History of Political Economy 22:2

CCC 0018–2702/90/$1.50

Walras on temporary equilibrium and dynamics

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1. *Introduction*

In economic textbooks Walras is regarded as the inventor of the theory of a *static* general equilibrium in a *nonmonetary* economy based upon the essential assumptions of marginalism *without* uncertainty. Although scholars like Jaffé (1980, 1981), Morishima (1977, 1980) and Walker (1987a, 1987b) time and again have sought to correct this caricature, this image of Walras's contribution to economics still remains in the minds of many economists. The "textbook" interpretation of Walras's theory runs, by and large, as follows. The most important feature of his analysis is the marginalist foundation in the behavior of agents in the context of a general market-clearing framework. As Hicks puts it: "the discovery of the conditions of static equilibrium under perfect competition was his central achievement" (1934, 347).[1] In his seminal *Value and capital* (1939, 57–61), Hicks confirms this interpretation of Walras's theory. Producers maximize their profits subject to technical restrictions, whereas consumers maximize their utility conditional upon budgetary constraints. Prices are determined outside the sphere of influence of individual agents. They do not decide on price setting but instead act as price takers. An auctioneer announces prices in a manner such that planned supply and planned demand converge. Exchange only occurs after the declaration of market-clearing prices. The prevalence of a market-clearing equilibrium is a sequel to a price *tâtonnement*. The analysis captures n commodities and n markets. One commodity operates as *numéraire*. Neither uncertainty nor time plays a role in the theory. The proof of the existence of an equilibrium follows from counting the number of equations and unknowns in the model.

In line with this common "textbook" interpretation of Walras's contribution to economic science, his *Eléments d'économie politique pure* (1874) is regarded as the first formal static general equilibrium analysis,

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1. Moreover, Hicks indicates that Walras's "position with Jevons and Menger as one of the independent discoverers of the Marginal Utility principle is generally regarded as Leon Walras' chief title to fame; and this no doubt justly enough" (1934, 338). Here the history and achievements of marginalism remain undiscussed (see, for example, Schumpeter 1954, Jaffé 1972, and van Witteloostuijn 1988). In van Witteloostuijn and Maks (1989) the resemblance of Walras's concepts with Hicks's work is outlined explicitly.

since up until the 1870s only partial equilibrium analyses were available (for example, Cournot 1838). This image of Walras's theory has been reinforced by his successors, who by and large have focused on the static part of his framework. Economists like Wald (1936), von Neumann (1937), and Hicks (1939) in the 1930s and Arrow and Debreu (1954; also Debreu 1959) and McKenzie (1959) in the 1950s have contributed to this stereotyping by assigning themselves the task of shaping Walras's static general nonmonetary equilibrium model in a way that answers questions involving the existence, stability, and uniqueness of an equilibrium.

For decades now eminent scholars have sought to enrich the traditional interpretation of Walras's theory. For example, Jaffé pointed to Walras's notions of capital formation (1942, 1953) and *tâtonnement* (1967, 1981), Morishima indicated his theory of money and capital (1977, 1980), and Walker stipulated the role of the entrepreneur in dynamic *tâtonnement* in the Walrasian framework (1986, 1987a, 1987b).[2]

This paper adds to this literature in two ways. First, the elaborate analysis of a state of *temporary* equilibrium in the context of a *dynamic* framework, as attributed to Walras by the scholars mentioned above, is gone through systematically. We argue that Walras (1) analyzed a *temporary* equilibrium (section III); (2) considered several additional complexities and insights which interfered with proving the *existence* and/or *stability* of equilibrium (section IV); and (3) pointed to the importance of *dynamics* (section V). Second, it is stipulated that Walras had an open eye for the role of *uncertainty* and *disequilibrium* in dynamic economics (section VI). The core of the paper is preceded by a brief outline of Hicks's (1939) well-known elaboration on a dynamic Walrasian system (section II) and concluded with some final remarks (section VII).

II. *Hicks on a dynamic Walrasian framework*

In *Value and capital* (1939) Hicks introduces the temporary equilibrium method to include the element of time in Walras's analysis of general equilibrium. In effect Hicks adopts a Swedish notion of period analysis

2. For example, Morishima (1977) expresses the opinion that "although Walras is celebrated for his studies in the utility theory of exchange and the general equilibrium theory of production, these are, in the full analysis which he offers, no more than an *hors d'oeuvre* to introduce the main dishes: the theory of capital formation and credit and the theory of circulation and money. Sometimes, *hors d'oeuvres* are more delicious and more substantial than the main dishes. But not in Walras's case; contrary to the vulgar view, I do rate his growth and money theory much more highly than his exchange and production theory" (1977, 70). As Jaffé (1942, 139) notes, Walras follows Marshall's period structure. Morishima even argues that Walras's treatment of saving and investment decisions offers the opportunity to introduce Keynesian results into Walras's framework (cf., e.g., Jaffé 1942 and Schumpeter 1954), in particular because Walras stipulated that decisions to save and invest are, in general, *separate* decisions made by *different* groups of agents (Walras 1954, 270). Therefore, outside the market-clearing equilibrium the equality ex ante between savings and investments does not necessarily hold.

(Lindahl 1929) by introducing a description of the economy as a sequence of discrete periods. During a period of time "variations in prices can be neglected. For theoretical purposes this means that prices will be supposed to change, not continuously, but at . . . intervals" (1939, 122). Within each period a price *tâtonnement* (131) yields a temporary market-clearing equilibrium (123). Essential is an assumption of temporary certainty, which "assume[s] perfect contemporaneous knowledge—that everyone knows the current prices in all those markets which concern him" (123). Hicks introduces dynamics by arguing that "we become able to treat a process of change consisting of a series of temporary equilibria; this enables us still to use equilibrium analysis in a dynamic field" (127). Intertemporal equilibrium occurs as a result of disappointed expectations over future events (intertemporal uncertainty), where "the degree of disequilibrium marks the extent to which expectations are cheated, and plans go astray" (132).

III. *Walras on temporary equilibrium*

It is particularly in part 4 of his book, "Theory of circulation and money," that Walras indicates his concern with *temporary* market-clearing equilibrium. A market-clearing equilibrium "will be established *effectively* by the reciprocal exchange between savings to be accumulated and new capital goods to be supplied *within a given period of time,* during which *no change in the data is allowed*" (Walras 1954, 282–83, emphasis his; also quoted in Morishima 1977, 79 and Jaffé 1980, 358).[3] Hence Walras restricts the equilibrium method to a given period of time during which changes are, by definition, conspicuous by their absence.

The constancy of the data during the given period allows for a further abstraction: temporary perfect foresight (see also Walras's correspondence with Poincaré, reported in Jaffé 1977). Walras argues that

> there may be a small element of uncertainty which is due solely to the difficulty of foreseeing possible changes in the data of the problem. If, however, we suppose these data constant for a given period of time and if we suppose the prices of goods and services and also the dates of their purchase and sale to be known for the whole period, there will be no occasion for uncertainty. (1954, 317, see also 318; quoted in Morishima 1980, 557–58)

Walras's introduction of equilibrium and his abstraction of uncertainty *for a given period of time* bear close resemblance to Hicks's temporary

3. All quotations from Walras are from Jaffé's English translation (1954) of the definitive (1926) edition of the *Eléments d'économie politique pure.* Additional references are included to other scholars who have quoted the essential passages elsewhere. References to Jaffé's publications give the original date but pagination from Walker's collected edition (Jaffé 1983).

equilibrium method (cf. Morishima 1977, 80; 1980, 552–54). During a given period of time, perfect foresight assures the confirmation of expectations of current variables and conditions.

The next question to ask is whether Walras takes the position that the temporary equilibrium is a real or an ideal state. Here both opposing views prevail, as is clearly manifest in the disagreement between Jaffé and Morishima (see especially their discussion in the *Journal of Economic Literature* 1980). On the one hand, Jaffé argues that

> the *Eléments*, instead of aiming to delineate a theory of the working of any real capitalistic system, was designed to portray how an imaginary system *might* work. . . . The *Eléments* was intended to be and is, in all but the name, a realistic utopia, i.e., a delineation of a state of affairs nowhere to be found in the actual world, independent of time and place, ideally perfect in certain respects, and yet composed of realistic psychological and material ingredients. (1980, 345, emphasis his; see also Walker 1984)

On the other hand, Morishima states that "the ultimate aim of the book was to construct a model, by the use of which we can examine how the capitalist system works" (1977, 4; see also Schumpeter 1954). A quotation from Walras himself supports Jaffé's interpretation:

> Equilibrium in production, like equilibrium in exchange, is an *ideal* and *not* a *real* state. It never happens in the real world that the selling price of any product is absolutely equal to the cost of the productive services that enter into that product, or that the effective demand and supply of services or products are absolutely equal. Yet equilibrium is the normal state, in the sense that it is the state towards which things spontaneously tend under a regime of free competition in exchange and production. (1954, 224, emphasis ours)

IV. *Walras on existence and stability*

Before turning to Walras's notions of dynamics, it is worthwhile to stipulate that his analysis of a temporary equilibrium moves far beyond the elementary level suggested by the "textbook" interpretation, since he considers several additional complexities and insights that impede proving the existence and/or stability of equilibrium. Hence the charge that Walras's proofs amount to no more than counting the number of equations and unknowns in a model is unjustified (see, e.g., Schumpeter 1954, Jaffé 1971, Morishima 1977). For illustrative purposes we present seven examples.

(1) Walras introduces a price *tâtonnement* to smooth temporary adjustment to equilibrium. In this respect he moves beyond the static

notion of *tâtonnement* adjustment to equilibrium by stressing the *dynamic* aspects of the equilibrating *process* towards temporary equilibrium (see, e.g., Walras 1954, 242). The dynamic aspects of Walras's *tâtonnement* have already been pointed out by Schumpeter (1954), Patinkin (1956), and Jaffé (1967, 1981). Recently, Walker (1986, 1987a, 1987b) has offered excellent elaborations on this subject which include a treatment of the role played by entrepreneurs.[4]

(2) Walras justifies the use of the *tâtonnement* process in his models by pointing to a method of analysis analogous with pure mechanics in his application of a method of *decreasing abstraction* (see, e.g., Schumpeter 1954, Morishima 1977). He argues that

> the more perfectly competition functions, the more rigorous is the manner of arriving at value in exchange. . . . Our task is to discover the laws to which . . . purchases and sales tend to conform automatically. To this end, we shall suppose the market is perfectly competitive, just as in pure mechanics we suppose, to start with, that machines are perfectly frictionless. (1954, 83–84)

(3) Walras recognizes that the introduction of production raises two additional difficulties in the context of price *tâtonnements* that must be bypassed in order to exclude the possibility of *false trading* (see, e.g., Schumpeter 1954, Morishima 1977, Jaffé 1981, Walker 1987a, 1987b). First, following the announcement of disequilibrium prices, production cannot be permitted, since production reflects an irreversible process. Therefore Walras does not permit production to proceed during the *tâtonnement* process. Instead, "tickets" (Jaffé) or "pledges" (Walker) replace anticipative production and, thereby, reflect planned supply (e.g., Walras 1954, viii). Second, Walras sidesteps the observation that production takes time by arguing that "production . . . requires a certain lapse of time. We shall resolve the . . . difficulty purely and simply by ignoring the time element at this point" (1954, 242, see also 282).

(4) Walras considers the special position of *free* goods (see, e.g., Schumpeter 1954). Free goods lie outside the sphere of influence of the economic process, because they are not scarce and therefore have no value in exchange. Walras argues that

> the *rareté* defined . . . as the intensity of the last want satisfied is precisely the same thing as the *scarcity* we have defined earlier . . . in terms of the twin conditions of utility and limitation in quantity.

4. Walker mentions the remarkable similarity between Walras's and Schumpeter's (1943) notions of the entrepreneurial role. Moreover, he argues that "Schumpeter added a special type of entrepreneur who initiates disequilibrium by the activity of innovating . . . and conceived of that activity as constituting another explanation of true profits" (Walker 1986, 21). Of course, in equilibrium entrepreneurs face zero profits.

> There could not possibly be a last want satisfied if there were no want, that is to say, if a commodity had neither extensive nor intensive utility, or if it were useless. Moreover, the intensity of the last want satisfied would be zero if a commodity which possessed a utility curve were so plentiful that its quantity exceeded its extensive utility, as would be the case, for example, if it were unlimited in quantity. (1954, 145, emphasis his)

(5) Walras admits that the ideal of free competition is only relevant in the case of private goods and loses its significance when *public* goods enter the picture.

> Once a principle has been scientifically established, the first thing that one can do is to distinguish immediately between the cases to which the principle applies and those to which it does not apply. Conversely, the fact that economists have often extended the principle of free competition beyond the limits of its true applicability is proof positive that the principle has not been demonstrated. For example . . . our proof implies a fundamental distinction between individual wants, i.e., private utility which the individual is capable of estimating, and social wants or public utility which is estimated in an entirely different way. Therefore, the principle of free competition, which is applicable to the production of things for private demand, is not applicable to the production of things where public interest is involved. (1954, 256–57)

Two other subjects which, according to Walras, lie beyond the scope of free competition are "natural and necessary monopolies" (1954, 257) and "the question of justice" (257).

(6) Walras attempts to show that a *monetary* equilibrium exists (see, e.g., Schumpeter 1954, Morishima 1977, Jaffé 1980) in which money has a positive price (the well-known Hahn problem; see Hahn 1965). He discerns a money market in which demand for money (desired cash balance) meets supply of money (quantity of money) (Walras 1954, 321). The money market is also subject to Walras's law of demand and supply, so that "the price of the service of money is established through its rise or fall according as the desired cash balance is greater or less than the quantity of money" (327). Walras classifies the service of money with the smoothing, over time, of the pattern of expenditures and receipts and the transference of purchasing power over time (e.g., 317).

(7) Walras pays attention to the *stability* features of equilibrium (see, e.g., Schumpeter 1954, Jaffé 1971, Morishima 1977). In respect to the n-market equilibrium he argues that

> such an equilibrium is exactly similar to that of a suspended body of which the centre of gravity lies directly beneath the point of sus-

> pension, so that if this centre of gravity were displaced from the vertical line beneath the point of suspension, it would automatically return to its original position through the force of gravitation. This equilibrium is, therefore, *stable*. (Walras 1954, 109, 112, emphasis his)

Morishima points out that "Walras discussed the stability of equilibrium in the framework of a many-goods economy. In spite of this fact, many economists credit Hicks with the first attempt to generalize to any number of markets the stability condition of a single market" (1977, 35).

V. *Walras on dynamics*

It was noted above that Walras had an understanding of the dynamics of *tâtonnement* processes (section IV, example 1). But dynamics within a period during which a temporary equilibrium is reached occur very differently from intertemporal dynamics over a series of such periods. Walras recognized the importance of the dynamics of a sequence of temporary equilibria (see, e.g., Morishima 1977, 1980; Jaffé 1980, 1981). In order to move into *intertemporal dynamics* he considered the changes as they occurred in a subsequent period by arguing that "although the economy is becoming *progressive*[5] it remains *static* because of the fact that the new capital goods play no part in the economy until later, in a period subsequent to the one under consideration" (1954, 378, emphasis his; also quoted in Morishima 1977, 79 and Jaffé 1980, 358). Hence in a subsequent period the quantities may change, leading to a new and different temporary equilibrium. The static analysis is dropped in favor of dynamic economics. Changes appear at the junction of subsequent periods. Equilibrium is a reference point that permanently moves.

> Our economy will then be ready to function and we shall be in a position, if we so desire, to pass from the static to the *dynamic* point of view. In order to make this point of transition we need only suppose the data of the problem, viz. the quantities possessed, the utility or want curves, etc., to vary as a function of *time*. The *fixed* equilibrium will then be transformed into a *variable* or *moving* equilibrium, which reestablishes itself automatically, as soon as it is

5. Another important aspect of dynamic economics, technological innovation, is also discussed by Walras: "It should, therefore, be clearly understood that every time the production function itself undergoes a change, we have a case of technical progress brought about by science and that every time the coefficients of production made up of land-services increase without any change in the production function, we have a case of economic progress resulting from saving. In reality, both kinds of progress may take place simultaneously. . . . But in this discussion we shall abstract from technical progress and consider economic progress only" (1954, 386). Here it should be noted that Walras owes his theory of marginal productivity to his correspondence with Barone (Jaffé 1964).

disturbed. (Walras 1954, 318, emphasis his; also quoted in Morishima 1977, 79–80 and 1980, 553)[6]

Thus Walras's analysis in entirety contains three phases:

(1) the phase of *preliminary gropings* towards the establishment of equilibrium in principle;
(2) the *static* phase in which equilibrium is effectively established *ab ovo* as regards the quantity of productive services and products made available during the period considered, under the stipulated conditions, and without any changes in the data of the problem;
(3) a *dynamic* phase in which the equilibrium is constantly being disturbed by changes in the data and is constantly being reestablished. (1954, 319, emphasis his; also quoted in Morishima 1980, 553)

Apart from saying that an economy can be described as a sequence of moving temporary equilibria, Walras concluded that a close approximation of economic reality requires the analysis of the *continuous market*. An economy continually needs to adjust to changing circumstances (data) without equilibrium ever being reached.

> In order to come still more closely to reality, we must drop the hypothesis of an annual market period and adopt in its place the hypothesis of a continuous market. Thus, we pass from the static to the dynamic state. . . . Such is the continuous market, which is perpetually tending towards equilibrium without ever actually attaining it, because the market has no other way of approaching equilibrium except by groping, and, before the goal is reached, it has to renew its efforts and start over again, all the basic data of the problem, e.g., the initial quantities possessed, the utilities of goods and services, the technical coefficients, the excess of income over consumption, the working capital requirements, etc., having changed in the meantime. (1954, 380; also quoted in Morishima 1977, 80–81 and Jaffé 1980, 366)[7]

6. With respect to changing data Walras even mentions the time variability of utility (see, e.g., Schumpeter 1954), although he does not elaborate this point further. He is "assuming that, during this interval [the given period], the utility, both extensive and intensive, remains *fixed* for each party, which makes it possible for me to include time implicitly in the expression of utility. Were this is not the case and had I supposed utility to be a *variable* functionally related to time, then time would have had to figure explicitly in the problem. And we should then have passed from economic *statics* to economic *dynamics*" (1954, 117, emphasis his).

7. Walras's arguments in favor of the continuous market show a resemblance to neo-Austrian theory. Hayek, for instance, argues that competition "is thus a process which involves a continuous change in the data and whose significance must therefore be

In light of his general equilibrium framework, a last remark is in order with respect to the significance that is to be attributed to Walras's notions of dynamics. Here Jaffé and Morishima (again) support different interpretations. On the one hand, Jaffé argues that Walras's insights on dynamics are only an afterthought: "Walras's strict adherence to the statical hypothesis on which the whole general equilibrium edifice rested led him to relegate his discussion of the 'Conditions and Consequences of Economic Progress' to part VII, toward the end of the *Eléments,* where it was tacked on, along with part VIII, as a *coda* structurally separate from the preceding self-contained pure theory" (1980, 365–66). On the other, Morishima holds the opinion that Walras's dynamics completes his general equilibrium framework: "Neither Schumpeter, Jaffé nor Blaug [1973] recognizes the importance of Part VII, entitled 'Conditions and Consequences of Economic Progress' " (1977, 5). Morishima (1980, 552) offers a convincing argument in favor of this interpretation, which implies that "the equilibrium established by Walras's theory of capital formation is no more than a temporary equilibrium. . . . but in Part VII, he . . . obtains a sequence of temporary equilibria for years 1, 2, 3, . . . , which gives 'a variable or moving equilibrium' through time" (552–53).

VI. *Walras on uncertainty and disequilibrium*

A sequence of moving temporary equilibria in the case of intertemporal perfect foresight guarantees the existence of an intertemporal equilibrium (Hicks 1939, 132; 1965, 24), as shown by Debreu (1959), Arrow and Hahn (1971), Radner (1974), and Grandmont (1983), among many others. An essential feature of Hicks's method of temporary equilibrium is the recognition of uncertainty and disequilibrium in a dynamic context. Disappointment in expectations over future conditions and events provokes disequilibrium. In Walras's general equilibrium model, the future inevitably enters the picture as soon as investment is introduced.

> Let P be the price of a capital good. Let p be its *gross* income, that is, the price of its service inclusive of both depreciation charge and the insurance premium. Let $u \cdot P$ be the portion of this income representing the depreciation charge and $v \cdot P$ the portion representing the insurance premium. What remains of the gross income after both charges have been deducted, $\pi = p - (u + v) \cdot P$, is the *net* income. . . . Under equilibrium conditions the ratio $[p - (u +$

completely missed by any theory which treats these data as constant" (1949, 106). Of course there are important differences between the neo-Austrian approach and Walras's theory. To mention two: (1) the neo-Austrian emphasis on the entrepreneurial role in the competitive discovery process and (2) the neo-Austrian opposition against the use of formal modeling (e.g., Kirzner 1973, 1985).

$v) \cdot P] / P$, or the rate of net income, is the same for all capital goods. Let i be this [common] ratio. When we determine i, we also determine the prices of all landed capital, personal capital and capital goods proper by virtue of the equation

$$p - (u + v) \cdot P = i \cdot P$$

or

$$P = p / (i + u + v).$$

(1954, 268–69, emphasis his)

The net income derived from a capital good is the present value of a capital good discounted with factor $1 / i$ over an infinite number of time periods where prices are *constant* (cf. Jaffé 1953, 164). From Walras's argument it is clear that "the [v] in Walras's equations of capital formation shows that he admitted *insurable risks* into his static system. . . . Measurable risks are not incompatible with static assumptions when the insurance of these risks relieves the insured" (Jaffé 1980, 352, emphasis his). Morishima expresses the same view by arguing that Walras's "theories of capital and money obviously allow for insurable risks. The rates of insurance premiums, denoted by him as [v], take on some given values reflecting the states of expectation concerning sudden and unforeseen destruction of capital goods by accident. Walras does not rule out uncertainties and risks of this kind, but assumes a given state of expectation of them" (1980, 558). The question, though, is whether the net present value of income from capital goods follows from an assumption of perfect foresight or from uncertainty. Is the assumption of an infinite sequence of constant prices (or net incomes) the result of uncertain expectations?

Although some scholars argue that Walras took notice of uncertainty as an afterthought, economists' general feeling is that an essential feature of Walras's formal analysis is an abstraction from uncertainty. For example, Jaffé argues that "the [v] in Walras's equations, far from symbolizing the presence of uncertainty in Walras's model, turns out to be a device for eliminating any vestige of uncertainty" (1980, 352); that "Walras failed to incorporate the concept of expectations in his theoretical structure" (1942, 150); and that "none of the uncertainties and unpredictable fluctuations, and none of the speculative expectations growing out of irremediable ignorance of the future are admissible within the bounds of [Walras's] '. . . static point of view' " (1980, 361). Morishima confirms Jaffé's interpretation by noting that "there is no analysis of . . . uncertainty in the *Eléments*" (1977, 205). This interpretation is not the only possible one.

The fact of the matter is that Walras only explicitly eliminates the

uncertainty of *current* variables. That is, events *within* a given period of time are not subject to uncertainty. Here his assumption does not deviate from Hicks's, since when "we remove this possibility of change for a given period of time, and if we suppose the prices of goods and services and also the dates of their purchase and sale to be known for the entire period, we eliminate all occasion for uncertainty" (Walras 1954, 318; also quoted in Morishima 1980, 558). However, this does not mean that the perfect-foresight assumption extends to events in *subsequent future* periods, when the data of the economy are no longer fixed. Capital goods yield income precisely in those future periods. As Schumpeter argues, "the only motive that capitalists can have in *his* [Walras's] *set-up* for demanding capital goods is in the net revenue *expected* from them" (1954, 1017, first emphasis his, second emphasis ours).[8] Hicks, above all, points out that "people expect the prices of products to remain unchanged in the future (as Walras tacitly assumes they do)" (1934, 346).[9]

While of course this expectation may be correct, there is no reason to support this hypothesis on a priori grounds. In effect Walras argues that "we must remember that the prices of capital goods vary not only by reason of past changes but also by reasons of *expected* changes either in gross income or in rates of depreciation and insurance; and that, especially with regard to future changes, expectations *differ* from individual

8. Walker argues that Walras "mentioned the uncertainty of receiving profits and the risk of incurring losses, although he did not develop that line of thought" (1986, 20). The lack of clarity that Walras displays in assuming temporary perfect foresight, along with (tacitly) intertemporal uncertainty, gives birth to a confusion that is manifest in Schumpeter's and Morishima's ambiguity. On the one hand, Schumpeter argues that "since the existence of . . . inventories [in Walras's system] presupposes a certain past behavior of the people concerned and since their current reproduction presupposes certain *expectations*, the system—even if perfectly stationary—still depicts a process in time and might therefore be called 'implicitly dynamic' " (1954, 1002, emphasis his), whereas, on the other hand, he states that Walras "is *excluding* uncertainty by special hypothesis" (1022, emphasis ours). In the same fashion, Morishima argues that "there is no analysis of risk and uncertainty in the *Eléments*" (1977, 205), as well as that "this does not mean that Walras has confined himself in a perfect world with no uncertainty and no risk" (1980, 558). The confusion surrounding the interpretation of Walras's treatment of dynamics (and uncertainty) may be due to the varying definitions of dynamics that are in use. Cf. Jolink and van Daal (1988).

9. Apparently agents have no reason to expect a change in the relevant data and project the current situation into an infinite future. Hence Walras's assumption of intertemporal uncertainty is remarkably similar to Keynes's famous convention that "it would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain. It is reasonable, therefore, to be guided to a considerable degree by the facts about which we feel somewhat confident, even though they may be less decisively relevant to the issue than other facts about which our knowledge is vague and scanty. For this reason the facts of the existing situation enter, in a sense disproportionately, into the formation of our long-term expectations; our usual practice being to take the existing situation and to project it into the future, modified only to the extent that we have more or less definite reasons for expecting a change" (1936, 148).

to individual" (1954, 310, emphasis his; also quoted in Jaffé 1942, 150).[10] He recognizes that expectations differ from individual to individual, a fact that resembles one of Hicks's causes of *intertemporal* disequilibrium. Hicks argues that "our analysis suggests several possible causes of disequilibrium. One (perhaps the least important) arises when different people's price-expectations are inconsistent" (1939, 133). (Intertemporal) uncertainty and (intertemporal) disequilibrium are but two sides of the same coin. Walras points to the relevance of disequilibrium by noting that

> viewed in this way, the market is like a lake agitated by the wind where the water is incessantly seeking its level without ever reaching it. But whereas there are days when the surface of a lake is almost smooth, there never is a day when the effective demand for products and services equals their effective supply and when the selling price of products equals the cost of the productive services used in making them. . . . It can happen and frequently does happen in the real world, that under some circumstances a selling price will remain for long periods of time above cost of production and continue to rise in spite of increase in output, while under other circumstances, a fall in price, following upon this rise, will *suddenly* bring the selling price below cost of production and force entrepreneurs to reverse their production policies. For, just as a lake is, at times, stirred to its very depths by a storm, so also the market is sometimes thrown into *violent confusion* by crises, which are *sudden* and general disturbances of equilibrium. (1954, 380–81, emphasis ours)[11]

VII. *Economists on Walras*

In contrast with the "textbook" interpretation, it appears that Walras was engaged in the analysis of a *temporary* market-clearing equilibrium. He stipulated that the economy can be described as a *sequence* of temporary equilibria reflecting a *moving reference point*. He regarded the states of free competition and market clearing only as *ideal* rather than real. In effect he concluded that a close approximation of reality

10. The argument proceeds by saying that "in any case, once the decision has been made on the basis of the rate of net income to exchange capital goods, both new and old, the exchange takes place in conformity with the mechanism of free competition and the law of offer and demand" (Walras 1954, 311). That is to say, although uncertainty in the future may yield intertemporal disequilibrium, it is no barrier to reaching temporary equilibrium. It should be noted that Jaffé argues that Walras only recognizes the importance of uncertainty as an afterthought (1942, 150). Cf. the discussion between Jaffé and Morishima in the *Journal of Economic Literature* 1980.

11. Schumpeter points out that "the analogy with the 'level' of a lake in order to convey his idea [follows from] the old idea of A. Smith" (1954, 999).

requires an analysis of a *continuous* market. In order to avoid a guaranteed intertemporal equilibrium, he adopted intertemporal *uncertainty* in the form of the projection of the current situation into an infinite future. In line with the role played by uncertainty, he pointed to the relevance of *disequilibrium*.

However, with respect to many aspects of his theory Walras did not himself develop a formal model of the conceptual framework. Some points have been picked up by his successors. For example, Hicks introduced an explicit treatment of uncertainty and expectations (1939) and a fixed-price method of temporary equilibrium (1965). Some of Walras's other suggestions are still difficult to explore formally. Noteworthy of these is a formal analysis of disequilibrium dynamics.[12] Nevertheless, the enriched interpretation of Walras's contribution to economic science reinforces Schumpeter's well-known evaluation that "so far as pure theory is concerned, Walras is in my opinion the greatest of all economists" (1954, 827).

The authors would like to thank two anonymous referees for their valuable comments. Of course, they cannot be blamed for remaining errors.

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12. Insofar as disequilibrium dynamics are offered in the economic literature, they are based upon employing simulative techniques. See, e.g., Nelson and Winter (1982), Duménil and Lévy (1987) and van Witteloostuijn and Maks (1988).

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