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CAPITAL REGULATION, LIQUIDITY REQUIREMENTS AND TAXATION IN A DYNAMIC MODEL OF BANKING

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Capital Regulation, Liquidity Requirements and Taxation in a Dynamic Model of Banking

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ABSTRACT

This paper formulates a dynamic model of a bank exposed to both credit and liquidity risk, which can resolve financial distress in three costly forms: fire sales, bond issuance and equity issuance. We use the model to analyze the impact of capital regulation, liquidity requirements and taxation on banks’ optimal policies and metrics of efficiency of intermediation and social value. We obtain three main results. First, mild capital requirements increase bank lending, bank efficiency and social value relative to an unregulated bank, but these benefits turn into costs if capital requirements are too stringent. Second, liquidity requirements reduce bank lending, efficiency and social value significantly, they nullify the benefits of mild capital requirements, and their private and social costs increase monotonically with their stringency. Third, increases in corporate income and bank liabilities taxes reduce bank lending, bank efficiency and social value, with tax receipts increasing with the former but decreasing with the latter. Moreover, the effects of an increase in both forms of taxation are dampened if they are jointly implemented with increases in capital and liquidity requirements.

\textsuperscript{*}The views expressed in this paper are those of the authors and do not necessarily represent those of the IMF or IMF policy.
I. Introduction

The 2007–2008 financial crisis has been a catalyst for significant bank regulation reforms. The regulatory framework embedded in the Basel II capital accord has been judged inadequate to cope with large financial shocks. As a result, the proposed new Basel III framework envisions a significant raise in bank capital requirements and the introduction of new liquidity requirements.\(^1\) At the same time, several proposals have been advanced to use forms of taxation with the twin objectives of raising funding to pay for resolution costs in stressed times, as well as a way to control risk behavior at large and complex financial institutions.\(^2\)

Assessing the \emph{joint} impact of these regulations and tax proposals on bank behavior is a difficult task. Recent central banks’ efforts to quantify the impact of capital and liquidity requirements on both banks’ behavior and the economy at large have produced mixed results.\(^3\) One reason for the difficulty is that the relatively large literature on bank regulation offers few formal analyses where a joint assessment of these policies can be made in a \emph{dynamic} context.

To our knowledge, none of the existing work considers a bank exposed to both credit and liquidity risk, and evaluates the implications of capital regulation, liquidity requirements, and taxation on bank optimal policies, and metrics of efficiency of intermediation and social costs.

The formulation of a dynamic model with all these features capable to address these important policy questions is the main contribution of this paper.

\(^1\)According to the Basel Committee on Banking Supervision (\emph{An Assessment of the Long–Term Economic Impact of Stronger Capital and Liquidity Requirements}, Bank for International Settlements, Basel, August 2010), the reform would increase the minimum common equity requirement from 2\% to 4.5\%. The Tier 1 capital requirement will increase from 4\% to 6\%. In addition, banks will be required to hold a “capital conservation buffer” of 2.5\% to withstand future periods of stress bringing the total common equity requirements to 7\%. Two new liquidity requirements are planned to be introduced: a short term liquidity coverage ratio, meant to ensure the survival of a bank for one month under stressed funding conditions, and a long-term so-called net stable funding ratio, designed to limit asset and liabilities mismatches.

\(^2\)See e.g.Acharya, Pedersen, Philippon, and Richardson (2010) and \emph{Financial Sector Taxation}, International Monetary Fund, Washington D.C., September 2010.

\(^3\)Basel Committee on Banking Supervision (2010) evaluates the long–term economic impact of the proposed capital and liquidity reforms using a variety of models, including dynamic stochastic general equilibrium models. It finds that the net economic benefits of these reforms, as measured as a reduction in the expected yearly output losses associated with a lower frequency of banking crises, are positive for a broad range of capital ratios, but become negative beyond a certain range. However, the relationship between higher capital levels and higher liquidity requirements on the probability of crises remains highly uncertain. Moreover, measurement errors in the computation of net economic benefits also arise from the use of banking crises classifications which record government responses to crises rather than adverse shocks to the banking system (see Boyd, De Nicolò, and Loukoianova (2010)).
Our model is novel in three important dimensions. First, we analyze a bank which dynamically transforms short term liabilities into longer-term illiquid assets whose returns are uncertain. This feature is consistent with banks’ special role in liquidity transformation emphasized in the literature (see e.g. Diamond and Dybvig (1983) and Allen and Gale (2007)). Yet, most existing models analyze bank liquidity transformation using simple three-periods set-ups: these are informative in capturing key static trade-offs, but are silent by construction on important dynamic trade-offs, including banks’ choices and constraints on whether, when, and how to continue operations in the face of financial distress.

Second, we model bank’s financial distress explicitly. The bank in our model invests in risky loans and risk-less bonds financed by (random) government-insured deposits and short term fully collateralized debt. Financial distress occurs when the bank is unable to honor part or all of its debt and tax obligations for given realizations of credit and liquidity shocks. The bank has the option to resolve distress in three costly forms: by liquidating assets at a cost, by issuing fully collateralized bonds, or by issuing equity. The liquidation costs of assets are interpreted as fire sale costs, and modeled introducing asymmetric costs of adjustment of the bank’s risky asset portfolio. The importance of fire sale costs in amplifying systemic banking distress has been brought to the fore in the recent crisis (for a recent contribution, see Acharya, Shin, and Yorulmazer (2010)).

Third, we evaluate the impact of bank regulations and taxation not only on bank optimal policies, but also in terms of three valuation metrics that identify how the value created by the bank accrues to the relevant stakeholders. The first one is the standard market value of equity, which measures shareholder’s value. The second is the enterprise value of the bank, which can be interpreted as the efficiency with which the bank carries out its maturity transformation function. The third one, termed the “social value” of the bank, is shareholders’ value less net bond holdings, plus the value of (insured) deposits, plus the value to Government tax receipts net of the value of default costs. To our knowledge, and with the exception of Van den Heuvel (2008), who focuses only on the welfare costs of capital regulation, this is the first study that evaluates stakeholders’ net benefits of capital regulation, liquidity requirements and taxation jointly.

Our benchmark bank is unregulated, but its (random) deposits are fully insured. A key feature of such a bank is that its default decision is endogenous and based on market valuation,
which is a standard feature of dynamic models of a firm (see e.g. Cooley and Quadrini (2001)).

By contrast, the regulated bank is subject to a bank closure rule based on the book value of equity, which triggers regulatory intervention. Thus, for the regulated bank default is *exogenous*, which is a standard assumption in models of the banking firm (see e.g. Zhu (2008)).

We consider the unregulated bank as the appropriate benchmark, since one of the asserted key roles of capital regulation and liquidity requirements is the abatement of the excessive bank risk-taking arising from moral hazard under partial or total insurance of its liabilities. Using a standard calibration of the parameters of the model, regulatory and tax parameters mimicking current capital regulation, liquidity requirement and tax proposals, we solve for the optimal policies and the metrics of interest for given state realizations, and present statistics of the steady state distribution of these policies and metrics by numerical simulation.

We obtain three sets of results. First, if capital requirements are mild, a bank subject only to capital regulation invests more in lending and its probability of default is lower than its unregulated counterpart. This additional lending is financed by higher levels of retained earnings or equity issuance, and its leverage is correspondingly lower than its unregulated peer. Our metrics of bank efficiency and social values indicate higher values of the bank subject to mild capital regulation relative to the unregulated bank. However, if capital requirements become too stringent, then the efficiency and social benefits of capital regulation disappear and turn into costs, even though default risk remains subdued: lending declines, and our metrics of bank efficiency and social value drop below those of the unregulated bank. These findings suggest the existence of an optimal level of bank-specific regulatory capital under deposit insurance.

Second, the impact of liquidity requirements on bank lending, efficiency and social value is significantly negative. When we consider liquidity requirements for a hypothetical bank not subject to capital regulation, lending, efficiency and social values are all well below those of an unregulated bank. Interestingly, the observed book capital ratio is relatively high, even though the bank has a probability of default slightly larger than that of an unregulated bank: this indicates that observed book capital ratios are not necessarily related one–to–one with probabilities of default. In the realistic case in which liquidity requirements are added to capital requirements, such requirements eliminate the benefits of mild capital requirements, since bank lending, efficiency and social values are reduced relative to the bank subject to
capital regulation only. In addition, the costs of these liquidity requirements, in terms of reductions in lending, efficiency and social values, increase monotonically with their stringency.

Lastly, an increase in corporate income taxes reduces lending, bank efficiency and social values due to standard negative income effects. However, tax receipts increase, generating higher government revenues. By contrast, the introduction of taxes on insured or uninsured liabilities, while decreasing bank lending, efficiency and social values to an extent higher than an increase in corporate income taxes, do not generate an increase in government tax receipts owing to substitution effects. Interestingly, when an increase in both corporate income taxes and taxes on bank liabilities are implemented in conjunction with increases in capital and liquidity requirements, their negative effects on bank lending, efficiency and social values are significantly dampened.

The remainder of this paper is composed of four sections. Section II presents a brief review of the relevant literature. Section III describes the benchmark model of an unregulated bank subject to standard corporate taxation. Section IV introduces capital regulation and liquidity requirements. Section V details the impact of bank regulation, while Section VI carries out a comparative dynamics exercise with respect to bank regulation and taxation. Section VII concludes with a brief discussion of the policy implications of the results. The Appendix describes the computational procedures for the simulation of the model.

II. A brief literature review

The literature on bank regulation is large, but it offers few formal analyses of the impact of regulatory constraints on bank optimal policies in a dynamic framework. The great majority of studies have focused on capital regulation. Capital requirements has been typically justified by their role in curbing excessive risk-taking (risk-shifting or asset substitution) induced by moral hazard of banks whose deposit are insured (for a review, see Freixas and Rochet (2008)). Based on the experience of the 2007-2008 financial crisis and on several of their influential theoretical contributions, Brunnermeier, Crockett, Goodhart, Pessard, and Shin (2009) have recently advocated the use of capital requirements designed to reduce the pro-cyclicality of financial institutions’ leverage. Such pro-cyclicality has been identified as an important factor underlying the amplification of credit cycles, and likely to increase the probability of so-called
illiquidity spirals, in which the entire banking system liquidates its assets at fire sale prices in a down-turn with adverse systemic consequences.\textsuperscript{4}

However, whether an increase in capital requirements unambiguously reduces banks’ incentives to take on more risk appears an unsettled issue even in the context of static models of banking. While several studies using partial equilibrium set-ups show that an increase in capital results in less risk (see, e.g. Besanko and Kanatas (1996), Hellmann, Murdock, and Stiglitz (2000), and Repullo (2004)), in other models of this type this conclusion can be reversed (see e.g. Blum (1999), and Calem and Rob (1999)), and such reversal can also occur in general equilibrium set-ups (see e.g. Gale and Ö zgür (2005), and De Nicolò and Lucchetta (2009) and Gale (2010)).

The relatively sparse literature of dynamic models of banking has exclusively focused on the pro-cyclicality aspects of bank capital regulation: capital requirements may increase in a recession and become less stringent in an expansion, thus amplifying fluctuations in lending and real activity. Even in this dimension, results are mixed depending on the details of the modeling set-ups. Estrella (2004) and Repullo and Suarez (2008) find that capital requirements are indeed pro-cyclical, while Peura and Keppo (2006) and Zhu (2008) find that this is not necessarily the case. The papers presenting a modeling approach closest to ours are Van den Heuvel (2009) and Zhu (2008). Van den Heuvel (2009) focuses on bank responses to monetary shocks. He presents a dynamic model of a bank which invests in risky loans and risk-free securities, its deposits are government-insured, and it is subject to capital requirements, and finds such requirements are pro-cyclical. Extending the model by Cooley and Quadrini (2001), Zhu (2008) considers a bank that invests in a risky decreasing return to scale technology, its sole source of financing are uninsured (and fairly priced) deposits, faces linear equity issuance costs, and it is subject to minimum capital requirements.

Summing up, none of the papers we have reviewed consider a bank subject to both credit and liquidity risk, where financial distress can be resolved in three costly forms (fire sales, bond issuance and equity issuance). Next, we detail our model, which embeds all these features, and allows us to explore the joint impact of capital regulation, liquidity requirements and taxation.\textsuperscript{4}

\textsuperscript{4}For a review of the literature on pro-cyclicality as related to capital regulation and some empirical evidence, see Zhu (2008) and Panetta and Angelini (2009)).
III. The model

Time is discrete and the horizon is infinite. We consider a bank that receives a random stream of short term deposits, can issue risk–free short term debt, and invests in longer-term assets and short term bonds. The bank manager maximizes shareholders’ value, so there are no managerial agency conflicts, and bank’s shareholders are risk–neutral.

A. Bank’s balance sheet

On the asset side, the bank can invest in a liquid, one–period bond (a T-bill), which yields a risk–free rate $r$, and in a portfolio of risky assets, called loans. We denote with $B_t$ the face value of the risk–free bond, and with $L_t \geq 0$ the nominal value of the stock of loans outstanding in period $t$ (i.e., in the time interval $(t-1,t]$).

Similarly to Zhu (2008), we make the following

Assumption 1 (Revenue function). The total revenue from loan investment is given by $Z_t \pi(L_t)$, where $\pi(L_t)$ satisfies conditions $\pi(0) = 0$, $\pi > 0$, $\pi' > 0$, and $\pi'' < 0$.

This assumption is empirically supported, as there is evidence of decreasing return to scale of bank investments.\(^5\) Loans may be viewed as including traditional loans as well as risky securities. $Z_t$ is a random credit shock realized on loans in the same time period, which can be viewed as capturing variations in banks’ total revenues as determined, for example, by business cycle conditions. Note that the choice variables $B_t$ and $L_t$ are set at the beginning of the period, while $Z_t$ is realized only at the end of the period.

The maturity of deposits is set to one period. Bank maturity transformation is introduced with the following

Assumption 2. (Loan reimbursement) A constant proportion $\delta \in (0,1/2)$ of the existing stock of loans at $t$, $L_t$, becomes due at $t + 1$.

The parameter \( \delta < 1/2 \) gauges the average maturity of the existing stock of loans, which is \((1 - \delta)/\delta > 1.\)\(^6\) Thus, the bank is engaging in maturity transformation of short term liabilities into longer-term investments. Under Assumption 2, the law of motion of \( L_t \) is

\[
L_t = L_{t-1}(1 - \delta) + I_t, \tag{1}
\]

where \( I_t \) is the investment in new loans if it is positive, or the amount of cash obtained by liquidating loans if it is negative.

To capture bank’s liquidation costs, we introduce convex adjustment costs as in the standard Q-theory of investment (see e.g. Abel and Eberly (1994)), with the following Assumption 3 (Loan Adjustment Costs). In increasing its investment in loans the bank incurs monitoring costs, whereas in decreasing the loans the bank pays liquidation costs. The adjustment costs function for loans is quadratic:

\[
m(I_t) = |I_t|^2 \left( \chi_{\{I_t > 0\}} \cdot m^+ + \chi_{\{I_t < 0\}} \cdot m^- \right), \tag{2}
\]

where \( \chi_{\{A\}} \) is the indicator of event \( A, \) and \( m^+ > m^- > 0 \) are the unit cost parameters.

Adjustment costs are deducted from profit. The asymmetry in the adjustment costs \( (m^- > m^+) \) captures costly reversibility: the bank faces higher costs to liquidate investments rather than expanding them. As detailed below, the higher costs of reducing the stock of loans can be interpreted as capturing fire sales costs incurred in financial distress.

On the liability side, at the beginning of period \( t, \) the bank receives a random amount of one-period deposits \( D_t, \) and this amount remains outstanding during the period. The stochastic process followed by \( D_t \) is detailed below. Deposits are insured according to the following

\(^6\)The (weighted) average maturity of existing loans at date \( t, \) assuming the bank does not default nor it makes any adjustments on current the investment in loans, is

\[
M_t = \sum_{s=0}^{\infty} s \frac{\delta^{t+s} L_t}{L_t} = \sum_{s=0}^{\infty} s \delta(1 - \delta)^s = \frac{1 - \delta}{\delta},
\]

as the residual loans outstanding at date \( t + s, s \geq 0, \) is \( L_{t+s} = L_t(1 - \delta)^s. \)
**Assumption 4 (Deposit insurance).** The deposit insurance agency insures all deposits. In the event the bank defaults on deposits and on the related interest payments, depositors are paid interest and principal by the deposit insurance agency, which absorbs the relevant loss.

Under this assumption, with no change in the model, the depositor is the deposit insurance agency itself, and its claims are risky, while deposits are risk–free from depositors’ standpoint.\(^7\) As in Zhu (2008), depositors’ supply of funds is assumed to be perfectly elastic, and depositors’ reservation rate is the risk–free rate, \(r\). The difference between the ex–ante yield on risky deposits and the risk–free rate is a subsidy that the agency provides to the bank, as the cost of this insurance is not charged to either banks or depositors.

To fund operations, the bank can issue a one–period bond. The bank is constrained to issue fully collateralized bonds, so that their return is the risk–free rate. We denote \(B_t < 0\) the notional amount of the bond issued at \(t – 1\) and outstanding until \(t\). The collateral constraint is detailed below.

To summarize, at \(t – 1\) (or at the beginning of period \(t\)), after the investment and financing decisions have been made, the balance sheet equation is

\[
L_t + B_t = D_t + K_t, \tag{3}
\]

where \(K\) denotes the (ex–ante) book value of equity, or bank capital. In this equation, \(B\) denotes the face value of a risk–free investment when \(B > 0\), and the face value of issued bond when \(B < 0\).

**B. Bank’s cash flow**

Once \(Z_t\) and \(D_{t+1}\) are realized at \(t\), the current state (before a decision is made) is summarized by the vector \(x_t = (L_t, B_t, D_t, Z_t, D_{t+1})\) as the bank enters date \(t\) with loans, bonds and deposits in amounts \(L_t, B_t,\) and \(D_t,\) respectively. Prior to its investment, financing and cash distribution decisions, the total internal cash available to the bank is

\[
w_t = w(x_t) = y_t - \tau(y_t) + B_t + \delta L_t + (D_{t+1} - D_t). \tag{4}
\]

\(^7\)This assumption is similar to Van den Heuvel (2009), but differs from Zhu (2008), who assumes that the bank will reward uninsured depositors with a risk premium.
Equation (4) says that total internal cash \( w_t \) equals bank’s earnings before taxes (EBT),

\[
y_t = y(x_t) = \pi(L_t)Z_t + r(B_t - D_t),
\]

(5)

minus corporate taxes \( \tau(y_t) \), plus the principal of one-year investment in bond maturing at \( t \), \( B_t > 0 \) (or alternatively the amount of maturing one-year debt, \( B_t < 0 \)) and from loans that are repaid, \( \delta L_t \), plus the net change in deposits, \( D_{t+1} - D_t \).

Corporate taxation is introduced consistently with current dynamic models of a non-financial firm (see e.g. Hennessy and Whited (2007)) with the following

**Assumption 5 (Corporate Taxation).** Corporate taxes are paid according to the following convex function of EBT:

\[
\tau(y) = \tau^+ \max\{y, 0\} + \tau^- \min\{y, 0\},
\]

(6)

where \( \tau^- \) and \( \tau^+ \), \( 0 \leq \tau^- \leq \tau^+ < 1 \), are the marginal corporate tax rates in case of negative and positive EBT, respectively.

The assumption \( \tau^- \leq \tau^+ \) is standard in the literature, as it captures a reduced tax benefit from loss carryforward or carrybacks. Note that convexity of the corporate tax function creates an incentive to manage risk, as noted by Stulz (1984).

Given the available cash \( w_t \) as defined in Equation (3) and the residual loans, \( L_t(1-\delta) \), bank’s managers choose the new level of investment in loans, \( L_{t+1} \) and the amount of risk-free bonds \( B_{t+1} \) (purchased if positive, issued if negative), with Equation (3) applying to \( B_{t+1}, L_{t+1}, D_{t+1} \). Both \( L_{t+1} \) and \( B_{t+1} \) remain constant until the next decision date, \( t+1 \).

These choices may differ according to whether the bank is or is not in financial distress. If total internal cash \( w_t \) is *positive*, it can be retained to change the investment in loans, it can be invested in one period risk-free bonds, or paid out to shareholders. However, if \( w_t \) is *negative*, the bank is in *financial distress*, since absent any action, the bank would be unable to honor part, or all, of its obligations towards either the tax authority, or depositors, or bondholders. When in financial distress, the bank can finance the shortfall \( w_t \) either by selling loans at fire sale prices, or by issuing bonds \( B_{t+1} < 0 \), or by injecting equity capital. However, overcoming this shortage of liquidity is expensive, because all these transactions generate (either explicit
of implicit) costs: in a fire sale, the bank incurs the downward adjustment cost defined in
Equation (2); bond issuance is subject to a collateral restriction; and floatation costs are paid
when seasoned equity are offered. We now detail these latter two restrictions on the banks’
financing channels.

Bank’s issuance of bonds is subject to a collateral constraint described by
Assumption 6 (Collateral constraint). If $B_t < 0$, the amount of bond issued by the bank is
constrained to be fully collateralized. In particular, the collateral constraint is

$$
L_t - m(-L_t(1 - \delta)) + \pi(L_t)Z_d - \tau(y_{t+1}^{\min}) + (B_t - D_t)(1 + r) + D_d \geq 0,
$$

where $Z_d$ is the worst possible credit shock (i.e., the lower bound of the support of $Z$), $D_d$ is
the worst case scenario flow of deposits, and $y_{t+1}^{\min} = \pi(L_t)Z_d + (B_t - D_t)r$ is the EBT in
the worst case end–of–period scenario for current $L_t$, $B_t$ and $D_t$.

The constraint in (7) reads as follows: the end–of–period amount $B_t(1+r) < 0$ that the bank
has to repay must not be higher than the after–tax operating income, $\pi(L_t)Z_d - rD_t - \tau(y_{t+1}^{\min})$,
in the worst case scenario, plus the total available cash obtained by liquidating the loans,
$L_t - m(-L_t(1 - \delta))$, plus the flow of new deposits in the worst case, $D_d$, net of the claim of
current depositors, $D_t$. The proceeds from loans liquidation are the sum of the loans that will
become due, $L_t\delta$, plus the amount that can be obtained by a forced liquidation of the loans,
$L_t(1 - \delta)$ net of the adjustment cost $m(-L_t(1 - \delta))$ as from Equation (2).

We denote with $\Gamma(D_t)$ the feasible set for the bank when the current deposit is $D_t$; i.e.,
the set of $(L_t, B_t)$ such that condition (7) is satisfied, if $B_t < 0$, no restrictions being imposed
when $B_t \geq 0$:

$$
\Gamma(D_t) = \left\{(L_t, B_t) \mid \frac{L_t - m(-L_t(1 - \delta)) + D_d + \pi(L_t)Z_d(1 - \tau(y_{t+1}^{\min}))}{1 + r(1 - \tau(y_{t+1}^{\min}))} + B_t \geq D_t, B_t < 0 \right\} \cup \{B_t \geq 0\}.
$$

(8)

In the plane $(L_t, B_t)$, the lower boundary of $\Gamma(D_t)$, when $B_t < 0$, is convex due to concavity
of the revenue function. This means that a bank can fund more investment in risky loans by
issuing more risk-free one period bonds. However, this investment has decreasing return to scale, and at some point the net return of a dollar raised by issuing bonds and invested in loans becomes negative. Figure 2 shows this for a given set of parameters.

Bank’s costs on equity issuance are due to information asymmetry and underwriting fees, and are modelled in a standard fashion (see e.g. Cooley and Quadrini (2001)) with the following Assumption 7 (Equity flotation costs). The bank raises capital by issuing seasoned shares incurring a proportional floatation cost $\lambda > 0$ on new equity issued.

As a result of the choice of $(L_{t+1}, B_{t+1})$, the residual cash flow to shareholders at date $t$ is

$$u_t = u(x_t, L_{t+1}, B_{t+1}) = w_t - B_{t+1} - L_{t+1} + L_t(1 - \delta) - m(I_{t+1}).$$

When $u_t$ is positive, it is distributed to shareholders (either as dividends or stock repurchases). If $u_t$ is negative, this amount is raised by issuing new equity. Hence, the actual cash flow to equity holders is

$$e_t = e(x_t, L_{t+1}, B_{t+1}) = \max\{u_t, 0\} + \min\{u_t, 0\}(1 + \lambda).$$

A description of the evolution of the state variables and of the related bank’s decisions in case the bank is solvent is in Figure 1.

Lastly, bank’s insolvency is modelled with the following Assumption 8 (Default). The bank defaults at time $t$ if the market value of bank’s equity is negative. In case of default, shareholders exercise the limited liability option (i.e., equity value is zero), and the assets are transferred to the deposit insurance agency, net of verification and bankruptcy costs in proportion $\gamma > 0$ of the face value of deposits, $\gamma D_t$. Then, the bank is restructured as a new entity with a minimum endowment of deposits, $D_d$, and an equivalent investment in risk-free bonds, $B_{t+1} = D_d$, so that $L_{t+1} = 0$ and $K_{t+1} = 0$.

The probabilistic assumptions of our model are as follows. There are two exogenous sources of uncertainty: the credit shock on the loan portfolio, $Z$, and the funding available from deposits, $D$. Denote with $s = (Z, D)$ the pair of state variables, and with $S$ the state space.
Assumption 9. The state space $S$, is compact. The random vector $s$ evolves according to a stationary and monotone (risk–neutral) Markov transition function $Q(s_{t+1} \mid s_t)$ defined as

$$Z_t - Z_{t-1} = (1 - \kappa_Z) (Z - Z_{t-1}) + \sigma_Z \varepsilon^Z_t$$

$$\log D_t - \log D_{t-1} = (1 - \kappa_D) (\log D - \log D_{t-1}) + \sigma_D \varepsilon^D_t.$$  

The error terms $\varepsilon^Z_t$ and $\varepsilon^D_t$ are i.i.d and have jointly normal distribution with correlation coefficient $\rho$.

In the above equation, $\kappa_Z$ is the persistence parameter, $\sigma_Z$ is the conditional volatility, and $Z$ is the long–term average of the credit shock; $\kappa_D$ is the persistence parameter for the deposit process, $\sigma_D$ is the conditional volatility, and $D$ is the long–term level of deposits.

C. The unregulated bank program and the valuation of securities

As noted, at date $t$, soon after observing the current state, $x_t$, the bank decides the new levels of loans and of risk–free investment with the goal of maximizing shareholders’ value. Here we consider an unregulated bank. Regulatory restrictions are introduced in the next section.

We denote $E$ the market value of bank’s equity. Given the state, $x_t = (L_t, B_t, D_t, Z_t, D_{t+1})$, bank’s equity is the result of the following program

$$E_t = E(x_t) = \max_{\{(L_{i+1}, B_{i+1}) \in \Gamma(D_{i+1}), i=t,...,T\}} \mathbb{E}_t \left[ \sum_{i=t}^T \beta^{i-t} e(x_i, L_{i+1}, B_{i+1}) \right],$$

where $\mathbb{E}_t[\cdot]$ is the expectation operator conditional on $D_t$, on the state variables at $t$, $(Z_t, D_{t+1})$, and on the decision $(L_{t+1}, B_{t+1})$; $\beta$ is a discount factor (assumed constant for simplicity); $(L_{i+1}, B_{i+1})$ is the decision at date $i$, for $i = t, \ldots$.

According to Assumption 8, shareholders choose to default if $e_t$ plus the continuation value of their claim is negative. In this case, they exercise the limited liability option (i.e., at default the value of equity is set to zero) and the bank is transferred to deposit insurance agency, net of verification and bankruptcy costs in proportion of $\gamma > 0$ of the value of the assets. In this case, the bank is valued as a going concern at the current state but assuming it is unlevered
(i.e., with \( D = 0 \)). We denote \( T \) the default date, which is a stopping time with respect to the information generated by the processes of \( Z_t \) and \( D_t \).

The corresponding Bellman equation for optimality of the solution when the bank is solvent is

\[
E(x_t) = \max_{(L_{t+1}, B_{t+1}) \in \Gamma(D_{t+1})} \left\{ e(x_t, L_{t+1}, B_{t+1}) + \beta E_t \left[ E(x_{t+1}) \right] \right\}.
\]

Because the model is stationary and the Bellman equation involves only two dates (the current, \( t \), and the next one, \( t+1 \)), we can drop the time index \( t \) and use the notation without a prime to denote the current value of the variables, and with a prime the future value of the variables. Hence, the above equation simplifies to

\[
E(x) = \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \beta E \left[ E(x') \right] \right\},
\]  

(13)

We denote the optimal policy when the firm is solvent, \((L^*(x), B^*(x))\).

When the bank is insolvent, the right-hand side of the above equation is negative. Hence, shareholders exercise the limited liability option and decide to default on their obligations. This puts a lower bound on \( E \) at zero. The default indicator function is denoted \( \Delta(x) \).

In conclusion, the Bellman equation that the value of equity satisfies is

\[
E(x) = \max \left\{ 0, \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L, B) + \beta E \left[ E(x') \right] \right\} \right\}.
\]  

(14)

Compactness of the feasible set of the bank can be shown as follows. Given the strict concavity of \( \pi(L) \), there exists a level \( L_u \) such that \( \pi(L_u) Z_u - r L_u = 0 \), where \( r \) is the cost of capital of the marginal dollar raised either through deposits or short term financing.\(^8\) Thus, any investment \( L > L_u \) would be unprofitable. This establishes an upper bound on the feasible set of \( L \), given by \([0, L_u]\) for some \( L_u \). With an upper bound on \( L \), and because the stochastic process \( D \) has compact support, the collateral constraint sets an lower bound \( B_d \) (i.e., an upper bound on bond issuance). Specifically, this is obtained by putting \( D_d \) in place of \( D_t \) and \( L_u \) in place of \( L_t \) in equation (7). Lastly, an upper bound on \( B \) can be obtained assuming

\(^8\)Deposits and short term bonds are the cheapest form financing. If the same dollar were raised by issuing equity, the cost would be higher due to both the higher cost of equity capital and to floatation costs. In this case the upper bound would be even lower.
that the proceeds from risk–free investments made by the bank are taxed at a rate not higher
than the personal tax rate. In conclusion, the feasible set of the bank can be assumed to be
\[ [0, L_u] \times [B_d, B_u] \].

**Proposition 1.** There exists a unique value function \( E(x) = E(L, B, D, Z, D') \) that satisfies (13) and is continuous, increasing, and differentiable in all its arguments, and concave in \( L \).

We solve equation (13) to determine the value of equity, the optimal policy when the bank
is solvent, and the optimal default policy, \( \Delta \), as a function of the current state, \( x \). We denote \( \varphi \), the state transition function based on the optimal policy:

\[
\varphi(x) = \begin{pmatrix} L^* \\ B^* \\ D' \end{pmatrix} (1 - \Delta) + \begin{pmatrix} 0 \\ D_d \\ D_d \end{pmatrix} \Delta, \tag{15}
\]

meaning that the new state is \((L^*, B^*, D')\) if the bank is solvent, and \((0, D_d, D_d)\) if the bank
defaults and a new bank is started endowed with seed deposits, \( D_d \), and equal amount of
cash balance, and no loans. The restructured bank is assumed to be endowed with the
minimum level of deposits, \( D_d \), which is momentarily invested in bonds. This bank will revise
its investment (together with the financing) policy in the subsequent decision dates.

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9Specifically, assume that the current deposits \( D \) are all invested in short term bonds, \( B \), with no investments in loans. To further increase the investment in bonds of one dollar, the bank must raise equity capital. A shareholder thus incurs a cost \( 1 + \lambda \), where \( \lambda \) is the floatation cost. This additional dollar is invested at the risk–free rate, so that at the end of the year, the proceeds of this investment that can be distributed are \((1 + r(1 - \tau^+))\). Alternatively, the shareholder can invest \( 1 + \lambda \) in a risk–free bond, obtaining \((1 + \lambda)(1 + r)\). Because \( \tau^+ \geq 0 \) and \( \lambda \geq 0 \), then \((1 + \lambda)(1 + r) \geq (1 + r(1 - \tau^+))\), there is no incentive of the bank to have a cash balance that is larger than \( D \) as long as either \( \lambda \) or \( \tau^+ \) are strictly positive. The foregoing argument is made for simplicity. If the shareholders are taxed on their investment proceeds at a rate \( \tau_p \), they obtain \((1 + \lambda)(1 + r(1 - \tau_p))\) from their investment in the risk–free asset. If \( \tau_p \leq \tau^+ \), then \((1 + \lambda)(1 + r(1 - \tau_p)) > (1 + r(1 - \tau^+))\), and the bank has no incentive to increase the investment in risk–free bonds beyond \( D \). Moreover, if floatation costs associated with equity issuance are strictly increasing in the amount issued, no assumption about differential tax rates are needed to establish an upper bound on \( B \).

10The proof of this proposition follows standard arguments. The existence and uniqueness of the value function \( E \) follow from the Contraction Mapping Theorem (Theorem 3.2 in Stokey and Lucas (1989)). The continuity, monotonicity, and concavity of the value function \( E \) in the argument \( L \) follow from Theorem 3.2 and Lemma 9.5 in Stokey and Lucas (1989). The continuity and monotonicity of \( E \) in \( B, D, Z, \) and \( D' \) follow from the continuity and monotonicity of \( e \) in \( Z \) and \( D' \) and the monotonicity of the Markov transition function of the process \((Z, D)\).

11Computationally, since default is irreversible, we must allow a new bank to re-enter otherwise there would eventually be no banks in the simulated sample.
The end–of–year cash flow from current deposits, $D_{t+1}$, for a given realization of the exogenous state variables, $(Z_{t+1}, D_{t+2})$, and on the related optimal policy, is

$$f(x_{t+1} \mid \varphi(x_{t+1})) = D_{t+1} (1 + r)(1 - \gamma \Delta(x_{t+1})).$$

Hence, the ex–ante fair value of newly issued deposits at $t$, from the viewpoint of the deposit insurance agency (i.e, incorporating the risk of bank’s default), is

$$F(x_t) = \beta E_t[f(x_{t+1} \mid \varphi(x_{t+1}))] = \beta D_{t+1} (1 + r)(1 - \gamma E_t[\Delta(x_{t+1})]) = \beta D_{t+1} (1 + r)(1 - \gamma P(x_t)),$$

where $P(x_t) = E_t[\Delta(x_{t+1})]$ is the conditional default probability. Dropping the dependence on the calendar date,

$$F(x) = \beta D'(1 + r)(1 - \gamma P(x)).$$

D. Efficiency and social cost metrics

Given the value of bank’s securities, our analysis of the impact of bank regulation and taxation focuses on three metrics. The first metric is the market value of equity $E(x)$, which represents the optimized value of current and the future private benefits accruing to shareholders at $x$.

A standard valuation concept is the market value of bank assets $E(x) + F(x)$, which includes current cash holdings, $B$. Given the maturity mis–match between the loan investment and the one–period deposit contracts, the one–period risk–free bond has in fact a dual role in our model. On the one hand, since it is a cash–equivalent asset, it is a risk–management tool suited to face sudden end–of–period deposit withdrawals and bad performances on loans. Because the bank faces distress and bankruptcy costs, and because of a convex corporate tax function, excessive fluctuations of the cash flow to shareholders is undesirable. For this reason, the availability of cash at the end of the period mitigates the incidence of these losses. On the other hand, positive bond holdings can be a “safe harbour” investment when expectations on loan performance are gloomy.

Yet, the market value of bank’s assets does not necessarily capture the role of banks as maturity transformers of liquid liabilities into longer term productive assets (loans). One of the key economic contributions of banks identified in the literature (see e.g. Diamond (1984)}
and Boyd and Prescott (1986)) is their role in efficiently intermediating funds toward their best productive use. But banks play no such role if they just raise funds to acquire risk–less (cash–equivalent) bonds. Thus, in our set up the value of the loans portfolio is an appropriate measure of the ability of the bank to create “productive” intermediation. we use the enterprise value of the bank as such a metric, defined as \( V(x) = E(x) + F(x) - B. \) Using the enterprise value is important because deposits and equity capital can be invested also in cash–equivalent assets (i.e., one–period risk–free bonds). While, as noted, the investment in risk–free bonds helps reducing the costs triggered by high cash flows volatility, thus increasing shareholders’ value, it is not creating necessarily efficient intermediation in the sense explained above. Hence, we need to subtract cash \( (B > 0) \) to obtain the actual value of the loans portfolio. On the other hand, the bank can issue a one–period risk–free bond to raise funds to invest in loans. Also in this case, the actual value of the bank’s loans is obtained by subtracting \( B < 0 \).

The third metric is what we term the “social” value of the bank, which is constructed as follows. The value of default costs on bank’s current deposits is:

\[
DC(x) = \beta D'(1 + r) \gamma P(x).
\]

This is a measure of the expected losses suffered by the deposit insurance agency. It is a linear function of the loss–given–default parameter, \( \gamma \), and of the default probability, \( P(x) \). We use this metric (or equivalently, the default probability) to measure the expected cost of bailing out a bank in financial crisis. To gauge the (social) impact of the moral hazard issue generated by the deposit insurance and the trade–offs between value creation through maturity transformation and the limitations imposed by capital and liquidity requirements, we introduce a function representing the value of the tax payoff to the Government. This is defined by the recursive equation

\[
G(x) = \tau(y') (1 - \Delta(x)) + \beta \mathbb{E} \left[ G(x') \right].
\]

Hence, the social, or total, value of the bank is the sum of values to all the stakeholders in the model: shareholders’ value (net of cash balance or plus the short term financing), plus the

\[12\] For the use of enterprise value in the context of dynamic models of non–financial firms, see e.g. Gamba and Triantis (2008) and Bolton, Chen, and Wang (2009).
value to (insured) depositors, plus the value to the Government, net of the value of default costs:

\[ SV(x) = E(x) - B + \beta D'(1 + r) + G(x) - DC(x). \]

Given the above definition and the fact that \( F(x) = \beta D'(1 + r) - DC(x) \), the social value is \( SV(x) = V(x) + G(x) \); that is, the social value of the bank is given by the market value of the loans (enterprise value) plus the value of taxes. We can use \( SV(x) \) to capture the tradeoff between a stricter requirement, which in general reduces the value of the bank’s loans and the flow of corporate taxes, and a looser regulation, which increases the expected bailout cost. Therefore, our measure of social value is a tool to describe stakeholders trade–offs.

IV. Bank regulation

In this section we define the restrictions implied by bank regulation on banks feasible choices of \( L \) and \( B \), and we discuss briefly the likely impact of these restrictions – together with those arising from the collateral constraint defined with Assumption 6 – on optimal bank policies in a static context. In the sequel, we illustrate the impact of bank regulation in a dynamic context by numerical simulation, where banks’ optimal dynamic decisions are constrained by the (dynamic) shadow costs associated with these restrictions.

Importantly, key bank regulations, such as capital requirements, are typically associated with specific bank closure rules that differ from those that would arise in an unfettered setting. Specifically, capital regulations are universally based on measures of accounting capital, rather than economic net worth (see e.g. Saunders and Cornett (2003)). As a result, bank closure rules differ from those of an unregulated bank, whose shareholders are allowed to determine voluntarily when to default and close the bank. In other words, in the unregulated case, default is endogenous, whereas under regulation it is exogenous. This motivates our treatment of bank closure rules as a feature of bank regulation.

A. Bank closure

Bank’s insolvency under regulation is defined by the following
Assumption 10 (Bank Closure Rule). The bank is closed at time $t$ if the ex-post net asset value (i.e., ex-post bank capital, as opposed to $K_t$) is negative:

$$v_t = L_t + B_t - D_t + y_t - \tau(y_t) = K_t + y_t - \tau(y_t) < 0. \quad (16)$$

After the closure, the market value of equity is set to zero (absolute priority rule), the assets are transferred to the deposit insurance agency, net of verification and bankruptcy costs, and the bank is restructured as after the standard default (Assumption 8.)

This default mechanism corresponds to a condition of ex-post negative book equity, as in Zhu (2008). However, differently from Zhu, we assume that the bank, after restructuring, continues to operate as a new entity.

The Bellman equation for optimality of the solution of the bank’s program, when the bank is solvent (i.e., $v_t \geq 0$ as in Assumption 10), is as in equation (13). When the bank is insolvent (i.e., $v < 0$), the value of equity, $E$, is zero.

B. Capital requirement

The first pillar of Basel II regulation establishes a lower bound $K_d$ on the book value of equity $K$, set by the regulator as a function on bank’s risk exposure at the beginning of the period. In particular, this requirement is a weighted average of banks risks. Since our model has just one composite risky asset, we set the weight applied to loans equal to 100%.

Thus, in our setting the capital requirement is a level of capital that is at least a proportion $k$ of the principal of the loans at the beginning of the period, $L$. Thus, $K_d = kL$, and this requirement is equivalent to constraining net worth to be positive ex-ante. Given the definition of the banks capital in (3), under this capital requirement the bank’s feasible choice set is

$$\Theta(D) = \{(L, B) \mid (1 - k)L + B \geq D\}. \quad (17)$$

---

When we relate the feasible choice set under the collateral constraint in Equation (8) to the feasible set under the capital requirement, in general neither $\Gamma(D) \subset \Theta(D)$ nor $\Theta(D) \subset \Gamma(D)$ in a proper sense. Hence, the capital requirement may (or may not) restrict the bank’s feasible policies, depending on the values of the parameters.

If the bank is short term borrowing, $B < 0$, the capital constraint restricts the bank’s feasible choice set for a given $D$ if $\Theta(D) \subset \Gamma(D)$. This is equivalent to

$$\frac{L - m(-L(1-\delta)) + D_d + \pi(L)Z_d(1 - \tau(y^{\text{min}}))}{1 + r(1 - \tau(y^{\text{min}}))} \geq (1 - k)L.$$  

Since the inequality is independent of $D$, what follows holds for any current $D$. If we assume a constant corporate tax rate (in place of two tax rates: $\tau^+$ and $\tau^-$) for the sake of simplicity, for a large range of values of the model parameters and of $L$, the above inequality is satisfied. This means that the capital requirement in fact restricts the bank’s feasible choice set. On the other hand, only for a large $L$ the capital constraint is not restricting the feasible choice set when $B < 0$.

Alternatively, if the bank is short term lending, $B \geq 0$, then the capital requirement restrict the feasible choice set, since it imposes the bank to have a fairly large cash balance $B$ if $L < D/(1-k)$, while the constraint is not restricting the feasible choice set or $L \geq D/(1-k)$. Figure 2 shows how the capital requirement is related to the collateral constraint for a specific set of parameters.

In this case, the Bellman equation of the equity value of a bank under a capital requirement is given by Equation (13) with a feasible set for $(L', B')$ equal to $\Gamma(D') \cap \Theta(D')$. Hence, the bank is forced to comply ex-ante with the capital requirement. However, at the end of the period, when the credit shock on existing loans, $Z'$, and the new deposit, $D'$, are realized, the bank may still face default risk if the innovations of the state variables are particularly unfavorable, and in particular, if the shock on loans is significantly negative.

**C. Liquidity requirement**

Bank supervisors around the world use a variety of quantitative measures to monitor the liquidity risk profiles of banks. The current Basel III regulatory proposals include the introduction
of a mandatory illiquidity coverage ratio: banks would be prescribed to hold a stock of high quality liquid assets such that the ratio of this stock over what is defined as a net cash outflows over a 30-day time period is not lower than 100%. In turn, the net cash outflow is expected to be determined by what would be required to face an acute short term stress scenario specified by supervisors. Banks would need to meet this requirement continuously as a defense against the potential onset of severe liquidity stress.\footnote{See Basel Committee on Banking Supervision, \textit{International Framework for Liquidity Risk Measurement, Standards and Monitoring}, Bank for International Settlements, Basel, December 2009.}

In our model, the stock of high quality liquid assets over the net cash outflows over a period is given by the total cash available at the end of the period over the total net cash flow in the worst case scenario for credit shocks and deposit flows. Formally, this illiquidity coverage ratio should be not lower than a level \( \ell \) defined by the regulator, or

\[
\frac{\delta L + Z_d \pi(L) - \tau(y^{\min}) + B(1 + r)}{D(1 + r) - D_d} \geq \ell.
\]

Hence, the feasible set for a bank complying with the liquidity requirement is

\[
\Lambda(D) = \left\{ (L, B) \mid \frac{\delta L + \ell D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{\ell(1 + r) - \tau(y^{\min}) r} + \frac{B(1 + r(1 - \tau(y^{\min})))}{\ell(1 + r) - \tau(y^{\min}) r} \geq D \right\}.
\]

Interestingly, for the case with \( \ell = 1 \), this simplifies to

\[
\Lambda(D) = \left\{ (L, B) \mid \frac{\delta L + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{(1 + r(1 - \tau(y^{\min})))} + B \geq D \right\},
\]

so that we can directly compare this to the collateral constraint, \( \Gamma(D) \). For a bank that is short term borrowing, \( B < 0 \), the liquidity constraint restricts the feasible choice set, or \( \Lambda(D) \subseteq \Gamma(D) \), if

\[
\frac{L - m(-L(1 - \delta)) + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{1 + r(1 - \tau(y^{\min}))} \geq \frac{\delta L + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{(1 + r(1 - \tau(y^{\min})))},
\]

or equivalently, if \( L(1 - \delta) \geq (L(1 - \delta))^2 m^- \). This is indeed the case for a wide range of parameters and a large set of values of \( L \). Moreover, the liquidity constraint always restricts the feasible choice set when the bank is short term lending, \( B > 0 \).
In conclusion, the liquidity constraint turns out to restrict the bank’s feasible choice set relative to the collateral constraint for a wide range of parameter values. Figure 2 shows a comparison of the liquidity requirement to the collateral constraint for a specific choice of parameters.

Lastly, we can compare the capital requirement with the liquidity constraint for \( \ell = 1 \). This allows us to see whether one turns out to be more restrictive than the other (in the region where also the collateral constraint is satisfied), and what is the effect on the feasible choice set of having these two requirement imposed at the same time.

The capital requirement turns out to be more restrictive than the liquidity requirement for a given \( D \) if \( \Theta(D) \subset \Lambda(D) \), or

\[
\delta L + D_d + Z_d \pi(L)(1 - \tau(y_{\text{min}})) \geq (1 - k)L.
\]

This can be easily seen on Figure 2. Assuming a constant tax rate (independent of the EBT), there is a threshold level \( \hat{L} \) where the above inequality holds as an equation. For \( L \) lower than \( \hat{L} \), the capital constraint is more restrictive than the liquidity constraint. Vice versa for \( L > \hat{L} \). Overall, when considered together, the two constraints create considerable restrictions on bank’s feasible choices.

V. The impact of bank regulation

In this section, we evaluate the impact of the introduction of bank regulations on banks’ optimal policies and the three metrics defined previously. We carry out this exercise under a set of benchmark parameters calibrated using selected statistics from U.S. banking data, some previous studies, and current regulatory and tax parameters, described in Subsection A. Optimal bank policies and relevant metrics under bank regulation for given realized states are presented in Subsection B. This discussion is informative in describing the main mechanisms through which regulatory constraints affect banks’ optimal choices and the implied metrics in a “static” context. However, being state-specific, it does not necessarily capture the actual likelihood of the different states and of the related policies. For this reason, in Subsection C
we detail the outcomes of these mechanisms in a dynamic context, analyzing some statistics of the steady state distributions of optimal policies and our metrics in a simulation exercise.

A. Calibration

Our benchmark calibration of the model pertains three sets of parameters, summarized in Table I. The first set comprises parameters of the two exogenous state variables. We estimate these parameters using statistics of yearly time series for the period 1980-2009 taken from the U.S. H-4 report on banks’ asset and liabilities from the Federal Reserve Board, and estimates of average loan rates reported in the IMF International Financial Statistics. Based on these data, we set the long term level of shock process to 0.0842 on an annual basis. The shock process has autocorrelation equal to 0.76 and conditional standard deviation 0.0164 per year. With these parameters, the unconditional standard deviation of the shock is 0.0252. The long term autocorrelation for the (log-)deposits process is 0.98, so it is highly persistent, and the conditional standard deviation is 0.0071 on an annual basis. Hence, the unconditional standard deviation of the (log-)deposit process is 0.0357. The historical annual correlation of the two processes is estimated −0.77.

The second set of parameters is taken from previous research. The annual discount factor $\beta$ is set to 0.95, equal to Zhu’s (2008) and similar to other authors’s (see e.g. Cooley and Quadrini (2001)). The risk–free rate is set to 3.5%, which falls between the one used by Zhu (2008), and the one in Cooley and Quadrini (2001).

With regard to corporate taxation, recall that the tax function is defined by the marginal tax rates, $\tau^+$ and $\tau^-$, for positive and negative income, respectively. Since we do not explicitly consider dividend and capital gain taxation for shareholders or interest taxation for depositors, the two marginal rates for corporate taxes are to be considered net of the effect of personal taxes. For this reason we choose $\tau^+ = 15\%$, which is close to the values determined by Graham (2000) for the marginal tax rate. The marginal tax rate for negative income is $\tau^- = 5\%$ to allow for convexity in the corporate tax schedule.

Furthermore, the proportional bankruptcy cost is $\gamma = 0.10$, Which is a value very close to what has been found by Hennessy and Whited (2007), who provide a (structural) estimate of 0.104 for this cost based on U.S. non–financial firms. The annual percentage of reimbursed
loan is 20%, so that the average maturity of outstanding loans is 4 years, in line with the assumption made by Van den Heuvel (2009). The floatation cost for seasoned equity issuance is 30%, as in Cooley and Quadrini (2001). This means the bank incurs a significant transaction cost any time it taps the market for equity capital, typically when it is in financial distress. Finally, we set our base case value for $\alpha$ to 0.85, which is slightly lower than the one used in other papers. Lastly, we set $m^+ = 0.03$ and $m^- = 0.04$. Since for $\alpha$ and $m$ we do not have a direct reference to previous research, we will do a robustness check on the selected values.

The third set of parameters is based on regulatory prescriptions. In our case, these are the ratio of capital to risk–weighted assets and the liquidity coverage ratio. The capital ratio $k$ is set to 8%, as in current Basel II regulation. The liquidity ratio is $\ell = 1$, based on current Basel III proposals.

B. State-dependent analysis

This section details the impact of bank regulations on optimal policies and our metrics of bank efficiency and social cost for given realization of the states.

B.1. Optimal policies

We illustrate optimal policies for four cases: the unregulated bank, the bank subject to capital regulation only, to liquidity requirements only, and to both capital regulation and liquidity requirements.

Figure 3 depicts the impact of regulatory restrictions on the bank’s policy related to loan investment and to short term investment and financing. The short term investment/financing policy is given by the optimal $B^*$ given the current state, averaged across all possible $Z$ in the left panel and averaged across all possible $D'$ in the right panel. The loan investment policy is represented by the ratio $L^*/L$ given the current state and averaged across $Z$ in the left panel and across $D'$ in the right panel. Tables II and III report average capital ratios and liquidity ratios for a solvent bank under the five cases as functions of different levels of credit shocks and different levels of bank capital at a point in time.
Consider first the unregulated bank. From Figure 3 we see that, when $D'$ is low (liquidity shock) or $Z$ is low (credit shock), the bank reduces short term debt and liquidates loans. Conversely, with an expansion both in liquidity (high $D'$) or in credit (high $Z$), there is an increase in loan investment funded by issuing short term bonds. It is important to note that the loan policy of an unregulated bank is *pro-cyclical*, since loans vary positively with credit and liquidity conditions. Thus, pro-cyclicality of lending is a feature of an optimal policy independently of capital requirements.

As shown in Table II, the resulting average capital and liquidity ratios are *negative* in all cases: the book value of equity is always lower than its market value. The average liquidity coverage ratio (see Table III) is also negative. This means that the unregulated bank takes an exposure in loans so that, in the worst case scenario for the credit and the liquidity shocks, a forced liquidation of loans is likely needed. In essence, the bank is trading off liquidation costs with the benefits of the larger investment in loans.

Consider now the bank subject to capital regulation only. Relative to the unregulated bank, this bank takes on less debt but invests more in loans than its unregulated counterpart for every realization of liquidity and credit shocks. Thus, by equation (17), the bank satisfies the capital requirement by increasing loans at a rate proportionally higher than the capital ratio coefficient. On the one hand, the capital requirement requires the bank to reduce its indebtedness relative to its loan investment. On the other hand, the capital requirement also reduces the risk that the bank may be closed under the bank closure rule: as a result, the expected returns on loans may increase, prompting a higher investment in loans. Importantly, loan increase with both higher levels of new deposits and more favorable credit shocks (see Figure 3). Thus, as in the case of the unregulated bank, loans vary positively with credit and liquidity conditions. However, under our parameterization, the pro-cyclicality of lending does not appear to be enhanced by capital requirements, as compared to the unregulated case, as the slope of loan policies in both cases are almost parallel.

Furthermore, the average capital ratio of a solvent bank subject to capital regulation increases monotonically in the level of $D$ (or decreases in $K$) for each level of the credit shock (Table II). Note that the capital ratio is constantly higher than the prescribed level of 0.08, due to the (shadow) cost of the capital requirement. As shown in Table III, the average liquidity coverage ratio of the bank under capital regulation is larger than its unregulated counterpart,
owing primarily to the larger level of loan investment and a less than proportional increase in short term financing, which raise the numerator of Equation (18). However, due to its reduced overall riskiness, the bank under capital regulation can afford to keep the level of loans high also under unfavorable credit shock realizations.

Next, consider the impact of liquidity requirements abstracting from capital requirements. As shown in Figure 3, the bank subject only to this requirement takes on less debt (or no debt at all) and its loan investment is significantly smaller not only relative to the its unregulated counterpart, but also with respect to the bank subject to capital regulation. This can be explained by noting that the constraint in equation (18) can be satisfied by raising $B$ more proportionally than a decline in $L$. Thus, the liquidity requirement implies optimal bank policies that move the control variables in exactly the opposite direction of the moves induced by capital requirements. Furthermore, the optimal level of net bond holdings of the bank under a liquidity requirement seems scarcely sensitive to different levels of new deposits, while investment in loans increase as the liquidity constraint (as well the collateral constraint) becomes less binding. Larger levels of investment in loans are also associated with more favorable credit shocks, and their sensitivity to these shocks is similar to that of the unregulated bank. Hence, pro-cyclicality in lending does not seem to be enhanced by liquidity requirements considered in isolation.

As shown in Table II, the average capital ratio under a liquidity requirement is large relative to the unregulated bank and, more interestingly, it is also larger than the ratio of the bank under capital regulation. In this case the capital ratio is pushed up by a relatively large net bond holding (the numerator) and lower investment in loans (the denominator). Note that this mechanism is totally different from that induced by capital regulation, since in that case the capital ratio is ultimately pushed up by equity issuance, as investment in loans and indebtedness increase relative to the unregulated case. Not surprisingly, the average liquidity coverage ratio is generally positive and higher than the prescribed level ($\ell = 1$), given the (shadow) cost related to the liquidity constraint.

By adding a liquidity requirement to a capital requirement, the two different mechanisms underlying the impact of each constraint described above in principle offset each other. For our parameter values, however, the effects of the liquidity requirement turn out to dominate on net the optimal policies. As shown in Figure 3, both bond holdings and investment lev-
els under capital and liquidity requirements are very close to those with liquidity regulation only for each level of new deposits and credit shocks, with the residual impact of the capital requirement reflected in a slightly higher level of investment in loans. The dominance of liquidity requirement is particularly self-evident in Figure 2, where we can see that the liquidity requirement is far more restrictive than the capital requirement for large enough $L$, and is also reflected in the average capital ratios and the liquidity coverage ratios reported in Tables II and III respectively, which exhibit values very close to the case of liquidity requirements only.

B.2. Metrics

Figure 4 shows the impact of capital and liquidity requirements on enterprise, government and social values as a function of a set of realizations of credit shocks and liquidity risk with reference to the steady state value of the credit shock and the level of deposits. These values are plotted in the left panels against the level of (new) deposits, $D'$, while keeping $Z$ at the steady state, and in the right panels against the credit shock parameter, $Z$, while holding $D'$ fixed at the steady state.

With regard to capital regulation (red line), there is a value loss in all dimensions, since the relevant ratios are all below one. Equity and enterprise values are smaller than in the unregulated case because the bank satisfies the capital requirement by expanding its investment in loans while reducing its indebtedness. However, because of decreasing returns in loan investment, the return on the overall loan portfolio is lower.

Moreover, the value losses associated with capital regulation are more severe the lower are the levels of new deposits and the credit shock. This is because in a downturn, during which credit quality deteriorates, the bank is forced to liquidate more loans, thereby incurring in significant liquidation (fire sale) costs. Thus, these results suggest that the losses in values associated with capital regulation appear to be largest for adverse realizations of both credit and liquidity risk parameters. Here the efficiency and social costs of “pro-cyclicality” take the form of relative over-investment in loans in an upswing, and relative high liquidation costs in a downturn. This result is somewhat at variance with the conjecture that capital regulation may abate both the private or social costs of bank distress, since the cost it imposes on the
bank and its intermediation capacity are highest precisely when the bank is likely to be in financial distress.

With regard to liquidity requirements (blue line in Figure 4), the loss in equity and enterprise value is generally larger than that associated with the capital requirement for relatively large levels of new deposits and credit shocks, while social value is uniformly lower for any level of credit shock. In order to satisfy the liquidity requirement, the bank over–invests in bonds and under–invests in loans relative to the unregulated case. On the other hand, in a downturn the bank reduces its position on risky investments quite similarly to what we have seen in the unregulated case, thus resulting in a lower reduction in value with respect to the case of capital requirement.

Lastly, the value losses under both capital regulation and liquidity requirements (green line) are the highest for virtually all levels of credit and liquidity shocks, and turn out higher than those associated with each of the constraints considered separately. Thus, the inefficiencies associated with each of these regulations are amplified when these regulations are introduced jointly.

C. Dynamics

Recall that in our model a bank is represented by the set of parameters governing credit and liquidity shocks, the curvature of its revenue function $\alpha$, and the maturity of its loan portfolio, $\delta$. Here we subject this bank to a large set of shocks and compute statistics of its optimal choices and our metrics. Since the bank’s optimal policies are path-dependent, we simulate a random sample of 10,000 paths of the exogenous shocks (or 10,000 possible scenarios this bank may face) of 100 annual periods each, and apply the optimal policy to these random paths. To capture the steady–state behavior of the bank and reduce the dependence on the initial state, we drop the first 50 periods. Hence, we obtain a panel of 50 steps for the 10,000 scenarios of the bank. We summarize the distribution of the relevant metrics by their mean and standard deviation.

Table IV illustrates these statistics when the bank is unregulated, when it is subject to either capital requirements only, or to liquidity requirements only, and when it is subject to both requirements.
Comparing the bank under capital regulation with the unregulated bank captures the net impact of the capital constraint on the bank’s dynamic choices. As illustrated in Section IV, the capital requirements is likely to be less binding with lower levels of bond holdings and higher loan investment. Ceteris paribus, the shadow price associated with the capital requirement becomes relatively smaller the more the bank invests in loans financed in part by short term risk–free market debt. Indeed the bank appears to behave so as to minimize the impact of such constraint: compared to the unregulated case, it has a larger loan portfolio (at book value) and a slightly lower level of debt on average.

Specifically, the regulated bank invests more in loans ($4.80 compared to $3.58 of the unregulated). Since deposits are not a control variable and follow the same exogenous process of the unregulated case, the regulated bank can fund this additional investment either with higher debt or increasing retained earnings and equity issuance. An increase in loan investment is profitable as long as the marginal cost of funding is lower than the expected return on investment. However, because loan investment has decreasing return to scale (i.e., the credit quality of borrowers decreases), the collateral constraint becomes increasingly binding (i.e., the lower boundary or $\Gamma$ is convex, as can be seen in Figure 2). Hence the regulated bank chooses a slightly lower level of debt. Therefore, the bank needs to increase retained earnings or issue equity to fund the additional loan investment. Specifically, from equations (9) and (10), more earnings are retained from $w_t$ or shares (incurring floatation costs $\lambda$) are issued if $w_t$ is negative.

As a result of these optimal policies, the bank holds a higher capital ratio than that prescribed by regulation, as the shadow price of violating that constraint is still positive, and the bank manages its earnings and investments so as to maintain a capital buffer to ensure that the constraint is not violated. This is a result consistent with the empirical evidence regarding banks holding (ex–ante) capital larger than required by regulations (see Flannery and Kasturi (2008)). Furthermore, capital regulation results in a bank with a lower probability of default than in the unregulated case. Under our parametrization, the probability of default is actually zero. As a result, the market value of deposits is (slightly) higher than in the unregulated

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15Recall that the restriction on $L$ and $B$ derived from capital regulation is such that $L + B - D \geq kL$, and the default condition is $L + B - D > -(y - \tau(y)) = -(\pi(L)Z + r(B - D))(1 - \tau)$ from equation (16). Under our parametrization, the lower bound on $Z$ is $Z_d = 0.0085 > 0$. As long as $\pi(L)Z_d + r(B - D) > 0$, the right–hand–side of the last inequality is negative, meaning that the capital condition is enough to prevent default in the worst case scenario, and therefore in any other scenario. However, $\pi(L)Z_d + r(B - D)$ can be negative if
case. Thus, capital regulation is successful in abating the probability of default under deposit insurance.

Remarkably, for the chosen set of parameters, capital regulation implies an increase in the efficiency of intermediation, since the enterprise value is larger than the unregulated case. This result suggest that the unregulated bank, relative to the bank subject to capital regulation, is under-investing in loans, is (perhaps) not financing all projects with positive NPV, preferring to distribute earnings as dividends rather than retaining them to support higher loan investment. However, for these effects to take place, capital regulation needs to be “mild” in the sense defined below.

The social value of the bank (i.e., the overall value created by the bank to all stakeholders including the Government) as well as Government Value, are also larger than in the unregulated case. For this calibration, the perceived costs of capital regulation possibly associated with “pro-cyclicality” seem to be compensated by the gains in efficiency of intermediation and social value.

Turning to the liquidity requirement, results are quite different and somewhat striking. When considered in isolation, the liquidity requirement severely hampers the ability of the bank to carry out efficient intermediation through maturity transformation and, notably, it appears to reduce the risk of default only slightly. As shown in Table IV, the bank subject only to these requirements has a significantly smaller loan portfolio than in the unregulated case ($0.78 vs $3.58), and instead invests a significative amount in liquid assets ($1.14). The capital ratio is higher than in the unregulated case, but this does not translate into a reduction of the default probability relative to the unregulated case. On the contrary, such probability increases slightly (from 0.01 to 0.26). This result is at variance with the common belief that higher book capital ratios are generally associated with lower default probabilities. What seems to matter is not the level of book capital ratio per se, but how it is generated. In the return on loan investment is not enough to pay short term net financing cost, \( r(B - D) \). But as long as \( \pi(L)Z_d + r(B - D) > -kL \), the bank is solvent. This is equivalent to writing \( \pi(L)Z_d + kL + r(B - D) > 0 \), or \( \pi(L)Z_d + (k - r)L + r(L + B - D) > 0 \), and this is true in our case, because \( k > r \), and \( K = L + B - D > 0 \) under the capital constraint.

Note that these results differ from the ranking of the metrics obtained in the context of the static results discussed in the previous section, since the latter do not take fully into account banks’ dynamic choices.

Our results are different from Zhu (2008), who shows that under its “flat” capital requirement, investment in lending decreases relative to the unregulated case.
this hypothetical case, all metrics indicate an inferior outcome relative to both the unregulated and the capital requirement case.

But an even more somber picture emerges in the realistic case in which the liquidity requirements are added to capital regulation. Relative to the bank subject only to capital requirements, lending is significantly reduced, and enterprise, government and social values worsen. In particular, the additional liquidity requirement yields an over-bloated book capital ratio. Such a ratio may be viewed as an indication of a safe but very inefficient bank. As we have seen, the capital requirement is the only tool instrumental in reducing the default probability relative to the unregulated as well as the liquidity constraint only case. However, with the additional liquidity requirement, the costs of reducing the default probability appear large, since the social value under both requirements is about 60% lower than in the case the bank is subject to capital regulation only.

The two key results of this section can be summarized as follows. Capital requirements can achieve the twin objectives of abating the incentives for higher risk-taking induced by deposit insurance and limited liability, and inefficient investment that characterize an unregulated bank. However, as we show below, if these requirements are too strict, then their efficiency and social costs may be high. By contrast, liquidity requirements appear an extremely inefficient way of attaining a balance between efficiency, social value and risk-taking in the banking system.

VI. Comparative dynamics of bank regulation and taxation

In this section we examine the impact of changes in bank regulations and taxation on the optimal policies of a bank subject to both capital regulation and liquidity requirements, and the relevant three main metrics, by means of the steady state distributions as in Table IV. The objective is to assess the sensitivity of banks’ optimal policies and the three metrics to stricter bank regulations and higher taxation embedded in current regulatory reform and some tax policy proposals.
A. Bank regulation

The results of changes in bank regulation parameters are presented in Table V. We consider the impact of an increase in the required capital ratio (from 8% to 12%, column $k$) and an increase in the liquidity ratio (from 1 to 1.2, column $\ell$).

For the bank under capital regulation, an increase in the capital ratio implies now a reduction in loans (from $4.80$ to $4.41$) and a corresponding reduction in indebtedness (from $2.29$ to $1.76$). In the previous section we saw that under a lower capital ratio, the bank was satisfying the capital requirement by increasing loans at a rate proportionally higher than the capital ratio coefficient, financing them with higher retained earnings or equity issuance. However, when the capital ratio coefficient becomes too high, such a strategy becomes too costly. Thus, the bank is compelled to reduce both loan investments and further reduce debt, as the collateral constraint is even more stringent.\footnote{For our parameterization, the simulation results suggest that the threshold value over which loan investment and indebtedness start to decrease is in the range (0.08,0.12).}

The increase in the required capital ratio impact negatively on bank’s efficiency and on social value. The bank enterprise value declines significantly (from $6.63$ to $6.11$) and such decline accounts for the bulk of the decline in bank’s social value. These results suggest that the benefits of a mild capital regulation (relative to the unregulated case) rapidly disappear and its cost increases more than proportionally with the stringency of such regulation. In other words, an optimal level of regulatory capital may exist as a function of banks’ characteristics.

For the bank subject to capital and liquidity requirements, an increase in the capital ratio implies a small increase in loans (from $2.70$ to $2.79$) matched by a small increase in indebtedness (from $0.07$ to $0.10$). The increase in capital requirement allows the bank to increase slightly its short term debt to finance new loans, since it has a relatively low leverage. However, the dominance of the liquidity requirement in constraining bank’s debt discussed previously allows the bank to respond to an increase in the capital ratio only in a limited way. As loan slightly increase, all metrics also increase moderately. On the other hand, an increase in the liquidity requirement (from $\ell = 1$ to $\ell = 1.2$) again reduces loans (from $2.70$ to $2.58$), and reduces short term financing (from $0.07$ to $0$), due to the mechanisms already discussed. All metrics witness a significant reduction.
In sum, increases in capital over a certain threshold reduce bank efficiency and increase social costs. When an increase in liquidity requirements is superimposed on a more stringent capital regulation, then the decline in enterprise and social values is even more pronounced.

B. Corporate income taxes

Turning to corporate taxation, the results of an increase of taxation of bank’s income (from $\tau^+ = 15\%$ and $\tau^- = 5\%$ to $\tau^+ = 20\%$ and $\tau^- = 7.5\%$) are in Table VI.

When we consider both the bank subject to capital regulation only, as the after–tax loan return is lowered with higher taxes, the investment in loans is reduced (from $4.80$ to $4.73$). On the liability side, the corresponding reduction in book value leaves bank capital largely unaffected (from $0.51$ to $0.50$), and reduces short term financing (from $-2.29$ to $-0.23$). With a higher tax bill, both the enterprise value and the social value of the bank are reduced when the tax rate is increased, although increased tax receipts increase government value (from $0.48$ to $0.61$).

When we consider the bank subject to both capital regulation and liquidity requirements, the income effect is still visible, as the bank under capital requirement lowers its loan investment from (from $2.70$ to $2.54$), as well as its indebtedness. As in the previous case, both enterprise value and social values are reduced and government value increases, although to a lower extent than in the previous case.

Of some interest is the case when the bank under capital restriction is subject simultaneously to a higher taxation ($\tau^+ = 20\%$ and $\tau^- = 7.5\%$), and a higher capital ratio ($k = 0.12$). The reduction in loan investment and short term financing is significant but less than proportional. As for the bank subject to both capital and liquidity restrictions, the above increase, together with an increase in the liquidity ratio to $\ell = 1.2$, produces effects similar to the ones due to an increase of the liquidity ratio only. This confirms that the liquidity requirement is the most demanding of the regulatory restrictions.

Summing up, an increase in corporate income taxes results in a reduction in lending and indebtedness due to income effects, and an attendant reduction in bank efficiency and social value. Interestingly, when the bank is subject to both tightening in capital regulation and
higher income taxation, the effects of taxation are dampened, and are almost nullified when the bank is also subject to an increase in liquidity requirements. Finally, in all cases considered, government value increases as a result of an increase in tax receipts.

C. Taxation of bank liabilities

As noted, in the aftermath of the recent financial crisis a variety of additional taxes on financial institutions have been proposed or enacted.\textsuperscript{19} The justifications of these taxes are several: to address the budgetary costs of the crisis (ex-post), to create resolution funds to address future distress (ex-ante), to better align bank managers’ incentives to target levels of bank risks, and to even control systemic risk in the banking system.\textsuperscript{20} The latter are typically addressed with corrective Pigouvian taxation, and current proposals of systemic risk levies are designed to mimic such Pigouvian levies (see, for example, Acharya and Richardson (2009), Perotti and Suarez (2009), and for a critical evaluation see Shackelford, Shaviro, and Slemrod (2010)).

Particular emphasis has been placed on taxes on bank liabilities, with some stressing the role of these taxes as a potential complementary tool for prudential regulation. On the one hand, Shin (2010) has argued that a well designed taxation of so-called “non-core” bank liabilities (excluding deposits and other stable sources of funding) would be easier to implement than other regulations, such as time varying capital requirements or expected loss provisioning, and would be more effective in dampening pro-cyclicality. On the other hand, the IMF advocated a Financial Activities Tax that is a flat-rate tax imposed on total bank liabilities, on the ground that total liabilities are those that would need to be supported should a resolution of distress arise.\textsuperscript{21}

Here we consider three simple liability taxation schemes that mimic taxes either already in place in some countries, or that are part of some current proposals. The first one imposes a flat tax rate $\tau_B$ on banks’ uninsured liabilities. The tax revenue is therefore $\tau_B \min\{0, B\}$. The second one imposes a flat-tax rate $\tau_D$ on insured deposits. In this case the tax rate can

\textsuperscript{19}See \textit{Financial Sector Taxation}, International Monetary Fund, Washington D.C., September 2010.
\textsuperscript{20}The negative externalities arising from collective bank failures resulting in heightened systemic risk have been typically associated indirectly or directly with environmental externalities.
\textsuperscript{21}These proposals also mention the possibility of further designing taxation of bank liabilities by imposing different tax rates either based on banks’ different risk profiles, or bank size, or both, in manners resembling risk-based deposit insurance charges. On the problems associated with implementing risk-based deposit insurance owing to asymmetric information, see Chan, Greenbaum, and Thakor (1992)
be also interpreted as a flat-rate deposit insurance premium, and the tax revenue is \( \tau_D D \). The third one combines the two schemes imposing the same tax rate on both insured and uninsured liabilities, i.e. \( \tau_B = \tau_D \). In all cases, we assume that these taxes are deductible from earnings for income taxation purposes.

Table VII reports the impact on bank optimal choices and relevant metrics of these taxes for a bank subject to capital regulation, and one subject to both capital regulation and liquidity requirements. We set \( \tau_B = \tau_D = 0.0075 \), which is a value in the low range of taxes currently in place or currently proposed.\(^{22}\)

Consider first a bank subject to capital regulation. A tax on uninsured liabilities results in a substantial decline in lending (from $4.80 to $3.94) and indebtedness (from $-2.29 to $-1.48), owing to the combination of a negative income effect, and a substitution effect due to an increase in the cost of uninsured debt relative to deposits. The substitution effect prompts a reduction of loans to support a lower level of debt, since deposits are exogenous. Both enterprise and social values decrease relative to the base case, indicating non-trivial efficiency and social costs associated with this form of taxation. Importantly, government value slightly declines (from $0.48 to $0.41), suggesting that the total tax revenue may not increase because of the reduction in the tax base implied by a relatively strong substitution effect.

By contrast, a tax on insured liabilities (deposits) induces an increase in loans and debt as the substitution effect works in the opposite direction of the previous case, although such increase is of negligible magnitude for our parameter configurations. Yet, both enterprise and social values decline, although the magnitude of such decline is smaller than the decline implied by the tax on uninsured liabilities. On the other hand, government value remains basically unchanged. Finally, under both taxes, the changes in loans and debt induces by the tax on uninsured liabilities seem to dominate, as loans and debt remain at levels set under such taxation. However, enterprise and social values further decline relative to the cases of imposition of each of the taxes individually.

When we consider a bank subject to both capital regulation and liquidity requirements, we obtain essentially the same qualitative results we have described for the case of capital regulation only. The key difference in this case is that all changes in policies and metrics are

\(^{22}\)See the relevant tables in *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010.
significantly smaller than those witnessed when capital regulation is in place. In other words, the strength of the liquidity constraint is such that the cumulative effects of these taxes are partly “neutralized”.

In sum, our model yields three main results concerning the impact of liability taxation. First, taxes on uninsured liabilities have a significant negative impact on lending, while the impact on lending of taxes on insured liabilities (deposits) goes in the opposite direction, but is quantitatively negligible under our parameterization. Second, under all three taxation schemes we have considered, bank efficiency and social values decline and, importantly, and differing from the effects of increases in corporate income taxation, total tax receipts decline owing to substitution effects. Third, the quantitative effects of these taxes are substantially reduced under both capital regulation and liquidity requirements, similarly to what we have seen for corporate income taxation.

VII. Conclusions

This paper has formulated a dynamic model of a bank exposed to credit risk, liquidity risk, and financial distress, and gauged the joint impact of capital regulation, liquidity requirements and taxation on banks’ optimal policies and metrics of efficiency of intermediation and social value.

The inverted U-shaped relationship between bank lending, bank efficiency, social value and regulatory capital ratios that we have uncovered suggests the existence of optimal levels of regulatory capital. In a policy perspective, this result implies that the determination of “efficient” regulatory capital ratios is generally more complicated than usually thought, as such levels are likely to be highly bank-specific, depending crucially on the configuration of risks a bank is exposed to. Similarly, our results on the high costs of liquidity requirements point out the consequences of the repression of the key maturity transformation role of bank intermediation.

On taxation, we highlighted the implications of basic trade-off between income and substitution effects. For the purpose of rising tax revenues, corporate income taxation seems to be preferable to taxes on bank liabilities, since the associated substitution effects of the latter
may imply at the same time higher bank efficiency and social costs and lower tax receipts. This is an under-researched area where more could be gained by the explicit modeling of the trade-offs of different tax schemes in the context of bank regulation.

Overall, our results suggest that prudence should be exercised in implementing non-trivial changes in capital and liquidity requirements, both at an individual and “systemic” level, before having gained a clear grasp of the associated private and social costs: these costs could be significantly larger than originally thought.

Under a modeling perspective, a key issue is the extent to which a bank can choose different risk profiles. Our model does not capture this feature by design, as a bank is characterized by “exogenous” sources of credit and liquidity risk. Similarly, the liability structure of our bank is highly stylized, as we do not allow for either risky debt or longer maturity debt. Extending the model along these dimensions is already part of our research agenda.
References


A. Algorithm

The solution of the Bellman equation in (13) is obtained numerically by a value iteration algorithm. The valuation model for bank’s equity is a continuous–decision and infinite–horizon Markov Decision Processes. The solution method is based on successive approximations of the fixed point solution of the Bellman equation. Numerically, we apply this method to an approximate discrete state-space and discrete decision valuation operator.\textsuperscript{23}

Given the dynamics of $Z_t$ in (11) and of $\log D_t$ in (12), using a vector notation $\xi(t) = (Z(t), \log D(t))$, we have a VAR of the form

$$\xi(t) = c + \mathcal{K} \xi(t - 1) + \varepsilon(t) \quad (20)$$

where $\varepsilon = (\varepsilon^Z, \varepsilon^D)$ is a bivariate Normal variate with zero mean and covariance matrix $\Sigma$,

$$c = \begin{pmatrix} (1 - \kappa_1)\xi_1 \\ (1 - \kappa_2)\xi_2 \end{pmatrix}, \quad \mathcal{K} = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix},$$

where, differently from the approach proposed in Tauchen (1986), $\Sigma$ may be non–diagonal and singular (when $|\rho| = 1$).

Given this alternative assumption for the error covariance matrix, we follow an efficient approach first proposed by Knotek and Terry (2008) based on numerical integration of the multivariate Normal distribution. In particular, the continuous–state process $\xi(t)$ is approximated by a discrete–state process $\hat{\xi}(t)$, which varies on the finite grid $\{X_1, X_2, \ldots, X_n\} \subset \mathbb{R}^2$. Define a partition of $\mathbb{R}^2$ made of $n$ non–overlapping 2–dimensional intervals $\{X'_1, X'_2, \ldots, X'_n\}$ such that $X_i \in X'_i$ for all $i = 1, \ldots, n$ and $\bigcup_{i=1}^n X'_i = \mathbb{R}^2$. The $n \times n$ transition matrix $\Pi$ is defined as

$$\Pi_{i,j} = \text{Prob} \left\{ \hat{\xi}(t + 1) \in X'_j \mid \hat{\xi}(t) = X'_i \right\}$$

$$= \text{Prob} \{ c + \mathcal{K} \xi(t) + \varepsilon(t + 1) \in X'_j \}$$

$$= \text{Prob} \{ \varepsilon(t + 1) \in X'_j \} = \int_{X'_j} \phi(\xi, 0, \Sigma) d\xi$$

\textsuperscript{23}See Rust (1996) or Burnside (1999) for a survey on numerical methods for continuous decision infinite horizon Markov Decision Processes.
where \( X_j' = X_j - c - K \xi(t) \), and \( \phi(\xi, 0, \Sigma) \) is the density of the bivariate Normal with mean zero and covariance matrix \( \Sigma \). The integral on the last line is computed numerically by Monte–Carlo integration. As for the choice of the grid, we follow Tauchen (1986) using a uniformly spaced scheme, restricting the support of each state variable within three times the standard deviation of each marginal distribution around the long–term average.

The feasible interval for loans, \([0, L_u]\), and for the face value of bonds, \([B_d, B_u]\) (with \(B_d < 0 < B_u\)), is set so that they are never binding for the equity maximizing program. We discretize \([L_d, L_u]\), to obtain a grid of \(N_L\) points

\[
\tilde{L} = \{ \tilde{L}_j = L_u(1 - \delta)^j \mid j = 1, \ldots, N_L \} \cup \{L_{N_L} = 0\}
\]

such that, if the bank choose inaction, the level of loan is what remains after the portion \(\delta L\) has been repaid. The interval \([B_d, B_u]\) is discretized into \(N_B\) equally–spaced values, making up the set \(\tilde{B}\). To keep the notation simple, we denote \(x = (\xi, L, B)\) the generic element of the discretized state.

We solve the problem

\[
E(x) = \max \left\{ 0, \max_{(L', B') \in A(D)} \left\{ e(x, L', B') + \beta \mathbb{E} \left[ E(x') \right] \right\} \right\},
\]

where function \(e(x, L, B)\), is defined in equation (10), and \(A(D)\) is the case specific feasible set defined differently for the unregulated and the regulated case, in all points of the discrete state space. The solution is found by successive approximations, starting from a guess function \(E_0(\cdot)\), putting it on the right–hand–side of the Bellman equation obtaining \(E_1(\cdot)\) and than by iterating on the same procedure obtaining the sequence \(\{E_n(\cdot), n = 0, 1, \ldots\}\). The procedure is terminated when the error \(\|E_{j+1} - E_j\|\) is lower than the desired tolerance.

For the set of parameters in Table I, we use \(L_u = 10, B_d = -5\) and \(B_u = 5\). Given the properties of the quadrature scheme, we solve the model using only 9 points for \(Z\), 9 points for \(D_t\). However, we need to allow for many more points when discretizing the control variables, so we choose \(N_L = 27\), and \(N_B = 27\). The tolerance for termination of the iterative procedure is set at \(10^{-4}\).
Given the optimal solution, we can determine the optimal policy and the transition function $\varphi(x)$ in (15) based on the arg-max of equity value at the discrete states $x$. We use Monte Carlo simulation to generate a sample of $\Omega$ possible future paths (or scenarios) for the bank. In particular, we obtain the simulated dynamics of the state variable $\xi = (Z, D)$ by application of the recursive formula in (20), starting from $Z(0) = \mathcal{Z}$ and $D(0) = \mathcal{D}$. Then, setting a feasible initial choice $L(0) = 0$ and $B(0)\mathcal{D}$ (so that the initial bank capital is null), we apply the transition function $\varphi$ along each simulated path in a recursive fashion. If a bank defaults at a given step, then the current depositors receive the full value of their claim, while the deposit insurance agency pays the bankruptcy cost. Afterwards, a seed deposit $D_d$ is invested in the bank, which is immediately invested in bonds, $B = D_d$, while $L = 0$. Then the “new” bank follows on the same path by applying the optimal policy. In our numerical experiments, we generate simulated samples with $\Omega = 10,000$ paths and $T = 100$ years (steps). To limit the dependence of our results on the initial conditions, we drop the first 50 steps.
Figure 1: **Bank’s dynamic.** Evolution of the state variables (credit shock, $Z$, and deposits, $D$) and of the bank’s control variables (cash and liquid investments, $B$, and loans, $L$) assuming the bank is solvent at each date.
Figure 2: **Comparison of constraints.** This figure presents the three feasible regions of \((L, B)\) defined by the collateral constraint, \(\Gamma(D)\) in Equation (8), the capital requirement, \(\Theta(D)\) from (17), and by the liquidity requirement, \(\Lambda(D)\) from Equation (19). The plot is based on the parameter values in Table I, for a current \(D = 2\).
\[ \begin{array}{ll}
\kappa_Z & \text{annual persistence of the credit shock} \\
\sigma_Z & \text{annual volatility of the credit shock} \\
\bar{Z} & \text{long term value of the credit shock} \\
\kappa_D & \text{annual persistence of the log of deposit} \\
\sigma_D & \text{annual volatility of the log of deposit} \\
\bar{D} & \text{long term value of deposit} \\
\rho & \text{correlation between log–deposit and credit shock} \\
\beta & \text{annual discount factor} \\
\tau^+ & \text{corporate tax rate for positive earnings} \\
\tau^- & \text{corporate tax rate for negative earnings} \\
\delta & \text{annual percentage of reimbursed loan} \\
\gamma & \text{bankruptcy costs} \\
\lambda & \text{flotation cost for equity} \\
\alpha & \text{return to scale for loan investment} \\
m^+ & \text{unit price for loan investment} \\
m^- & \text{unit price for loan fire sales} \\
k & \text{percentage of loans for capital regulation} \\
\ell & \text{liquidity coverage ratio} \\
\tau_B & \text{tax rate on uninsured liabilities} \\
\tau_D & \text{tax rate on insured liabilities} \\
\end{array} \]

Table I: Model parameters
Figure 3: Bank’s policy. This figure illustrates the impact of regulatory restrictions on the bank’s policy related to loan investment and to short term investment and financing, for the non–regulated case, and for the cases with capital constraint, with a liquidity constraint, and with both capital and liquidity constraints. The short term investment/financing policy is given by the optimal $B^*$ given the current state, averaged across all possible $Z$ in the left panel and averaged across all possible $D'$ in the right panel. The loan investment policy is represented by the ratio $L^*/L$ given the current state and averaged across $Z$ in the left panel and across $D'$ in the right panel. These values are plotted against the liquidity shock, $D' - D$, in the left panels and the credit shock, $Z$, on loans in the right panels, and are obtained assuming that the bank is currently at the steady–term state (so that the credit shock is 0.085, and the deposits from the previous date are $D = $2, respectively), while $B = $0, and $L = $4.1.

The values are from the numerical solution of the model using 9 points for $Z$, 9 points for $D$, 27 points for $L$, and 36 points for $C$, based on the parameter values in Table I.
Table II: Capital ratio. Average capital ratio (i.e., bank capital over loans, or \((L^* + B^* - D')/L^*\), where \((L^*, B^*)\) is the optimal solution for a solvent bank and \(D'\) is the new possible level of deposits) as a function of the credit shock, \(Z\), at different levels of current bank capital, \(K = L + B - D\). The average is computed over the different possible levels of \(D'\). The average capital ratio is computed at \(B = 0\). The current level of \(L\) is 4.1. The different levels of \(K\) are obtained by changing \(D\). These results are based on the numerical solution of the valuation problem in (13) with the parameters in Table I.

<table>
<thead>
<tr>
<th>(Z)</th>
<th>0.039</th>
<th>0.054</th>
<th>0.069</th>
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Table III: Liquidity coverage ratio. Average liquidity coverage ratio (i.e., end-of-period total cash available in the worst case scenario over the end-of-period net cash outflows due to a variation in deposits, or \( \delta L^* + \pi (L^*)Z_d - \tau(y_{min}) + B^*(1 + r)\)\(/(D'(1 + r) - D_d) \), where \((L^*, B^*)\) is the optimal solution for a solvent bank and \(D'\) is the new possible level of deposits) as a function of the credit shock, \(Z\), at different levels of current bank capital, \(K = L + B - D\). The average is computed over the different possible levels of \(D'\). The average capital ratio is computed at \(B = 0\). The current level of \(L\) is 41. The different levels of \(K\) are obtained by changing \(D\). These results are based on the numerical solution of the valuation problem in (13) with the parameters in Table I.
Figure 4: Value loss associated with regulatory restrictions. This figure illustrates the impact of regulatory restrictions by comparing the value of equity, the enterprise value (i.e., market value of deposits plus market value of equity net of cash balance, or plus short term debt), and the social value (i.e., Enterprise value of the bank plus the value of taxes) of the bank, for the case with a capital constraint, with a liquidity constraint, and with both capital and liquidity constraints as a proportion of the value from the non-regulated case. These values are plotted against the shock on deposits, $D'_e - D$, in the left panels and the credit shock, $Z$, on loans in the right panels, and are obtained assuming that the bank is currently at the steady-state (so that the credit shock is 0.085, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.1m$ so that bank capital is $K = 2.1$. The values are from the numerical solution of the model using 9 points for $Z$, 9 points for $D$, 27 points for $L$, and 36 points for $C$, based on the parameter values in Table I.
Table IV: The impact of bank regulation. These are obtained from the solution of the bank valuation problem using the parameters in Table I for 10,000 random paths of 50 periods (years) length. Capital Ratio is the Bank's Capital over the principal of Loans \((K/L)\). Book Leverage is the book value of Deposits plus the short term financing over the book value of Loans plus the short term investment in bonds \(((D - \min\{B, 0\})/(L + \max\{B, 0\}))\). Market Leverage is the book value of Deposits plus the short term financing over the quasi–market value of the assets \(((D - \min\{B, 0\})/(D - \min\{B, 0\} + E))\).

The results are presented for the unregulated case, the case with capital ratio restrictions, the case with liquidity restrictions, and the case with both capital and liquidity restrictions.

Table V: Increases in capital and liquidity requirements. Obtained from the solution of the bank valuation problem using the parameters in Table I for 10,000 random paths of 50 periods (years) length. The table shows, respectively, from left to right, the cases of the bank with capital requirement, and the bank subject to both capital and liquidity restrictions. The table presents the average values (computed on non–defaulted banks) of the different metrics. The columns represent different choices of parameters: the column denoted “base”, is the base case, with the parameters in Table I. The others are obtained by changing only the parameter(s) we use to denominate the column: in “\(k\)” we set the capital ratio to 0.12; in “\(\ell\)” is with \(\ell = 1.2\).
Table VI: Increases in corporate income taxes. Obtained from the solution of the bank valuation problem using the parameters in Table I for 10,000 random paths of 50 periods (years) length. The table shows, respectively, from left to right the bank with capital requirement, and the bank subject to both capital and liquidity restrictions. The table presents the average values (computed on non–defaulted banks) of the different metrics. The columns represent different choices of parameters: the column denoted “base”, is the base case, with the parameters in Table I. The others are obtained by changing only the parameter(s) we use to denominate the column: “τ” is with $\tau^+ = 20\%$ and $\tau^- = 7.5\%$; in “$k$” we set the capital ratio to 0.12; “ℓ” is with $\ell = 1.2$; and in “all” we set $\tau^+ = 20\%$, $\tau^- = 7.5\%$, $k = 0.12$, and $\ell = 1.2$.

Table VII: The impact of taxation of liabilities. Obtained from the solution of the bank valuation problem using the base case parameters, for 10,000 random paths of 50 periods (years) length. The table presents the average values (computed on non–defaulted banks) of the different metrics. The table presents three different choices of parameters for taxation of liabilities. The first has $\tau_B = 0.0075$ and $\tau_D = 0$; the second has $\tau_B = 0$ and $\tau_D = 0.0075$; the third has $\tau_B = 0.0075$ and $\tau_D = 0.0075$. For each choice, we consider the cases with capital requirement, “Cap.”, and with capital and liquidity requirements, “C&L”. These cases are compared with the base.