ defects.

At stage 1 of the game, when the matched partners have to decide about transacting with each other, the analysis is also similar to the information intermediary case. In equilibrium the binding scope of cooperation is the smaller one of the individual scopes:

$$X^*_A = \min \{X^*_A,i(L(\kappa_x), M_x \tau_A d^*(\kappa_x, \sigma_{t+1}, M, F)), X^*_A,x(L(\kappa_i), M_i \tau_A d^*(\kappa_i, \sigma_{t+1}, M, F))\}$$ (16)

The following proposition, related to Proposition 2, follows.

**Proposition 6 (The scope of cooperation with arbitration association)** If all other players $-i$ play the AA strategy, player $i$ transacts at stage 1 if and only if $X \leq X^*_A$. Otherwise, $i$ does not transact.

Comparing the scopes of cooperation of both types of associations, I can state the following proposition, which I prove in the appendix.

**Proposition 7 (Comparing the scopes of cooperation)** Without loss of generality, consider $X^*_I = X^*_I,i(\kappa_x, \cdot)$ and $X^*_A = X^*_A,i(\kappa_x, \cdot)$. The scope of cooperation in the arbitrator case is larger [smaller] than in the information intermediary case if $(\tau_A - \lambda)I > [<] M_i \tau_A \delta F$.

This proposition shows under which conditions the information intermediary and arbitration functions of associations, respectively, are more valuable for members. If the credibility of unverified information is higher than the competence of the arbitration tribunal, it is efficient
for associations to restrict themselves to the information intermediary function. This is even more valid if the cost of setting up an arbitration tribunal is high. In turn, if this cost is moderate and the credibility of unverified information is low as compared to the competence of arbitrators, it pays to set up an arbitration tribunal.

At stage 0, players decide about membership in the association. The analysis follows the information intermediary case and is conducted in the appendix. The expected net payoff for player $i$ from joining an association that offers arbitration services is:

$$B_{AA} = \beta_{AA} \int_{X_{NW,x}}^{X_{AA,x}} \frac{e^{-X}}{2(1-e^{-1})} h e^{\theta X} dX - F$$

I prove the following proposition in the appendix.

**Proposition 8 (Membership decisions with arbitrator)** (i) Given that $F$ is not prohibitively high, if all other players $-i$ play the AA strategy, player $i$ joins the association if and only if $\kappa_i \leq \kappa_{AA}^*$. (ii) Depending on the membership fee $F$, there are two possible equilibrium membership sizes, characterized by $\kappa_{AA}(F)$. For small levels of $F$, there are two $\kappa_{AA}(F)$, called $\kappa_{AA,1}(F)$ and $\kappa_{AA,2}(F)$, $\kappa_{AA,1}(F) < \kappa_{AA,2}(F)$. For large levels of $F$, $\kappa_{AA}(F) = 0$.

The intuition of Proposition 8 is the same as of Proposition 3: players who are badly connected benefit most from association membership. The main difference between both propositions is a technical one: due to the additional constraint that an arbitration association can only support cooperation if the equilibrium damage payment is not too low ($d^* \geq d$), it is ineffective for too high or too low $\kappa_{AA}^*$. One consequence of this is that, depending on $F$, there are two equilibrium membership sizes (or none) but not one.\(^{35}\)

Finally, given everybody’s perfect foresight, at stage 0 the arbitrator announces the damage payment determination rule $d = d^*$, as specified in Proposition 4. Announcing a tougher rule, $d > d^*$, would not be credible. Announcing a softer rule, $d < d^*$, would reduce the effectiveness of the arbitrator (see Proposition 4.(ii)).

4 Results, Explanations, and Hypotheses

4.1 Theory: associations and other governance institutions

Based on their empirical findings, Johnson et al. (2002:252) state, “Trade associations providing arbitration services may perform a similar role to the courts. Trade associations may have an

\(^{35}\)These results imply that no general statement is possible about the relative membership sizes $\kappa_{TA}(f)$ and $\kappa_{AA}(F)$. As long as $F$ is not prohibitive, multiple equilibria exist in both cases, which cannot be ranked and interpreted in an economically meaningful (or even testable) way. See Figures 4 and 5 in Appendix B for the intuition.
additional effect as well, through information services that they provide their members.” If a player is an association member, the value of cooperation for his matched partner is larger than the value with enforcement by social networks only (Lemmas 1 and 3). The intuition is that centralized organizations such as associations spread news—in the case of an arbitration association even qualified news—about opportunistic behavior in the economy and thereby strengthen the collective enforcement mechanism of social networks.

Masten and Prüfer (2011) show that enforcement by social networks and courts is complementary: whereas social networks support cooperation in low-value/short-distance transactions, courts support cooperation in high-value/long-distance transactions. Comparing the scopes of cooperation, I conclude that associations are a hybrid between social networks and courts:

\[ X_{NW}^* < \max\{X_{II}^*, X_{AA}^*\} < X_{Court}^* \]  

The left inequality builds on Lemmas 1 and 3, the \( \max \)-condition depends on Proposition 7. The right inequality captures that courts have access to the coercive powers of the state. Given that they are sufficiently competent, they can enforce transactions with very high value, which coincides with long-distance transactions in this model. In contrast, all (lawful) private organizations, including associations offering arbitration services, are restricted by the economic requirement that the maximum damage payment they decide has to be self-enforcing. If it is too high, convicted players will refuse to pay it.

When should we expect to observe which kind of association? First, given that both types of associations modeled here support cooperation by coordinating a boycott of defectors, it is ineffective for a single association to offer both services, given that the costs of operating them would add up. Comparing both functions from the perspective of a single player, \( i \), the information repository function dominates the arbitration function if \( B_{II} > B_{AA} \). This boils down to a comparison of the scopes of cooperation and the associated costs, \( f \) and \( F \). Since costs are independent of the players’ individual characteristics, all players unanimously prefer the one or the other association function, depending on Proposition 7. Therefore, we can expect associations to adjust the function offered quickly if the parameters of the environment change.

What are the cross effects if several institutions exist? Both social networks and associations rely on ostracism as an enforcement tool. Therefore, as demonstrated above for the case where social networks exist and associations are offered on top, the scope of cooperation strictly increases if an additional institution is available to the players and affordable. This does not hold with respect to communities and courts. As Masten and Prüfer (2011) demonstrate, whereas the existence of social networks supplements court enforcement, the existence of courts diminishes the effectiveness of social network enforcement. The intuition of the latter effect is
that, if court enforcement is available on top of network enforcement, some traders who would have otherwise refused to trade with a defector next period will trade because they know that the court protects them from being cheated. Therefore, the expected network sanction in the next period is lower than it otherwise would be, which diminishes the players’ incentives to cooperate because of fear of ostracism in the current period. Given that associations also rely on ostracism, they suffer from the same crowding out effect if filing a suit with the courts is an option available to the players.\textsuperscript{36}

Importantly, existence of an association does not only benefit its members: if a nonmember’s partner joins an association, the nonmember’s individual scope of cooperation increases because now defection is more costly to him (Propositions 1 and 5). This improves the nonmember’s ability to commit to cooperative behavior and, in expectation, increases the scope of cooperation of the entire partnership (Propositions 2 and 6).\textsuperscript{37}

### 4.2 Empirics: explaining puzzles and constructing hypotheses

A key result of this model is that the additional value of association membership decreases if transactors are well connected informally. This result leads to the empirically testable hypothesis that we should observe less associations in small and dense economies such as villages or clans, where most players know each other. The hypothesis is confirmed by Pyle (2005), who reports that business associations are perceived less valuable in local trade in Eastern Europe. Similarly, Casari (2007) finds that informal, decentralized institutions (social networks) were more likely to be used than formal, centralized institutions (so-called “chapters”) by communities that were remote and small in the medieval Italian Alps. The intuition is that enforcement by associations and social networks are substitutes, which work by the same mechanism: ruining a defector’s reputation and, thereby, his business opportunities in the future. If information exchange via social networks is already very effective (high average $\kappa_i$), the additional share of the population that can be informed by an association decreases.

In this model, if $\theta$ decreases, the potential value of any transaction, $a e^{\theta X}$, decreases. Therefore, $\theta$ can be interpreted as a proxy for the level of competition: the lower $\theta$ is, the higher is $\textsuperscript{36}$In reality some association tribunals can have their judgements enforced by public courts (Bernstein, 2001:1737). This makes them “quasi-courts” in the sense of Masten and Prüfer (2011). See also the Conclusion.

$\textsuperscript{37}$This result extends Richman (2004:2346), who states that only “participating long-term players” would have incentives to cooperate and newcomers to a certain community without a reputation would be unable to commit credibly to uphold their contractual promises. My analysis shows that the commitment ability of a player depends on his partner’s punishment options, not on his own embeddedness.
the degree of competition.\textsuperscript{38} If $\theta$ decreases, the value of association membership also decreases (Lemmas 1 and 3). The intuition is that, if $\theta$ is small, the expected damage that a defector who is reported to the association suffers from by losing trade in the next period is relatively low. Consequently, the deterrence effect of joining an association against would-be defectors is rather limited, which explains why fewer members are willing to pay the membership fee. Rephrased, my model suggests the hypothesis that more intense competition leads to less (closely organized) associations. This hypothesis is confirmed by Pyle (2005), who reports that business associations are perceived to be less valuable to members in more competitive industries. Similarly, Grafe and Gelderblom (2010) find that intense competition between medieval mercantile groups and local merchants is associated with lower degrees of control delegation to the groups. Casari (2007) finds that the lower the value of a common resource is (proxied by $\theta$ in my model), the smaller is the potential gain from adopting a formal, centralized institution.

The analysis of endogenous membership decisions has produced two additional testable hypotheses that have not been studied empirically, yet. First, I expect players who are less connected informally to join an association and well connected players not to join.\textsuperscript{39} Second, I expect the average damage payment determined by arbitrators against nonmembers to be higher than the average payment determined against members.\textsuperscript{40} The reason for this difference is not that the arbitrator has a bias against nonmembers but that he has to make sure that members who defected find it worthwhile to pay the judgement \textit{and} the membership fee of the next period. Nonmembers do not have to account for future membership fees and hence would be willing to pay an even higher damage payment to avoid ostracism by the association.\textsuperscript{41}

\textsuperscript{38}In any imperfectly competitive market with downward-sloping demand, if competition gets more intense, the marginal buyer moves to the right. So does the average buyer. Assuming constant or increasing marginal cost, this is associated with a decrease in the surplus of the average transaction, being defined as the difference between the average buyer’s willingness-to-pay and the seller’s corresponding marginal cost. Alternatively, let us re-specify the baseline value of a transaction as $ae^{(\theta+\omega)X}$, where $\omega$ measures product differentiation. If products traded are less differentiated/more homogeneous, $\omega$ and the baseline value decrease. In both interpretations, the limit case is given by Bertrand competition with homogeneous goods ($\omega = 0$), where the price equals marginal cost and the average buyer is located as far to the right as possible in equilibrium and the surplus of the average transaction is minimized.

\textsuperscript{39}In empirical work, individual connectedness could be proxied by the number of years an individual has been working in a certain industry or, on the firm level, by the number of years a firm has been active, controlling for total revenues. Tests should take care of the fact that long-lasting associations may exhibit unobserved internal dynamics that can discriminate against newcomers. Hence, the cleanest test would be to study who joins a newly founded association.

\textsuperscript{40}Bernstein (2001:1727) documents by-laws of a U.S. cotton industry association which explicitly handle arbitration cases involving members and nonmembers.

\textsuperscript{41}For an empirical test, it would be necessary to control for the severity of a transgression, which may be
Another field where the theoretical insights could be applied is the interaction of association membership and political competition. Based on survey data from Russia, Pyle (2011:27) wonders: “it is less than clear why we would not observe higher membership rates in associations if indeed they offer services that secure property rights. Our surveys suggest, after all, that the associations are open and nonexclusive. In some settings, as shown previously, macrolevel political institutions may produce similar outcomes. But to point this out does not clarify why membership rates are not higher in regions with limited political competition. Perhaps the benefits of membership are not widely recognized or understood.” Propositions 3 and 8 suggest an alternative explanation: membership is more attractive for less connected players. If in the regions with less political competition potential members are highly connected on average, for instance because these regions may be located rather remotely, highly connected players may also be expected to be well connected to the autocratic ruler and hence do not need to join an association to protect their property rights. If this holds, the less connected players may share the belief that membership is only attractive for few other players. The association and its members would be trapped in the $\kappa^*_1$-equilibrium and remain small.

The model also generates the insight that a larger variance of the distribution of connectedness, $Z$, leads to less network supported cooperation on average. The intuition is that, if $Z$ is more dispersed, the probability that one player in a partnership has a low connectedness ($\kappa_i$) is high. By Propositions 2 and 6, this decreases the binding scope of cooperation of the entire partnership. In turn, it implies that the additional value of formal associations is relatively large. Hence, we obtain the testable hypothesis that in dynamic industries with many entrants (e.g. in the life sciences or Internet technologies/services) many players would join an association, whereas in less dynamic industries with rather homogeneous levels of connectedness the share of association members should be smaller.42

5 Conclusion

Business associations—private, formal, noncommercial organizations designed to promote the common business interests of their members—have assumed many functions throughout history and all over the world. Two functions that are particularly valuable for association members if classified by arbitration tribunals.

42In empirical work, it would be helpful to obtain data on the main functions of a given association. In the case of dynamic industries, the model predicts that members yield high benefits from association functions that support cooperation in bilateral dealings/honest trade. Contrary, in stable industries with a few long-term players, it might also be that a high share of industry participants are association members—but for collusive reasons rather than to promote bilateral trade, which is already secured by informal ties.
the public legal system is ineffective or public authorities are even corrupt and exploitative are serving as a repository for member-supplied information about the conduct of their business partners and offering arbitration in the case of disputes involving their members.

Recent empirical research into the functioning of modern business associations and merchant guilds and related institutions in the European middle ages has produced fascinating new results, which stand to be explained by theory. In this paper, I have constructed a theoretical framework that connects the organizational and institutional features of formal and informal business organization with socioeconomic distance. This approach has enabled me to suggest explanations for several empirical puzzles, to put forward novel testable hypotheses, and to relate business associations to alternative governance institutions (see Section 4).

More research, both empirical and theoretical, is needed. In the models of this paper, two partners play the central transaction and a multitude of other players, which can be coordinated by a centralized organization, or not, rests in the back, ready to support the auxiliary transaction (punishment of defectors). Going a step further, several real-world applications introduce a second layer of institutions, where associations are the players in the central transaction and a higher-order association exists to mediate conflicts between the players and coordinate collective action. In the European middle ages, the German Hanse, which was an association of cities dominated by merchant guilds, adopted such a role (Ogilvie, 2011:20). In modern times, the “New York (Arbitration) Convention” supports the enforcement of arbitration judgments by foreign courts, an important institution of international trade.43 Both applications can be informed by the findings of this paper but it is unclear how the results change if different governance structures among the players exist, for instance if the second-layer association is dominated by a subset of the players.

An important subject of future research is the interdependence of business associations and the level of political competition. Contributing to this issue, Olson (2000), Frye (2006), and Pyle (2011) represent intriguing studies into the overlap of organizational economics, economic governance, and political economy. A related normative question is, under which circumstances might associations increase the surplus of the players involved and when might there be negative spillover effects on nonmembers that outweigh the positive effects?44

Finally, studying all institutions in the typology of commitment mechanisms of Masten and Prüfer (2011) has yet to be completed. Specifically, the analysis of models of “first-party systems” (Dixit, 2009:10), that is, institutions such as social norms that support cooperation by

43See Leeson (2008a) and http://www.newyorkconvention.org/.

44The former approach is underlined by the private ordering literature, including this paper. The latter approach is followed by several scholars from industrial organization, public choice, and law & economics.
directly affecting the pro-social preferences of players, has just begun. In addition, those private-ordering institutions that share some characteristics with associations and some with public courts, for instance specialized courts (family and juvenile courts, etc.) and criminal organizations (gangs, Mafia) are understudied. These institutions are both community-embedded, that is, they send and receive informal information about the conduct of players, and have access to coercion to enforce their judgements. Given this apparent combination of the best of both worlds (of private and public enforcement), why are these “quasi-courts” not more dominant in the economy?

\[45\] Tabellini (2008), Dixit (2009), and Baron (2010) are applauseable pioneering papers.
Appendix

A Technical Discussion of the Model

My model builds on Dixit (2003b), who constructs a two-period model, where the Prisoner’s Dilemma payoff from mutual defection (corresponding to \( d \) in Figure 1) is positive. He assumes the existence of two behavioral types, Normal (N) and Macchiavellian (M), and specifies that a N type’s payoff is negative either if his partner unilaterally defects (corresponding to \( l \) in Figure 1) or if he is matched to a M type, an “especially skillful cheater” (1299) whose actions are not modeled. Dixit specifies a candidate equilibrium strategy (related to my II- and AA-strategies), which rests on the assumption that a player who receives a message from his current partner’s former partner learns the payoff of the former partner and thereby can condition his strategy on that payoff. He uses Perfect Bayesian Equilibrium as solution concept.

In Dixit’s finite-period model, \( d > 0 \) to incentivize the players to interact at all with an unknown partner. As a result, however, players would always cooperate with another N type because the unique one-shot equilibrium, mutual defection, yields positive payoffs. This is why he introduces the M types: “Without them, there would be no cheating in equilibrium [...]. [T]he N types’ behavior is driven by their fear of being confused with the M types that are known to lurk in the population” (1301).

By contrast, in my model the game is repeated infinitely many times. Therefore, I can assume \( d < 0 \) and do not need to model behavioral types. Players interact with unknown partners if they expect not to be cheated, which depends on the information received about their current partner’s previous action (not a specific payoff). They do not cheat (for \( X \leq X^* \)) because they fear to lose the payoff from mutual cooperation (\( he^{\theta X} \)) in the next period, not the payoff from mutual defection (\( de^{\theta X} \)), as in Dixit’s model. A Perfect Bayesian Equilibrium would have to be conditioned on the complete information that a player has on his partner’s history. In contrast, the concept of stationary Markov-perfectness allows me to ignore all details except the current state of a player when choosing an action (Mailath and Samuelson, 2006:177). In line with empirical findings, e.g. Bernstein (1992, 2001) or Greif et al. (1994), I defined the state space to be a binary variable here: either a player broke the rules and his new partner heard about it, or not. For tractability, strategies in period \( t \) only depend on behavior in the previous period, \( t - 1 \), but not in earlier periods. This structure is in line with the empirical observation (Ostrom, 1990) and the theoretical finding that optimal punishment periods are finite (Greif, 2006, Appendix C)—although not necessarily at a length of one period.

An additional key difference to Dixit (2003b) is that he focuses on the social network model
and only offers a reduced form model of external enforcement, where “at a cost \( c \) per unit of arc length along the circle, any cheating can be detected and the information is made available to [all] future traders” (1311; emphasis added). He recommends that “external enforcement [...] can be modeled more explicitly” (1312), which is what I have done in this paper, apart from connecting the empirical and theoretical literatures on associations.

*Equilibrium uniqueness:* Dixit comments: “As usual in such games, there is a multiplicity of equilibria, each sustained by its own expectations. But I shall give this system its best shot by looking at the best possible \( X \)” (2003b:1302). I follow a similar approach to study the (maximum) scope of cooperation under which associations assume information intermediary or arbitration functions in an equilibrium. As is clear from the quotation, introducing behavioral types would not solve the multiple equilibrium issue.

### B Proofs

#### B.1 Proof of Proposition 1

To prove Proposition 1 I have to show two characteristics: (i) Monotonicity of \( V_{II,i} \) in \( X \) and (ii) the conditions under which \( V_{II,i} \geq 0 \).\(^{46}\)

On (i): Differentiating \( V_{II,i} \) with respect to \( X \) shows that

\[
\frac{\partial V_{II,i}}{\partial X} = \frac{((e-1)e^{1+X+2X\theta}(w-h)(2-\theta)\theta^2+e^{2X+\theta}(e-1)(\theta-1)(1-\sigma_t+1)\kappa_x\delta_h)^2}{e^{1+X+\theta X}(e-1)(\theta-2)\theta}.
\]

(B.1)

Evaluating (B.1), I get

\[
\frac{\partial V_{II,i}}{\partial X} < 0 \quad \forall \theta > 0 \text{ and for } \theta = 0 \land X \in (0,1).
\]

\[
\frac{\partial V_{II,i}}{\partial X} = 0, \text{ for } \theta = 0 \land X \in \{0,1\}.
\]

It follows that \( V_{II,i} \) is monotonic in \( X \) for \( \theta > 0 \).

On (ii): Substituting values in (7) shows that \( V_{II,i} \geq 0 \) if, and only if:

\[
\frac{h}{w-h} \geq \frac{(e-1)}{e(\theta-2)} + \frac{(e\theta-e)\sigma_{t+1}}{\theta-1} \quad \text{for } X = 0,
\]

(B.2)

\[
\frac{h}{w-h} \geq \frac{e\theta(e-1)}{\theta(1-\sigma_{t+1})\kappa_x} + \frac{(e\theta-e)\sigma_{t+1}}{\theta-1} \quad \text{for } X = 1.
\]

(B.3)

Define \( X_{II,i}^* = \{X|V_{II,i} = 0\} \). Because \( \frac{\partial V_{II,i}}{\partial X} < 0 \), \( X_{II,i}^* \) characterizes an upper bound on \( i \)'s incentive to cooperate in X-space. I have to distinguish among three subcases. First, if (B.2) does not hold, \( V_{II,i} < 0 \) for all \( X \). Hence, player \( i \) has no incentive to cooperate; \( X_{II,i}^* = 0 \). Second, if (B.3)—and necessarily (B.2)—holds, \( V_{II,i} \geq 0 \) for all \( X \). Hence, player \( i \) has an incentive to always cooperate; \( X_{II,i}^* = 1 \). Third, if (B.2) holds but (B.3) does not hold, there is

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\(^{46}\)This proof builds on the proof of Proposition 2 (for the scope of cooperation supported by social networks) in Masten and Prüfer (2011).
a unique $X_{II,i}^{*} \in [0, 1)$, such that $V_{II,i}(X \leq X_{II,i}^{*}) \geq 0 > V_{II,i}(X > X_{II,i}^{*})$. Hence, player $i$ has an incentive to cooperate for $X \leq X_{II,i}^{*}$ but not for $X > X_{II,i}^{*}$. Q.E.D.

B.2 Derivation of the probability $\beta_{II}$

By definition:

$$\beta_{II} \equiv \text{prob}(X_{II,i}^{*}(L(\kappa_x), M_i I(\kappa_x, \sigma_{t+1})) \geq X_{II,x}(L(\kappa_i), M_i I(\kappa_i, \sigma_{t+1})|M_i = 0)$$

(B.4)

$$\Leftrightarrow \text{prob}(V_{II,i}(L(\kappa_x), M_i I(\kappa_x, \sigma_{t+1})) > V_{II,x}(L(\kappa_i), M_i I(\kappa_i, \sigma_{t+1})|M_i = 0)$$

(B.5)

From the perspective of $i$ at stage 0, $\kappa_x$ is unknown and $E(M_x) = \sigma_t$, where the realized equilibrium share of members in the population $\sigma_t$ must correspond to this belief. In any stationary equilibrium, $\sigma_t = \sigma_{t+1}$. Hence, I can rewrite (B.5):

$$\text{prob}(L(\kappa_i) < L(E(\kappa_x)) + \sigma_t I(E(\kappa_x), \sigma_t))$$

(B.6)

Define $\hat{L} \equiv L(E(\kappa_x))/\delta E(\kappa_x)$, such that $\hat{L}$ only depends on $\theta$ and $X$. Define $\hat{I}(E(\kappa_x)) \equiv I(E(\kappa_x))/\delta \sigma_t$, such that $\hat{I}(E(\kappa_x))$ only depends on $\theta$, $X$, and negatively on $E(\kappa_x)$. Hence, I can rewrite (B.6):

$$\text{prob}(\delta \kappa_i h \hat{L} < \delta E(\kappa_x) h \hat{L} + \delta \sigma_t ^2 \hat{I}(E(\kappa_x)))$$

(B.7)

$$\Leftrightarrow \text{prob}(\kappa_i < E(\kappa_x) + \sigma_t ^2 \hat{I}(E(\kappa_x))/\hat{L})$$

(B.8)

It follows that, for $\sigma_t = 0$:

$$\beta_{II} = \text{prob}(\kappa_i < E(\kappa_x)) = (1 - Z(\kappa_i))$$

(B.9)

For $\sigma_t > 0$:

$$\beta_{II} = (1 - Z(\kappa_i)) + Y(E(\kappa_x)),$$

(B.10)

where $Y(E(\kappa_x))$ is a positive number that increases in $\sigma_t$ and decreases in $E(\kappa_x)$. From the perspective of $i$, (B.9) is known and decreasing in $\kappa_i$ and (B.10) can be calculated by resubstituting $\hat{L}$ and $\hat{I}$ and using (3) and (4), all of which depend on model fundamentals.

B.3 Proof of Proposition 3

(i): Consider equation (9). First, recall from Section B.2 that $\partial \beta_{II} / \partial \kappa_i < 0$. Second, ignoring the effect of $I(\kappa_i, \sigma_{t+1})$ on $X_{II,x}^{*}$ and recalling that $L$ and $I$ are additively separable, $\frac{\partial X_{NW,x}^{*}}{\partial \kappa_i} = \frac{\partial X_{II,x}^{*}}{\partial \kappa_i}$: both boundaries of the interval defined in $B_{II}$ shift by the same rate if $\kappa_i$ is changed. Now, considering the effect of $I(\kappa_i, \sigma_{t+1})$ on $X_{II,x}^{*}$, recall that Lemma 1.(ii) states
that \( \frac{\partial I(\kappa, \sigma_{t+1})}{\partial \kappa} < 0 \). Hence, the interval \( (X_{I,x}^* - X_{NW,x}^*) \) decreases in \( \kappa_i \). Because the association membership fee \( f \) is nondiscriminatory with respect to the connectedness of individuals, by definition, it follows that \( B_{II} \) is monotonically decreasing in \( \kappa_i \). As long as \( f \) is not prohibitively high and hence \( B_{II} \geq 0 \) for some players, the marginal player, who has connectedness \( \kappa_{II}^* \), is indifferent between joining and not joining the association: \( B_{II}(\kappa_i = \kappa_{II}^*) = 0 \). It follows that only players with connectedness \( \kappa_i \in [0, \kappa_{II}^*] \) join the association.

(ii): **Equilibrium characterization**: In a stationary equilibrium of a repeated game, \( \sigma_t = \sigma_{t+1} \), by definition. The realized membership share in the total population must be the same in every period and it must correspond to the players’ beliefs about the membership share in the next period. As shown in Section B.2, \( \beta_{II} \) increases in \( \sigma_{t+1} \). Because \( I \) increases in \( \sigma_{t+1} \), \( X_{II,x}^* \) also increases in \( \sigma_{t+1} \). Therefore, \( B_{II} \) also increases in \( \sigma_{t+1} \): from any player’s perspective, if the expected membership size is larger, the value of being able to inform all members about one partner’s behavior increases monotonically.

The fact that \( B_{II} \) monotonically increases in the belief \( \sigma_{t+1} \) but monotonically decreases in a member’s own connectedness, \( \kappa_i \), creates a fixed-point problem. An equilibrium is characterized by a player’s level of connectedness \( \kappa_{II}^* = \sigma_{t+1} \), where the player’s belief about the membership-size corresponds to his own connectedness and he is indifferent between joining and not joining the association, that is where \( B_{II}(\kappa_{II}^*) = 0 \). Then, for every belief \( \sigma_{t+1} \), all players with connectedness \( \kappa_i \leq \kappa_{II}^* = \sigma_{t+1} \) will join the association, whereas all players with \( \kappa_i > \kappa_{II}^* = \sigma_{t+1} \) will not join.

**Existence proof**: Substituting \( \kappa_{II}^* \) for \( \kappa_x \) and \( \sigma_{t+1} \) in (4) yields that \( I(\kappa_{II}^* = 0) = 0 \), \( I(\kappa_{II}^* = 1) > 0 \) and that, for \( \kappa_{II}^* \in (0, 1) \), \( I(\kappa_{II}^*) \) is hump-shaped in \( \kappa_{II}^* \). It follows that \( X_{II,x}^* \) is also hump-shaped in \( \kappa_{II}^* \), the interval \( (X_{II,x}^* - X_{NW,x}^*) \) is hump-shaped in \( \kappa_{II}^* \), and \( B_{II}(\kappa_{II}^*) \) is hump-shaped in \( \kappa_{II}^* \) for \( \kappa_{II}^* \in (0, 1) \), whereas \( B_{II}(\kappa_{II}^* = 0) = -f \) and \( B_{II}(\kappa_{II}^* = 1) > -f \). Given that \( f > 0 \) but sufficiently small, there is at least one level of \( \kappa_{II}^* \) for which \( B_{II}(\kappa_{II}^*) = 0 \), by the intermediate value theorem. Hence, there are three possible equilibrium membership sizes, characterized by \( \kappa_{II}^*(f) \):

1. For **small** levels of \( f \), there is a unique \( \kappa_{II}^*(f) \).

2. For **intermediate** levels of \( f \), there are two \( \kappa_{II}^*(f) \)—denoted by \( \kappa_{II,1} \) and \( \kappa_{II,2} \). The hump-shape of \( B_{II}(\kappa_{II}^*) \) implies that \( \kappa_{II,1} = \{ \kappa_{II}^* \mid \frac{\partial B_{II}(\kappa_{II}^*)}{\partial \kappa_{II}^*} > 0 \} \) and \( \kappa_{II,2} = \{ \kappa_{II}^* \mid \frac{\partial B_{II}(\kappa_{II}^*)}{\partial \kappa_{II}^*} < 0 \} \):

   \[ \kappa_{II,1} < \kappa_{II,2} \]

3. For **large** levels of \( f \), there is no \( \kappa_{II}^*(f) \) for which \( B_{II}(\kappa_{II}^*) = 0 \). Hence, the association has no members and \( \kappa_{II}^*(f) = 0 \).  

Q.E.D.
Illustration: Figure 3 illustrates the dependence of $B_{II}$ from $\kappa_i$ and $\sigma_{t+1}$. As is visible from equation (4), which shapes $B_{II}$, $B_{II} = -f$ if the players hold the belief $\sigma_{t+1} = 0$, independent of $\kappa_i$. For all beliefs $\sigma_{t+1} > 0$, $B_{II}$ is monotonically decreasing in $\kappa_i$. If $\sigma_{t+1} = 1$, $B_{II} > 0$ for all $\kappa_i$. In between these two extreme beliefs, there may be zero, one, or two levels of $\kappa_i$ where $B_{II}(\kappa_i = \sigma_{t+1}) = 0$.

Figure 3: Membership benefit, beliefs, and individual connectedness for intermediate $f$.

Figure 4 exemplifies the specification of $\kappa_{II,1}^*$ and $\kappa_{II,2}^*$. The gross benefit from membership is zero if $\kappa_{II}^* = 0$ (because here everybody would expect that nobody joins the association). It is slightly positive if $\kappa_{II}^* = 1$ (because $I$ is still positive at $\kappa_x = 0$). As long as the fee $f > 0$, the net benefit from association membership, $B_{II}$, is negative at $\kappa_{II}^* = 0$. For intermediate $f$, $B_{II}$ is also negative at $\kappa_{II}^* = 1$ and intersects the horizontal axis twice, at $\kappa_{II,1}^*$ and $\kappa_{II,2}^*$. This case is displayed in Figure 4.

B.4 Proof of Proposition 7

I defined $G$ as a player’s expected per-period gain from mutual cooperation before the identity of his partner is known. If cooperation can be expected for all distances, this implies:

$$G = \int_0^1 \frac{e^{-X}}{2(1-e^{-1})} he^{\theta X} dX = \frac{L}{\delta} + \frac{I}{\delta \sigma_{t+1}} \quad (B.11)$$

$^{47}$See equation (4). This characteristic is due to the imperfect information transmission mechanism of social networks assumed in this model: even if a player is perfectly connected, there is still a positive probability that his message will not be received by another player located at distance $X > 0$. The association does not suffer from this problem, by assumption.

$^{48}$If cooperation can only be expected along the distances $X \in [0,X^*]$, where $X^* < 1$, then $G = \int_0^{X^*} \frac{e^{-X}}{2(1-e^{-1})} he^{\theta X} dX$. However, it still holds that $G(X^* < 1) = L(X^* < 1)/\delta + I(X^* < 1)/(\delta \sigma_{t+1})$. 
Figure 4: Specification of $\kappa^*_{II,1}$ and $\kappa^*_{II,2}$ (for intermediate $f$).

If player $x$’s connectedness and association membership determines the binding scope of cooperation in his partnership with $i$—i.e. if $X^*_I = X^*_{II,i}(\kappa_x, \cdot)$ and $X^*_A = X^*_{AA,i}(\kappa_x, \cdot)$—then $X^*_I < X^*_A$ if, and only if, $V^*_I < V^*_A$. This condition equals (assuming $M_x = 1$, otherwise the comparison resembles the social network case):

$$\lambda I(\kappa_x, \sigma_{t+1}) < \tau_A(\sigma_{t+1}(\delta G - L(\kappa_x)) - M_i \delta F)$$  \hspace{1cm} (B.12)

Substituting (B.11) into (B.12), I get:

$$(\tau_A - \lambda)I > M_i \tau_A \delta F \text{ Q.E.D.} \hspace{1cm} (B.13)$$

**B.5 Proof of Proposition 8**

**Preliminaries:** As the proof is strongly related to the proof of Proposition 3, here I focus on the main differences between the two cases. I first have to specify the dependence of $\beta_{AA}$, the probability with which $M_i$ has an effect on $X^*_A$. By definition:

$$\beta_{AA} \equiv \text{prob}(X^*_{A,i}(L(\kappa_x), M_x \tau_A d^*(\kappa_x, \sigma_{t+1}, M_x, F)) > X^*_{AA,i}(L(\kappa_i), M_i \tau_A d^*(\kappa_i, \sigma_{t+1}, M_x, F)|M_i \{B.10\}))$$

$$\Leftrightarrow \text{prob}(V_{A,i}(L(\kappa_x), M_x \tau_A d^*(\kappa_x, \sigma_{t+1}, M_x, F)) > V_{AA,i}(L(\kappa_i), M_i \tau_A d^*(\kappa_i, \sigma_{t+1}, M_x, F)|M_i \{B.15\}))$$

$$\Leftrightarrow \text{prob}(L(\kappa_i) < L(E(\kappa_x)) + \sigma^2_{t} \tau_A (\delta G - L(E(\kappa_x))))$$  \hspace{1cm} (B.16)

Recall that $E(M_x) = \sigma_t$, $\sigma_t = \sigma_{t+1}$, and $\hat{L} \equiv L(E(\kappa_x))/\delta E(\kappa_x)h$. Define $\hat{G} \equiv G/h$, such that $\hat{G}$ only depends on $\theta$. Then, I can rewrite (B.16):

$$\beta_{AA} = \text{prob}(\kappa_i < E(\kappa_x) + \sigma^2_{t} \tau_A (\frac{\hat{G}}{L} - E(\kappa_x)))$$  \hspace{1cm} (B.17)
Recall that \( \text{prob}(\kappa_i < E(\kappa_x)) = (1 - Z(\kappa_i)) \), which is decreasing in \( \kappa_i \). It follows that \( \beta_{AA} \) is decreasing in \( \kappa_i \) and increasing in \( \sigma_t \), just as \( \beta_{II} \).

**Proof:** (i) The proof of Proposition 3.(i) applies if I substitute \( X^*_{AA,x} \) for \( X^*_{II,x} \), and \( \tau_A d^*(\cdot) \) for \( I(\cdot) \), \( F \) for \( f \), and \( B_{AA} \) for \( B_{II} \). As a result, \( B_{AA} \) monotonically increases in the belief \( \sigma_{t+1} \) and monotonically decreases in \( \kappa_i \).

(ii): Substituting \( \kappa^*_{AA} \) for \( \kappa_x \) and \( \sigma_{t+1} \) in (10) yields that \( \tau_A d^*(\kappa^*_{AA} = 0) = 0, \tau_A d^*(\kappa^*_{AA} = 1) = \tau_A(\delta G - L(\kappa^*_{AA})) > 0 \), whereas, for \( \kappa^*_{AA} \in (0,1), \tau_A d^*(\kappa^*_{AA}) \) is hump-shaped in \( \kappa^*_{AA} \). However, \( d > 0 \) (see (12)) implies that the set of \( \kappa^*_{AA} \)-values for which \( d^* \) can sustain an equilibrium based on the AA Strategy is censored at some \( \kappa > 0 \) and some \( \kappa < 1; \kappa \leq \bar{\kappa} \). It follows that \( X^*_{AA,x} \) is also hump-shaped and censored in \( \kappa^*_{AA} \), the interval \( (X^*_{AA,x} - X^*_{NW,x}) \) is hump-shaped and censored in \( \kappa^*_{AA} \), and \( B_{AA}(\kappa^*_{AA}) \) is hump-shaped and censored in \( \kappa^*_{AA} \). Figure 5 displays this case. By the intermediate value theorem, it follows that:

1. For small \( F \), there are two \( \kappa^*_{AA}(F) \), denoted by \( \kappa^*_{AA,1}(F) \) and \( \kappa^*_{AA,2}(F) \), \( \kappa^*_{AA,1}(F) < \kappa^*_{AA,2}(F) \).

2. For large \( F \), there is no \( \kappa^*_{AA}(F) > 0 \) for which \( B_{AA}(\kappa^*_{AA}) = 0 \). Hence, the association has no members and \( \kappa^*_{AA}(F) = 0 \). Q.E.D.

![Figure 5: Specification of \( \kappa^*_{AA,1} \) and \( \kappa^*_{AA,2} \) (for small \( F \)).](image-url)
References


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