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Published in:
The American Economic Review

Publication date:
1965

Link to publication

Citation for published version (APA):

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Download date: 01. Dec. 2019
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Published by: American Economic Association
Stable URL: http://www.jstor.org/stable/1814555
BIASED EFFICIENCY GROWTH AND CAPITAL-LABOR SUBSTITUTION IN THE U.S., 1899–1960

By Paul A. David and Th. van de Klundert*

It is by now generally conceded that the presence of a large residual element in the growth of aggregate output, an element that is not accounted for by the growth of inputs of capital and labor measured in a more or less conventional manner, is aesthetically unsatisfying in explanations of the supply side of economic growth. Moreover, the simple labeling of that residual element as the consequence of “technical progress,” or an equivalently broad and imperfectly understood phenomenon, does not prove practically helpful in guiding decisions about policies aimed at influencing the aggregate growth rate. So long as the residual is no more than “a measure of our ignorance,” a substantial portion of the observed rate of growth of output presents no handles for control.

In this situation it is hardly surprising that the notion that there is some sense in which “inputs” just equal output should acquire strong appeal. Indeed, this has been the line taken by much of the recent interesting work with aggregate production functions. The “residual” has been treated as the consequence of the mismeasurement of the inputs; conventional measures of inputs of labor and capital are regarded as inadequate because they fail to reflect alterations in the economic quality of physical units of the factors of production. As this approach is currently being pursued, the object of the game is to make the offending residual disappear by contriving new (and more appropriate) measures of the growth of labor and capital inputs which will, between them, fully account for the observed growth of output. (Cf. e.g., Denison [7], Domar [8], Griliches [12].) It leads to the “embodying” of “technical change” in capital inputs (cf., e.g., Solow [32] [34]), on the one hand, and, on the other, to the “embodying” of ostensibly superior technical

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knowledge and skill in the labor force through the agency of education.\(^1\)

Since everyone can make his own inputs and the only restriction imposed on a player is that he shall not personally overexhaust the growth of output, this is generally considered to be a particularly good game; it affords considerable opportunity for the exercise of ingenuity and offers a wide variety of results, each of which is at least internally quite consistent.

While it might be appropriate at some levels of discussion to raise objections to specific devices employed in carrying through the embodying operations, and to attempt to suggest refinements, it does seem more important to ask first how useful it is to continue playing the game under the present rules. Granting that conventional measures of inputs may be inappropriate because they fail to capture quality changes or, to put it differently, because one suspects that a broad array of secular developments has had "input-augmenting" effects, would it not be preferable to begin by establishing the magnitude of those effects before attempting to identify them with specific secular developments? In other words, would it not be sensible to start by trying to identify the form which the growth of conventional input efficiency has taken and then proceed to tackle the intriguing, but quite distinct, question of the sources of such growth?

As it is now, the typical modus operandi implicitly involves an effort to dispose of both issues by a single stroke. The generalized embodiment approach attempts to account for the growth of the residual by hypothesizing that various factors, such as technological advance in the design of capital, increased formal education, shortened working hours, have been responsible for the augmentation of one or another of the conventional inputs. The nature and magnitude of factor augmentation derived from each source considered is tacitly specified in the course of adjusting the conventional measures of labor and capital inputs. For example, having postulated that the spread and lengthening of formal education has augmented labor (and not capital), one might follow Denison [7] in correcting a man-hour input series so that it reflects the movements of an index of the (weighted) educational level of the work force. Although a check on the procedure would be afforded by comparison of the results of such adjustments for change in labor quality with the actual increase occurring in the efficiency of man-hour inputs, the empirical question of the form taken by total conventional factor-efficiency growth is suppressed; ultimate vindication of the hypotheses underlying the refashioned input measures is left to turn on their performance, as a combined package, in explaining the growth of output.

\(^1\) Richard Nelson [26] has given neat expression to the formal symmetry between improvements in the quality of the capital stock, as a consequence of technical advances embodied in the design of new capital and alterations in the age distribution of the capital stock, and improvements in the quality of the labor force.
So long as there is no way of telling how increases in conventional total factor efficiency actually have been distributed between labor-augmentation and capital-augmentation, the present approach seems reasonable. But, as shall be seen, it is possible to infer the rate of labor-augmentation and the rate of capital-augmentation from conventional measures of inputs and output. Once such information is available, it may be used to place prior (not a priori) restrictions upon attempts at empirical identification of the sources of factor-efficiency growth. This would, at the very least, have the virtue of stabilizing the distribution of the residual between labor-associated and capital-associated improvements in factor efficiency and so prevent the disquieting pronounced shifts in the imputation of the residual to various causes proposed by each new statistical study.

With this providing one source of motivation, the present paper ventures an initial investigation of aggregate production-function relationships allowing for the possibility that the growth of the efficiency of conventional inputs may be “nonneutral,” in the sense that the marginal productivities of those inputs do not increase at the same rate through time. Factor-efficiency growth so conceived may be nonneutral because technical innovations have a labor-augmenting or a capital-augmenting bias, or because unmeasured quality improvements in one of the inputs have taken place with relative rapidity. For the moment, however, the reasons will have to remain a subsidiary concern, and we shall, for the sake of simplicity, adopt the convention of regarding labor and capital inputs as being augmented by “technical change.” Such contribution to the understanding of aggregate productivity growth as we can hope to make here will be confined simply to establishing the form taken by factor-efficiency growth in the private domestic sector of the U.S. economy during the present century, rather than identifying the sources from which it has flowed.

Of course, the implications of the form in which technical change has occurred extend beyond the sphere of current preoccupation with the sources of conventional productivity increase. In the context of the analysis of growth models the nature of technical change carries significance for the existence of an equilibrium growth path (cf. Solow [30], Uzawa [37], Amano [2]), while for students of the inventive and innovative process the question of the historical bias of technological advances has formed a subject of no little concern. (Cf., e.g., [25], especially paper by W. Fellner; Habakkuk [14].) Thus, an answer to the question of whether there has been any bias in the direction of technical change, and a measurement of such bias as has been experienced in the United States since the beginning of the twentieth century, are of interest in their own right.

Yet another matter of interest falls within the purview of this
paper, in part because it is one upon which the question of the neutrality of technical change has some bearing. Much of the research into productivity growth has been concerned with the algebraic form of the production function, especially with the magnitude of the elasticity of substitution between labor and capital.\(^2\) In addition, substitution possibilities are an important determinant of the properties of dynamic two-factor models and are of significance in the operation of the price mechanism in free-market economies.\(^3\) The assumption that technical change is Hicks-neutral (leaving relative marginal productivities of the factors undisturbed) is usually explicitly invoked by studies undertaking the estimation of the elasticity of substitution from time-series data.\(^4\) While this assumption is not required for proper estimation of the substitution parameter, some estimation methods that have been used (cf., e.g., Kendrick and Sato [19], Kravis [21]) do demand Hicks-neutrality and are, consequently, misleading when technical change happens to be biased towards saving either labor or capital. The present study makes use of a general aggregate production function of the now familiar constant elasticity of substitution (CES) form, doing so in a manner that permits estimation of the substitution parameter while allowing for the possibility of nonneutral technical change.

The principal conclusions reached via this route may be anticipated briefly here. It is found that during the twentieth century technical change in the U.S. Private Domestic Economy has not been Hicks-neutral, nor, for that matter, has it been Harrod-neutral; although both labor-augmentation and capital-augmentation have been going on since 1900, these changes have been biased in a labor-saving direction. It is estimated that over this period capital-augmentation has proceeded at the rate of approximately 1.5 per cent per annum, while the annual rate of labor-augmentation has exceeded that by roughly 0.7 of a percentage point. Initial abandonment of the assumption that technical change was Hicks-neutral also leads to an estimated long-run elasticity of substitution in the neighborhood of 0.32, a value that casts very serious doubt

\(^1\) This question may be regarded as one which involves the determination of the appropriate form for indexes of total factor input employed in the construction of measures of output per unit of total input. It is, then, in principle closely connected with the broad issue of the measurement and explanation of conventional total productivity change. As a practical matter, however, the form of the production function does not appear to make a great deal of difference in the calculation of total productivity indexes; within the range of variation of estimates that have been secured for the elasticity of substitution, productivity indexes are found to be rather insensitive to corresponding variations in the weighting schemes used to combine labor and capital inputs. Cf., e.g., Nelson [26, pp. 577–78].

\(^2\) On two-factor growth models, cf., Solow [30], Eitsner [9], Pitchford [29]. For other implications of the curvature of production-function isoquants, cf. ACMS [3].

\(^4\) Hicks-neutrality is also commonly assumed for the purpose of estimating the rate of shift of the aggregate production function through time, even when the function is specified as being of the Cobb-Douglas form.
on the appropriateness of the (unitary elasticity of substitution) Cobb-Douglas form for aggregate production function analyses of the U.S. economy. The latter finding obviously heightens the attractiveness of the unrestricted input-augmentation concept adopted here, for if the production function were actually of the Cobb-Douglas form, Hicks-neutral technical change would not be less general than input-augmentation.5

I. An Aggregate Production Function with Labor- and Capital-Augmenting Technical Change

We begin by assuming that the aggregate production function is homogeneous of the first degree in inputs of capital and labor measured in efficiency terms, rather than in conventional units, and that it is characterized by constant elasticity of substitution between the “inputs” thus defined.

\[ V = \left[ (E_L L)^{-\rho} + (E_K K)^{-\rho} \right]^{-1/\rho} \]

fulfills these requirements, and the first and second derivatives of the volume of output, \( V \), with respect to the inputs of labor and capital services (\( E_L L \) and \( E_K K \), respectively) obey all the normal conditions for a production function. In this notation, \( L \) and \( K \) represent conventional measures of the physical flow of labor and capital inputs, although in using the model we shall follow the common procedure of taking a conventional measure of the capital stock as a proxy for the flow of constant efficiency services it renders.6 The coefficients \( E_L \) and \( E_K \) then represent the levels of efficiency of the conventional inputs of labor (measured as man-hours employed) and capital (measured, in principle, in terms of machines of a constant kind).7 Alterations in \( E_L \) and \( E_K \) through time are to be interpreted as labor-augmenting and capital-augmenting “technical changes,” although this says nothing about the sources of such efficiency growth. An increase in \( E_L \) is designated as labor-augmenting change, or labor-associated efficiency growth, despite the fact that it may be a consequence of the introduction of better machines.

The remaining parameter in the production function, \( \rho \), is related to \( \sigma \),
the elasticity of substitution; \( \rho = (1 - \sigma) / \sigma \), as is shown by Arrow, Chenery, Minhas, and Solow (ACMS) [3]. Since the elasticity parameter itself must be positive, the function given in equation (1) is only defined for \( \rho \geq -1 \). We assume, as has already been stated, that the substitution parameter, and hence \( \rho \), is constant through time.8

The concept of factor-augmenting technical changes defined in this model (and the corollary notion of biases in the direction taken by technical change toward either relative labor-augmentation or relative capital-augmentation) can be related to the more familiar Hicksian concepts of neutral, labor-saving, and capital-saving technological progress. Following Hicks's [15, p. 122] definitions, “inventions” are to be classified as labor-saving, neutral, or capital-saving according to whether—given a constant capital-labor ratio—they lower the marginal productivity of labor relative to the marginal productivity of capital, leave the relative marginal productivities unaltered, or raise the marginal productivity of labor relative to that of capital. The equality of the rates of growth of labor and capital efficiency in the present model is exactly equivalent to neutrality in Hicks's sense. However, what we designate as a labor-augmenting bias in technical change \((\dot{E}_L/E_L > \dot{E}_K/E_K)\) amounts to the same thing as a labor-saving innovation if, and only if, the elasticity of substitution is less than unity. Similarly, for a capital-augmenting bias in technical change to satisfy Hicks's definition of capital-saving, the elasticity of substitution must be less than unity.9

8 Cf. Brown and De Cani [5] [6], for an attempt to allow for variations in the elasticity of substitution by the contrivance of defining “technological epochs.”

9 Cf. Solow [31]. To derive the foregoing set of equivalences, (1) may first be differentiated partially with respect to \( L \) and \( K \), which yields:

\[ m_L = \frac{\partial V}{\partial L} = E_L \left( \frac{V}{L} \right)^{1+\rho} \]

and

\[ m_K = \frac{\partial V}{\partial K} = E_K \left( \frac{V}{K} \right)^{1+\rho} \]

Now, differentiating (1), (2), and (3) with respect to time, we obtain

\[ \frac{\dot{V}}{V} = \alpha \left( \frac{L}{E_L} + \frac{\dot{E}_L}{E_L} \right) + \beta \left( \frac{K}{E_K} + \frac{\dot{E}_K}{E_K} \right) \]

where \( \alpha \) and \( \beta \) represent the elasticities of output, \( V \), with respect to \( L \) and \( K \), respectively. We also have

\[ \frac{m_L}{m_L} = \frac{1}{\sigma} \left( \frac{\dot{V}}{V} - \frac{\dot{L}}{L} \right) + \frac{\sigma - 1}{\sigma} \frac{\dot{E}_L}{E_L} \]

and
Establishing these correspondent relationships not only serves to connect the concept of factor-augmentation with older and more familiar approaches to the problem of bias in technical change, it also has the virtue of simplifying our terminology. In anticipation of the finding that the elasticity of substitution is less than unity, we shall feel free to speak of technical change as labor-saving where $\frac{\dot{E}_L}{E_K} > \frac{\dot{E}_K}{E_K}$, and as capital-saving where the reverse is the case.

In order to utilize the production model which has been set forth here in empirical work, we assume perfect competition in all markets and, further, hypothesize that the observable relationships among $V, L,$ and $K$ can be regarded as the result of profit maximization subject to the constraint of the function given by (1). With this justification, the real wage, $w$, may be taken as equal to the marginal product of labor, $m_L$; the real rate of return on capital, $r$, can be set equal to $m_K$; and in place of the elasticity of output with respect to $L$, $(\alpha)$, and the elasticity of output with respect to $K$, $(\beta)$, we may write the share of total output received by labor $\pi_L$ and the share going to capital $\pi_K$, respectively. Dividing equation (2) by equation (3)—these equations are given in footnote 9—and making the appropriate substitutions for the marginal product terms, leads to the expression for the capital-labor ratio,

$$\frac{\dot{m}_K}{m_K} = \frac{1}{\sigma} \left( \frac{\dot{V}}{V} - \frac{\dot{K}}{K} \right) + \frac{\sigma - 1}{\sigma} \frac{\dot{E}_K}{E_K}.$$

Substituting from expression (1a) into (2a) and (3a) leads to the following relationships:

$$\frac{\dot{m}_L}{m_L} = \frac{\beta}{\sigma} \left( \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \right) + \frac{1}{\sigma} \frac{\dot{E}}{E} + \frac{\sigma - 1}{\sigma} \frac{\dot{E}_L}{E_L};$$

$$\frac{\dot{m}_K}{m_K} = -\frac{\alpha}{\sigma} \left( \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \right) + \frac{1}{\sigma} \frac{\dot{E}}{E} + \frac{\sigma - 1}{\sigma} \frac{\dot{E}_K}{E_K},$$

where the total efficiency growth rate, $\dot{E}/E$, is defined as

$$\frac{\dot{E}}{E} = \alpha \frac{\dot{E}_L}{E_L} + \beta \frac{\dot{E}_K}{E_K}.$$

From the last two equations it can readily be seen that if $\sigma < 1$, the Hicksian definition of a labor-saving change in technology,

$$\left( \frac{\dot{m}_L}{m_L} - \frac{\dot{m}_K}{m_K} \right) = \frac{\sigma - 1}{\sigma} \left( \frac{\dot{E}_L}{E_L} - \frac{\dot{E}_K}{E_K} \right) < 0,$$

is satisfied when $\dot{E}_L/E_L > \dot{E}_K/E_K$, and the Hicksian definition of a capital-saving change is satisfied when $\dot{E}_L/E_L < \dot{E}_K/E_K$. The absence of any difference in the rates of labor and capital augmentation $\dot{E}_L/E_L = \dot{E}_K/E_K$ meets the requirement for Hicks-neutrality $\dot{m}_L/m_L = \pi/m_K$ with all nonnegative values of $\sigma$. 
(4) \[
\frac{K}{L} = \left( \frac{w}{r} \right)^\sigma \left( \frac{E_L}{E_K} \right)^{1-\sigma},
\]
which, upon differentiation with respect to time, yields
(4a) \[
\frac{K}{K} - \frac{L}{L} = \sigma \left( \frac{\dot{w}}{w} - \frac{\dot{r}}{r} \right) + (1 - \sigma) \left( \frac{E_L}{E_L} - \frac{E_K}{E_K} \right),
\]
or, simply rearranging the terms,
(4b) \[
\frac{\dot{w}}{w} - \frac{\dot{r}}{r} = \frac{1}{\sigma} \left( \frac{K}{K} - \frac{L}{L} \right) - \left( \frac{1 - \sigma}{\sigma} \right) \left( \frac{E_L}{E_L} - \frac{E_K}{E_K} \right).
\]

Equation (4b) suggests two reasons for alterations in relative rates of factor remuneration: should capital services become more abundant than labor services, either because physical capital of a constant kind increases more rapidly than man-hour inputs or because capital-saving technical change renders labor services measured in efficiency terms comparatively less abundant, relative wage rates (per man-hour) will tend to rise. In contrast to this macroeconomic view, the form of equation (4a) suggests an essentially microeconomic explanation of the connections among these variables: increases in the wage rate relative to the return on capital induce substitution for labor, producing a rise in the capital-labor ratio expressed in conventional terms, while technical change of a labor-saving sort has precisely the same influence.

It will readily be seen that by multiplying both sides of equation (4) by \((L/K)^\sigma\), we obtain an expression relating the capital-labor ratio to the relative factor shares \((\pi_L/\pi_K)\) and the ratio of the efficiency levels of labor and capital:
(5) \[
\frac{K}{L} = \left( \frac{\pi_L}{\pi_K} \right)^{\sigma/(1-\sigma)} \left( \frac{E_L}{E_K} \right).
\]

Now, if it is assumed that any changes in the relative efficiency level of labor which take place over the course of time do so at a constant geometric rate \((\lambda_L - \lambda_K)\) given by
(6) \[
\left( \frac{E_L}{E_K} \right)_t = \frac{E_L(0)}{E_K(0)} e^{(\lambda_L - \lambda_K)t},
\]
substitution of this condition into (5) leads to the following (natural) logarithmic relationship:
(7) \[
\ln \left( \frac{K}{L} \right) = \left( \frac{\sigma}{1 - \sigma} \right) \ln \left( \frac{\pi_L}{\pi_K} \right) + (\lambda_L - \lambda_K)t + \ln \left( \frac{E_L(0)}{E_K(0)} \right).
\]

Although the stipulation that biased factor-augmentation, either labor-
saving or capital-saving, can only proceed at a steady rate through time is admittedly quite restrictive, it does provide, in equation (7), the basis for a least-squares regression model that can be employed in estimating the elasticity of substitution without making the much stronger assumption of neutrality of technical change.10 By the same token, this model permits estimation of the magnitude of the (exponential) bias in efficiency growth.

The implication of using equation (7) to estimate the elasticity of substitution \( \sigma \) under an a priori specification of Hicks-neutral technical change becomes fully apparent when one returns for a moment to consider equation (4a). From the latter it is immediately seen that, *ceteris paribus*, assuming Hicks-neutrality when \( [(\ell_L/E_L) - (\ell_K/E_K)] \) is positive must lead to an overestimate of the elasticity parameter, whereas, when \( [(\ell_L/E_K) - (\ell_K/E_K)] \) is actually negative, the estimate of \( \sigma \) will be biased downward. In a recent article Kendrick and Sato [19, pp. 980–81] present an estimate of the elasticity of substitution obtained as “the difference between the growth rate of capital and labor inputs, divided by the difference between the growth rates of the real prices of labor and capital”—a procedure which, as (4a) makes evident, rests upon the assumption of Hicks-neutrality.11 The Kendrick-Sato estimate is \( \sigma = 0.58 \) for the U.S. Private Domestic Economy, 1919–60; if, as is the case, technical change during that period has been labor-saving rather than neutral (i.e., if \( (\lambda_L - \lambda_K) > 0 \), and \( \sigma < 1 \)), it is only to be expected that an estimation procedure allowing for nonneutrality in technical change will result in a smaller value being obtained for the elasticity of substitution.

It should be clear that from a theoretical viewpoint selection of the conventional capital-labor ratio as the dependent variable, in equation (7), for purposes of regression analysis is an arbitrary choice. On practical grounds, however, there is something to be said in its favor: the nature of the available data makes it quite likely that the ratio of real capital stock estimates to the estimates of man-hours employed will contain substantial year-to-year errors of measurement, errors which it will

10 Throughout this paper we omit the stochastic term in presenting regression models.

11 Kendrick [18, pp. 120–21] discusses the possibilities of labor-saving technical change, but then proceeds to compute estimates of the arc-elasticity of substitution from the observed changes in the capital-labor ratio and an index of relative factor prices, without mentioning the tacit assumption of neutral technical change involved in this calculation. The necessity of the neutrality assumption for this method of estimation is made clear by its appearance at the beginning of the formal derivation supplied in Kendrick and Sato [19, Appendix A], but it is nowhere referred to in the text of the article. I. B. Kravis, in his 1959 article [21, pp. 940–41] which anticipated the current interest in measuring the elasticity of substitution, took the same approach, but presented his findings in a more cautious manner: “The doubling of the quantity ratio \( [K/L] \), in our notation] and the drastic decline of the price ratio \( [r/w] \), in our notation] imply an ‘historical’ elasticity of substitution of .64, but the mechanism underlying these changes is far from clear.”
not be feasible to correct. Such being the case, econometric considerations indicate the desirability of having the errors of measurement occur in the dependent variable, and not among the explanatory variables of the regression. Nevertheless, the macroeconomic interpretation offered for equation (4b) provides a forcible reminder that a single-equation estimation procedure such as that proposed here must go forward under the burden of a simultaneous-equations bias of unknown dimensions. At present we are not prepared to remove this encumbrance by undertaking the simultaneous estimation of a complete model.

II. Parameter Estimates for the U.S. Private Domestic Economy, 1899–1960

The basic regression model developed in the preceding section will, in this section of the paper, be elaborated and fitted to observations on the U.S. Private Domestic Economy for the period 1899–1960. A description of the sources of the data employed, together with an all-too-cursory set of comments on the problems they pose, is to be found in Appendix C. To put the matter here most concisely, we make use of statistics provided by J. W. Kendrick [18] for man-hours employed and the value of the net capital stock in 1929 prices as measures of $L$ and $K$, respectively. As an appropriate measure of $\pi_L$ we have, with all due trepidation, fixed upon the proportion of employee compensation in Gross (Private) Business Product.

However, in Part A of this section data for the Private Domestic Economy in the period 1899–1960 will first be used to fit a regression model which does not require information about the real capital stock, but which nonetheless yields estimates of the elasticity of substitution and the rate of technical change—the latter on the assumption that technical change is neutral. This preliminary empirical step has a double purpose; it provides a basis for comparing our subsequent statistical results with those obtainable for the same social accounting entity and time period by the application of a more commonly employed estimation method, and, secondly, it serves to bring into focus some serious difficulties encountered by this well-established approach to estimating the elasticity of substitution. Then, in Part B, a regression model derived from equation (7), permitting estimation of a constant rate of change in relative conventional input efficiency, is fitted to the data after the introduction of a number of modifications designed to cope with problems

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12 Cf. footnote 18 below. The problem of errors of measurement associated with the ratio $K/L$ is all the more serious in light of the fact that the regression model ultimately to be fitted to the data includes a lagged value of $K/L$. (Cf. Section II, equation (II).) This is equivalent to introducing a first-difference between the successive $K/L$ ratios as one of the variables of the equation, and, given the likelihood of even greater measurement errors in the first-differences than exist in the original $K/L$ series, it is especially desirable to arrange the regression model so that the first-difference term would appear as the dependent variable.
posed by cyclical variations in the rate of utilization of the capital stock and lags in the response of the capital-labor ratio to alterations in relative factor prices. In the final section (Section III) of the paper, the parameter estimates derived in Part II.B are used first to investigate the behavior of relative factor efficiency levels over time, and then to compute rates of growth of labor- and capital-efficiency and the contributions made by labor-augmenting and capital-augmenting improvements to the long-run rate of growth of total conventional factor productivity.

A. Estimating the Elasticity of Substitution Without Capital Stock Data

Although the preceding discussion has considered means of estimating the elasticity of substitution which involved relationships between the conventional capital-labor ratio and relative factor prices (or shares), an alternative approach to estimating parameters of the CES function is available. Dividing both sides of equation (2) by \( w \), and rearranging terms, we have

\[
\pi_L = w^{1-\sigma} E_L^{\sigma-1}.
\]

If it is then specified that the efficiency of conventional labor inputs, \( E_L \), grows through time at the exponential rate \( \lambda_L \), equation (8) leads immediately to a natural logarithmic expression from which the parameters \( \sigma \) and \( \lambda_L \) may be estimated without any information regarding the growth of conventional capital inputs:

\[
\ln \pi_L = \ln E_L(0) + (1 - \sigma) \ln w + \lambda_L(\sigma - 1)t.
\]

In their pathbreaking 1961 article ACMS [3, p. 244] essentially fitted equation (9) to data for the U.S. Private Nonfarm sector during the period 1909–49 and obtained \( \sigma = 0.569 \) as an estimate of the elasticity of substitution, and \( \lambda_L = 0.0183 \). It should be observed that although ACMS started by considering an aggregate production model in which technical change was Hicks-neutral, rather than the more general input-augmentation form here given by (1), their approach to the estimation of \( \sigma \) does not depend upon any assumption of Hicks-neutrality; the latter need only be invoked by ACMS \textit{ex post} to justify interpreting the estimated rate of growth of labor efficiency as being identical to the rate of growth of capital efficiency, \( \lambda_K \), and hence equal to the rate of neutral technical change, \( \lambda = \lambda_L = \lambda_K \).

Fitting the same equation (9) by least-squares regression to the data

\footnote{Because this approach avoids the use of any information regarding the real capital stock, it is of considerable convenience in empirical work in areas where such data are scarce. Cf., P. A. David, "Economic History Through the Looking-Glass," a paper read at the Boston meeting of the Econometric Society, December 1963, abstracted in \textit{Econometrica}, October 1964.}
described in Appendix C for the Private Domestic Economy in the period 1899–1960,\textsuperscript{14} yields the following result:

\begin{equation}
\ln \pi_L = 16.1915 + 0.3815 \ln w - 0.00721t; \quad R = .922
\end{equation}

(9.54) \hspace{1cm} (-6.54) \hspace{1cm} d = 1.113.

The figures appearing in parentheses below the regression coefficients are the values of the $t$-statistics derived from tests of the null hypotheses that the respective coefficients are not significantly different from zero.\textsuperscript{15} From (I) it is seen that the coefficient of the real wage variable is significantly different from zero at virtually any level of confidence one might care to name, implying that $\sigma$ is less than unity and that the production function is therefore not of the Cobb-Douglas form. As an estimate of the elasticity of substitution we have $\sigma = .619$. The coefficient of $t$ is also highly significant, and taken in conjunction with that of $\ln w$ provides an estimate of $\lambda_L = .0190$, or 1.90 per cent per annum.

These statistical results are in close agreement with those of ACMS [3], save for the interpretation placed upon the estimated rate of growth of labor efficiency. Although ACMS do not offer evidence in support of their reading of $\lambda_L$ as the rate of neutral technical change, they do advance a test of neutrality [3, pp. 235–36, 245] which, in terms of the notation used in stating the production function (1), would amount to computing the values of $E_L/E_K$ through time from the estimates of $\sigma$ and $\lambda_L$ and from a formula derived by writing (1) as:

\begin{equation}
\left( \frac{E_L}{E_K} \right)^{\sigma} = \left[ E_L \left( \frac{K}{V} \right) \right]^{\sigma} - \left( \frac{K}{L} \right)^{\sigma},
\end{equation}

where $E_L$ is to be calculated from $E_L = (E_L(0)) e^{(\lambda_L)t}$. The absence of a trend in the time path of $E_L/E_K$ would speak in favor of the assumption of neutral technical change and the identification of $\lambda_L$ with $\lambda$. However, if the estimate of the elasticity of substitution employed in making the computation is for any reason biased, the proposed test can be quite misleading. Should the estimate of $\sigma$ be too large, its use in (10) biases the test against finding a significant upward trend in the relative efficiency of labor.\textsuperscript{16} To put it simply, the degree of conviction with which this

\textsuperscript{14} Average annual real wages, $w$, were derived for the purpose of fitting model (I) from the wage share $\pi_L$ and real gross private domestic product per man-hour employed, since $w = \pi_L (V/L)$.

\textsuperscript{15} The convention of reporting such $t$-statistics rather than the standard errors of the regression coefficients is adhered to throughout the presentation of our statistical findings; the letter $d$ represents, as usual, the Durbin-Watson statistic, and $R$ is the coefficient of multiple correlation adjusted for degrees of freedom.

\textsuperscript{16} If $\sigma$ is overstated by $\hat{\sigma}$, then $\beta$ is biased downward, and the derived estimate $\hat{\lambda}_L$ will also be biased downward. The computed values of $E_L$ will then fail to reflect the actual extent of the rise in $E_L$ over time, and the time path of $E_L/E_K$ calculated from (10) will be biased downward.
test of neutrality can be accepted hinges on the strength of one's trust in the estimated value for the elasticity parameter.

There are rather persuasive reasons for suspecting that the regression model provided by (9) leads to estimates of \( \sigma \) that are, in fact, too high.\(^{17}\) Since it has been seen that this method of estimating the elasticity of substitution does not depend upon the assumption of neutral technical change, it should be clear that any such bias must arise from a quite different source. Specifically, we suggest that there is a problem of bias occasioned by the presence of errors of observation in the (explanatory) real wage variable appearing in (9). The adjustment of the capital-intensity of production in response to relative factor prices that underlies the regression model is not a short-run process; it may therefore be argued that the capital-labor ratios desired at various points in time are not significantly affected by transitory movements in real wage rates which are merely reflections of short-period variations in the level of aggregate demand. At any moment, then, the choice of technique is influenced not by the prevailing actual real wage which includes a transitory, cyclical component, but by what may be called the "structural" or "secular" real wage rate. The line of argument is now already so reminiscent of the statistical underpinnings of Friedman's [10] Permanent Income Hypothesis that it should not be necessary to elaborate it further in the text: the presence of a theoretically extraneous, transitory component in observed average annual real wage rates would produce a downward bias in the simple least-squares estimate of the coefficient of \( \ln w \) in (9), which would in turn yield an upward biased estimate of \( \sigma.\)^{18} 

Essentially the same result may be seen immediately if equation (4) were to be used to compute \((E_L/E_L) - (E_K/E_K) = (\lambda_L - \lambda_K)\): given the rate of growth of \( K/L \) and of relative factor prices (or shares), employing too large an estimate of \( \sigma \) in equation (4) introduces a downward bias in the computed value of \((\lambda_L - \lambda_K)\).

\(^{17}\) Cf. ACMS [3, p. 245] for discussion of the simultaneous-equations-bias problem raised by the time-series estimation of (9). There is no mention of other sources of bias or of the direction in which they might work.

\(^{18}\) McKinnon [24, p. 514], has also pointed out that business-cycle phenomena may be a possible source of bias in time-series estimates of the elasticity of substitution, but his argument is rather different from that advanced here. Instead of (9) we start with the functional relationship

\[(i) \quad \ln \left( \frac{L}{V} \right)^* = \ln E_L(0) - \sigma \ln w^* + (\sigma - 1)L_A^t,\]

where \(w^*\) is the structural real wage rate and \((V/L)^*\) is the productivity of labor as determined by \(E_L\) and \((K/L)^*\), the latter being determined—for a given state of technology—by \((w/r)^*\). Denote the actual average real wage by \(w = w^*(1 + \omega)\), where \(w^\omega\) is the transitory, cyclical component, and actual \((L/V) = (L/V)^*(1 + \mu)\), where \((L/V)^*\mu\) is a transitory component of actual labor requirements per unit of output arising from unplanned cyclical variations in the utilization of fixed capital. (On the short-run behavior of \((L/V)\) in U. S. manufacturing, cf. Wilson and Eckstein [38].) Then, substituting for \((L/V)^*\) and \(w^*\) in (i), and adding \(\ln w\) to both sides, we have:
As an alternative route to the conclusion that the least-squares estimate of $\sigma$ from equation (9) is likely to be biased upward, it may be argued that there is a specification error in the model. Indeed, the presence of significant autocorrelation of the residuals in (I) does lend a measure of support to the notion that (9) contains a specification error of some sort.\textsuperscript{19} It is therefore worth observing that in the short run $\pi_L$ is influenced by the level of effective demand, tending to fall in booms, when the unemployment rate ($U$) is declining, and to rise in the downswing of the cycle. This occurs because labor productivity tends to rise more rapidly than real wages when unemployment is falling while the reverse happens at the stage of the cycle at which unemployment is beginning to rise. If $\ln \pi_L$ and $\ln U$ exhibit this positive relationship and $\ln w$ and $\ln U$ are negatively related, it can be shown through the application of Theil's [35, p. 43] formula for specification errors that the coefficient of $\ln w$ in (9) will be underestimated by simple least squares.\textsuperscript{20}

In the light of such considerations one is inclined to question the validity of the test of neutrality suggested by ACMS [3] and to suspect that the method of estimating the elasticity of substitution and the rate of growth of labor efficiency employed in (I) results in an overestimate of the former and, consequently, in an understatement of the latter. Rather than explore further in this direction, we proceed to a more direct statistical test of the neutrality assumption which also attempts to provide a measure of the long-run elasticity of substitution free from the distorting influence of business-cycle phenomena.

B. A Model Allowing Exponential Bias in Efficiency Growth

The basic regression model given by (7), being derived from the production function (1), should be interpreted as indicating the relation

\[ \ln \pi_L = \theta_0 + \theta_1 \ln w + \theta_2 \delta + \eta, \]

where $\theta_1 = (1 - \sigma)$, and the stochastic term $\eta$ is,

\[ \eta = \ln (1 + \mu) - \sigma \ln (1 + \omega). \]

\textsuperscript{19} With $d = 1.113$, the hypothesis of no serial correlation must be rejected at the 1 per cent level. Cf. Theil [36].

\textsuperscript{20} Writing $a_1$ for the regression coefficient of $\ln \pi_L$ on $\ln w$, $a_2$ for the regression coefficient of $\ln \pi_L$ on $\ln U$, and $a_3$ for the regression coefficient of $\ln U$ on $\ln w$, then

\[ \varepsilon(a_1) = (1 - \sigma) + a_2a_3. \]

If $a_2 < 0$ and $a_3 > 0$, then $\varepsilon(a_1) = (1 - \delta) < (1 - \sigma)$. \vspace{12pt}
ship between the desired capital-intensity of production and the expected or "normal" long-run level of relative factor shares, rather than between actual levels of capital-intensity \( (k = K/L) \) and relative factor shares \( (\pi = \pi_L/\pi_K) \) at every point in time. Defining desired capital-intensity as \( k^* \) and the expected, long-run ratio of factor shares as \( \pi^* \), (7) can be rewritten explicitly as:

\[
(7a) \quad \ln k_t^* = \frac{E_L(0)}{E_K(0)} + \left( \frac{\sigma}{1 - \sigma} \right) \ln \pi_t^* + (\lambda_L - \lambda_K)t.
\]

It would then seem reasonable to assume that expectations regarding the "normal" level of relative shares are formed on the basis of the past history of their actual levels, and that recent history leaves a stronger imprint upon such expectations than do events farther removed in time. Following along these lines, \( \pi^* \) might be specified to be the exponentially weighted geometric average of all previous actual \( (\pi) \) values,

\[
(11) \quad \pi_t^* = \prod_{j=0}^{\infty} \pi_{t-j}^{(1 - \phi)} \quad \text{where} \quad 0 < \phi < 1,
\]

or, taking logarithms,

\[
(11a) \quad \ln \pi_t^* = \sum_{j=0}^{\infty} \phi(1 - \phi)^j \ln \pi_{t-j},
\]

implying that the exponential weights increase geometrically as the actual \( (\pi) \) values considered approach the present. The distributed-lag specification given in (11a) is equivalent to

\[
(11b) \quad \ln \pi_t^* - \ln \pi_{t-1}^* = \phi(\ln \pi_t - \ln \pi_{t-1}),
\]

or, taking antilogarithms,

\[
(11c) \quad \frac{\pi_t^*}{\pi_{t-1}^*} = \left( \frac{\pi_t}{\pi_{t-1}} \right)^\phi.
\]

It is simply the ratio version of the familiar Nerlove [27] distributed-lag form, chosen here for its convenience in the estimation of regressions involving the logarithms of variables rather than the variables themselves.\(^{22}\)

\(^{21}\) The notion of producers forming expectations as to the behavior of relative factor shares is perhaps less intuitively appealing than the idea that they have expectations regarding "normal" or long-run factor prices. The two approaches may be logically equivalent, but it should be recognized that they may lead to different empirical results. While we have chosen here to avoid the complexities of introducing separate corrections for wage-rate and rental-rate expectations, this is clearly a matter calling for further investigation.

\(^{22}\) For an extensive discussion of distributed-lag techniques, cf. Nerlove [27].
Although (7a) says that the desired capital-intensity is adjusted in response to the influence of $\pi^*$, it need not, indeed, it should not be assumed that such adjustments are completed so that the desired capital-intensity is always identical to the actual capital-intensity of production in each period of time. Since we are dealing with an adjustment process involving fixed capital of durability considerably greater than the length of annual periods of observation, and a capital stock comprised of some indivisible elements whose presence militates against complete, instantaneous alterations in factor proportions, the total adjustment of the actual capital-labor ratio to the desired level may be presumed to take place only with some lag. Once again employing the ratio form of the lag specification suggested by Nerlove, it may be postulated that:

\[
\frac{k_t}{k_{t-1}} = \left(\frac{k_t^*}{k_{t-1}^*}\right)\gamma, \quad 0 < \gamma < 1.
\]

This expression says that changes in the ratio of actual capital-intensities in all pairs of consecutive periods are a constant fraction of the changes in the ratio of the capital-intensity currently desired to the actual capital-intensity of production in the previous period. The parameter $\gamma$ may, therefore, be regarded as the elasticity of adjustment, indicating the fraction of the desired adjustment that is completed in the course of a single year.\(^{23}\)

Let us consider the regression model that incorporates the foregoing corrections for the transitory component in the actual level of relative factor shares and for lags in the adjustment of the actual capital-intensity of production to the desired level. Taking (natural) logarithms of equation (12), we have

\[
\ln k_t = \gamma \ln k_t^* + (1 - \gamma) \ln k_{t-1},
\]

and, substituting from (7a) for $\ln k_t^*$:

\[
\ln k_t = \gamma \alpha_0 + \left(\frac{\gamma \sigma}{1 - \sigma}\right) \ln \pi_t^* + \gamma (\lambda_L - \lambda_K) t + (1 - \gamma) \ln k_{t-1},
\]

\(^{23}\) Equation (12) is equivalent to the assumption that the actual values of $k$ are exponentially weighted products of all previous desired values of $k^*$, with the weights rising geometrically as the values of $k^*$ approach the present. Symbolically,

\[
\ln k_t = \sum_{r=0}^{\infty} \gamma(1 - \gamma)^r \ln k_{t-r}, \quad 0 < \gamma < 1.
\]

This form is not superior on any a priori grounds to other conceivable specifications of the adjustment process. We will subsequently make use of a form which implies that past values of the capital stock in existence, rather than the capital stock in use, are considered in adjusting to the desired degree of capital-intensity.
where, for notational convenience, \( \alpha_0 = E_L(0)/E_K(0) \). From (13) we progress, via some intermediate steps, to the regression model:

\[
(15) \quad \ln k_t = v_0 + v_1 \ln \pi_t + v_2 t + v_3 \ln k_{t-1} + v_4 \ln k_{t-2},
\]

where,

\[
v_0 = \gamma(\lambda_L - \lambda_K)(1 - \phi) + \phi \gamma \alpha_0
\]

\[
v_1 = \phi \gamma \left( \frac{\sigma}{1 - \sigma} \right)
\]

\[
v_2 = \phi \gamma (\lambda_L - \lambda_K)
\]

\[
v_3 = [(1 - \phi) + (1 - \gamma)]
\]

\[
v_4 = (1 - \phi)(\gamma - 1)
\]

As it presently stands, the regression model given by (15) suffers from several deficiencies which militate against its immediate application to the available data. In the first place, inspection of the relationships among the coefficients \( (v_0, \ldots, v_4) \) reveals that while it is possible to estimate the sum and the product of the parameters \( \phi \) and \( \gamma \), and, hence, to estimate \( \sigma \) and \( (\lambda_L - \lambda_K) \), one would still be unable to say which was \( \phi \) and which was the elasticity of adjustment, \( \gamma \). This in turn would prevent estimation of \( \alpha_0 \). Secondly, as a rather more serious practical obstacle, the presence on the "explanatory" side of the equation of two previous values of the capital-intensity variable \( k_{t-1} \) and \( k_{t-2} \)—which exhibits a strong upward trend—as well as the time variable, \( t \), creates a virtually overwhelming multicollinearity problem, vitiating useful application of the regression model in this context.25 Thirdly, the development of the model has glossed over a difficulty that almost always plagues empirical implementation of models that call for information

\[\text{Substitution in (13) from (11b) leads to:}\]

\[
(14) \quad \ln k_t = \gamma \alpha_0 + \left( \frac{\gamma \sigma}{1 - \sigma} \right) \phi \ln \pi_t + \left( \frac{\gamma \sigma}{1 - \sigma} \right)(1 - \phi) \ln \pi_{t-1} + \gamma(\lambda_L - \lambda_K) t + \ln k_{t-1}.
\]

But from (13) we also have,

\[
\ln \pi_{t-1} = \frac{(1 - \sigma)}{\gamma \sigma} \ln k_{t-1} - \frac{(1 - \sigma)(1 - \gamma)}{\gamma \sigma} \ln k_{t-2} - \frac{(\lambda_L - \lambda_K)(1 - \sigma)}{\sigma} t + \frac{(1 - \sigma)[(\lambda_L - \lambda_K) - \alpha_0]}{\sigma},
\]

which, upon substitution into (14), yields (15).

\[\text{The real villain of the piece is } k_{t-2}, \text{ which entered as a consequence of the replacement of } \pi \text{ in equation (7) by } \pi^*, \text{ and the form of the distributed-lag specification given for } \pi^*. \text{ To avoid the severe multicollinearity resulting from inclusion of } k_{t-2} \text{ among the independent variables, it will be necessary to handle the correction for transitory movements in relative factor shares in a less straightforward manner.}\]
about the input of conventional capital services: the existence of temporal variations in the proportion of the physical capital stock actually employed in production. Since we have already gone as far as to take a stock concept of capital as a proxy for the flow of capital services, thereby disregarding possible secular alterations in durability, the least that must be done to bring the data into line with the conceptual requirements of the production function is to introduce a means of adjusting the available statistics on the existing capital stock for short-run changes in the rate at which it is utilized.

It is possible to remove these deficiencies while retaining some form of correction for differences between actual and expected or “normal” relative factor shares, as well as an adjustment for lags in the response of the actual capital-labor ratio. But to do so, one must pursue a slightly different and somewhat cruder line of attack. We proceed by first reformulating the correction for the transitory component in the movements of observed relative factor shares. Next, a device to correct the data for variations in the rate of utilization of the existing, or nominal, capital stock will be explicitly introduced into the regression model. Finally, we shall modify the hypothesis about the way in which lagged adjustments are made in the degree of capital-intensity.

If, as has been previously argued, short-run fluctuations in relative (factor prices and hence in) factor shares are discounted as reflecting the transitory influence of business-cycle conditions, an alternate approach would try to take account of this discounting procedure in an explicit fashion, instead of doing so implicitly through the device of a distributed-lag specification for \( \pi^* \). Since labor's share displays a tendency to move inversely to the rate of unemployment \( U \) over the course of the business cycle, it may be hypothesized that \( \pi_L^* \) is higher than actual \( \pi_L \) when the rate of unemployment is abnormally low, while it lies below actual \( \pi_L \) during abnormally high periods of unemployment. On this line of reasoning, it may be postulated that the ratio of “normal” to actual relative factor shares varies positively with the rate of employment, so that a 1 per cent change in the latter is always accompanied by a constant, but unspecified, per cent change \( (\delta) \) of the former in the same direction. Symbolically, in place of equation (11), we assume that:

\[
(16) \quad \left( \frac{\pi_L}{\pi_K} \right)^* = \left( \frac{\pi_L}{\pi_K} \right) c_0 (1 - U)^\delta, \quad c_0 > 0, \quad \delta \geq 0,
\]

or, taking (natural) logarithms,

\[
(16a) \quad \ln \pi^* = \ln \pi + \delta \ln (1 - U) + \ln c_0.
\]

Then, substituting (16a) in (7a), we obtain the revised basic equation for the desired capital-intensity of production:
\[\ln k^* = \left(\frac{\sigma}{1 - \sigma}\right) \ln \pi_t + \left(\frac{\sigma \delta}{1 - \sigma}\right) \ln (1 - U)t + (\lambda_L - \lambda_R)t + \alpha_0'\]

where, for convenience,

\[\alpha_0' = \left[\left(\frac{\sigma}{1 - \sigma}\right) \ln c_0 + \alpha_0\right].\]

There is no prescribed way to correct statistics relating to the real capital stock in existence so that they will reflect the movements of that part of the stock which is actually being utilized. (Such a correction is necessary to transform the data on the observed conventional capital-labor ratio \(\hat{k}\) into a series describing the behavior of the actual degree of conventional capital-intensity \(k\), which is the variable that is called for by the model.) Yet it does seem sensible to relate variations in the utilization of capital to concurrent variations in the rate at which the labor force is utilized or, in other words, to the employment rate. Alternative specifications for this relationship have been advanced in the literature, but, supported by the results of some experimentation which is reported in Appendix A, we shall make use of the form

\[(18) \quad k = \hat{k}d_0(1 - U)t, \quad t > 0,\]

where \(d_0\) is some positive constant, and \(U\) is the proportion of the labor force unemployed. Equation (18) says simply that percentage changes in the rate of utilization of the existing capital stock are a constant fraction or multiple of the concurrent percentage changes in the labor force employment rate. Instead of prespecifying the magnitude of this constant fraction or multiple \((\zeta)\), we shall leave it to be determined in the course of fitting the complete model.

Having distinguished the observed or nominal capital-labor ratio \(\hat{k}\) from the actual degree of conventional capital-intensity \(k\), it is now necessary to reconsider the process through which actual capital-intensity of production is brought into line with the desired degree of capital-intensity \(k^*\). The description of that process provided by equation (12) assumes that past values of the actual capital-intensity of production are compared with the level currently desired. However, it appears no less reasonable to suppose that, in adjusting the actual technique of production to the desired technique, the capital stock in existence in preceding periods is what is considered by decision-makers, rather than just that part of the stock which had actually been in use. Because of fixed costs of capital, firms may well tend to compensate for periods of unplanned underutilization of existing plant and equipment by moving, in the next period, to a level of actual capital-intensity somewhat higher than would have been selected in the absence of previous sub-
normal utilization. Keynes's [20, pp. 69–71] argument, that user cost on equipment tends to fall if capital is redundant and entrepreneurs anticipate that the redundancy will not be removed quickly in the future, would seem to support this line of reasoning. Abnormal underutilization, reflected in \((k > k)\), would lower calculated costs of using equipment rather than leaving it idle, and thus tend to raise actual capital-intensity \((k)\) relative to desired capital-intensity \((k^*)\) in the following period.26

The foregoing considerations provide one type of justification for formally hypothesizing that the process of adjusting the actual capital-intensity of production is perhaps better described by

\[
\frac{k_t}{k_{t-1}} = \left(\frac{k^*_t}{k^*_{t-1}}\right)^\gamma,
\]

than by equation (12). There is another, and possibly more persuasive, reason for working with (19): retaining the form given in (12) would, in conjunction with (18), introduce both current and lagged values of the employment variable, \(\ln(1-U)\), into the regression analysis. This would only lead one back into a serious multicollinearity problem. (Cf. Appendix A.)

Therefore, taking (natural) logarithms of (19), and substituting for \(k^*\) from (17), we arrive at the expression:

\[
\ln k_t = \gamma \left(\frac{\sigma}{1 - \sigma}\right) \ln \pi_t + \delta \left(\frac{\gamma \sigma}{1 - \sigma}\right) \ln (1 - U)_t + \gamma (\lambda_L - \lambda_K)_t + (1 - \gamma) \ln k_{t-1} + \gamma \alpha'_0.
\]

Substitution for \(k\) from (18) then leads to the revised regression model:

\[
\ln k_t = v' + v'^t \ln \pi_t + v'^2 t + v'^3 \ln k_{t-1} + v'^4 \ln (1 - U)_t
\]

\[
v' = \left[ \left(\frac{\gamma \sigma}{1 - \sigma}\right) \ln c_0 + \gamma \left(\frac{E_L(0)}{E_K(0)}\right) + \ln d_0 \right]
\]

\[
v'^t = \gamma \sigma/(1 - \sigma)
\]

\[
v'^2 = \gamma (\lambda_L - \lambda_K)
\]

\[
v'^3 = (1 - \gamma)
\]

\[
v'^4 = \left[ \left(\frac{\gamma \sigma \delta}{1 - \sigma}\right) - \zeta \right]
\]

One implication of such a situation is that actual rates of utilization of capital would not tend to fall as sharply as rates of employment during cyclical contractions; in terms of (18), the elasticity parameter \(\zeta\) ought to be found to be less than unity. Cf. below, for evidence pointing in this direction and for a discussion of the difference between the concept of capital underutilization adopted here and the more commonly held notion of "excess capacity."
From the viewpoint of the desirability of providing estimates of all the parameters of the model, equation (21) is unfortunately no more satisfactory than (15), inasmuch as separate estimates for only three of the eight parameters can be obtained directly from the coefficients fitted for (21) by simple least-squares regression; values for \( E_L(0)/E_K(0) \), \( \sigma_0, d_0, \delta \) and \( \xi \), unfortunately cannot be shaken out of the vector of estimates \((\sigma'_0, \ldots, v'_4)\). However, the parameters \( \sigma, (\lambda_L - \lambda_K), \gamma \), which can be estimated, are those of major interest here. It is also possible to go a bit further and compute a lower-bound estimate of \( \gamma \), the elasticity of the utilized capital stock with respect to the labor force employment rate.\(^{27}\)

The result of fitting equation (21) to observations on \((\hat{K}/L), (\pi_L/(1 - \pi_L))\), and \((1 - U)\) for the U.S. Private Domestic Economy during the period 1899–1960,\(^{28}\) by simple least squares is shown by

\[
\ln \left( \frac{\hat{K}}{L} \right)_t = -2.1670 + 0.1285 \ln \left( \frac{\pi_L}{1 - \pi_L} \right)_t + 0.0020t \\
+ 0.7225 \ln \left( \frac{\hat{K}}{L} \right)_{t-1} - 0.3153 \ln (1 - U)_t, \\
R = .980, \quad d = 1.252.
\]

Making use of relationships (21a)-(21d), in footnote 27, the estimated parameters implied by (II) are:

\[
\hat{\gamma} = 0.2775, \\
(\lambda_L - \lambda_K) = 0.0072, \\
\hat{\delta} = 0.3165, \\
\hat{\xi} \geq 0.3153
\]

These estimates are obtained from (21) as:

\[
(21a) \quad \hat{\gamma} = (1 - \hat{\delta}), \\
(21b) \quad (\lambda_L - \lambda_K) = \frac{\hat{\delta}'}{(1 - \hat{\delta}')}, \\
(21c) \quad \hat{\delta} = \left[ \frac{\hat{\delta}'}{(1 - \hat{\delta}')} \right] / \left[ 1 + \frac{\hat{\delta}'}{(1 - \hat{\delta}')} \right] = \frac{\hat{\delta}'}{1 + \hat{\delta}' + \hat{\delta}'}, \\
\]

and, since

\[
(21d) \quad \hat{\xi} = v_1 \hat{\delta} - v_4', \\
\]

and \( \delta > 0 \), if it is to have the meaning assigned to it in (16), we have the lower-bound estimate \( \hat{\xi} \geq -\hat{\delta}' \), if \( \hat{\delta}' \geq 0 \).

\(^{27}\) Cf. Appendix C for source of data on the rate of unemployment in the civilian labor force, used in fitting (II).
From the $t$-values for the $H_0: \beta_i = 0$ (one-tail) significance tests shown below the coefficients ($\hat{\beta}_1, \cdots, \hat{\beta}_n$), it may be observed that, since there are 56 degrees of freedom, the null hypotheses can be rejected with 95 per cent confidence in each of the four tests.

As far as the elasticity of adjustment estimate ($\hat{\gamma}$) is concerned, this amounts to the finding that the hypothesis of complete adjustment ($\gamma=1$) can be rejected. Indeed, we note from $\hat{\gamma}$ that in the course of a single year a 10 per cent change in the ratio between current desired capital-intensity and the previous existing capital-labor ratio leads to but a 2.8 per cent alteration in the ratio of the current actual capital-intensity to the previous existing capital-labor ratio.29

Given the considerable lag in the adjustment of the capital-labor ratio, the estimated long-run elasticity of substitution $\delta = 0.316$ is arresting small. Yet this should occasion little surprise, it was anticipated that if the coefficient ($\lambda_L \sim \lambda_R$) was found to be significantly above zero, the estimate obtained for the elasticity of substitution would lie below the values that have been derived on the assumption of neutral technical change by Kendrick and Sato [19], and Kravis [21], i.e., $\delta = 0.58$ and $\delta = 0.64$, respectively. It may also be observed that the substitution parameter estimated from regression (II) is roughly half as large as the estimate provided by regression (I) in Part II.A, and is only a bit more than half that obtained with the same regression model by ACMS [3]. We are inclined to regard the latter comparisons as empirical support for the earlier argument that the usefulness of the regression procedure indicated by equation (9) is impaired by a mis-specification of the wage-rate variable—equivalent to the presence of errors of observation in that variable as employed in (I)—which produces upward-biased estimates of $\sigma$. The short-run elasticity estimate ($\hat{\delta} = 0.088$) provided by regression (II) is truly minute, but it is nonetheless significantly greater than zero—as the statistical significance of $\hat{\delta}$ implies.

Even as a lower bound, the estimate that appears above for $\gamma$ is also strikingly small; it suggests that a 1 per cent fall (or rise) in the employment rate is accompanied by only a 0.3 per cent drop (or rise) in the rate of utilization of the existing stock of capital. Moreover, if we were to inquire how far above zero the estimated value of $\delta$ would have to lie before it could be said that percentage changes in the rate of capital-utilization were equal to those in the employment rate—an assumption

29 Although $\hat{\gamma}$ is low, it is significantly greater than zero; the $t$-statistic for $H_0: \gamma = 0$ is $t = 3.09$. Cf. Appendix B on possible downward bias in $\hat{\gamma}$.

30 This estimate does not seem out of line with the results of several recent studies: Brown and De Cani [5] [6], Lucas [23], McKinnon [24]. However, the statistical procedures used in the foregoing studies may very well have led to more severe downward biases in the estimates of $\sigma$—for reasons cited in Appendix B—than the present study encountered.
sometimes made in aggregate production function analysis—the relationship given in (21d) implies that if \( \xi = 1 \), \( \delta \) is at the implausibly high level \( \delta = 5.33 \). For \( \delta = 2.0 \), which might be appropriate to the experience of the mid-1950’s but seems to be still rather high for the 1899–1960 period as a whole, the corresponding estimate \( \xi = .5718 \) remains well below unity.\(^{31}\)

On first consideration, these findings might appear to fly in the face of the common observation that variations in the rate of employment in the U.S. economy are accompanied by more than proportional changes in measured rates of “capacity-utilization.” However, the concept underlying reported rates of capacity-utilization is rather different from the concept of the rate of capital-utilization adopted here. The most familiar indices of capacity-utilization (and of excess capacity) are measures of the relationship between actual output and full-capacity output, the latter being defined either as maximum output obtainable under normal work scheduling or as that level of output observed during periods of peak production.\(^{32}\) By contrast, full-capital-utilization, as it is defined here, refers to that flow of capital services yielded by the capital stocks of firms operating at optimum points on their cost curves; it is that flow which, with product and factor prices remaining unchanged, would occasion neither net investment nor disinvestment. Since our concept

\[ \begin{align*}
\text{Interval of Rising} & \quad \text{Arc-Estimates of} \\
\text{Unemployment} & \quad \delta = - \frac{(1 - U)}{\pi} \left( \frac{\Delta \pi}{\Delta(1 - U)} \right) \\
\text{Rate} & \\
1919-22 & 1.2 \\
1929-32 & 0.5 \\
1955-58 & 1.9
\end{align*} \]

The sample is obviously restricted, but from the above estimates and the general behavior of the data, it does seem difficult not to conclude that \( \xi = 1 \) implies an estimate of \( \delta (= 5.33) \) that is clearly too high.\(^{32}\)

\(^{31}\) The statements regarding the plausibility of implied estimates of the parameter \( \delta \) are based on the following considerations. From (21d) and (II) we have the relation: \( \xi = .12855 + 0.3153 \). In other words, for \( \xi = 1 \), a 1 per cent drop in the employment rate would mean that the ratio of the “normal,” or long-run, to the actual relative shares would fall by more than 5 per cent. If (in the very short run) \( \pi^* \) were taken as constant, this means that the actual relative share of labor \( \pi \) would rise by 5 per cent, roughly, when the employment rate dropped by 1 per cent. The latter hardly appears to be reasonable in light of the short-period movements of the available data on factor shares and the employment rate. From the data described in Appendix C one can compute the following estimates of \( \delta \) over three-year intervals of rising unemployment, on the strong assumption that \( \pi^* \) is constant within each of those short intervals:

\(^{32}\) Cf. Phillips [28]. The technique of selecting peak production years as full-capacity-utilization reference points and assuming, in constructing measures of capacity-utilization for purposes of business-cycle analysis, that the relationship between the capital stock and output changes only very slowly, if at all, seems quite inadmissible in the context of the present study. Emphasis here is placed upon discovering the breadth of factor-substitution possibilities and the long-run rates of factor-augmentation.
of the rate of capital-utilization does not involve comparisons of actual and full-capacity output, there is no reason to suppose that the estimate of \( \xi \) should coincide with inferences based upon the observed relationship between fluctuations in the usual capacity-utilization measures and those in the rate of employment.

The implication of the fractional upper- and lower-bound estimates of \( \xi \), rough as they are, is that, over the course of the business cycle, firms collectively act to reduce the impact of demand fluctuations upon the rate of utilization of the existing capital stock, thus shifting the burden of adjustments to deficiencies in effective demand onto the labor force. This jibes with the inference one draws from Keynes's discussion of the behavior of user cost over the cycle, and might therefore be interpreted as indirect support for specifying (19) in a manner in which the existence of idle capacity in the preceding period is allowed to raise the ratio of current actual capital-intensity to current desired capital-intensity. It should, of course, be recognized that the production model with which we are working strictly does not admit the possibility of departures from the full-capital-utilization optimum; idle capital, whose marginal productivity is zero, simply doesn't fit into the present theoretical framework. This may result in some downward bias in the estimate of \( \xi \) when the model is fitted to data generated by a world where capital sometimes stands idle.

Perhaps the most intriguing result provided by (II), and certainly the most novel finding, is the estimate obtained for \( (\lambda_L - \lambda_K) \), the rate of bias in the growth of conventional input efficiencies. Discussion of this parameter estimate has, like all good things, been saved 'til last. The fact that \( \xi^*_U \) is positive and significantly greater than zero leads immediately to the conclusion that over the period 1899–1960 technical change has not been neutral, but has instead increased conventional labor-input efficiency more rapidly than the efficiency of conventional capital inputs. To restate the point in Hicks's terminology (noting that \( 0 < \delta < 1 \)): technical progress in the Private Domestic Sector of the U.S. economy has been labor-saving during the present century. As for the mag-

---

33 The short-run relation between changes in the measures of capacity-utilization and the employment rate could reflect variations in output per man employed with constant capital service inputs, due to changes in employment and the number of hours worked per man employed, assuming the possibility of varying factor proportions on old equipment. (Cf. Wilson and Eckstein [38].)

It may be noted that the small values implied for \( \xi \) do not result from the fact that the employment rate is defined in terms of persons employed, while the measure of (constant efficiency) labor input used in (II) is defined in terms of man-hours employed. Since the variance of an employment rate \( (1-U') \) defined as the proportion of potential full-employment man-hours actually worked would be greater than that of the employment rate as it is conventionally defined, the regression coefficient of \( \ln(\hat{\xi}/L) \) on \( \ln(1-U') \) would be smaller, neglecting sign, than that obtained in (II); the implied estimates of \( \xi \) would, consequently, lie even further below unity than those we have found.
magnitude of the differentially faster rate of labor-augmentation, the estimated rate is 0.72 per cent per annum; over the course of the six decades since 1900, the efficiency of labor has increased by roughly 54 per cent more than the efficiency of capital. These findings are sufficiently striking in their import to warrant their further consideration in Section III.

But, before examining the question of the apparent nonneutrality of efficiency growth more closely, it is necessary to enter an explicit caveat regarding the parameter estimates just discussed. The estimates offered here are, at best, tentative, for the estimation procedure has several potentially serious sources of bias. As the existence of a simultaneous-equations bias of unknown dimension has already been pointed out, it is not necessary to dwell longer on that unpleasant vision; two other econometric horrors must be confronted—lagged values of the dependent variables and serial correlation of the disturbance terms. A glance at (II) will suffice to confirm that both are indeed present, although the serial correlation of disturbances is not terribly severe ($d = 1.252$) and is slightly less pronounced than in regression (I).

The conflicting effects of these sources of bias upon the parameter estimates receive some further consideration in Appendix B. While one will conclude from that discussion that nothing very concrete can be asserted about the existence, direction, and magnitude of over-all biases in the estimates derived from regression (II), it is found that the inclusion of the lagged dependent variable among the explanatory variables in (II) does not in itself result in the estimate of $(\lambda_L - \lambda_K)$ which supports the inference that technical change has been labor-saving.

III. Time Paths of Labor and Capital Efficiency

Despite the fact that technical change may be characterized by a fairly steady, persistent labor-saving bias, at least over the very long run, it is rather farfetched to imagine that either the magnitude or the direction of that bias will necessarily be the same in all short intervals of time or that such departures from the long-term trend as do occur will be of a random sort. Quite the contrary, if one thinks of technical change in terms of the introduction, extension, and diffusion of discrete innovations in organization and production techniques, and quite possibly also in the content of labor force training, one is dealing with a set of processes that are played out over time rather than being instantaneously completed. It might therefore be expected that the absorption of one major innovation (or a limited number) may well dominate the technological scene during an extended period of time; and that while this absorption is going on, the impact of such dominant innovations will impress itself upon the direction and rate of change of relative factor efficiency in the economy as a whole. With this as the paradigm of tech-
nical progress, departures from the long-run trend in relative factor efficiency \(E_L/E_K\) would be anomalous if they turned out to be serially independent rather than autocorrelated.4

It is therefore of some interest to inquire what the data and the parameter estimates obtained in (II) imply about the time path of \(E_L/E_K\) in the Private Domestic Sector of the U.S. economy during the present century. Answering this question by calculating the time profile of \(E_L/E_K\) proves to be a comparatively easy matter. From (5), (6), (18) and (19) we have—noting that \(\rho = [(1 - \sigma)/\sigma]\)—the following expression:

\[
(22) \quad k_id_0(1 - U)^\gamma_t = \left(\pi_t c_0(1 - U)^\gamma_t\right)^{\gamma/\rho} k_{t-1}^{(1-\gamma)} \left(\frac{E_L(0)}{E_K(0)} e^{(\lambda_l - \lambda_K) t}\right)^{\gamma},
\]

which may be rewritten as,

\[
(23) \quad \left(\frac{E_L}{E_K}\right)_t = \left[\frac{1/\rho}{\pi_t (1 - U)^\gamma_t} \left(\frac{E_L}{E_K}_t\right)\right] c_0^{1/\rho} d_0^{1/\gamma}. \tag{23}
\]

Now, making use of the relationships between the regression coefficients in (II) and the structural parameters of the model, \(E_L/E_K\) can be computed in index form from:

\[
(24) \quad \left(\frac{\tilde{E}_L}{E_K}\right)_t = \left(\frac{E_L/E_K}{E_L/E_K}_0\right) = \left[\left(\frac{\lambda_L}{L}\right)_t \left(\frac{K}{L}\right)^{\xi_t^t} \left(1 - U\right)^{\xi_t^t} \left(\frac{\lambda_K}{K}\right)^{\xi_t^t} \right]^{1/(1 - \xi_t^t)}. \tag{24}
\]

Applying the estimates \(\xi_t^t, \xi_t^t, \text{ and } \xi_t^t\) from (II) and the corresponding time-series observations, equation (24) generates the time path of \(E_L/E_K\) plotted on semilogarithmic scales in Chart 1.

Quite apart from the sawtooth short-period movements of the index that appear in the Chart, as a consequence of having forced the production function to account for the data exactly rather than stochastically, the time path of \(E_L/E_K\) reveals the deficiencies of the assumption of a constant rate of bias in the direction of technical change.56 Indeed,

4 These observations have some implications for the econometric difficulties encountered in estimating the model considered here, particularly the problem of autocorrelation in the disturbance terms. Although artificial statistical contrivance may be employed in an effort to render residuals serially independent, it does seem sensible to view the element of serial correlation that persists in the residuals of regression (II) as the consequence of the incomplete or incorrect specification of the regression model. (Cf. [12, pp. 71-72].) Thus regarded, the problem of serial correlation in the disturbances of (II) may be ultimately intractable, for one of the most likely sources of mis-specification that comes to mind is the assumption of a constant geometric rate of bias in the growth of factor efficiencies, and upon that assumption the entire approach pursued here must rely.

56 It should be clear that the computed index of \(E_L/E_K\) simply represents the estimated trend values (given by \(e^{-0.072t}\)) plus the residuals of regression (II). The pronounced short-run
visual inspection of Chart 1 suggests that in the United States the six decades since 1899 might be thought of as encompassing three major periods, each characterized by a rather different experience with regard to the relative growth of labor and capital efficiency:

(a) 1900–1918, in which labor-saving technical changes took place more rapidly than the long-term trend rate of bias (0.72 per cent per annum);

(b) 1919–45—a longer interval over whose entire course no significant labor- or capital-saving bias emerged, despite marked internal changes of some duration in the direction of movements in $E_L/E_K$;

(c) 1946–60, the postwar period, during which the rise in relative labor efficiency was resumed at a rate even faster than that experienced prior to 1919.

Any serious effort to account for the appearance of such alterations in the drift of "technical change" or to relate them to other economic events distinguishing the three periods clearly lies beyond the scope of the present study. It may, nonetheless, be observed that the decline in the level of $E_L/E_K$ during the latter part of the second of these periods movements of the residuals around the trend line probably result in part from errors of measurement in the capital-labor ratios, i.e., in the dependent variable of the regression. So long as we retain the basic assumptions that the elasticity of substitution is constant, that there are constant returns to scale, and that markets are free from monopoly elements, the more sustained departures of $E_L/E_K$ from its trend line in Chart 1 are attributable to the unsteady character of technical progress. Were one prone to discard the foregoing basic assumptions of the analysis, other possible explanations would spring to mind: changes in the elasticity of substitution, changes in the degree of monopoly in product and factor markets, etc.
(i.e., during 1933–43) may well be a reflection of a sustained rise in the rate of utilization of the existing stock of capital, brought about initially by the slow recovery from the depression of the 1930’s and then extended by the pressure upon available capacity during World War II. The reversal of this movement and the extremely rapid rise of \( E_L/E_K \) in Chart 1 during the late 1940’s would, on this line of explanation, reflect the return to lower normal rates of capital-utilization following the war. This is, however, no more than speculation intended to provoke further inquiry.

Although a direct estimate of the (constant) long-run disparity between the rate of growth of labor efficiency and the rate of growth of capital efficiency has been secured from regression (II)—as well as an alternative (larger) estimate of \( \lambda_L - \lambda_K \) from regression (III) in Appendix B—nothing has been said about the magnitudes of the actual rates of labor- and capital-augmentation over the period 1899–1960. Yet, answers to the question of just how rapidly labor efficiency and capital efficiency have grown, and an assessment of the importance of the contributions made to the rate of growth of conventional total factor productivity, lie within reach.

Going back to equation (1a), and noting that \( \alpha \) and \( \beta \) may be replaced by the shares of labor and capital in output, respectively, we have the relationship

\[
(25) \quad \pi_L \frac{E_L}{E_L} + (1 - \pi_L) \frac{E_K}{E_K} = \left( \frac{V}{V} - \frac{L}{L} \right) - (1 - \pi_L) \left( \frac{K}{K} - \frac{L}{L} \right)
\]

which, recalling the definition of \( \lambda \equiv \dot{E}/E = \alpha \lambda_L + \beta \lambda_K \), is equivalent to

\[
(25a) \quad \lambda = \left( \frac{V}{V} - \frac{L}{L} \right) - (1 - \pi_L) \left( \frac{K}{K} - \frac{L}{L} \right).
\]

In addition, from the definition of \( \lambda \), it is known that

\[
(25b) \quad \lambda_K = \lambda - \pi_L (\lambda_L - \lambda_K).
\]

More sophisticated alternatives are available, but it is sufficient for the present purpose to proceed by estimating the long-term rate of total conventional input efficiency growth (\( \lambda \)) from the estimated trend rates of growth of \( V, K, \) and \( L \) in the Private Domestic Economy during the period 1899–1960, using the arithmetic average of labor’s share in that interval as the weight for the \( [(\dot{K}/K) - (\dot{L}/L)] \) term in (25a).\(^36\) The estimate previously obtained for \( \lambda_L - \lambda_K \) can then be used with \( \lambda \) and \( \pi_L \)

\(^36\) It is not necessary to correct the existing capital stock (\( \dot{K} \)) data for variations in the rate of utilization, since the rate of exponential growth of \( \dot{K} \) is obtained by regressing observations of the natural logarithm of \( \dot{K} \) on time for the entire long period 1899–1960. See Appendix C for regression equations used to obtain estimates of \( \dot{V}/V \), \( \dot{K}/K \), and \( \dot{L}/L \).
### Table 1—Estimates of Average Annual Compound Rates of Growth of Factor Efficiencies, U.S. Private Domestic Economy, 1899–1960

(in percentages)

<table>
<thead>
<tr>
<th>Trend Estimates of:</th>
<th>(Average) Estimate</th>
<th>Estimate Implied from (25b) of:</th>
<th>Relative “Contribution” of $\lambda_L$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V \quad L \quad K \quad L$</td>
<td>$\hat{\pi}_L$</td>
<td>$\lambda$</td>
<td>$\hat{\lambda}_L$</td>
</tr>
<tr>
<td>$V \quad L$</td>
<td>$K \quad L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Regression Model II Estimate ($\lambda_L-\lambda_K$) = 0.0072</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.38</td>
<td>1.01</td>
<td>.476$^a$</td>
<td>1.85</td>
</tr>
<tr>
<td>2.38</td>
<td>1.01</td>
<td>.751$^b$</td>
<td>2.13</td>
</tr>
<tr>
<td>B. Regression Model III Estimate ($\lambda_L-\lambda_K$) = 0.0086</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.38</td>
<td>1.01</td>
<td>.476$^a$</td>
<td>1.85</td>
</tr>
<tr>
<td>2.38</td>
<td>1.01</td>
<td>.751$^b$</td>
<td>2.13</td>
</tr>
</tbody>
</table>

$^a$ Employee compensation in Gross (Private) Business Product, Appendix B.


to calculate $\hat{\lambda}_K$ from (25b), and, therefore, to compute $\hat{\lambda}_L$. The results of the computations just described are presented in Table 1.

Since the average level of labor's share is quite sensitive to variations in the definition of payments for labor services and the inclusiveness of the aggregate output measure, these computations are affected by the particular concept of labor's share that is employed.$^{57}$ Panel A of Table 1 therefore presents two sets of factor-efficiency growth rate estimates; these have been computed using the estimate ($\lambda_L-\lambda_K$) = 0.0072 from regression model (II) and alternative averages for labor's share over the 1899–1960 period. The first line shows the results obtained with the share concept adopted in the present study—essentially, employee compensation exclusive of entrepreneurial income as a proportion of gross private business product. The second line shows the same calculation with the much higher average value of labor's share in national income, defined to include an estimate of the wage component of entrepreneurial incomes; this definition and measure of labor's share are adopted by Kendrick [18, Tables 25, 28, pp. 112, 179]. It should be observed that the estimates of $\lambda_L$ and $\lambda_K$ are really quite

$^{57}$ Cf. Kravis [21] and Grant [11] for a recent survey of alternative measures of labor's share and a discussion of the problems of consistent measurement. Cf. also notes in Appendix C. We are concerned here only with the mean level of $\pi_L$, implicitly basing this concern on the observation that differences in the definition of $\pi_L$ produce greater alterations in the average levels of the shares than in their long-run growth rates. Marked differences between trends in alternative factor share measures would lead to regression estimates of ($\lambda_L-\lambda_K$) other than those found here.
insensitive to the very marked difference between the average values used for $\pi_L$, although, as would be expected, the estimate of $\lambda$ the rate of growth of total (conventional) input productivity is more profoundly affected.

Alternative estimates, showing the effects of the same variation in the magnitude of the average share going to labor, are also given in Panel B of Table 1, where the calculation is repeated using the higher estimate of $(\lambda_L - \lambda_K)$ obtained with regression model (III). Comparison of $\lambda_L$ and $\lambda_K$ values in Panel B with those in Panel A permits an assessment of their rather low degree of sensitivity to possible bias in the estimate of $(\lambda_L - \lambda_K)$ accepted from regression (II), upon which the computations in Panel A are based.

The rates of growth in Table 1 speak fairly well for themselves. Focusing on the first line in Panel A, it is found that the efficiency of labor has grown at an annual rate of approximately 2.2 per cent per year, and the efficiency of capital has increased at an annual rate of roughly 1.5 per cent.\footnote{The direct estimate furnished for $\lambda_L$ by regression model (I) is, by contrast, 1.9 per cent per annum. It will be recalled that we argued in Section II.A that the latter estimate was likely to be subject to a downward bias.} The estimated percentage rate of growth of (weighted) total factor efficiency is 1.85 per cent per annum; it is strikingly close to the 1.83 per cent per annum estimate provided for the total productivity growth rate during 1909–49 by ACMS [3] and agrees fairly well with the findings of other comparable studies.\footnote{Kendrick [18, Table 25, p. 113] gives total factor productivity growth rates for the Private Domestic Economy which average out to 1.77 per cent per annum for the period 1899–1957. Cf. also, Abramovitz [1, p. 11] and Solow [33, p. 316].} There is a touch of irony here: since it has been found that technical change is not neutral, the 1.83 per cent estimate presented by ACMS is, strictly speaking, not an estimate of $\lambda$, but of $\lambda_L$, and regarded as such, it appears to fall rather to the low side of the truth. (Cf., above, Section II.A.) Further, when one uses a figure for $\pi_L$ that corresponds to the more usual labor-share concept adopted by the other aggregate productivity studies cited, i.e., $\pi_L = .75$, the second line of Table 1 shows that the resulting estimate for $\lambda$ would be 2.1 per cent per annum—a good bit higher than the general run of long-run total productivity growth rates found in those studies!

The last column of Table 1 offers the results of a computation designed to provide an answer to the question of the importance of the contribution made to the rate of total factor efficiency growth ($\lambda$) by the annual rate of increase in the efficiency of labor. It is evident that the particular way one defines labor’s share can make a considerable difference here. With $\pi_L = .476$ it appears that something less than 60 per cent of the rate of growth of factor productivity is accounted for by labor-augmenting technical changes, whereas with $\pi_L = .751$ it would seem that...
labor-augmentation accounts for roughly 80 per cent of the annual total productivity growth rate. Although it can be said with reasonable assurance that more than half of the "residual," or economically unexplained, component of the output growth rate of the Private Domestic Economy has come in the form of labor-augmenting improvements, the spread between 57 per cent and 81 per cent (compare Panel A) seems too wide to be very useful in guiding research into the sources of aggregate productivity growth. In the latter connection it would certainly be better to rely on the estimates of $\lambda_L$ and $\lambda_K$ instead; to work towards identification of the secular developments which have been responsible for something like a 2.2 or 2.3 per cent annual rate of increase in the efficiency of conventionally measured labor inputs on the one hand, and a 1.5 or 1.6 per cent annual rate of growth of the efficiency of conventionally measured capital inputs on the other hand.

Of course, if one were tempted by the thought that the search for the sources of productivity growth could be called off completely, simply by defining the inputs in the aggregate production function in terms of efficiency units and thereby doing away with the very notion of total productivity change, it could be said that "capital inputs"—$(E_KK)$ in the production function (1)—have been growing at the rate of approximately 3.23 per cent per annum, while "labor inputs" $(E_LL)$ have grown at 2.98 per cent per annum. Since the annual rate of growth of real output in the Private Domestic Economy has been about 3.04 per cent during the present century, one is left with the following very simple characterization of the long-run pattern of growth: there has been a tendency for output per unit of "labor input" to rise slightly in consequence of the increasing "capital"-intensity of the aggregate production process, while, on the other side of the coin, the increasing relative abundance of "capital" and the rather restricted scope for input substitution have led to a moderate rise in the share of real output received by labor.

Whatever the virtues of simplification, this vignette unfortunately brings us no closer to fully understanding the mechanism underlying the growth of effective labor inputs and the rising relative abundance of effective capital inputs in the United States; but, as was announced at the outset, such questions are quite distinct from those we have attempted to answer here and we are content for the moment to leave them to others.

**APPENDIX A: CORRECTING THE NOMINAL CAPITAL STOCK FOR VARIATIONS IN THE RATE OF UTILIZATION**

Capital stock data can be corrected for changes in the rate of utilization prior to its use in the regression analysis by assuming that the rate of utilization is equal to the rate of employment. This procedure was adopted by
Solow [31]. When this device is used, in place of (II) we obtain:

\[
\ln \left( \frac{\hat{K}(1 - U)}{L} \right)_t = 4.0256 + 0.1886 \ln \left( \frac{\pi_L}{\pi_K} \right)_t + 0.0051t
\]
\[
+ 0.3484 \ln \left( \frac{\hat{K}}{L} \right)_{t-1}, \quad R = .963, \quad d = .376.
\]

The implied parameter estimates, compared to those derived from (II), show a higher elasticity of adjustment and, therefore, a lower elasticity of substitution \( \eta = 0.22 \). The estimated bias toward labor-saving in technical change is somewhat more pronounced: \( \lambda_L - \lambda_K = 0.0078 \), compared to 0.0072 found with (II). These estimates are not in conflict with the general conclusions of the text based on (II), but the proportion of the total variance explained by (IIA) is not as large as that explained in (II) and, more important, despite the use of a lagged dependent-variable serial correlation of the disturbance terms in (IIA) is very pronounced—indicating some serious specification error in the model.

It is sometimes argued (cf., e.g., [34], [4]) that the correction for under-utilization of capital should be made in a more flexible manner than that used in (IIA), on the grounds that when the rate of unemployment is already low, further reductions in \( U \) will have a smaller influence on the rate of utilization of capital. On this argument the effective or utilized capital stock \( K \) could be related to the nominal stock as:

\[
K = K_e - (z_1U + z_2U^2).
\]

Substitution of this expression in place of the specification given by equation (18) leads to the fitted regression equation:

\[
\ln \left( \frac{\hat{K}}{L} \right)_t = -3.6223 + 0.1294 \ln \left( \frac{\pi_L}{\pi_K} \right)_t + 0.0020t
\]
\[
+ 0.7216 \ln \left( \frac{\hat{K}}{L} \right)_{t-1} + 0.0027U_t
\]
\[
+ 0.0004U^2_t; \quad R = .980, \quad d = 1.255.
\]

While there is virtually no difference between the estimates of \( \sigma \) and \( \lambda_L - \lambda_K \) obtained with (IIB) and those secured with (II), as a consequence of the greater degree of multicollinearity among the explanatory variables in (IIB), the standard errors of the regression coefficients are greater, and the \( t \)-statistics shown in parentheses below the coefficients are in every case smaller than those given by (II). Moreover, neither the regression coefficient of \( U \) nor that of \( U^2 \) in (IIB) is significantly different from zero. Thus, compared to (II), (IIB) fails to offer an improvement in the reliability of the results.
It was argued in the text (Section II. B) that application of the specification of the lagged adjustment process given by (12) to the capital-labor ratio adjusted for underutilization in the preceding period, as well as in the current period, creates some statistical problems which make the adjustment specification given by (19) an attractive alternative. For the satisfaction of the curious, we present the outcome of taking the route suggested by (12). Using the form for the underutilization correction given by (18) in conjunction with (12) leads to the addition of a term \[ v_t \ln(1 - U)_{t-1} \] to the RHS of (21). Fitting this amended model yields, in place of (II):

\[
\ln \left( \frac{K}{L} \right)_t = -1.9624 - 0.0379 \ln \left( \frac{\pi_L}{\pi_K} \right)_t + 0.0007t \\
= (-1.08) \quad (1.57)
\]

\[
+ 0.9741 \ln \left( \frac{K}{L} \right)_{t-1} - 1.1349 \ln (1 - U)_t \\
= (20.47) \quad (-13.11)
\]

\[
+ 1.2569 \ln (1 - U)_{t-1}; \quad R = .995, \quad d = 2.321.
\]

As noted in the text, multicollinearity is quite serious in this case. In addition, the coefficients of the first two independent variables turn out not to be significantly different from zero, and the estimate of the elasticity of adjustment (\( \gamma \)) from \( 1 - \theta'_t = .0259 \) is implausibly low.

**Appendix B: Parameter Estimation Biases**

Hurwicz [16] has shown that for small sample sizes the classical least-squares estimate of the regression coefficient of a lagged dependent variable, such as \( \theta'_t \) in (II), will be subject to a downward bias. Although the 61 observations used in fitting the model do not constitute a "large sample" as these things go, a sample of this size does tend to lessen the problem of extreme bias of this sort.\(^{40}\) Yet this affords but cold comfort: Griliches [12] shows that if there is serial correlation in the disturbances, least-squares estimates of the coefficient of a lagged endogenous variable (\( \theta'_t \) again) will be biased even in the case of large samples. In contrast to the case considered by Hurwicz, the bias in \( \theta'_t \) will be upwards when the disturbances are autocorrelated. What must be hoped for, then, is that the opposing small sample and autocorrelation biases, whose magnitudes we do not know, tend to cancel each other. If, however, the net result is that an upward bias persists in \( \theta'_t \), the estimate \( \hat{\gamma} \) will be biased downward; \( \sigma \) will,

\(^{40}\) Cf. McKinnon [24]. This consideration constitutes an argument against following the lead provided by Brown and De Cani [5] [6], who subdivided a roughly equivalent number of time series observations into groups corresponding to comparatively short 'epochs' and then ran separate regressions for these groups to allow for secular variations in the elasticity of substitution. Despite the fact that the above authors work with a distributed-lag specification to obtain estimates of the long-run elasticity of substitution, they fail to note the effect that breaking their observations into shorter series has in this connection.
therefore, have been overestimated and \( \lambda_L - \lambda_K \) will also be biased upward. As far as the strength of the inferences drawn from (II) regarding the nonneutrality of technical change is concerned, it would clearly be comforting to believe that if the biases arising from the Hurwicz and Griliches cases failed to cancel out, the effect of downward bias due to small sample size dominated. The estimate of \( \gamma \) would then be upward biased and \( \lambda_L - \lambda_K \) = 0.0072, would lie below the true parameter value.

Although nothing definite can be said about the actual size of the biases that may exist in the estimates derived from (II), it is possible to provide some indication that the inclusion of the lagged dependent variable \( \ln k_{t-1} \) in (II) does not in itself produce an estimate of \( \lambda_L - \lambda_K \) which is biased upward, i.e., one which favors the conclusion that technical change is labor-saving. If it is assumed that the elasticity of adjustment defined in (19), i.e., \( \gamma \), is unity, the worrisome lagged dependent variable drops out of regression model (21). Fitting the resulting lagless version of (21) with the same data as was employed in (II) yields the following regression equation:

\[
\text{(III)} \quad \ln \left( \frac{K}{L} \right)_t = -10.5476 + 0.1942 \ln \left( \frac{\pi L}{1 - \pi L} \right)_t + 0.0086 t \\
- 1.0882 \ln (1 - U)_t; \quad R = .958, \quad d = 0.194. \\
\text{(2.05)} \quad \text{(12.01)} \quad \text{(-9.37)}
\]

Denoting the vector of regression coefficients, in order of their appearance in (III) by \( (v_0', v_0'', v_3') \), the estimates of the relevant production-function parameters are \( \lambda_L - \lambda_K = v_0'' = 0.0086 \), and \( \delta = v_3'' / (1 + v_3'') = 0.1626 \), the latter being interpreted as the elasticity of substitution under the assumption of complete adjustment within a single year. In the absence of simultaneous-equations bias, the estimate \( v_0'' = 0.0086 \) should be unbiased. We note, therefore, that it is larger than the corresponding estimate, \( \lambda_L - \lambda_K = 0.0072 \), obtained with the distributed-lag model (II).

As a final comment in this cautionary vein, it should be remarked that the presence of some serial correlation in the disturbances of (II) makes it likely that application of the usual least-squares formula in computing the sampling variances of the regression coefficients leads to underestimates of the standard errors, jeopardizing the strict validity of the usual \( t \)-tests. (Cf., e.g., Johnston [17, pp. 179 ff].) Yet, in contrast with the very high degree of autocorrelation of disturbances in (III), where \( d = 0.194 \) and the \( t \)-statistics given in parentheses below the regression coefficients appear spuriously large, this problem is much less serious in the case of (II). The latter constitutes a clear advantage of the inclusion of the lagged dependent variable in (II), but it is an advantage gained, as has been seen, at the cost of possible biases in the parameter estimates.

Lucas [23], by application of two-stage least squares to the estimation of the elasticity of substitution, has found the simultaneous-equations bias not to be very serious, but his results relate to the case of a single industry.
APPENDIX C: SOURCES OF THE DATA

1. $\dot{K}$: Unweighted Real Capital Input in the U.S. Private Domestic Economy (PDE), in 1929 Dollar Millions.
   a. Data for 1899–1953 from Kendrick [18, Table A-XXII: Supplement, pp. 336–37 and Table A-XV, p. 321]. Extensions for 1954–60 based on revisions and further data kindly supplied by J. W. Kendrick and Maude Pech. The deflation of gross capital formation data, from which these real capital stock figures are derived [18, p. 35] is intended to provide a base-period resource cost measure of capital in which increases in productive efficiency of comparable items of the stock are not reflected unless more resources are used in their production. However, in cumulating real capital formation figures into stock estimates, Kendrick has adopted a real stock concept net of depreciation allowances (rather than gross of depreciation and net of replacements) as a “better measure of a basic capacity to contribute to production.” (Ibid.) Since the latter allows for obsolescence in the measure of capital, it conflicts with the stated intent of the deflation operation and creates an ambiguity as to the precise meaning of the figures.
   b. Exponential trend rate of growth in $\dot{K}$, estimated from:
   \[
   \ln \dot{K}_t = 3.9807 + 0.0167t; \quad R = .971 
   \]
   (31.3)

2. $L$: Unweighted Input of Labor in the U.S. PDE, in Millions of Man-hours Employed.
   b. Exponential trend rate of growth in $L$, estimated from:
   \[
   \ln L_t = 4.2936 + 0.0066t; \quad R = .816 
   \]
   (10.9)

   b. Exponential trend rate of growth in $V$ estimated from:
   \[
   \ln V_t = 3.6174 + 0.0304t; \quad R = .980 
   \]
   (38.2)

and partnerships (line 16), other private business (line 26). Wage payments to employees in general government, in government business, in households and institutions are omitted and no allowance is included for wage payments in proprietors' incomes. Data for 1899–1928 from extrapolation of later series on the share of employee compensation in Gross Private Domestic Business Product computed from Grant [11, Table 2, Col. (1) and Table 3, p. 279] for 1899–1929. The two series are virtually identical in the overlap year, 1929.

b. Apart from the exclusion of wage payments to workers in the government sector, the major difference between the numerator of the labor share measure used here and the more conventional measures lies in the exclusion of any imputation of entrepreneurial income to labor. As Lebergott [22, pp. 190–219] points out in a critical survey of the controversy over the stability of labor's share, virtually any attempt to split up entrepreneurial income between labor service and capital service payments will necessarily be arbitrary and, for the early years of the twentieth century, will rest on extremely treacherous data. Lebergott suggests that studies of factor substitution be limited, therefore, to those sectors of the economy in which entrepreneurial income is insignificant. Acceptable as this recommendation is, continuous time series for aggregate inputs of labor and capital in the private nonentrepreneurial sector of the economy are not yet readily available for analysis of the kind pursued here.

As for the denominator in the share measure, Grant [11] has advanced persuasive reasons in support of his contention that use of the Commerce national-income concept poses problems of inconsistencies among the components of the denominator which distort the picture of the movements in income distribution over time. He also points out that distortions have been introduced by the arbitrary reconciliation of national income estimates for the pre-1929 period with the Commerce concept. In preparing estimates of Gross Private Domestic Business Product as an alternative and preferable denominator, Grant takes Kendrick's [18] annual figures for GNP on a Commerce basis as a starting point and proceeds by deducting "irrelevant" nonbusiness items.

Data for 1899–1960 from Lebergott [22, Tables A-3 and A-15].

REFERENCES


