MONETARY POLICY RULES, ADVERSE SELECTION AND LONG-RUN FINANCIAL RISK

BY

HANS BLOMMESTEIN, SYLVESTER EIJFFINGER, ZONGXIN QIAN

November 2011

European Banking Center Discussion Paper
No. 2011-032

This is also a CentER Discussion Paper
No. 2011-121

ISSN 0924-7815
Monetary Policy Rules, Adverse Selection
and Long-Run Financial Risk*

Hans Blommestein† Sylvester Eijffinger‡ Zongxin Qian§

November 11, 2011

Abstract

This paper constructs a macro-finance model with two types of borrowers: entrepreneurs who engage in productive activities and gamblers who play in lotteries. It links a central bank’s interest rate policy to expected cash flows of both types of borrowers. Via this link we study how the interactions between various shocks and different monetary policy rules affect the quality of the borrower pool faced by financial intermediaries. We find that if the economy is hit by an expansionary monetary policy shock, in the long run the proportion of entrepreneurs in the borrower pool will be persistently lower than the steady state level. This worsening of the borrower pool is more serious if the central bank does not react to output fluctuations. By contrast, not reacting to output fluctuations in case of a negative productivity shock avoids a persistent worsening of the borrower pool in the long run.

Keywords: Monetary Policy, Adverse Selection, Financial Crisis

JEL codes: E44 E52 G01

*We are grateful to Klaus Desmet, Benedikt Goderis, Jenny Ligthart, Kebin Ma, Rob Nijskens, Peter van Oudheusden, Maria Fabiana Penas, Damjan Pfajfar, Louis Raes, Sjak Smulders, Roberto Rigobon, Harald Uhlig, Burak Uras, Gonzaque Vannoorenberghe, Huaxiang Yin and other seminar participants at Tilburg University for helpful discussion.
†CentER, Tilburg University; OECD.
‡CentER and EBC, Tilburg University; CEPR.
§CentER and EBC, Tilburg University. Corresponding author, email: z.qian@uvt.nl
1 Introduction

Taylor (2009) suggests that government policies could be sources of financial crises. In this paper, we focus on the impact of one particular type of government policy, central bank’s interest rate policy, on financial stability. More specifically, we investigate how the interactions between various shocks and central bank’s interest rate rules dynamically affect the adverse selection problem faced by financial intermediaries.

To that end, we build a dynamic general equilibrium model with two types of borrowers. One is a gambler who borrows to invest in a fixed-supply gambling asset. The other is an entrepreneur who borrows to pay the set-up cost for production. Borrowers are protected by the limited liability law. Limited liability together with fixed-supply can generate a bubble in the gambling asset market (Allen and Gale, 2000). When there is a bubble, lending to gamblers generates expected losses.1 In this case, there are two reasons why the gamblers still get loans from financial intermediaries (Barlevy, 2008). First, lending to entrepreneurs generates expected profits. Second, there is no screening between gamblers and entrepreneurs.2 Without screening in the financial intermediation sector, a persistent decrease in the proportion of entrepreneurs in the borrower pool can accumulate huge losses for financial intermediaries. This paper links the central bank’s interest rate policy to changes in the borrower pool in a general equilibrium framework. More specifically, a change in the interest rate affects liabilities of both types of borrowers in the same way, but affects the payoffs of the assets they buy in a different way. On the one hand, the payoff of the gambling asset is exogenously determined by the lotteries. On the other hand, the payoffs of the firms set up by the entrepreneurs are endogenously determined by a number of factors including the interest rate. Therefore, a change in the interest rate can disproportionately change

---

1See proof in section 2.2.

2Empirical studies (Giot and Schwienbacher, 2003; Bertoni et al., 2011) suggest that even venture capital firms which are supposed to have a strong ability to select good borrowers do not really select good firms. Reinhart and Rogoff (2009) suggest that the expansion of the financial intermediation sector in the run-up to crises causes overcapacity in the financial industry. Since many new intermediaries enter the market with less experience during the expansionary period, one should expect a weaker average ability to screen the borrowers. Moreover, the theoretical model of Dell’Ariccia and Marquez (2006) suggests that financial intermediaries will optimally choose not to screen the borrowers if the number of new loan applicants is sufficiently large.
the expected cash flows for gamblers and entrepreneurs. The difference in the changes of expected cash flows leads to a difference in the entry decisions which changes the proportion of entrepreneurs in the borrower pool.

The key result of our model is that the central bank’s interest rate policy can reduce the riskiness of the loan portfolio in the short run, while persistently increase the riskiness in the long run. More specifically, by lowering the interest rate, the central bank makes debt repayment easier for both entrepreneurs and gamblers. This encourages entry of both types of investors. Our quantitative analysis suggests that entry of entrepreneurs may increase more than entry of gamblers in the short run, which means that the proportion of entrepreneurs in the pool of new loan applicants increases in the short run. Since loans to entrepreneurs are less risky than loans to gamblers, the loan portfolio becomes less risky in the short run. However, more entry of entrepreneurs intensifies competition in the production sector and reduces future profits of the producers. This deters entrepreneurial entry in the long run. By contrast, future payoffs of the lotteries are exogenously determined and are not affected by the current-period entry of gamblers.\footnote{Competition can push up the price of the gambling asset. However, it also pushes up the cost of entering the production sector, which increases the value of the firms. Therefore, competition-induced changes in asset prices are limited in relative terms. As a result, the effect of such changes in relative asset prices on the proportion of entrepreneurs is also limited.} Therefore, the proportion of entrepreneurs persistently stays at low levels in the long run.

Taylor (2009) argues that deviations from the Taylor rule can be a source of financial crisis. We find that expansionary monetary policy shocks can persistently worsen the borrower pool faced by financial intermediaries in the long run. This is consistent with Taylor’s argument. However, quantitative results of our model also suggest that sticking to the Taylor rule is not sufficient to eliminate financial crises. Actually, if the economy is hit by a negative productivity shock, a central bank which deviates from the Taylor rule by not reacting to output fluctuations can reduce the long-run financial risk.

The financial accelerator model (Bernanke et al., 1999) also links productivity shocks to financial intermediation in a general equilibrium macroeconomic framework.\footnote{See Allen and Gale (2007) for a survey of partial equilibrium models which also link real shocks to financial stability.} However,
there is no distinctive difference between the short-run and long-run effects of shocks in the financial accelerator model. As we discussed, the distinction between the short run and long run is important. Moreover, the financial accelerator model considers only one type of borrower (the entrepreneur) and assumes that the number of entrepreneurs is fixed. Our model instead features endogenous entry of both gamblers and entrepreneurs. It enables us to study the dynamic changes in the borrower pool faced by financial intermediaries.

Our modeling of the production sector is related to the macroeconomic model with endogenous firm entry by Ghironi and Melitz (2005) and Bilbiie et al. (2008), but differs in several important respects. In Ghironi and Melitz (2005) and Bilbiie et al. (2008), there is no financial friction and therefore no role for financial intermediation. In our model, there is financial friction and firms must rely on financial intermediaries to buy goods necessary to start their business. This enables us to study the feedback from the real sector to the financial sector. In Ghironi and Melitz (2005) and Bilbiie et al. (2008), firms exit exogenously at a constant rate. In our model, the exit of firms is endogenously determined by their ability to repay their debt. One particularly important difference is that nominal wage is sticky in our model whereas it is flexible in Ghironi and Melitz (2005) and Bilbiie et al. (2008). Bilbiie et al. (2008) find that entrepreneurial entry initially decreases after an expansionary monetary policy shock. This is because output expansion created by the interest rate shock pushes up the real wage. The higher real wage not only makes entrepreneurial entry more costly but also decreases profits after entry. Therefore, fewer entrepreneurs want to enter. However, as suggested by Rotemberg (2008), if the nominal wage is sticky, the rise in the real wage will be more modest. As a result, profits could rise rather than fall. Therefore, entrepreneurial entry can rise despite the increase in entry cost. Since entrepreneurial entry affects the borrower pool faced by financial intermediaries, it is crucial to model wage setting in a more realistic way.

A popular claim in media and policy discussions is that speculation in the secondary financial market is a source of financial crisis. Very often this claim is used to justify financial suppression, for example through restrictions on secondary market trading. While fire sales
in the secondary market can trigger a financial crisis (Allen and Gale, 2004, 2007), banning secondary market trading is far from a justified solution to prevent financial crises. Our model suggests that productivity shocks originating from the real sector can lead to a financial crisis even if there is a ban on secondary market trading of the gambling asset.

We proceed as follows. Section 2 introduces the model; Section 3 solves the model; Section 4 displays and discusses impulse responses of the model under different shocks; Section 5 studies the robustness of the qualitative results to sticky interest rate passthrough. Section 6 concludes.

2 The Model

To facilitate the comparison of impulse responses in our model to those in the standard literature, we incorporate nominal and real frictions (price stickiness, wage stickiness, habit formation) in the standard new Keynesian models into our model. Aggregation is very difficult if we have both price stickiness and endogenous entry and exit of firms in one sector. Therefore, we separate those two features into two different sectors. First, we have a consumption goods sector with sticky prices and fixed mass of firms. Second, we have an intermediate goods sector with flexible prices and endogenous entering and exiting of firms. The consumption goods producers use intermediate goods for production while the intermediate goods producers use labor for production. Entering entrepreneurs in the intermediate goods sector must hire labor and buy consumption goods to set up their firms. The entry cost in terms of wage payment is covered by the shareholders or households. The entry cost in terms of consumption goods is covered by loans from the financial intermediaries. Besides entrepreneurs, financial intermediaries also face another type of borrower, the gambler. Gamblers use the borrowed amount to buy an asset of which the payoff is completely exogenously determined by lotteries. In each period, financial intermediaries receive an installment repayment from the borrowers if there is no default. Households make decisions on labor supply, consumption, investment in deposits and new stocks of firms in the intermediate goods sec-
tor. They receive profits and wage payments from the firms and interest payments from the financial intermediaries. Finally, there is a central bank which sets nominal money market interest rates. Figure 1 summarizes the interrelationships between the agents.

2.1 Firms

2.1.1 Consumption Goods Producers

There is a continuum of symmetric monopolistically competitive producers for the consumption goods, each producing a differentiated variety \( z \in (0,1) \).\(^5\) The production function of firm \( z \) is \( y_t(z) = X_t(z) \), where \( X_t(z) \) is the amount of aggregate intermediate goods employed in the consumption goods production process of firm \( z \).

The consumption basket \( C_t \) takes the constant elasticity of substitution (CES) form: \( C_t = \left[ \int_0^1 c_t(z)^{\frac{2-1}{\gamma}} dz \right]^{\frac{\gamma}{\gamma-1}} \), where \( \gamma \in (1, \infty) \) is the elasticity of substitution across the consumption goods, \( c_t(z) \) is the demand for individual firm \( z \)'s goods. It follows from the CES consumption basket assumption that the household demand for firm \( z \)'s goods is \( c_t(z) = \left[ \frac{p_t(z)}{P_t^C} \right]^{-\gamma} C_t \), where \( p_t(z) \) is the price of firm \( z \)'s good, and \( P_t^C \equiv \left[ \int_0^1 p_t(z)^{1-\gamma} dz \right]^{1\over 1-\gamma} \) is the ideal consumer price index (CPI).

We assume there is price inertia in the consumption goods sector. More specifically, we follow Rotemberg (1982) and Bilbiie et al. (2008) to assume that the consumption goods producer \( z \) has to pay a price setting cost of the form \( pac_t(z) = \frac{\eta}{2} \left[ \Phi_t(z) - 1 \right]^2 p_t(z) y_t(z) \), where \( \eta \in [0, \infty) \) and \( \Phi_t(z) \equiv \frac{p_t(z)}{p_t(z-1)} \) is firm \( z \)'s gross price inflation. Following Erceg et al. (2000), we assume that there is a subsidy \( \tau_c = \frac{1}{\gamma-1} \) to the firm’s output so that the distortion from monopolistic competition in the consumption goods sector is eliminated.\(^6\) Therefore,

\(^{5}\)The fixed number of varieties has been normalized to unity.

\(^{6}\)One purpose of introducing the subsidies in the model is to facilitate comparison of the quantitative results with those of Bilbiie et al. (2008) since they introduce a government subsidy to eliminate distortion from monopolistic competition in their model. Moreover, given that the distortion from monopolistic competition is eliminated, the central bank’s monetary policy only has to concern about frictions from nominal rigidities and the financial sector.
firm $z$’s periodic real profit is

$$m_t^c(z) = \{(1 + \tau_c)p_t(z)y_t(z) - P_t^M X_t(z) - \frac{\eta}{2} [\Phi_t(z) - 1]^2 p_t(z)y_t(z)\}/P_t^C$$

$$= \{(1 + \tau_c)p_t(z) - P_t^M - \frac{\eta}{2} [\Phi_t(z) - 1]^2 p_t(z)\}y_t(z)/P_t^C,$$

where $P_t^M$ is the price index of the aggregate intermediate goods, and the second equality comes from our specification of the consumption goods production function.

Firm $z$ chooses a price level to maximize the net present value (NPV) of the profit flows $E_t \Sigma_{s=t}^{\infty}[\Lambda_{t,s} m_s^c(z)]$, where $\Lambda_{t,s} \equiv \beta^{s-t}(U_{Cs}/U_{Ct})$ is the stochastic discount factor, $\beta$ is the subjective discount factor and $U_{Cs}$ is the marginal utility of consumption in period $s$. Following Bilbiie et al. (2008), we interpret the real price setting cost as the amount of marketing materials needed to set the price and assume that the basket of the marketing materials has the same composition as the consumption basket. Therefore, the demand function for firm $z$’s goods is $y_t(z) = \left[\frac{p_t(z)}{P_t^C}\right]^{-\gamma} Y_t$. Maximizing the NPV of firm $z$’s profit flows subject to the demand function, we obtain the optimal pricing condition for the consumption good producer firm $z$: $p_t(z) = \mu_t(z)P_t^M$, where $\mu_t(z)$ is the markup over marginal cost defined as

$$\mu_t(z) = \frac{\gamma}{(\gamma-1)\left\{1+\tau_c - \frac{\eta}{2} [\Phi_t(z) - 1]^2\right\} + \Gamma_t(z)},$$

$$\Gamma_t(z) \equiv \eta \left\{\Phi_t(z)[\Phi_t(z) - 1] - \beta E_t \left[\frac{U_{Ct+1}}{U_{Ct}} \frac{Y_{t+1}}{Y_t} \left(\frac{P_t^C}{P_{t-1}^C}\right)^{\gamma-1} \Phi_{t+1}(z)^{2-\gamma} (\Phi_{t+1}(z) - 1)\right]\right\},$$

where $\Phi_t^C \equiv \frac{P_t^C}{P_{t-1}^C}$ is the gross consumer price inflation rate. Note that in the steady state with no price adjustment, the markup is one. This is because the monopolistic distortion is eliminated by the production subsidy.

Imposing symmetry, it is easy to see that the producer price inflation rate of the consumption goods sector is also the CPI inflation rate. More specifically, when producers are symmetric, the aggregate pricing equation of the consumption goods sector reduces to $P_t^C = \left[\int_0^1 p_t^{1-\gamma} dz\right]^{1-\gamma} = p_t$, where $p_t = p_t(z)$ is the average producer price in the consumption goods sector.
2.1.2 Intermediate Goods Producers

Similar to the consumption goods production sector, the intermediate goods production sector also features monopolistic competition. However, different from the consumption goods sector, we assume that the number of varieties of the intermediate goods can change over time due to free entry and exit. More specifically, we assume that there is a continuum of intermediate goods producers, each producing a different variety \( \omega \in \Omega \). A basket of the intermediate goods is produced according to \( X_t = \left[ \int_{\Omega} x_t(\omega) \frac{1}{\epsilon} d\omega \right]^{\frac{1}{\epsilon-1}} \), where \( \epsilon \in (1, \infty) \) is the elasticity of substitution across intermediate goods. Hence, the individual intermediate goods firm \( \omega \)'s demand function is \( x_t(\omega) = \left[ \frac{p_{t}^{m}(\omega)}{P_{t}^{M}} \right]^{-\epsilon} X_t \), where \( p_{t}^{m}(\omega) \) is the price of firm \( \omega \)'s good, \( P_{t}^{M} = \left[ \int_{\Omega} p_{t}^{m}(\omega) \frac{1}{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \) is the aggregate price index of the intermediate goods.

Following Ghironi and Melitz (2005), we assume that the production function of the intermediate goods firm \( \omega \) is \( x_t(\omega) = \omega Z_t l_t(\omega) \), where \( l_t(\omega) \) is the labor input for production, \( Z_t \) is the stochastic aggregate productivity level, \( \omega \) is the individual productivity level which is drawn after entry and remains fixed thereafter. Hence, the unit labor cost for intermediate goods production is \( w_t/\omega Z_t \), where \( w_t \) is the aggregate real wage rate. We assume that there is no price adjustment cost in the intermediate goods sector. Similar to the consumption goods sector, there is an output subsidy \( \tau_m = \frac{1}{\epsilon-1} \) so that distortion from monopolistic competition is eliminated. Given those assumptions, the real gross profit function of the intermediate goods firm \( \omega \) is

\[
m_t(\omega) = \left[ (1 + \tau_m) \frac{p_{t}^{m}(\omega)}{P_{t}^{C}} - w_t/\omega Z_t \right] x_t(\omega)
\]

\[
= \left[ (1 + \tau_m) p_{t}^{m}(\omega) - p_{t}^{m}(\omega) \right] x_t(\omega)/P_{t}^{C}
\]

\[
= \tau_m \frac{P_{t}^{M}}{P_{t}^{C}} \left[ \frac{p_{t}^{m}(\omega)}{P_{t}^{M}} \right]^{1-\epsilon} X_t,
\]

where the second equation follows from firms setting their price equal to the marginal cost, \( p_{t}^{m}(\omega) = \frac{w_t P_{t}^{C}}{\omega Z_t} \), due to the subsidy; the third equation is the result of substituting in the demand function \( x_t(\omega) = \left[ \frac{p_{t}^{m}(\omega)}{P_{t}^{M}} \right]^{-\epsilon} X_t \). Obviously, firms with a higher individual productivity \( \omega \) earn more profit.
2.1.3 Aggregation, Entry and Exit of Intermediate Goods Producers

To enter the market, the intermediate goods producers have to pay a sunk cost. The sunk cost is composed of two parts. One part is an amount of effective labor cost \((\frac{w}{Z} f_{ew})\) covered by the firm’s own money.\(^7\) The other part is the cost of purchasing a fixed amount \((f^c)\) of aggregate consumption goods covered by loans from financial intermediaries. The loan is then repaid by installments in each period. As we shall see in subsection 2.2, the periodic installment \((f_t)\) is predetermined and unaffected by an individual firm’s productivity. This suggests that the probability that firm \(\omega\) is able to pay the full amount of installment is higher when its individual productivity \(\omega\) is higher. Therefore, there is a cutoff individual productivity level \(\omega^*_t\) which satisfies \(m_t(\omega^*_t) = f_t\). Note that we add a time subscript to the cutoff individual productivity as the cutoff level varies with aggregate productivity. We assume that firms that fail to pay the full amount of the installment will go bankrupt. This assumption implies that the bankruptcy law imposes a strict solvency constraint on the borrowers so that all defaulting borrowers will be forced to go bankrupt even if some of them may be able to repay the debt in the future, once the aggregate economic situation has become more favorable.

In practice, bankruptcy laws differ across countries. For example, bankruptcy laws in the UK are much stricter than in the US. In the US, there is Chapter 11 which allows the firms in financial distress to reorganize and continue to operate afterward. We do not model this for tractability reasons. However, the existence of a soft budget constraint is likely to deter the entry of entrepreneurs and worsen the borrower pool faced by financial intermediaries, since keeping more firms in the market could reduce expected profit flows for an entering entrepreneur. Moreover, a soft budget constraint could encourage gambling since it gives gamblers a better chance to survive longer and benefit more from taking the gamble. In this sense, introducing a soft budget constraint may strengthen rather than weaken the results of the current model. We further assume that there is limited liability which means that the firms do not have to pay an amount more than its profit to the lender when bankrupt.

Therefore, those firms which expect to earn profits less than the installment will exit the

\(^7\)More precisely, it is indirectly covered by the households owning the firms.
market without production since they can earn nothing from producing. This is different from the traditional financial accelerator model (Bernanke et al., 1999) in which the firms’ current period profit is modeled as a collateral for the loan. However, it is consistent with the theoretical model of Dell’Ariccia and Marquez (2006) which suggests that if the number of new loan applications is sufficiently large, financial intermediaries will optimally choose not to screen the borrowers and require no collateral from them. With the assumptions we introduced, we can aggregate the intermediate goods production sector in the same way as Melitz (2003) and Ghironi and Melitz (2005) have done. More specifically, we assume that the intermediate goods producers draw their individual productivity levels from a Pareto distribution \( G(\omega) = 1 - (1/\omega)^k \) over \([1, \infty)\). Then an average productivity level defined as

\[
\omega^m_t \equiv \left[ \frac{1}{1-G(\omega^*_t)} \int_{\omega^*_t}^{\infty} \omega^{\epsilon-1} dG(\omega) \right]^{1/(\epsilon-1)} = \left[ k/(k - \epsilon + 1) \right]^{1/(\epsilon-1)} \omega^*_t
\]

can summarize all the information on the individual productivity distributions relevant for all macroeconomic variables. Essentially, the intermediate goods producer block of our model with \( N_t \) firms with heterogeneous productivity is isomorphic to one where \( N_t \) representative firms with productivity \( \omega^m_t \) produce the intermediate goods. Particularly, we have \( P_t^M = N_t^{1-\epsilon} p^m_t(\omega^m_t) \), which is a result of Melitz (2003).

Following Ghironi and Melitz (2005) and Bilbiie et al. (2008), we assume that there is a time-to-build lag such that the firms start producing only one period after paying the sunk cost. Firms with an individual productivity level higher than \( \omega^*_t \) will not go bankrupt, so the firm survival rate in period \( t \) is \( \theta_t \equiv 1 - G(\omega^*_t) = (1/\omega^*_t)^k \). Therefore, an entering firm’s average value in period \( t \) is \( v_t = E_t \sum_{s=t+1}^{\infty} \{ \Lambda_t, \theta_s \} \{ m_s(\omega^m_s) - f_s \} \}. Free entry in the intermediate goods production sector requires that the average value of the firm equals the sunk cost paid with own funds:

\[
v_t = \frac{w_t}{Z_t} f^{cw}.
\]

Denoting the number of new entrants in the intermediate goods sector by \( N_t^e \), we get the
dynamic equation for the number of producing firms: \( N_t = \theta_t(N_{t-1} + N^e_{t-1}) \).

## 2.2 Financial Intermediation

In each period, there are a number \( (N_t^r) \) of investors who come to the financial intermediaries for funding. A proportion \( \phi_t = N_t^c / N_t^r \) of those investors are entrepreneurs who will invest the borrowed money in the intermediate goods sector to start their business. The other \( (1 - \phi_t)N_t^r \) investors are gamblers who will invest the borrowed money on a pure gambling asset of which the supply is fixed for each period. The loan from the intermediaries is paid back by a periodic installment \( (f_s) \) from one period after the borrowing. We introduce the one-period lag here to allow for a time-to-build lag in the real sector. We assume that the financial intermediaries do not screen out gamblers from the borrower pool. As a result, the borrowing amount and periodic repayment will be the same for both entrepreneurs and gamblers. Therefore, the borrowed money of a gambler in period \( t \) is \( f^e \) which is equal to the part of sunk cost of an intermediate goods producer covered by a loan from the financial intermediaries.

One period after purchase, the buyer of the gambling asset can participate in a lottery, which gives a payoff \( g \) with probability \( \lambda \) and a payoff zero with probability \( 1 - \lambda \). Conditional on winning the lottery, the owner of the gambling asset can participate in the same lottery again in the next period. The gambler can keep participating in the lottery until he fails to win the lottery. Denote the real gambling asset price by \( p_t^r \). Then the number of gambling asset bought by a gambler is \( \frac{f^e}{p_t^r} \). Similar to the entrepreneurs, gamblers will go bankrupt if they cannot pay the full amount of installment, and their profit is zero when bankrupt due to protection by the limited liability law. Therefore, the expected payoff of a gambler is

\[
E_t \{ \sum_{s=t+1}^{\infty} [\Lambda_s^{s-t} \prod_{s=t+1}^{s} \text{Prob}(g \frac{f^e}{p_t^r} \geq f_s)] (g \frac{f^e}{p_t^r} - f_s) \},
\]

where \( \text{Prob}(x) \) denotes the probability
of event $x$.\footnote{Here the analysis is simplified by assuming that the investors cannot sell the assets in a secondary market. In other words, they are locked up after purchasing. Ofek and Richardson (2003) provide evidence that lockup agreements are responsible for the buildup of the internet bubble. In practice, the gamblers could be the existing business owners who starts excessively risky new projects with easy money from the financial intermediaries. Typically, selling of the projects involves very high liquidation costs. Therefore, it is reasonable to assume no resale of those assets. The lockup assumption is a very stringent form of short sale constraint. Our intuition is that a less stringent form of short sale constraint should be enough to keep the bubble. Kocherlakota (2008) shows that short sale constraints can arise endogenously. Hence, the model’s results could be more general.} Assuming that the gamblers have to pay an entrance fee\footnote{This could be the searching cost for the gambling opportunity, for instance.} ($f^g$) for the gambling market with own money, we can write the free entry condition of the gambling market as

$$E_t\{\sum_{s=t+1}^{\infty}[\Lambda_{t,s}\lambda^{s-t}\prod_{t+1}^{s}Prob(\frac{f^e}{p_t^e} \geq f^s)(\frac{f^e}{p_t^e} - f^s)]\} = f^g.$$ 

Assuming that $g$ is large enough so that $\frac{f^e}{p_t^e} \geq f^s$ always holds, the above equation reduces to

$$E_t\{\sum_{s=t+1}^{\infty}[\Lambda_{t,s}\lambda^{s-t}(\frac{f^e}{p_t^e} - f^s)]\} = f^g$$

which implies that the real asset price is

$$p_t^r = \frac{f^e g E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t})]}{f^g + E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t} f^s)]}.$$ 

Following Allen and Gale (2000), we define the fundamental value of the gambling asset as the NPV of the returns from the gambling asset when the gamblers have to buy it with their own money. More specifically, the fundamental value is $g E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t})]$. It is easy to see that when

$$\frac{f^e}{f^g + E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t} f^s)]} > 1,$$

the real asset price is larger than its fundamental value. This reflects the idea of Allen and Gale (2000) that excessive risk-taking behavior induced by the limited liability law can create asset price bubbles. More specifically, for bubbles to exist, entry into the real sector must be more difficult than entry into the gambling sector, i.e., $f^e > f^g$ must hold. Additionally, the NPV of expected repayments from the gambler ($E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t} f^s)]$) must be relatively small compared to the amount borrowed ($f^e$). This suggests that lending to a gambler cannot be good business for the financial intermediaries if there is a bubble in the gambling asset price. However, even in this case, the financial intermediaries may still be willing to lend to loan applicants because expected returns from lending to the entrepreneurs could cover the expected losses from lending to the gamblers.

To facilitate impulse response analysis later, we define $DV_{1t} \equiv E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t})]$ and $DV_{2t} \equiv E_t[\sum_{s=t+1}^{\infty}(\Lambda_{t,s}\lambda^{s-t} f^s)]$. These two definitions can be written in recursive forms:
\[ DV_{1t} = E_t(\Lambda_{t,t+1}) + E_t(\Lambda_{t,t+1}\lambda DV_{1t+1}), \quad DV_{2t} = E_t(\Lambda_{t,t+1}\lambda f_{t+1}) + E_t(\Lambda_{t,t+1}\lambda DV_{2t+1}). \]

Denote the fundamental value of the gambling asset by \( f v_t \) and define the bubble size as

\[ bb_t \equiv \frac{f v_t}{f^s + E_t[\sum_{s=t+1}^{\infty} (\Lambda_{s,t} \lambda^s - f_s)]}, \]

then \( f v_t = g DV_{1t}, \) \( bb_t = \frac{f v_t}{f^s + DV_{2t}}, \) \( p^*_t = bb_t f v_t. \)

Rather than explicitly modeling the pricing behavior of the financial intermediaries, we adopt the reduced form specification of Chowdhury et al. (2006):

\[ r_{t+1}^b = (1 + \phi_r) r_t^m \]

where \( r_t^m \) is the real gross money market interest rate, and \( r_{t+1}^b \) is the real gross loan rate which satisfies \( (r_{t+1}^b - 1) f^c = f_{t+1}. \) Note that we use the beginning of the period timing for \( r_{t+1}^b, \) so the above equation actually describes the evolution of current period credit spread. \( (1 + \phi_r) \) captures the interest rate passthrough which can be determined by various factors.\(^{10}\) When \( \phi_r = -1, \) the interest rate passthrough is zero.

### 2.3 Labor Market Structure and Wage Setting

Following Erceg et al. (2000), we assume that there is monopolistic competition in the labor market. Each household \( j \in (0, 1) \) supplies a differentiated labor type \( H(j) \) to the market and the aggregate labor demand is \( L_t = \left[ \int_0^1 H_t(j) \frac{\epsilon_w - 1}{\epsilon_w} dj \right]^{\frac{1}{\epsilon_w - 1}}, \) where \( \epsilon_w \in (1, \infty). \) The demand for each labor type \( j \) is then

\[ H_t(j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\epsilon_w} L_t, \]

where \( W_t(j) \) is the nominal wage of the \( jth \) household, \( W_t \equiv \left[ \int_0^1 W_t(j)^{1-\epsilon_w} dj \right]^{\frac{1}{1-\epsilon_w}} \) is the aggregate nominal wage rate.

\(^{10}\)See Ravenna and Walsh (2006), Nabar et al. (1993), Sander and Kleimeier (2004) for summaries of theoretical discussions.
There is nominal wage rigidity. More specifically, a household can reset its nominal wage rate with a fixed probability $1 - \eta_w$ in each period, where $\eta_w \in (0, 1)$. The nominal wage rate of those who cannot reoptimize face the wage rate from the last period, that is, $W_t(j) = W_{t-1}(j)$ if $j$ cannot reset its wage. The real wage is defined as $w_t(j) \equiv \frac{W_t(j)}{P_t}$.

2.4 Households

In each period, the household $j$ gets a working salary (in real terms) from the firms $w_t(j)H_t(j)$. Following Erceg et al. (2000), we assume that the government subsidizes the workers with a subsidy rate $\tau_l = \frac{1}{\epsilon_w - 1}$ to eliminate monopolistic distortion from the labor market, so the actual real labor income is $(1 + \tau_l)w_t(j)H_t(j)$. The households also get profits from the firms. More specifically, they get profits $m^c_t$ from the consumption goods producers and profits $N_t[m^m_t(\omega^m_t) - f_t]$ from the intermediate goods producers. Here we make use of the result of Melitz (2003) that the firm with the average productivity $\omega^m_t$ earns the average profit in the market. Besides the labor income and profit dividends from the firms, the households also get the repayment of their deposits from the financial intermediaries $r^m_t S_t$, where $S_t$ is the amount of deposits in period $t$. Because of nominal wage rigidity, it is uncertain whether the household $j$ could reoptimize its wage. This could generate discrepancy in labor incomes between those who can reset their wage rates and those who cannot. Hence, the decision on saving and spending could differ across households. Following Christiano et al. (2005), we assume that there are short-term securities with payoffs contingent on whether households can reset their nominal wage. This ensures that the households are homogeneous in terms of consumption, investment and deposit, though they are heterogeneous in terms of wage setting and labor supply. Therefore, we can treat the household $j$ as a representative household in terms of consumption, investment, deposit and claims on profit. In sum, the household $j$’s wealth in each period is $(1 + \tau_l)w_t(j)H_t(j) + m^c_t + N_t[m^m_t(\omega^m_t) - f_t] + r^m_t S_t + A_t(j)$, where $A_t(j)$ is the payoff from the state-contingent securities. The households use their wealth to consume $C_t$, invest $N^c_t v_t$ to build new production lines in the intermediate goods sector, deposit $S_{t+1}$ to the financial intermediaries and pay a lump-sum tax $T^L_t$ (defined in real terms).
to the government. Therefore, the household budget constraint is
\[ C_t + N_e v_t + S_{t+1} + T_t^L = (1 + \tau_t) w_t(j) H_t(j) + m^c_t + N_t[\omega^m_t] - f_t] + r^m_t S_t + A_t(j). \]

The household chooses deposits \((S_{t+1})\) and labor supply \((H_t)\) to maximize its expected intertemporal utility \(E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, H_s)\), where \(\beta\) is the subjective discount factor, \(U(C_s, H_s)\) is the periodic utility function in period \(s\), \(C_s\) is the aggregate consumption, \(H_s\) is the labor supply. Christiano et al. (2005) suggest that it is necessary to model habit formation to capture the hump-shaped response of consumption to the monetary policy shock. Following them, we model habit formation as the dependence of the current period’s utility on the last period’s consumption. More specifically, we have
\[ U(C_s, H_s) = \ln(C_s - b C_{s-1}) - \chi H^{1+1/\phi_l}_{s=1+1/\phi_l}, \]
where \(b\) is the parameter governing the relative importance of habit formation, \(\phi_l\) is the Frisch elasticity. The maximization problem gives the first-order condition (FOC) for deposit:
\[ U_{C_t} = \beta E_t(r^m_t U_{C_{t+1}}), \]
where \(U_{C_t} \equiv (C_t - b C_{t-1})^{-1} - \beta b[E_t(C_{t+1}) - b C_t]^{-1}\) is the marginal utility of consumption. The FOC for deposit suggests that the marginal disutility of giving up current consumption must be equal to the expected utility gain from the corresponding increase in next period’s consumption.

The households that can reset their wage rate choose the reset wage rate \((W^*_t)\) to maximize
\[ E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s|t, H_s|t(j)), \]
where \(X_{s|t}\) denotes the value of variable \(X\) in period \(s\) for the households which last reset their wage rates in period \(t\). The corresponding FOC is:
\[ \sum_{s=t}^{\infty} \beta^{s-t} E_t \left\{ H_s|t(j) \left( U_{C_s|t} \frac{W^*_t}{P^*_t} - \chi H_s|t(j) \right)^{1/\phi_l} \right\} = 0. \]
It determines the optimal reset wage rate and labor supply. In case there is no nominal wage rigidity, the FOC for labor supply is \(U_{C_t} \frac{W_t(j)}{P_t} = \chi H_t(j)^{1/\phi_l}\), which suggests that the marginal disutility from working must be equal to the utility gain from the corresponding increase in consumption. Note that this FOC is the same as the one in a perfectly competitive labor market, as the distortion from labor market monopoly power is eliminated by the labor
subsidy.

2.5 Market Clearing and Aggregate Accounting

Consumption goods market clearing requires that the output of consumption goods equals its demand from consumption \((C_t)\), investment \((N_t^if^e)\), and marketing \((PAC_t)\), where \(PAC_t \equiv pac_t(z)/P_t^C\) is both the average and aggregate real price setting cost since the number of consumption goods producers is normalized to one and the consumption goods producers are symmetric. More specifically, we have \(Y_t = C_t + N_t^if^e + PAC_t\). Substituting the definition of \(pac_t(z)\) into the market clearing condition, we get \(Y_t = C_t + N_t^if^e + \eta^2(\Phi_t - 1)^2 p_t y_t/P_t^C\), where we have omitted the index \(z\) by applying the symmetry assumption across consumption goods producers. Combining this equation with the demand function of the consumption good producer \(y_t = (p_t/P_t^C)^{-\gamma}Y_t\), we obtain

\[
C_t + N_t^if^e = [1 - \frac{\eta}{2}(\Phi_t - 1)^2] p_t^{1-\gamma}(P_t^C)^{\gamma-1}Y_t.
\]

Each consumption goods producer demands \(X_t(z) = y_t(z) = [p_t(z)/P_t^C]^{-\gamma}Y_t\) amount of aggregate intermediate goods and the number of consumption goods producers is normalized to one, so \([p_t(z)/P_t^C]^{-\gamma}Y_t\) is the total demand for the aggregate intermediate goods. Intermediate goods market clearing then requires \([p_t(z)/P_t^C]^{-\gamma}Y_t = X_t\).

The government budget constraint requires that the tax revenue equals the sum of all subsidies, that is, \(T_t^L = \tau_l w_t L_t + \tau_c Y_t + \tau_m P_t^M X_t/P_t^C\). Here we use the result that total production subsidies (in real terms) to the consumption goods sector and intermediate goods sector are respectively \(\int_0^1 \tau_c p_t(z) y_t(z) dz/P_t^C = \tau_c Y_t\) and \(\int_\Omega \tau_m p_t^m(\omega) x_t(\omega) d\omega/P_t^C = \tau_m P_t^M X_t/P_t^C\). Combining the government and household budget constraint, we get the aggregate accounting identity \(C_t + N_t^e v_t + S_{t+1} = w_t L_t + m_c^e - \tau_c Y_t - N_t^f_t + \tau_m S_t\).\(^{11}\) Note that the total profit in the intermediate goods sector net of subsidy is zero because the price is set to marginal cost when there is a production subsidy. The total profit in the consumption goods sector net of

\(^{11}\)The aggregate payoff from the state-contingent securities is zero.
subsidy is \((m^c_t - \tau_c Y_t)\). In the steady state without price adjustment it is also zero because pricing markup is driven to one by the subsidy. However, the markup can deviate from one if there is nominal price adjustment. In that case, \((m^c_t - \tau_c Y_t)\) will be different from zero.

Gambling asset demand is equal to the number of gamblers multiplied by the per gambler purchase, that is, \((N^r_t - N^e_t)\frac{f^e_t}{f^e_t}\). Denote the periodically fixed supply of the gambling asset by \(GS\). Then the gambling market clearing condition is \((N^r_t - N^e_t)\frac{f^e_t}{f^e_t} = GS\). Finally, loan market equilibrium requires total saving equal to total lending: \(S_{t+1} = N^r_t f^e\).

### 2.6 Monetary Policy Rules

Following Bilbiie et al. (2008), we define the gross real money market interest rate by \(r^m_t \equiv \hat{i}^m_t/\Phi^C_t\), where \(\hat{i}^m_t\) is the gross nominal money market interest rate. The nominal money market interest rate is set by the central bank according to a specific feedback rule. We consider three different monetary policy rules in our analysis. The first two rules involve interest rate smoothing. One of those two rules does not react to output while the other one does. More specifically, one rule has the following form

\[
\hat{i}^m_t = \rho \hat{i}^m_{t-1} + (1 - \rho)(1.5\pi_{t+1}),
\]

while the other rule is

\[
\hat{i}^m_t = \rho \hat{i}^m_{t-1} + (1 - \rho)(1.5\pi_{t+1} + 0.1\hat{y}^a_{t+1}),
\]

where the smoothing parameter \(\rho\) is set to 0.8 so that the first interest rate smoothing rule is identical to the one used by Bilbiie et al. (2008), while the second rule is identical to the one used by Christiano et al. (2005). The third monetary policy rule is a forward-looking Taylor rule without interest rate smoothing:

\[
\hat{i}^m_t = 1.5\pi_{t+1} + (0.5/4)\hat{y}^a_{t+1},
\]
where the 0.5 coefficient of Taylor’s original specification (Taylor, 1993) is divided by 4 since the annualized inflation and interest rate in Taylor’s original specification are replaced by quarterly inflation and interest rate in the current paper.

Note that $\hat{y}_t^a$ is the deviation of GDP from its flexible-price steady-state level. It is equal to the theoretical output gap in case there is no technology shock, but will diverge from the theoretical output gap if there is a technology shock to the economy. However, as noted by Woodford (2003), the widely used empirical output gap estimated as the deviation of output from a smooth trend can be very different from the theoretical output gap. Neiss and Nelson (2005) estimate the theoretical output gap for the US, UK and Australia and find that the empirical output gap estimates from detrending methods are very different from the theoretical output gap. Further, troughs in the HP-filtered output gap accord well with the recessions documented by the NBER (Rudd and Whelan, 2007), which suggests that by targeting the output gap generated by detrending methods such as the HP filter, central banks are actually targeting output fluctuations ($\hat{y}_t^a$) rather than the theory-consistent output gap.

### 2.7 Model Summary

Table 1 summarizes the main equations of the model. The infinite sum $V_t$ defined in the text is rewritten in recursive form in the table. The real profit equation of the consumption goods sector in the table is the result of substituting the pricing equation and demand function of the consumption goods sector into the real profit function in the text. The model can be simplified by using the aggregate pricing equation of the consumption goods sector $P_t^C = p_t$ to substitute for $P_t^C$ in the other equations of the system. Moreover, the price levels $p_t, P_t^M, p_t^{m(\omega_t^m)}, p_t^{m(\omega_t^*)}$ are not stationary in the model. To simulate the model, we have to transform it to make all the variables in the model stationary. This is done by defining the real price of aggregate intermediate goods by $q_t = P_t^M / p_t$ and using it to substitute the nominal price levels in the model. The transformed model is summarized in Table 2. Note

---

12 We do not include the specification of the monetary policy rule in the summary table to save space.
that we follow Bilbiie et al. (2008) by using beginning of the period timing, so $r_{t+1}^h, f_{t+1}, S_{t+1}$ are actually determined in period $t$. The model can be closed by specifying a process of the exogenous variable $Z_t$ and the parameters: $N^r, \beta, b, f^e, f^{ew}, \gamma, \epsilon, \epsilon_w, \eta, \eta_w, k, \chi, \phi_l, \phi_r$, which we will do in the next section.
Table 1: Model Summary

<table>
<thead>
<tr>
<th>Parameter / Condition</th>
<th>Mathematical Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental value (gamble)</td>
<td>( f_{vt} = gDV_{it} )</td>
</tr>
<tr>
<td>Bubble size</td>
<td>( bb_t = \frac{f_t}{f_{t+1} + DV_{si}} )</td>
</tr>
<tr>
<td>Asset price (gamble)</td>
<td>( p_m^i = bb_t f_{vt} )</td>
</tr>
<tr>
<td>Gambling asset market clearing</td>
<td>( (N_t^i - N_t^e) \frac{r_t^m}{r_t} = GS )</td>
</tr>
<tr>
<td>Proportion of entrepreneur</td>
<td>( \phi_t = N_t^e/N_t^i )</td>
</tr>
<tr>
<td>Definition of loan rate</td>
<td>( (r_t^b - 1) r_t^c = f_{t+1} )</td>
</tr>
<tr>
<td>Evolution of loan rate</td>
<td>( r_t^b = (1 + \phi_t) r_t^c )</td>
</tr>
<tr>
<td>Pricing (consumption goods)</td>
<td>( p_t = \mu M_t )</td>
</tr>
<tr>
<td>Markup (consumption goods)</td>
<td>( \mu_t = (\gamma - 1) \frac{1 + \tau_t - \frac{\tau_t}{1 + \tau_t}}{2(\phi_t - 1)^2} + \Gamma_t(z) )</td>
</tr>
<tr>
<td>Real profit (consumption goods)</td>
<td>( m_t^c = { 1 + \tau_t - 1/\mu_t - \frac{2}{\phi_t - 1} } p_t^c (P_t^r)^{\gamma - 1} Y_t )</td>
</tr>
<tr>
<td>Aggregate pricing (consumption goods)</td>
<td>( P_t^r = p_t )</td>
</tr>
<tr>
<td>Average individual productivity</td>
<td>( \omega_t^m = k / (k - \epsilon + 1)^{1/(\gamma - 1)} \omega_t^* )</td>
</tr>
<tr>
<td>Pricing (intermediate goods)</td>
<td>( p_t^m (\omega_t^m) = \frac{u_t p_t^m}{\omega_t^m} )</td>
</tr>
<tr>
<td>Real Profit (intermediate goods)</td>
<td>( m_t^i (\omega_t^m) = \tau_t p_t^m [p_t^m (\omega_t^m)]^{\gamma - 1} X_t )</td>
</tr>
<tr>
<td>Aggregate pricing (intermediate goods)</td>
<td>( P_t^M = N_t^\gamma \frac{1}{\frac{1}{\phi_t^M} f_t^m (\omega_t^m)} )</td>
</tr>
<tr>
<td>Firm value (intermediate goods)</td>
<td>( v_t = E_t \left[ \beta \frac{U_{Ct+1}}{U_{Ct}} \theta_{t+1} [m_{t+1} (\omega_t^m) - f_{t+1}] \right] + E_t \left[ \beta \frac{U_{Ct+1}}{U_{Ct}} \theta_{t+1} v_{t+1} \right] )</td>
</tr>
<tr>
<td>Free entry (intermediate goods)</td>
<td>( v_t = \frac{u_t}{\phi_t^M} )</td>
</tr>
<tr>
<td>Cutoff condition (intermediate goods)</td>
<td>( m_t^i (\omega_t^m) = f_t )</td>
</tr>
<tr>
<td>Survival rate</td>
<td>( \theta_t = (1/\omega_t^m)^k )</td>
</tr>
<tr>
<td>Number of firms</td>
<td>( N_t = \theta_t (N_{t-1} + N_{t-1}^e) )</td>
</tr>
<tr>
<td>Euler equation (deposit)</td>
<td>( U_{Ct} = \beta E_t (r_{t+1} U_{Ct+1}) )</td>
</tr>
<tr>
<td>Labor supply</td>
<td>( \sum_{s=t}^{\infty} \beta (\theta_s) s^{-\epsilon} E_t \left[ H_{s,t}(j) \left[ U_{Ct} \frac{W_t^*}{P_t} - \chi H_{s,t}(j)^{1/\phi_t} \right] \right] = 0 )</td>
</tr>
<tr>
<td>Good market clearing (consumption)</td>
<td>( \sum_{s=t}^{\infty} \beta (\theta_s) s^{-\epsilon} \left[ 1 - \frac{2}{\phi_t - 1} (\phi_t - 1)^2 p_t^c (P_t^r)^{\gamma - 1} Y_t = C_t + N_t^e f_t^c \right] )</td>
</tr>
<tr>
<td>Good market clearing (intermediate)</td>
<td>( \left[ \frac{P_t^M}{P_t^r} \right]^{\gamma - 1} Y_t = X_t )</td>
</tr>
<tr>
<td>Loan market clearing</td>
<td>( S_{t+1} = N_t^e f_t^c )</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>( C_t + N_t^e v_t + S_{t+1} = w_t L_t + m_t^i - \tau_t Y_t - N_t f_t + \tau_t^m S_t )</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>( \Phi_t^C = P_t^C / P_t^{C-1} )</td>
</tr>
<tr>
<td>Producer price inflation (consumption goods)</td>
<td>( \Phi_t = pt / pt-1 )</td>
</tr>
</tbody>
</table>

---

1. \( DV_{it} = E_t \left[ \sum_{s=t+1}^{\infty} (\Lambda_{t,s} \lambda^{s-t}) \right] = E_t (\Lambda_{t,t+1} \lambda) + E_t (\Lambda_{t,t+1} \lambda DV_{it+1}) \).
2. \( DV_{2t} = E_t \left[ \sum_{s=t+1}^{\infty} (\Lambda_{t,s} \lambda^{s-t} f_s) \right] = E_t (\Lambda_{t,t+1} \lambda f_{t+1}) + E_t (\Lambda_{t,t+1} \lambda DV_{2t+1}) \).
3. \( \Gamma_t(z) = E_t \left[ \Phi_t(z) [\Phi_t(z) - 1] - \beta E_t \left[ \frac{U_{Ct+1}}{U_{Ct}} \frac{Y_t}{\gamma} \Phi_t^C (\Phi_t(z) - 1)^{\gamma - 1} [\Phi_t(z) - 1] \right] \right] \).
4. \( U_{Ct} = (C_t - bC_{t-1})^{-1} - \beta b [E_t (C_{t+1}) - bC_{t}]^{-1} \) is the marginal utility of consumption.
Table 2: Transformed Model Summary

<table>
<thead>
<tr>
<th>Fundamental value (gambles)</th>
<th>( f v_t = g D V_{1t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble size</td>
<td>( b b_t = \frac{f^\gamma}{P_t + D V_{2t}} )</td>
</tr>
<tr>
<td>Asset price (gambles)</td>
<td>( p_t^* = b b_t f v_t )</td>
</tr>
<tr>
<td>Gambling asset market clearing</td>
<td>( (N_t^c - N_t^g) \frac{f^\gamma}{P_t} = GS )</td>
</tr>
<tr>
<td>Proportion of entrepreneur</td>
<td>( \phi_t = N_t^c / N_t^g )</td>
</tr>
<tr>
<td>Definition of loan rate</td>
<td>( (r_{t+1}^b - 1) f^e = f_{t+1} )</td>
</tr>
<tr>
<td>Evolution of loan rate</td>
<td>( r_{t+1}^b = (1 + \phi_t) f_{t+1}^m )</td>
</tr>
<tr>
<td>Pricing (consumption goods)</td>
<td>( \mu_t q_t = 1 )</td>
</tr>
<tr>
<td>Markup (consumption goods)</td>
<td>( \mu_t^\gamma = (\gamma - 1)[1 + \tau_c - \frac{q}{2} (\Phi_t - 1)^2] + \Gamma_t(z) )</td>
</tr>
<tr>
<td>Real profit (consumption goods)</td>
<td>( m_t^c = {1 + \tau_c - 1/\mu_t - \frac{q}{2} (\Phi_t - 1)^2} Y_t )</td>
</tr>
<tr>
<td>Average individual productivity</td>
<td>( \omega_t^m = [k/(k - \epsilon + 1)]^{1/(\epsilon - 1)} \omega_t^* )</td>
</tr>
<tr>
<td>Real Profit (intermediate goods)</td>
<td>( m_t(\omega_t^m) = \tau_m(\frac{u}{\omega_t^m})^{1-\epsilon} q_t^m X_t )</td>
</tr>
<tr>
<td>Aggregate pricing (intermediate goods)</td>
<td>( q_t = \frac{u}{\omega_t^m} N_t^1 - \epsilon )</td>
</tr>
<tr>
<td>Firm value (intermediate goods)</td>
<td>( v_t = E_t[\beta U_{t+1}^{C_t} \theta_{t+1} [m_{t+1}(\omega_{t+1}^m) - f_{t+1}]] + E_t \left( \beta \frac{U_{t+1}^{C_t}}{U_{t}^{C_t}} \theta_{t+1} v_{t+1} \right) )</td>
</tr>
<tr>
<td>Free entry (intermediate goods)</td>
<td>( v_t = \frac{u}{\omega_t^m} f_c )</td>
</tr>
<tr>
<td>Cutoff condition (intermediate goods)</td>
<td>( m_t(\omega_t^c) = f_t )</td>
</tr>
<tr>
<td>Survival rate</td>
<td>( \theta_t = (1/\omega_t^c)^c )</td>
</tr>
<tr>
<td>Number of firms</td>
<td>( N_t = \theta_t(N_{t-1} + N_{t-1}^c) )</td>
</tr>
<tr>
<td>Euler equation (deposit)</td>
<td>( U_{C_t} = \beta E_t(r_{t+1}^m U_{C_{t+1}}) )</td>
</tr>
<tr>
<td>Labor supply</td>
<td>( \Sigma_{s \in t}[\beta \eta_w]^{s-t} E_t \left{ H_{s</td>
</tr>
<tr>
<td>Good market clearing (consumption)</td>
<td>( [1 - \frac{1}{2} (\Phi_t - 1)^2] Y_t = C_t + N_t^c f^e )</td>
</tr>
<tr>
<td>Good market clearing (intermediate)</td>
<td>( Y_t = C_t )</td>
</tr>
<tr>
<td>Loan market clearing</td>
<td>( S_{t+1} = N_t^c f^e )</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>( C_t + N_t^c v_t + S_{t+1} = w_t L_t + m_t^c - \tau_c Y_t - N_t f_t + r_{t+1}^m S_t )</td>
</tr>
</tbody>
</table>

1. \( DV_{1t} = E_t[\Sigma_{s=t+1}(\Lambda_{t,s} x^{s-t})] = E_t(\Lambda_{t,t+1} \lambda) + E_t(\Lambda_{t,t+1} \lambda D V_{1t+1}) \).
2. \( DV_{2t} = E_t[\Sigma_{s=t+1}(\Lambda_{t,s} x^{s-t} f_s)] = E_t(\Lambda_{t,t+1} \lambda f_{t+1}) + E_t(\Lambda_{t,t+1} \lambda D V_{2t+1}) \).
3. \( \Gamma_t(z) = \eta \left\{ \Phi_t(z) [\Phi_t(z) - 1] - \beta E_t \left[ \frac{U_{C_{t+1}}^{C_t} Y_{t+1}}{U_{C_t}} \frac{W_{t}^e}{P_t} (\Phi_t(z))^{\gamma - 1} \Phi_t(z)^{2-\gamma} (\Phi_t(z) - 1) \right] \right\} \).
4. \( U_{C_t} = (C_t - b C_{t-1})^{-1} - \beta b [E_t(C_{t+1}) - b C_t]^{-1} \) is the marginal utility of consumption.
3 Model Solution

3.1 Log-Linearization

We linearize the model in Table 2 by the method of Uhlig (1999). The result is summarized in Table 3, where \( \pi_t \equiv \frac{p_t - p_{t-1}}{p_{t-1}} \) is the CPI inflation rate. Due to the symmetry assumption in the consumption goods production sector, individual producer price inflation is equal to the average producer price inflation. Therefore, we omit the index \( z \) in the notation. We omit \( z \) in the notation of other variables in the consumption goods production sector for the same reason. The labor supply equation in the nonlinear model is substituted by two equations. One is the definition of nominal wage inflation \( \pi^w_t \). The other equation captures the wage inflation dynamics.\(^{13}\)

\(^{13}\)See Gali (2008) for derivation. The only difference is that consumption utility in our model involves habit formation and wage markup is driven to one by the subsidy.
Table 3: Log Linear Model Summary

<table>
<thead>
<tr>
<th>Fundamental value (gambles)</th>
<th>( f v_t = DV_{1t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble size</td>
<td>( (f^g + DV_2) \hat{\beta} b_t + DV_2DV_{2t} = 0 )</td>
</tr>
<tr>
<td>Asset price (gambles)</td>
<td>( \hat{\rho} v_t = \hat{\beta} b_t + \hat{f} v_t )</td>
</tr>
<tr>
<td>Gambling asset market clearing</td>
<td>( f^c \bar{N}_t^c - f^c \hat{\phi} b_t = f^c (1 - \hat{\phi}) \hat{P}_t^r )</td>
</tr>
<tr>
<td>Proportion of entrepreneur</td>
<td>( \hat{\phi}_t = \bar{N}_t^c - \bar{N}_t^r )</td>
</tr>
<tr>
<td>Definition of loan rate</td>
<td>( \hat{r}^b f^r \hat{b}_t = \hat{f} \hat{r}_t + 1 )</td>
</tr>
<tr>
<td>Evolution of loan rate</td>
<td>( \hat{r}_t = (1 + \phi_t) \hat{r}_t^m )</td>
</tr>
<tr>
<td>Pricing (consumption goods)</td>
<td>( \hat{q}_t = -\mu_t )</td>
</tr>
<tr>
<td>Markup (consumption goods)</td>
<td>( \hat{\pi}_t = \beta \bar{E}<em>t (\hat{\tau}</em>{t+1} - \frac{2}{\phi} \hat{\mu}_t )</td>
</tr>
<tr>
<td>Real profit (consumption goods)</td>
<td>( \hat{m}_t^c = (\gamma - 1) \hat{\mu}_t + \hat{Y}_t )</td>
</tr>
<tr>
<td>Average individual productivity</td>
<td>( \hat{\omega}_t^m = \hat{\omega}_t^m )</td>
</tr>
<tr>
<td>Real Profit (intermediate goods)</td>
<td>( \hat{m}_t (\hat{\omega}_t^m) = c \hat{q}_t + (1 - \epsilon) (\hat{w}_t - \hat{\omega}_t^m - \hat{Z}_t) + \hat{X}_t )</td>
</tr>
<tr>
<td>Aggregate pricing (intermediate goods)</td>
<td>( \hat{m}_t (\hat{\omega}_t^* = c \hat{q}_t + (1 - \epsilon) (\hat{w}_t - \hat{\omega}_t^* - \hat{Z}_t) + \hat{X}_t )</td>
</tr>
<tr>
<td>Firm value (intermediate goods)</td>
<td>( \hat{U}<em>{Ct} + \hat{v}<em>t = E_t (\hat{U}</em>{Ct+1}) + E_t (\hat{\theta}</em>{t+1}) + \beta \hat{E}_{t} \hat{m}<em>t (\hat{\omega}<em>t^m) E_t [\hat{m}</em>{t+1} (\hat{\omega}</em>{t+1}^m)] )</td>
</tr>
<tr>
<td>Free entry (intermediate goods)</td>
<td>( \hat{U}<em>{Ct} + \hat{v}<em>t = E_t (\hat{U}</em>{Ct+1}) + E_t (\hat{\theta}</em>{t+1}) + \beta \hat{E}_{t} \hat{m}<em>t (\hat{\omega}<em>t^m) E_t [\hat{m}</em>{t+1} (\hat{\omega}</em>{t+1}^m)] )</td>
</tr>
<tr>
<td>Cutoff condition (intermediate goods)</td>
<td>( \hat{m}_t (\hat{\omega}_t^*) = \hat{f}_t )</td>
</tr>
<tr>
<td>Survival rate</td>
<td>( \hat{\theta}_t = -k \hat{\omega}_t^* )</td>
</tr>
<tr>
<td>Number of firms</td>
<td>( \bar{N}<em>t = \hat{\theta}<em>t + \beta \bar{N}</em>{t-1} + (1 - \beta) \bar{N}</em>{t-1} )</td>
</tr>
<tr>
<td>Euler equation (deposit)</td>
<td>( \hat{U}<em>{Ct} = E_t (\hat{U}</em>{Ct+1}) + E_t (\hat{r}_t^m) )</td>
</tr>
<tr>
<td>Labor supply</td>
<td>( \bar{\pi}_t^w - \pi_t \hat{w}_t = \hat{w}<em>t - \hat{w}</em>{t-1} )</td>
</tr>
<tr>
<td>Good market clearing (consumption)</td>
<td>( \bar{Y}_t = \bar{C} \hat{C}_t + \bar{N} \hat{C}_t )</td>
</tr>
<tr>
<td>Good market clearing (intermediate)</td>
<td>( \bar{X}_t = \bar{Y}_t )</td>
</tr>
<tr>
<td>Loan market clearing</td>
<td>( \hat{S}_{t+1} = \bar{N}_t )</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>( \bar{C} \hat{C}_t + \bar{N} \hat{C}<em>t + \bar{S} \hat{S}</em>{t+1} = \bar{w} \hat{L}_t + \bar{L}_t )</td>
</tr>
</tbody>
</table>

\[ \hat{f} v_t = \frac{1}{1 + \frac{C}{\theta}} \hat{f} v_t \]

\[ \hat{f}^b f^r \hat{b}_t = \hat{f} \hat{r}_t + 1 \]

\[ \hat{r}_t = (1 + \phi_t) \hat{r}_t^m \]

\[ \hat{q}_t = -\mu_t \]

\[ \hat{\pi}_t = \beta \bar{E}_t (\hat{\tau}_{t+1} + \frac{2}{\phi} \hat{\mu}_t \)

\[ \hat{m}_t (\hat{\omega}_t^m) = c \hat{q}_t + (1 - \epsilon) (\hat{w}_t - \hat{\omega}_t^m - \hat{Z}_t) + \hat{X}_t \]

\[ \hat{m}_t (\hat{\omega}_t^*) = c \hat{q}_t + (1 - \epsilon) (\hat{w}_t - \hat{\omega}_t^* - \hat{Z}_t) + \hat{X}_t \]

\[ \hat{U}_{Ct} + \hat{v}_t = E_t (\hat{U}_{Ct+1}) + E_t (\hat{\theta}_{t+1}) + \beta \hat{E}_{t} \hat{m}_t (\hat{\omega}_t^m) E_t [\hat{m}_{t+1} (\hat{\omega}_{t+1}^m)] \]

\[ \hat{U}_{Ct} + \hat{v}_t = E_t (\hat{U}_{Ct+1}) + E_t (\hat{\theta}_{t+1}) + \beta \hat{E}_{t} \hat{m}_t (\hat{\omega}_t^m) E_t [\hat{m}_{t+1} (\hat{\omega}_{t+1}^m)] \]

\[ \hat{m}_t (\hat{\omega}_t^*) = \hat{f}_t \]

\[ \hat{\theta}_t = -k \hat{\omega}_t^* \]

\[ \bar{N}_t = \hat{\theta}_t + \beta \bar{N}_{t-1} + (1 - \beta) \bar{N}_{t-1} \]

\[ \bar{\pi}_t^w - \pi_t = \hat{w}_t - \hat{w}_{t-1} \]

\[ \bar{\pi}_t^w = \beta \bar{E}_t \bar{\pi}_{t+1} - \lambda_w (\hat{w}_t - \frac{1}{\bar{\pi}} \hat{L}_t + \bar{U}_{Ct}) \]

\[ \bar{Y}_t = \bar{C} \hat{C}_t + \bar{N} \hat{C}_t \]

\[ \bar{X}_t = \bar{Y}_t \]

\[ \bar{S}_{t+1} = \bar{N}_t \]

\[ \bar{C} \hat{C}_t + \bar{N} \hat{C}_t + \bar{S} \hat{S}_{t+1} = \bar{w} \hat{L}_t + \bar{L}_t \]

\[ \hat{m}_t (\hat{\omega}_t^m) = c \hat{q}_t + (1 - \epsilon) (\hat{w}_t - \hat{\omega}_t^m - \hat{Z}_t) + \hat{X}_t \]

\[ \hat{m}_t (\hat{\omega}_t^*) = c \hat{q}_t + (1 - \epsilon) (\hat{w}_t - \hat{\omega}_t^* - \hat{Z}_t) + \hat{X}_t \]

3.2 Calibration

As in the standard business cycle model, the periods are interpreted as quarters. The household discount factor \( \beta \) is set to 1/1.0025, which implies that the US steady-state monetary policy rate is 1% per annum (Goodfriend and McCallum, 2007; Curdia and Woodford, 2010). Habit formation parameter \( b \) is set to 0.65, the value estimated by Christiano et al. (2005).

We set elasticities \( \gamma = \epsilon = 3.8 \) to fit the U.S. plant and macro trade data (Ghironi and
Melitz, 2005; Bilbiie et al., 2008). Following Ghironi and Melitz (2005), we calibrate the Pareto distribution shape parameter $k$ to match the standard deviation of log US plant sales which is 1.67 according to Bernard et al. (2003). We follow Bilbiie et al. (2008) by setting the Frisch elasticity to $\phi_l = 2$ and the price stickiness parameter to $\eta = 77$. The weight of labor disutility $\chi$ is calibrated to generate a steady-state labor effect level of one regardless of the Frisch elasticity. Following Erceg et al. (2000), we set elasticity of labor $\epsilon_w$ to 4 and sticky wage parameter $\eta_w$ to 0.75. Parameter governing interest rate passthrough, $\phi_r$, is set to 0.3, the value estimated for the US by Chowdhury et al. (2006). Steady-state lending rate is set to $(1.02)^{1/4}$ times the monetary policy rate, reflecting the 2% US steady-state annual credit spread (Curdia and Woodford, 2010). We normalize the sunk cost in consumption goods $f^e$ to 1 since its level does not affect the coefficients of the impulse response functions. Steady-state intermediate goods producer survival rate is set to 0.975, the same number as the one specified in Ghironi and Melitz (2005) and Bilbiie et al. (2008). The difference is that the firms’ survival rate is fixed in Ghironi and Melitz (2005) and Bilbiie et al. (2008) while it can deviate from the steady-state level in our model. We require the calibrated steady-state variables to capture the US private debt to GDP ratio ($S_t/y_a^t$), 80% per annum or 3.2 per quarter (Curdia and Woodford, 2010).$^{14}$ We set $\lambda = 0.9$, $f^g = 0.934363$, so that there is no bubble in the steady state. The periodic supply of the gambling asset $GS$ is normalized to 1 since it does not affect the impulse response function. $^{15}$ Following King and Rebelo (1999) and Bilbiie et al. (2008), we assume that the aggregate productivity evolves as follows: $\ln Z_t = 0.979\ln Z_{t-1} + e$, where $e$ is an i.i.d. random shock with variance $\sigma^2$.

$^{14}$The real GDP level $y_a^t$ is defined as the sum of consumption ($C_t$) and investment ($N_e^t v_t$).

$^{15}$What matters for the impulse responses is the product of steady-state price and periodic supply of the gambling asset. This product is determined by the steady-state gambling asset market clearing condition, given the numerical values of our other parameters.
We consider the impulse responses of the variables to two types of shocks: an expansionary monetary policy shock and a negative productivity shock. We focus on the variables related to the riskiness of loan portfolios for financial intermediaries. More specifically, we report and discuss the impulse responses of the number of entrepreneurial entry, the number of entering gamblers, the survival rate of entrepreneurs, the bubble size and the gambling asset price. We also report the impulse responses of two other variables (the average profit of intermediate goods producers and the required periodic repayment to financial intermediaries) since they are closely related to investors’ entry decisions. Interested readers can refer to the figures in the appendices for the impulse responses of all the other variables in the model.

4.1 Impulse Responses to An Expansionary Monetary Policy Shock

Figure 2 shows the impulse responses of the key variables (percentage deviations from the steady-state levels) to a one-percent unexpected decrease in the net nominal money market interest rate. The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.

The impulse responses are qualitatively similar under all three interest rate rules. More specifically, after the shock, entrepreneurial entry increases in the first few quarters, and then starts to decline and persistently stays below the steady-state level for a long period of time. This result is in sharp contrast to the result of Bilbiie et al. (2008). Bilbiie et al. (2008) who find that the expansionary monetary policy shock immediately reduces firm entry if entry

16See figure A1 for impulse responses of all variables.
incurs sunk investment in effective labor. Furthermore, they find firm entry persistently stays above the steady-state level after the first few periods. As noted by Rotemberg (2008), the initial decline in firm entry in Bilbiie et al. (2008) comes from the procyclical rise in the real wage, which makes entry more expensive and future returns less attractive. Particularly, average profit in the intermediate goods sector decreases after the shock despite the increase in demand. If the nominal wage is sticky, the real wage is less procyclical and entry is less costly. Additionally, average profits in the near future rise. This further encourages entry. Our result confirms Rotemberg’s conjecture that an expansionary monetary policy shock stimulates entry on impact when a realistic level of wage rigidity is introduced into the model. Holding the number of producers constant, the increase in demand also increases profitability in the current period. This makes debt repayment easier and raises the survival rate of intermediate goods firms. Both the increase in firm survival rate and entrepreneurial entry in the initial period increase the number of producers in the future. Intensified competition not only reduces sales of each individual intermediate goods producer but also reduces the price of intermediate goods relative to consumption goods. This is because a one-percent decrease in the price of aggregate intermediate goods only leads to a less-than-one-percent decrease in the price of aggregate consumption goods when prices in the consumption goods sector are sticky. As a result, future profits in real terms decrease, leading to lower levels of entrepreneurial entry.

The number of gamblers in the pool of new loan applicants is persistently higher than the steady-state level after the shock. This is because the unexpected decline in the nominal interest rate persistently reduces required periodic repayments to the financial intermediaries. Lower periodic repayments lead to higher expected cash flows from the gamble which attracts more gamblers. The increase in the number of gamblers pushes up the price of the gambling asset. Limited liability encourages excessive risk-taking behavior. Therefore, the rise in the gambling asset price is more than the rise in its fundamental value. In other words, the bubble size becomes larger than one. Recall that the expected repayment from gamblers

---

17 Required periodic repayments increase in the initial periods. However, the effect of the persistent reduction in future required periodic repayments dominates the changes in the net present value of cash flows.
to the financial intermediaries is inversely related to the bubble size. Therefore, after the expansionary monetary policy shock, the expected loss from lending to gamblers will be persistently higher than the steady-state level. Figure 2 suggests that the initial rise in the number of entrepreneurs quantitatively dominates the initial rise in the number of gamblers. Therefore, the proportion of entrepreneurs in the borrower pool initially increases. However, the initial increase in the proportion of entrepreneurs does not last long. Instead, the proportion of entrepreneurs persistently stays below the steady-state level in the long run. A persistently-higher-than-steady-state expected loss from lending to gamblers together with a persistently-higher-than-steady-state proportion of gamblers in the borrower pool accumulates a significant risk in the financial sector. Interestingly, the effect of the monetary policy shock on the accumulation of long run financial risk is quantitatively much more significant when the interest rate rule does not react to output fluctuations. This is because under the rules reacting to the output fluctuation, initial rise in entrepreneurial entry is reduced by the central bank’s action to cut aggregate demand. The lower initial rise in entrepreneurial entry reduces the future numbers of competitors in the market, making entry in the following periods more attractive. Taylor (2009) argues that keeping the policy interest rate persistently lower than the level implied by the Taylor rule may be a source of financial crisis. We find that if the economy is hit by an expansionary monetary policy shock and the central bank does not react to output fluctuations, the nominal money market interest rate will be persistently lower than the level implied by the forward-looking Taylor rule. As we discussed, not reacting to output fluctuations leads to a more significant long-run financial risk. In this sense, our findings in this section are consistent with Taylor’s argument. However, we shall see in the next subsection, sticking to the Taylor rule is not sufficient to eliminate financial crises.

\(^{18}\)See figure A1.
4.2 Impulse Responses to A Negative Productivity Shock

Figure 3 shows the impulse responses of the key variables (percentage deviations from the steady-state levels) to a one-standard-deviation\textsuperscript{19} decrease in aggregate productivity\textsuperscript{20}. The first observation is that impulse responses are very similar under the two interest rate rules reacting to output fluctuations. Secondly, impulse responses under the two rules reacting to output fluctuations are very different from the ones under the interest rate rule not reacting to output fluctuations. More specifically, we have the following key results.

When the interest rate rule reacts to output fluctuations, entrepreneurial entry initially increases. By contrast, entrepreneurial entry initially decreases when the interest rate rule does not react to output fluctuations. The initial decrease in aggregate productivity affects entrepreneurial entry through two channels. The first one is the \textit{direct profit channel}: persistently-lower-than-steady-state aggregate productivity can directly reduce future profitability of intermediate goods production, which deters entrepreneurial entry. The second channel is the \textit{interest rate channel}: the real money market interest rate decreases after the shock under all three interest rate rules. The reduction in the real money market rate reduces future real loan rates and required periodic repayments, making entrepreneurial entry more attractive. Additionally, lower real money market rates increase demand. This reduces the negative effect of the productivity shock on production and profits and further encourages entrepreneurial entry. The net effect of the negative aggregate productivity shock on entrepreneurial entry depends on the size of the offsetting effects. If the interest rate rules react to output fluctuations, the interest rate channel dominates on impact and the firm value exceeds the sunk cost of investment, which means that entrepreneurial entry must increase to preserve the free entry condition in the intermediate goods sector. By contrast, if the interest rate rule does not react to output fluctuations, the direct profit channel dominates on impact, leading to an immediate reduction in entrepreneurial entry.

The initial increases in entrepreneurial entry under the two interest rate rules reacting to

\textsuperscript{19}The standard deviation of aggregate productivity shock is set to 0.0012, the number used in King and Rebelo (1999).

\textsuperscript{20}See figure A2 for impulse responses of all variables.
output fluctuations do not last long and are followed by persistently-lower-than-steady-state numbers of entrepreneurial entry. This is because the initial rise in entrepreneurial entry makes the number of intermediate goods producers persistently higher than the steady-state number. Competition reduces future profitability and deters entry. By contrast, under the interest rate rule not reacting to output fluctuation, due to the initial decrease in entrepreneurial entry, the number of intermediate goods producers is persistently below the steady-state value. Less competition attracts entry, so entrepreneurial entry quickly recovers and remains at higher-than-steady-state values for a long period of time.

The firm survival rate initially increases after the negative aggregate productivity shock under the interest rate rules reacting to output fluctuations whereas it initially decreases under the interest rate rule not reacting to output fluctuations. The responses of the firm survival rate become quantitatively very small after five years under all interest rate rules. Similar to entrepreneurial entry, the firm survival rate is also affected by the bad productivity shock through two channels: the direct profit channel and the interest rate channel. Lower productivity reduces profits while the lower interest rate increases profits by increasing demand. If the interest rate rule reacts to output fluctuations, the interest rate channel initially dominates, leading to a higher firm survival rate. Conversely, if the interest rate rule does not react to output fluctuations, the direct profit channel dominates on impact. As a result, the firm survival rate decreases. A higher survival rate of the firms also increases the future number of competitors and deters entrepreneurial entry in the long run.

The number of gamblers initially increases under all three different monetary policy rules. However, the initial rise in the number of gamblers is small and transitory if the central bank does not react to output fluctuations. By contrast, the initial rise in the number of gamblers is large and persistent if the central bank does react to output fluctuations. Consequently, the bubble size is persistently higher if the central bank reacts to output fluctuations. The intuition is as follows. Reduction in real interest rates reduces future required repayments and increases cash flows from gambling. This attracts gamblers. Excessive risk-taking behavior

\[\text{Note that required repayment is predetermined when the shock hits, so the cut in real interest rate does not work through affecting the required repayment in the initial period.}\]
by the gamblers increases the bubble size. If the central bank tries to avoid the current recession by cutting the interest rate, it lowers the real interest rate more than when it does not care about output fluctuations. As a result, cash flows from gambling increase more, and more gamblers enter the market, leading to a larger size of the bubble. A larger bubble size suggests a higher expected loss from lending to gamblers. Together with a higher proportion of gamblers in the borrower pool, it imposes a significant risk to the financial sector. The results suggest that sticking to a Taylor rule is not sufficient to eliminate financial crises. Actually, in case the economy is hit by a negative productivity shock, deviating from the Taylor rule by not reacting to output fluctuations can reduce the long-run financial risk.

5 Sticky Interest Rate Passthrough

In our benchmark model, the passthrough from changes in money market rate to the loan rate is more than one. It is interesting to see what happens if we have a lower interest rate passthrough. Particularly, many studies find that the interest rate passthrough is sticky in Europe. In this section, we investigate the implication of sticky interest rate passthrough in our model. More specifically, we produce impulse responses of the variables to the shocks with \( \phi_r = -0.8 \) which implies an interest rate passthrough of 0.2, the value estimated by Chowdhury et al. (2006) for France.

5.1 Impulse Responses to An Expansionary Monetary Policy Shock

Figure 4 displays the impulse responses of key variables after a one-percent unexpected decrease in net nominal money market rate.\(^{22}\) The qualitative results are similar to the benchmark model. The number of entrepreneurial entry initially rises and then remains at levels lower than the steady-state value for a long time. Intermediate firm survival rate initially rises, followed by quantitatively negligible responses. The number of gamblers in the borrower pool increases and stays at levels higher than the steady-state level for a long time.

\(^{22}\)See figure A3 for impulse responses of all variables.
The proportion of entrepreneurs initially increases, starts to decline after a short period and remains at levels lower than the steady-state level for a long time. Bubble size and real asset price increase, and persistently stay at levels higher than the steady-state levels.

Two notable differences from the benchmark model are: variables converge to their steady-state levels faster than in the benchmark model; the quantitative responses are less different under the three interest rate rules than in the benchmark model. This is because now the differences in the effects of initial change in money market rate are narrowed down by the sticky interest passthrough when transmitted to the intermediate goods sector.

5.2 Impulse Responses to A Negative Productivity Shock

Figure 5 displays the impulse responses of key variables after a one-standard-deviation negative productivity shock.\textsuperscript{23} As in the benchmark model, entrepreneurial entry initially increases, then declines to a level lower than the steady-state level and slowly recovers when interest rate rules react to output fluctuation. The difference is that the initial increase in entrepreneurial entry is smaller, leading to smaller numbers of future competitors in the intermediate goods sector. Hence, the proportion of entrepreneurs in the borrower pool converges faster to the steady-state level than in the benchmark model. The bubble size remains above the steady state for more than five years if the interest rate rule reacts to output fluctuation. However, both size and duration of the bubble are smaller in magnitude than in the benchmark model. Therefore, when the interest rate passthrough is sticky, the economy shocked by a negative productivity shock is less prone to long run financial crash.

6 Conclusion

Our model demonstrates that large unexpected expansionary monetary policy shocks could trigger financial crises in the long run. Interestingly, the central bank’s reaction to output fluctuations can reduce the negative effect of the unexpected reduction in money market

\textsuperscript{23}See figure A4 for impulse responses of all variables.
interest rate on the long-run financial stability. As we know, the Taylor rule includes the central bank’s reaction to output fluctuations. In this sense, sticking to the Taylor rule can help reduce the long-run financial risk. However, a central bank’s monetary policy aimed at smoothing output fluctuations can persistently worsen the borrower pool faced by financial intermediaries in the long run if the economy is hit by a negative aggregate productivity shock. That is, it will persistently increase the proportion of gamblers in the pool of new loan applicants. Furthermore, the expected loss from lending to each gambler is persistently higher than the steady-state level under such a policy. If the central bank only responds to inflation, the negative effect of the aggregate productivity shock on the borrower pool is more transitory but larger in magnitude, which suggests that the financial intermediaries have to temporarily withstand higher pressure. As a tradeoff, they can avoid persistent future losses if they survive the current stress. The traditional business cycle view of financial crises\textsuperscript{24} suggests that a sharp drop in the productivity of the real sector could generate a bank run. Hence it is tempting for governments to intervene to avoid financial crises. However, our analysis suggests that policies that try to reduce the probability of a current crisis may create a future crisis in the long run.

\textsuperscript{24}See Allen and Gale (2007) for a summary.
Figure 1: Structure of the model

- **Consumer goods sector**
  - Sticky price
  - Fixed variety
  - Sells goods; Distributes profit
  - Buy goods

- **Intermediate goods sector**
  - Flexible price
  - Endogenous entry and exit
  - Supplies input
  - Pays wage; Distributes profit
  - Finance Set-up (share);
  - Supply labor

- **Households / Workers**
  - Sticky wage
  - Habit formation
  - Deposit
  - Pay interest
  - Sets interest rate policy

- **Financial intermediaries**
  - Pay debt
  - Pay interest
  - Finance (loan)

- **Central bank**
  - Pay debt
  - Finance (loan)

- **Gamblers**
  - Pay debt
  - Finance (loan)
Figure 2: Impulse Responses After An Expansionary Monetary Policy Shock, Key Variables

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Impulse Responses After An Expansionary Monetary Policy Shock, Key Variables (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Figure 3: Impulse Responses After A Negative Productivity Shock, Key Variables

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Impulse Responses After A Negative Productivity Shock, Key Variables (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Figure 4: Sticky Interest Rate Passthrough and Impulse Responses After An Expansionary Monetary Policy Shock, Key Variables

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Sticky Interest Rate Passthrough and Impulse Responses After An Expansionary Monetary Policy Shock, Key Variables (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Figure 5: Sticky Interest Rate Passthrough and Impulse Responses After A Negative Productivity Shock, Key Variables

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Sticky Interest Rate Passthrough and Impulse Responses After A Negative Productivity Shock, Key Variables (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
References


Figure A1: Impulse Responses After An Expansionary Monetary Policy Shock

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Figure A3: Sticky Interest Rate Passthrough and Impulse Responses After An Expansionary Monetary Policy Shock

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Sticky Interest Rate Passthrough and Impulse Responses After An Expansionary Monetary Policy Shock (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Sticky Interest Rate Passthrough and Impulse Responses After An Expansionary Monetary Policy Shock (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Figure A4: Sticky Interest Rate Passthrough and Impulse Responses After A Negative Aggregate Productivity Shock

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Sticky Interest Rate Passthrough and Impulse Responses After A Negative Aggregate Productivity Shock (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.
Sticky Interest Rate Passthrough and Impulse Responses After A Negative Aggregate Productivity Shock (Continued)

Notes: The variable on the horizontal axis is the number of years after the shock. The responses are normalized so that one denotes one percent deviation from the steady-state level. The dashed curves with square markers correspond to the responses to the shocks under the interest rate smoothing rule without reacting to output fluctuations. The solid curves with cross markers correspond to the responses under the interest smoothing rule reacting to output fluctuations. The dotted curves with round markers correspond to the responses under the forward-looking Taylor rule.